

# ALPS exercises in Quantum Computing

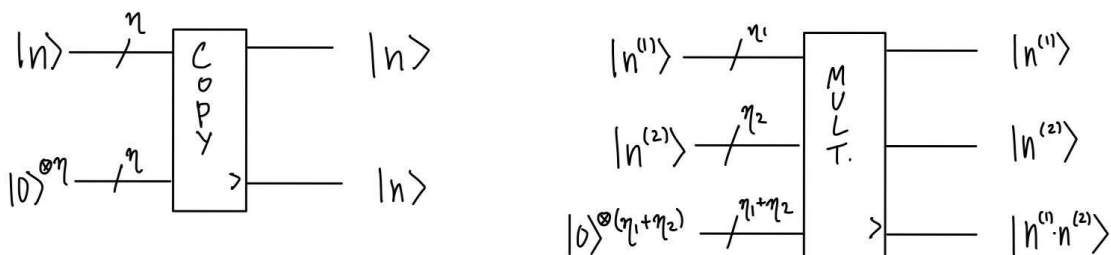
## Digital quantum simulation tutorial, I. Schwinger model electric propagator via phase kickback

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In this tutorial, we learn how to effect time evolution for the  $\hat{E}^2$  operator of a link (i.e., circuitize  $\exp(-i\phi\hat{E}^2)$ ) in the Schwinger model without applying phase rotations to the  $\hat{E}$  register itself. The electric energy  $\hat{E}^2$  is a rather simple function of  $\hat{E}$ , but this method becomes especially handy when one needs to deal with more complicated functions of  $\hat{E}$ .

1. Consider a single qubit with computational basis states  $|n\rangle$  ( $n = 0, 1$ ). If  $Z|0\rangle = |0\rangle$  and  $Z|1\rangle = -|1\rangle$ , give an expression for the number operator  $\hat{n}$  associated with this qubit ( $\hat{n}|n\rangle = n|n\rangle$ ) in terms of  $Z$ .
2. Now let  $|n\rangle = \otimes_{j=0}^{\eta-1} |n_j\rangle = |n_{\eta-1}n_{\eta-2}\cdots n_1n_0\rangle$  be a nonnegative integer expressed in binary using  $\eta$  qubits. ( $0 \leq n \leq 2^\eta - 1$ ). If  $Z_j$  is the  $Z$  operator corresponding to the  $j^{\text{th}}$  qubit in the register, write the expression for the number operator  $\hat{n}$  of the complete register in terms of the  $Z_j$ 's.
3. Suppose we want to circuitize  $\hat{n}$  as a Hamiltonian, i.e., find a circuit to effect  $\hat{V} = \exp(-i\phi\hat{n})$  for some angle  $\phi$ . Using the expression for  $\hat{n}$  found in (2), express  $\hat{V}$  as a function of the  $Z_j$ 's. "Trotterize" this expression by decomposing  $\hat{V}$  into a product of simpler exponentials. What kind of gates does your expression suggest for implementing  $\hat{V}$ ? Sketch the circuit, including rotation angles. (For single-qubit rotation gates, you can use the spin-1/2 definition for rotations about the  $z$ -axis:  $R_z(\phi) = \exp(-i\phi Z/2)$ .)
4. Next, suppose you have access to black-box circuits COPY and MULT(IPLY) (and their inverses) defined by the transformations



Write down a minimal circuit to effect  $|n\rangle|0\rangle^{\otimes\gamma} \rightarrow |n^2\rangle \otimes |\text{other output bits}\rangle$ . How many auxiliary qubits  $\gamma$  had to be introduced in order to obtain  $|n^2\rangle$ ? And what exactly is stored in the "other output" bits?

5. Using the results from (3), draw or explicitly describe a circuit that will effect

$$|n^2\rangle \rightarrow e^{-i\phi n^2} |n^2\rangle.$$

6. Put together the results of (4) and (5) to give a complete circuit that will effect

$$|n\rangle|0\rangle^{\otimes\gamma} \rightarrow e^{-i\phi n^2} |n\rangle|0\rangle^{\otimes\gamma}.$$

Remember to clean up or "uncompute" any intermediate results generated in step (4).

You should be convinced that steps (1)-(6) above have the same effect as having applied  $\exp(-i\phi\hat{n}^2)$  to the  $|n\rangle$  register itself. However, we instead used an ancilla register and rotations applied to the ancilla register to ultimately "kick back" the desired phase to  $|n\rangle$ .

7. Connecting to the lattice Schwinger model, one possibility is to take  $\hat{E} \equiv \hat{n} - \Lambda$ , where the cutoff  $\Lambda$  is related to the electric register size by  $2\Lambda = 2^\eta$ . Noting that  $\hat{E}^2 = \hat{n}^2 - 2\Lambda\hat{n} + \Lambda^2$ , explain how to simulate the electric propagator  $\exp(-i\phi\hat{E}^2)$  by taking advantage of the phase kickback method.