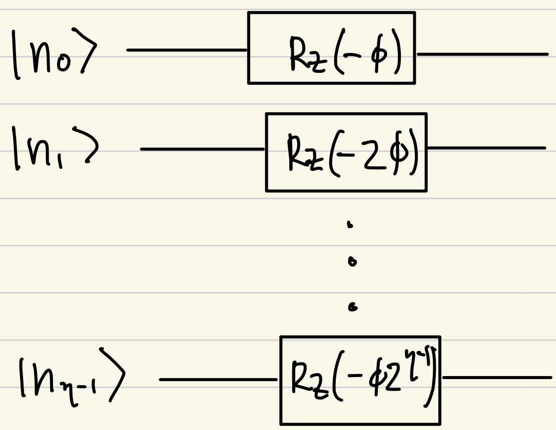
Electric propagator solution

Jesse Stryker

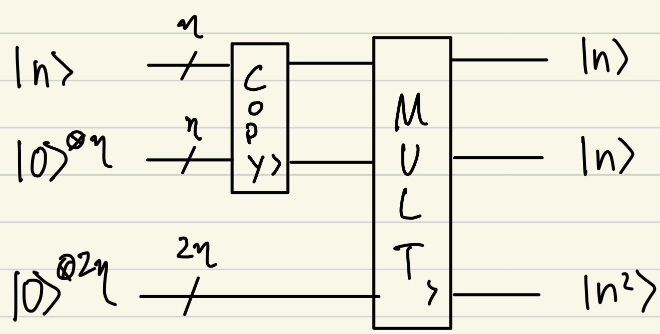
1. For one qubit, $\hat{n} = \frac{1}{2}(1 - Z)$

2. Multi-qubit: $\hat{n} = \sum_{j=0}^{\eta-1} 2^j \hat{n}_j$

3. $$\begin{aligned} \exp(-i\phi\hat{n}) &= \exp\left(-i\phi \tfrac{1}{2} \textstyle\sum_{j=0}^{\eta-1} 2^j (1 - Z_j)\right) \\ &= \exp\left(-i\phi \tfrac{1}{2} \textstyle\sum_{j=0}^{\eta-1} 2^j\right) \exp\left(\tfrac{i\phi}{2} \textstyle\sum_{j=0}^{\eta-1} Z_j 2^j\right) \\ &= (\text{global phase}) * \textstyle\prod_{j=0}^{\eta-1} \exp\left(\tfrac{i\phi}{2} 2^j Z_j\right) \\ &= (\text{global phase}) * \textstyle\prod_{j=0}^{\eta-1} R_{z_j}(-2^j\phi) \end{aligned}$$



4. $|n\rangle |0\rangle^{\otimes \gamma} \to |n^2\rangle \otimes$ something



$$|n\rangle \otimes |0\rangle^{\otimes 3\eta} \to \underbrace{|n\rangle \otimes |n\rangle}_{\text{other output}} \otimes |n^2\rangle$$

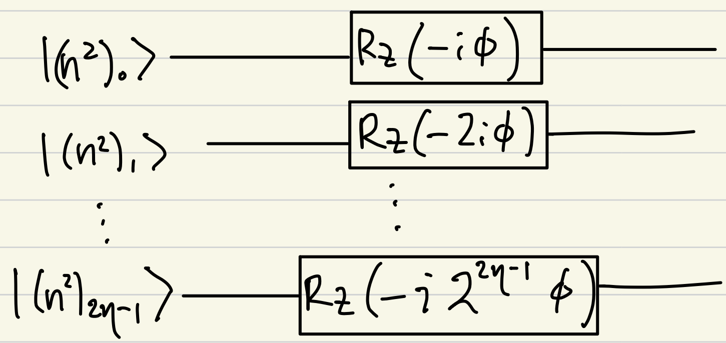
$\gamma = 3\eta$

5. $|n^2\rangle \to e^{-i\phi n^2}|n^2\rangle$

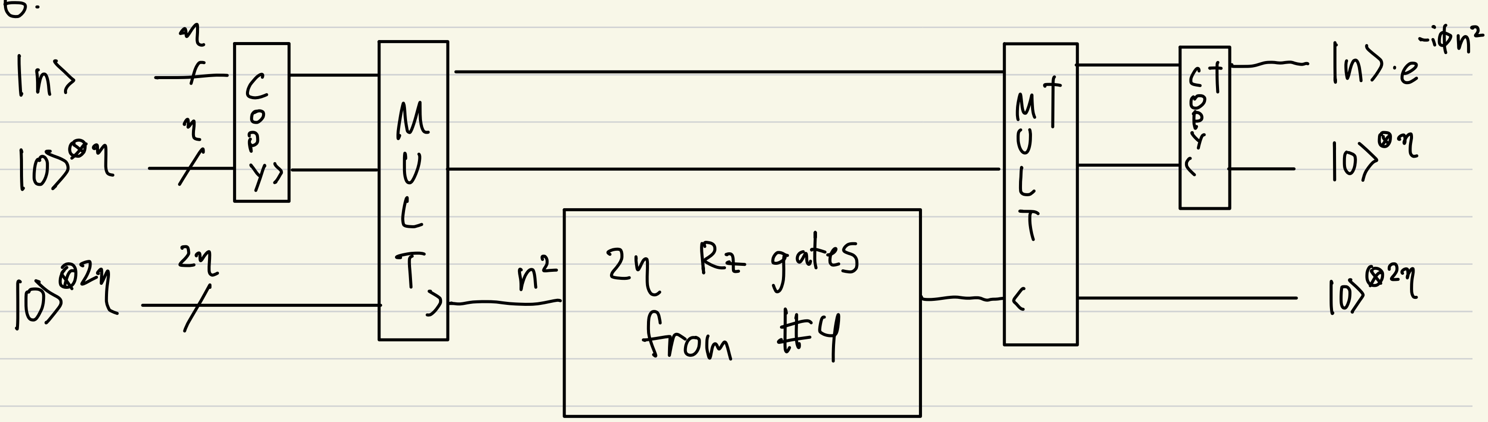
want: $e^{-i\phi n^2}$

$|n^2\rangle$

$e^{-i\phi\,(2\eta\text{-qubit number operator})}$



6.



7. $\hat{E} = \hat{n} - \Lambda$

$\Rightarrow \hat{E}^2 = (\hat{n}-\Lambda)^2 = \hat{n}^2 - 2\Lambda\hat{n} + \Lambda^2$

$\Rightarrow e^{-i\phi\hat{E}^2} = \underbrace{e^{-i\phi\hat{n}^2}}_{\text{Use \#6}}\ \underbrace{e^{i2\phi\Lambda\hat{n}}}_{\text{Use \#3}}$ * (global phase)

Hopping propagator solution

Jesse Stryker

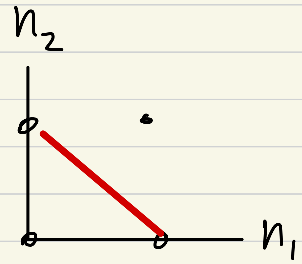
1.

$\sigma_1^- \sigma_2^+ \; |01\rangle = |10\rangle$

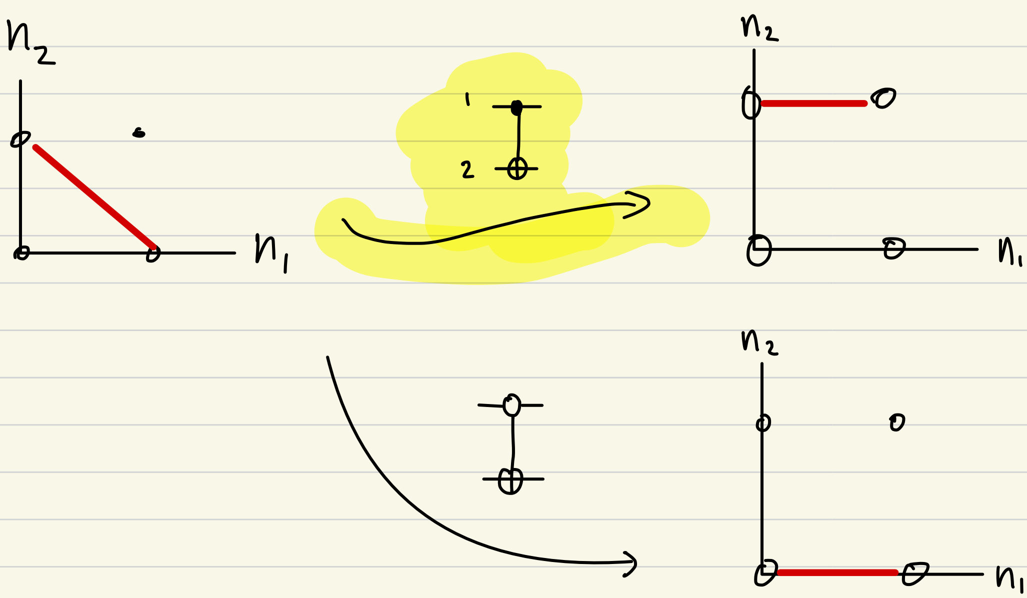
$\sigma_1^+ \sigma_2^- \; |10\rangle = |01\rangle$

All other matrix elements are zero.

$\Rightarrow$



Two possibilities for V':



CNOT controlled on qubit 1

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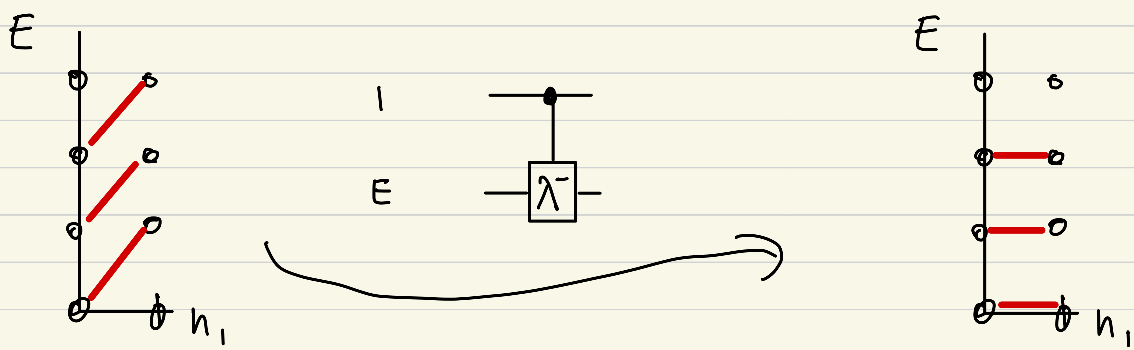
2. $H'' \sim \sigma_1^- U + \text{H.c.}$



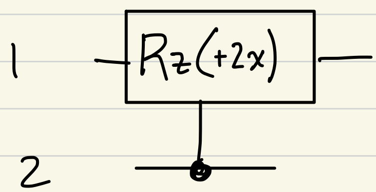
$V'' = \lambda^-$ controlled on qubit 1

3. $X_1 \, |1\rangle\langle 1|_2 \left(\mathbb{1} - |E_{max}\rangle\langle E_{max}| \right)$

$$\Big\downarrow H_1$$

$Z_1 \, |1\rangle\langle 1|_2 \left(\mathbb{1} - |E_{max}\rangle\langle E_{max}| \right)$

$Z_1 \, |1\rangle\langle 1|_2$



$- Z_1 \, |1\rangle\langle 1|_2 \, |E_{max}\rangle\langle E_{max}|$