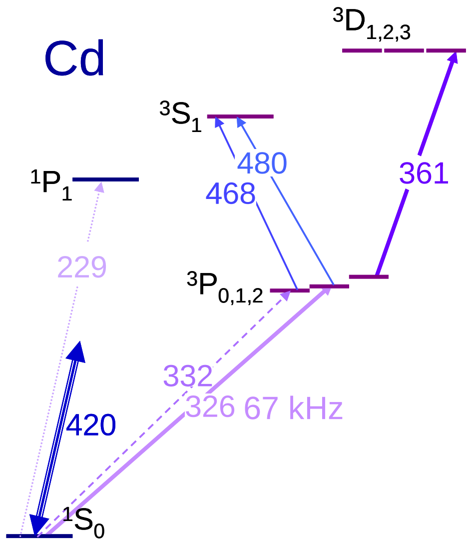
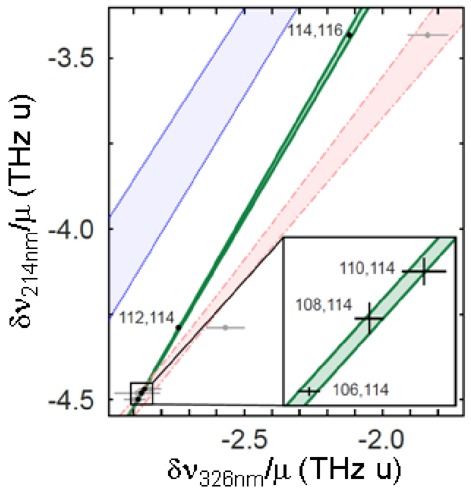
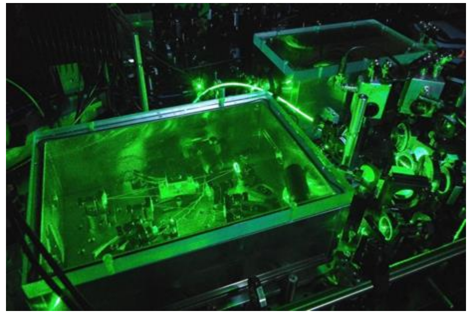
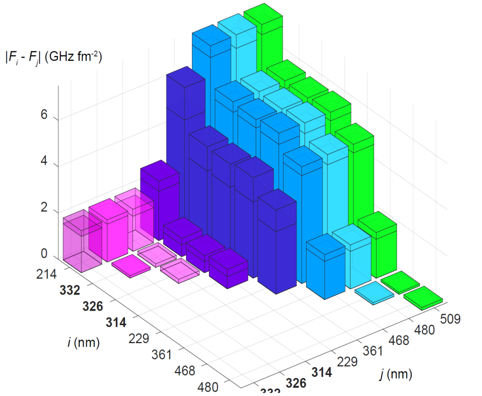
Laser-cooling cadmium with only UVA Triplet Excitations & Cd Isotope Shifts

- Cadmium optical lattice clocks
  - Small BBR sensitivity
  - Narrow-line cooling & magic wavelength
- Laser-cooling without UVC – UVA & blue triplet excitations
  - Only small MOT gradients are required
  - Low temperatures & 100% transfer to narrow-line MOT
- Cd fermions, ^{111}Cd & ^{113}Cd
  - Spin ½ & negative nuclear magnetic moments
  - No additional repumping required
- Isotope shifts & King plots for beyond SM Physics
  - 6 stable spin 0 isotopes, ^{106}Cd to ^{116}Cd
- UVA SFG lasers and FPGA control system
- Absolute frequency measurement of 361 nm $^3P_2 \rightarrow {}^3D_3$









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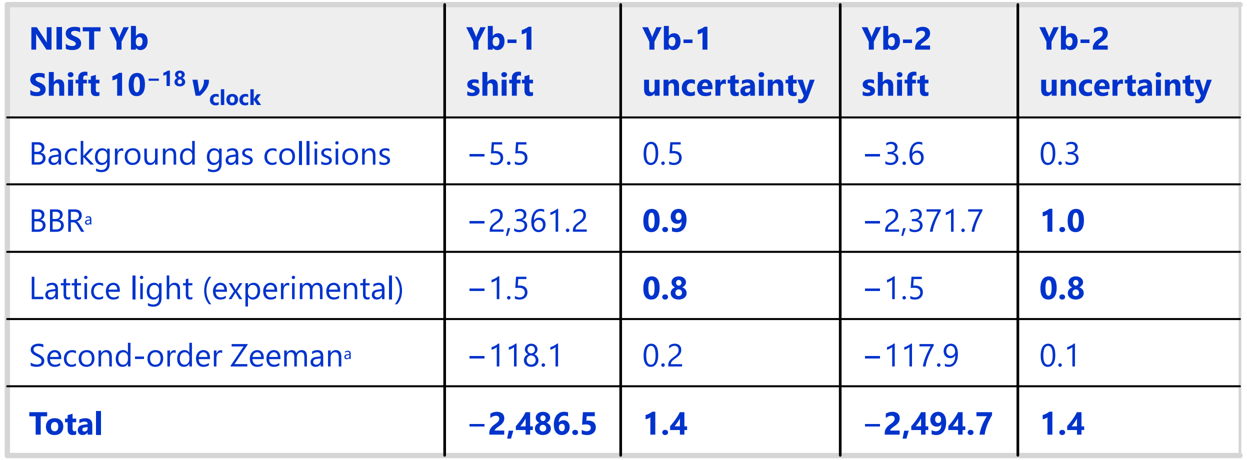
Support from NSF, Penn State and RIKEN.

Yamaguchi, Safronova, KG & Katori, PRL '19 Ohayon, Hofsäss, Padilla-Castillo, Wright, Meijer, Truppe, KG & Sahoo, New J. Phys '22

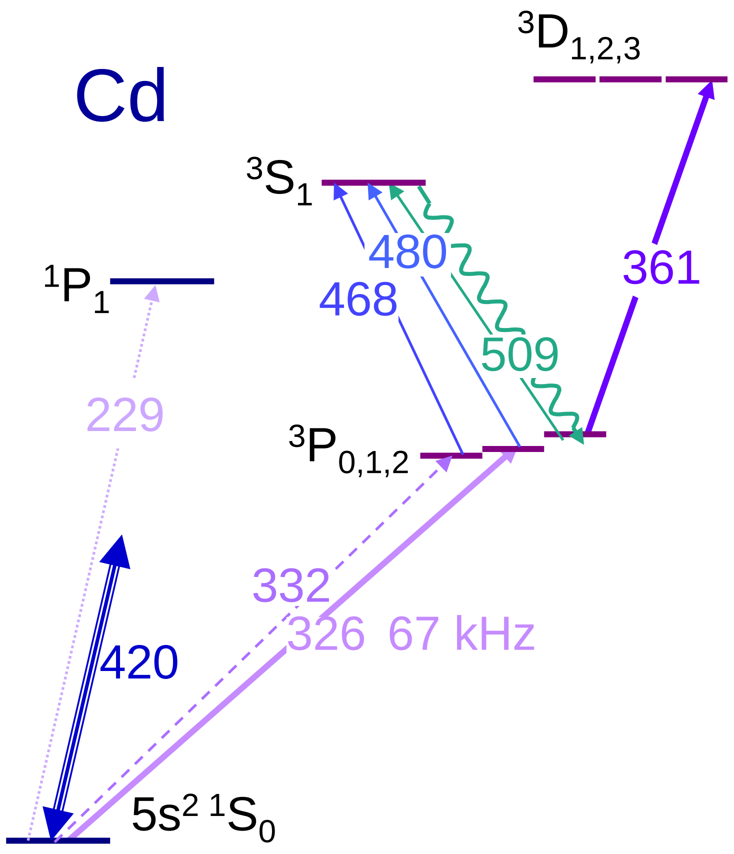
Atomic Clock Candidates

- Dominant lattice clock uncertainties have been AC Stark Shifts: from blackbody radiation and higher order shifts from the lattice light.
- Hg, Cd, Zn & Mg have small sensitivities to BBR.



NIST Yb Shift $10^{-18}\,\nu_{\mathrm{clock}}$	**Yb-1 shift**	**Yb-1 uncertainty**	**Yb-2 shift**	**Yb-2 uncertainty**
Background gas collisions	−5.5	0.5	−3.6	0.3
BBR[a]	−2,361.2	**0.9**	−2,371.7	**1.0**
Lattice light (experimental)	−1.5	**0.8**	−1.5	**0.8**
Second-order Zeeman[a]	−118.1	0.2	−117.9	0.1
Total	**−2,486.5**	**1.4**	**−2,494.7**	**1.4**

Clock	±0.1K BBR (10^{-18})	$^3P_1$ Γ (kHz)	$^3P_1$ λ (nm)	Magic λ_m (nm)	# of I=½ Fermions	I of Fermions	1 mTorr pressure (°C)
Cs	23						
Rb	16						
Sr	7.3	7	689	813		9/2	465
Yb	3.3	182	556	760	1	**1/2**,5/2	520
Ca	3.3	0.4	657	680			405
Mg	0.53	0.036	457	469		5/2	370
Cd	0.38	67	326	420	2	**1/2**, **1/2**	215
Hg	0.24	1,300	254	363	1	**1/2**, 3/2	<22
Al⁺	0.005						

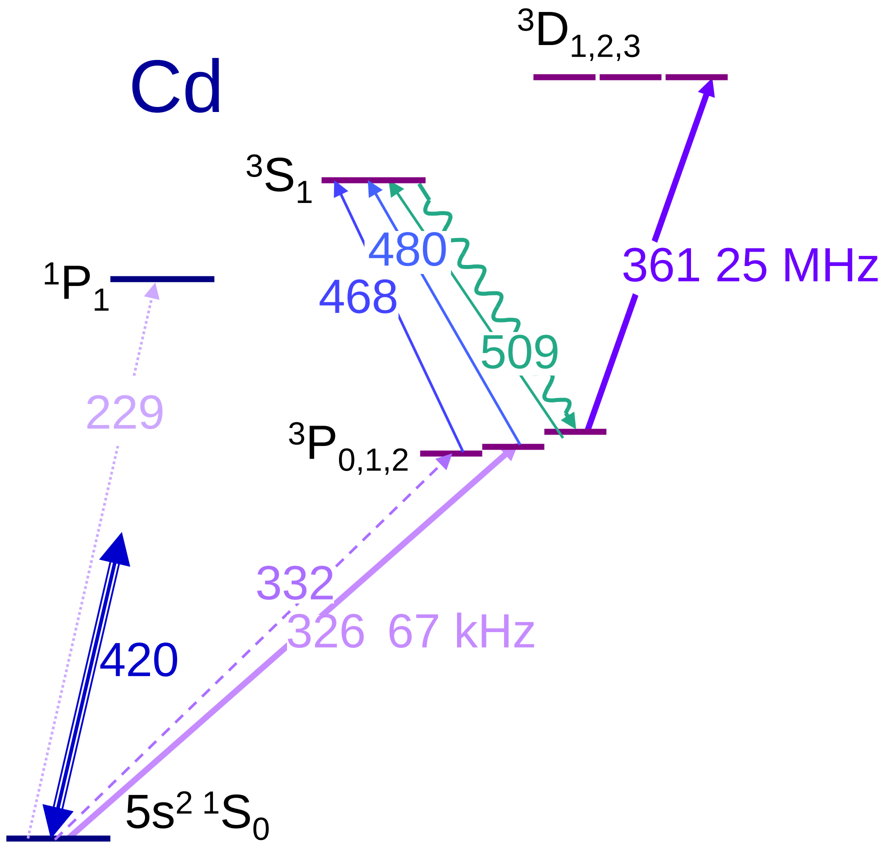


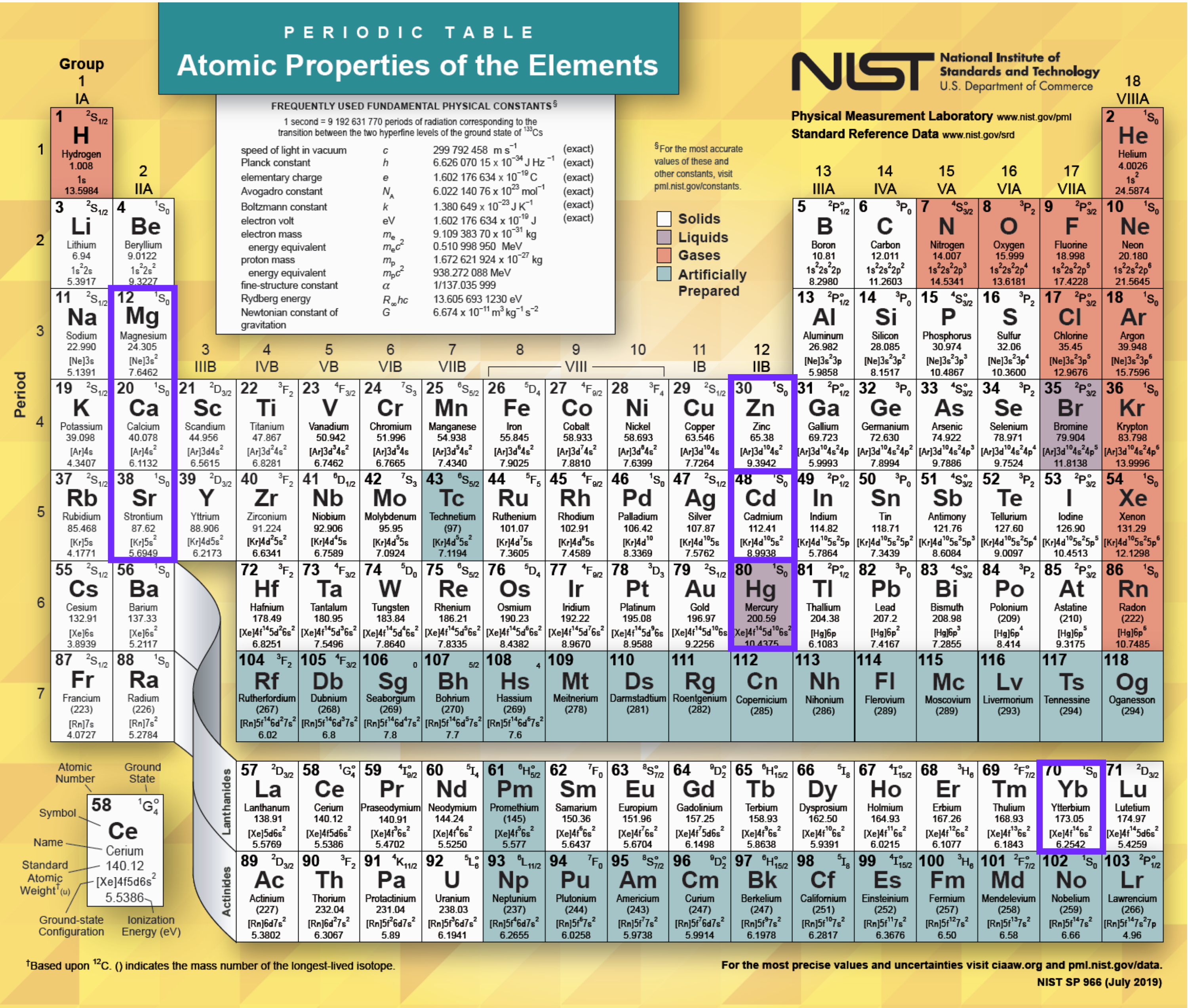
- Wavelengths are easier for Cd than Hg?
  - Cd 229 nm & Hg 1.3 MHz wide $^1S_0 \rightarrow {}^3P_1$ 254 nm are UV-C.
  - Trap Cd with only UVA.

McGrew ... Ludlow *et al.*, Nature ‘18

Optical Lattice Clocks

- Several choices are Sr, Yb, Hg, Mg, & Cd.
- 2 valence electrons and narrow ${}^1S_0 \rightarrow {}^3P_0$ clock transitions.




PERIODIC TABLE

Atomic Properties of the Elements

NIST National Institute of Standards and Technology U.S. Department of Commerce

Physical Measurement Laboratory www.nist.gov/pml

Standard Reference Data www.nist.gov/srd

FREQUENTLY USED FUNDAMENTAL PHYSICAL CONSTANTS§

1 second = 9 192 631 770 periods of radiation corresponding to the transition between the two hyperfine levels of the ground state of ^{133}Cs

speed of light in vacuum	c	299 792 458 m s^{-1}	(exact)
Planck constant	h	6.626 070 15 x 10^{-34} J Hz^{-1}	(exact)
elementary charge	e	1.602 176 634 x 10^{-19} C	(exact)
Avogadro constant	N_A	6.022 140 76 x 10^{23} mol^{-1}	(exact)
Boltzmann constant	k	1.380 649 x 10^{-23} J K^{-1}	(exact)
electron volt	eV	1.602 176 634 x 10^{-19} J	(exact)
electron mass	m_e	9.109 383 70 x 10^{-31} kg	
energy equivalent	m_ec^2	0.510 998 950 MeV	
proton mass	m_p	1.672 621 924 x 10^{-27} kg	
energy equivalent	m_pc^2	938.272 088 MeV	
fine-structure constant	α	1/137.035 999	
Rydberg energy	$R_\infty hc$	13.605 693 1230 eV	
Newtonian constant of gravitation	G	6.674 x 10^{-11} m^3 kg^{-1} s^{-2}	

§For the most accurate values of these and other constants, visit pml.nist.gov/constants.

☐ Solids
▨ Liquids
🟥 Gases
▨ Artificially Prepared

†Based upon ^{12}C. () indicates the mass number of the longest-lived isotope.

For the most precise values and uncertainties visit ciaaw.org and pml.nist.gov/data.

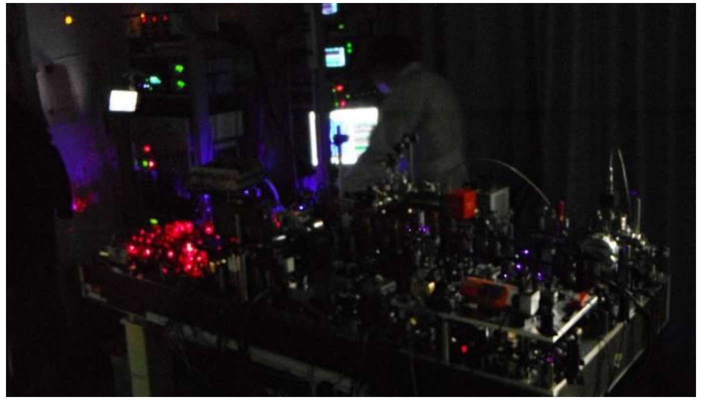
NIST SP 966 (July 2019)

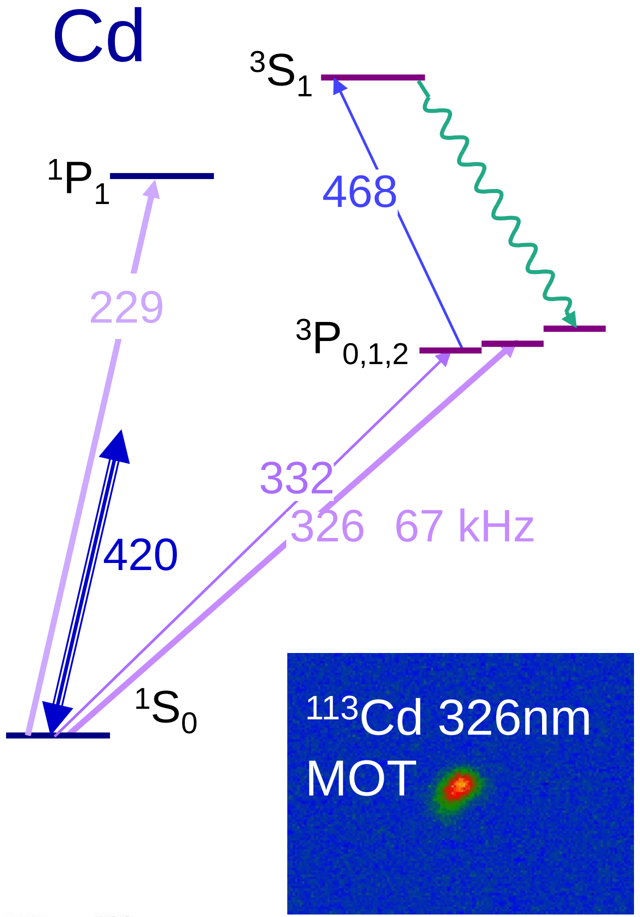
Kramida, Olsen & Ralchenko, NIST

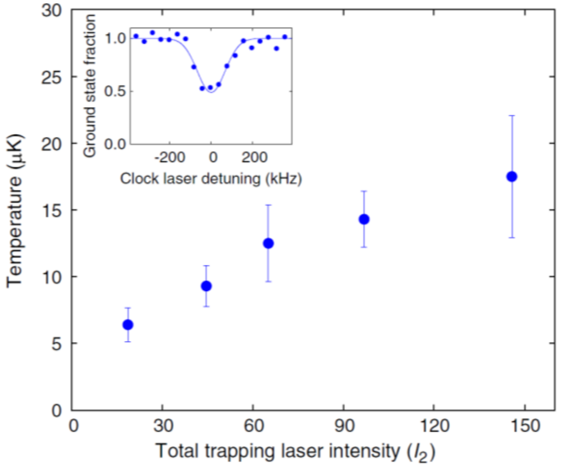
RIKEN Cd Optical Lattice Clock

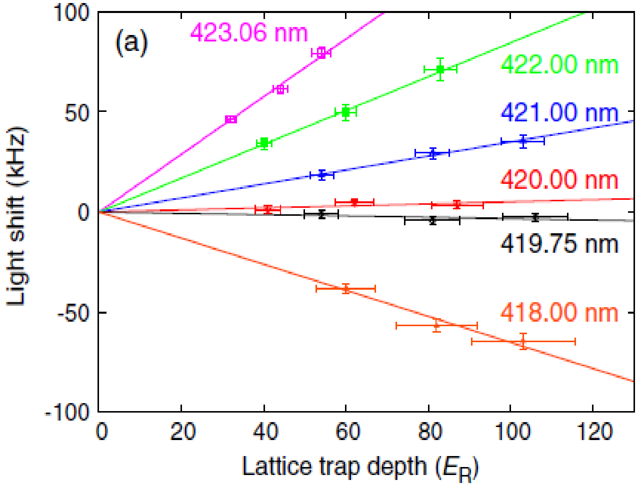
- Hg, Cd, & Mg have small sensitivities to BBR shifts.
  - 229 nm is UV-C.
  - ~10^6 atoms
- Wavelengths and Doppler and sideband cooling are easier for Cd than Hg.
  - $^1S_0 \rightarrow {}^3P_1$ Doppler cooling to 5.6 μK.
  - Magic wavelength is 419.88 nm.
  - MOT gradients: 170 G/cm & 1 G/cm.

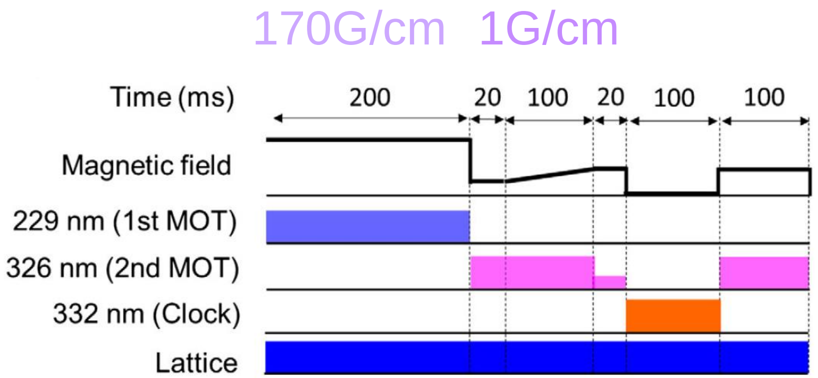
Transportable Cd clock:









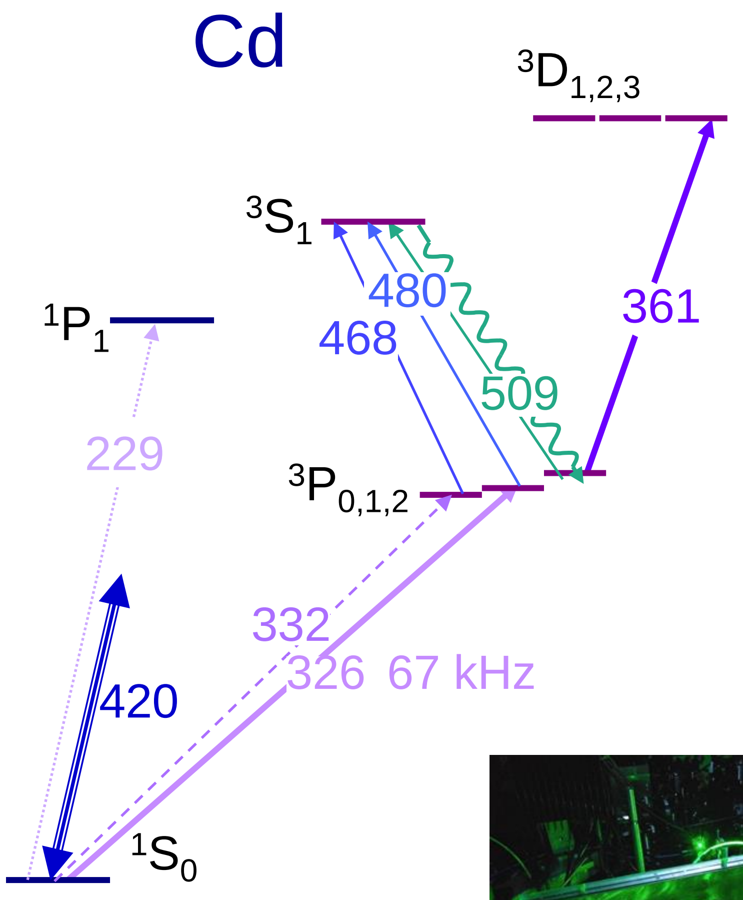


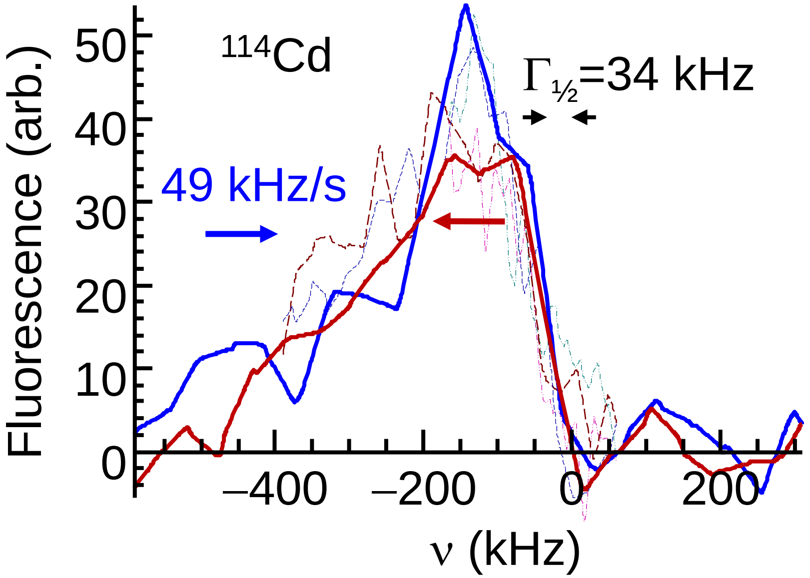
229nm: Kaneda, Yarborough, Merzlyak, Yamaguchi, Hayashida, Ohmae & Katori Opt. Lett '16

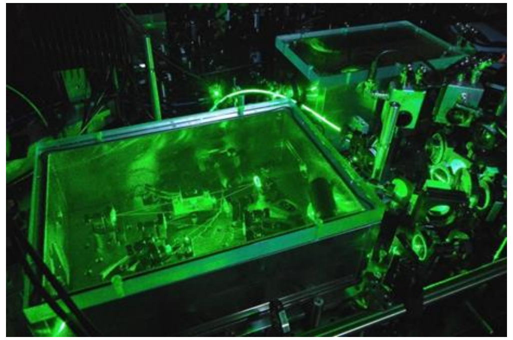
Yamaguchi, Safronova, KG & Katori, PRL '19

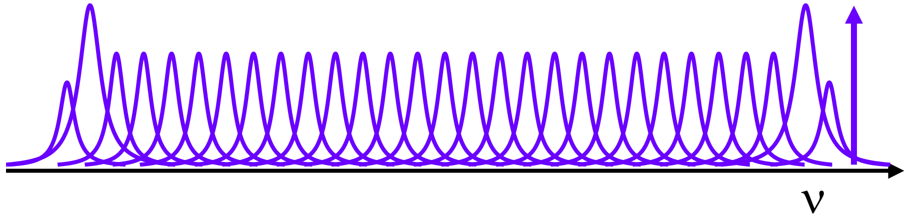
PSU Laser Cooling and Trapping of Cd

- Try to use only UVA - avoid 229 nm light.
- First, can only 326 nm light capture atoms with a 67 kHz linewidth transition from a room temperature source?
  - 6 Bosons: Trap & cool 4 of 6 bosonic (^{110}Cd, ^{112}Cd, ^{114}Cd & ^{116}Cd) and both fermionic isotopes (^{111}Cd & ^{113}Cd).
  - ^{106}Cd & ^{108}Cd have ~1% abundance
- 8.6 MHz FM of narrow 326 nm laser to broaden capture velocity.
  - 50 to 150 mW of 326 nm light
  - MOT gradient: 0.5 G/cm



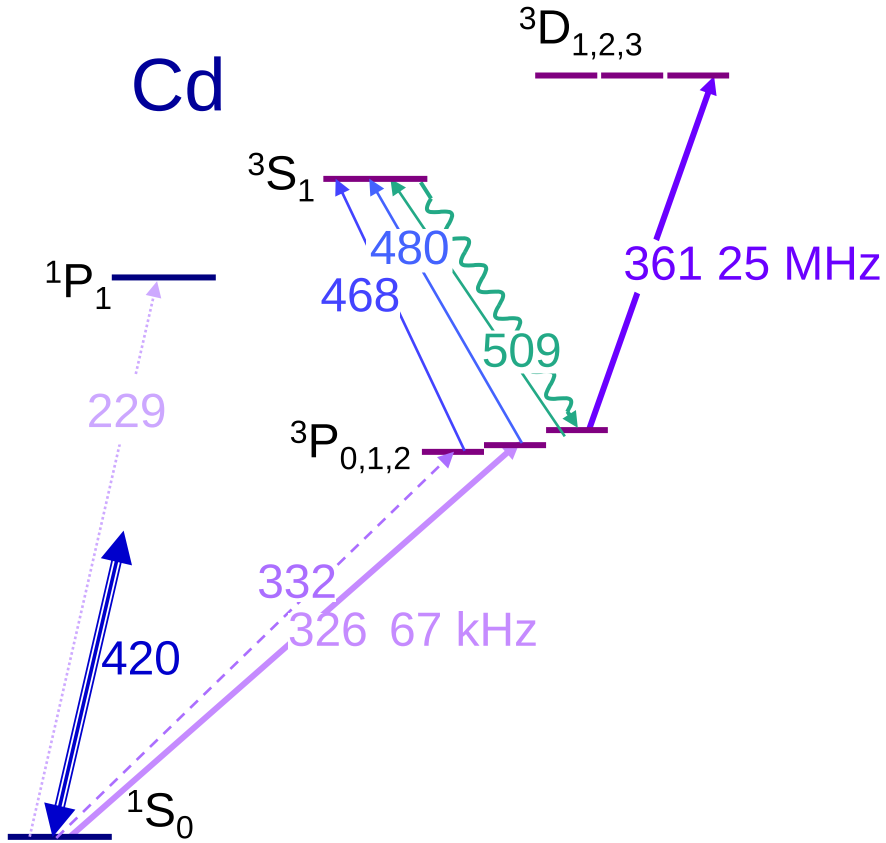


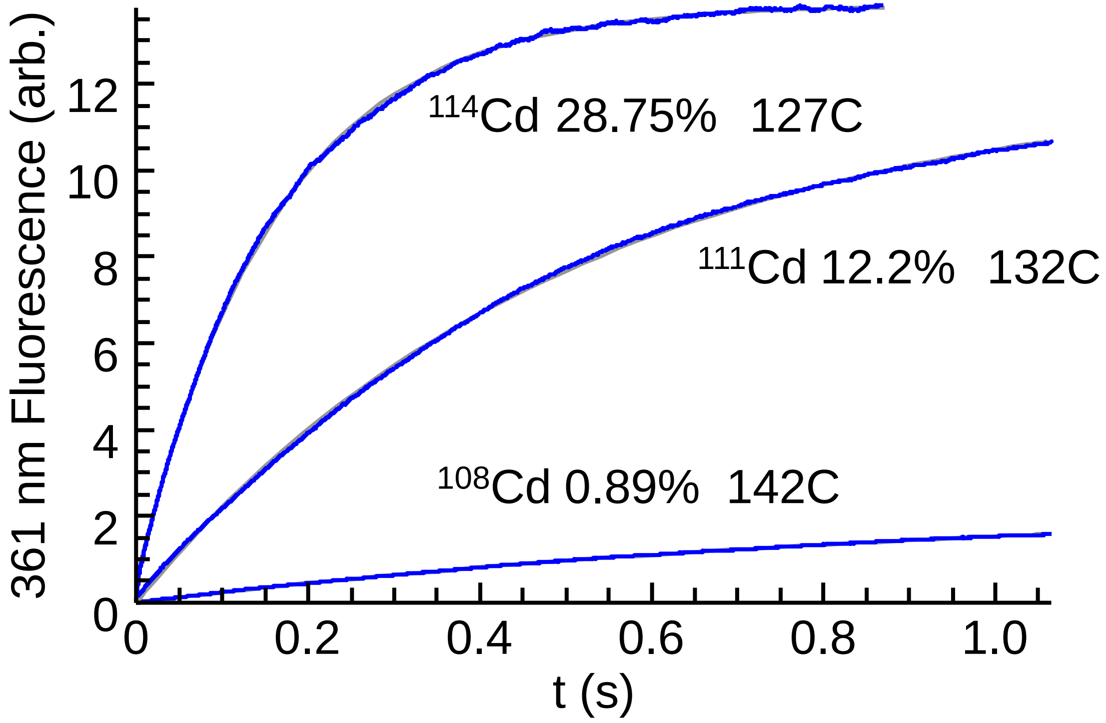


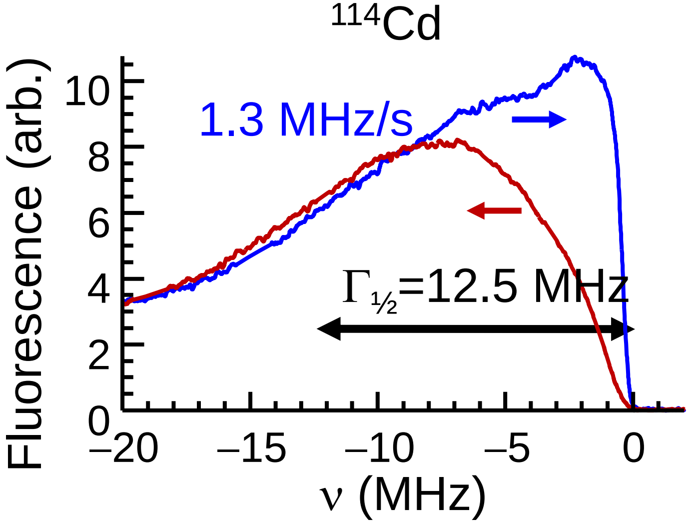


Enhanced Loading with a Cd Metastable MOT

- Use only UVA & blue triplet excitations - no 229 nm light.
- 361 nm meta-stable MOT loading:
  - $\sim 10^{10}$ atoms, 10× RIKEN 229 nm MOT
  - 361 nm also gives high S/N detection
- Trap all 6 Bosons: ^{106}Cd, ^{108}Cd, ^{110}Cd, ^{112}Cd, ^{114}Cd & ^{116}Cd
  - MOT gradient: 5.7 G/cm (170 to 400 G/cm for 229 nm singlet MOT).





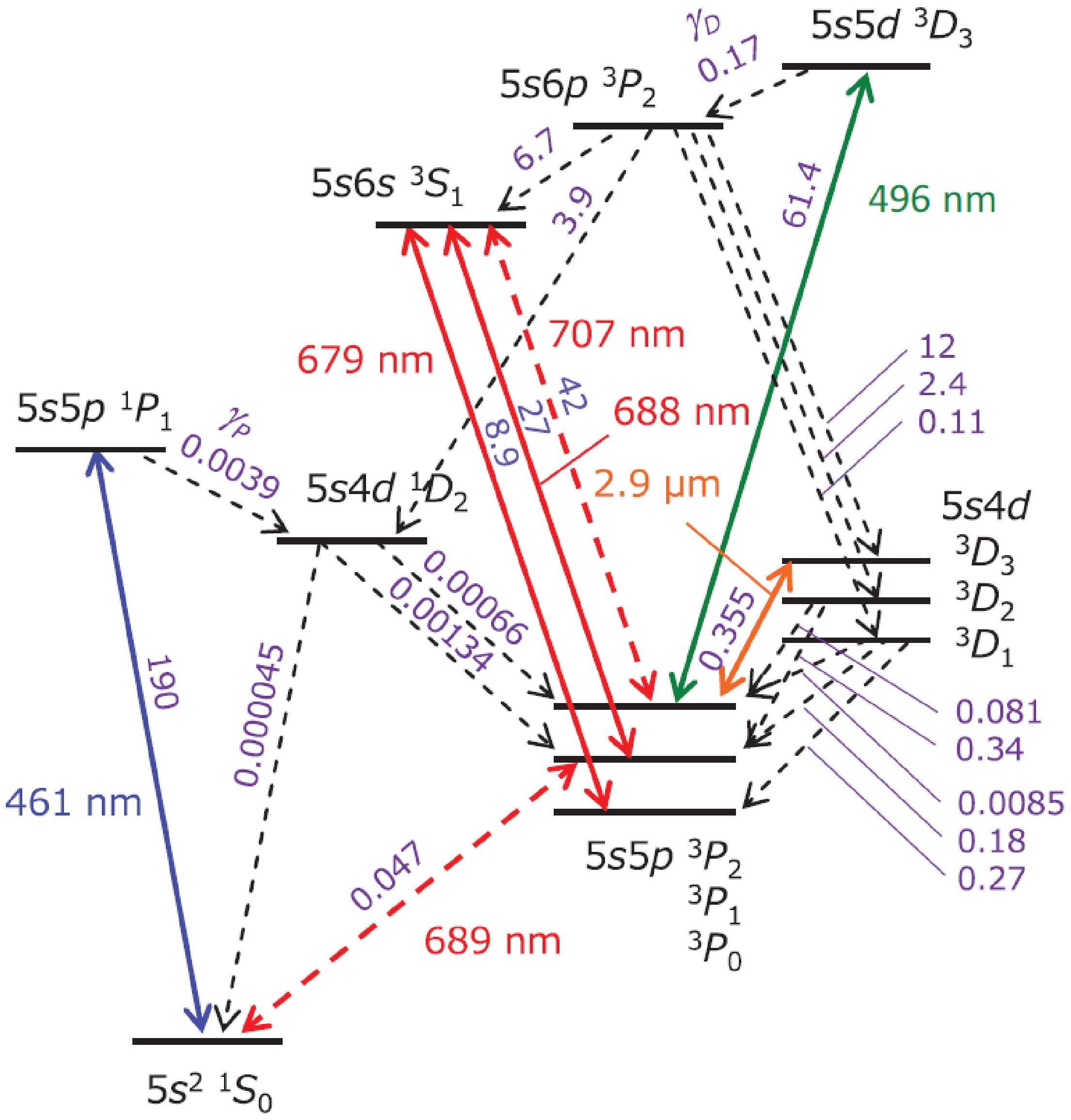


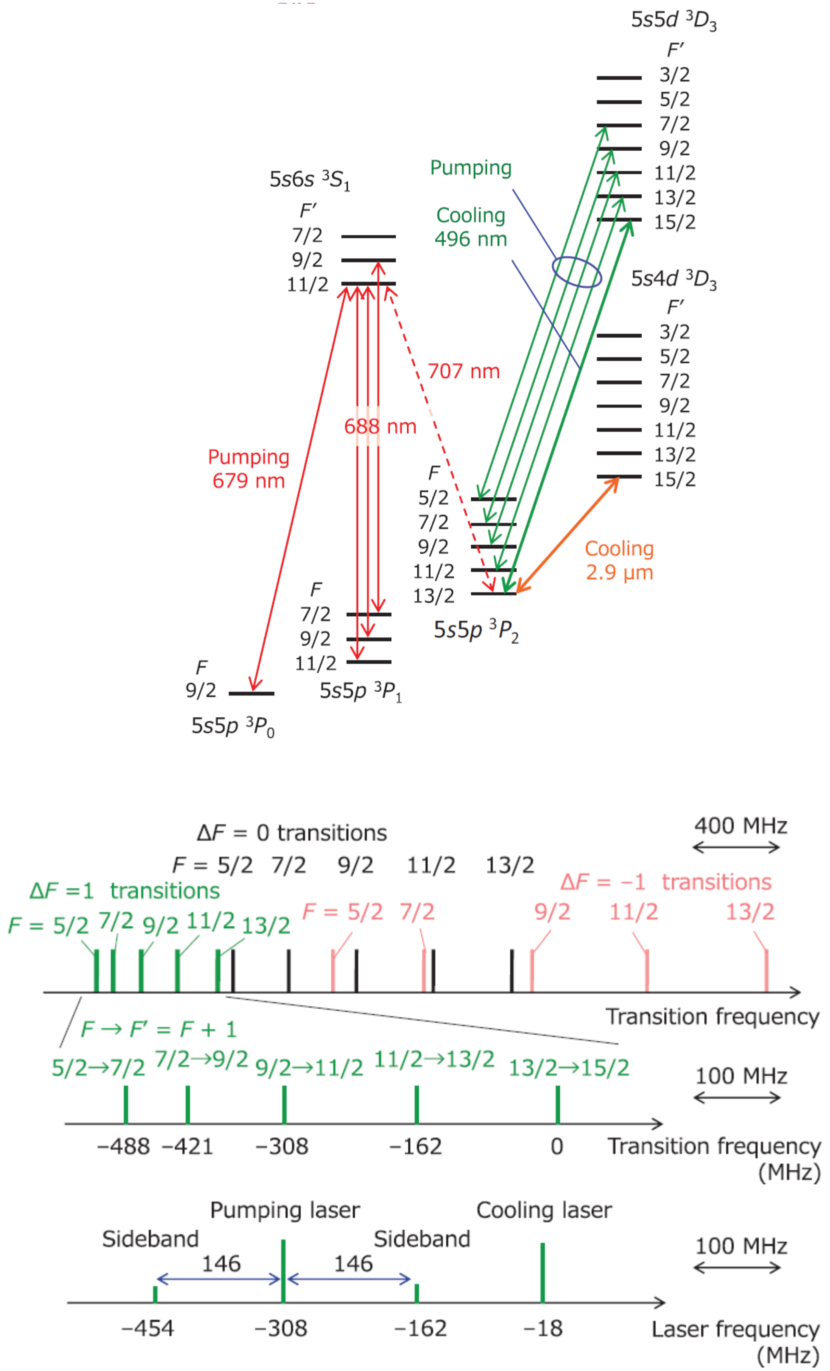
- 2 Fermion spin ½ isotopes: ^{111}Cd & ^{113}Cd
  - Spin ½ nuclei (& negative magnetic moments)

Mg: Kulosa, Fim, Zipfel, Rühmann, Sauer, Jha, KG, Ertmer, Rasel, Safronova, Porsev, PRL ’15

Metastable MOT of Sr Fermions

- Sr has spin 9/2 nucleus - $^3D_3$ splits into 7 hyperfine levels



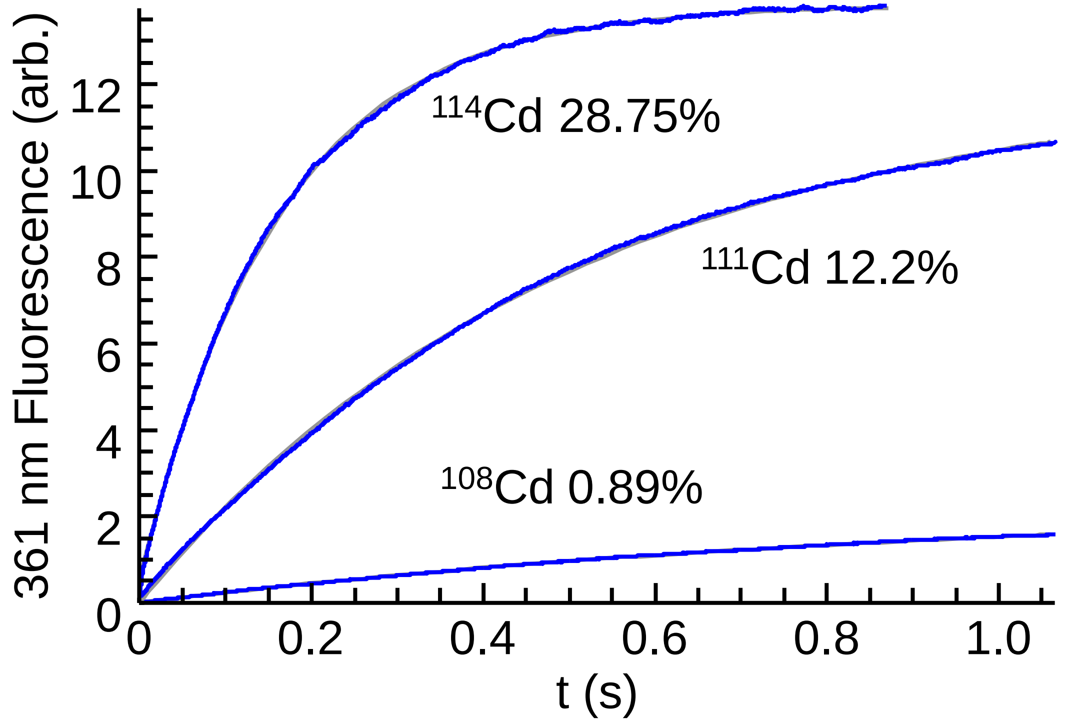


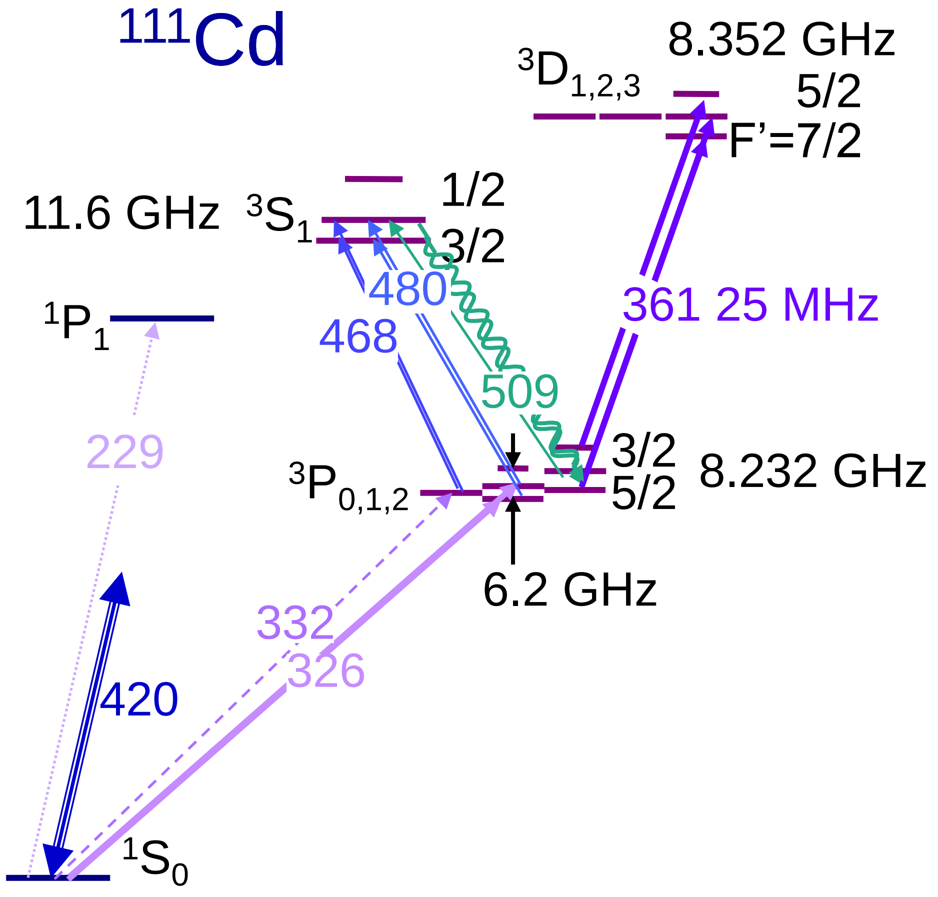
Akatsuka, Hashiguchi, Takahashi, Ohmae, Takamoto & Katori, PRA ‘21

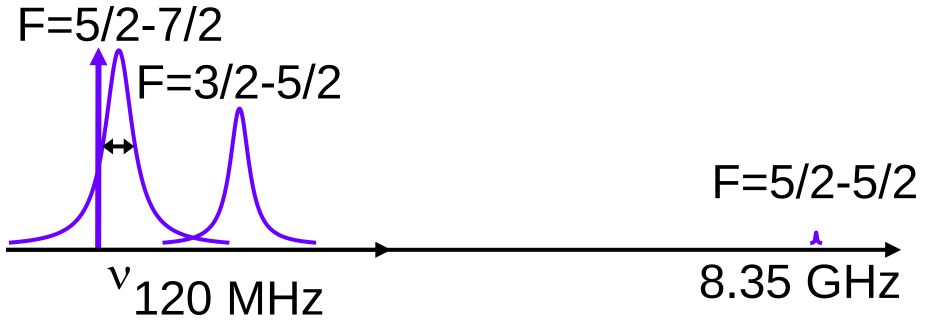
Metastable Trapping of Cd Fermions

- 2 Fermion isotopes, ^{111}Cd & ^{113}Cd, both with:
  - spin ½ nuclei → only 2 hyperfine sub-levels
  - negative nuclear magnetic moments → inverted hfs.

→ No additional lasers are needed to laser cool Cd fermions.



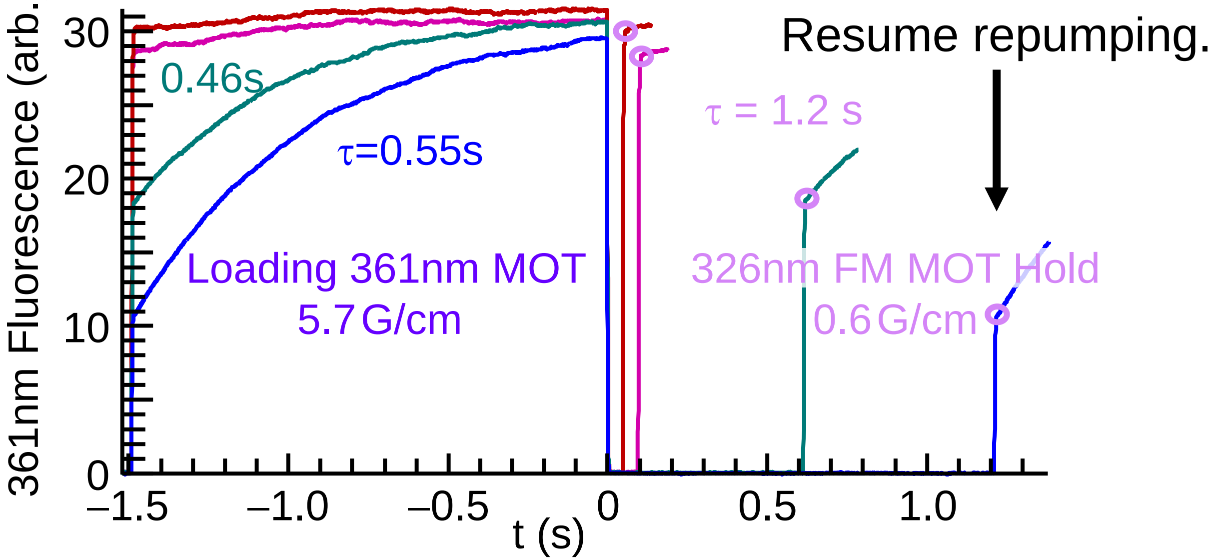


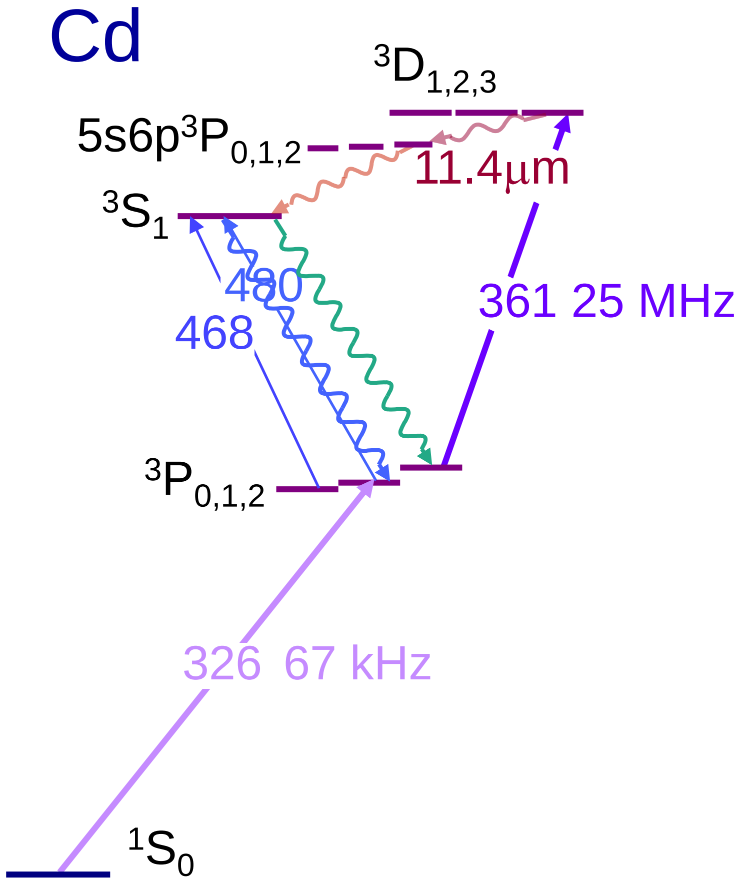


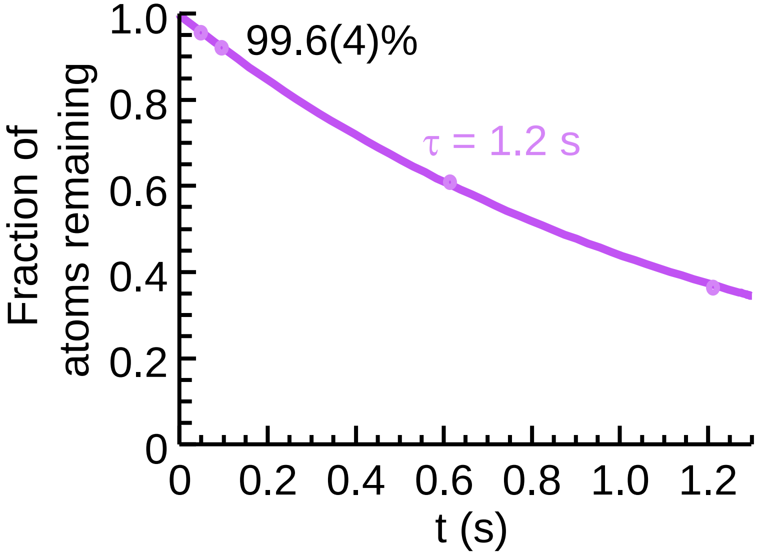
- Large loading rates of all 8 stable bosonic & fermionic isotopes.

Transfer from Metastable MOT to Narrow Line MOT

- Inhibit 480 nm $^3P_1$ repumping to transfer ^{114}Cd to 326 nm FM MOT.





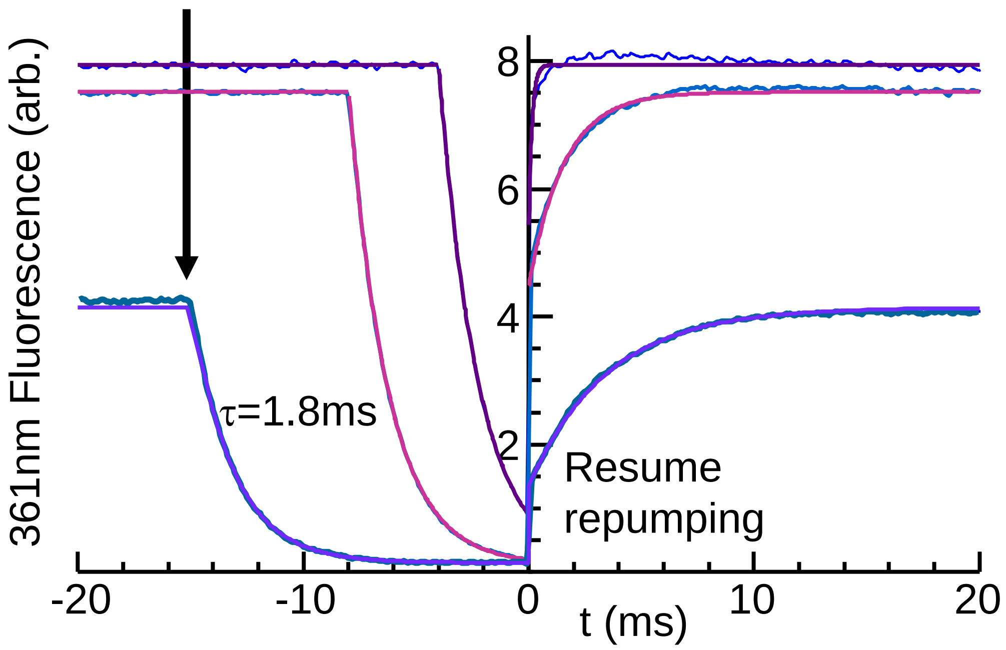


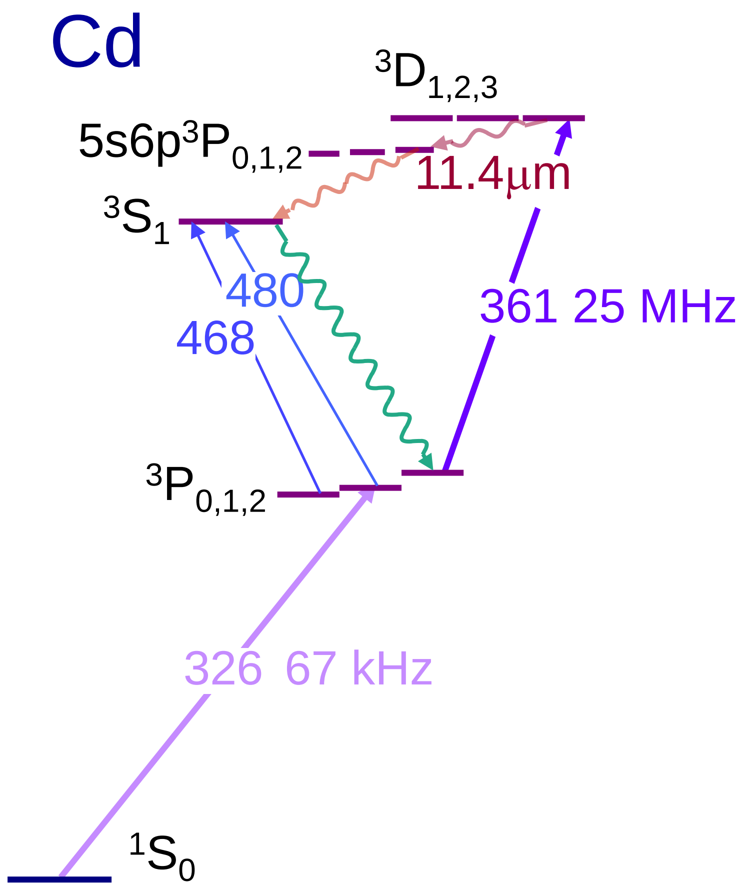
- All atoms are transferred from 361nm MOT to 326nm FM MOT.
- 326nm FM MOT holds atoms for a long time.
- Light assisted, fine structure changing, or energy pooling $^3P_2$ collisions?

Traverso, Chakraborty, Martinez … Killian, PRA ‘09

Metastable $^3P_2$ and $^3P_0$ collisions

- Inhibit 468 nm $^3P_0$ repumping to transfer ^{114}Cd to (untrapped) $^3P_0$ state.





- 361nm MOT continues to more slowly accumulate atoms, loading them into $^3P_0$.
- With atoms in $^3P_0$, more atoms are trapped than when all atoms are in $^3P_2$.
- More investigation needed to confirm $^3P_2$ fine-structure changing collisions are the dominant 361nm MOT loss mechanism.

Traverso, Chakraborty, Martinez … Killian, PRA '09

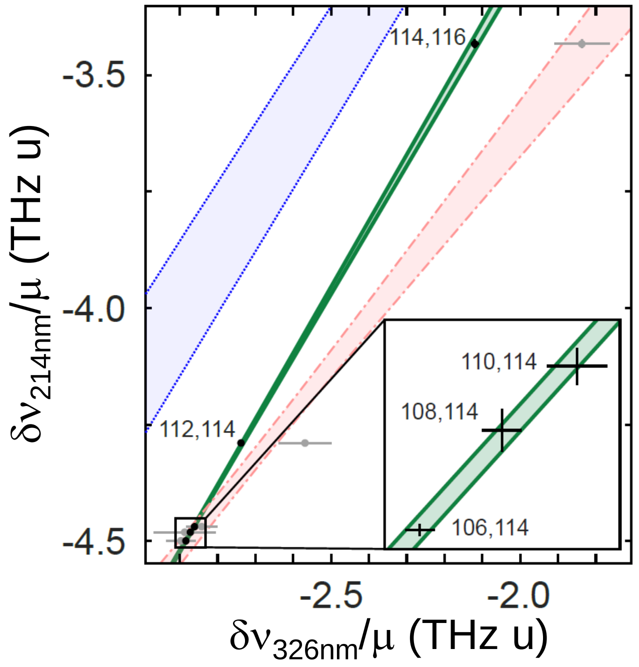
Isotope Shifts – a Probe of Beyond Standard Model Physics

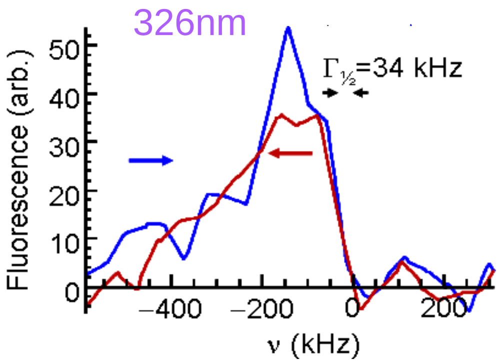
- "Normal Mass Shift" - the frequencies of deuterium transitions are higher than hydrogen's:

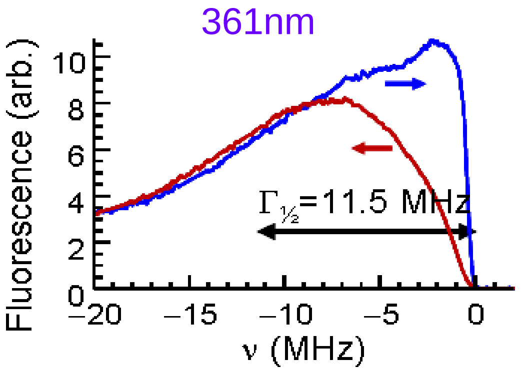
$$E_n = -\frac{Ry}{n^2}\frac{1}{1+\frac{m_e}{m_{nuc}}}$$

- There is also the "Specific Mass Shift" – electrons of multi-electron atoms, including relativistic shifts.
- The "Field Shift" is the overlap of electrons with the nuclear charge distribution. It can be large in heavy atoms.
- King Plots: (I=0) isotope shifts of frequencies of 2 transitions, divided by μ, the inverse nuclear mass difference.

$$\delta\nu_i^{A,A'} = F_i \delta\lambda^{A,A'} + K_i \mu^{A,A'}$$







Reference cavity:

905.03 MHz FSR,

140.84 MHz & 201.04 MHz transverse mode splittings.

Isotope	326nm (MHz)	Cd⁺ 214 nm	480nm (MHz)	361m (MHz)
106	1913.0	2991	-798.5	-607.6
108	1402.4	2194	-586.7	-447.4
110	914.7	1432	-383.4	-293.9
112	429.9	675	-183.1	-142.2
114	0	0	0	0
116	-321.5	-527	152.7	122.0

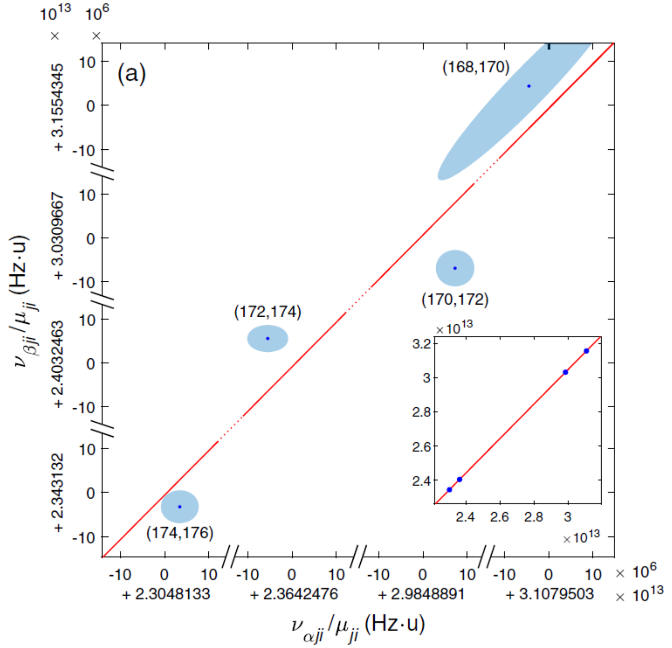
- 326 nm measurements clarified ISS & 4σ nuclear charge radius discrepancy from 1959: $F_{326}=-4354(62)$ MHz/fm^2, previously $-6200(500)$ MHz/fm^2.
- First measurements of 480nm & 361nm ISS's.

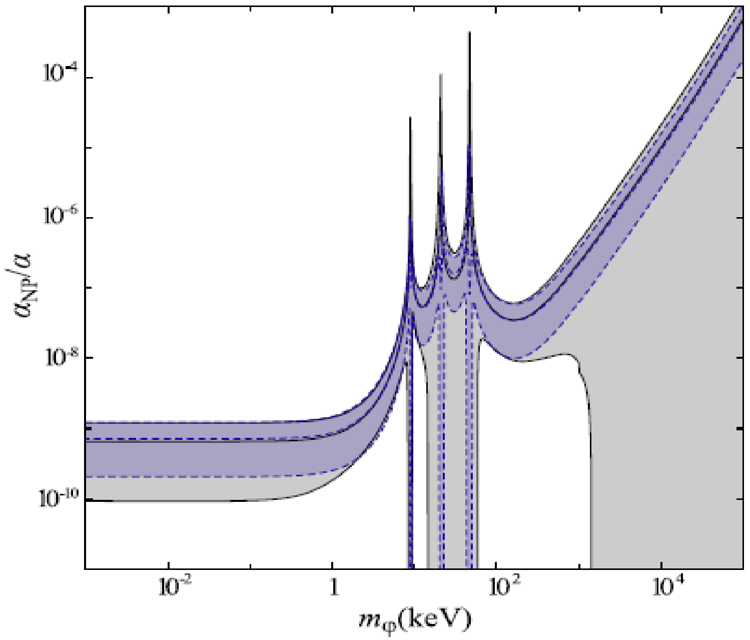
Ohayon, Hofsäss, Padilla-Castillo, Wright, Meijer, Truppe, KG & Sahoo, New J. Phys '22

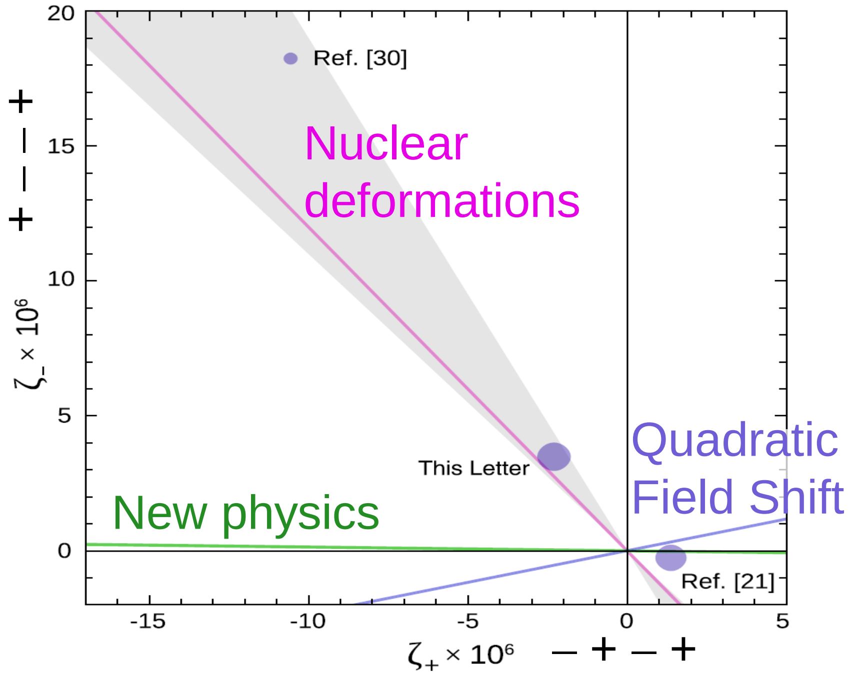
Cd⁺ 214 nm: Han, Pan … Wang, PRR '22. Schelfhout and McFerran, PRA '22 Kelly & Tomchuk, Proc. Phys. Soc. '59

Yb Isotope Shifts

- Frequency measurements of 5 Yb^+ & Yb isotopes give a pattern of King Plot non-linearities.
  - An important check of SM & for searches for BSM physics.
- 2 possible nonlinearities for 5 measurements: + – – + and – + – +.
  - Nuclear deformations may explain most of the deviations.
- Search for intermediate-mass bosonic candidate for Dark Matter.







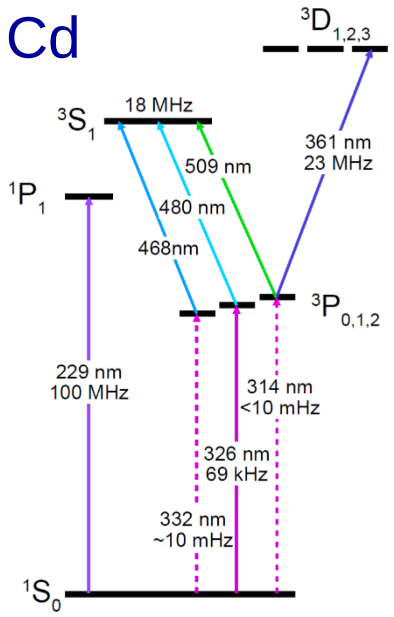
Counts … Berengut … Vuletić, PRL 2020
Hur … Berengut … Vuletić, PRL 2022

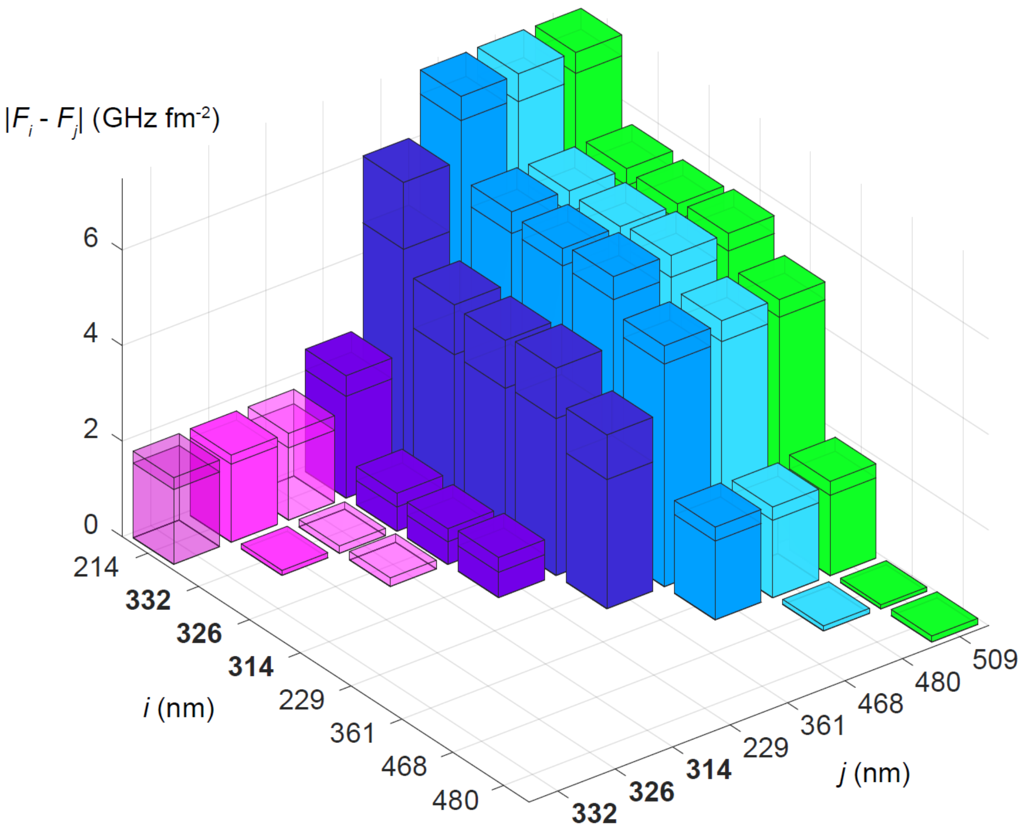
Figueroa, Berengut…Flambaum, Budker… PRL 2022
Ono… Yamamoto, Tanaka & Takahashi, PRX 2022

Ca^+: Solaro … Berengut, Fuchs… PRL 2020

Cd Isotope Shifts and Probing Beyond Standard Model Physics

- A King Plot nonlinearity from at least 4 I=0 isotopes.
  - More isotopes improves the check of the ISS pattern.
- Cd has 6 I=0 isotopes, moderately heavy, with smaller nuclear deformations than Yb.
  - Cd^{48} is close to the Z=50 proton shell gap.
- The sensitivity to new physics should be similar to the difference of the Field Shifts of two transitions.
- Two possibilities:
  - Precise measurements of 2 narrow transitions.
  - One narrow transition & one between higher states with a very different Field Shifts.
- ISs & precise structure calculations determine F_{D2} & K_{D2} for Cd+ (&D1). F_i & K_i for other transitions can then be bootstrapped from King Plots.
  - 60× more precise differences of charge radii than e^- & muonic x-ray scattering.

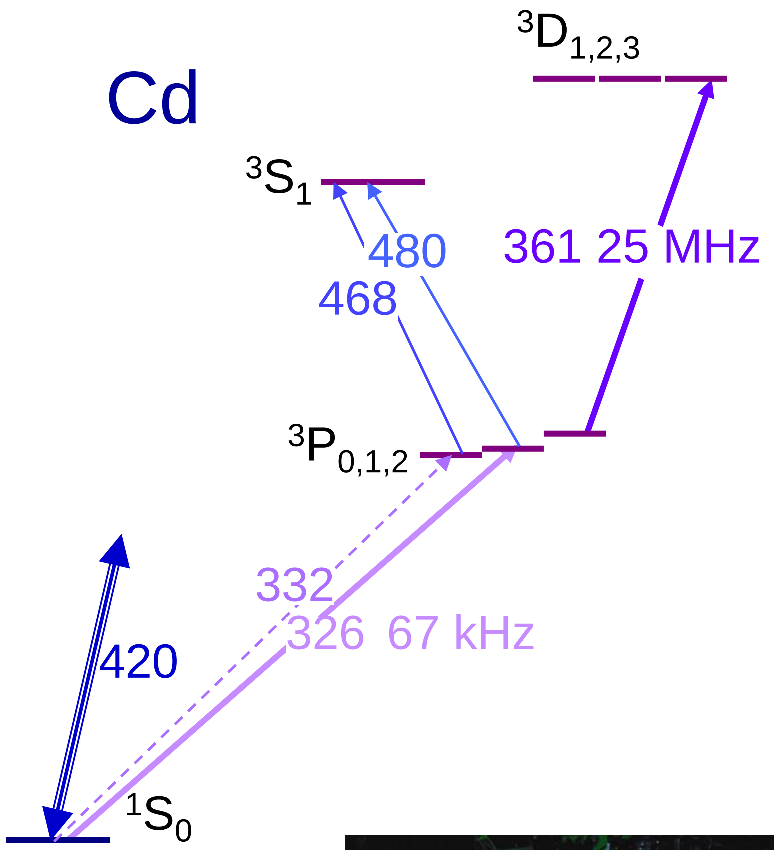


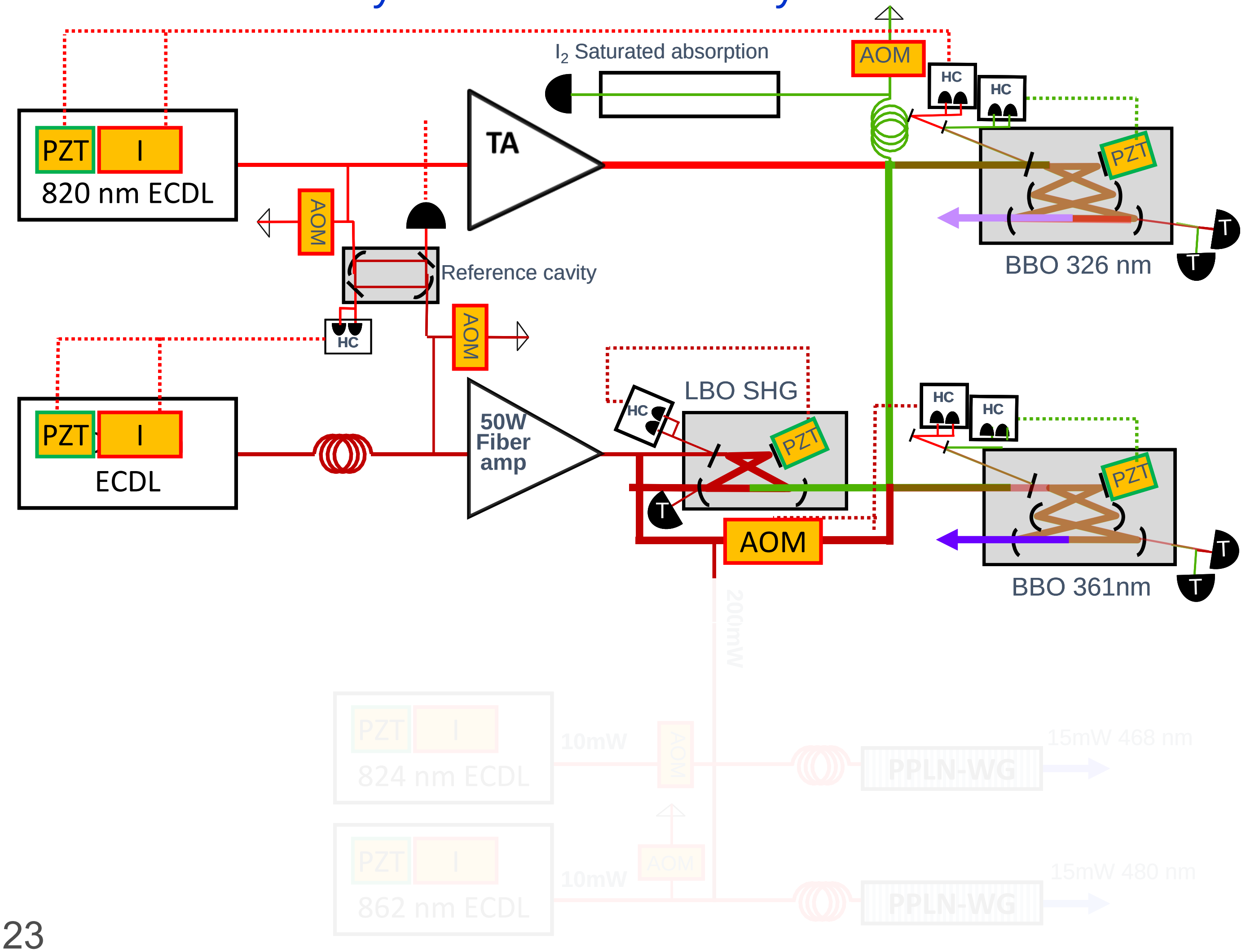


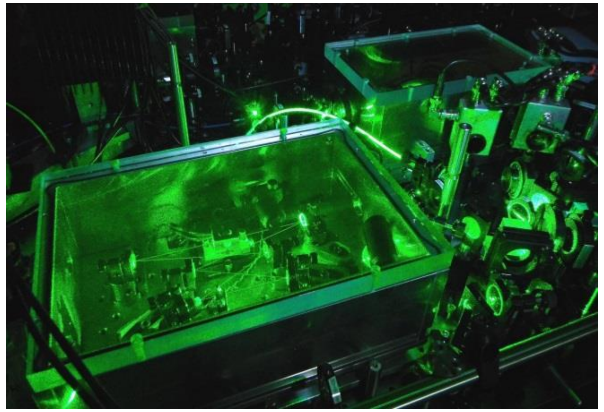
Ohayon, Hofsäss, Padilla-Castillo, Wright, Meijer, Truppe, KG & Sahoo, New J. Phys ‘22

(Too?) Elegant Scheme to Generate Cd Wavelengths

- 50 W Fiber Amplifier
- Single reference cavity for all lasers: 905.03 MHz FSR, 140.84 MHz & 201.04 MHz transverse mode splittings.
- Auto-locks with a single FPGA
- Switch to SHG vs. TA & doubly resonant SFG cavity?





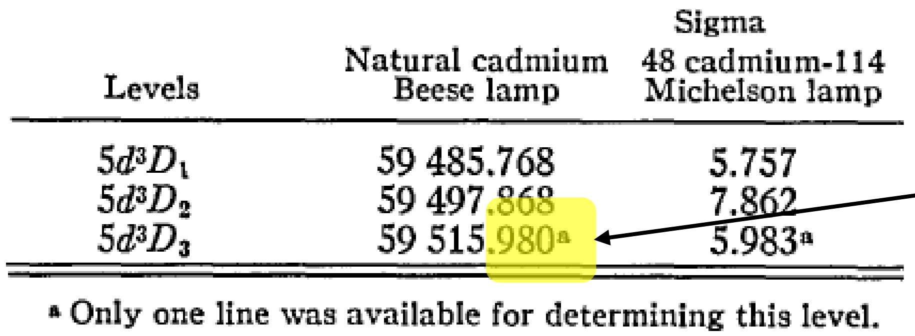


Schussheim & KG, Front. Opt. Sci. '18, Rev. Sci. Instr. '23

361 nm $^3P_2$ -$^3D_3$ Transition Frequency Measurement

- From I_2 coincidence we get: $\nu_{114Cd} = 3/2\ \nu_{R83\ 28\text{-}0\ a7} + 261(3)$ MHz.
- The previous measurement was Adam & Burns '56.
  - The paper had a typographical error of 0.010 cm^{-1} (300 MHz). (Alexander Kramida, NIST, database to be corrected)
- Many I_2 lines have been measured with frequency combs. A molecular model with hyperfine interactions gives nearby lines to 10 MHz (Tiemann, Hannover).

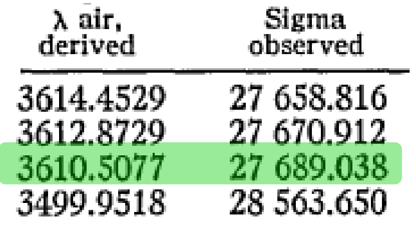
  $\nu_{R83\ 28\text{-}0\ a7}$= 553,397,541(10) MHz
  → Natural Cd $^3D_3$: 59515.990 cm^{-1}
  - Previous uncertainty of 300 MHz → 15 MHz from I_2.



Levels	Natural cadmium Beese lamp	Sigma
		48 cadmium-114 Michelson lamp
$5d^3D_1$	59 485.768	5.757
$5d^3D_2$	59 497.868	7.862
$5d^3D_3$	59 515.980[a]	5.983[a]

[a] Only one line was available for determining this level.

59515.9**9**0

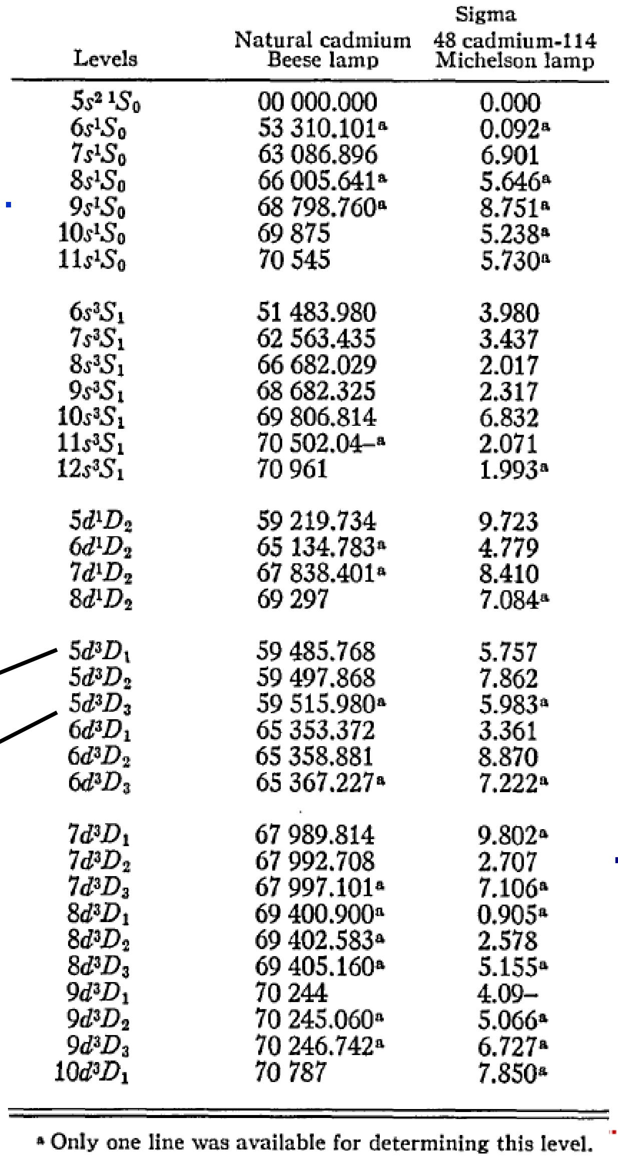


λ air, derived	Sigma observed
3614.4529	27 658.816
3612.8729	27 670.912
3610.5077	27 689.038
3499.9518	28 563.650

^{114}Cd: 27689.04124 cm^{-1}

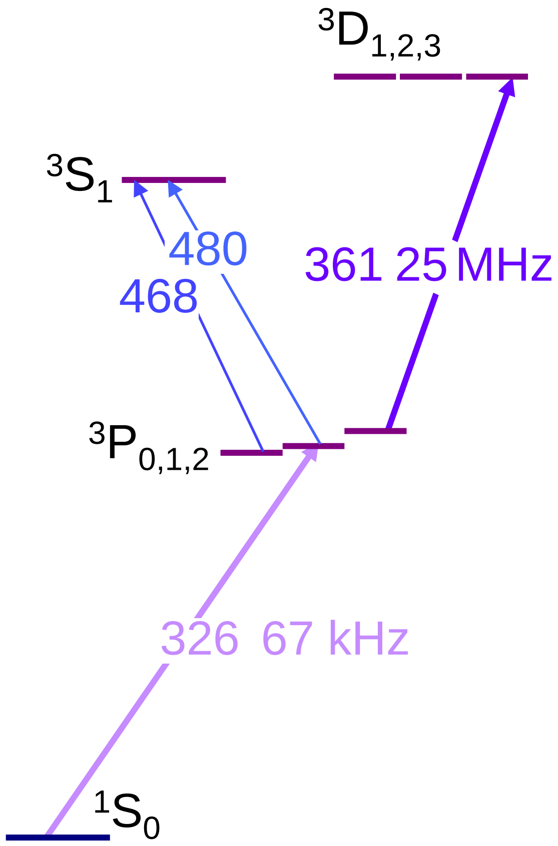
Adam & Burns, JOSA B '56

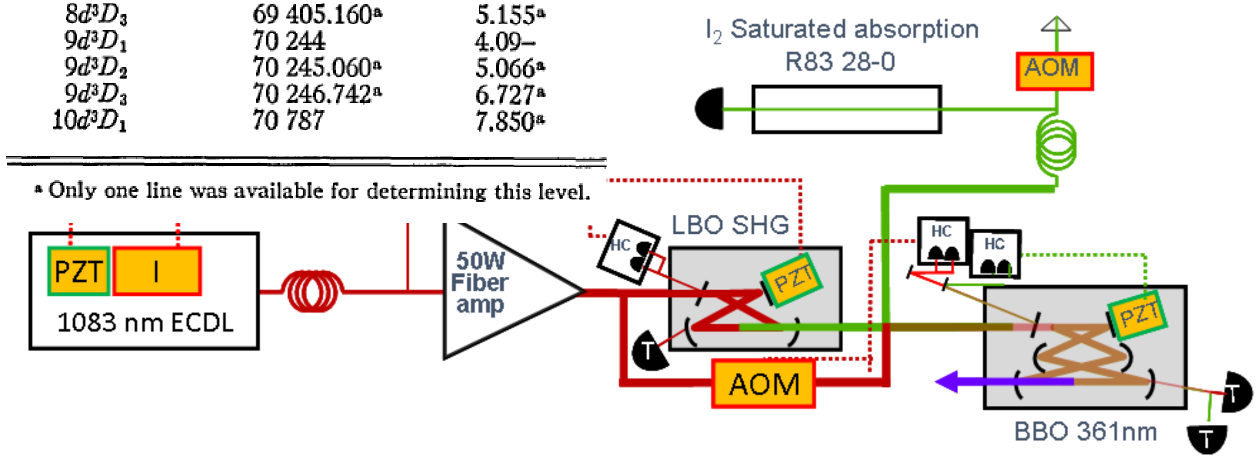
nist.gov cd I



Levels	Natural cadmium Beese lamp	Sigma
		48 cadmium-114 Michelson lamp
$5s^2\ ^1S_0$	00 000.000	0.000
$6s^1S_0$	53 310.101[a]	0.092[a]
$7s^1S_0$	63 086.896	6.901
$8s^1S_0$	66 005.641[a]	5.646[a]
$9s^1S_0$	68 798.760[a]	8.751[a]
$10s^1S_0$	69 875	5.238[a]
$11s^1S_0$	70 545	5.730[a]
$6s^3S_1$	51 483.980	3.980
$7s^3S_1$	62 563.435	3.437
$8s^3S_1$	66 682.029	2.017
$9s^3S_1$	68 682.325	2.317
$10s^3S_1$	69 806.814	6.832
$11s^3S_1$	70 502.04–[a]	2.071
$12s^3S_1$	70 961	1.993[a]
$5d^1D_2$	59 219.734	9.723
$6d^1D_2$	65 134.783[a]	4.779
$7d^1D_2$	67 838.401[a]	8.410
$8d^1D_2$	69 297	7.084[a]
$5d^3D_1$	59 485.768	5.757
$5d^3D_2$	59 497.868	7.862
$5d^3D_3$	59 515.980[a]	5.983[a]
$6d^3D_1$	65 353.372	3.361
$6d^3D_2$	65 358.881	8.870
$6d^3D_3$	65 367.227[a]	7.222[a]
$7d^3D_1$	67 989.814	9.802[a]
$7d^3D_2$	67 992.708	2.707
$7d^3D_3$	67 997.101[a]	7.106[a]
$8d^3D_1$	69 400.900[a]	0.905[a]
$8d^3D_2$	69 402.583[a]	2.578
$8d^3D_3$	69 405.160[a]	5.155[a]
$9d^3D_1$	70 244	4.09–
$9d^3D_2$	70 245.060[a]	5.066[a]
$9d^3D_3$	70 246.742[a]	6.727[a]
$10d^3D_1$	70 787	7.850[a]

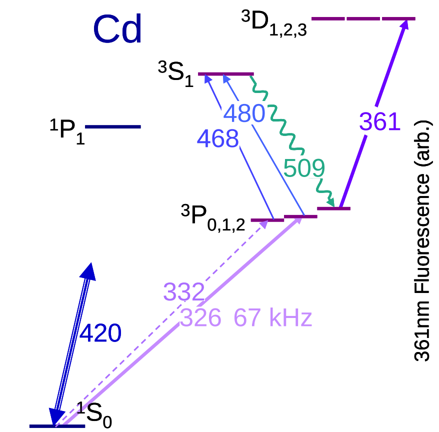
[a] Only one line was available for determining this level.

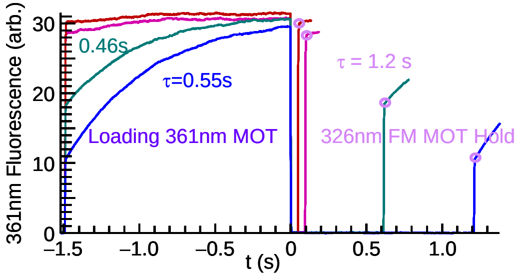


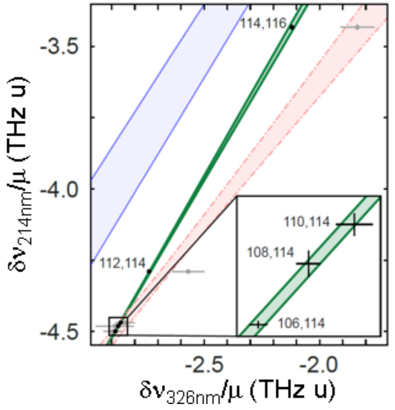


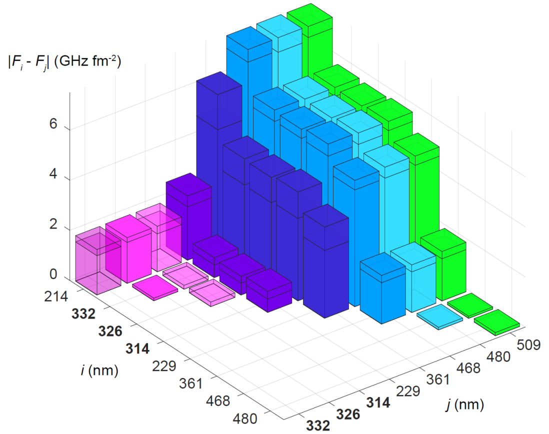
Summary

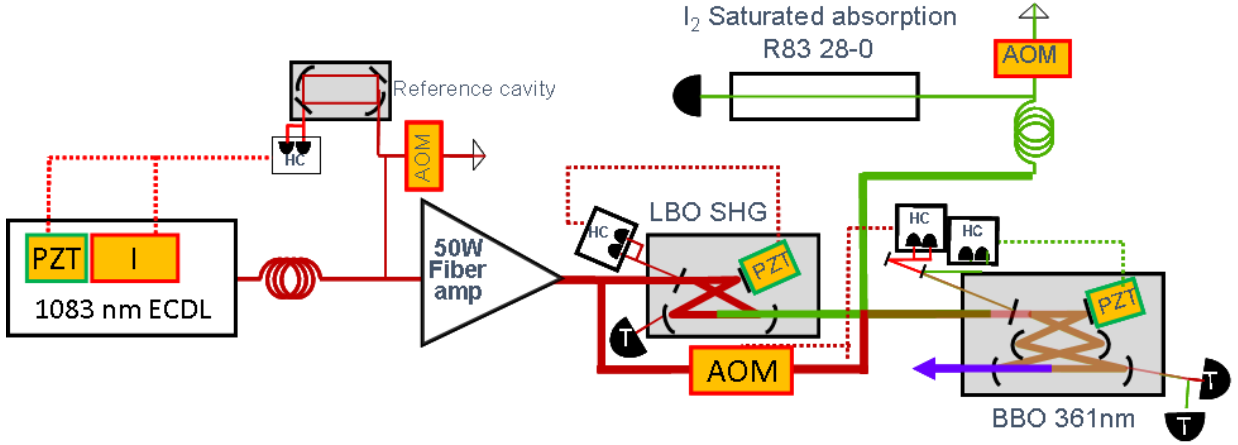
- Cadmium for optical lattice clocks
  - Small BBR sensitivity
  - Narrow-line cooling & magic wavelength
- MOT using only 50 mW of 326 nm light exciting the 67 kHz wide $^1S_0 \rightarrow {}^3P_1$ transition
- Large MOT without UVC light – UVA & blue triplet excitations
  - Only small MOT gradients are required, < 6 G/cm
  - Low temperatures & 100% transfer to narrow-line MOT
- Cd fermions, ^{111}Cd & ^{113}Cd
  - Spin ½ & negative nuclear magnetic moments
  - No additional repumping required
- Isotope shifts & King plots for beyond SM Physics
  - 6 stable spin 0 isotopes, ^{106}Cd to ^{116}Cd
- UVA SFG lasers with an auto-locking FPGA control system
- Absolute frequency measurement of 361 nm $^3P_2 \rightarrow {}^3D_3$
  - $f_{114Cd} = 3/2\ \nu_{R83\ 28\text{-}0\ a7} + 261(3)$ MHz











Many Channel FPGA System with Auto-Lock

- Auto-lock entire system with a single FPGA module
- 9 fast servos with MHz bandwidth, 3 AWG’s, 8 slow (temperature) servos, DSP …
- Touch-screen control:

- Simplified IIR PID’s:

$$y_0 = a_1 y_1 + b_0 x_0 + b_1 x_1$$

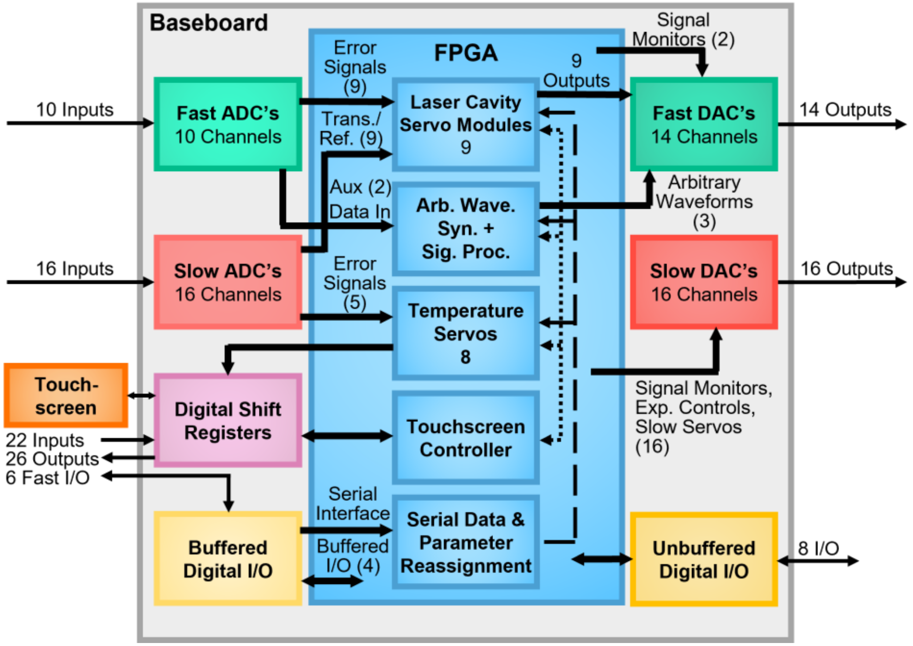
- Typical servo gain margins are ×2 or $2^{½}$
- Many servo’s have gain and frequency resolution of 1,2,5,10 …
- Other FPGA servos use large multipliers – timing can be challenging at ≥100 MS/s.
- Multiplications by 2^n are fast and efficient.

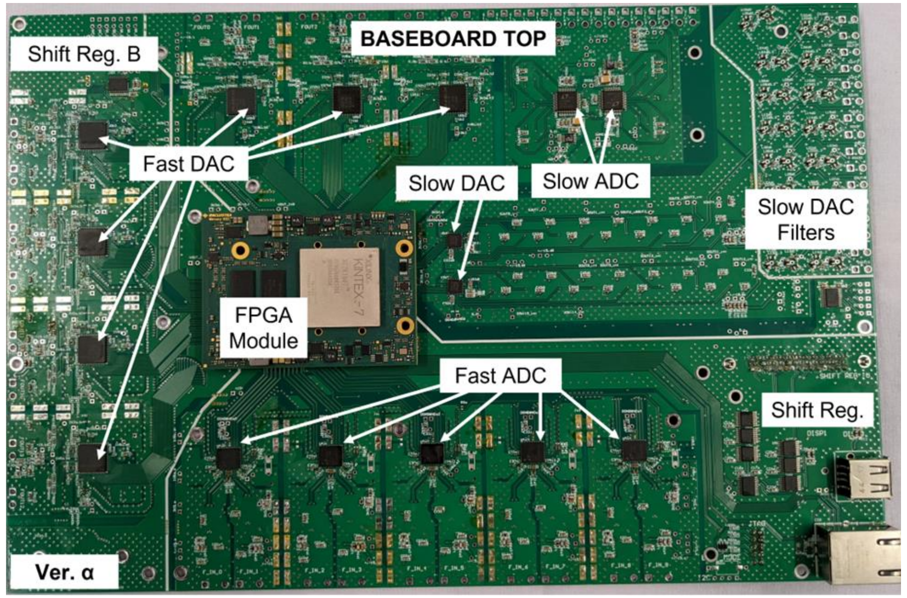
$$a_1 = 1 - \tilde{\omega}_H \qquad b_0 = b_1 = \tilde{G}/2$$

$$y_0 = y_1 - \tilde{\omega}_H y_1 + \frac{\tilde{G}}{2}\left(x_0 + x_1\right)$$

- We get 25% gain and frequency resolution with one optional addition: 0.875, 1, 1.25, 1.5, 1.75, 2…

$$\tilde{G}, \tilde{\omega}_H \ldots = 2^n\left(1 + \left\{-\tfrac{1}{8}, 0, \tfrac{1}{4}, \tfrac{1}{2}\right\}\right)$$

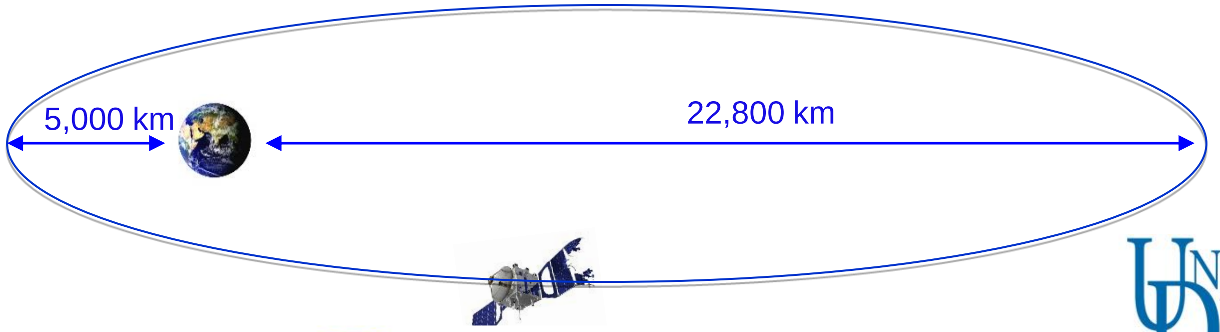


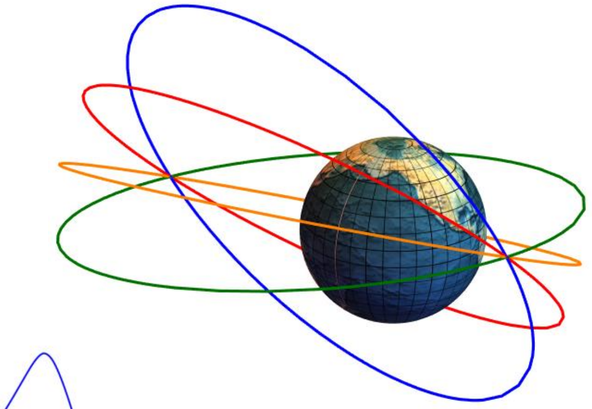


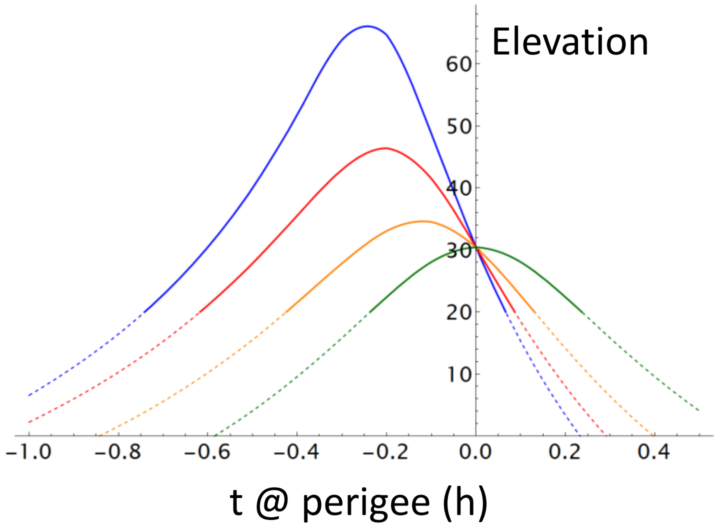
Leibrandt & Heidecker Rev. Sci. Iinstr.‘15 Schussheim & KG, Rev. Sci. Instr. ‘23

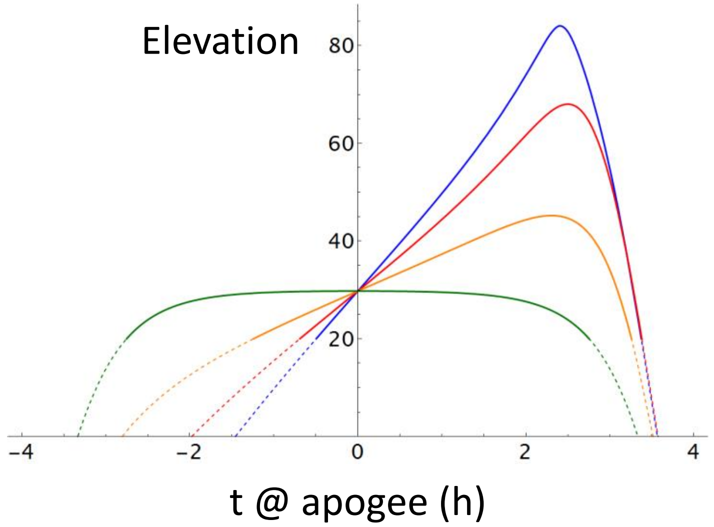
FOCOS – Fundamental Physics with an Optical Clock Orbiting in Space

- Yb lattice clock:
  1×10^{-18} accuracy & $1 \times 10^{-16}\ \tau^{-1/2}$ stability.
- Test redshift at 1×10^{-9} (30,000 × improvement)
- Test Local Lorentz Invariance
- World-wide network of atomic clocks
- Searches for Dark Matter on Earth and in space.
- Precision geodesic referencing at the mm-level
- Redefinition of the SI second & space-time reference.





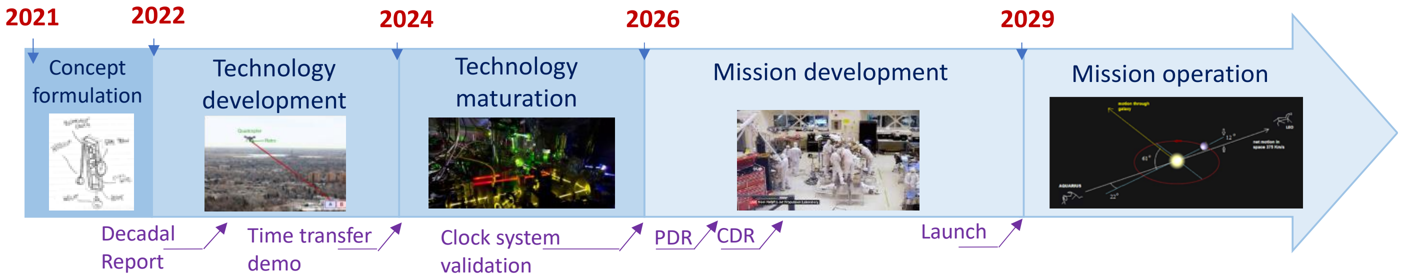




C. Oates, NIST
K. Gibble, PSU
L. Hollberg, Stanford
N. Newbury, NIST
N. Yu, CalTech-JPL
A. Derevianko, U. Nev, Reno
M. Safronova, U. Del.
L. Sinclair, NIST

An 8 hr. orbit inclination of 9.3° gives perigee and apogee maximum elevations of 30.4° at 40° N for a 5,000 km perigee altitude.

Quantum Science and Technology **7** 044002 ‘22

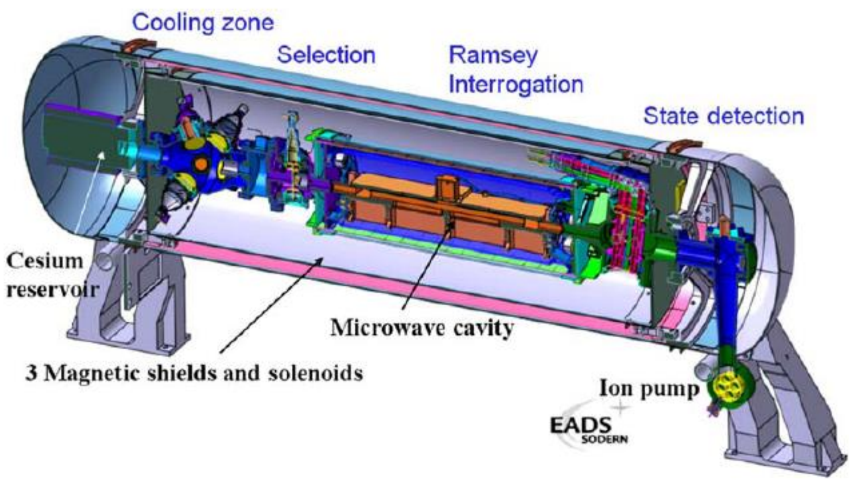


ACES: Atomic Clock Ensemble in Space

Laser-cooled (microwave) Cs clock on the International Space Station.

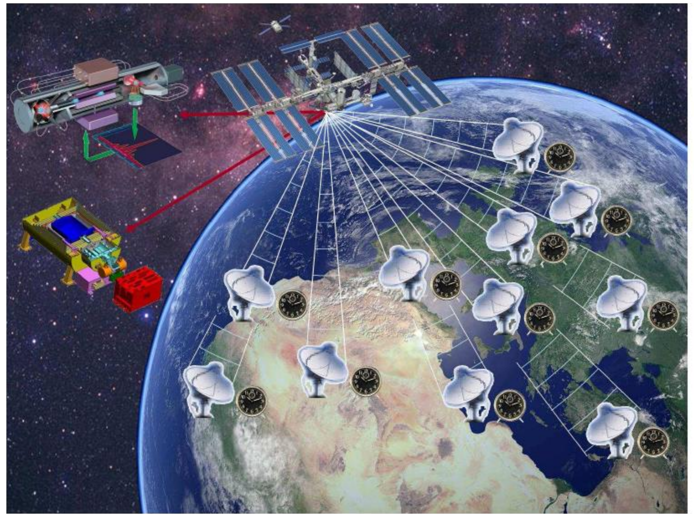
- 10^{-16} accuracy goal
- Time Transfer
- Gravitational Redshift
- Time Dilation
- Time variation of the fine structure constant
- Geodesy
- Launch in 2025.

PHARAO: Projet d'Horloge Atomique par Refroidissement d'Atomes en Orbit



PHARAO (10^{-16})	**Shift**	**Uncertainty**
Quadratic Zeeman	900	0.1
Blackbody radiation	-172	0.7
Ultracold collisions & cavity phase		$0.68\ \mathrm{yr}^{-1/2}$
Microwave Lensing	1.15	0.4
Total		**1.1**

ACES



Peterman, KG, Laurent, Salomon, Metrologia ’16

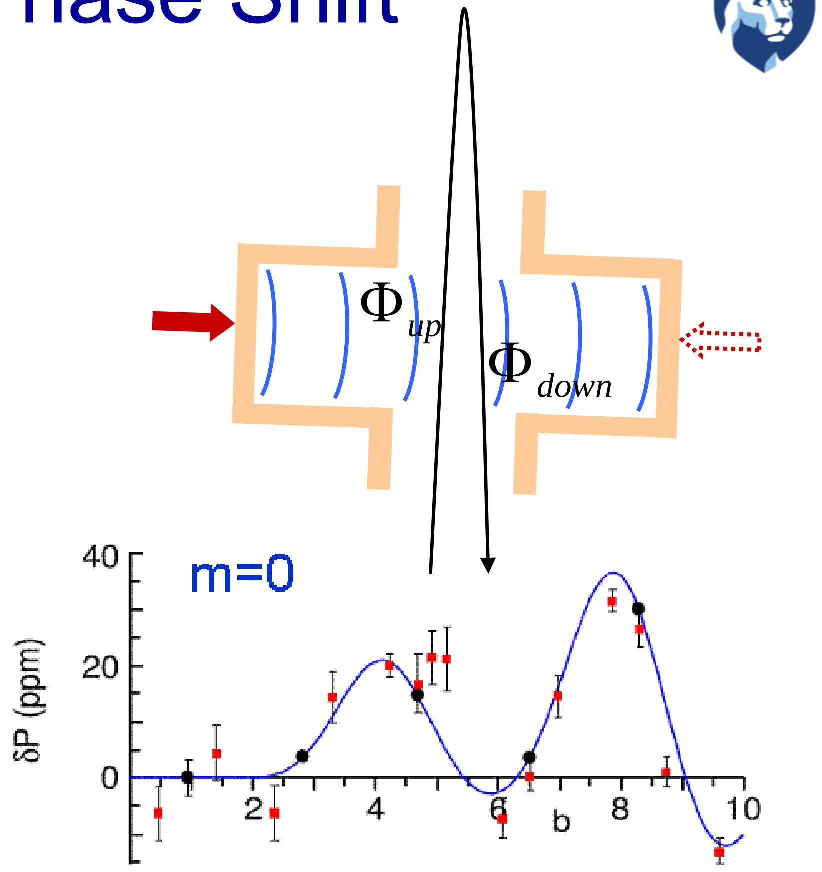
Laurent, Esnaut, KG, Peterman, Lévèque…Salomon, Metrologia ’20

(1^{st} order) Doppler Shift - “Distributed Cavity Phase Shift”

Atom motion plus a spatial phase variation of an electromagnetic field.

Standing wave + traveling wave→ spatially varying phase

$$\delta\nu = \frac{\Phi_{up} - \Phi_{down}}{\pi}\Delta\nu$$



Known since 1970’s – surprisingly, no prior calculations agreed with measurements.

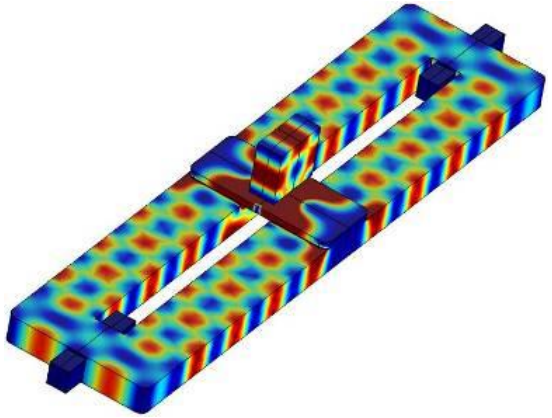
Large finite-element E&M calculations agree with no free parameters.

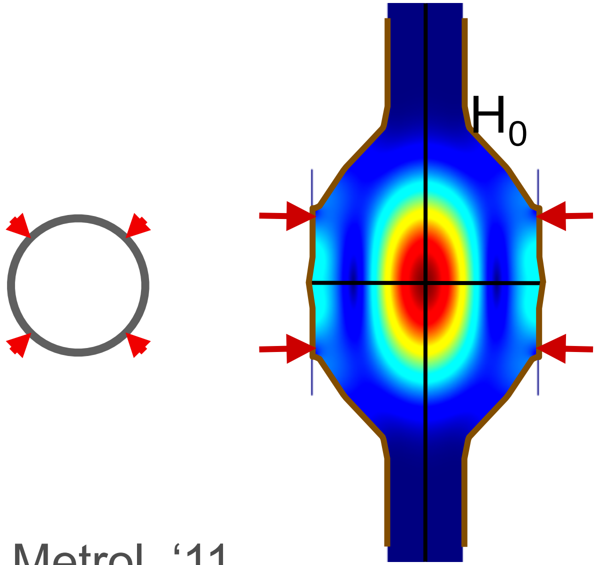
2D FEM for fountains – Fourier azimuthuthal decomposition in cos(mϕ)

3D FEM for PHARAO/ACES (1 TB RAM)

New fountain cavity design reduces Doppler uncertainty from ±3x10^{-16} to ±0.5x10^{-16}.

- NPL(UK), KRISS, NPL India, NRC, Poland, CERN





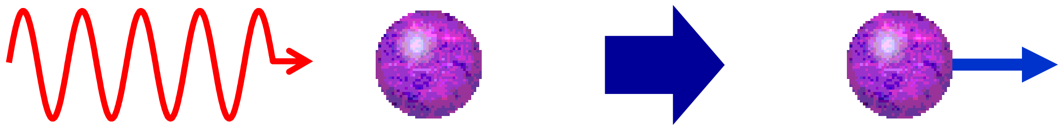
Li & KG, Metrologia ’04 & ’10,
Guéna, Li, KG, Bize, Clairon, PRL ‘10.

NPL & PTB, PSU, Metrol. ‘11
Szymaniec, Lea, KG, CPEM ‘12

Beattie, Jian, Szymaniec & KG, Metrologia ‘20

Microwave Photon Recoil?

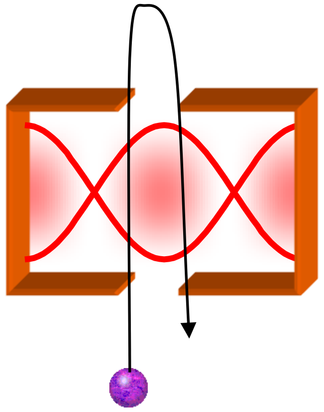
Infinite plane wave



$$\frac{hc}{\lambda} = \hbar k$$

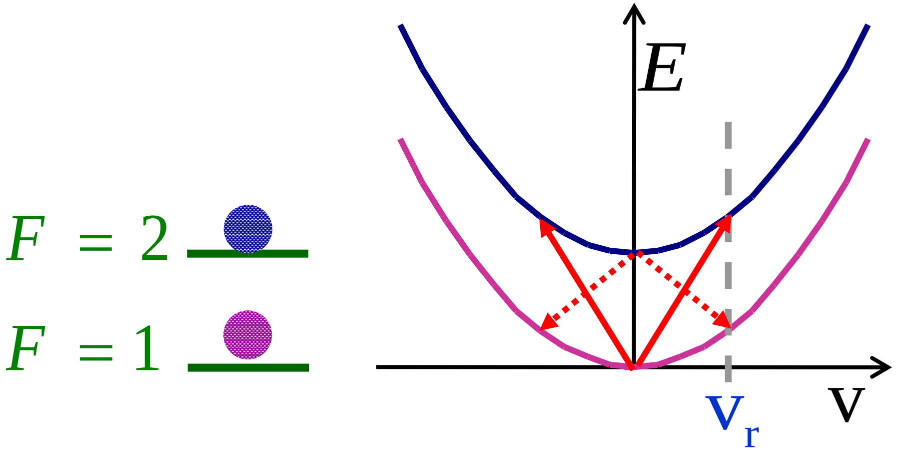
$$m v_x = \hbar k = \frac{hc}{\lambda}$$

$$v_x \approx 0.1 \mu m / s$$



Recoil Frequency Shift:

Conserve E & p.



$$\frac{\delta \upsilon}{\upsilon} = \frac{\hbar^2 k^2}{2m\hbar\omega} = \frac{\hbar\omega}{2mc^2}$$

$$= \pm 1.5 \times 10^{-16}$$

Is an atom's recoil equal to ħk?

Finite beam:

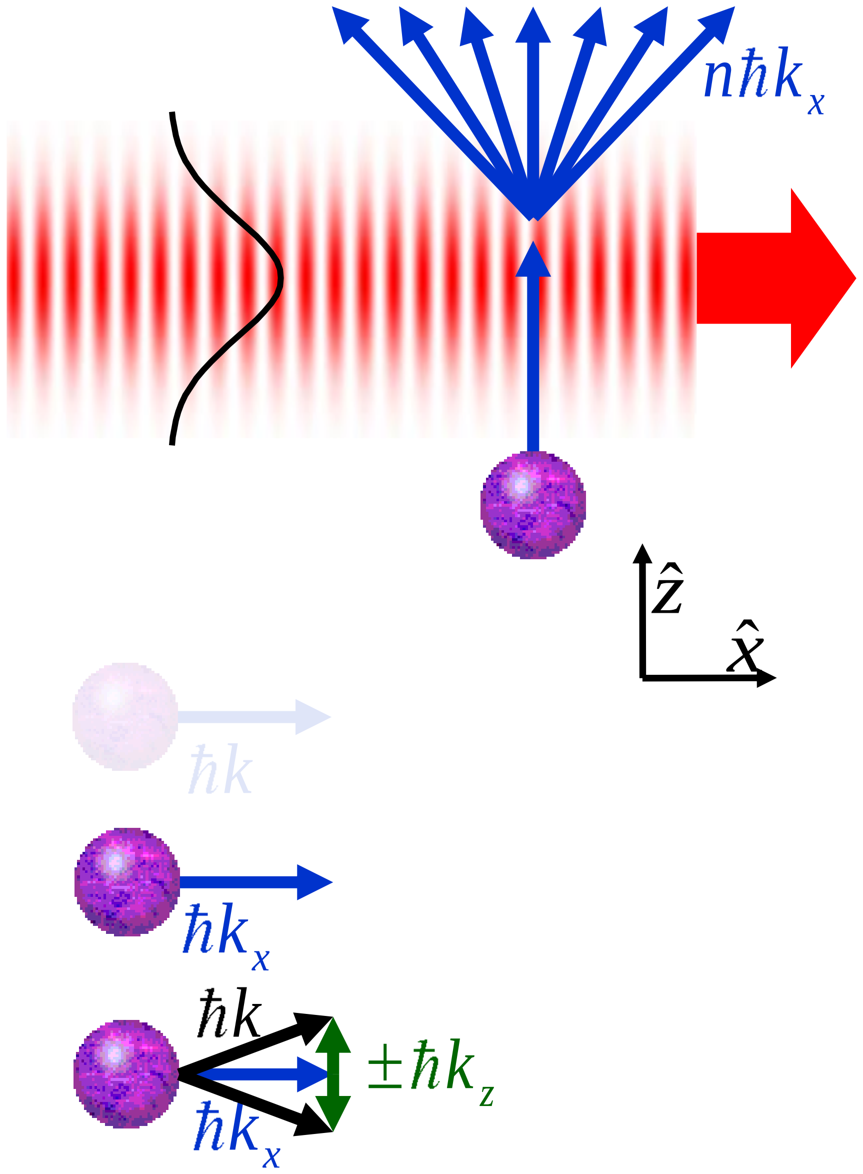
Maxwell: $$\left(\nabla^2 + k^2\right) E = 0$$

so $k_x^2 + k_y^2 + k_z^2 = k^2 \quad \& \quad k_x < k$

$$k_{y,z} \approx \frac{2}{w_0 = 2mm} \Rightarrow \delta k_x = -8\, ppb$$

Three appealing choices:

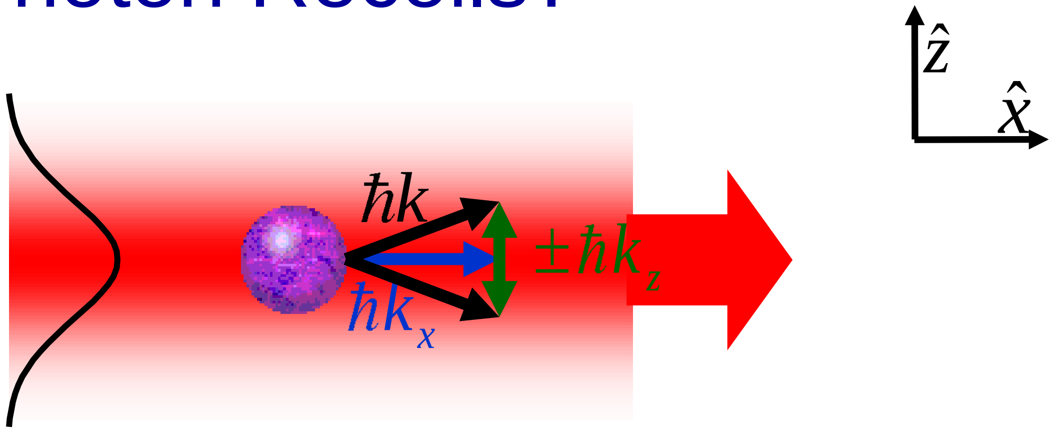
1. The photon momentum comes in discrete units of k, in the x direction.
2. The atom has a recoil of k_x in the x direction; v_y & v_z are unchanged.
3. The atom has a recoil of k_x in the x direction, and also $\pm k_y$ & $\pm k_z$.



Wicht…, Chu, Phys. Script '02
Cladé…, Biraben, PRL '06

Gould, Ruff, Pritchard, PRL '86
Wicht, … Chu, PRA '05

Transverse (Microwave) Photon Recoils?

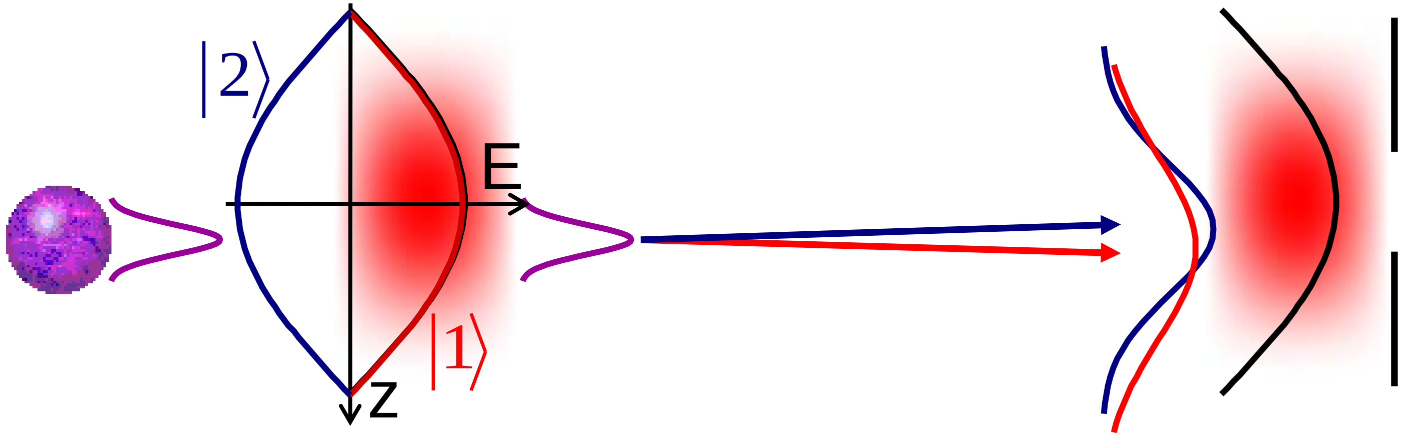


There is no grating in the z direction.

→ No recoil in z direction.

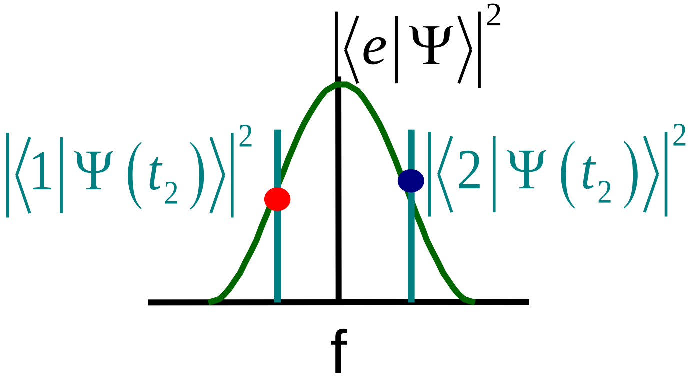
“Microwave” Stern-Gerlach regime

Same problem for microwave clocks: The dipole force of the microwave field acts as a lens on the atomic wavefunctions.



Shift: ≈ ±4nm

Δwidth: ≈ ±2nm



$$\delta P = \left[|\langle 2|\Psi(t_2)\rangle|^2 - |\langle 1|\Psi(t_2)\rangle|^2 \right] \sin\left[\frac{\pi}{2}\cos(k_x x) \right]$$

$$\delta\upsilon \neq \frac{\hbar k^2}{4\pi m}$$

$$\delta\upsilon = \frac{\pi}{2}\Delta\upsilon_R \frac{w_1}{w_2}$$

$$\frac{\delta\upsilon}{\upsilon} \neq 1.5\times10^{-16}$$

$$\frac{\delta\upsilon}{\upsilon} \approx 7\times10^{-17}$$

The majority of primary clocks correct the not-yet-observed bias that we calculate.

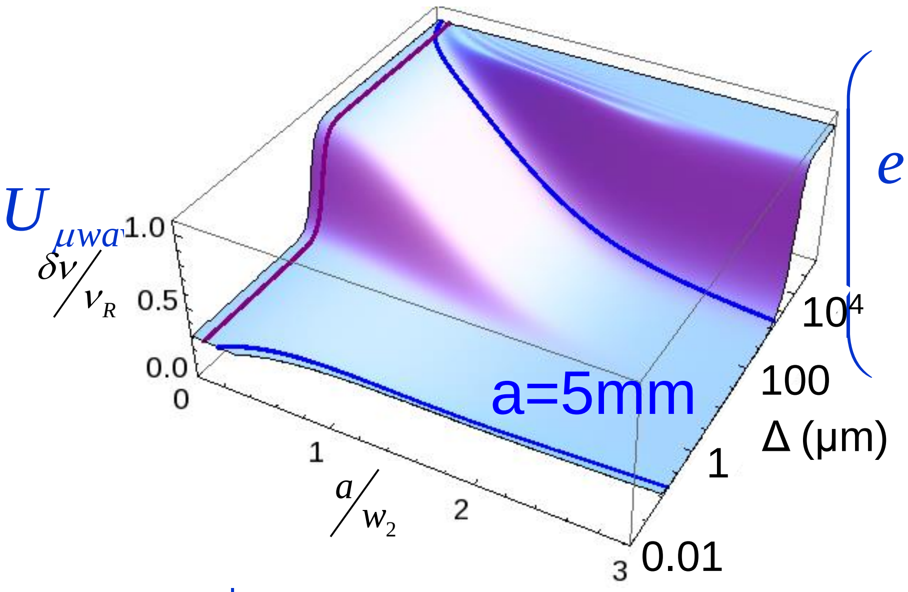
KG PRL ’06, PRA ’14 …

Microwave Lensing & Photon Recoil Shifts

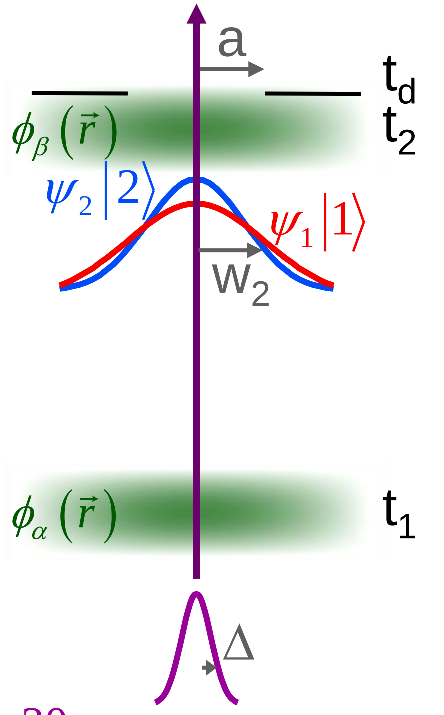
- Consider a single atom traveling on the fountain axis.
  - weak pulses ($\phi_{\alpha,\beta}\rightarrow 0$), 1D, detect right after 2nd interaction.
- Only single photon recoils contribute → analytic.

$$\delta\nu = \frac{1}{2\pi T}\frac{\phi_\alpha\phi_\beta\int_{-a}^{a}\cos\left(k_\beta x\right)\sum_{\pm}\pm e^{-\frac{x^2\pm v_R xT+\frac{1}{2}v_R{}^2T^2}{w_2^2}}\sin\left(w_{12}^2\frac{k_\alpha x\pm\omega_R T}{w_2^2}\right)dx}{\phi_\alpha\phi_\beta\int_{-a}^{a}\cos\left(k_\beta x\right)\sum_{\pm}e^{-\frac{x^2\pm v_R xT+\frac{1}{2}v_R{}^2T^2}{w_2^2}}\cos\left(w_{12}^2\frac{k_\alpha x\pm\omega_R T}{w_2^2}\right)dx}$$

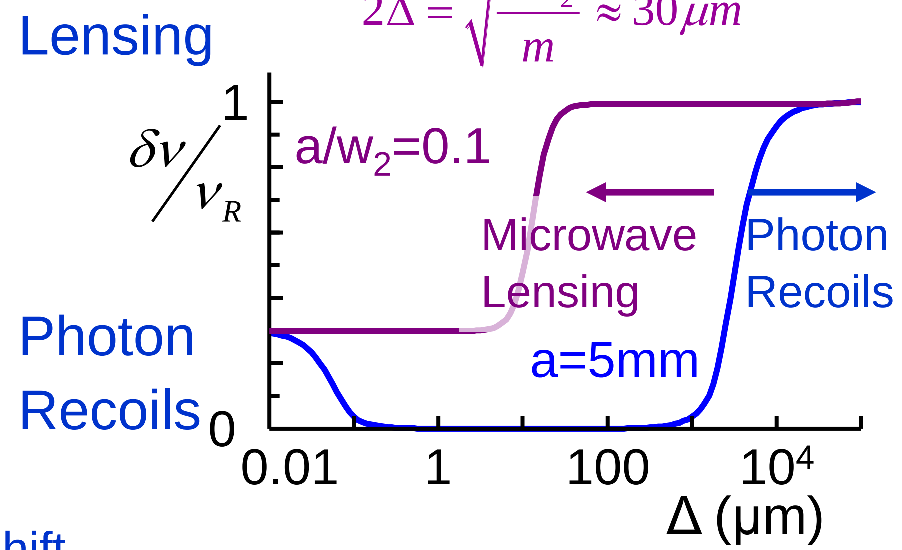
$$\frac{w_{12}^2}{w_2^2}=\frac{\frac{\hbar^2}{2m^2\Delta^2}t_1t_2+2\Delta^2}{\frac{\hbar^2}{2m^2\Delta^2}t_2^2+2\Delta^2}$$



$$U_{\mu wa}=\begin{pmatrix} e^{-\frac{i}{2}\varphi\left(1-\frac{k^2x^2}{2}+\dots\right)} & 0 \\ 0 & e^{\frac{i}{2}\varphi\left(1-\frac{k^2x^2}{2}+\dots\right)} \end{pmatrix}$$



$$2\Delta=\sqrt{\frac{2\hbar t_2}{m}}\approx 30\mu m$$

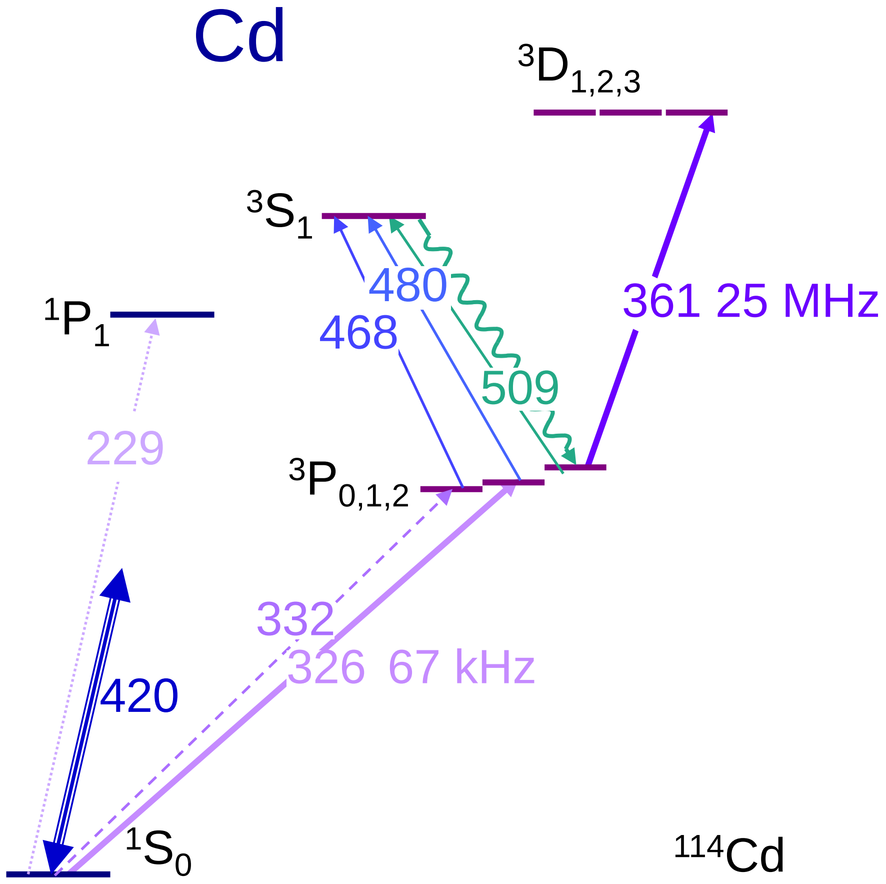


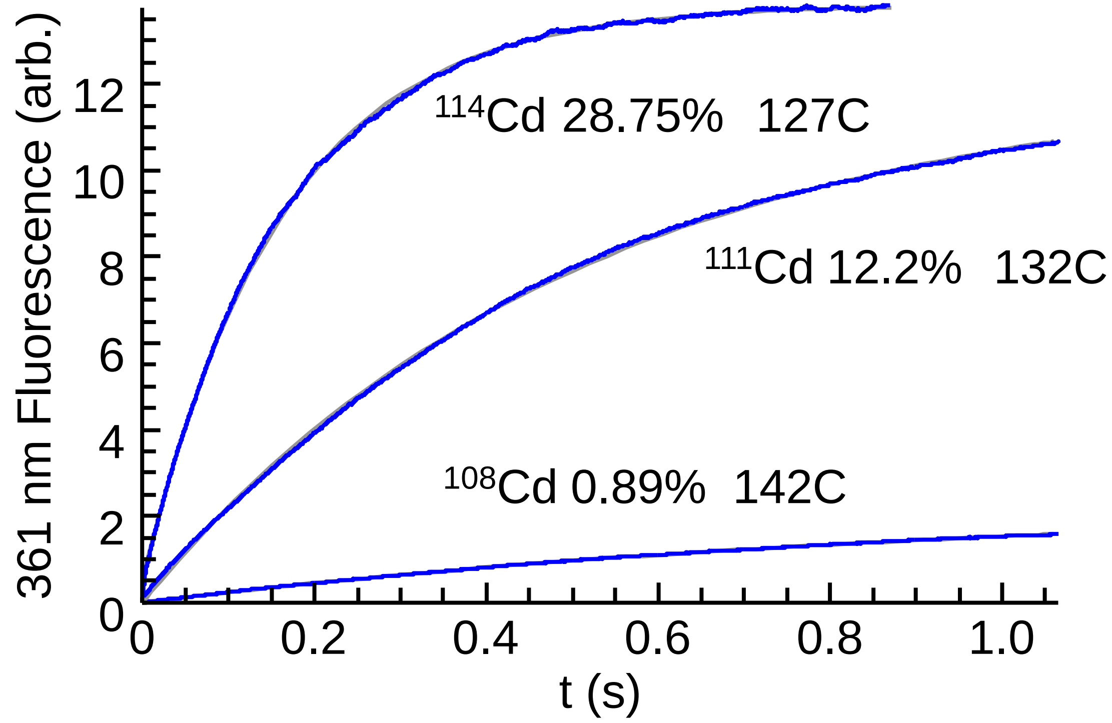
- As Δ→∞, microwave lensing → the recoil shift (& a→∞).
- Confirmation of sign & numerical factors of calculated microwave lensing shift.

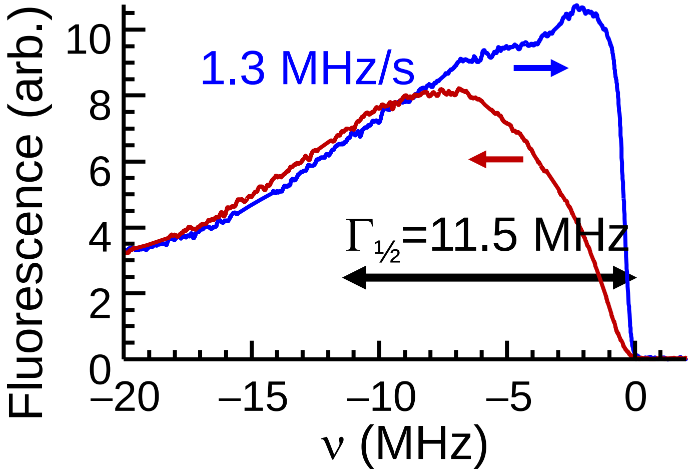
KG PRL '06, 8th SFSM J. Phys: Conf. Ser (2016)

Enhanced Loading with Cd Metastable MOT

- Use only UVA - avoid 229 nm light
- 361 nm meta-stable MOT loading
  - ~10^{10} atoms, ~10× RIKEN 229 nm MOT
  - 361 nm also gives high S/N detection
- Trap all 6 Bosons: ^{106}Cd, ^{108}Cd, ^{110}Cd, ^{112}Cd, ^{114}Cd & ^{116}Cd
  - MOT gradient: 5.7 G/cm (170 to 400 G/cm for 229 nm singlet MOT).





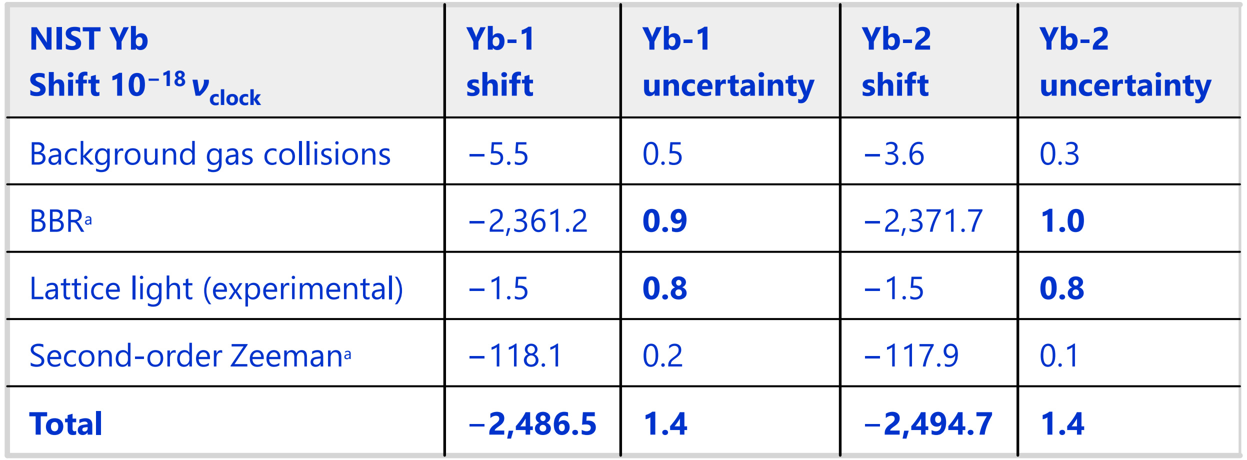


- 2 Fermion spin ½ isotopes: ^{111}Cd & ^{113}Cd
  - Spin ½ nuclei (& negative magnetic moments)

Mg: Kulosa, Fim, Zipfel, Rühmann, Sauer, Jha, KG, Ertmer, Rasel, Safronova, Porsev, PRL ‘15

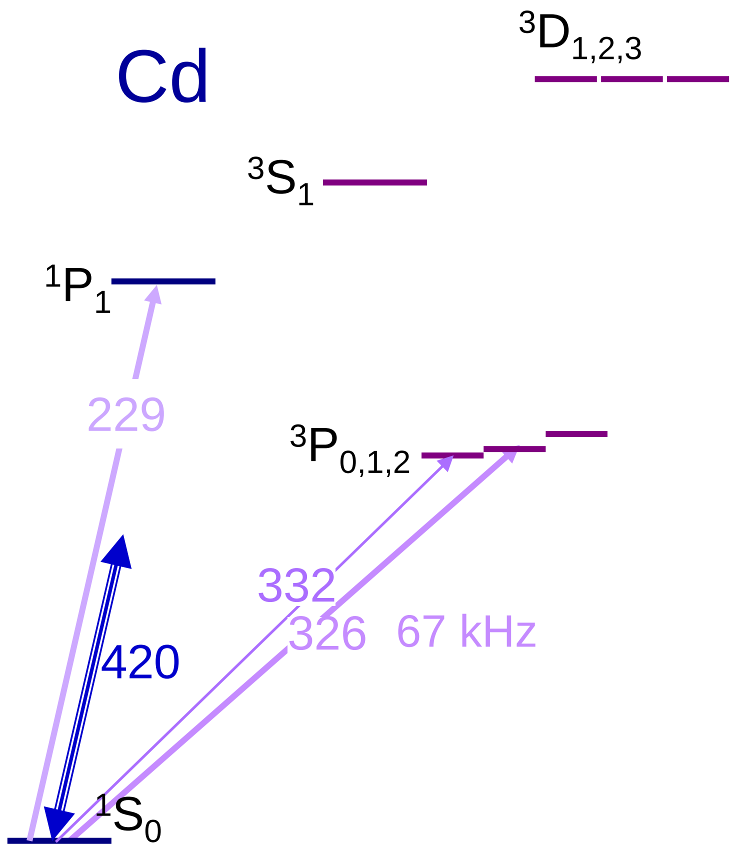
Atomic Clock Candidates

- Dominant lattice clock uncertainties have been AC Stark Shifts: from blackbody radiation and higher order shifts from the lattice light.
- Hg, Cd, Zn & Mg have small sensitivities to BBR.



NIST Yb Shift $10^{-18}\,\nu_{\text{clock}}$	**Yb-1 shift**	**Yb-1 uncertainty**	**Yb-2 shift**	**Yb-2 uncertainty**
Background gas collisions	−5.5	0.5	−3.6	0.3
BBR[a]	−2,361.2	**0.9**	−2,371.7	**1.0**
Lattice light (experimental)	−1.5	**0.8**	−1.5	**0.8**
Second-order Zeeman[a]	−118.1	0.2	−117.9	0.1
Total	**−2,486.5**	**1.4**	**−2,494.7**	**1.4**

Clock	±0.1K BBR (10^{-18})	$^3P_1$ Γ (kHz)	$^3P_1$ λ (nm)	Magic λ_m (nm)	# of I=½ Fermions	I of Fermions	1 mTorr pressure (°C)
Cs	23						
Rb	16						
Sr	7.3	7	689	813		9/2	465
Yb	3.3	182	556	760	1	**1/2**,5/2	520
Ca	3.3	0.4	657	680			405
Mg	0.53	0.036	457	469		5/2	370
Cd	0.38	67	326	420	2	**1/2**, **1/2**	215
Hg	0.24	1,300	254	363	1	**1/2**, 3/2	<22
Al⁺	0.005						



McGrew ... Ludlow *et al.*, Nature ‘18

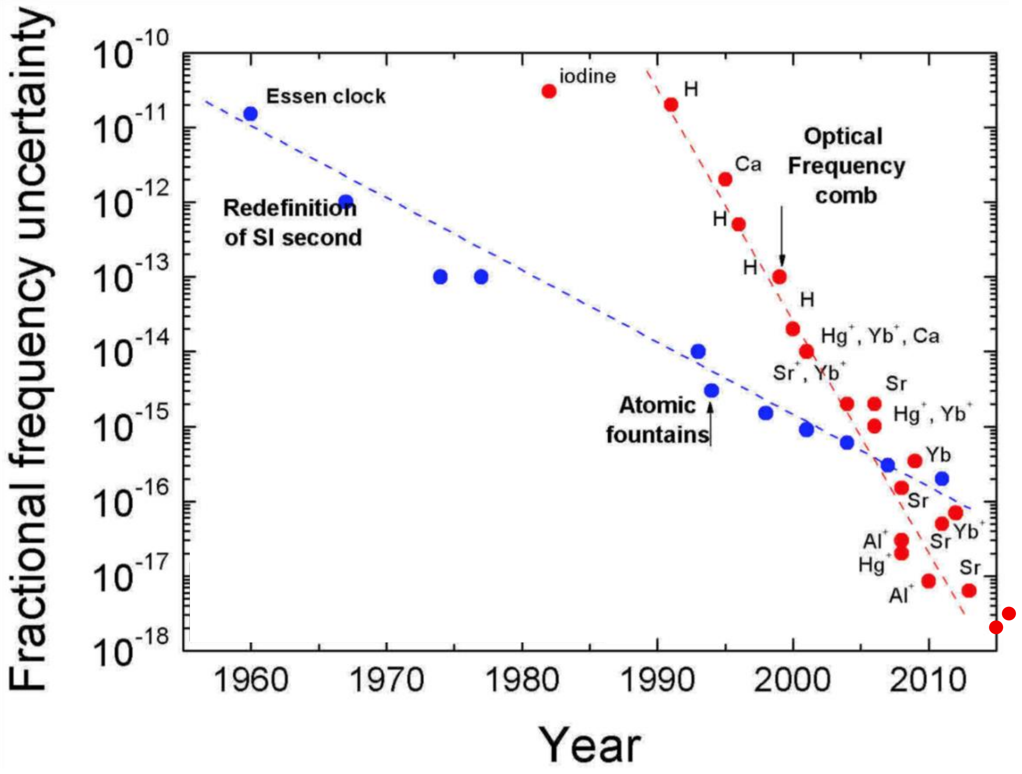
Optical Frequency Atomic Clocks

A comparable S/N and linewidth $\Delta\nu$ gives optical standards much higher stability.

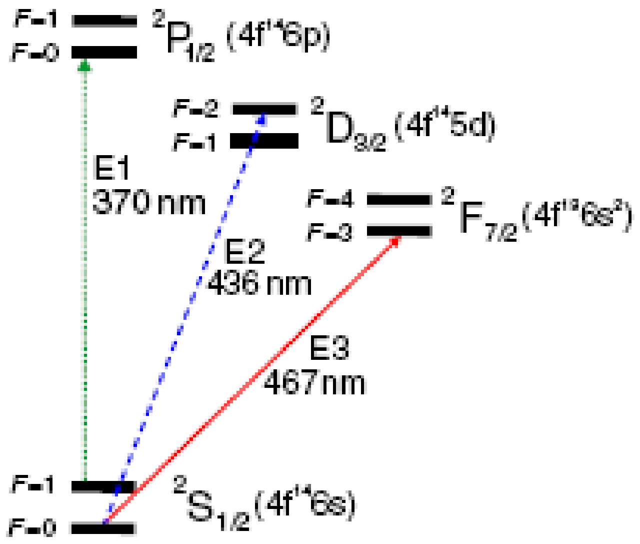
$$\frac{\delta\upsilon}{\upsilon} = \frac{\Delta\upsilon}{\pi\upsilon S/N}$$

It has been easier to improve the accuracy of optical clocks more rapidly than for μwave clocks.

  - Laser-cooling & fs comb lasers
  - Recently contributing to TAI
- Ions: Yb^+, Al^+, Hg^+, In^+
- ^{229}Thorium nuclear clock
- Optical lattice clocks.



Yb⁺: $\pm 3.2\times10^{-18}$



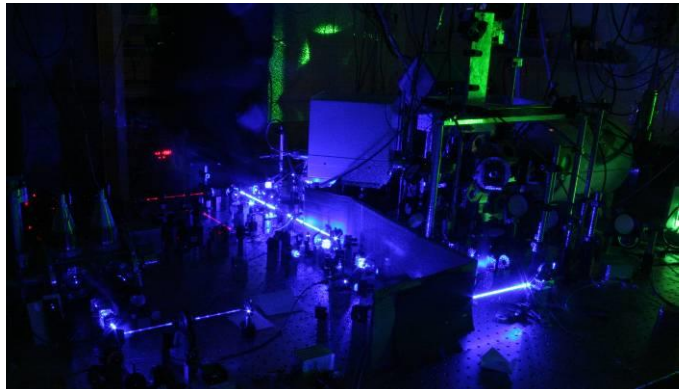
Poli *et al*., Nuovo Cimento ‘13

Ludlow *et al*., Rev. Mod. Phys. ‘15

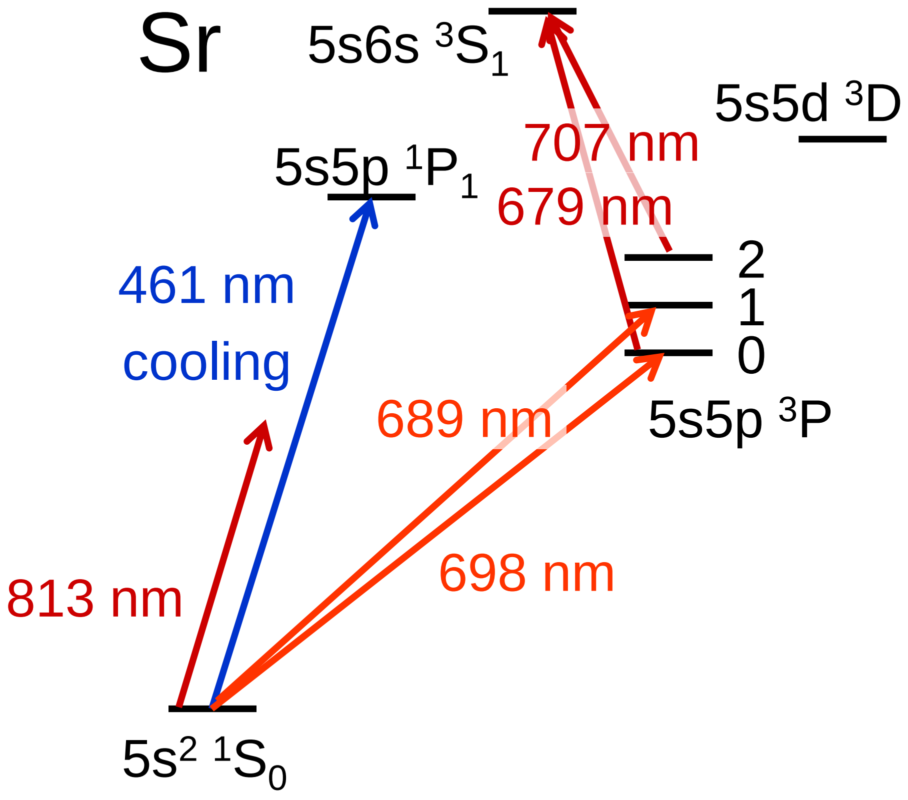
Huntemann *et al*., PRL ‘16

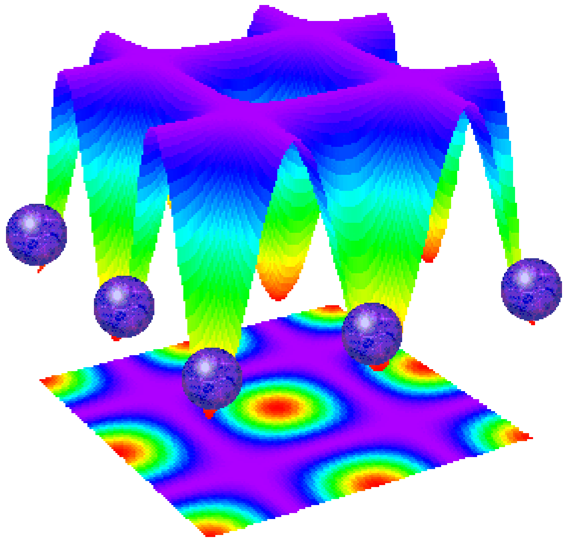
Optical Lattice Clocks

- Several choices: Sr, Yb, Hg, Mg, & Cd.
- Sr & Yb lattice clocks report accuracies of $\pm 10^{-18}$.
  - Narrow linewidth - 1mHz (0.1 Hz)
  - Blackbody shifts
  - Lattice light shifts
  - Fermionic isotopes
  - – suppressed collision shift



P. Lemonde



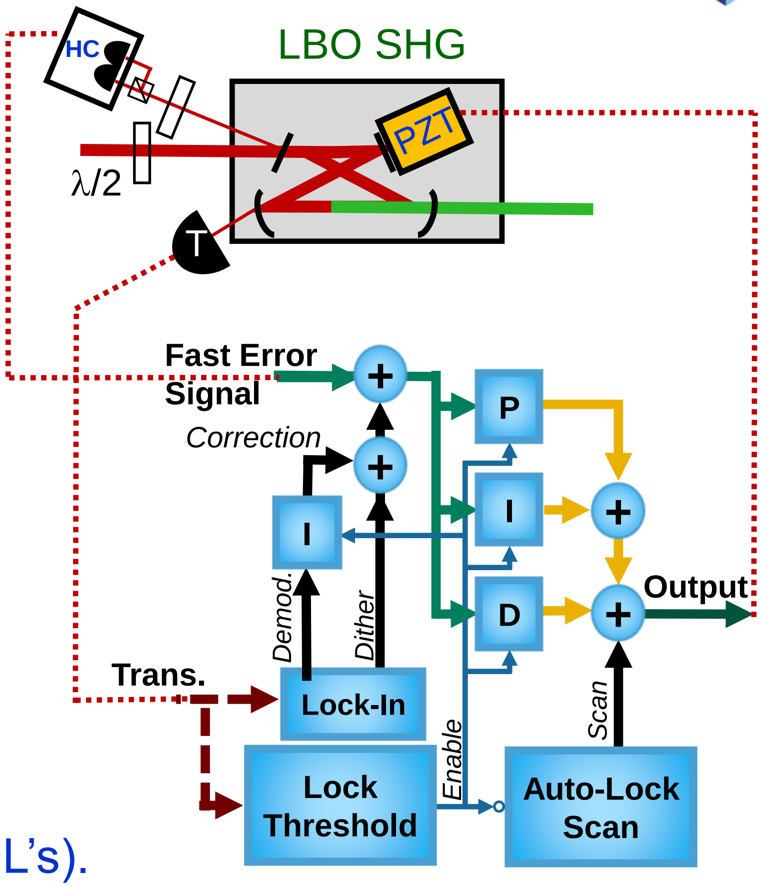


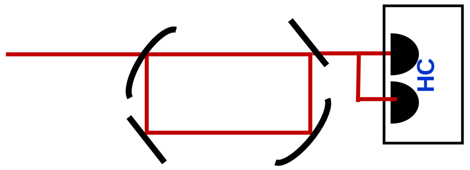
Katori *et al*., PRL ’03

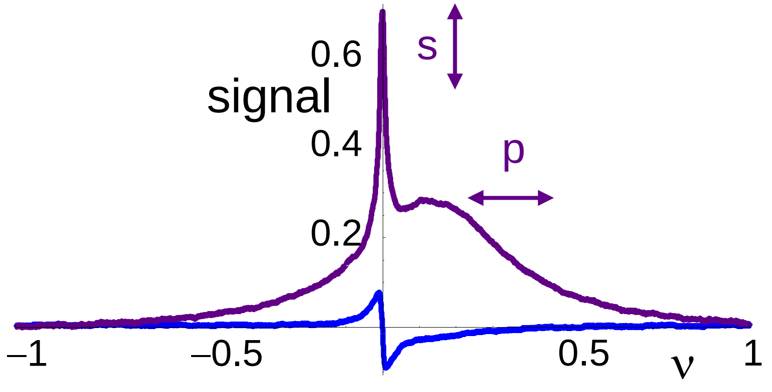
Nicholson … Ye, Nature Comm. ’15

Enhanced Hänsch-Couillaud Cavity Lock

- Polarization lock – compact & efficient with laser power.
  Auto-lock using cavity transmission.
- Cavity has to be birefringent.
- Polarization drifts, e.g. temperature dependent birefringence, can produce lock offsets.
- Add small amplitude dither with slow feedback
  (20 mHz) to correct offset error & drifts.
  - Dither amplitude can be well below the noise level with a kHz bandwith.
  - Inhibit dither for fluorescence detection.
- Natural to implement with an FPGA – inputs: HC fast error input & cavity transmission for auto-lock
- Rectangular reference cavity:
  - 8 input/output ports
  - birefringence from mirror reflectivities for s & p polarization
- Transmission H-C: spatially filters lasers with *rich* transverse mode structure (TA’s & ECDL’s).





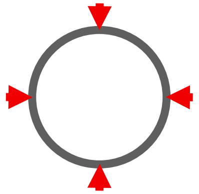


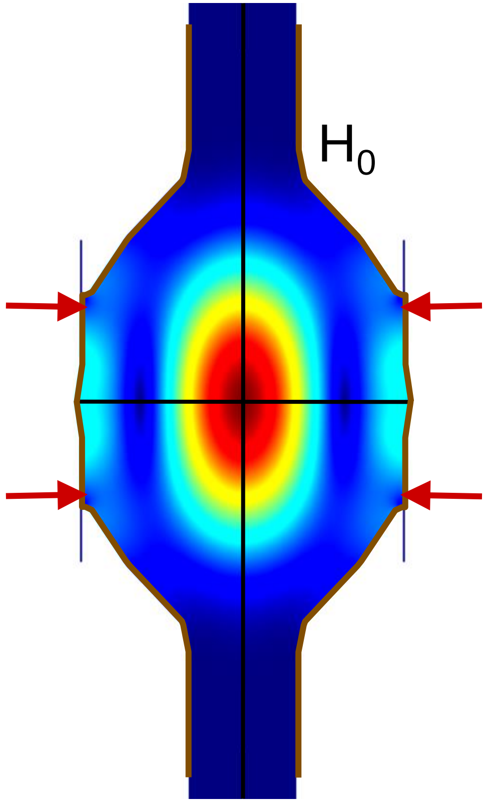
Hänsch & Couillaud, Opt. Comm. ‘80

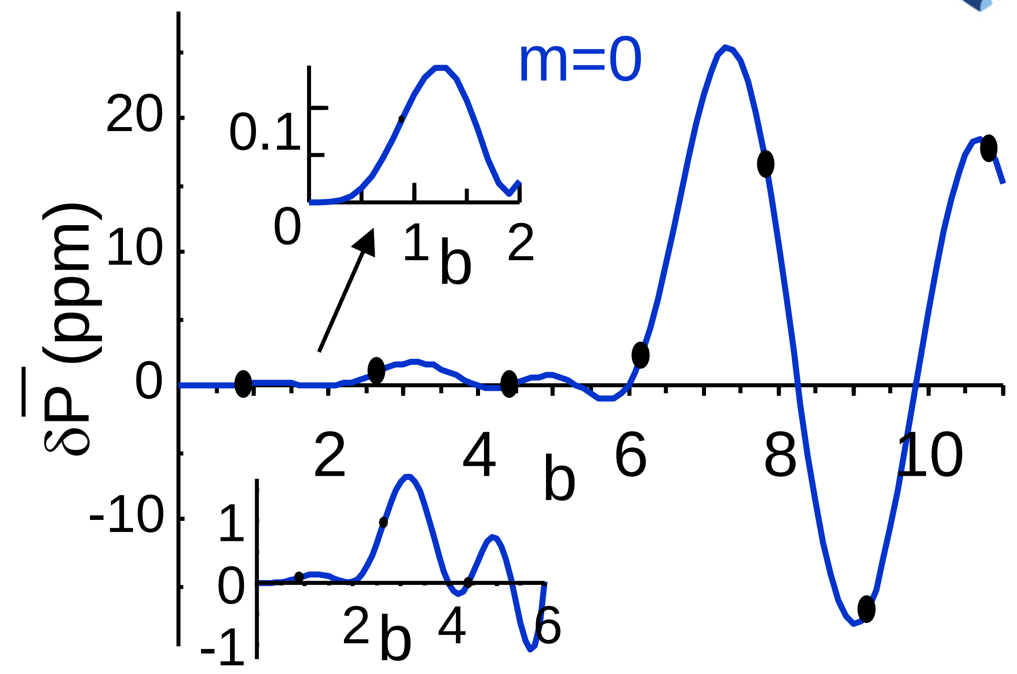
Schussheim & KG, Front. Opt. Sci. ’18 Vainio, Bernard, Marmet, App. Phys. B ‘11 Schussheim & KG, Rev. Sci. Instr. ‘23

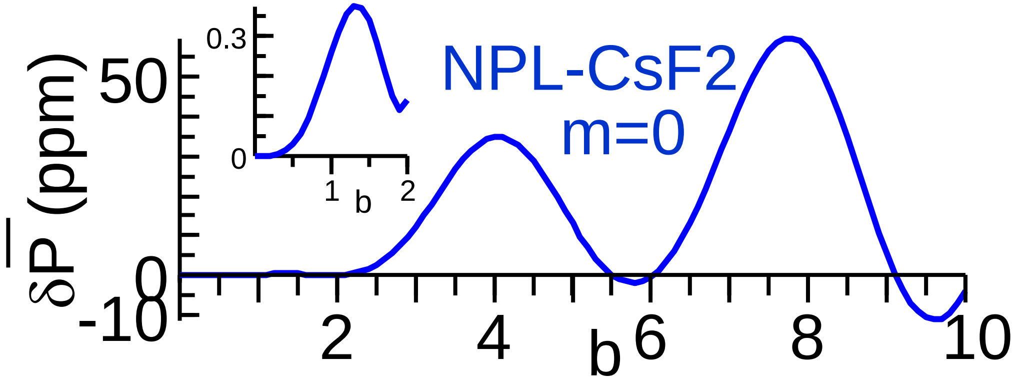
New Cavities of the NPL's - UK & India

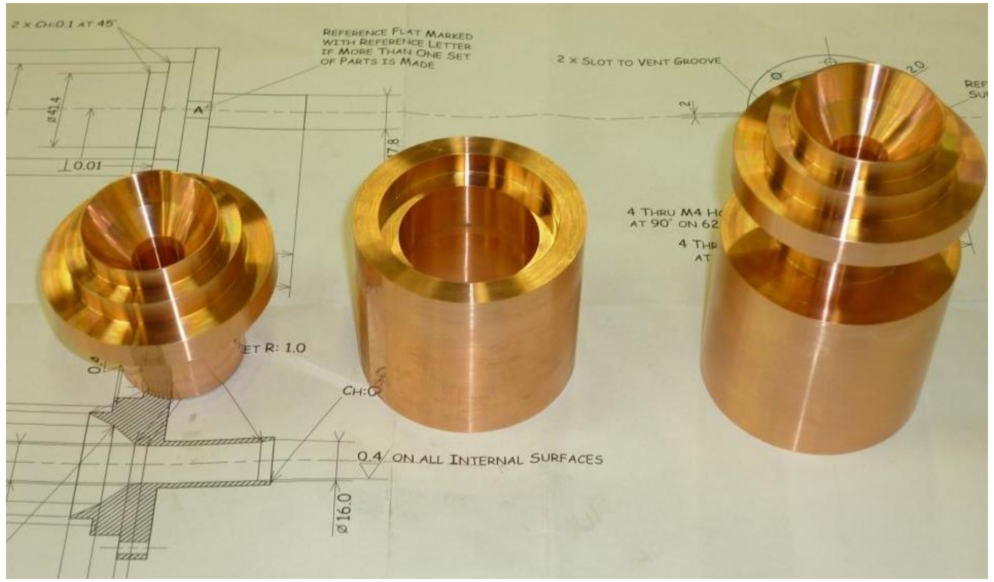
- NPL-CsF3 (UK), NRC, KRISS
- 14mm aperture
- 4 independent cables, $\Delta\phi=\pi/2$, allows accurate vertical alignment.
- Perturbed all boundaries.





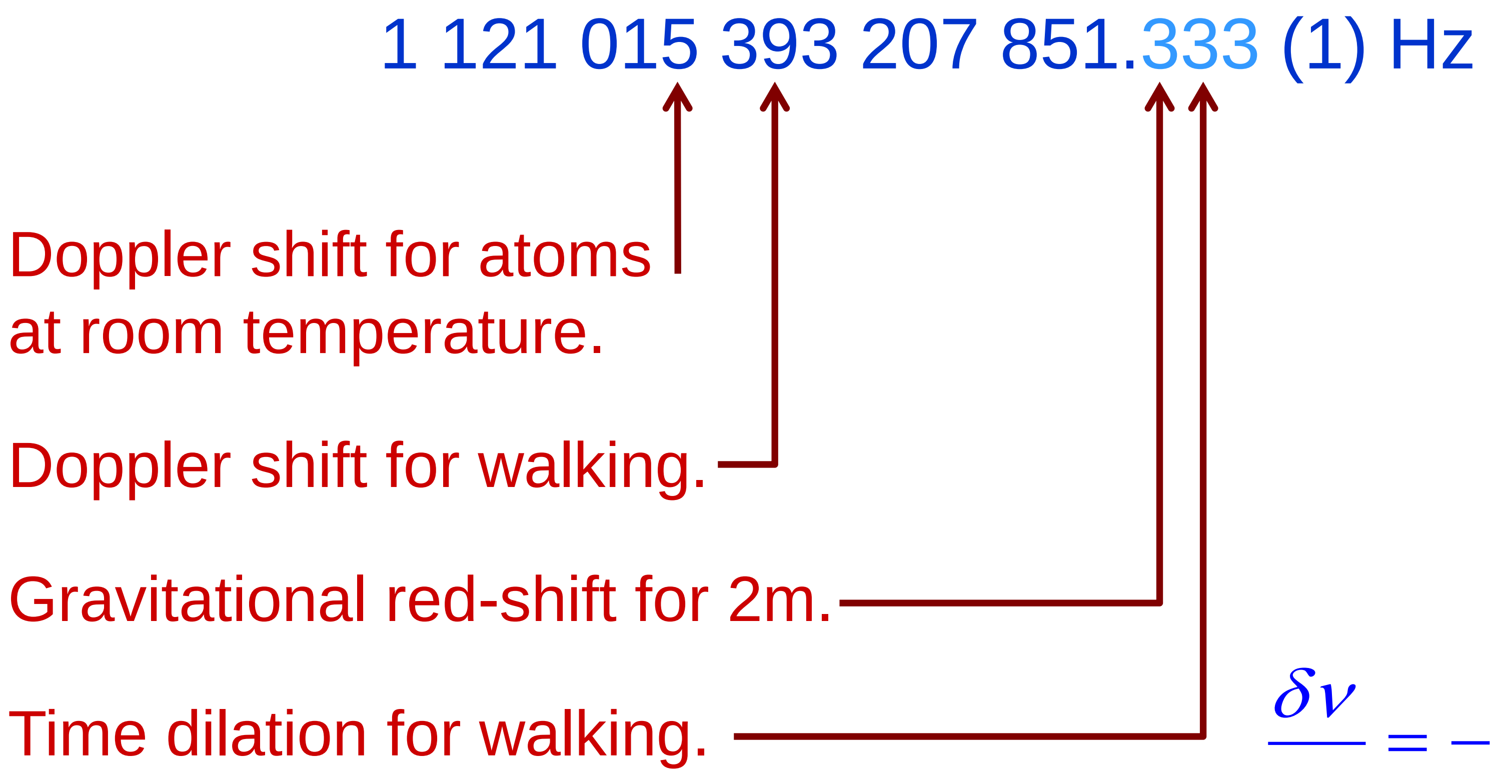






KG, Lea, Sen Gupta, & Szymaniec

Atomic Clocks



1 121 015 393 207 851.333 (1) Hz

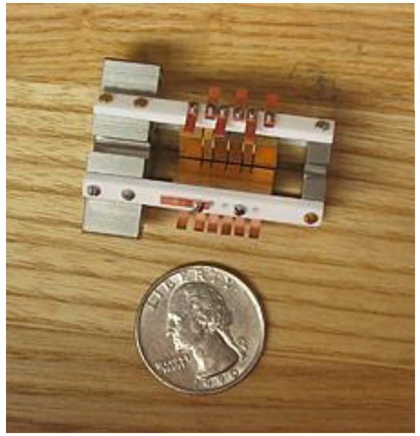
Doppler shift for atoms at room temperature.

Doppler shift for walking.

Gravitational red-shift for 2m.

Time dilation for walking.

$$\frac{\delta \nu}{\nu} = -\frac{1}{2}\frac{v^2}{c^2}$$



NIST
National Institute of
Standards and Technology

0.4s in the age of the universe!

$\delta\nu/\nu = \pm 9.4\times 10^{-19}$

NIST Al$^+$: Brewer, Chen, Hankin … Wineland, Hume, Leibrandt, PRL ‘19

International Atomic Time (TAI)

Uncertainty (10^{-16})	NPL CsF2/3		PTB CSF2		SYRTE FO2		NIST F1/4		IT F2
	2010	2015	2010	2018	2010	2014	2011	202?	2014
Doppler (**D**istributed **C**avity **P**hase)	3.0	0.5	1.5	1.5	3.0	0.9	0.2		0.2
Microwave lensing	1.5	0.3		0.2	1.4	0.7			1.5
Ultracold Collision shift	1.0	0.4	3.4	0.4	1.4	1.2	2.6		1.3
Background gas collisions	1.0	0.3	0.5	0.1	1.0	1.0	0.1		0.5
Spectrum, leakage	1.0	0.5	4.6	0.1	0.7	0.5	1.2		
Blackbody radiation shift	1.1	0.6	0.6	0.6	0.6	0.6	2.8		0.1
Quadratic Zeeman	0.8	0.5	0.6	0.1	0.3	0.3	0.3		0.8
Total	**4.1**	**1.4**	**6.0**	**1.7**	**3.8**	**2.1**	**4.0**		**2.4**

Li & KG, Metrologia ’04 & ’10

Guéna, Li, KG, Bize, Clairon, PRL ‘11

Li, KG, Szymaniec, Metrol. ‘11

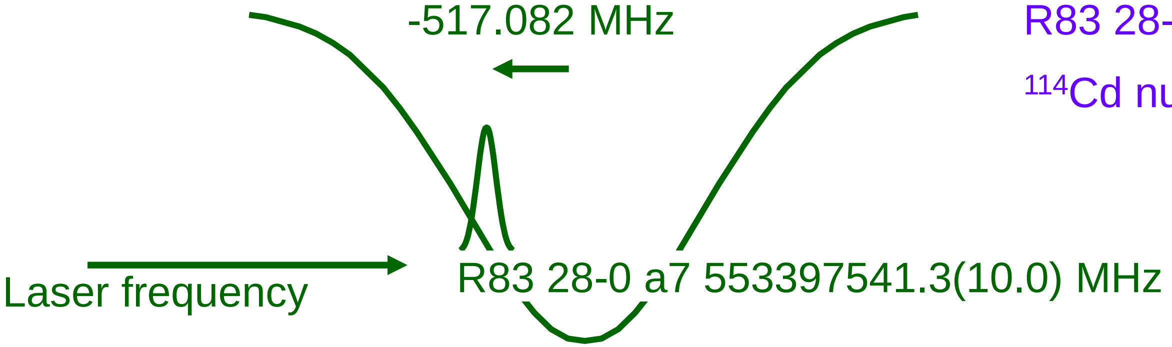
Guéna, … Li, KG, Clairon, Bize, IEEE TUFFC ‘12

Weyers, Gerginov, Kazda … KG, Metrologia ‘18

Beattie, Jian, Szymaniec & KG, Metrologia ‘20

NIST F4: Hoth, KG, Gerginov, EFTF-IFCS ‘23

KG, PRL ‘13

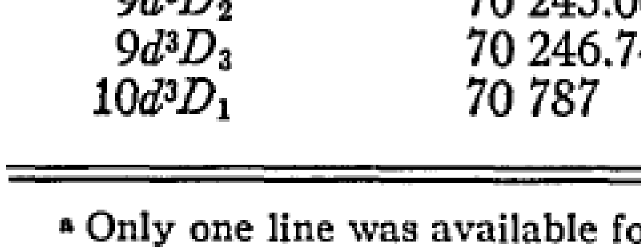

-517.082 MHz

Laser frequency

R83 28-0 a7 553397541.3(10.0) MHz

R83 28-0 a7 $\nu_{R83\ 28\text{-}0\ a7}$= 553397541.3(10.0) MHz

^{114}Cd ν_{114} = 3/2 ν_{a7}+261.48MHz

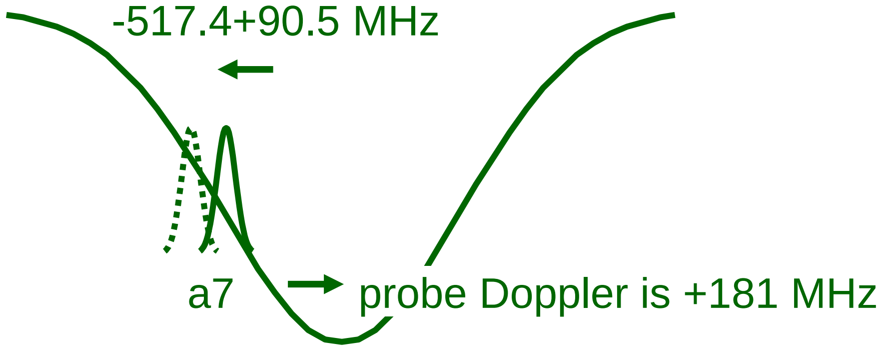

$9d^3D_2$ 70 243.0

$9d^3D_3$ 70 246.7

$10d^3D_1$ 70 787

ᵃ Only one line was available fo

I2 AOM shift is -90.5×2 MHz on probe, so +90.5 MHz


-517.4+90.5 MHz

a7

probe Doppler is +181 MHz

^{114}Cd:

Adams Burns Michelson lamp: 27689.034 - PSU ν_{114} : -217.2(300.) MHz
300 uncertainty from Kramida

MHz,-MHz}

Adams Burns Electrodeless lamp: 27689.040 (could be 41) - PSU ν_{114} : –37.(700.) MHz
700 uncertainty from Kramida

Adams Burns 3P2 E levels are different so that would change 3D3 E level – all uncertain – get better measurement of 3P2

1083 ref cavity AOM shift is negative, so +4×, +8×ν_{synth}

361 AOM shifts are positive, 1083 & MOT, +1×, +2×$\nu_{MOT\ synth}$

Example of 1 of 4 measurements:
nu_a1+ 3* 184.60 [a7-a1] + 12*(32.06-37.7) [ref cav] +2*55.5 + 82.45 + 3/2*90.5= nu_a1 + 3/2*90.5 MHz + 679.54 MHz

^{114}Cd is +679.52 MHz of observed a1, so +3/2 90.5 gives ν_{114} = 3/2 ν_{a1}+815.27MHz

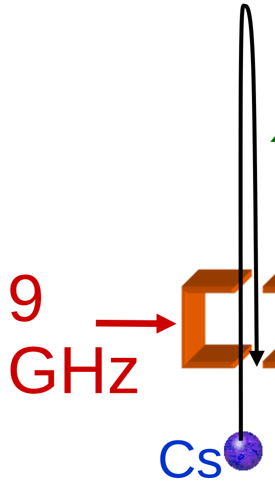
$\nu_{R83\ 28\text{-}0\ a1}$= 553,397,172.1(10) MHz

Natural Cd:

PSU measurement: ν_{nat} = ν_{114} -111.648 MHz
Adams Burns JOSA B 1956 air wavelength: 361.05077 – index 1.00028525
Adams Burns air wavelength - PSU: -0.32(285.)MHz
Adam & Burns energy levels for that wavelength: {corrected .980 typo}
59515.990 - 31826.952

Laser-Cooled Fountain Clocks & the Cold Collision Frequency Shift

- Microwave Spectroscopy
  - laser-cooling: Doppler shifts & narrow linewidth



$$\Psi = \frac{|3\rangle + e^{i2\pi(9GHz)t+i\varphi_c}|4\rangle}{\sqrt{2}}$$

Cs 6s ground state

$|F=4\rangle$

$|F=3\rangle$

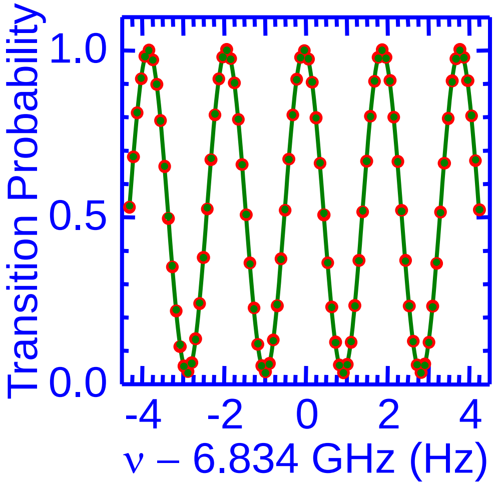
$\nu \equiv 9{,}192{,}631{,}77\underline{0}$ Hz

- Collisions can shift the phase of the atomic coherence

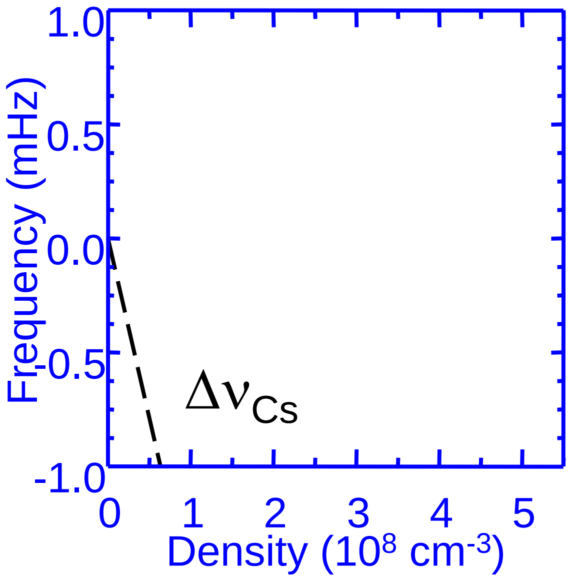
phase shifts coherence by φ_c

$$\sigma_0 = -10^6\,\mathring{A}^2 \approx \frac{\lambda_{dB}^2}{\pi}$$

$\sigma = -10^6\,\text{Å}^2$



The dominant problem for Cs fountain clocks.

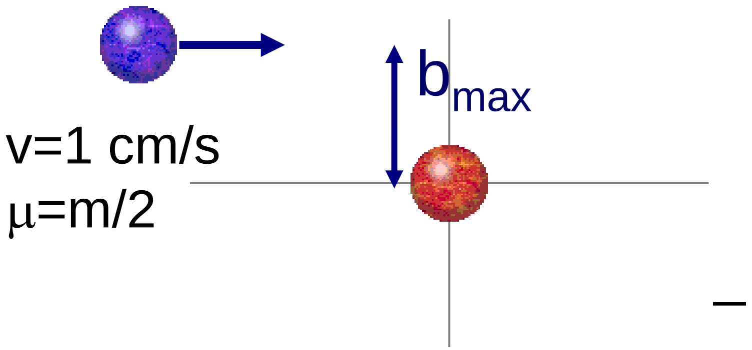


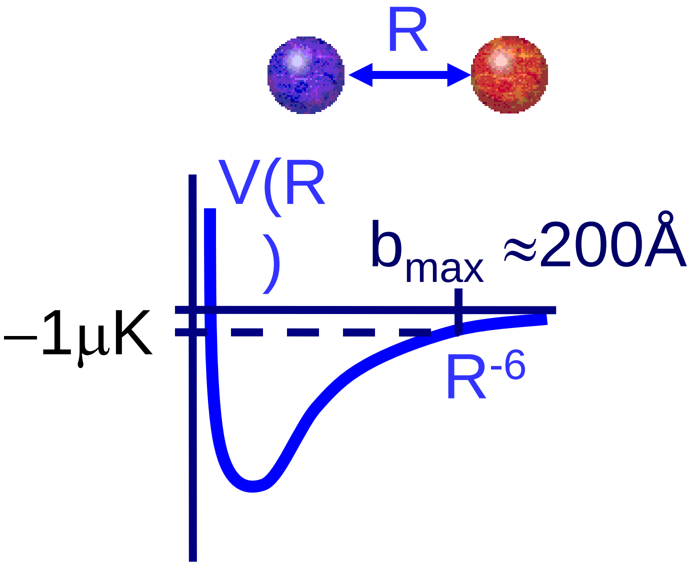
Theory: Tiesinga, Verhaar, Stoof, & van Bragt, PRA '92

Expt: Gibble & Chu, PRL '93 Gibble, Chang, Legere, PRL '95.

Ultra-Cold Atom Scattering

Semi-classically:
Cs-Cs at 1μK





$$L_{max} = \mu v b_{max} \approx 0.3 h$$

Quantum Mechanics

$$\lambda_{dB} = \frac{h}{\mu v} \approx 3{,}000 Å >> b_{max}$$

$$L = \sqrt{l(l+1)}h$$

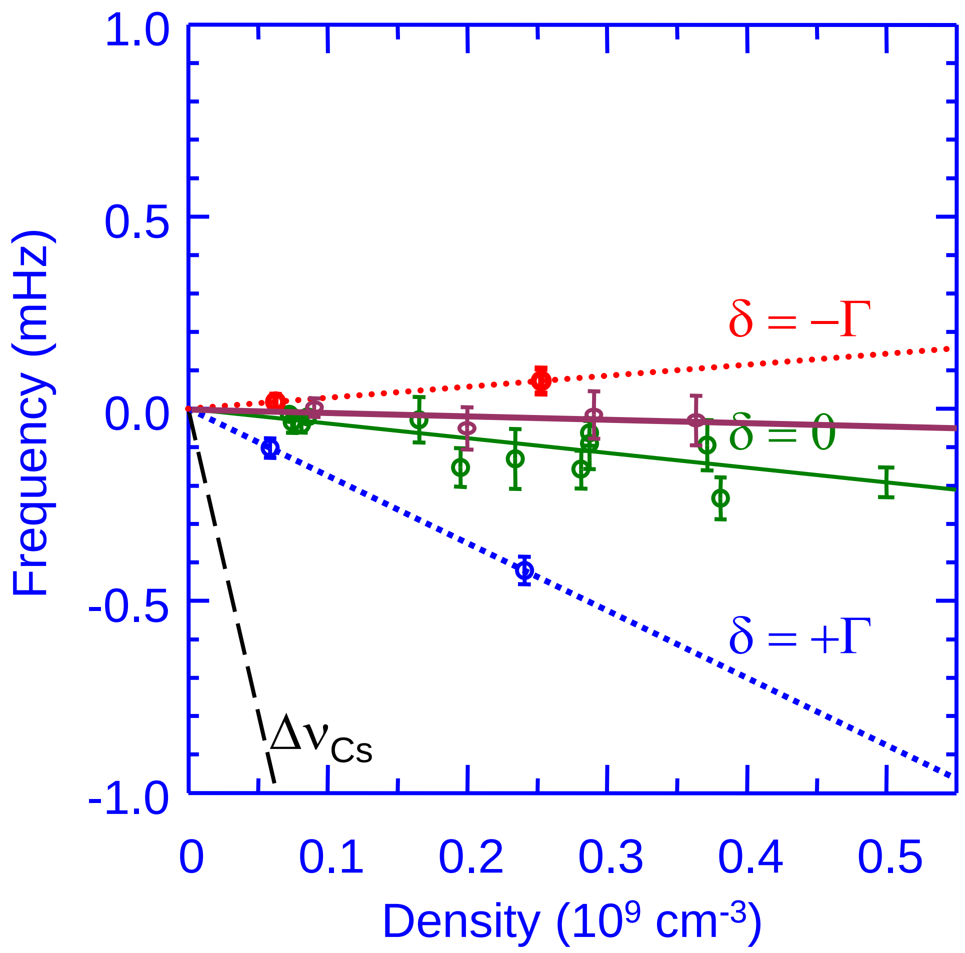
No p-wave ∴ $\ell = 0$

s-wave scattering for T → 0: $\sigma_0 = \frac{\lambda_{dB}^2}{\pi} \sin^2(\delta_0) \approx 10^6 Å^2$

Spherical outgoing waves → isotropic scattering

Cs-Cs - 99.9(1)% s-wave @ 0.89 μK
Gibble, Chang, Legere, PRL ‘95.

^{87}Rb Cold Collision Frequency Shift



- -0.38(8)mHz @ $n=1.0(6)10^9 \times \mathrm{cm}^{-3}$
- 50× less than ^{133}Cs
- Consistent with calculations
- GPS & SI second

RECOMMENDATION CCTF 1 (2004):

Concerning secondary representations of the second

The Consultative Committee for Time and Frequency,

…

recommends that the unperturbed ground-state hyperfine quantum transition of ^{87}Rb may be used as a secondary representation of the second with a frequency of f_{Rb} = 6 834 682 610.904 324 Hz and an estimated relative standard uncertainty (1σ) of 3×10^{-15}, … .

CCTF (2006): recommends

^{199}Hg$^+$ @ 282nm, ^{88}Sr$^+$ @ 674nm, ^{171}Yb$^+$ @ 436nm, & ^{87}Sr 698 nm.

Cs: KG & Chu, PRL '93

Rb: Fertig & KG, PRL '00

Sortais Bize, Nicolas, Clairon, Salomon, & Williams, PRL '00

Theory: Kokkelmans, Verhaar, KG, & Heinzen, PRA '97

Kempen, Kokkelmans, Heinzen, & Verhaar, PRL '02

Optical Clocks

- Fractional frequency is the key performance measure.

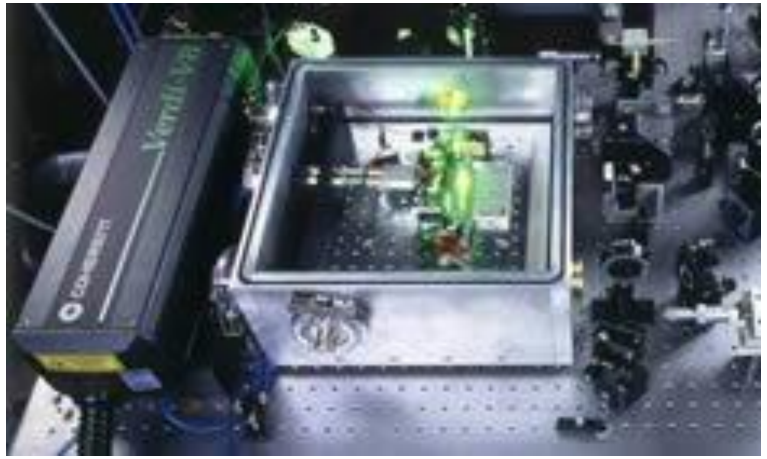
$$\upsilon_{opt} \approx 10^{15}\,Hz \quad 100{,}000\times$$

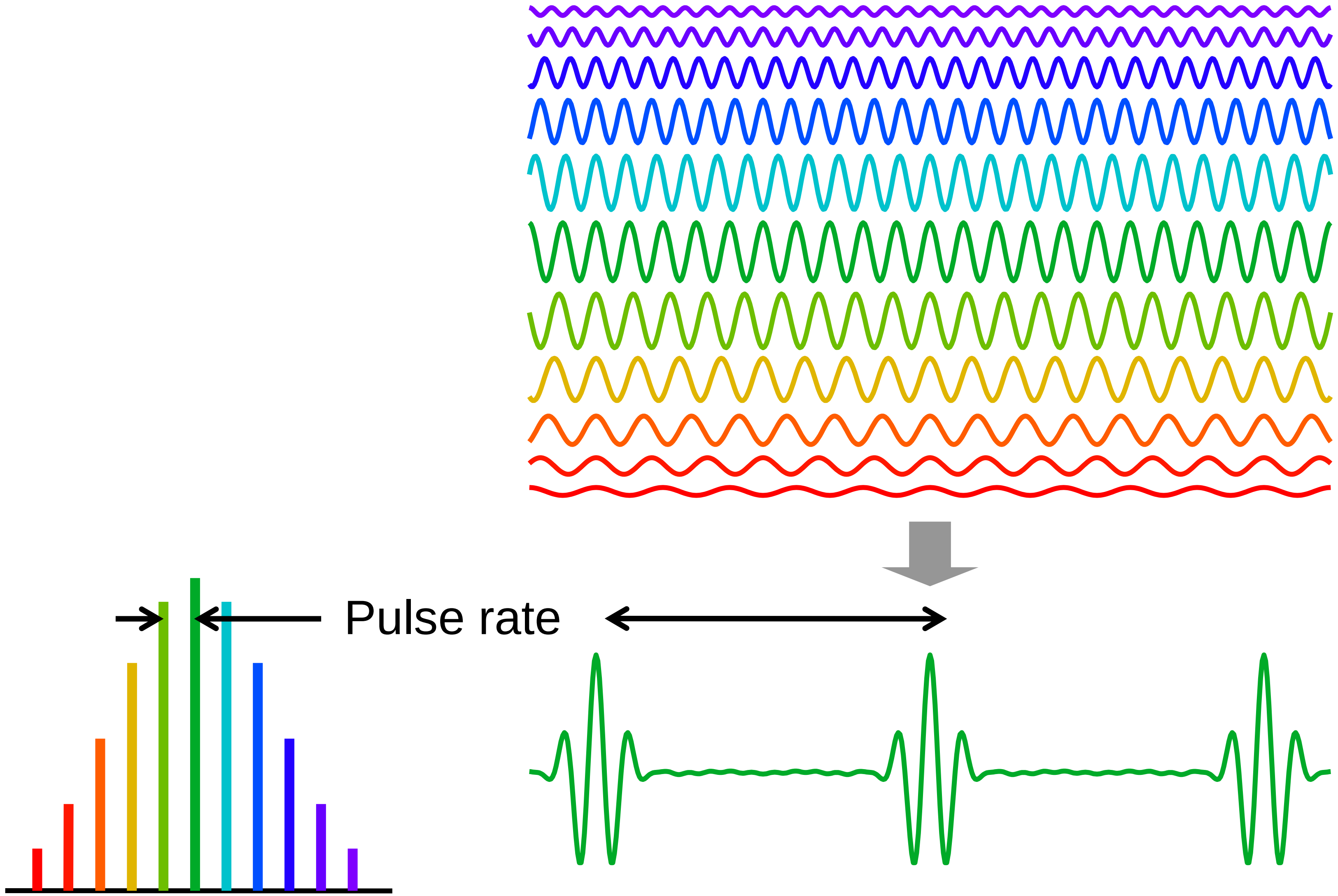
$$\frac{\delta\upsilon}{\upsilon} = \frac{\Delta\upsilon}{\pi\upsilon S/N}$$

Count optical cycles!

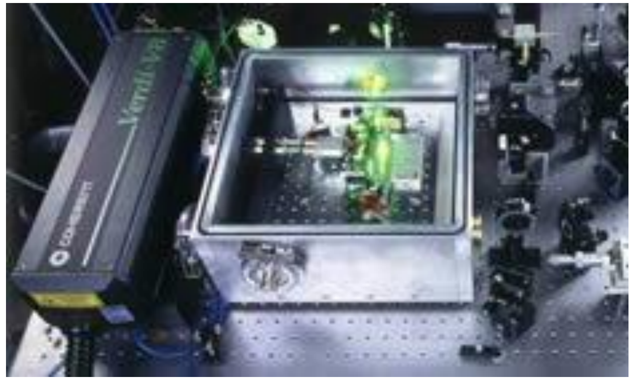
- How do you make a very short pulse of light (sound)?

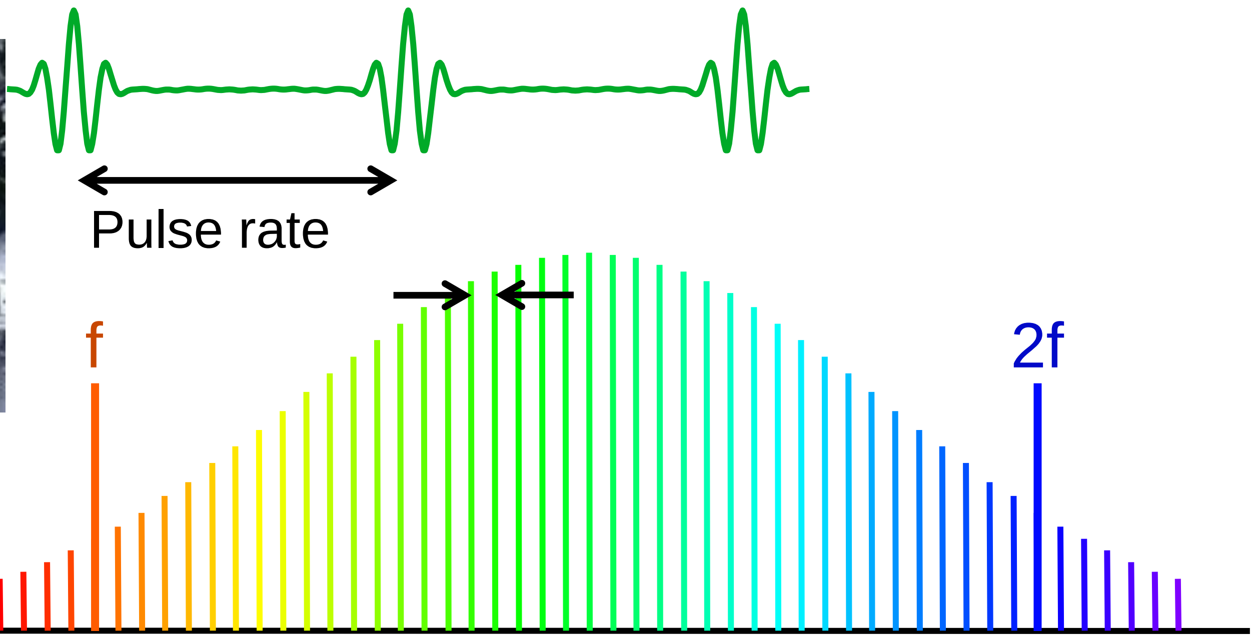
A laser with a million colors!





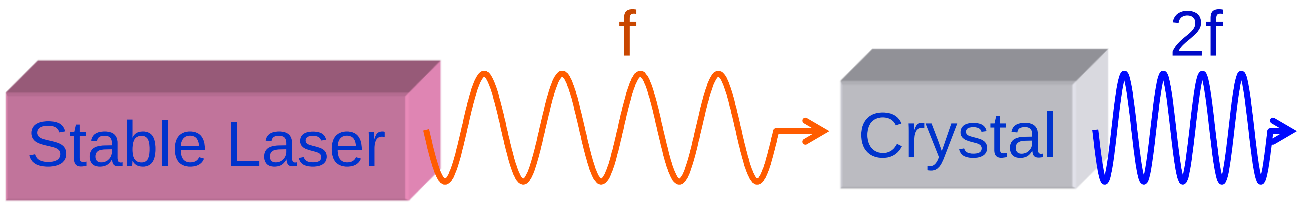
Counting the Ticks of Light




Pulse rate

f

2f


Stable Laser

f

Crystal

2f

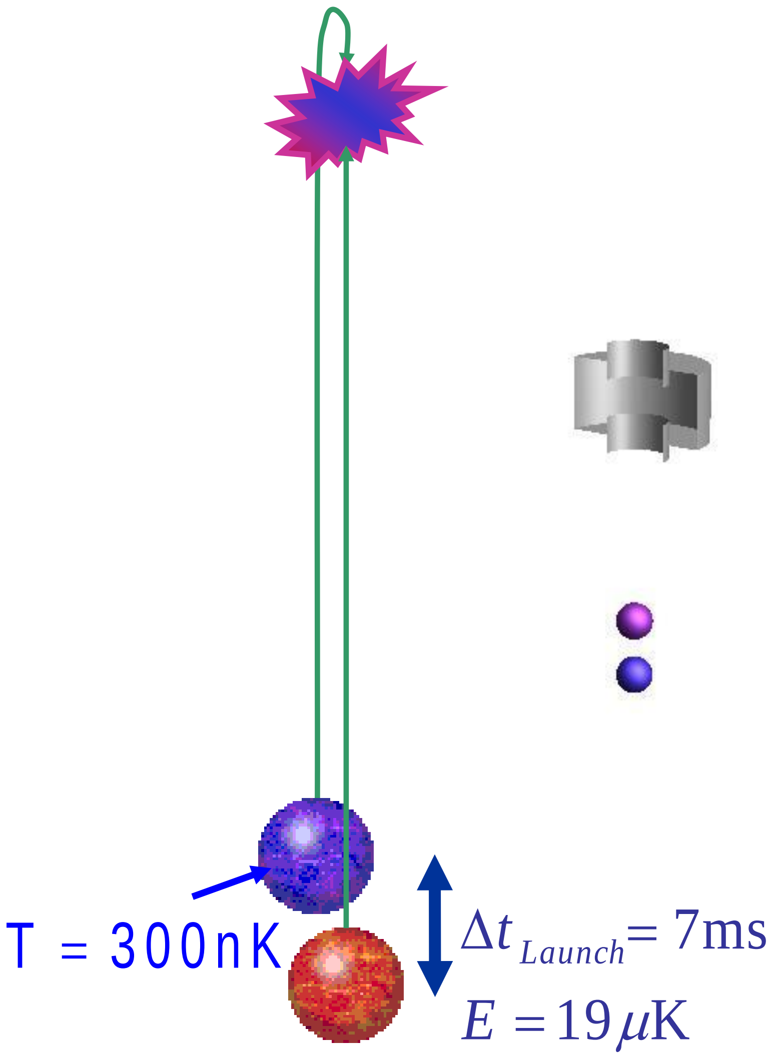
2f – f = (Pulse Rate) × 429,228 = f

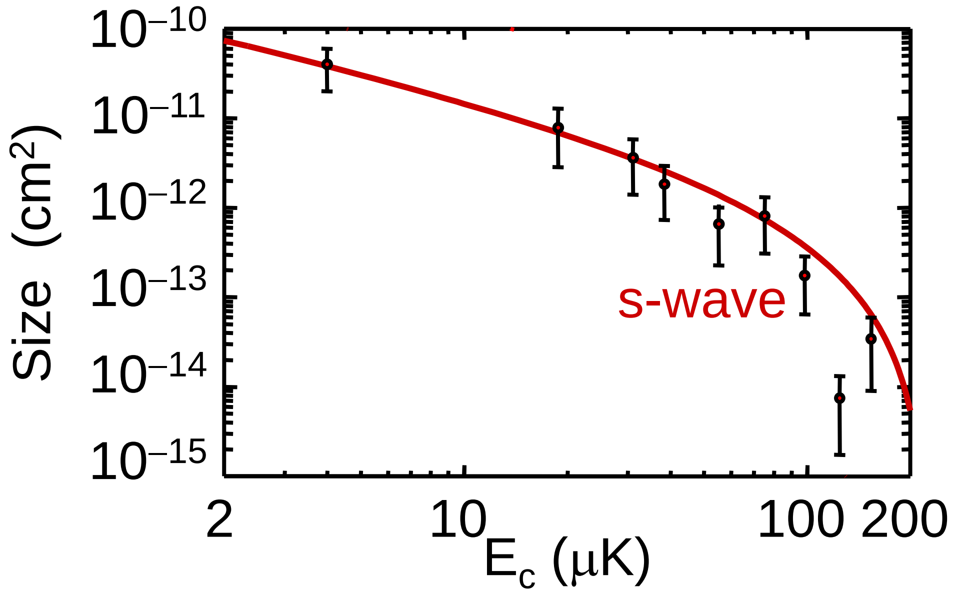
- Measure f by measuring the difference between f & 2f!
- Much better clocks - also large impact on chemistry, astro, …

Diddams *et al.*, PRL ’00

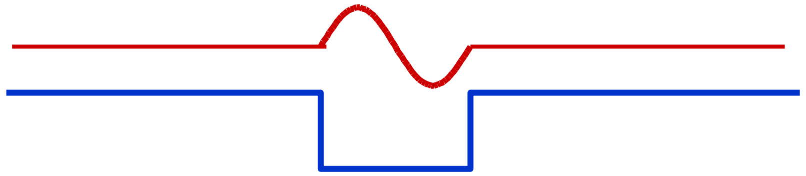
Rafac *et al.*, PRL ‘01

Juggling Atomic Fountains





$$E_c = \frac{mg^2\Delta t^2}{4}$$



- State-to-state velocity-selected differential crossed-beam scattering at μK energies.

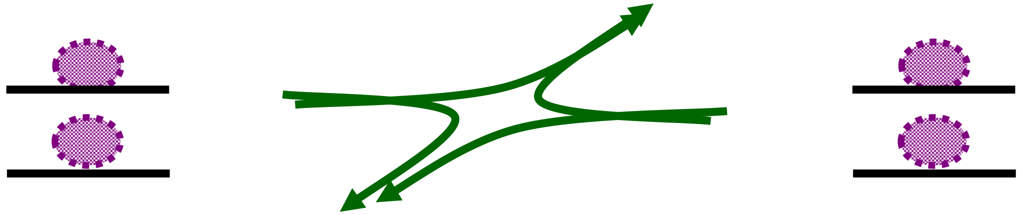
- Ramsauer-Townsend effect
- Large improvement in clock stability.

Legere & KG, PRL ‘98

Distinguishable Fermions?

Identical?

$\frac{|\uparrow\rangle+|\downarrow\rangle}{\sqrt{2}}$



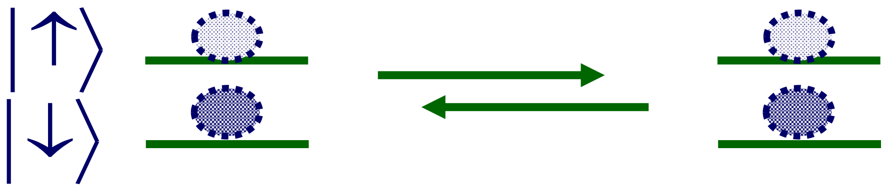
$\frac{|\uparrow\rangle-|\downarrow\rangle}{\sqrt{2}}$

Despite decoherence, Bosons and Fermions often appear to be quantum mechanically identical.

Ultracold Fermions

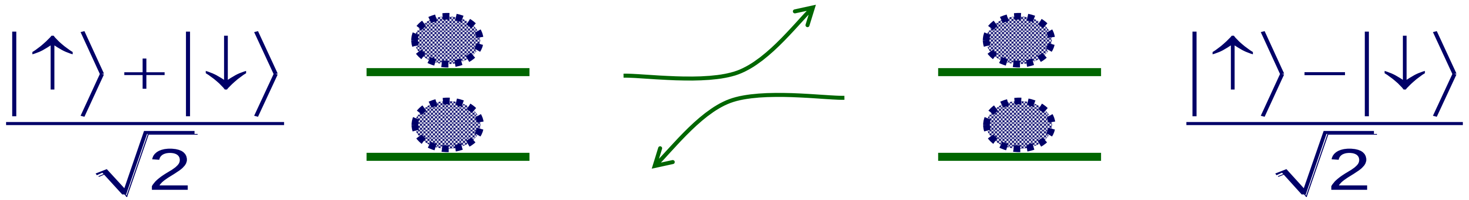
- At ultracold temperatures, only s-wave scattering.
- Antisymmetric wavefunction → no scattering of identical fermion superpositions!

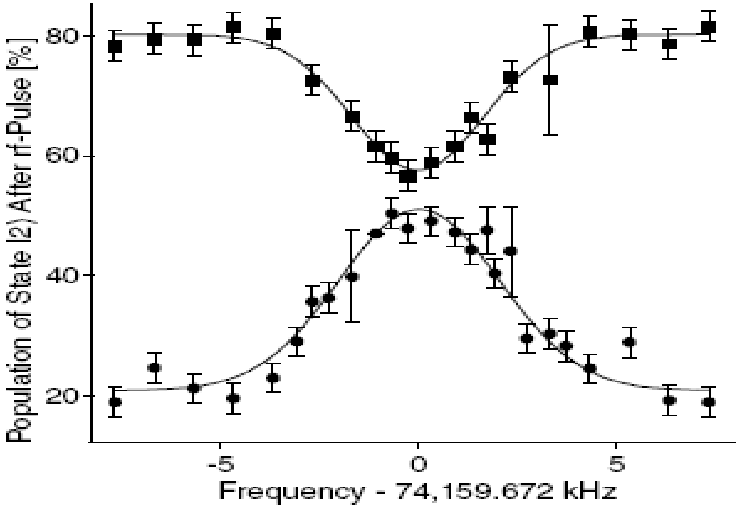
$$\sigma = 0$$



- Decoherence → distinguishable fermions → collisions

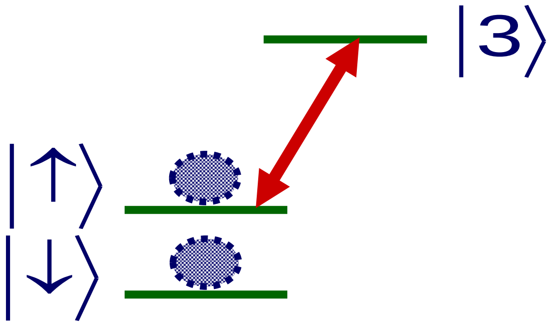
$$\sigma \neq 0$$





- No collision shift! Despite being distinguishable, fermions act as if they're indistinguishable!
- $[H_{light}, V_{\uparrow\downarrow}]=0$ →

Fermions are Universally Immune to Collisions!



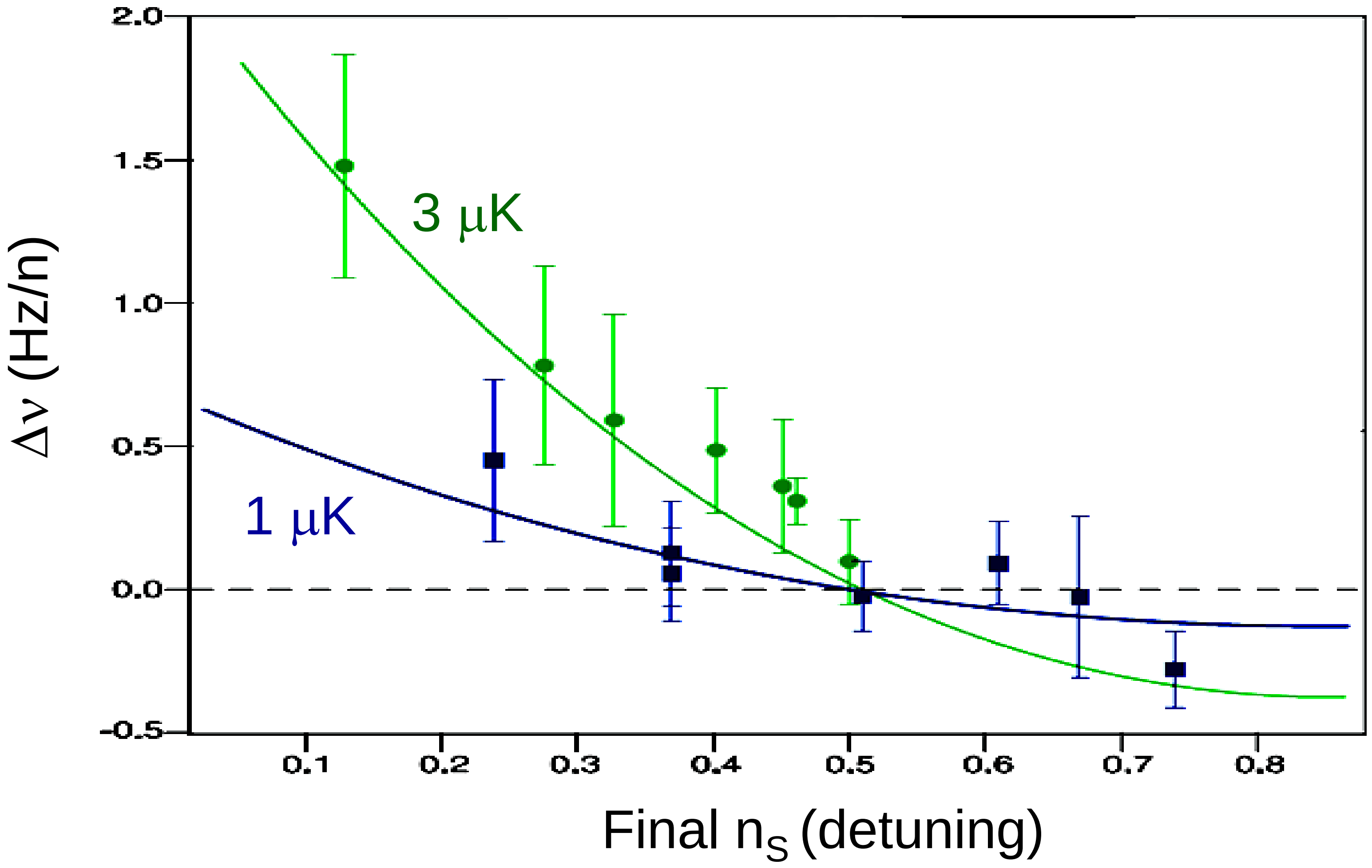
Zwierlein, Hadzibabic, Gupta, & Ketterle, PRL '03

Optical-Clock Fermion Collisions

- “Fermions are universally immune to collisions.”
- Campbell ... Ye observed a collision shift for fermions.
- Treated shift as ⍰ $n_\downarrow – n_\uparrow$.

$$\Delta\nu \overset{?}{=} \frac{2\hbar a_{\downarrow\uparrow}}{m} g^{(2)} \left(n_\downarrow - n_\uparrow\right)$$

- $[H_{light}, V_{\downarrow\uparrow}] \neq 0$ for optical fields with fast spatial variations.
- Our theoretical work showed that the Sr & Yb collision shifts are p-wave, not s-wave.



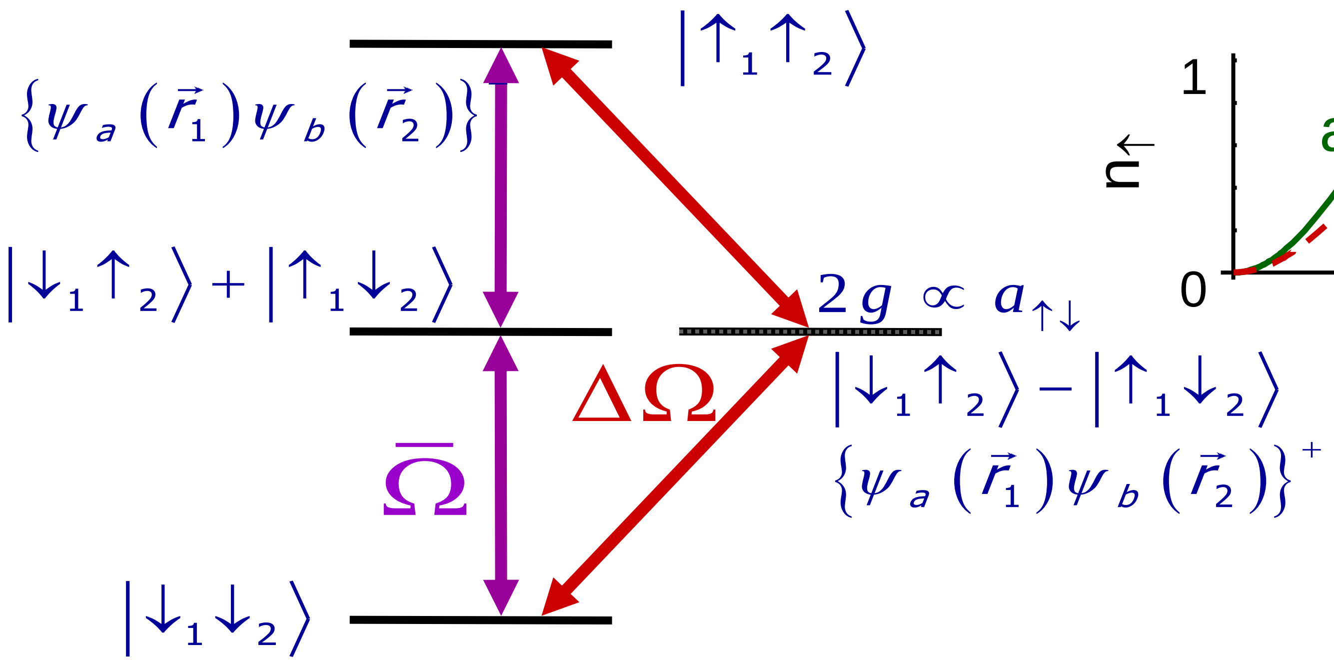
Zwierlein, Hadzibabic, Gupta, & Ketterle, PRL ‘03
Campbell, … Julienne, Ye, Science ‘09
KG, PRL ‘09

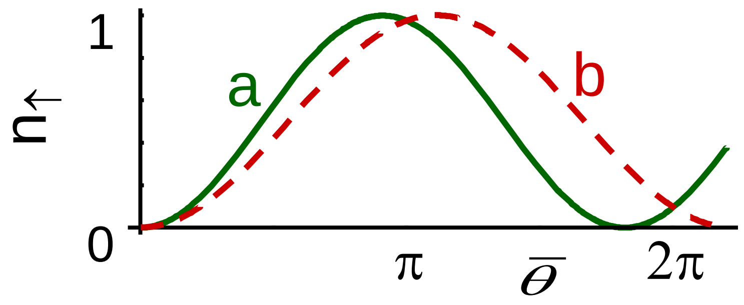
Lemke, … Oates, PRL ‘09
Lemke, … Oates, Ludlow, PRL ‘11

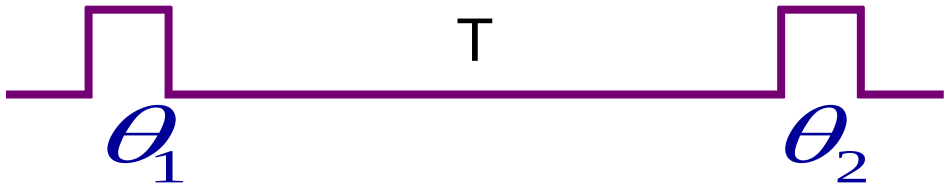
Fermion Clock Collision Shift

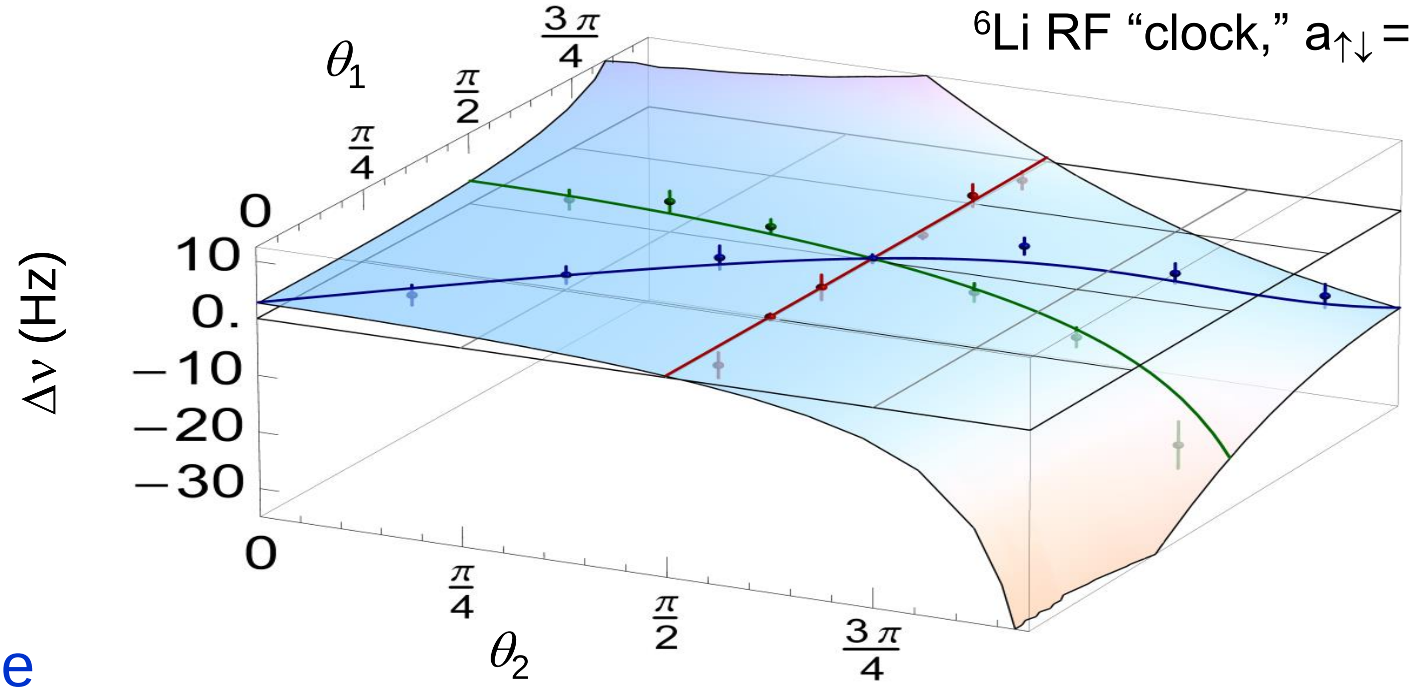
- Many particles are sum of pair-wise effects
- Basis – Singlet and Triplet states of 2 atoms:

$$\Omega = \Omega_a e^{i\Delta\omega_a t} \qquad \Omega' = \Omega_b e^{i\Delta\omega_b t}$$









^{6}Li RF “clock,” $a_{\uparrow\downarrow}=4a_0$

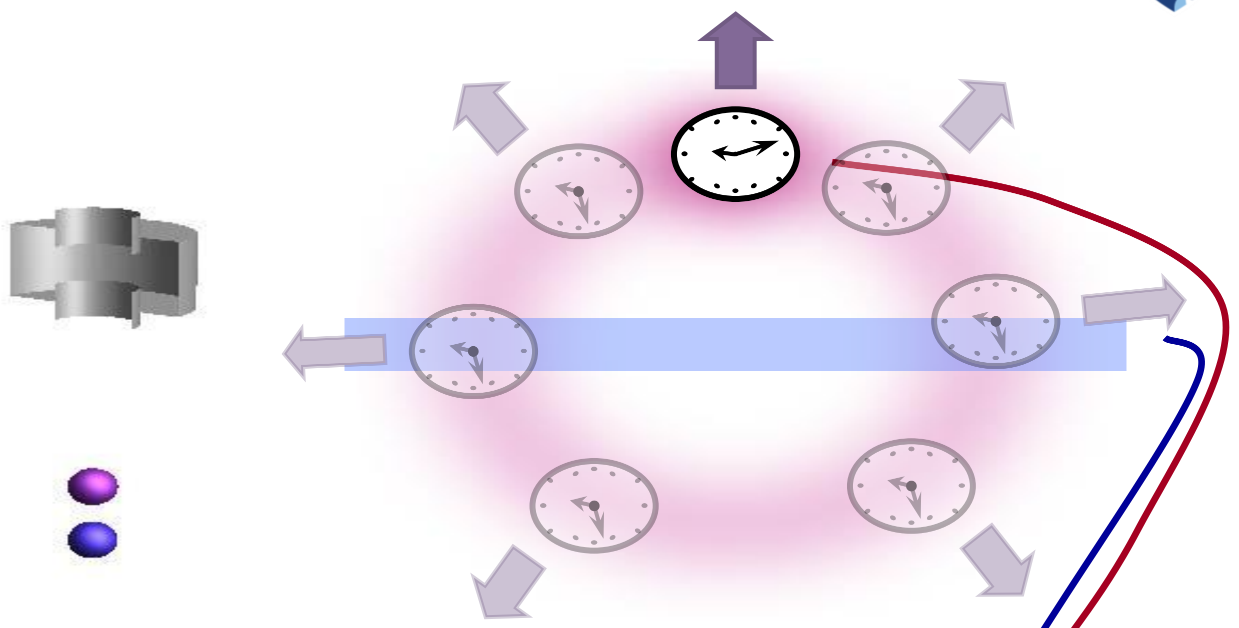
$$\Delta\nu = \frac{g}{2\pi}\frac{\sin(2\Delta\theta_1)\sin(\Delta\theta_2)\cos(\bar{\theta}_2)}{\sin(\bar{\theta}_1)\sin(\bar{\theta}_2)}$$

- The light only addresses atom pairs that are identical.

KG, PRL ‘09
Hazlett, Zhang, Stites, KG, O’Hara PRL ‘13

Precise Measurement of Scattering Phase Shifts

- Juggle atoms by tossing 2 laser-cooled clouds with short delay.
- Launch delays of 7 to 20 ms give ultra-cold scattering, 15 to 200μK.
- In a clock, a microwave cavity prepares atoms in a coherent superposition and enables a readout of the relative phase of those two clock states.

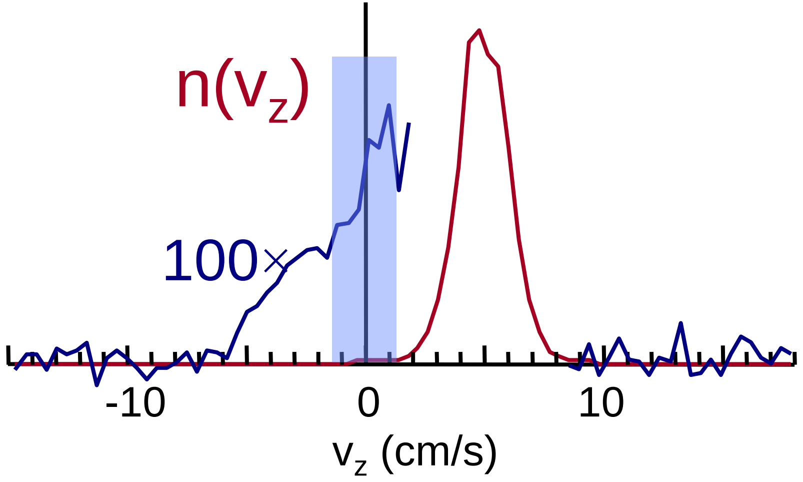


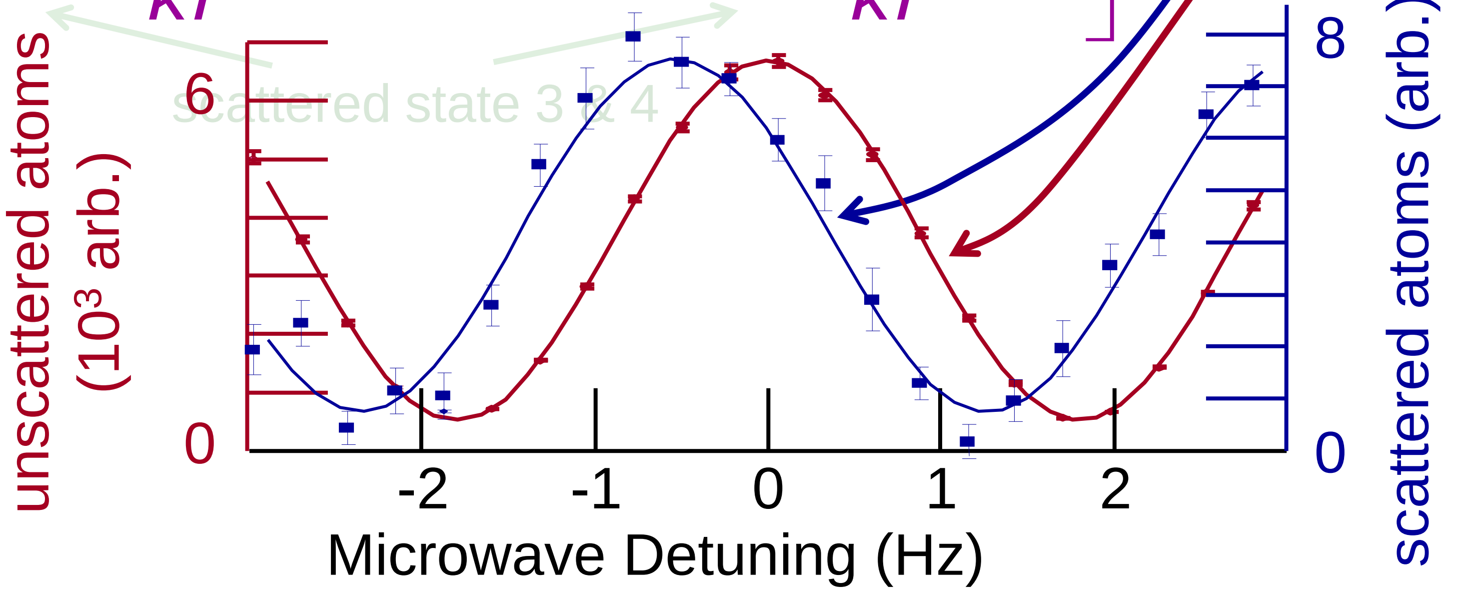
$$\psi^{+} = \frac{1}{\sqrt{2}}\Big[e^{+ikz}\left(|3\rangle + |4\rangle\right)\Big] + e^{i\delta_3}\sin\delta_3\,\frac{e^{ikr}}{kr}|3\rangle + e^{i\delta_4}\sin\delta_4\,\frac{e^{ikr}}{kr}|4\rangle\Big]$$

unscattered

scattered state 3 & 4

- Detect only scattered atoms.



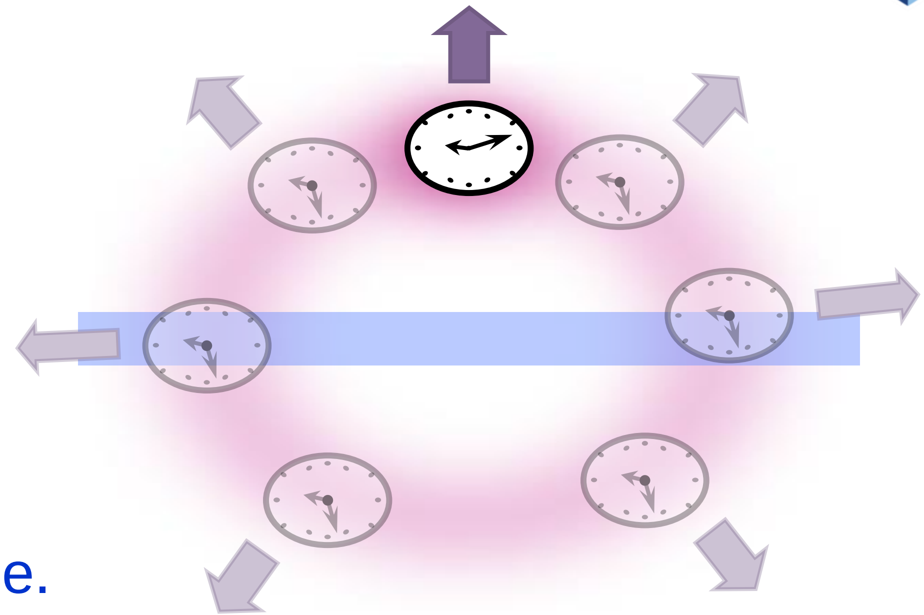


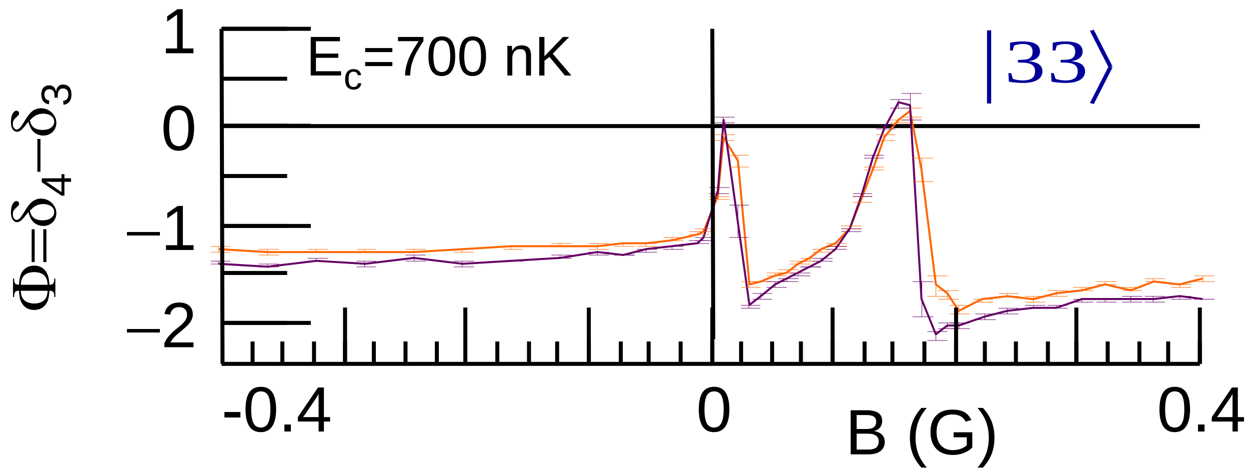
Hart, Xu, Legere, & KG, Nature ‘07

Precise Measurement of Scattering Phase Shifts

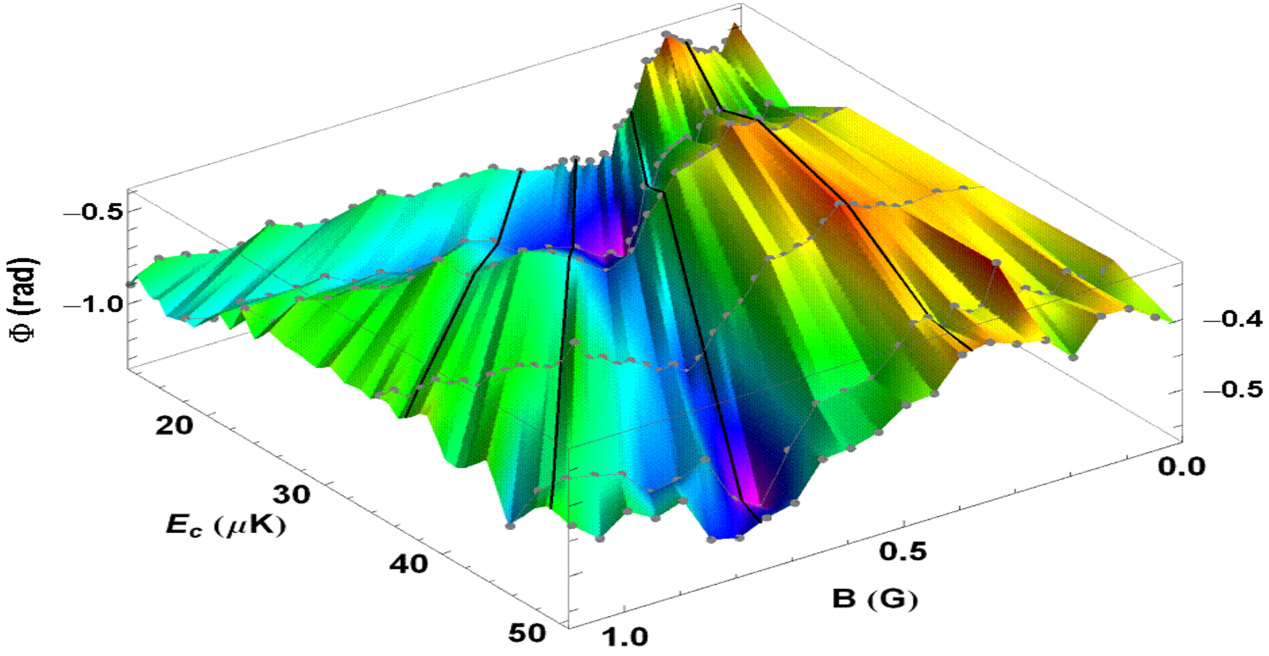
- In scattering measurements, effects are proportional to atomic density.
- Best density measurements are 10%.
- Key is that the relative phase of the clock coherence of the scattered atoms is independent of density.

- Clock accuracy: ppm scattering lengths.
- Sensitive to time variation of m_e/m_p & α at a Feshbach resonance.





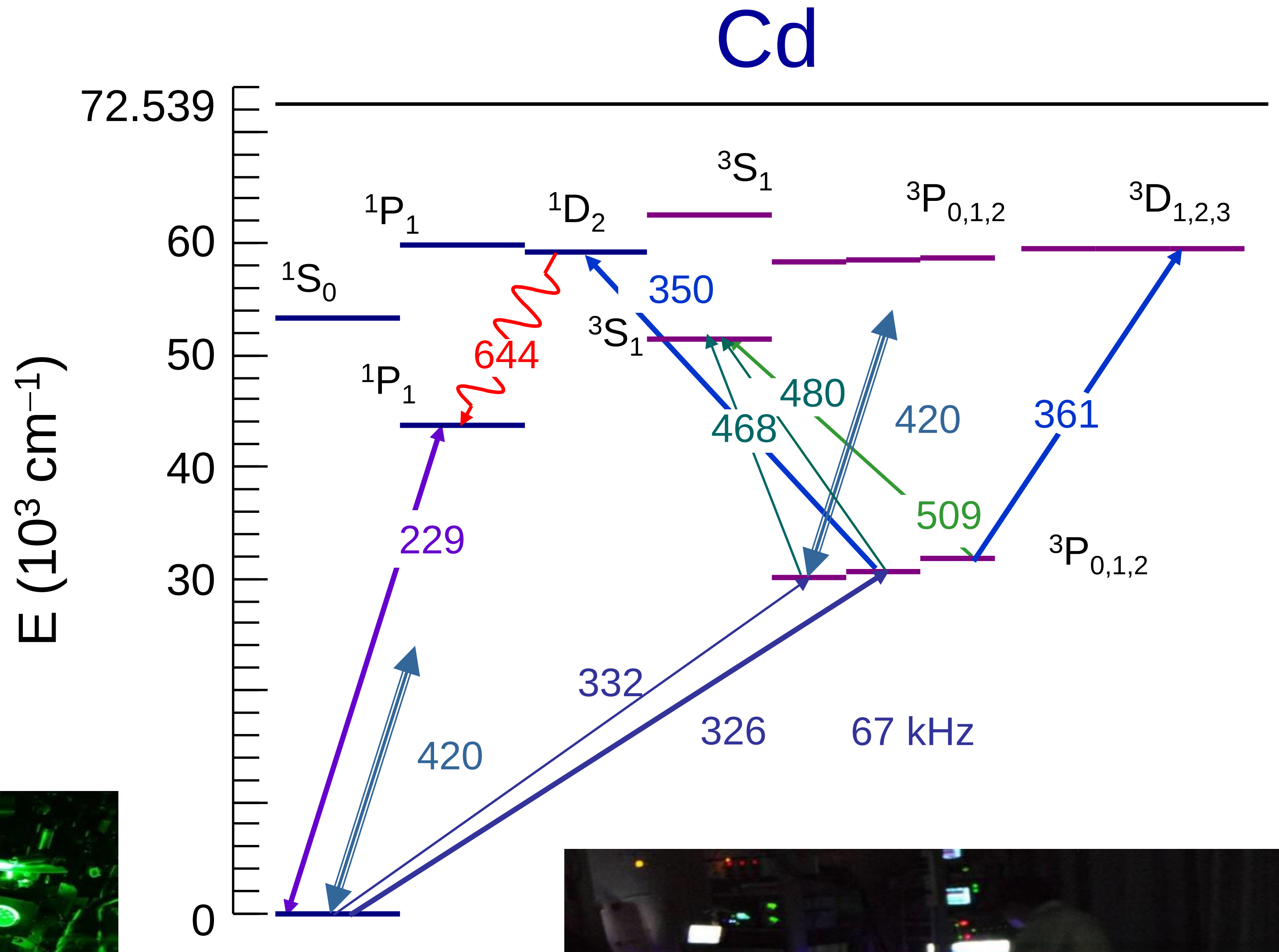
Bennett, KG, Kokkelmans, Hutson, PRL ‘17.
Chin & Flambaum, PRL ’06
Papoular … Shlyapnikov, PRA ‘12.



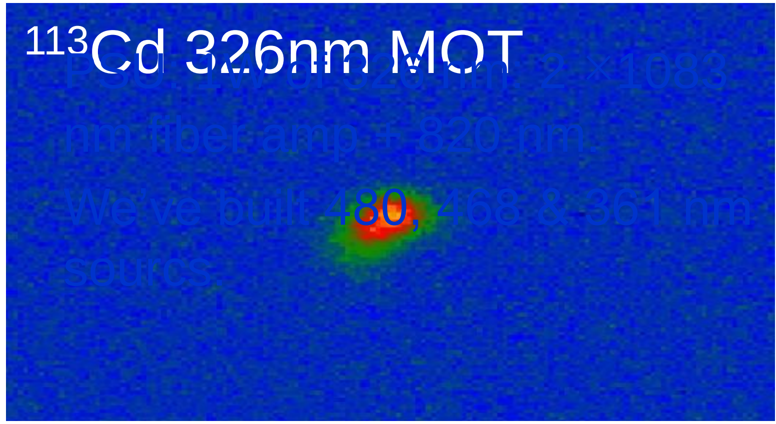
Hart, Xu, Legere, KG, Nature ‘07
Gensemer, Martin, Bennett, KG, PRL ‘12

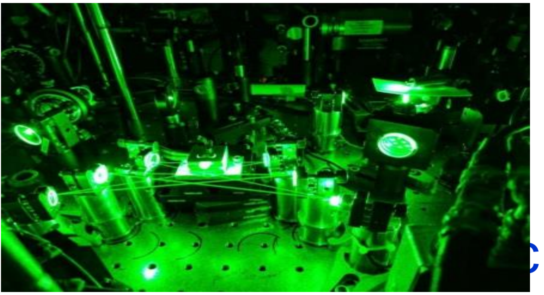
Cd Optical Lattice Clock

- Hg, Cd, & Mg have small sensitivities to BBR shifts.
- 2 Fermion spin ½ isotopes, ^{111}Cd & ^{113}Cd
  - select scattering lengths.
    - 6 Bosons: ^{106}Cd, ^{108}Cd, ^{110}Cd, ^{112}Cd, ^{114}Cd, ^{116}Cd.
- Wavelengths and Doppler/sideband cooling are easier for Cd than Hg.
- Elegant scheme to generate all wavelengths.

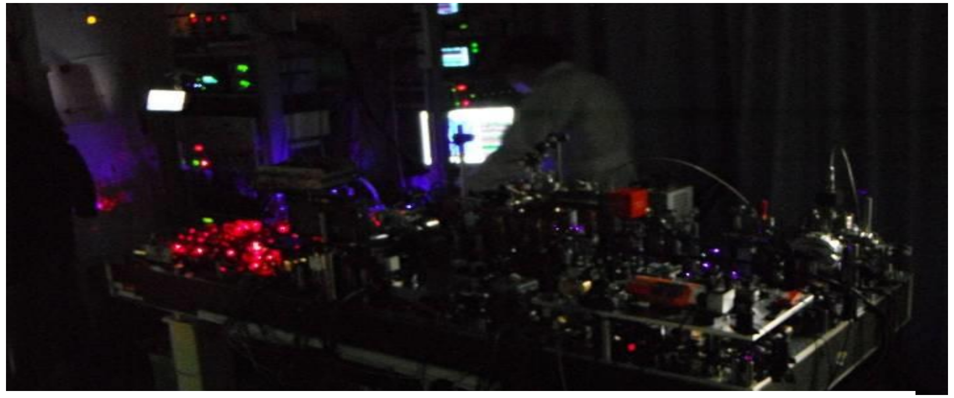


- PSU: 1W of 326 nm: 2 ×1083 nm fiber amp + 820 nm.
- We've built 480, 468 & 361 nm sources.


^{113}Cd 326nm MOT



clock:
A. Yamaguchi, KG, &
H. Katori



Mg: Kulosa, Fim, Zipfel, Rühmann, Sauer, Jha, KG, Ertmer, Rasel, Safronova's, Porsev PRL '15