# Time crystals as regenerative frequency dividers

## and other time and frequency applications of low noise photonic systems

**Andrey B. Matsko**

Jet Propulsion Laboratory, California Institute of Technology

# Contents

- Background on low loss systems and (some) of their uses at JPL
  - JPL’s Frequency and Timing Advanced Instrument Development group: main activities and interests
- Early studies of dissipative solitons and parametric instabilities in low loss photonic systems
  - Low loss resonant (micro)systems
  - Dissipative solitons in high-Q systems
  - Use of dissipative solitons (and not only the solitons) in clocks and oscillators
- Recent developments
  - Fundamental limitations of spectral purity of photonic oscillators: theory and experiments
  - <u>Photonic time crystals as ideal frequency dividers and their use in clocks</u>
- Outlook

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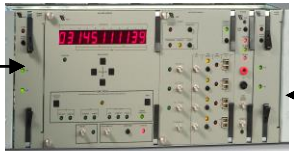
# Deep Space Network Clocks and Oscillators

H-Masers

Frequency Distribution

Photonic Frequency Distribution



Time Distribution

DSS-xx

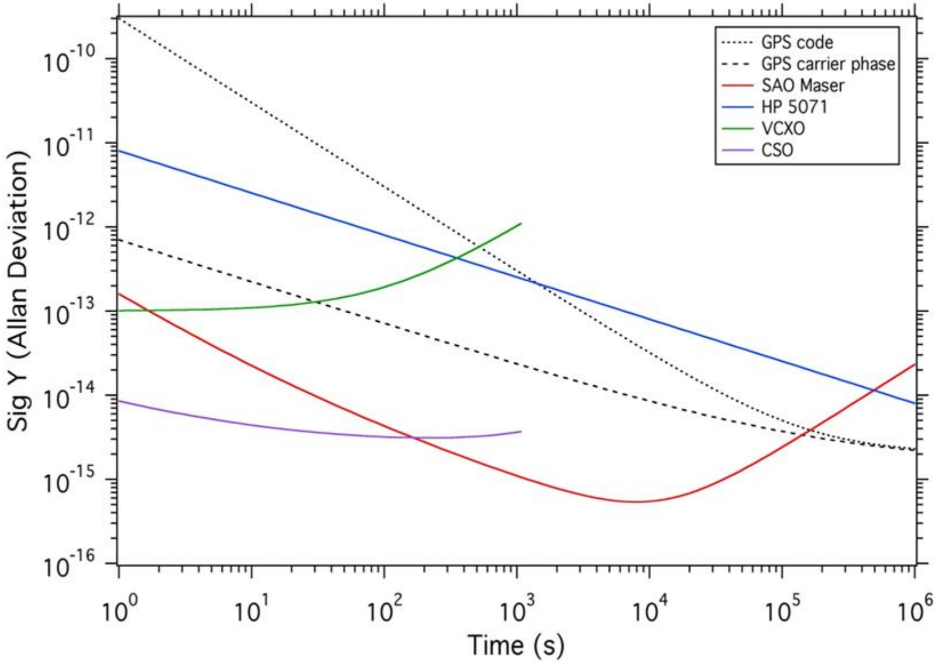
**SAO H-maser (Vintages 1974, 1979)**

M&C, GPS

Master Clock

Phase Noise Comparison @ 5 MHz

| Carrier Offset | DSN Requirement (dBc/Hz) | SAO H-maser + VCO (dBc/Hz) | SAO H-maser + CSO (dBc/Hz) |
|---|---|---|---|
| **1 Hz** | -118 | -123 to -130<br>-128 (typ.) | -151 to -161 |
| **10** | -129 | -147 (typ.) | |
| **100** | -141 | -152 (typ.) | |



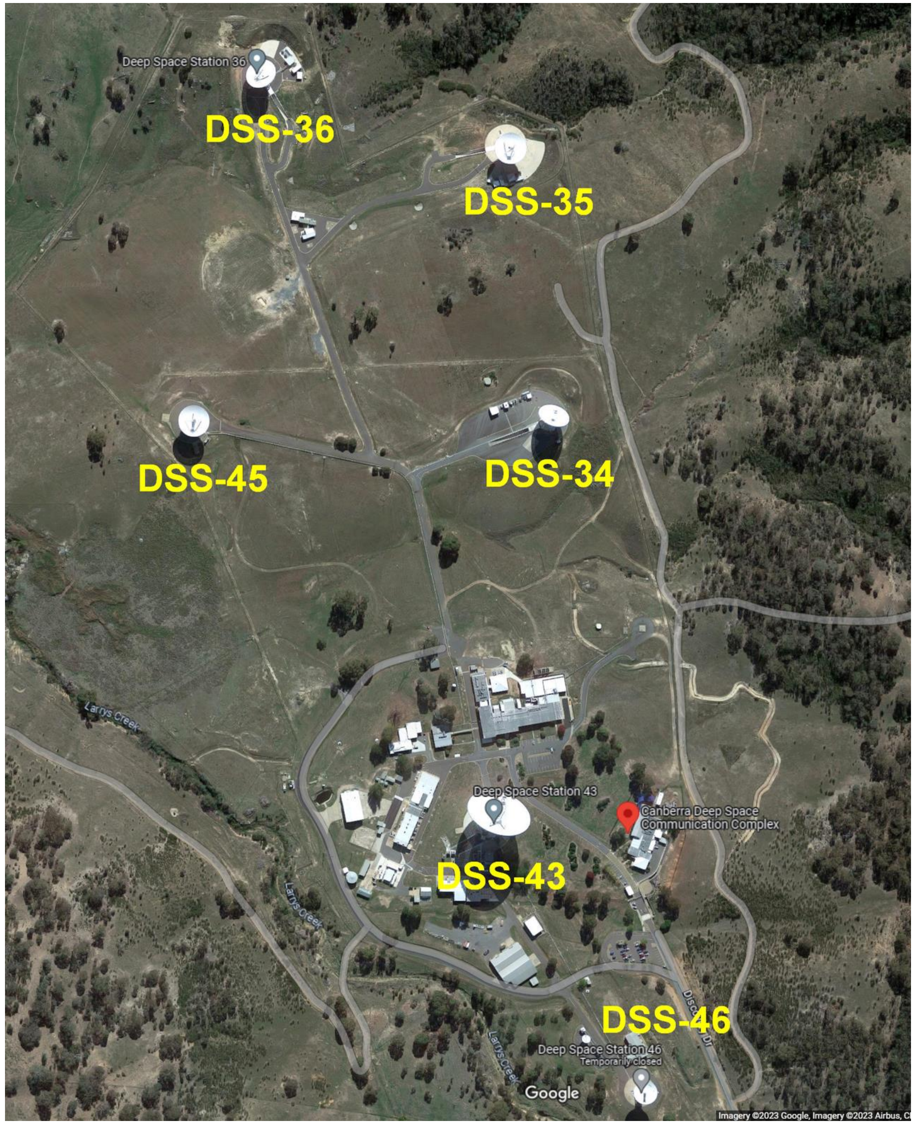

## FTS in each DSN Cite includes:

- Redundant atomic frequency standards
- Redundant clocks
- Signal distribution via fiber optic links
- S, X, Ka phase reference combs
- Performance measurement and calibration capabilities

C. Thornton and J. Border, 2003, Radiometric Tracking Techniques for Deep Space Navigation, JPL Deep Space Communications and Navigation Series (John Wiley & Sons, New York).

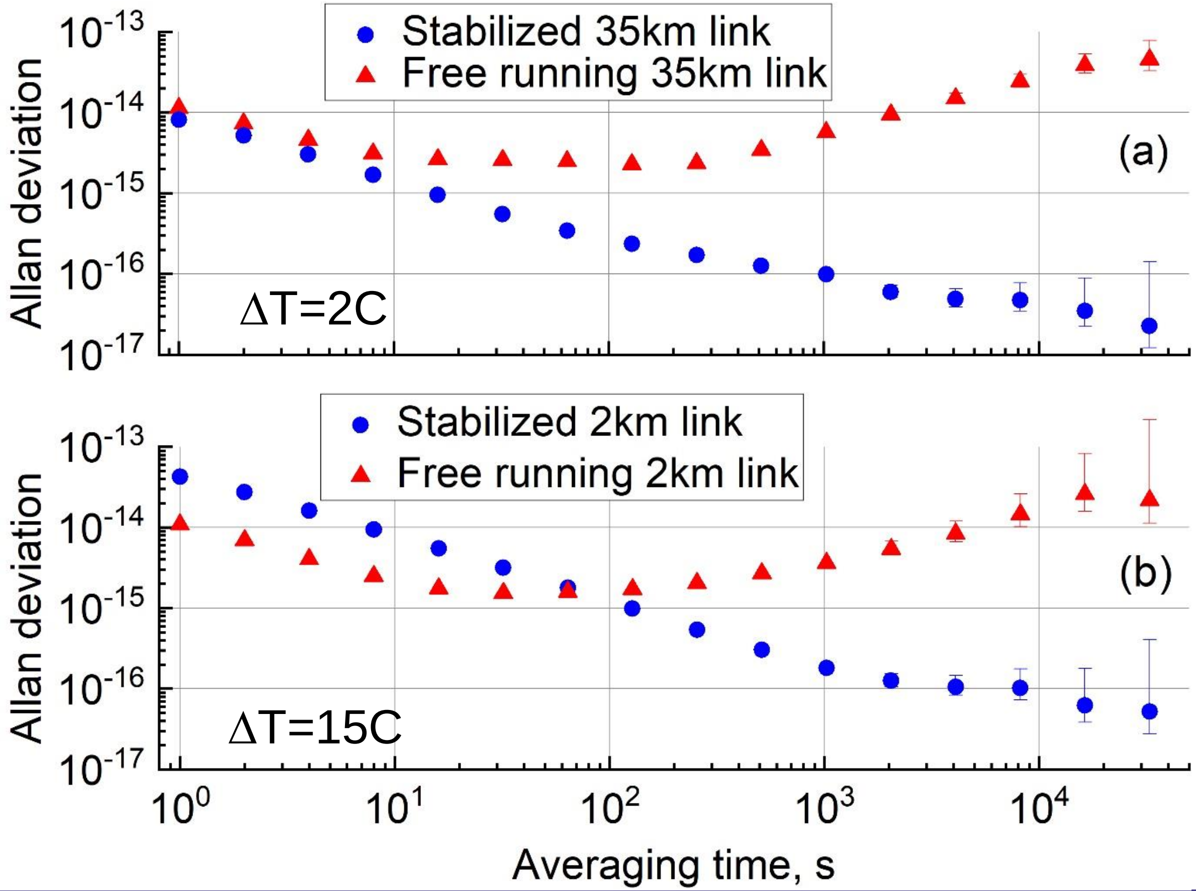
This is not research but hard-core engineering
Allowed operation disruption is 8 min per year

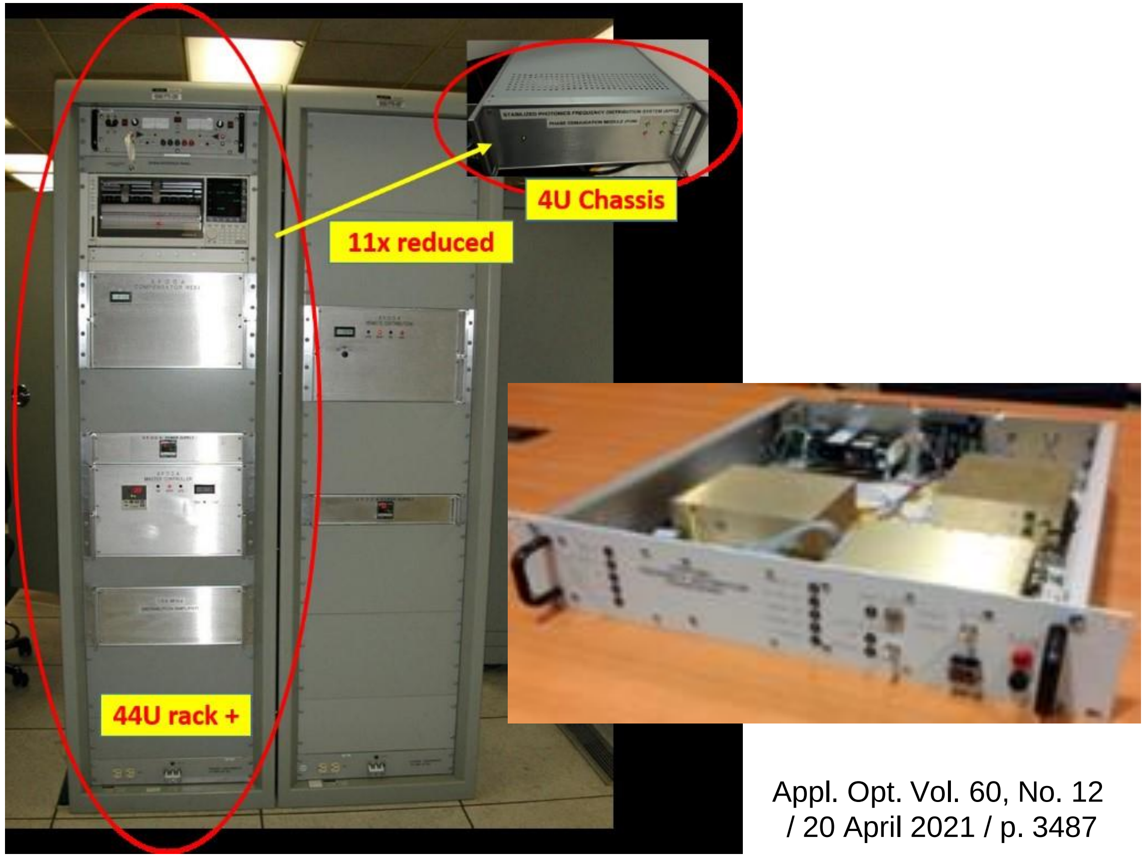
# Canberra Site a Week Ago



# DSN Low Loss Photonics Links

- JPL time and frequency group was the first one to utilize optical fibers for time and frequency transfer [G. Lutes, “Experimental optical fiber communications link,” TDA Prog. report 42-59 (Jet Propulsion Laboratory, 1980), pp. 77–85]
- Stability of existing links is better than the stability of a maser.

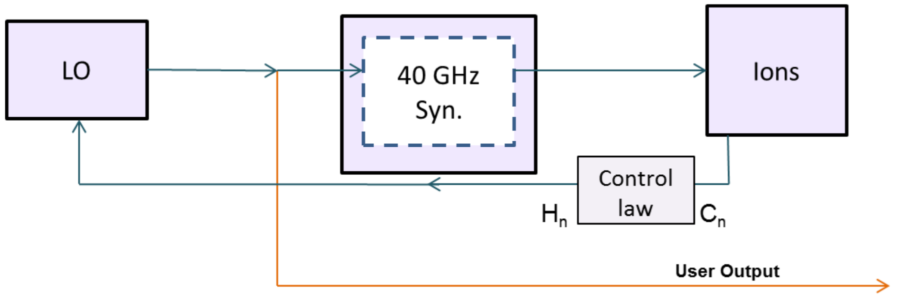


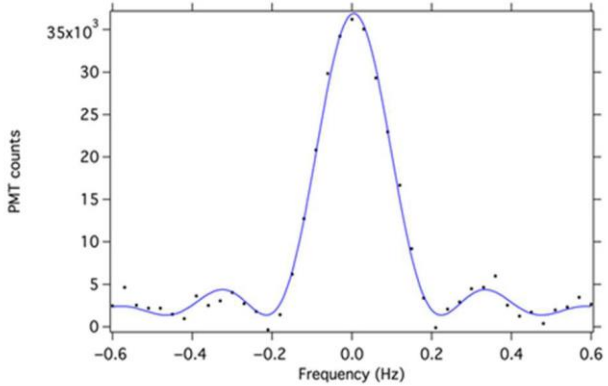




Appl. Opt. Vol. 60, No. 12 / 20 April 2021 / p. 3487

# More than 20 years history of Mercury Ion Clock development

## Many people have been involved

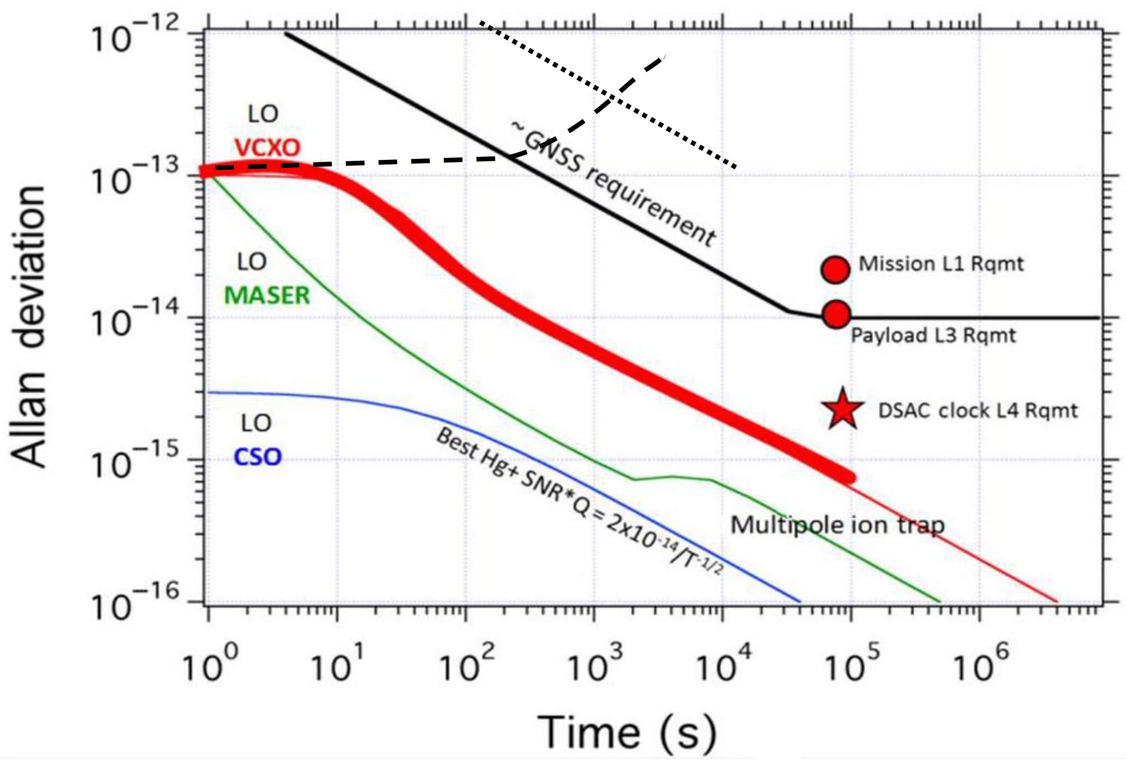



Achievable Stability:

- **Local Oscillator (< 10s)**
- SNR***Q** (~10s to 1 day)
- Systematic offsets/environment (> 1 day)
  - Magnetic Shifts
  - Number (Doppler Shifts) – QP only
  - Collision Shifts
  - Light Shifts – QP only

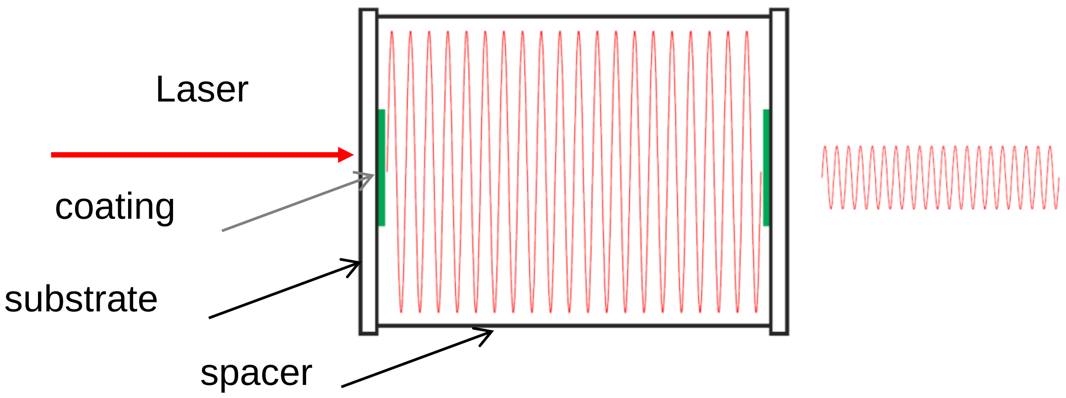
**Please note that Q is important here**




Mercury Ion Clock for a NASA Technology Demonstration Mission;
IEEE UFFC Transactions, Vol. 63, No.7, July 2016.

# Development of Ultrastable Fabry-Perot Cavities

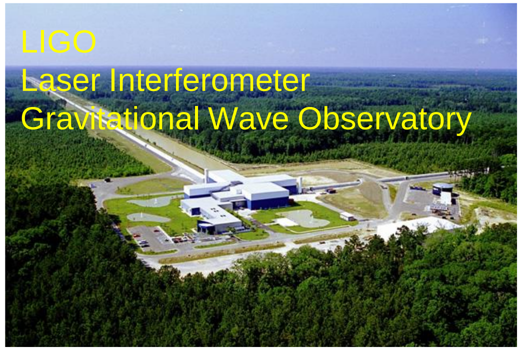
Stable laser based on Fabry Perot etalon:

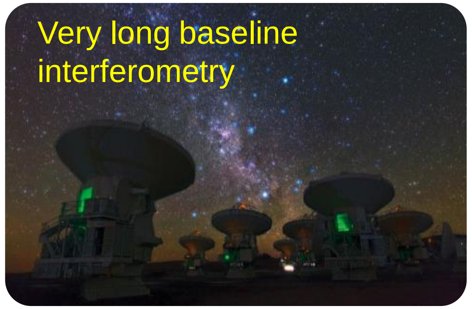


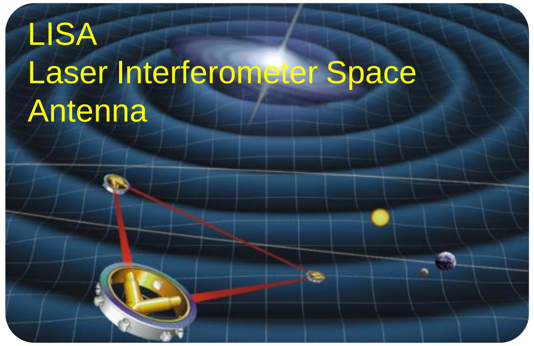

Laser frequency is referenced to cavity length

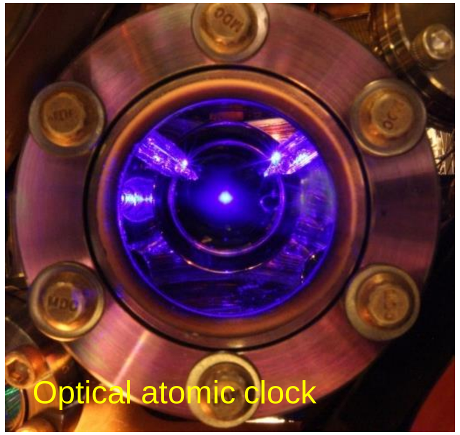
Optical frequency stability → Cavity length fluctuation

$$\frac{\Delta \upsilon}{\upsilon} = -\frac{\Delta L}{L}$$



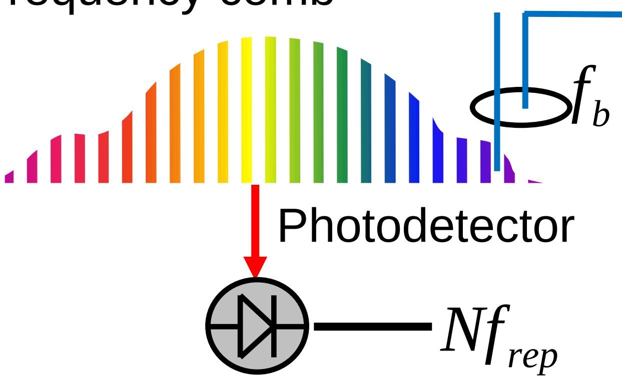




Highly stable optical frequency is critical in a variety of advanced research  direction: optical atomic clock,  gravational wave detection, very long base line interferometry, geodesy…

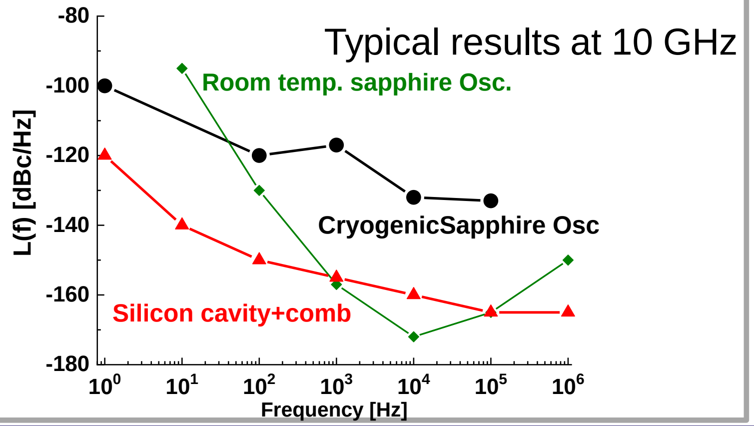
# An Ultrastable Optical Cavity System Can Be Better Than Quantum Clocks

## Photonics microwave generation

Frequency comb

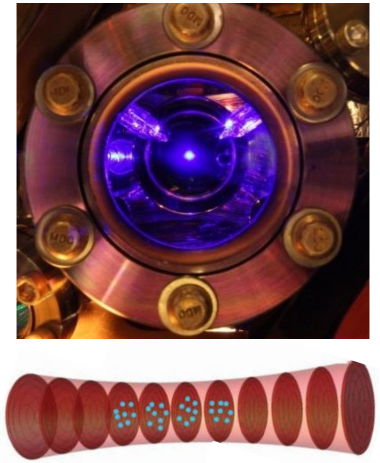


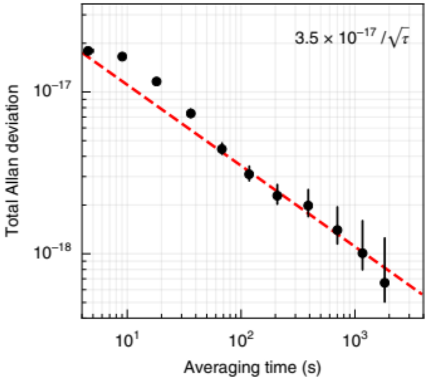

Low noise microwave reference
Cesium fountain clock (10 GHz)
Mercury clock (40 GHz)
VLBI, Radar…




A JILA silicon Fabry-Perot cavity at 4 Kelvin frequency stability $6 \times 10^{-17}$, drift $<5 \times 10^{-19}$/s

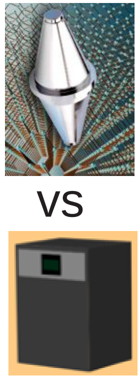
## LO for optical clock

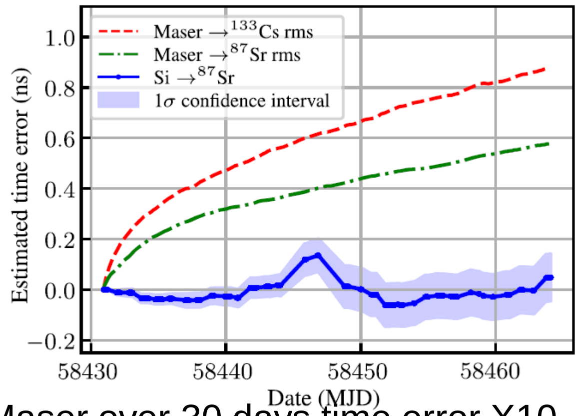





Sr atomic optical clock

## Stable Optical Carrier for time keeping



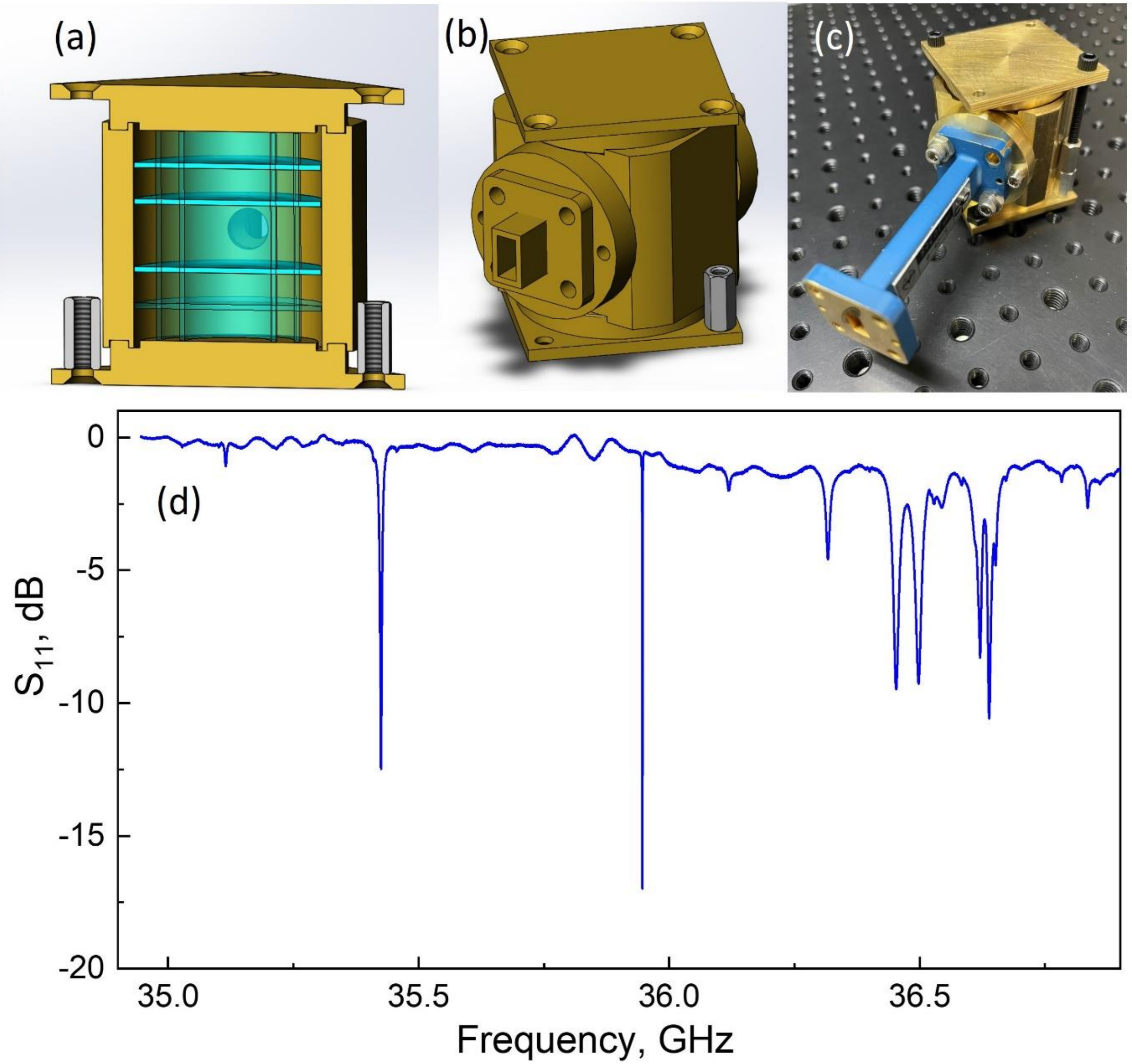




Cavity vs Maser over 30 days time error X10 improvement



Wei Zhang et al.

# Development of Sapphire (DBR) Resonator

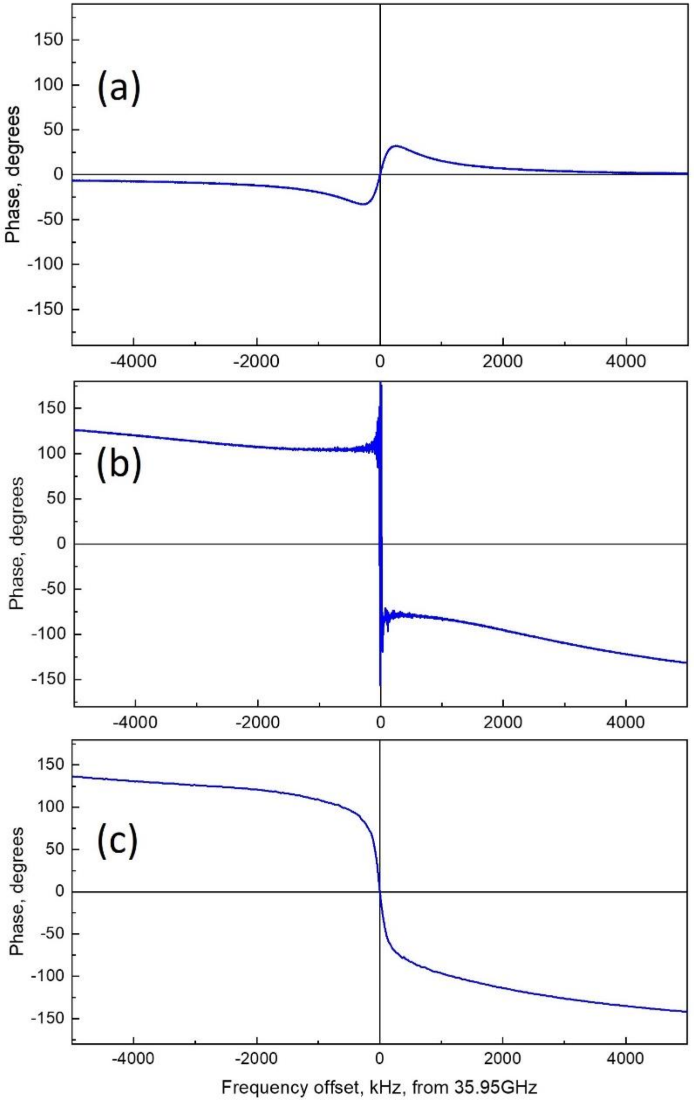



Goal: Support legacy of John Dick, Lute Maleki and others and develop RF oscillators based on low loss crystals.

An intrinsic Q-factor of $2 \times 10^5$ is demonstrated at 36 GHz for the lowest order TE-mode of a sapphire DBR.

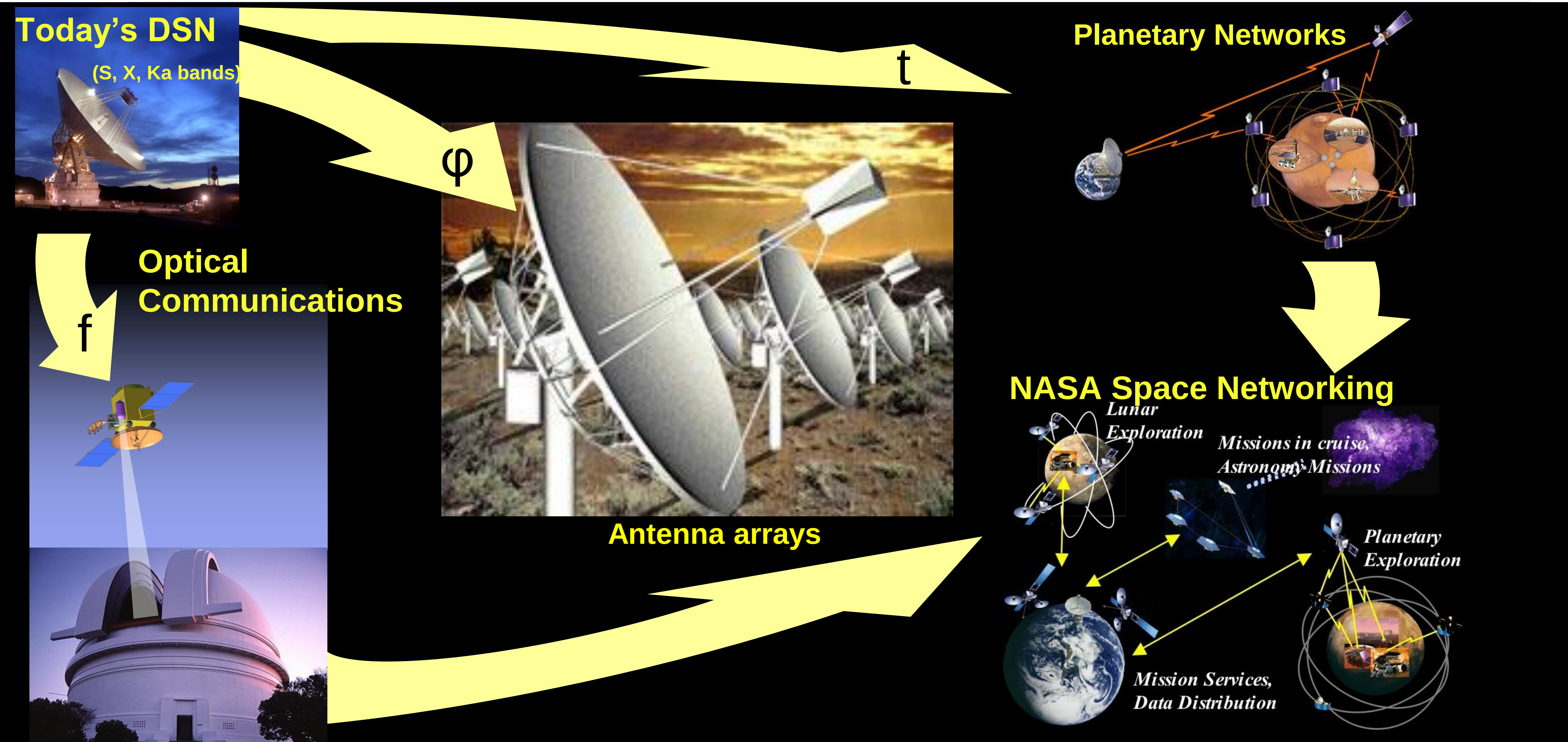
An oscillator suitable for the mercury clock operation is the next step of the development.

V. Iltchenko, et. al. TBP



Jet Propulsion Laboratory
California Institute of Technology

# Future Drivers of the Frequency & Timing Systems Development




**Reliability and SWaP are in the development focus (R. Tjoelker)**

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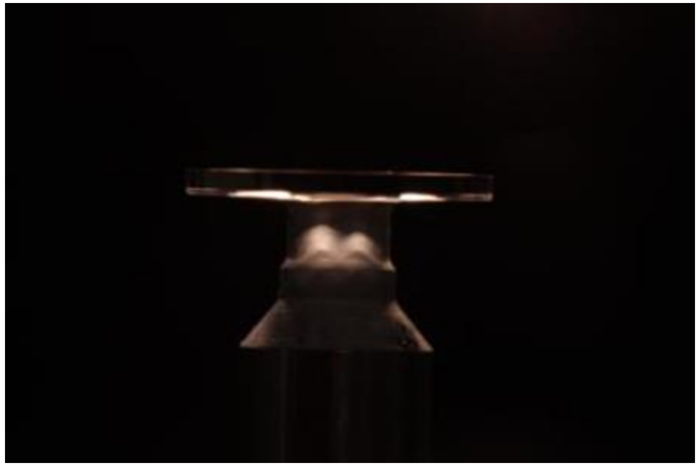
## How the modern (micro) photonics can help to push forward our time and frequency applications?

- **Stable and spectrally pure (UV) lasers**
- **Stable and spectrally pure photonic oscillators**

# Whispering Gallery Mode Resonators

Cavities enable stable oscillators. **Q-factor** of a cavity is ultimately determined by the material attenuation $\alpha$:



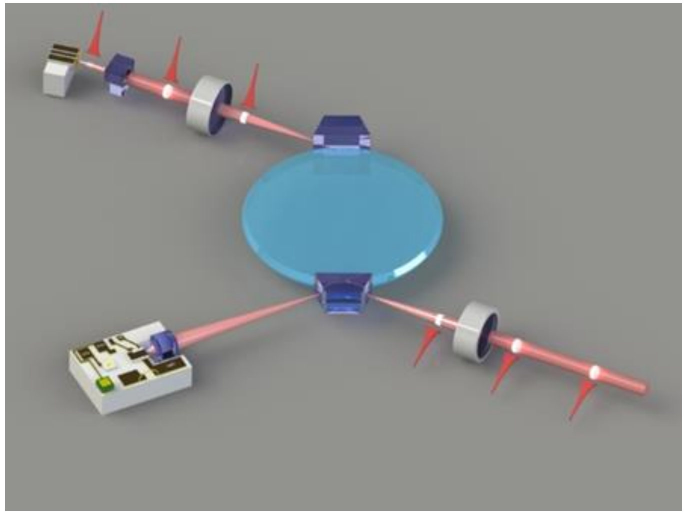
Solid state optical resonators with dimensions ranging from 10's of micron to a few mm, to support various applications

Extremely high Q's (record > $10^{11}$ in $CaF_2$) ~$1 \times 10^9$ $LiTaO_3$

Easy to T control/ encapsulate

Can support any broad range of wavelengths (unique feature)

A toolbox of low loss optical coupling solutions is developed



$$(2\gamma)^{-1} = n_0(\alpha c)^{-1} \qquad Q = \frac{2\pi n}{\alpha\lambda}$$

For $\alpha \simeq \alpha_{UV}\, e^{\lambda_{UV}/\lambda} + \alpha_R\, \lambda^{-4} + \alpha_{IR}\, e^{-\lambda_{IR}/\lambda}$

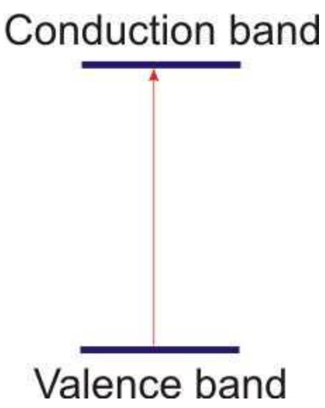
*E.D.Palik, "Handbook on optical constants of solids", Academic, NY, 1998*

| | |
|---|---|
| $CaF_2$ | $3 \times 10^{-7}$ cm$^{-1}$ |
| $SiO_2$ (optical fiber) | $5 \times 10^{-6}$ cm$^{-1}$ |
| $Al_2O_3$ | $1 \times 10^{-5}$ cm$^{-1}$ |
| $SiO_2$ (crystal) | $1 \times 10^{-5}$ cm$^{-1}$ |
| Si (crystal) | $8 \times 10^{-5}$ cm$^{-1}$ |
| $LiNbO_3/LiTaO_3$ | $1 \times 10^{-4}$ cm$^{-1}$ |
| $Si_3N_4$/Hydex | $1 \times 10^{-2}$ cm$^{-1}$ |
| Polymers | $1 \times 10^{-1}$ cm$^{-1}$ |

*Transparency of most of the optical materials is not well documented in literature and the documented values vary by an order of magnitude

Andrey Matsko, WMB6, IMS2019

# Attenuation of the material




$$\alpha(\lambda) \simeq \alpha_{UV} e^{\lambda_{UV}/\lambda}$$

The density of states is not zero at the band edge.
There are always band tails in non ideal crystals.

F. Urbach, *Phys. Rev.* 92, 1324 (1953)

$$\alpha(\lambda) \simeq \alpha_{IR} e^{-\lambda_{IR}/\lambda}$$

The multiphonon absorption occurs due to unharmonicity of internal vibrational modes of an ideal crystalline lattice

*M. Sparks and L. J. Sham, Phys. Rev. B 8, 3037 (1973)*

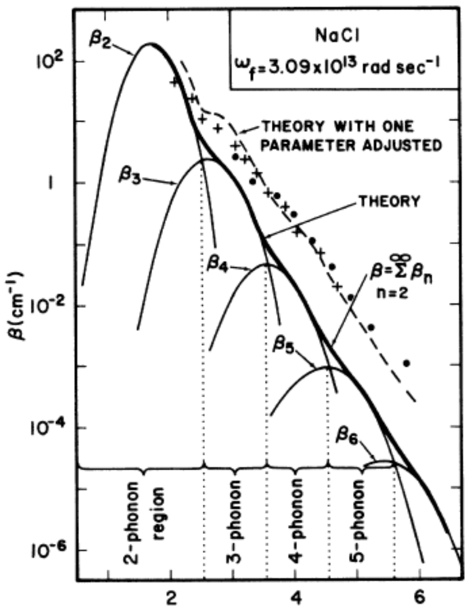
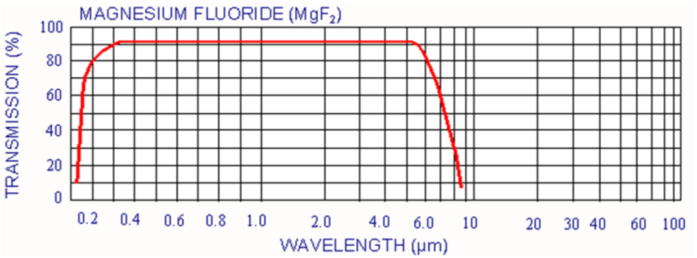



FIG. 10. Theoretical estimates of $\beta_n$ at room temperature for NaCl. Experimental points from Fig. 1 are shown for comparison.

In the center of the transparency window the attenuation is given by the impurities and scattering




Surface scattering is the leading effect in crystalline cavities

$$Q_{ss} = \frac{K_{TE}}{1 + K_{TE}} \frac{3\lambda^3 a}{8n\pi^2 B^2 \sigma^2}.$$

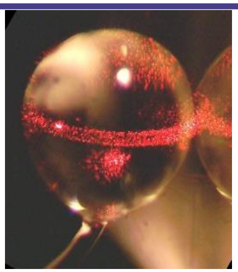
*M.L Gorodetsky et al., Opt. Lett. 21, 453 (1996);*

*D.W. Vernooy et al., Opt. Lett.23, 247 (1998);*

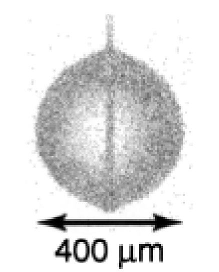
*L. Collot et al., Europhys. Lett. 23, 327 (1993);*

*V.B. Braginsky et al., Phys. Lett. A 137, 393 (1989).*

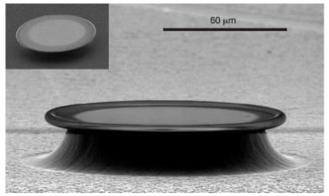
# Variety of the solid state WGMRs (early results)



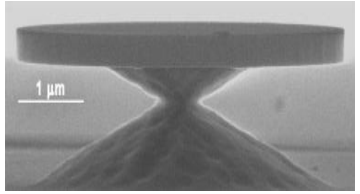
Fused silica: Gorodetsky et al., OL **21**, 453 (1996).




Solid $H_2$: K. Hakuta et al. OL **27** (2002)

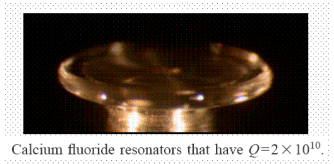



Fused silica: K. Vahala et al., Nature **421**, 925 (2003)

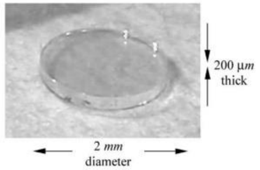



Si: Appl. Phys. Lett. **85** (2004)

Please note, the first crystalline WGMRs were created well before 1996.

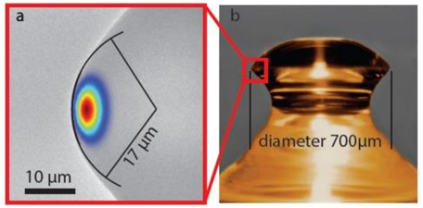



$CaF_2$: Savchenkov et al, PRA **70**, 051804 (2004); OE **15**, 6768 (2007).




LN WGM: D.A. Cohen and A.F.J. Levi, Electron. Lett. **37** (1) , 2001.

$CaF_2$ $Q_m>10^5$: J. Hofer, A. Schliesser, P. Del'Haye, and T. Kippenberg (CLEO'09)

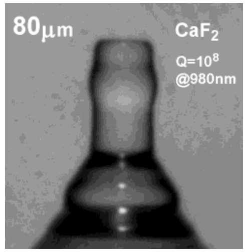



$MgF_2$ resonator, C. Y. Wang, T. Herr, P. Del'Haye, A. Schliesser, J. Hofer, R. Holzwarth, T. W. Hänsch, N. Picqué, T. J. Kippenberg, arXiv:1109.2716

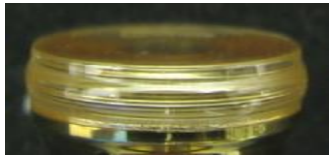
C. G. B. Garrett, W. Kaiser, and W. L. Bond, "Stimulated emission into optical whispering gallery modes of spheres", Phys. Rev. **124**, 1807-1809 (1961).

P. Walsh and G. Kemeny, "Laser operation without spikes in a ruby ring", J. Appl. Phys. **34**, 956-957 (1963).

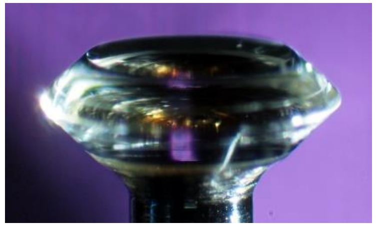
D. Roess and G. Gehrer, "Selection of discrete modes in toroidal lasers", Proc. of IEEE **52**, 1359-1360 (1964).



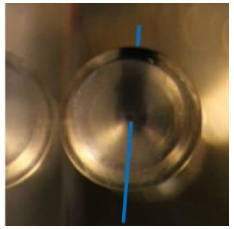

Grudinin et al Opt. Commun. 265, 33-38 (2006).



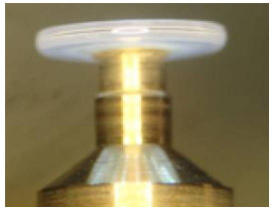
LN WGM resonator, D. Haertle, T. Beckmann, J. Schwesyg, S. Hermann, A. Zimmermann, K. Buse Photonics West 2009



$MgF_2$ resonator, OEwaves (2011)

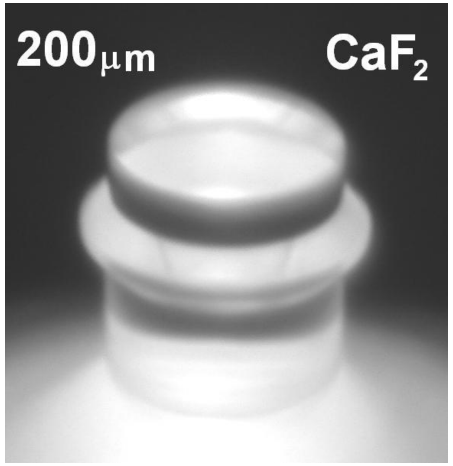


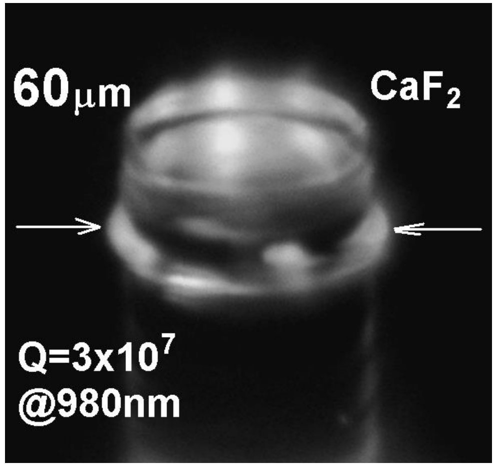
BBO resonator, JPL, 2012

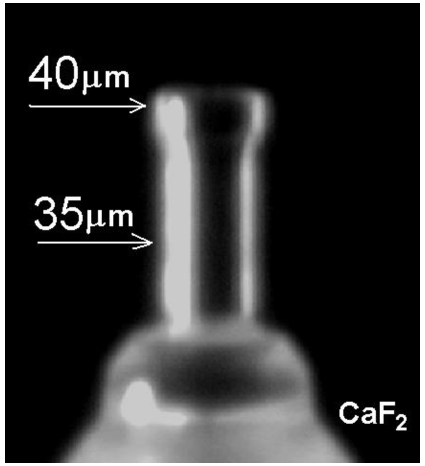


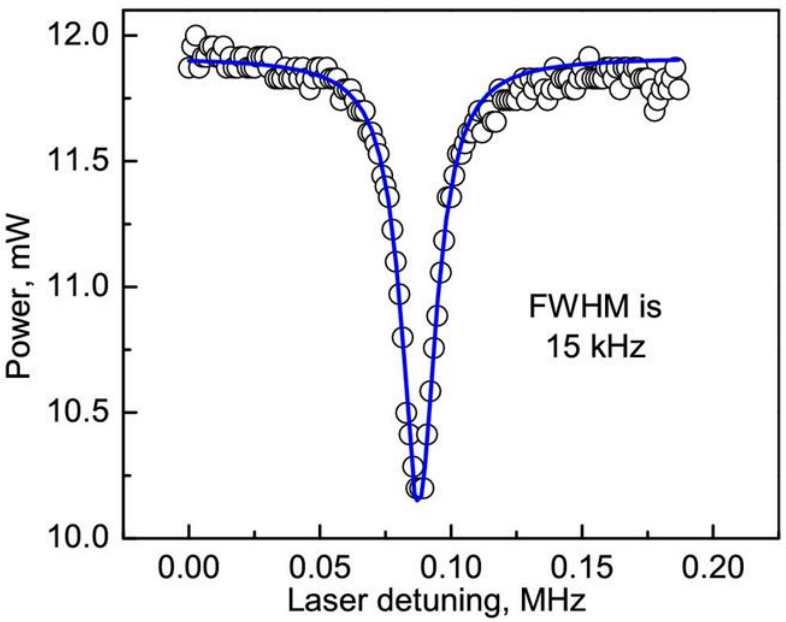
SBN resonator, OEwaves (2012)

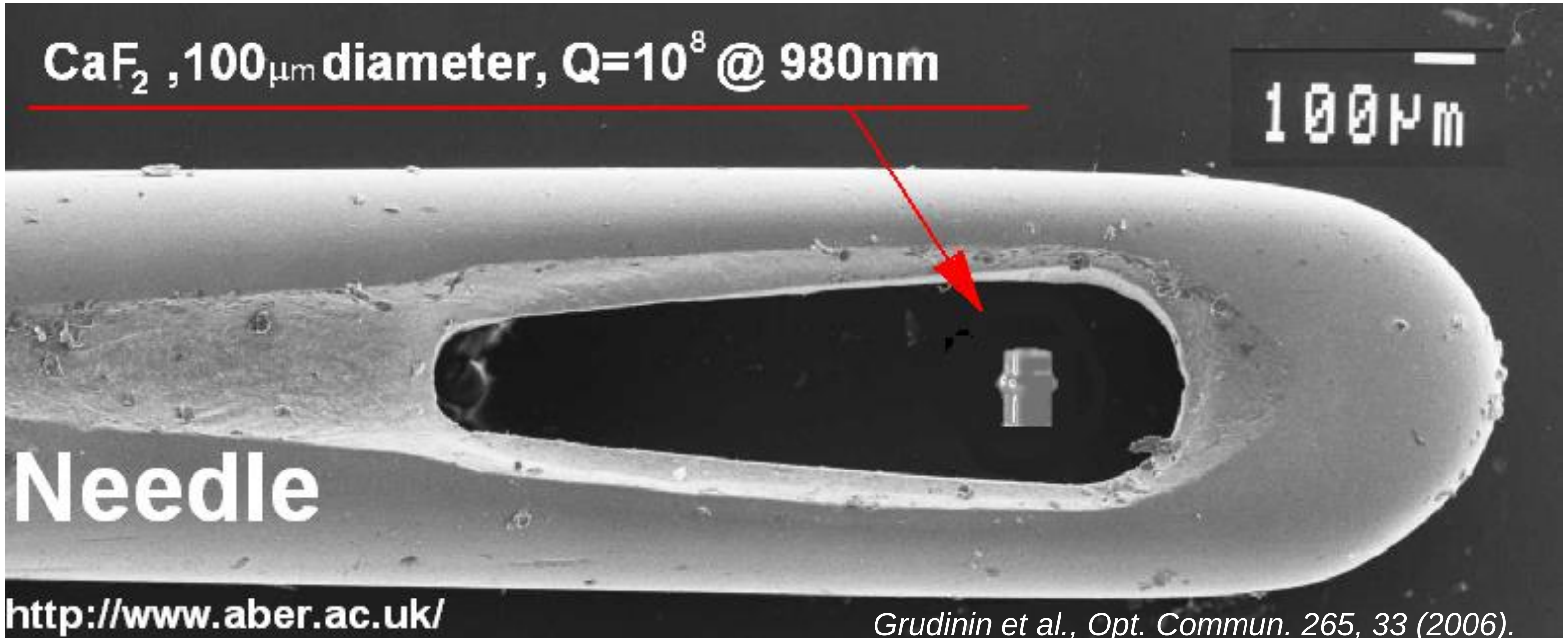
# Resonators of nearly any reasonable size can be created by mechanical polishing



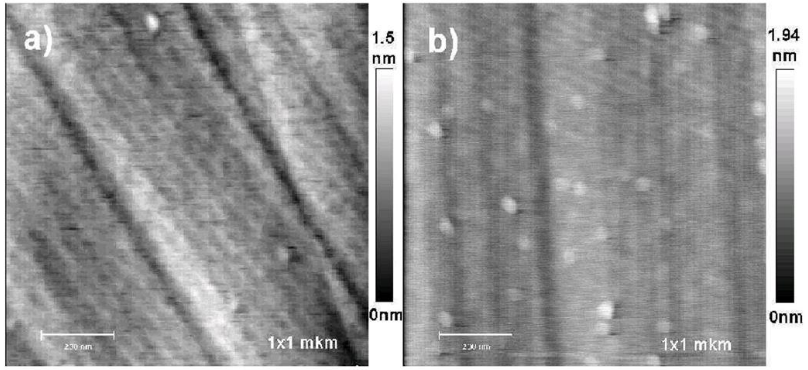




Grudinin et al., Opt. Commun. 265, 33 (2006).




$\sigma$ = 0.19 nm
B = 40 nm

$\sigma$ = 0.15 nm
B = 40 nm

# Importance of size and Q-factor for EM mixing

## Cavities enable efficient nonlinear interaction

$Q$ — Cavity field build-up factor and linewidth

$V$ — Small size and high optical energy density

$$\frac{Q^n}{V}$$

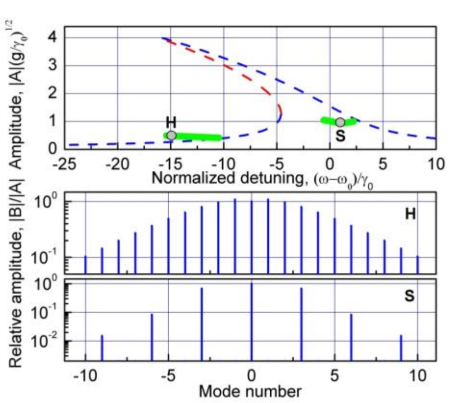
Figure of merit for nonlinear processes:

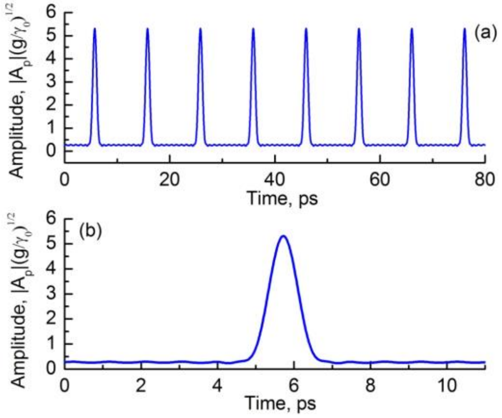
Purcell's factor: n=1

SRS and FWM: n=2

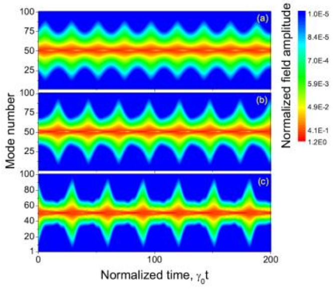
Frequency doubling and acousto-optics: n=3

# Microcavity-based dissipative solitons: some early studies

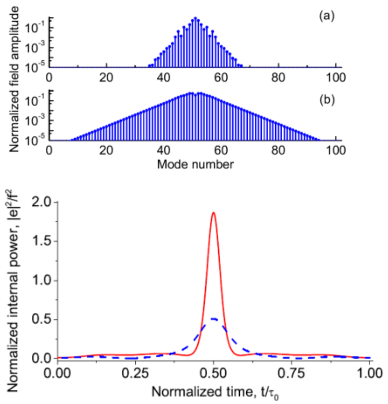


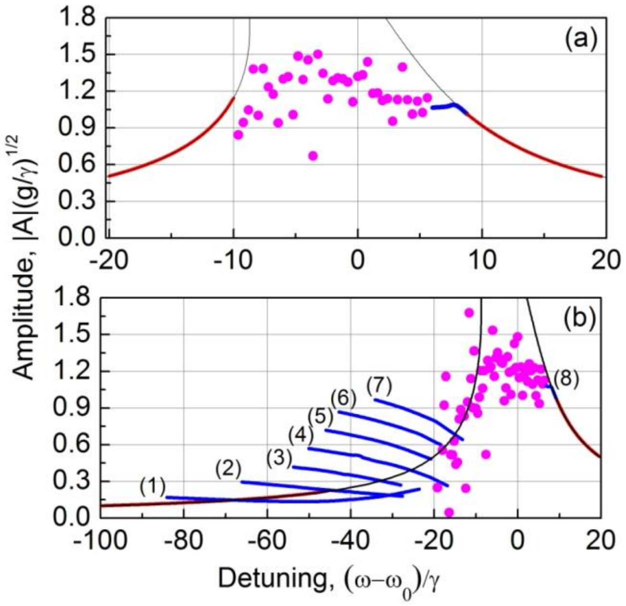




*A. B. Matsko et al., Phys. Rev. A* ***85****, 023830 (2012)*

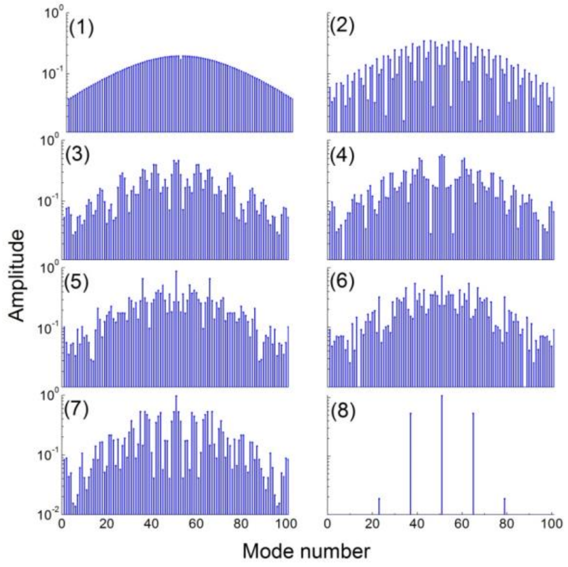



A. B. Matsko et al.
Opt. Lett. **37**, 4856
(2012)

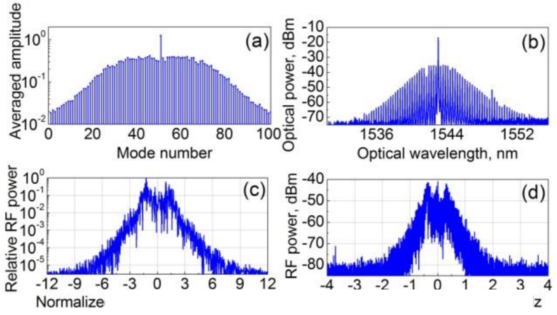


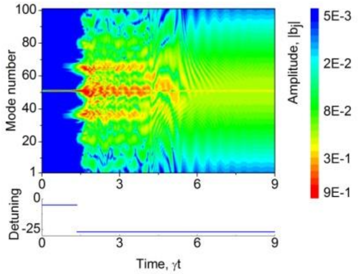




## *Multistability*




## *Chaos*



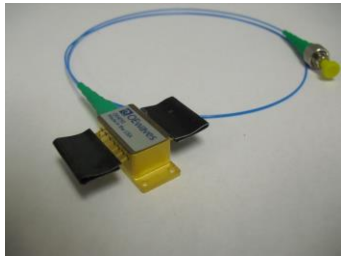

*A. B. Matsko et al., Opt. Lett.* ***38****, 525 (2013)*

# Optical clocks using high-Q microcavities

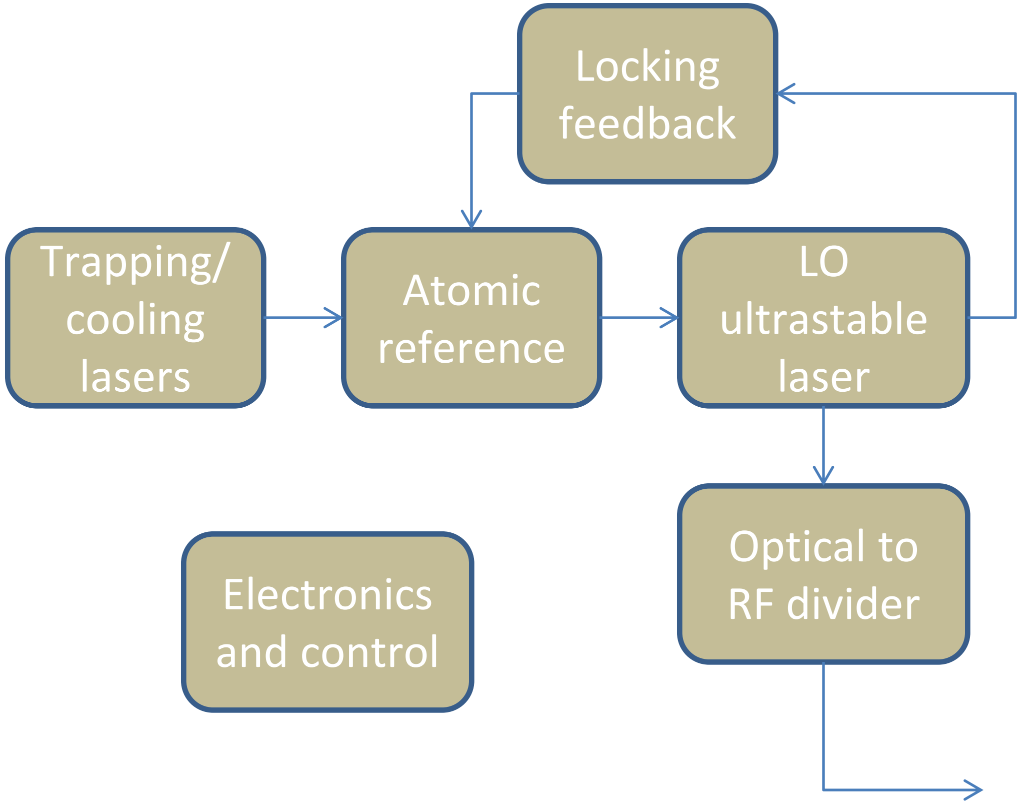
# Optical Clocks

## Now, using the high-Q cavities we can improve clocks

### Lasers



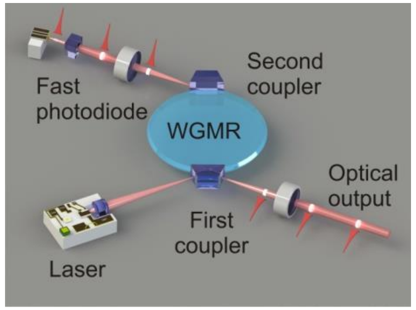
1. High power and frequency stability
2. Low RIN and frequency noise
3. Low vibration sensitivity
4. Low RAM
5. Compact




### Reference cavities

1. Low SWaP
2. High stability
3. High finesse
4. Low environmental sensitivity

### Kerr combs

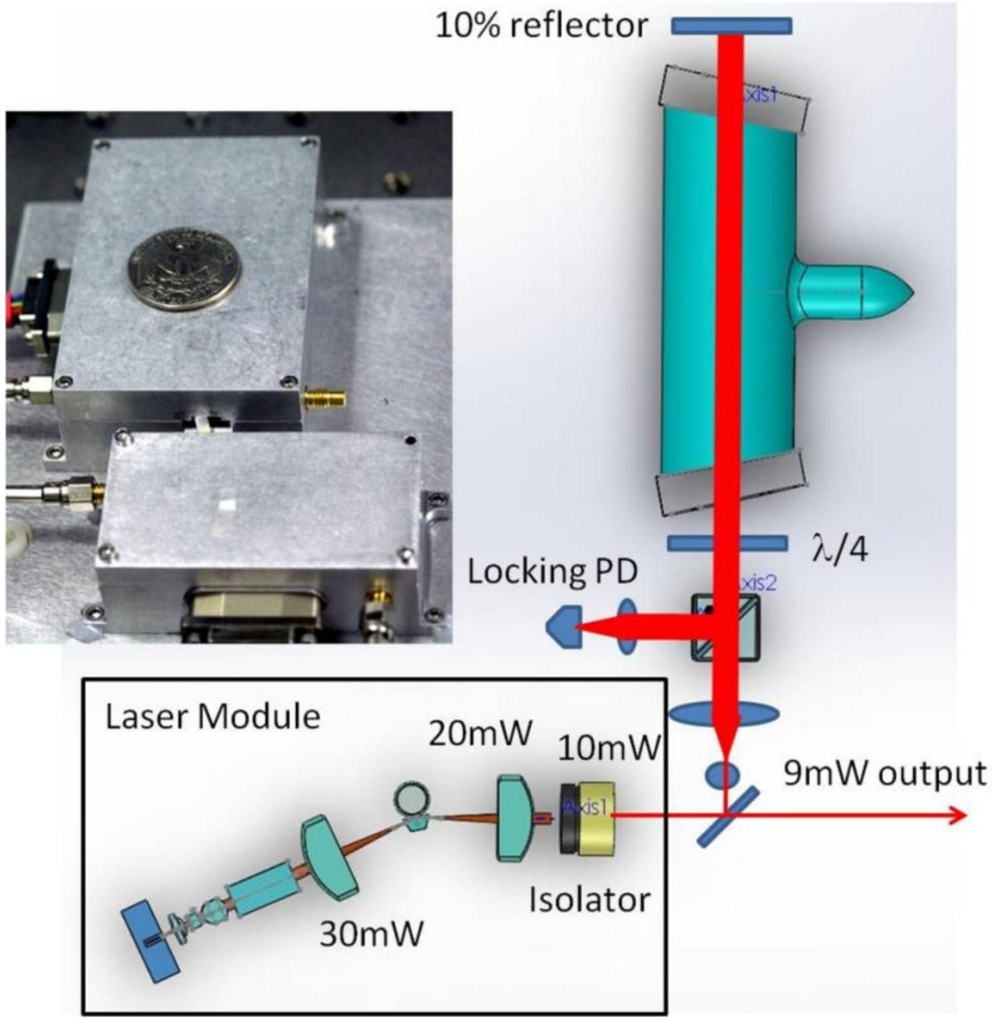


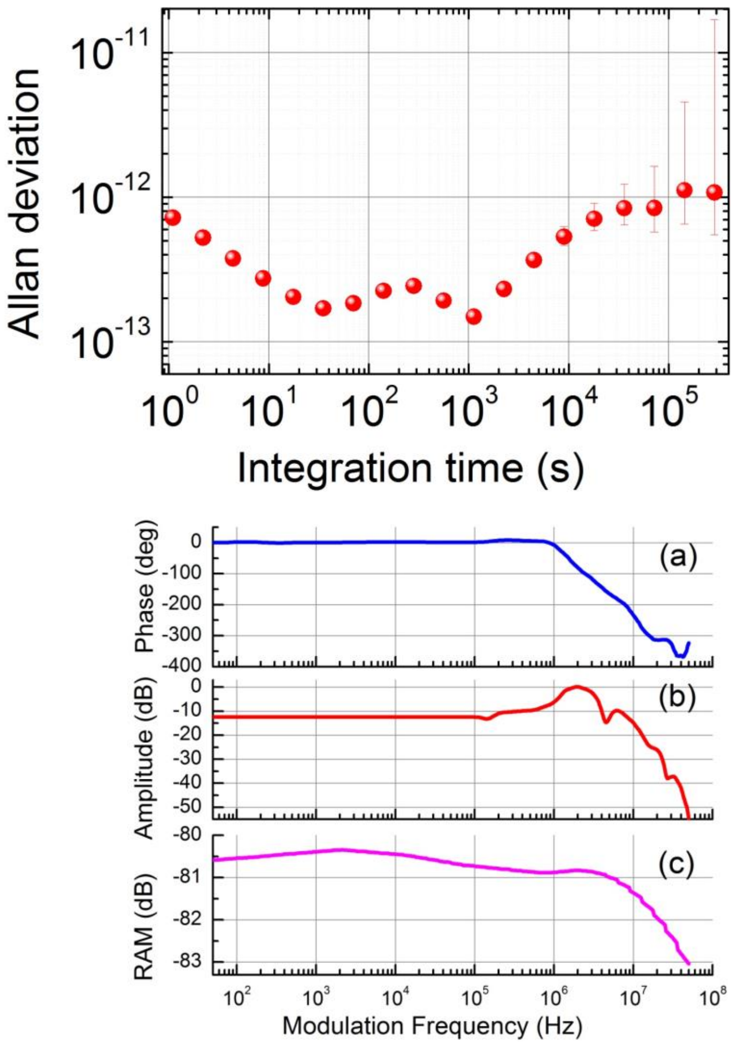

1. Low SWaP
2. High stability
3. Low environmental sensitivity



Supported by JPL, NASA, ONR, AFRL and DARPA, in collaboration with Nan Yu et al.

# Atomic Cell Stabilized Laser: Improved Locking Due to High-Q Cavity




A **frequency comb** is needed to make a clock

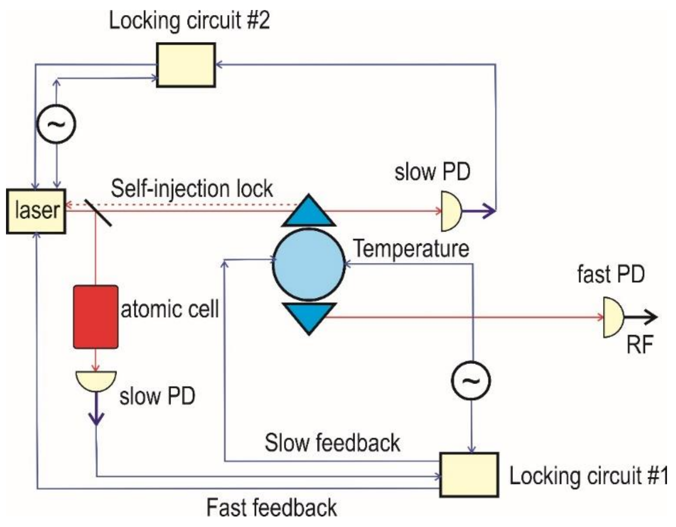
Low relative amplitude modulaton (RAM) is essential for the accurate locking to the atomic transition.

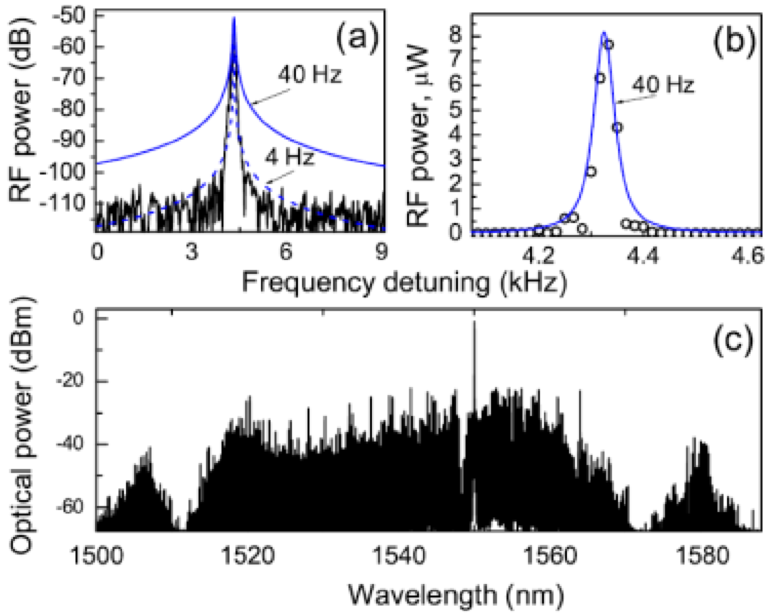
W. Liang et al., Appl. Opt. **54**, 3353-3359 (2015)

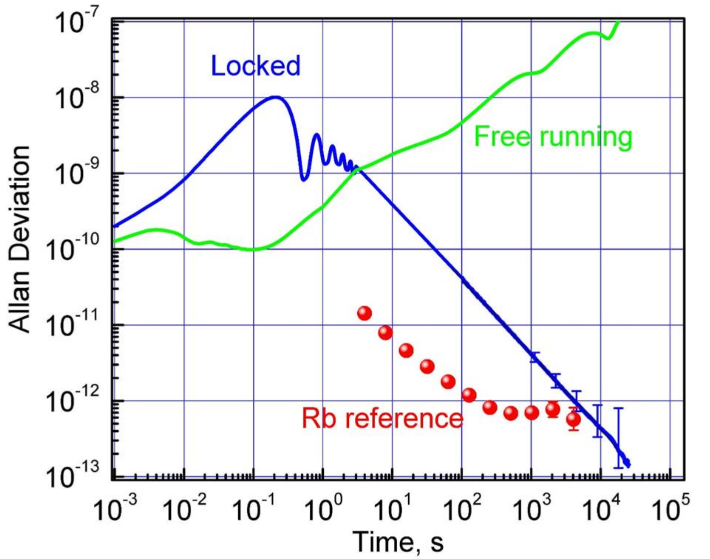


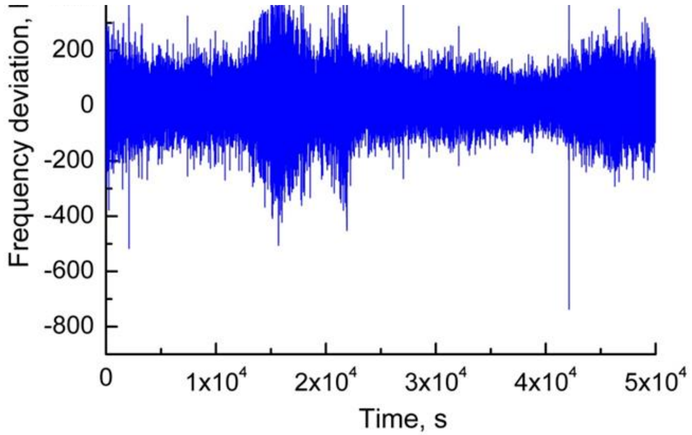
Was supported by DARPA

# Atomic Transition Stabilized Kerr Frequency Comb




Main idea: Frequency stability can be transferred from optical frequency to microwave frequency using an optical comb. The stabilization goes through the resonator stability. Does not works well because of the efficiency dips.

A microwave signal is generated by demodulating the Kerr comb with a high-frequency photodiode. (a) The signal on logarithmic scale; (b) the same signal on linear scale; (c) optical comb used in the experiment. Linear fit of the microwave line gives a 40 Hz linewidth.

The idea still can work IF/WHEN a good single mode cavity will be created



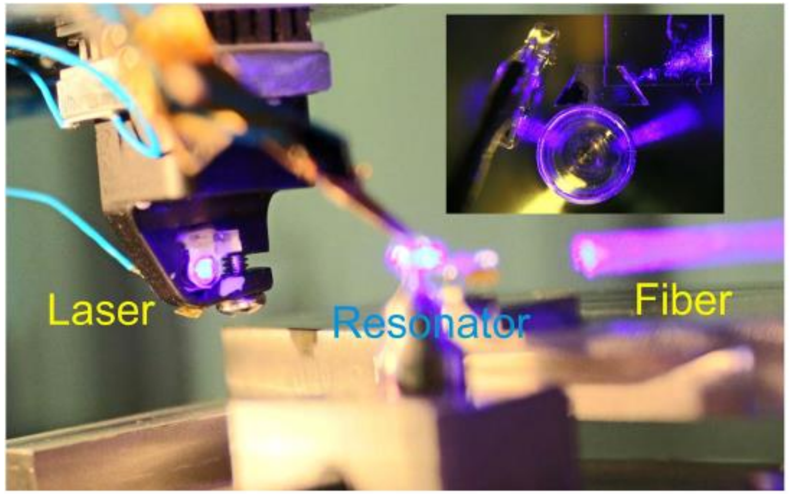
Was supported by DARPA

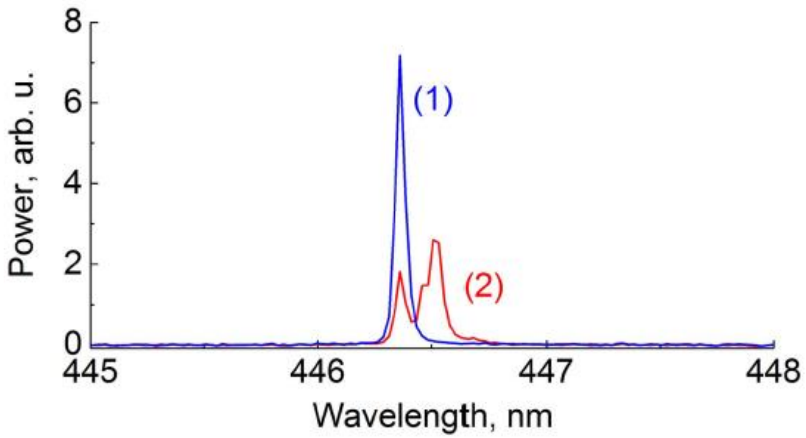
# Stabilized Blue Lasers

IOP Publishing | Journal of Optics
J. Opt. 20 (2018) 045801 (5pp) | https://doi.org/10.1088/2040-8986/aaae4f

## Self-injection locked blue laser

Prathamesh S Donvalkar, Anatoliy Savchenkov and Andrey Matsko

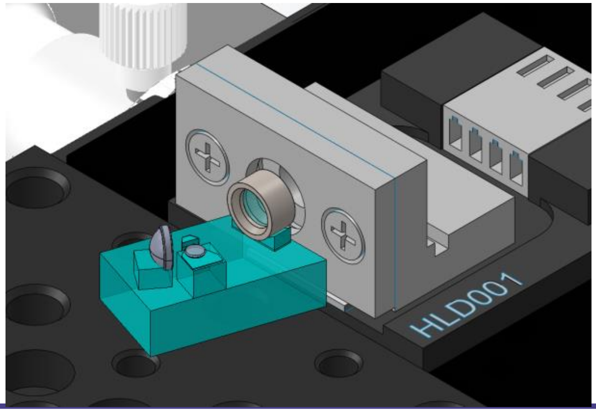


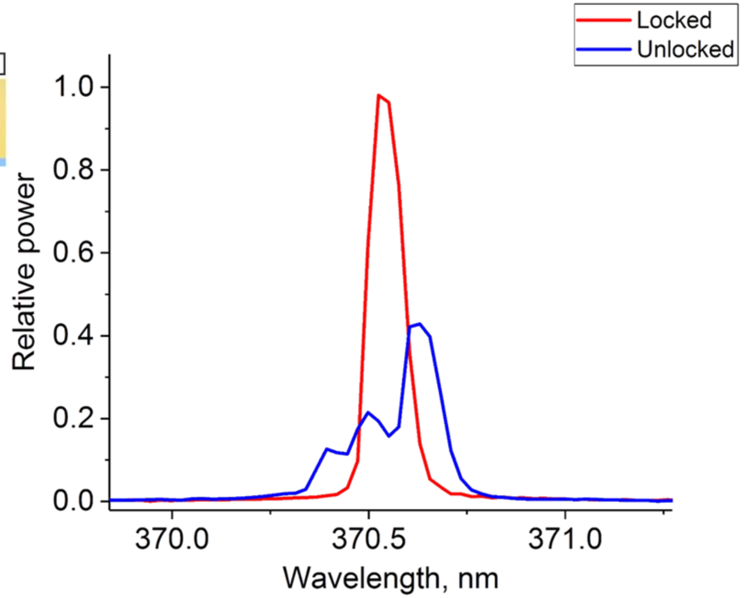



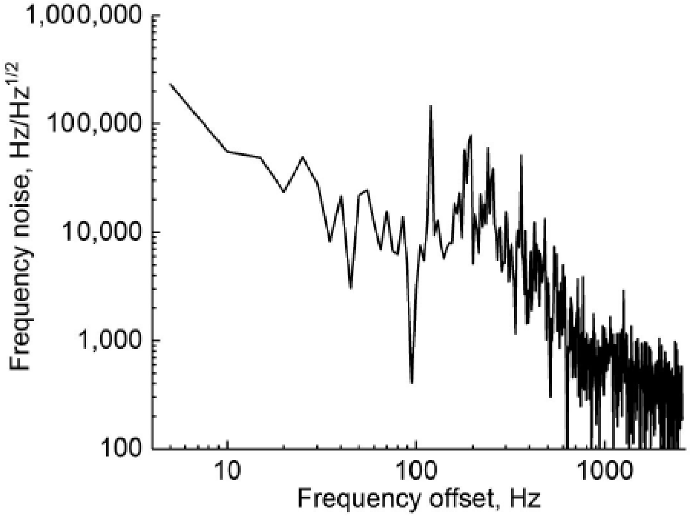

Letter | Vol. 44, No. 17 / 1 September 2019 / Optics Letters 4175
Optics Letters

## Self-injection locking efficiency of a UV Fabry–Perot laser diode

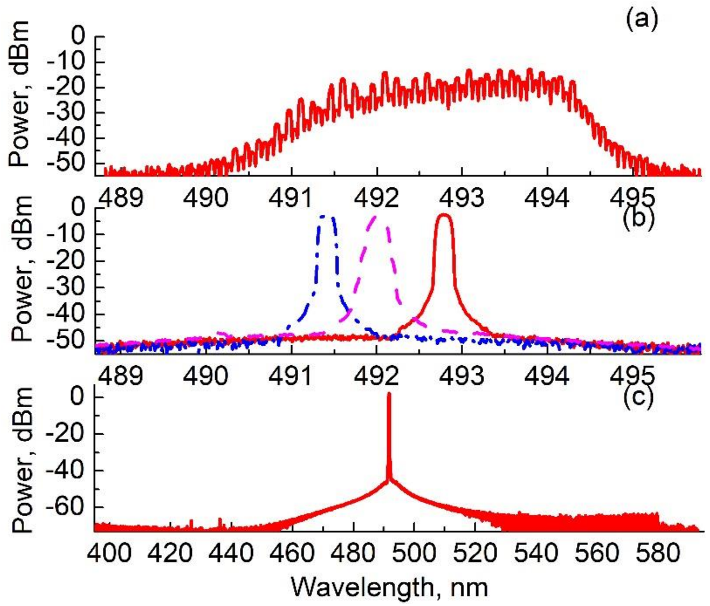
Anatoliy A. Savchenkov,[1] Sheng-Wey Chiow,[2] Mohammadreza Ghasemkhani,[3] Skip Williams,[1] Nan Yu,[2] Robert C. Stirbl,[2] and Andrey B. Matsko[1,2,*]

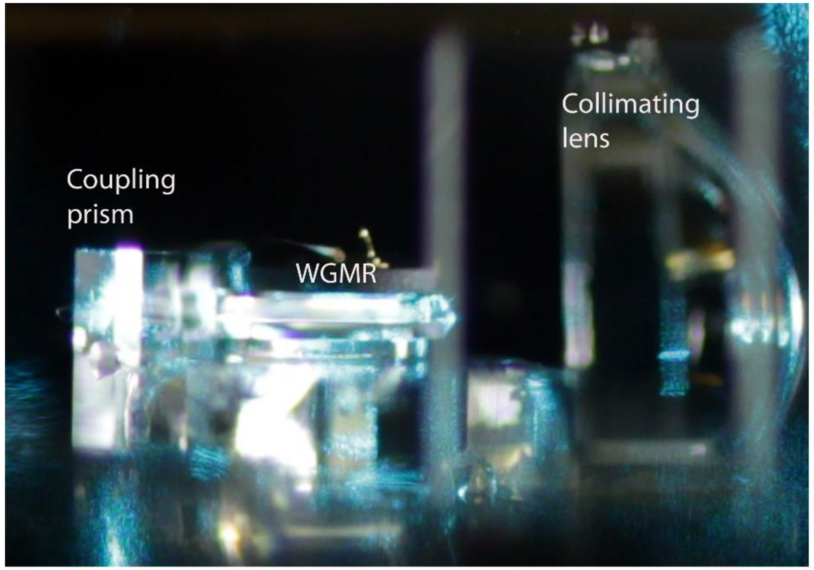


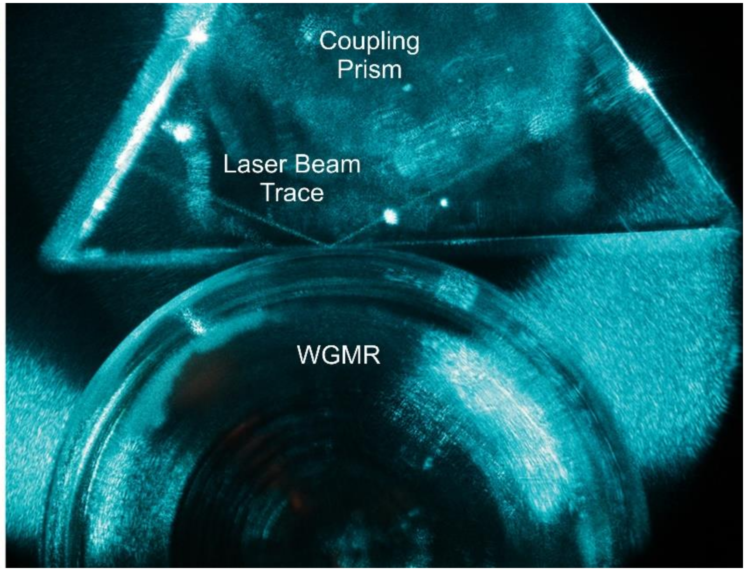



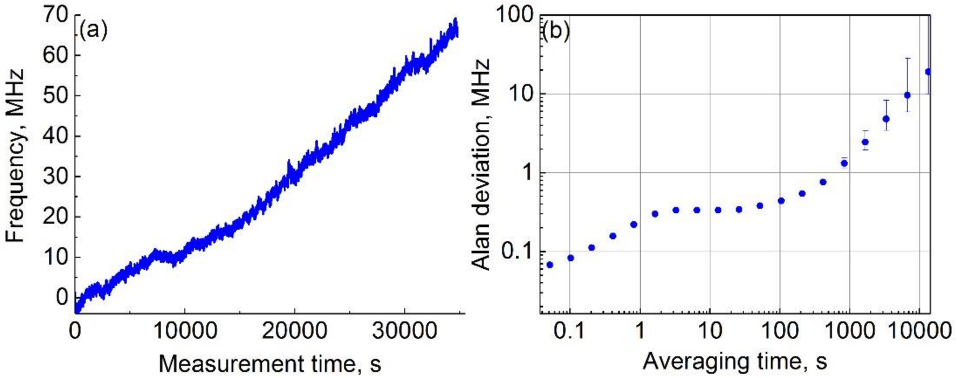



# Stabilized Magenta Lasers



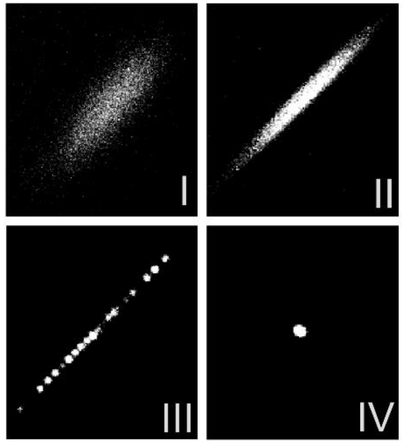



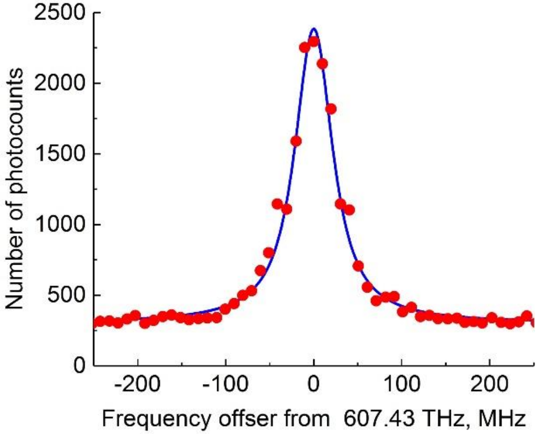




## Application of a self-injection locked cyan laser for Barium ion cooling and spectroscopy

Anatoliy A. Savchenkov[1], Justin E. Christensen[2], David Hucul[3,4], Wesley C. Campbell[2,5], Eric R. Hudson[2,5], Skip Williams[1]✉ & Andrey B. Matsko[6]

SCIENTIFIC REPORTS | (2020) 10:16494



# Problem to solve

- **Need to create a laser at 194 nm**

# RF oscillators based on dissipative solitons

# Spectrally Pure RF Oscillators

RF oscillators are needed in a variety of systems including:

- Radar
- Direction finding of signals of interest
- Applications related to coherent operation across multiple segmented platforms

Radar operational requirements include ever-increasing demands for lower-phase noise signal and waveform generation

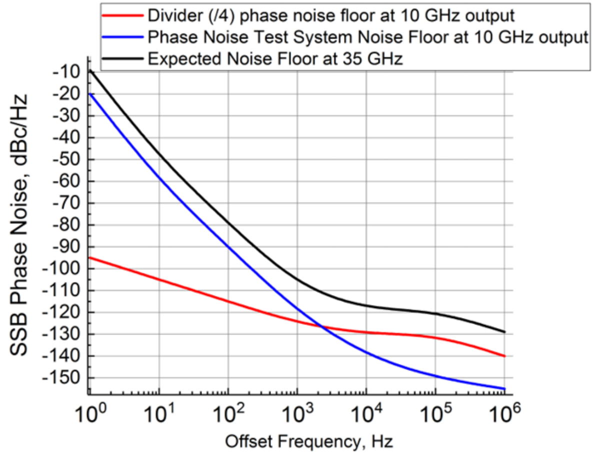
**Existing electronic technologies cannot meet these demands concurrently within environmental and tactical requirements**

- **RF *<u>photonic</u>* oscillators are characterized with**

✓ High Spectral Purity
✓ Small SWaP
✓ Low Environmental sensitivity
✓ Convenient Signal Distribution

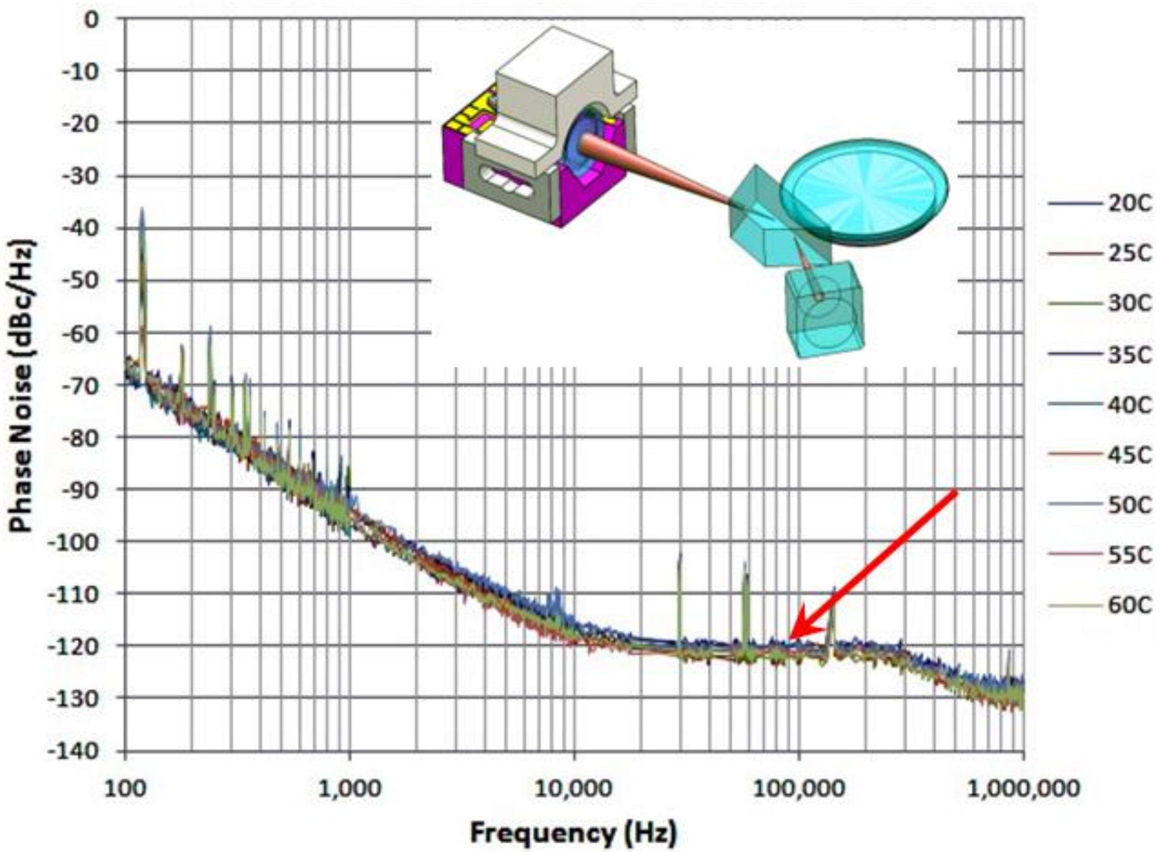
# Advantages of Nonlinear Microcavities

- **High transparency/Low loss materials**
- **Low bending and radiative losses**
  - **Allows for direct generation of high frequency**
  - **Small size**
- **Low vibration sensitivity**
  - **Small size**
- **Superior phase noise owing to high-Q**
- **Relatively simple design**
- **Division of the optical frequency noise**



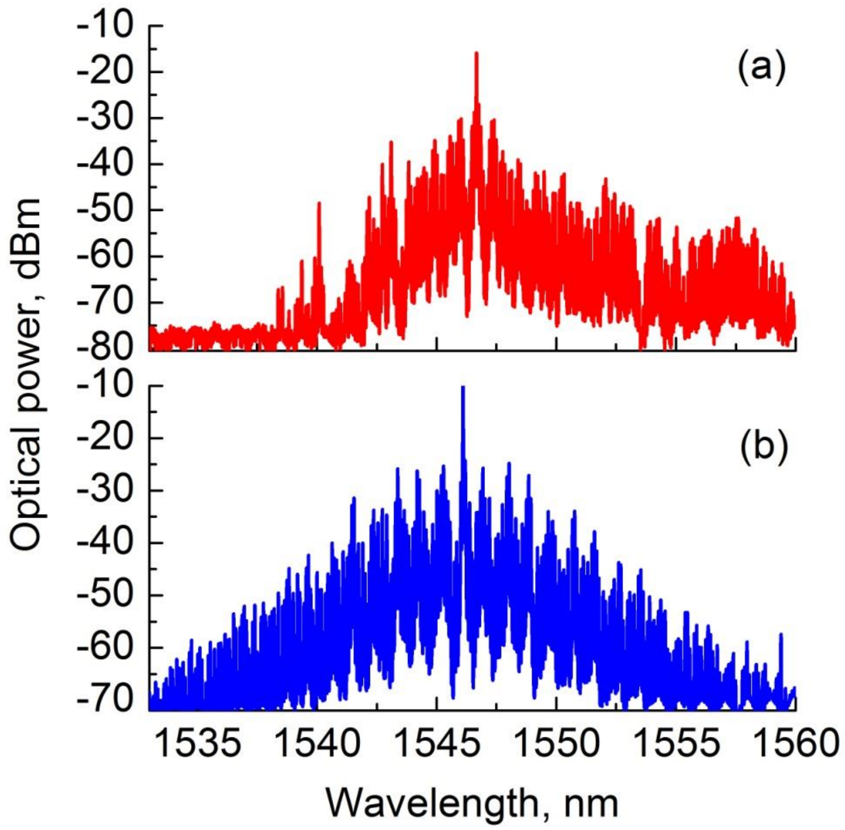

The phase noise measurement is limited by performance of the phase noise measurement setup.

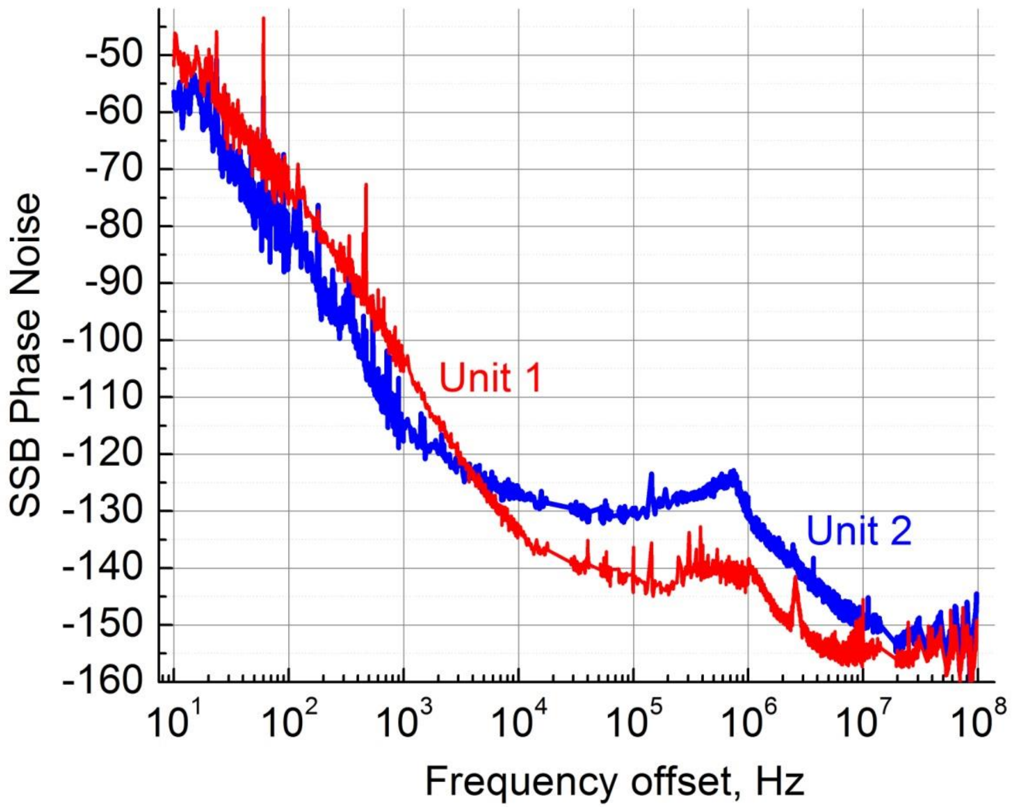
*D Seidel et al., "Minituarized Ka-band Photonic Oscillators", 2018 International Topical Meeting on Microwave Photonics*




Phase noise of the 28 GHz signal generated by the packaged photonic oscillator. Inset represents Laser, Resonator and Coupling optics layout. 28 GHz resonator ~ 2.5 mm in diameter. Non-soliton regime is utilized.

# Performance Optimization




Unit 1 produces irregular combs. Unit 2 produces symmetric multi-soliton combs.
RF phase noise is comparable -> solitons are unnecessary for good RF performance.

A. Matsko, et al. "Turn-key operation and stabilization of Kerr frequency combs" *2016 IEEE International Frequency Control Symposium (IFCS).*





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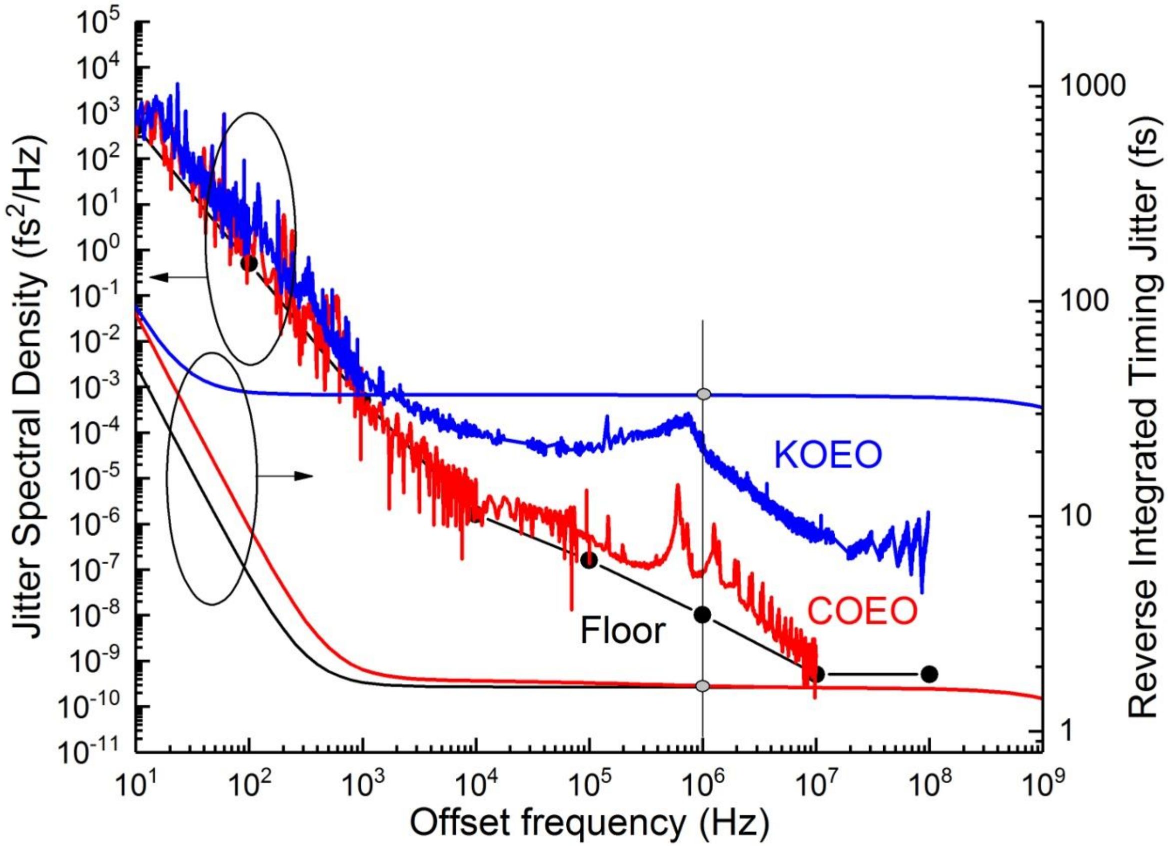
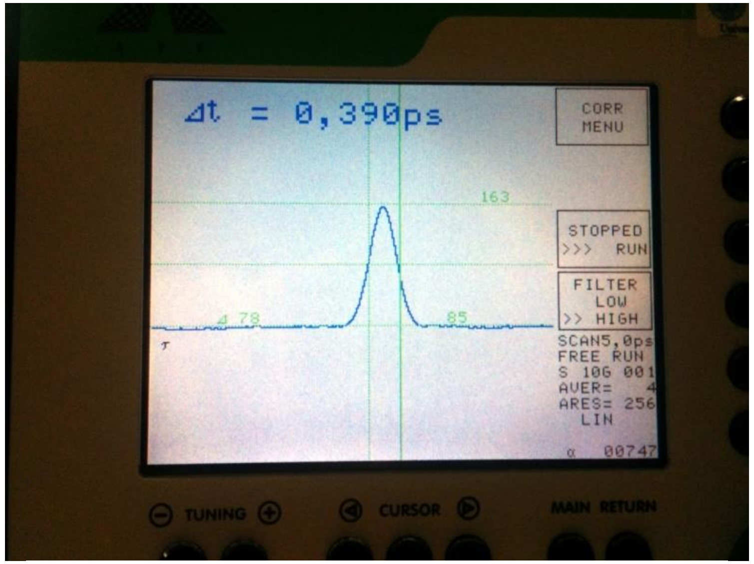
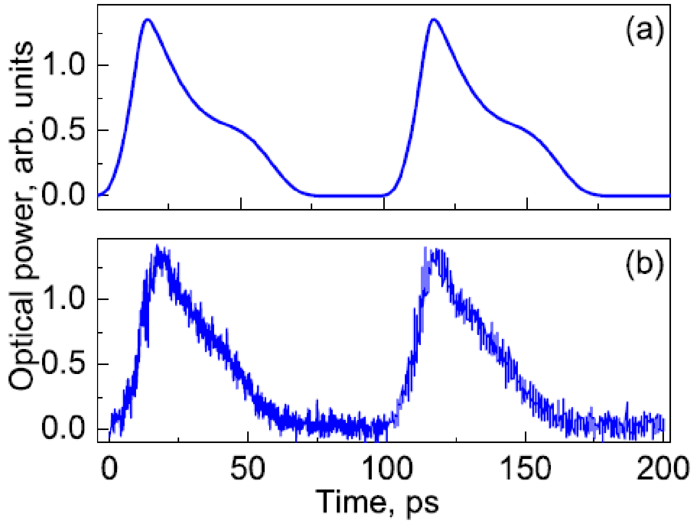
# Timing Jitter




Photonic oscillators are also ideal for 5G and 6G applications

KOEO – Kerr comb oscillator

COEO– Coupled Opto-Electronic Oscillator

W. Liang et al., Nature Commun. **6**, Article no: 7957 (2015)



Was supported by DARPA

# Problem to solve

- Need to reach AD~$10^{-13}$ at 1s

## Contents

- Background on low loss systems and (some) of their uses at JPL
  - JPL's Frequency and Timing Advanced Instrument Development group: main activities and interests
- Early studies of dissipative solitons and parametric instabilities in low loss photonic systems
  - Low loss resonant (micro)systems
  - Dissipative solitons in high-Q systems
  - Use of dissipative solitons (and not only the solitons) in clocks and oscillators
- Recent developments
  - Fundamental limitations of spectral purity of photonic oscillators: theory and experiments
  - Photonic time crystals as ideal frequency dividers and their use in clocks
- Outlook

## Fundamental limitations of spectral purity of Kerr comb oscillators: theory and experiment

# Analytical Theory of the Mode Locked Comb

Kerr frequency combs can be analyzed using Lugiato-Lefever (LL) equation accompanied by the standard high finesse cavity input-output equation:

$$T_R \frac{\partial A}{\partial T} + \frac{i}{2}\beta_{2\Sigma}\frac{\partial^2 A}{\partial t^2} - i\gamma_\Sigma |A|^2 A = -\left(\alpha_\Sigma + \frac{T_c}{2} + i\delta_0\right)A + i\sqrt{T_c P_{in}}e^{i\varphi_{in}},$$

$$A_{out} = \sqrt{P_{in}}e^{i\varphi_{in}} + i\sqrt{T_c}A;$$

or by two input-output equations, in the case of two optical couplers:

$$T_R \frac{\partial A}{\partial T} + \frac{i}{2}\beta_{2\Sigma}\frac{\partial^2 A}{\partial t^2} - i\gamma_\Sigma |A|^2 A = -\left(\alpha_\Sigma + \frac{T_{c1}+T_{c2}}{2} + i\delta_0\right)A + i\sqrt{T_c P_{in}}e^{i\varphi_{in}},$$

$$A_{out1} = \sqrt{P_{in}}e^{i\varphi_{in}} + i\sqrt{T_{c1}}A,$$

$$A_{out2} = i\sqrt{T_{c2}}A;$$

A. B. Matsko et al., “Whispering gallery mode oscillators and optical comb generators,” Proc. of 7th Symp. Frequency Standards and Metrology, ed. L. Maleki, pp. 539–558 (World Scientific, New Jersey, 2009).
A. B. Matsko et al., “Mode-locked Kerr frequency combs,” Opt. Lett. 36, 2845–2847 (2011).

A. B. Matsko and L. Maleki Optics Express **21** (23), 28862 (2013).

# An Analytic Solution

The solution is assumed in an autosoliton form

$$A(T,t) = A_c + A_p(T,t),$$

$$A_c = \sqrt{P_c}\, e^{i\varphi_c},$$

$$A_p(T,t) = \sqrt{\frac{P_p}{2}} \left[\operatorname{sech}\left(\frac{t-\xi}{\tau}\right)\right]^{1+iq} e^{i\Omega(t-\xi)+i\varphi_p}$$

Then the parameters of the solution are found using perturbed Lagrange equations derived from Lagrange operator:

$$L = \int_{-\infty}^{\infty} \left\{ \frac{T_R}{2}\left( A_p^* \frac{\partial A_p}{\partial T} - A_p \frac{\partial A_p^*}{\partial T} \right) - \frac{i}{2}\left( \beta_{2\Sigma} \left| \frac{\partial A_p}{\partial t} \right|^2 + \gamma_\Sigma \left| A_p \right|^4 \right) \right\} dt$$


H. A. Haus and A. Mecozzi, “Noise of mode-locked lasers”, IEEE J. Quantum. Electron. 29, 983–996 (1993).
A. Hasegawa, “Soliton-based optical communications: an overview,” IEEE J. Sel. Top. Quantum Electron. 6, 1161–1172 (2000).








A. B. Matsko and L. Maleki Optics Express **21** (23), 28862 (2013).

# Steady State Formalism

$$\tau^2 = -\frac{2\beta_{2\Sigma}}{\gamma_\Sigma P_p}; \quad P_p = \frac{4\delta_0}{\gamma_\Sigma};$$

$$\sin(\varphi_c - \varphi_p) = -\sqrt{\frac{8\delta_0(\alpha_\Sigma + T_c/2)^2}{\pi^2 T_c \gamma_\Sigma \mathrm{P}_{in}}} \Rightarrow \delta_0 \le \frac{\pi^2 T_c \gamma_\Sigma \mathrm{P}_{in}}{8(\alpha_\Sigma + T_c/2)^2}$$

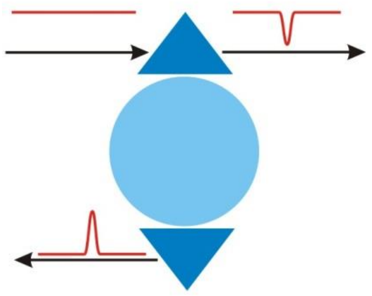
Unless the pump light is filtered out the output pulses are “dark” solitons

$$\frac{P_{out}}{P_{in}} \cong \left[1 - \frac{\pi}{2}\sin^2(\varphi_c - \varphi_p)\operatorname{sec}h\left(\frac{t}{\tau}\right)\right]^2$$

In the case of two couplers

$$\frac{P_{out}}{P_{in}} \cong \left[\frac{\pi}{2}\sin^2(\varphi_c - \varphi_p)\operatorname{sec}h\left(\frac{t}{\tau}\right)\right]^2$$

The peak power of the pulse is comparable with the cw pump power



*A. B. Matsko and L. Maleki, OE **21**, 28862 (2013)*





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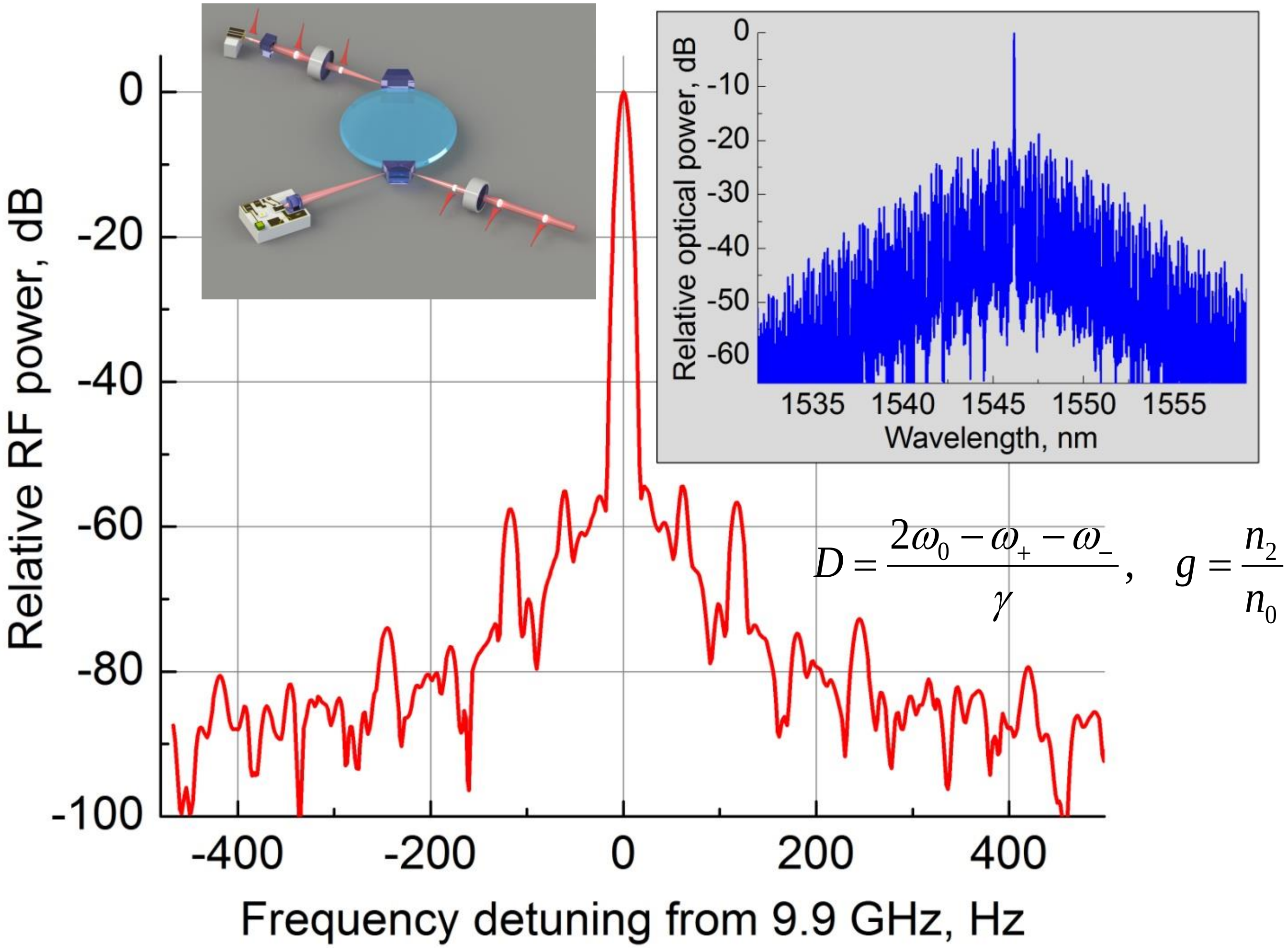
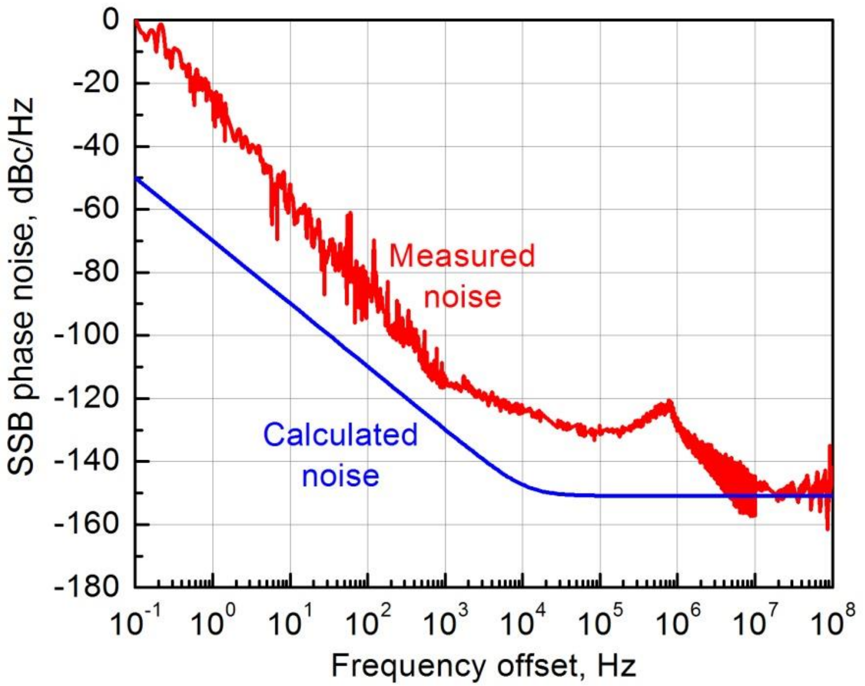
# Phase Noise Limit of Kerr Comb Oscillator




$$D = \frac{2\omega_0 - \omega_+ - \omega_-}{\gamma}, \quad g = \frac{n_2}{n_0}\frac{\hbar\omega_0^2 c}{V n_0}$$




$$L_\varphi \cong \frac{\pi}{\sqrt{2}}\sqrt{\frac{P_{th}}{P_{in}(-D)}}\frac{g}{\gamma^2}\left[1+\frac{1}{24}\left(1+\frac{\pi^2 f^2}{\gamma^2}\right)^{-1}\frac{\gamma^2}{\pi^2 f^2}\frac{P_{in}(-D)}{P_{th}}\right]$$

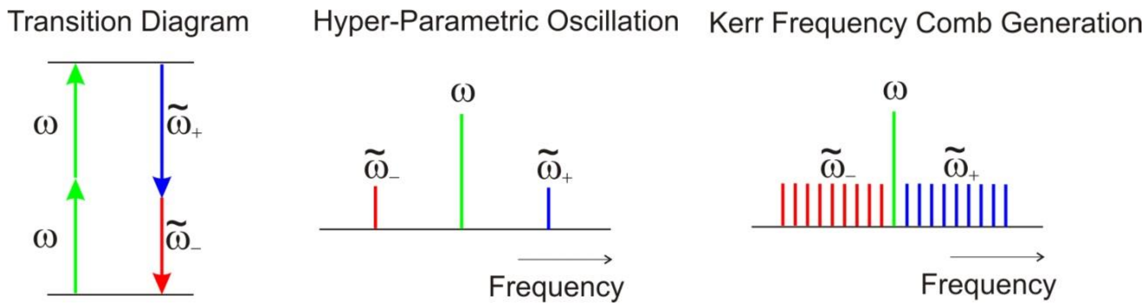
*A. B. Matsko and L. Maleki, OE **21**, 28862 (2013)*

Oscillators are useful for clocks as flywheels



Was supported by DARPA

# Fundamental Stability of the Comb Oscillator Does not Depend on Q




$$\tilde{\omega}_+ + \tilde{\omega}_- = 2\omega \qquad \tilde{\omega}_+ - \tilde{\omega}_- \approx 2\omega_{FSR}$$

Sidebands are always equidistant because of energy and photon number conservation, but the repetition rate of the comb can change.

$$\sigma \cong \frac{g^{1/2}}{\sqrt{2}\omega_{FSR}} \frac{1}{\sqrt{\tau}}$$

$$g = \frac{n_2}{n} \frac{\hbar \omega_0^2 c}{Vn},$$

$$\hat{V} = -\frac{g}{2} (\hat{e}^+)^2 \hat{e}^2,$$

$$\sigma \cong \frac{4(-D)^{1/4} g^{1/2}}{5\omega_{FSR}} \frac{1}{\sqrt{\tau}}$$

Minimum Allan deviation for two sidebands
*A. B. Matsko, et al., Phys. Rev. A* ***71****, 033804 (2005)*

Minimum Alan deviation for fundamental soliton Kerr comb
*A. B. Matsko and L. Maleki, OE* ***21****, 28862 (2013)*

$$\hat{e} = \sum_N \hat{a}_j$$

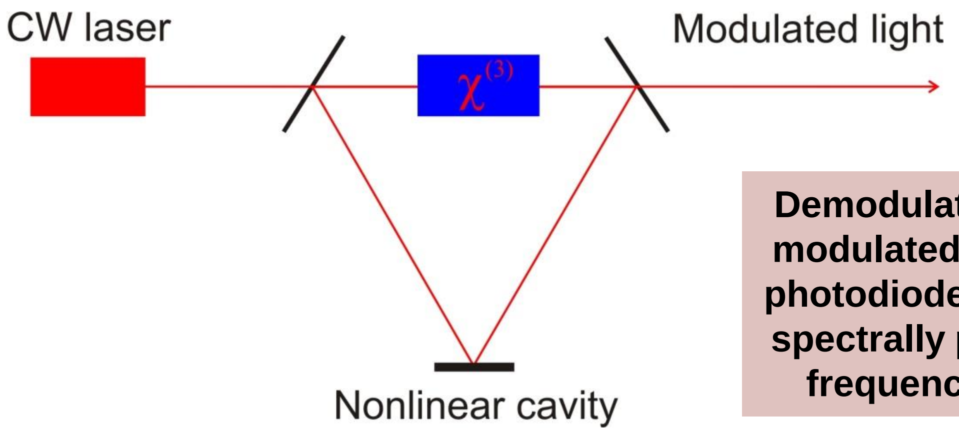
$$\frac{d}{dt}\hat{a}_j = -(\gamma + i\omega_j)\hat{a}_j + \frac{i}{\hbar}[\hat{V}, \hat{a}_j] + Fe^{-i\omega t}\delta_{(N+1)/2,j}$$

# Size and Power Dependence

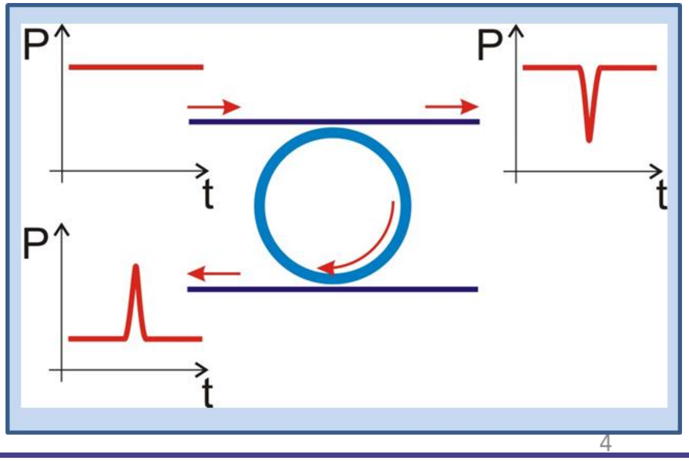
**For the same oscillation frequency the better performance has an oscillator with larger mode volume and smaller nonlinearity**

$$\sigma \cong \frac{g^{1/2}}{\sqrt{2}\omega_{FSR}} \frac{1}{\sqrt{\tau}} \qquad g = \frac{n_2}{n} \frac{\hbar \omega_0^2 c}{Vn},$$

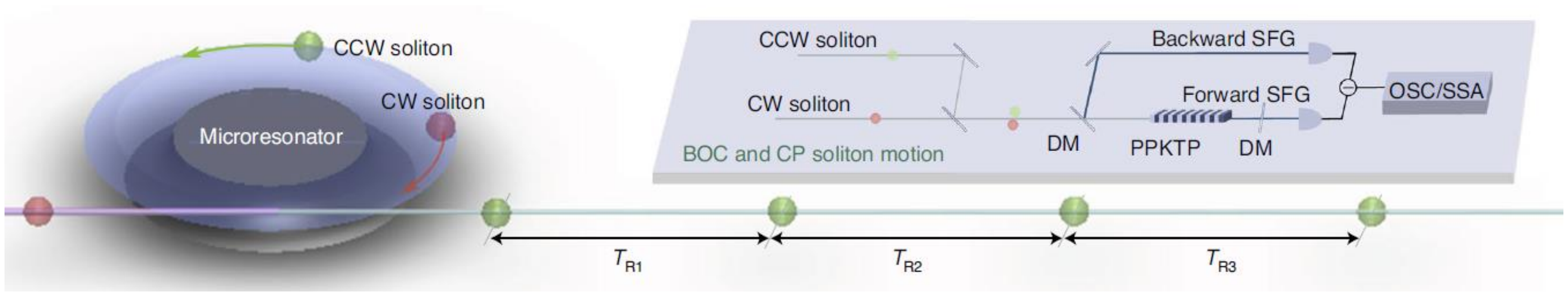
**The time averaged input power is equal to time averaged output power. The energy is conserved. No attenuation is present, in the global sense. The signal is generated at the photodiode that performs the measurement.**




**Demodulation of the modulated light on a photodiode produces spectrally pure radio frequency signal**



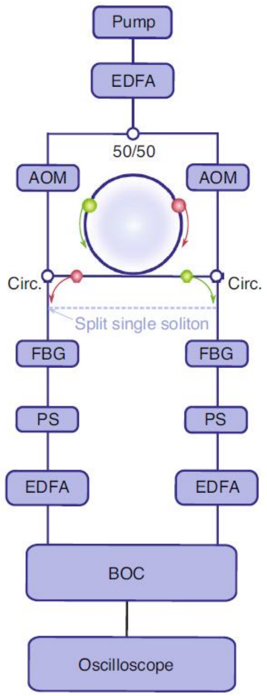
# Fundamental Limitations Verified Experimentally

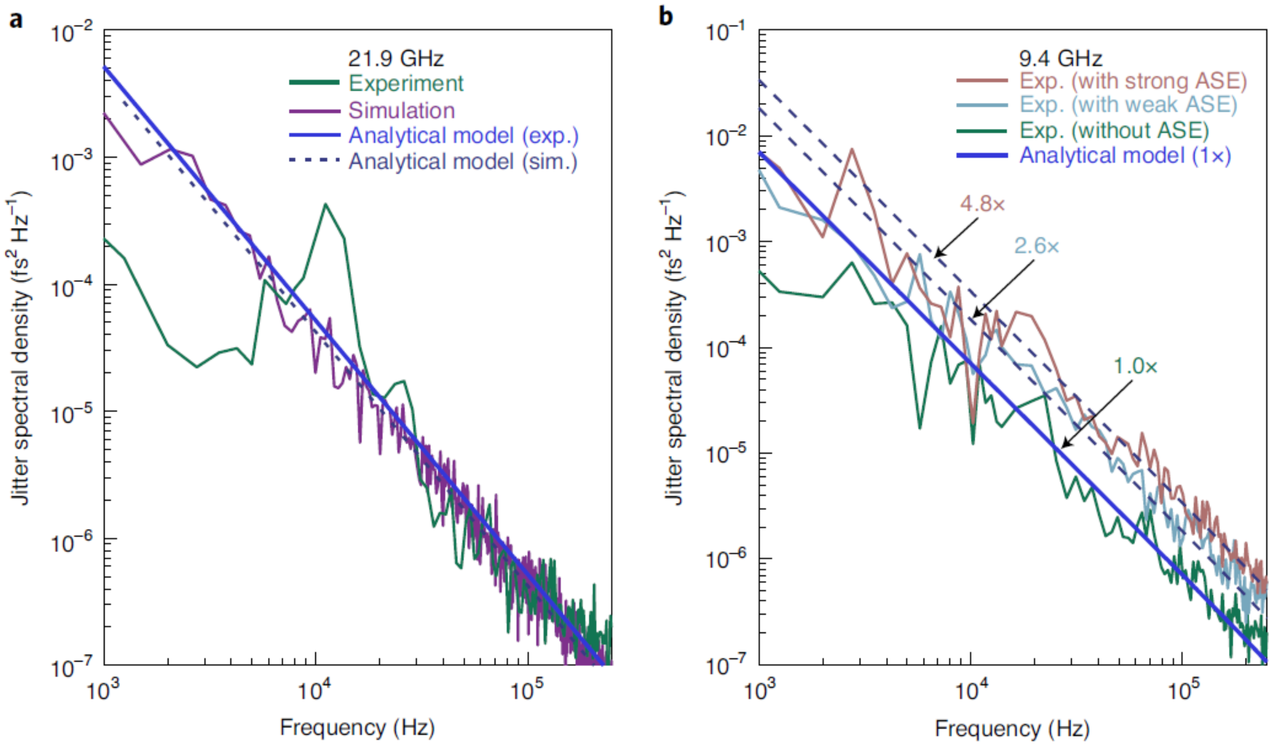



nature physics LETTERS
https://doi.org/10.1038/s41567-020-01152-5

## Quantum diffusion of microcavity solitons

Chengying Bao[1], Myoung-Gyun Suh[1,6], Boqiang Shen[1], Kemal Şafak[2], Anan Dai[2], Heming Wang[1], Lue Wu[1], Zhiquan Yuan[1], Qi-Fan Yang[1], Andrey B. Matsko[3], Franz X. Kärtner[4,5] and Kerry J. Vahala[1]

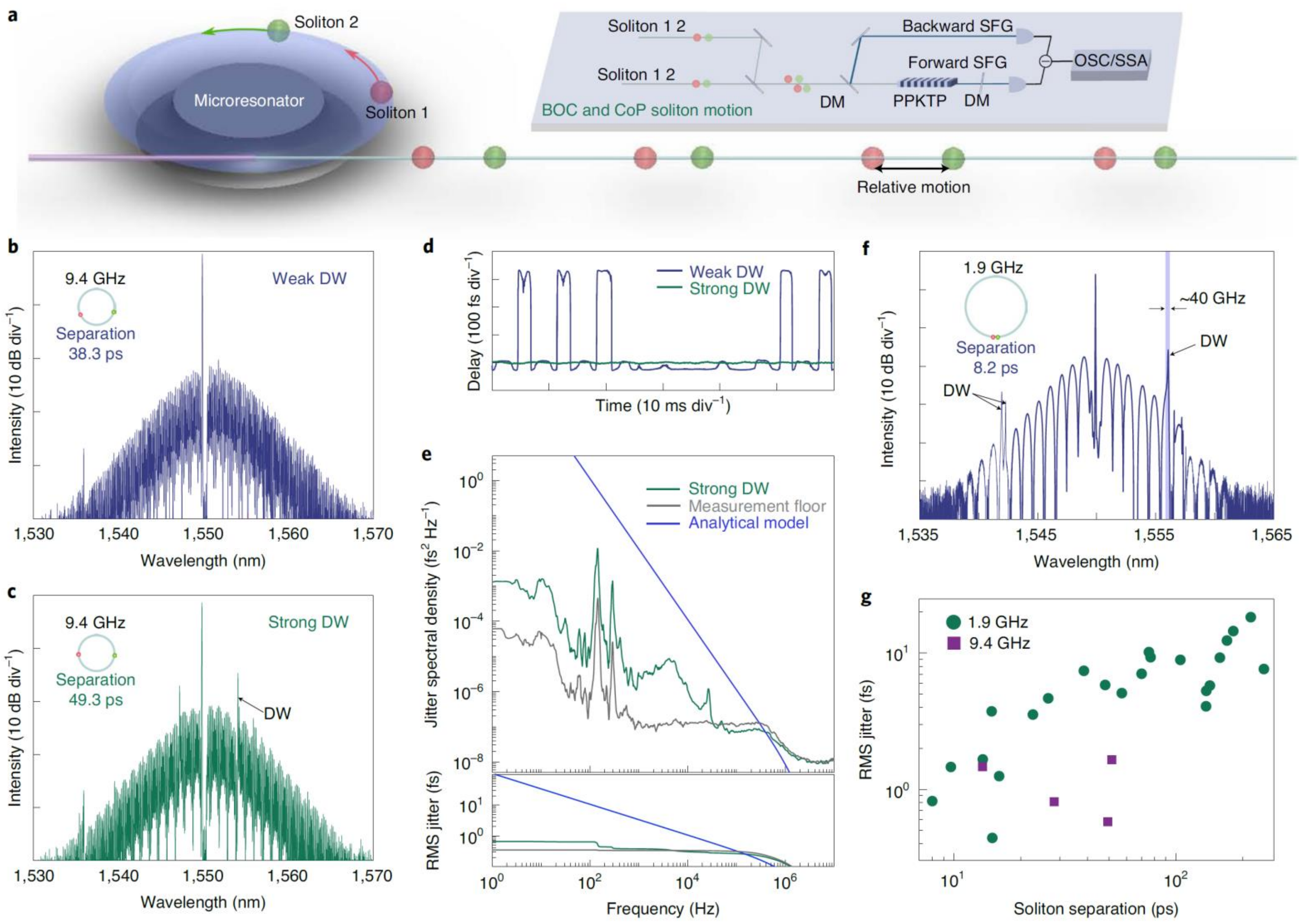




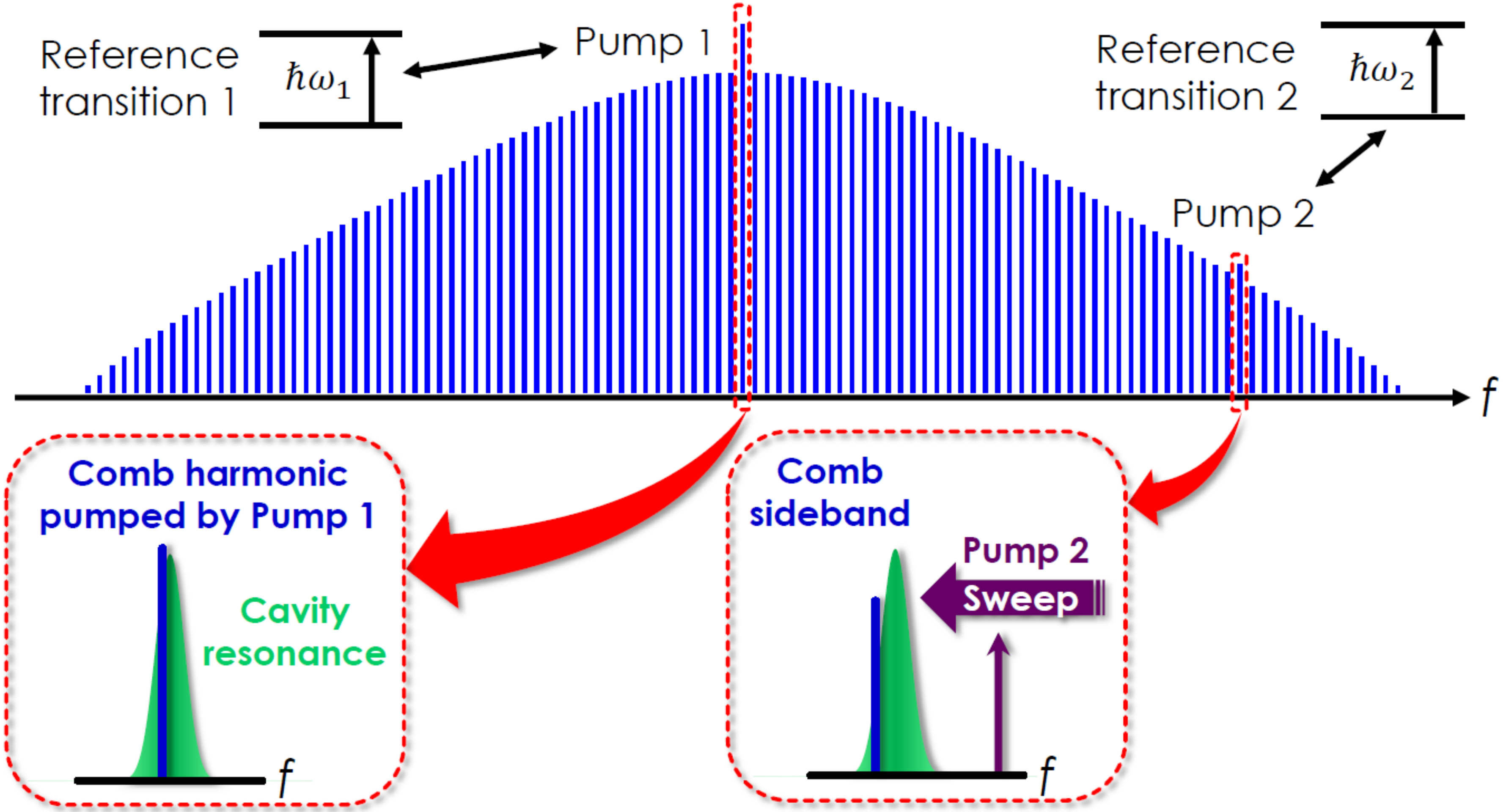

In collaboration with Caltech, supported by JPL/NASA

# Timing Jitter Correlation for Two Pulses in the Same Cavity



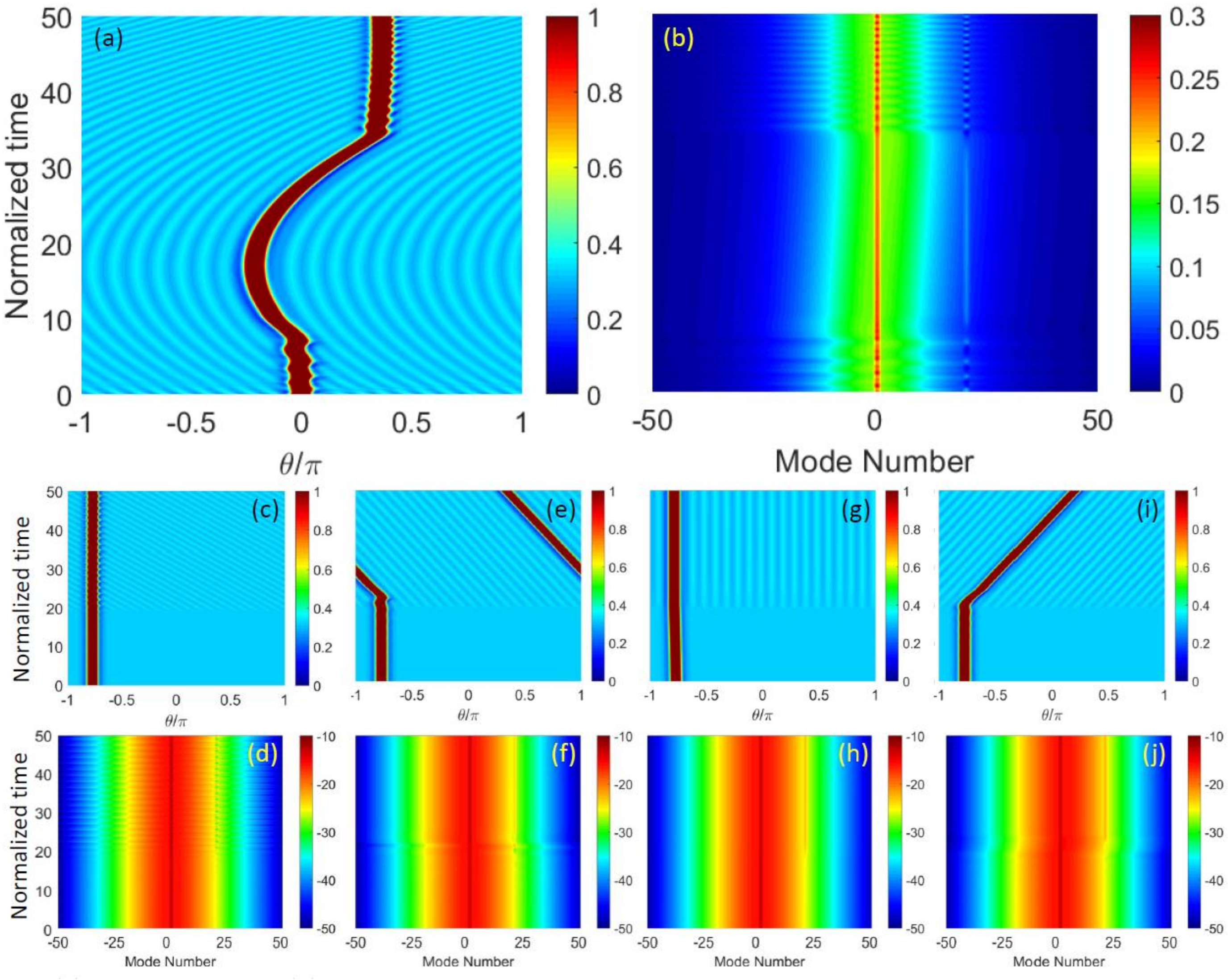
# Time crystals as ideal frequency dividers

# Second coherent pump for the soliton stabilization

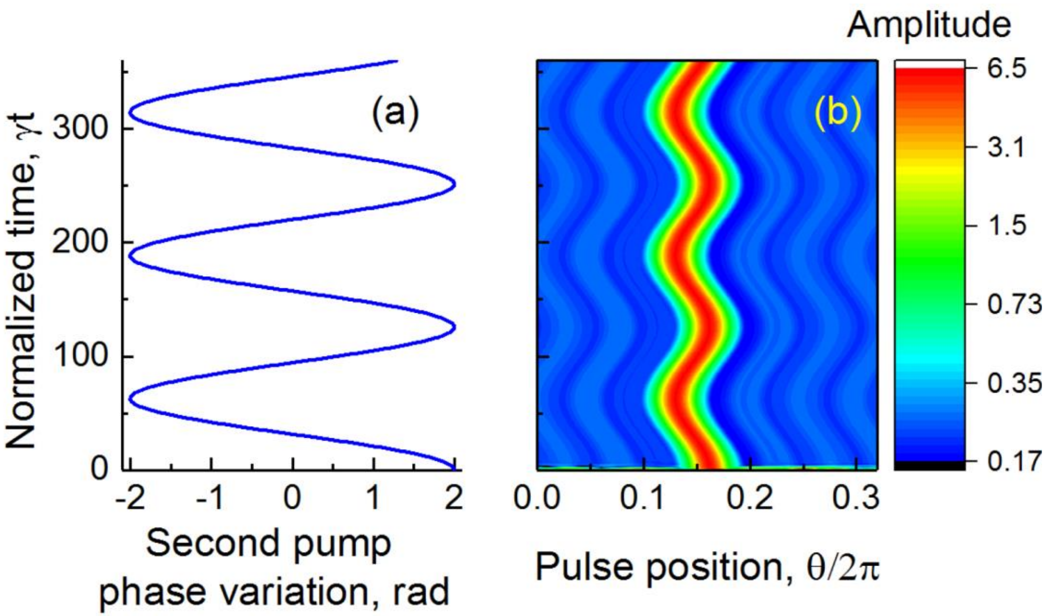



*H. Taheri et al., Optical lattice trap for Kerr solitons. The European Physical Journal D. 2017*

# A trap for solitons



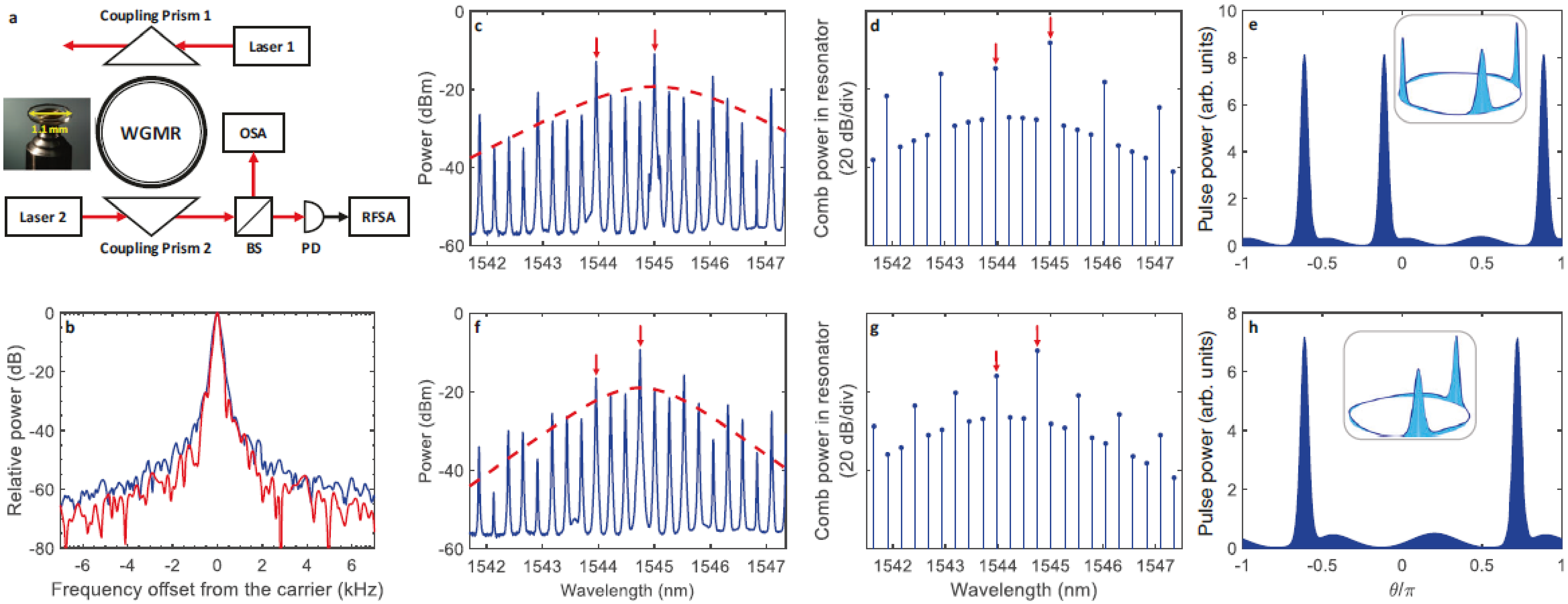

We manipulate by the pulse by tuning the second pump




*H. Taheri et al., Optical lattice trap for Kerr solitons. The European Physical Journal D. 2017*

# Discrete Time Crystals

**Time symmetry breaking, similar to a high order parametric oscillator. This is not a soliton trap!**

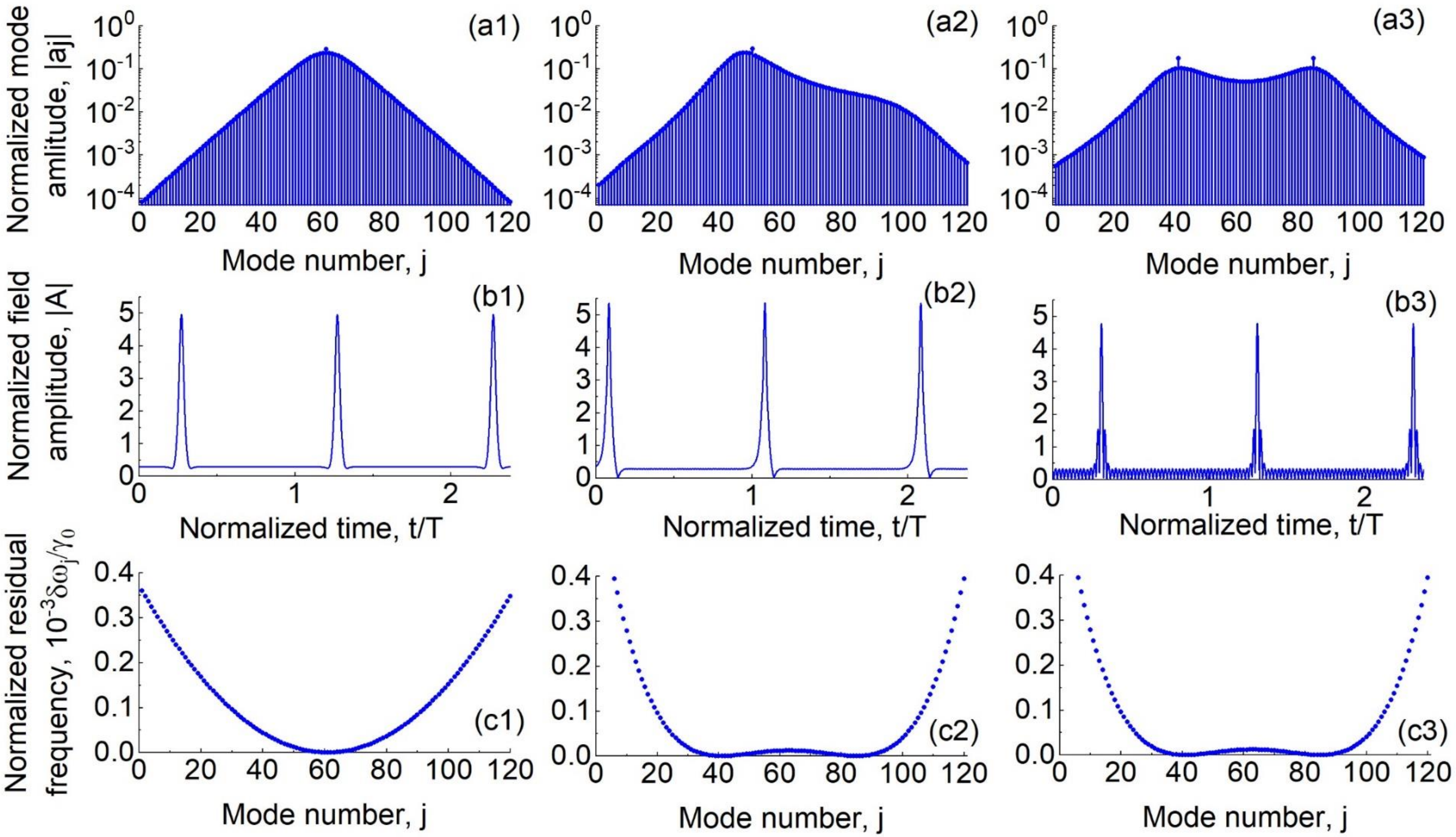



**The phase noise of the laser beat note is divided!**

H. Taheri at al., Nature Communications 13 (1), 848 (2022).

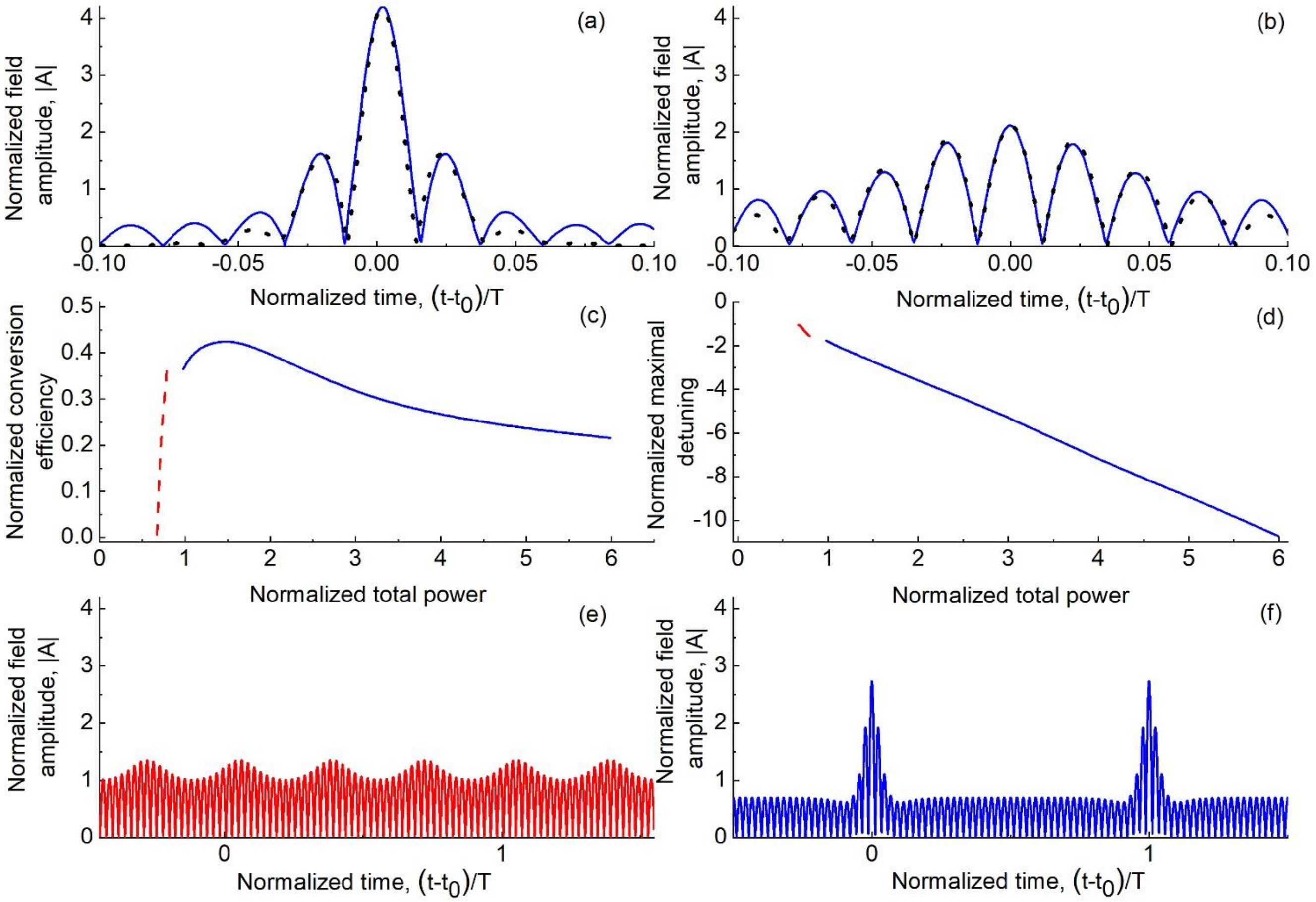
# Dichromatically Pumped Kerr Solitons (DTC-like)

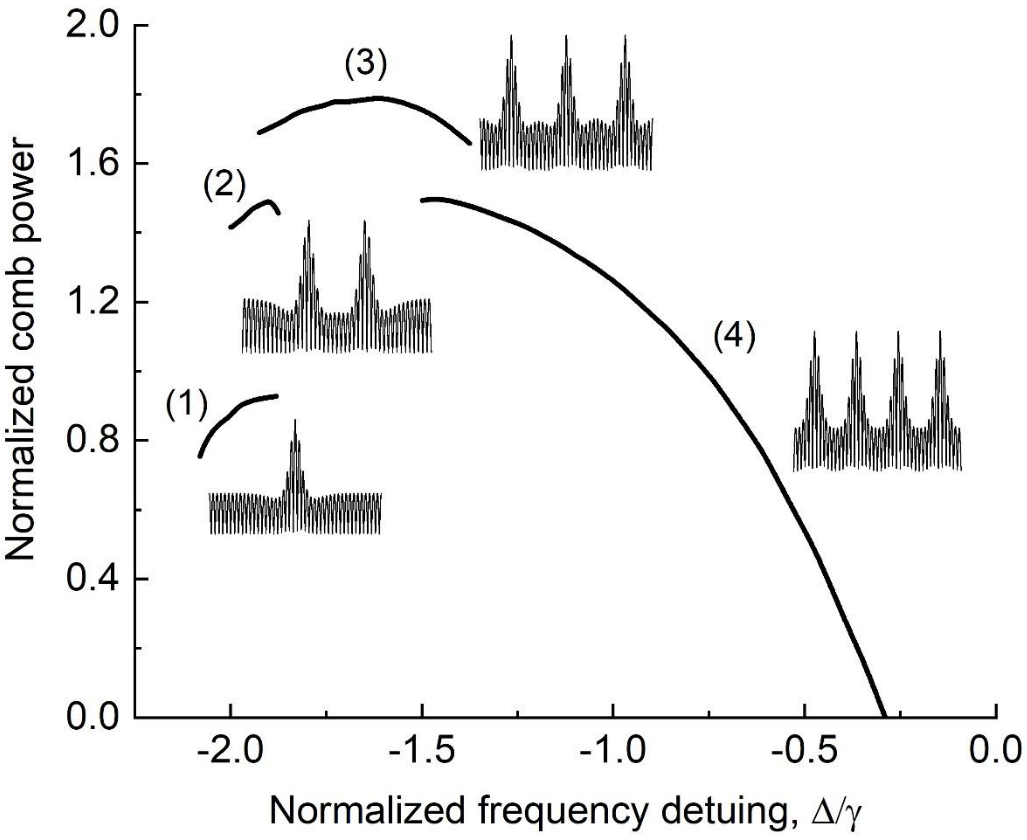
The main idea is to match the group velocity of the beat note of the two pumps and group velocity of the dissipative soliton




A. B. Matsko and L. Maleki, Optics Letters 48, 715 (2023)

# Time Symmetry Breaking in Dual Pumped Systems




A. B. Matsko and L. Maleki, Optics Letters 48, 715 (2023)

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