A computable Hydrogen optical Lattice Clock

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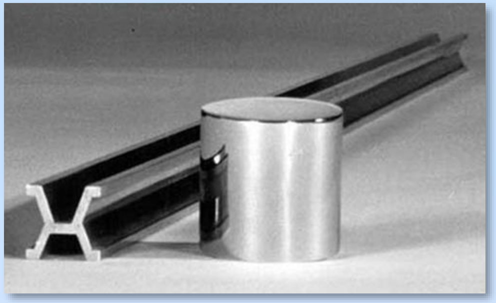
Units

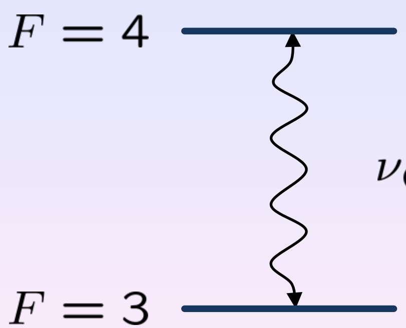
There are three ways to define units:

1) Based on an artifact, like the Kings foot or pieces of platinum-iridium



Artifact may get lost or damaged

2) Based on a natural occurring object, like the cesium atom



$\nu_{\mathrm{Cs}} \equiv 9\,192\,631\,770$ Hz

Fixing the dimensional Constants

3) By fixing the values of physical constants

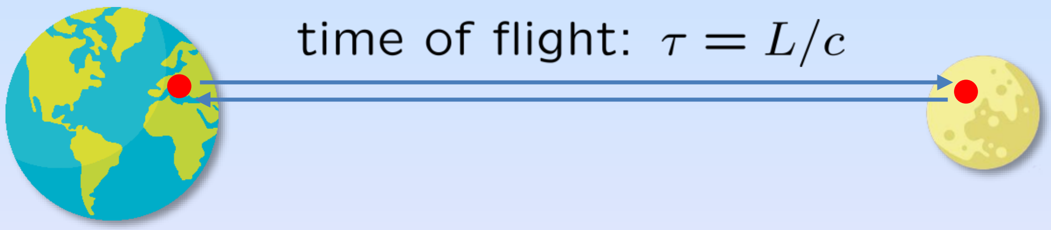
- Seperates the definition from the realization!
- Realization can be adapted to the scale.
- No longer need to improve defintiions, only the realization.
- Can define a system of units without reference to an existing object.

Example #1: Speed of Light

Definition (1983): $c \equiv 299\,792\,458$ m/s

a) Realization of the meter (large distance):

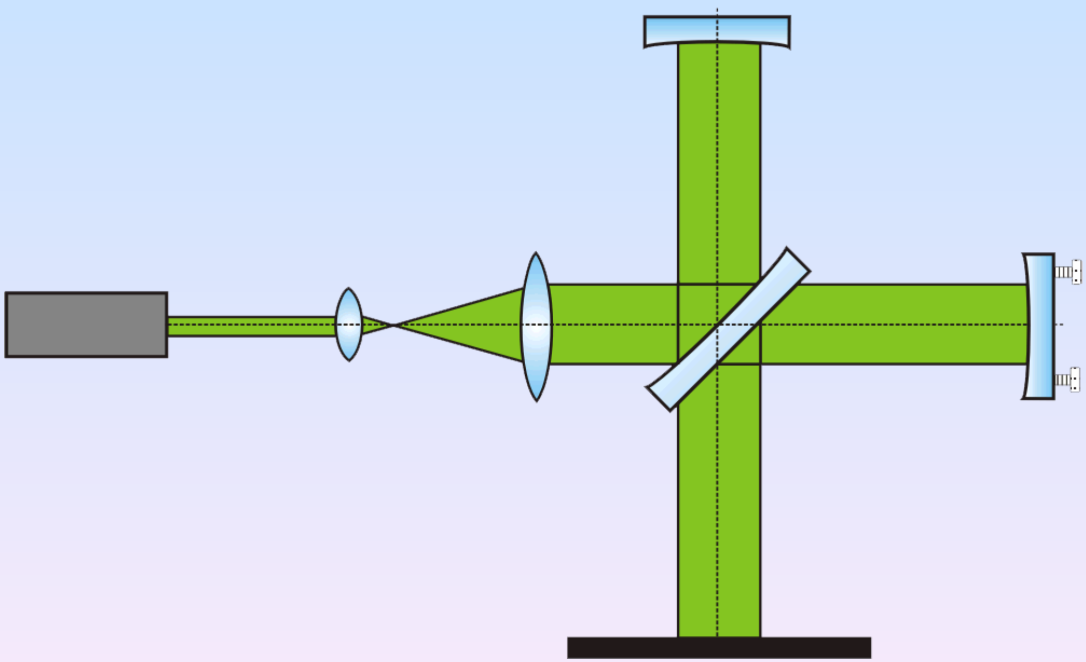
time of flight: $\tau = L/c$



Example #1: Speed of Light

Definition (1983): $c \equiv 299\,792\,458$ m/s

b) Realization of the meter (short distance):



path length: $\lambda = c/\nu$

Example #2: Planck's Constant

Definition (2019): $h \equiv 6.626\,070\,15$ Js

a) Realization of the kg (microscopic):

- mass of the "Cs-photon" :

$$m_{Cs} = \frac{h\nu_{\mathrm{Cs}}}{c^2} = 2.0317730264961724120491383 40\ldots \times 10^{-32}\ \mathrm{kg}$$

Example #2: Planck's Constant

Definition (2019): $h \equiv 6.626\,070\,15$ Js

b) Realization of the kg (macroscopic):

- the kilogram:

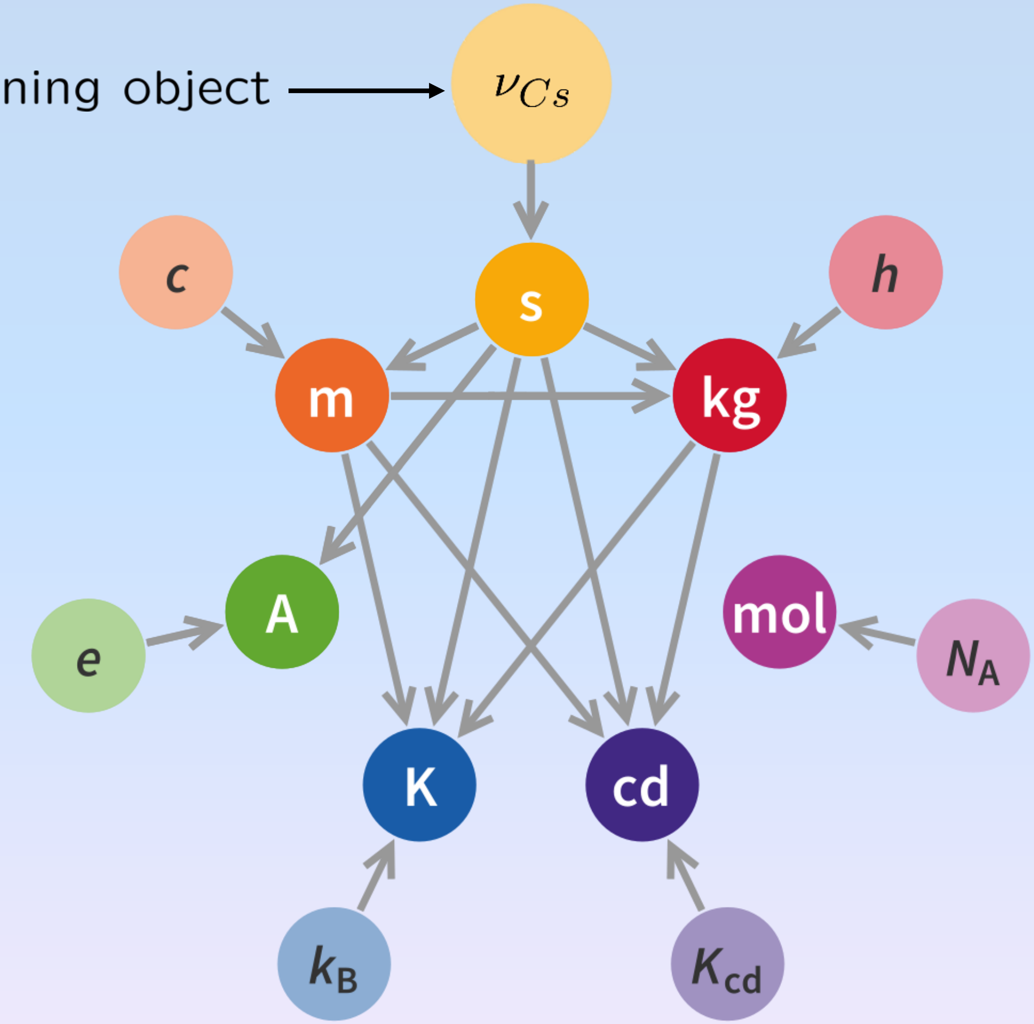
$$1 \text{ kg} = 49218096064820647224721909235 69 \text{ Cs photons}$$

➡ not very practical!

➡ current practical realizations by Watt-Balance or silicon sphere

Everything tied to the Second

the last remaining object →



will go away some day...

Energy Levels of atomic Hydrogen

(only S-states)

Bohr & Schrödinger & Dirac & recoil & relativistic recoil

$$E = \left(-\frac{1}{n^2} - \frac{4n-3}{4n^2}\alpha^2 - \frac{2n^3+6n^2-12n+5}{8n^6}\alpha^4 \ldots \right) \frac{1}{1+m_e/m_p}$$

$$+ \left(\frac{1}{n^4}\alpha^2 - \frac{4n-3}{8n^6}\alpha^4 + \frac{8n^3+40n^2-72n+29}{64n^6}\alpha^6 \ldots \right) \frac{m_e}{m_p}\frac{1}{(1+m_e/m_p)^3}$$

$$+ \frac{2\alpha^3 m_e}{\pi n^3 m_p (1+m_e/m_p)^3} \left(-\frac{2}{3}\ln(\alpha) - \frac{8}{3}\ln k_0(n) - \frac{1}{9} + \frac{14}{3}\left(\ln(\frac{2}{n}) + \sum_{m=1}^{n}\frac{1}{m} \right.\right.$$

$$\left.\left. +1 - \frac{1}{2n} \right) - \frac{2}{1-(m_e/m_p)^2}\ln(\frac{m_e}{m_p}+1) + \frac{2}{1-(m_p/m_e)^2}\ln(\frac{m_p}{m_e}+1) \right)$$

$$+ \frac{2\alpha^4 m_e}{m_p n^3}\left(4\ln(2) - \frac{7}{2} - \frac{44\alpha}{60\pi}\ln(\alpha)^2 \right)$$

Energy Levels of atomic Hydrogen

Full recoil and QED in SI units:

$$E_{nlj} = R_\infty \left(-\frac{1}{n^2} + f_{nlj}\left(\alpha, \frac{m_e}{m_p}, \ldots\right) + \frac{16\pi^2 m_e^2 c^2 \alpha^2}{3n^3h^2}\, r_p^2 \right)$$

➡ any other atomic or molecular transition frequency is written as:

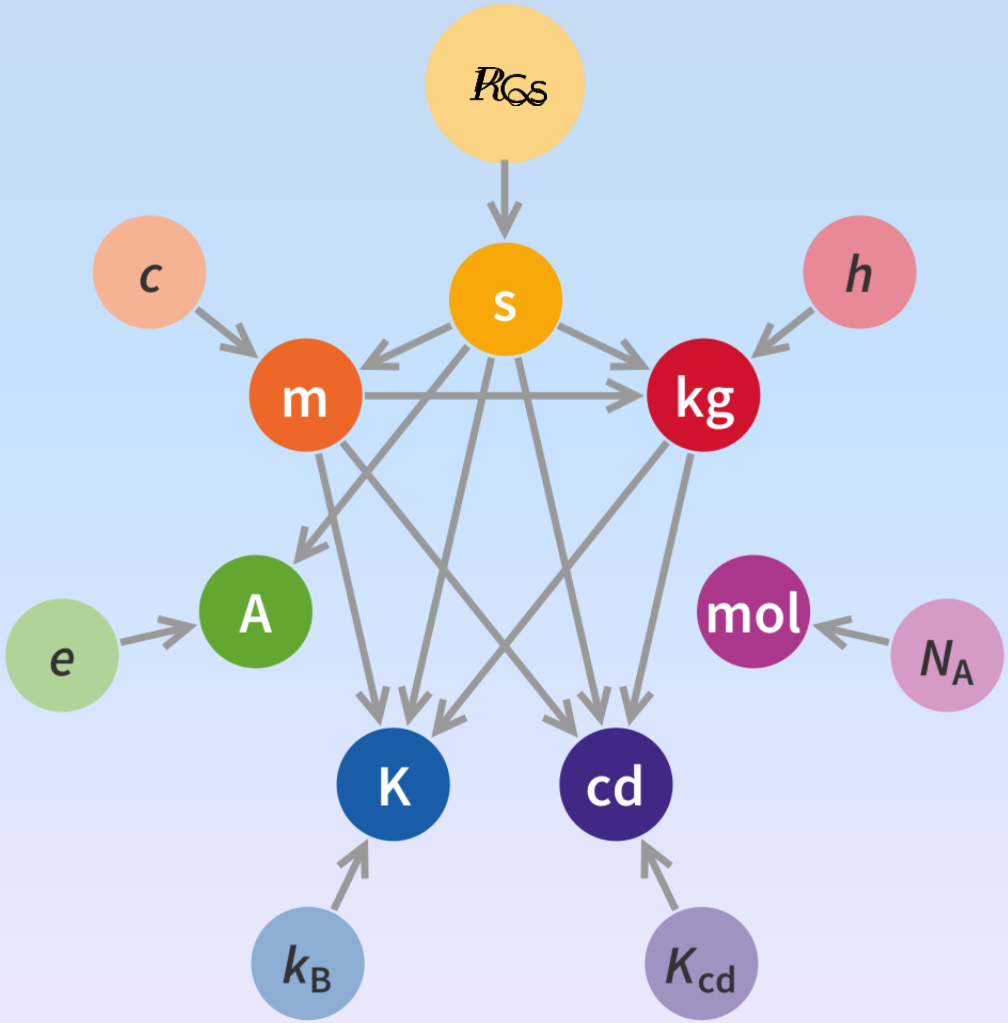
$$\Delta E = R_\infty \times \text{theory}$$

E. Tiesinga *et al.* Rev. Mod. Phys. 93, 025010 (2021) CODATA 2018

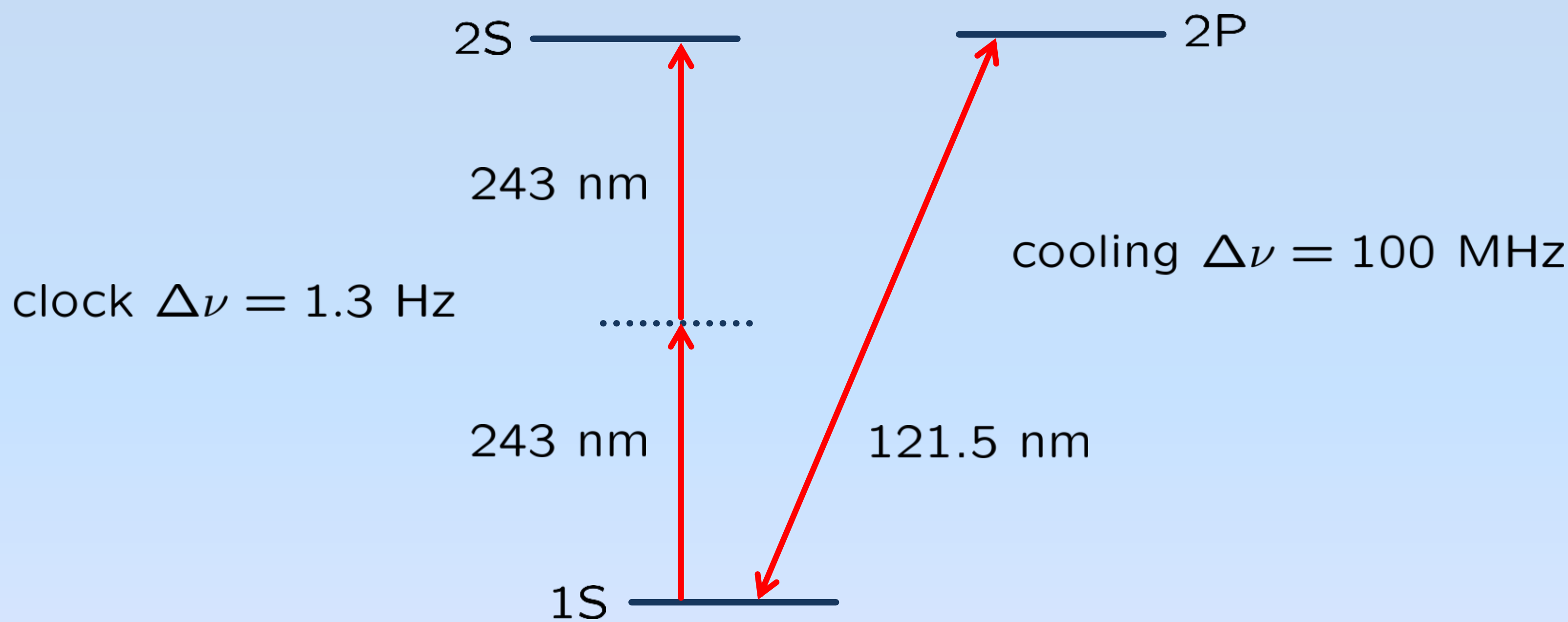
M. Horbatsch and E. A. Hessels, Phys. Rev. A 93, 022513 (2016)

M. Eides *et al.* Theory of Light Hydrogenic Bound States, Springer 2007

Everything tied to the Second

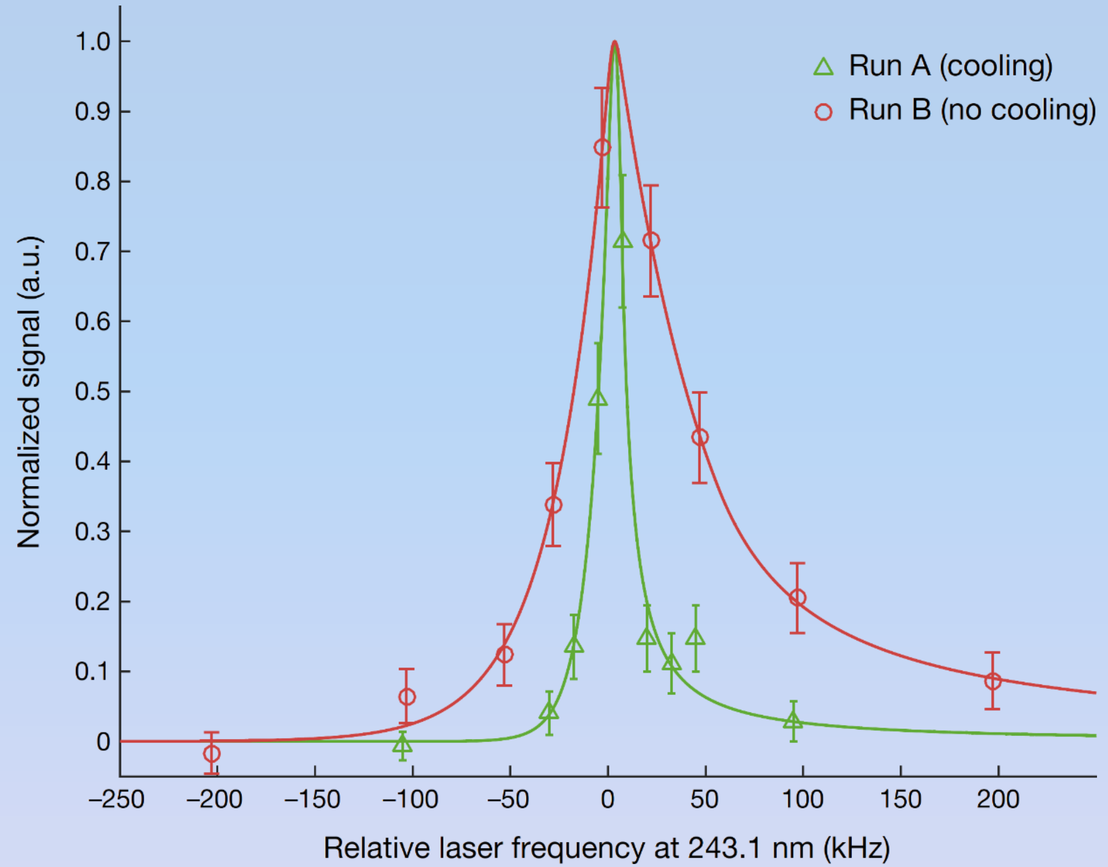


Laser cooling atomic Hydrogen ?



- Large photon recoil: $v_r = 3.26$ m/s
- Doppler limit $h\Delta\nu/k_B = 2.4$ mK
- Recoil limit $(h/\lambda)^2/mk_B = 1.3$ mK
- cw laser at 121.5 nm very difficult

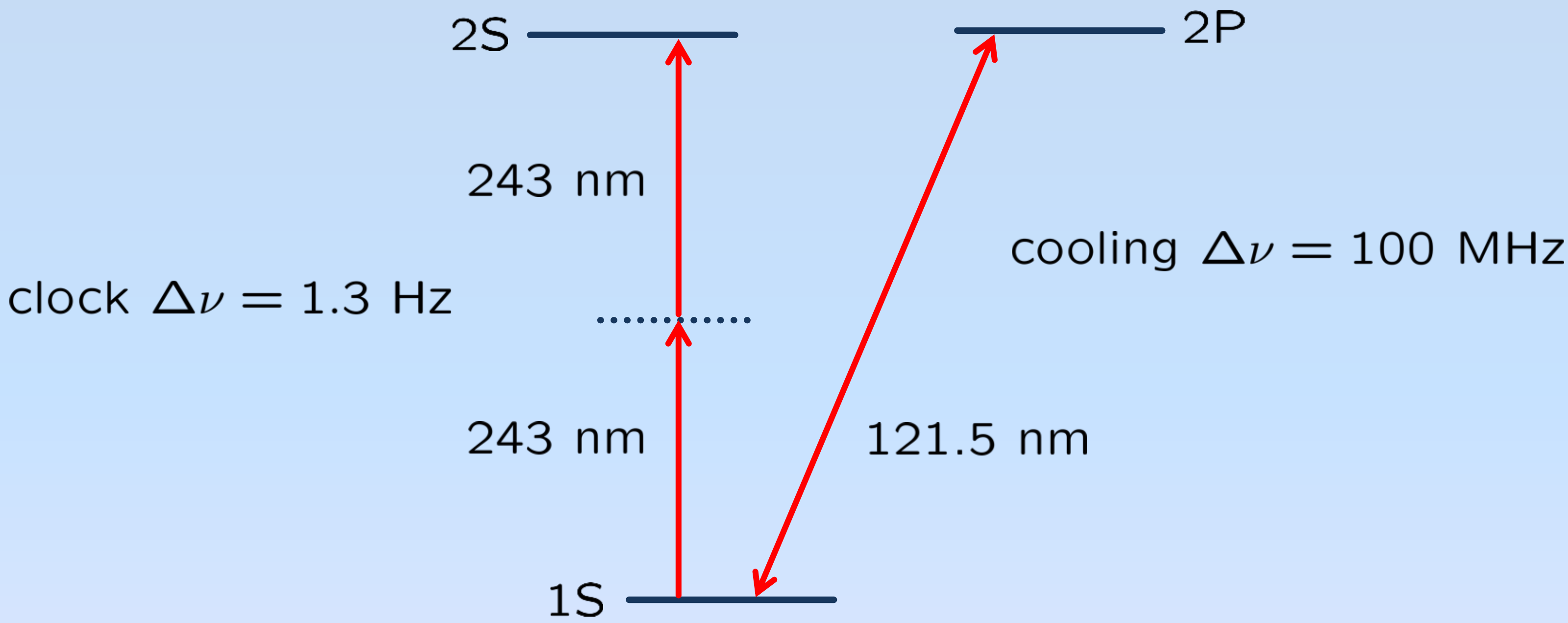
Laser cooling Antihydrogen !



The ALPHA collaboration, Nature 592, 35 (2021)

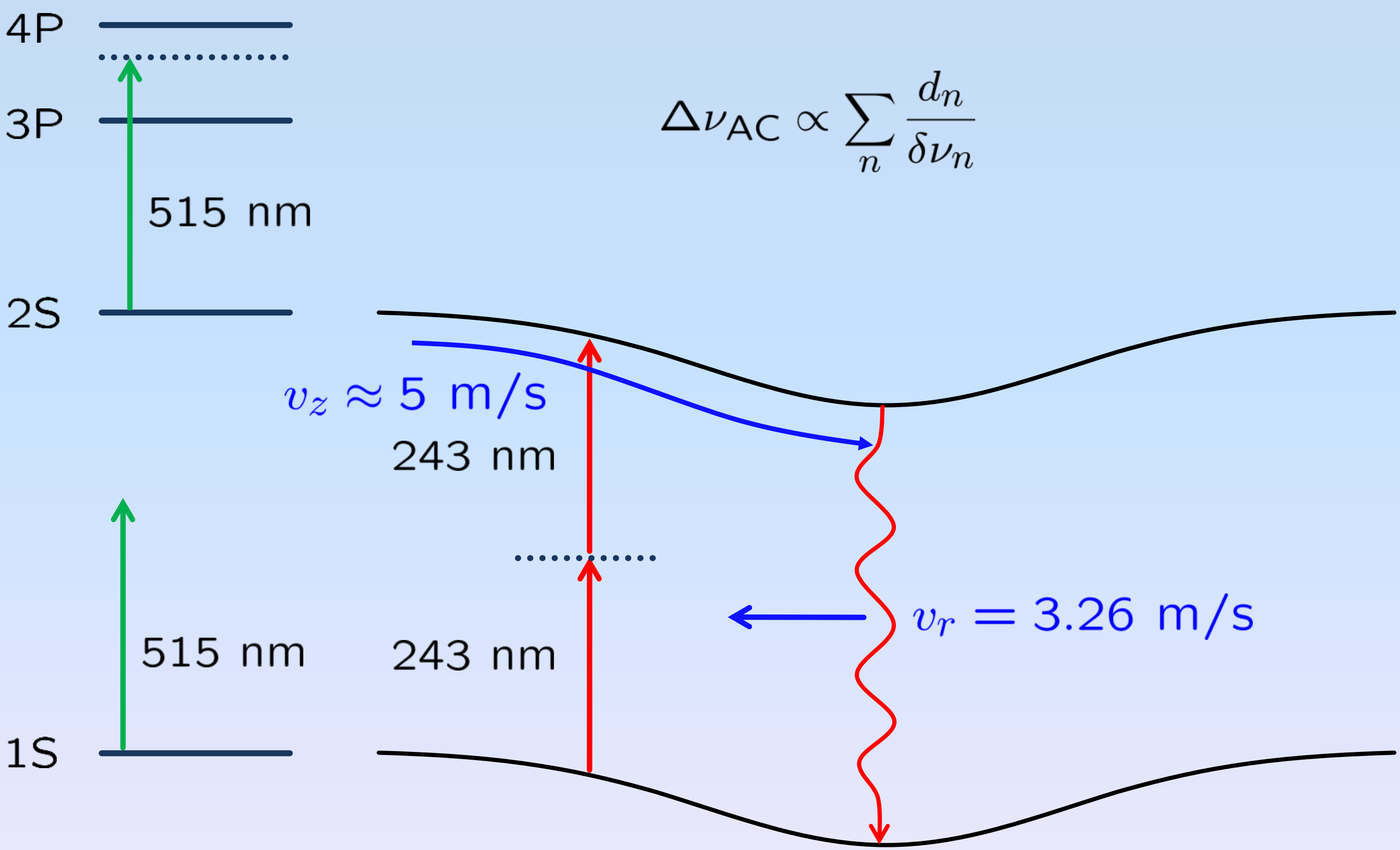
- 6 hours cooling time!
- Magnetic trapping: Zeeman shift by $\sim$18 GHz/Tesla

Laser cooling atomic Hydrogen ?



- Large photon recoil: $v_r = 3.26$ m/s
- Doppler limit $h\Delta\nu/k_B = 4.8$ mK
  Doppler free two-photon transition: don't need to be very cold!
- Recoil limit $(h/\lambda)^2/2mk_B = 0.64$ mK
- cw laser at 121.5 nm very difficult

Optical Dipole Trap

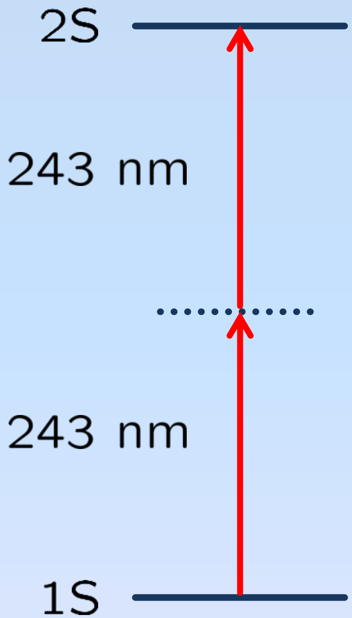


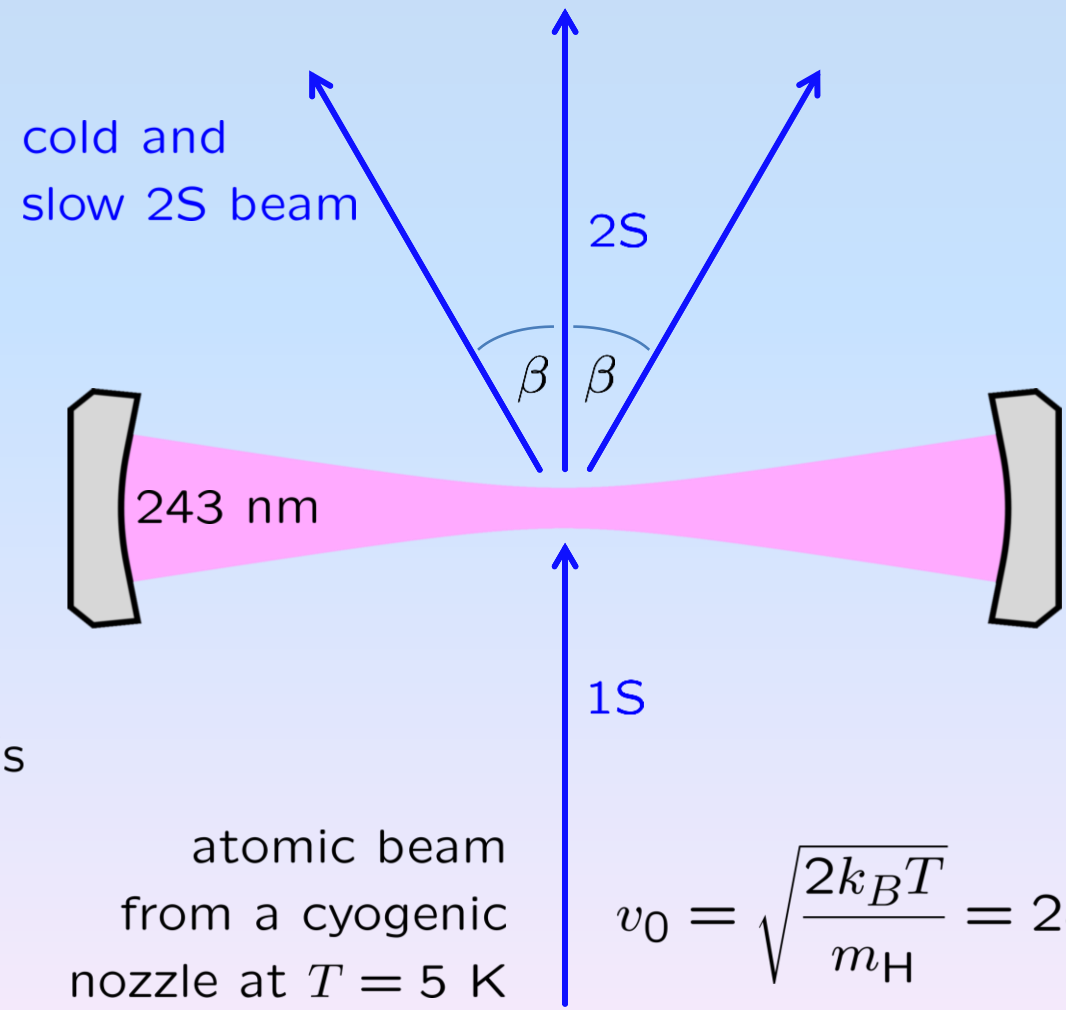
$$\Delta\nu_{AC} \propto \sum_n \frac{d_n}{\delta\nu_n}$$

- atoms recoil only inside the trap
- 515 nm easy (SHG Ytterbium fiber laser)

Velocity Selection

$$\beta = \arctan(v_r/v_z) = 33^\circ \quad \text{for } v_z = 5 \text{ m/s}$$

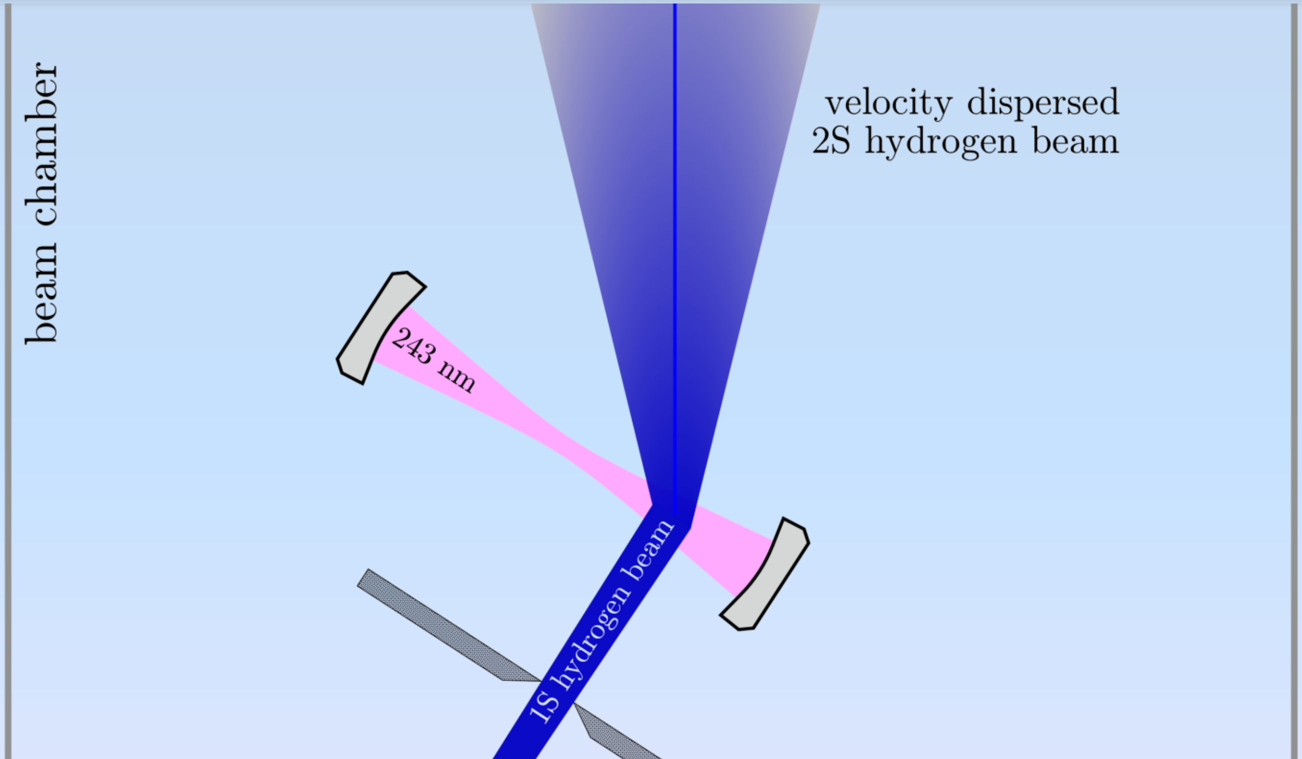




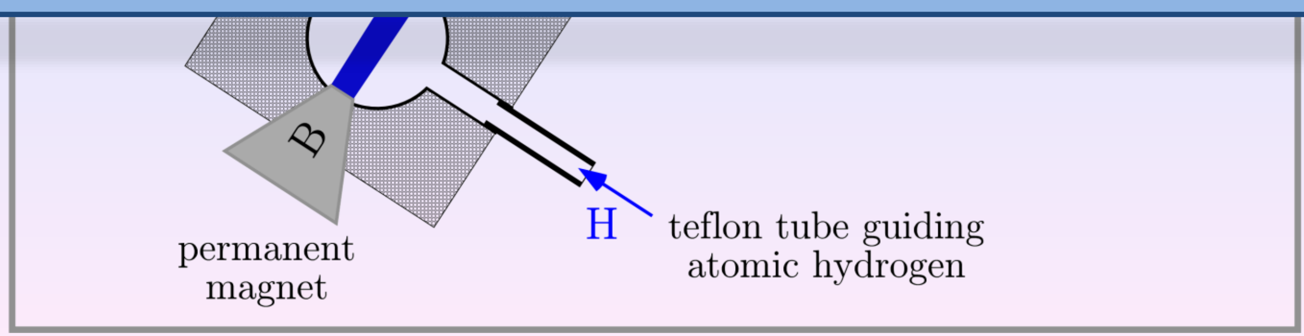
$$v_r = \frac{2\hbar k}{m_{\mathrm{H}}} = 3.26 \text{ m/s}$$

$$v_0 = \sqrt{\frac{2k_B T}{m_{\mathrm{H}}}} = 287 \text{ m/s}$$

Trapping Atomic Hydrogen

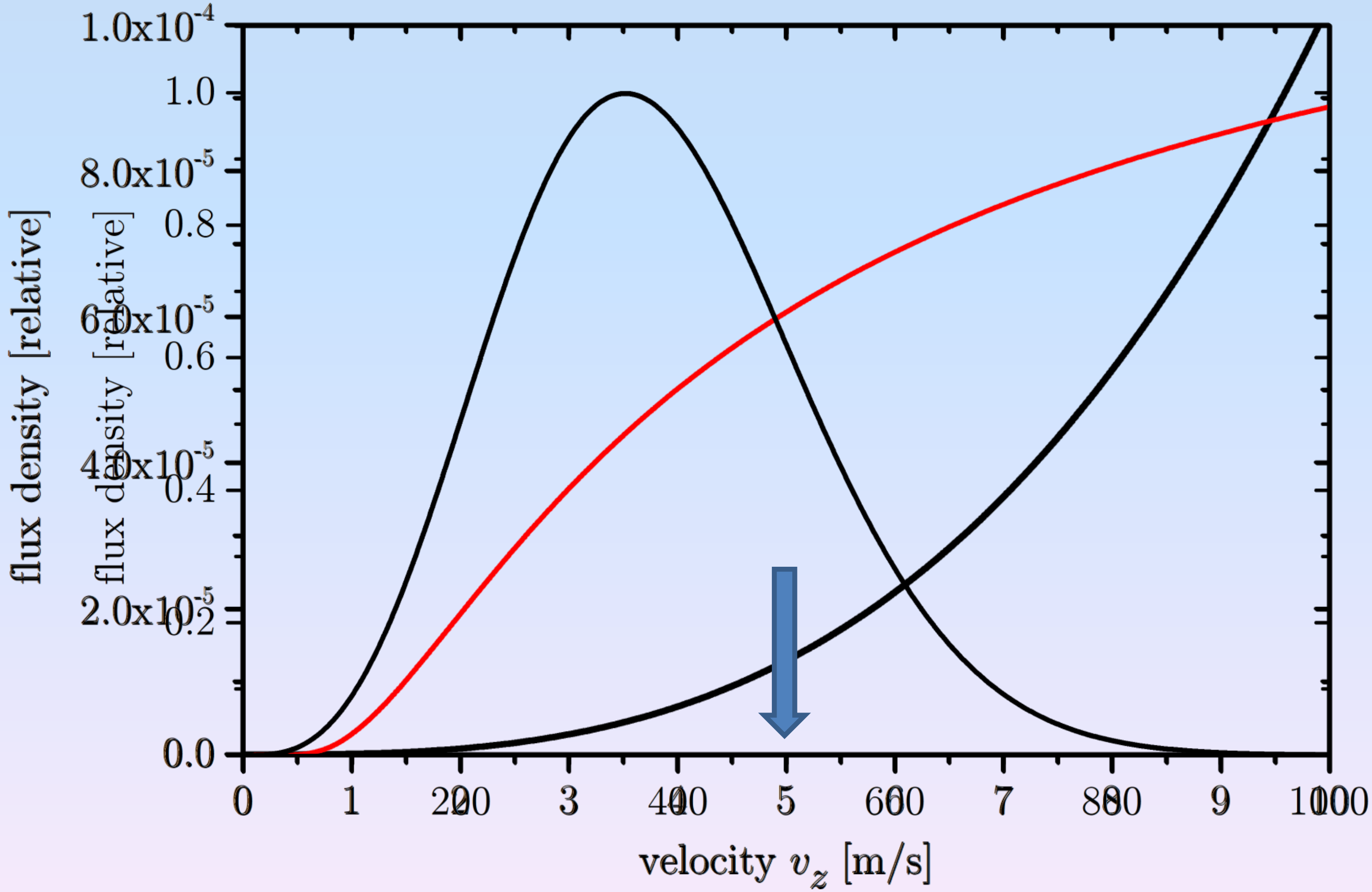


not more complex than a common lattice clock, no exotic lasers



Velocity Distributions

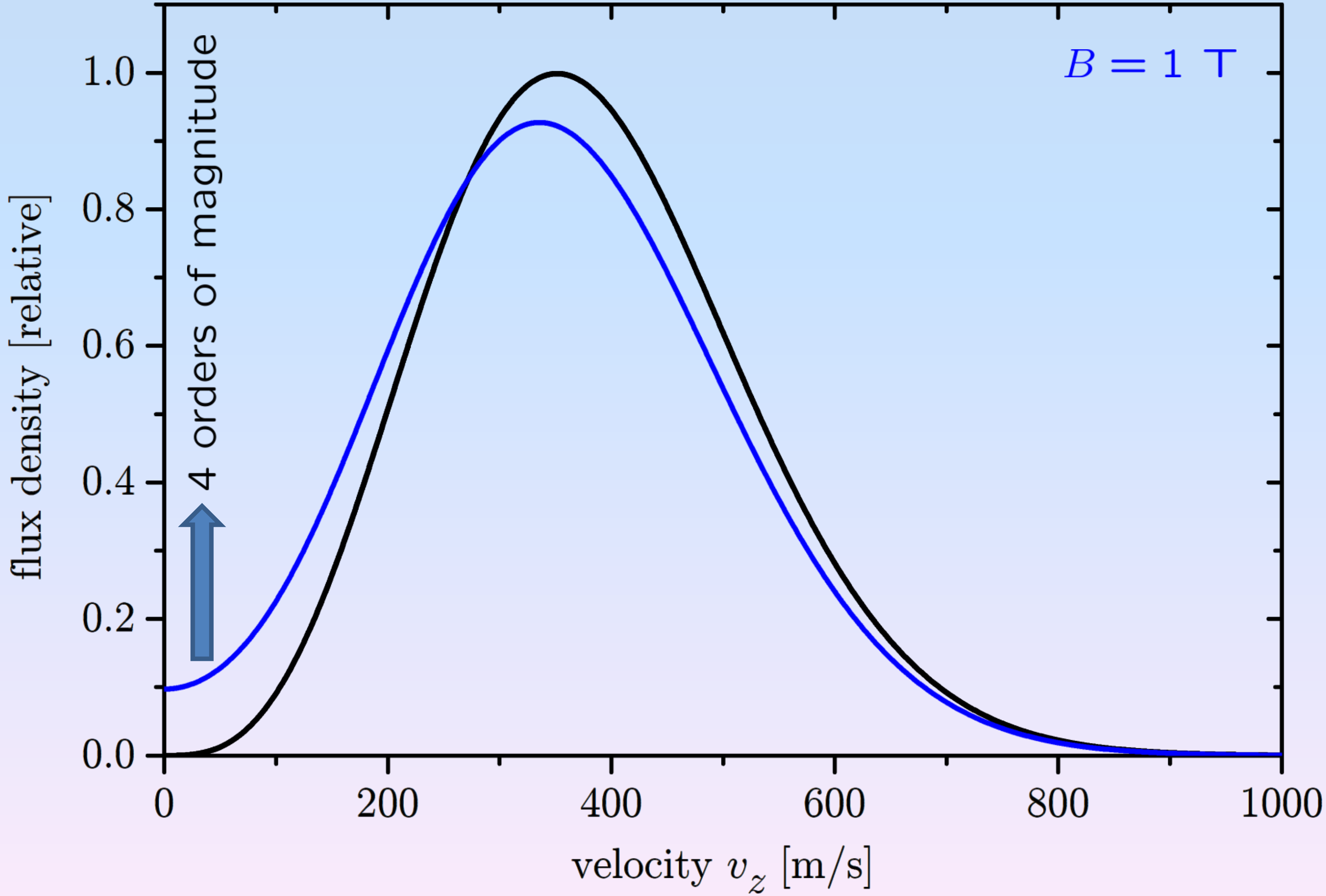
$$p(v_z) = \frac{4v_0^3}{\sqrt{\pi}} \, v_z^2 \, e^{-(v_z/v_0)^2} \times e^{-v_{\text{cut}}/v_z} \qquad v_{\text{cut}} = \sqrt{\pi}\sigma^2 r_n N_0 v_0 = 3.8 \text{ m/s}$$



I. Estermann, O.C. Simpson and O. Stern, Phys. Rev. 71, 238 (1947)

Velocity Distributions

High field seekers ($F = 1, M_F = -1$) run uphill to leave the nozzle

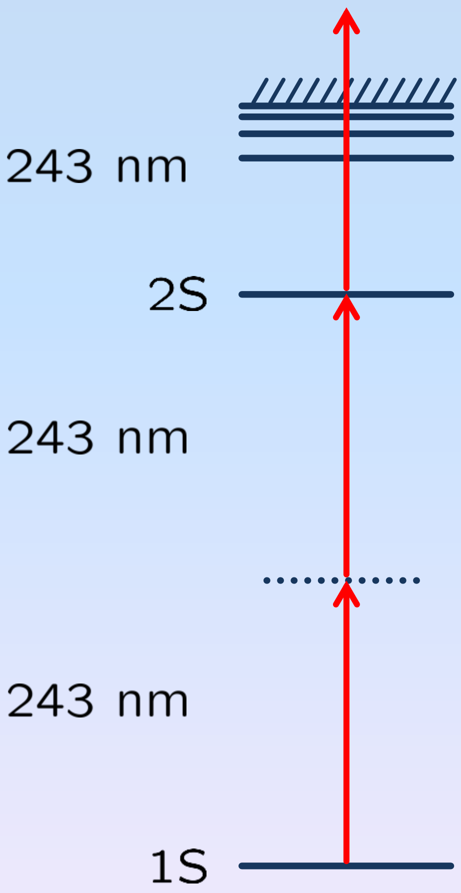


Expected Trap loading

velocity selection	1.5×10^{-6}	(4 orders gain with B-field)
solid angle	4.0×10^{-5}	
trapping probability	6.0×10^{-3}	
1S-2S excitation probability	0.14	(assuming 6 W at 243 nm)
intra-beam collissions	0.5	
2S life time	0.21	(natural and collisional)
hyperfine selection	0.25	(only $F = 1$, $M_F = -1$)
trapping probability	1.3×10^{-15}	
flux from the nozzle	1.0×10^{18} atoms/sec	
loss by collissions	10% per sec	

loading rate: 1000 atoms/sec; steady state: 10^4 atoms

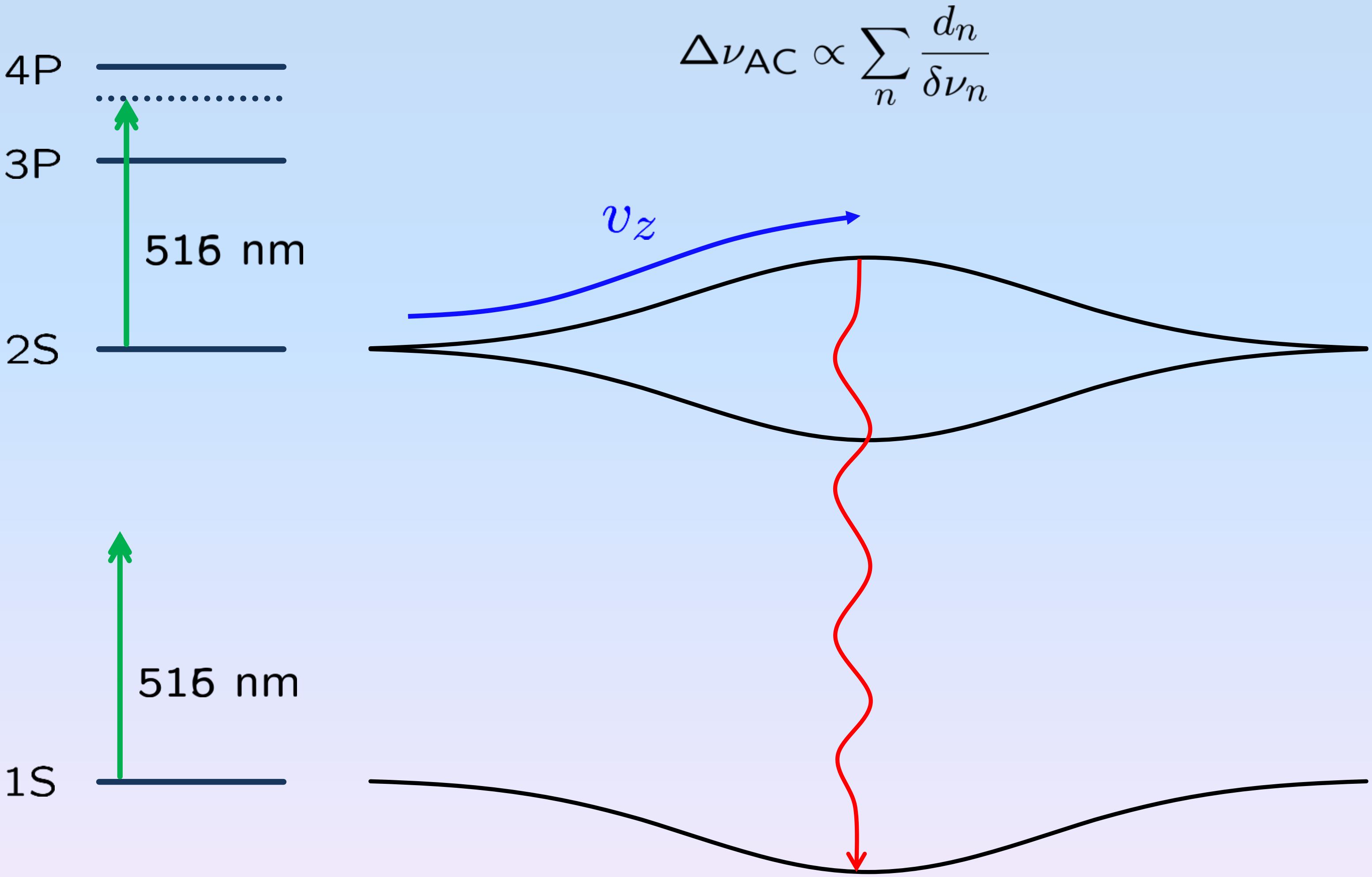
Ion Detection



Outlook/Prospects

- Non-destructive detection might be possible
- Several improvements of the loading scheme are possible ...

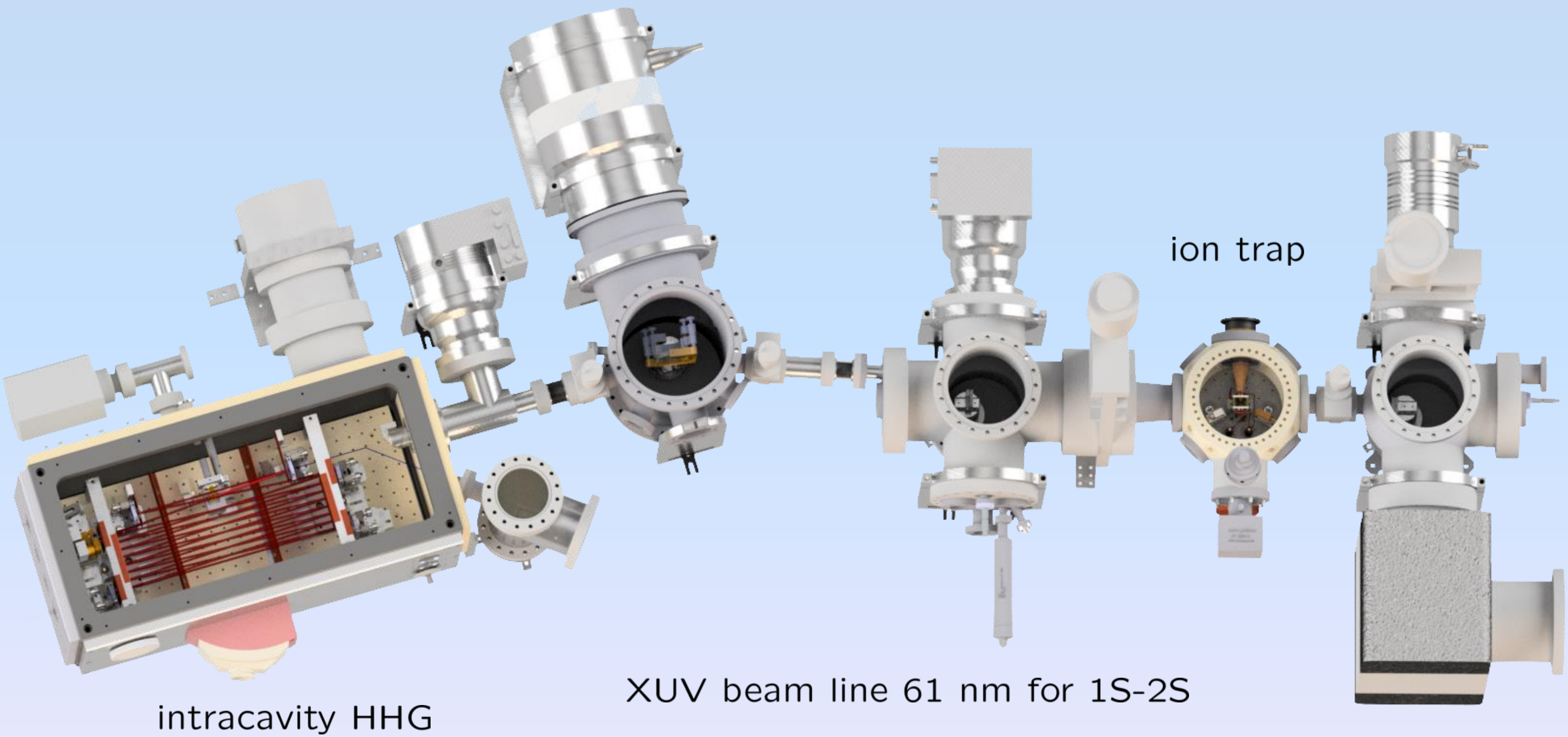
Sisyphus loading



Outlook/Prospects

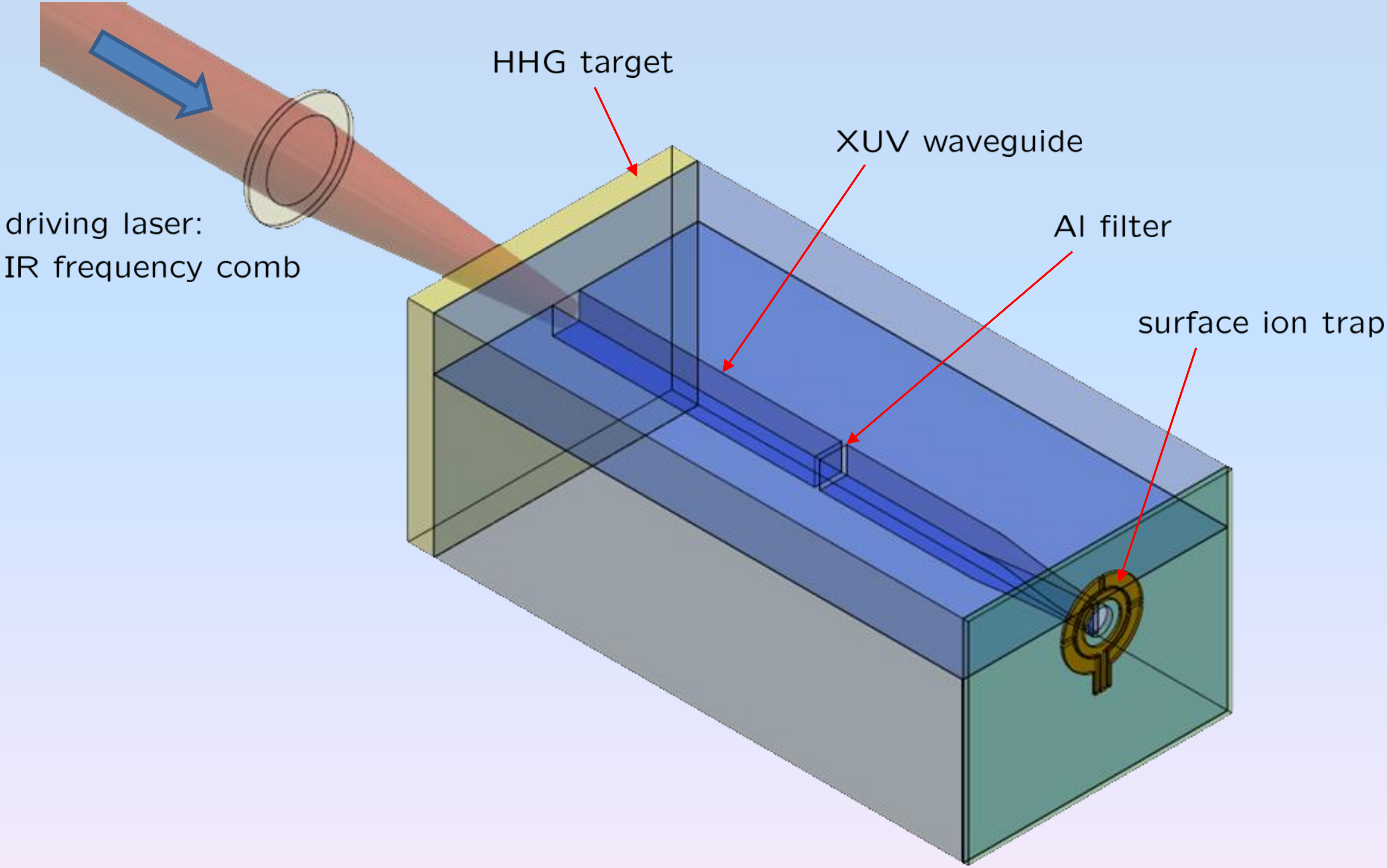
- Evaporative cooling
- Other hydrogen like systems conceivable ...

1S-2S in hydrogen like helium



J. Moreno, F. Schmid, J. Weitenberg, S.G. Karshenboim, T.W. Hänsch, Th. Udem and A. Ozawa
Eur. Phys. J. D, 77, 67 (2023)

Miniaturization



F. Canella, J. Weitenberg, M. Thariq, F. Schmid, P Dwivedi, G. Galzerano, T.W. Hänsch, Th. Udem, A. Ozawa
A low repetition rate optical frequency comb, submitted (arXiv:2309.09616)

Outlook/Prospects

- Velocity selector useful for other spectrocopy experiments
- Trap can be used to extract antihydrogen from magentic confinement

Problems:

- Nuclear charge radius: lattice QCD, He^+, leptonic system ...
- Magic wavelength very large slope: other QED test, tune-out ...

Thank you for your Attention

Fixing the Rydberg Constant

$$R_\infty = \frac{\alpha^2 m_e}{2h}$$

- This seem to fix the electron mass m_e.
- How can we? Is the kg not already fixed by h?
- No, it is fixed by h **and** c **and** ν_{Cs}. R_∞ takes over the role of ν_{Cs}.
- Knowing α, we would have another microsopic mass reference:

$$m_e = \frac{2hR_\infty}{\alpha^2}$$

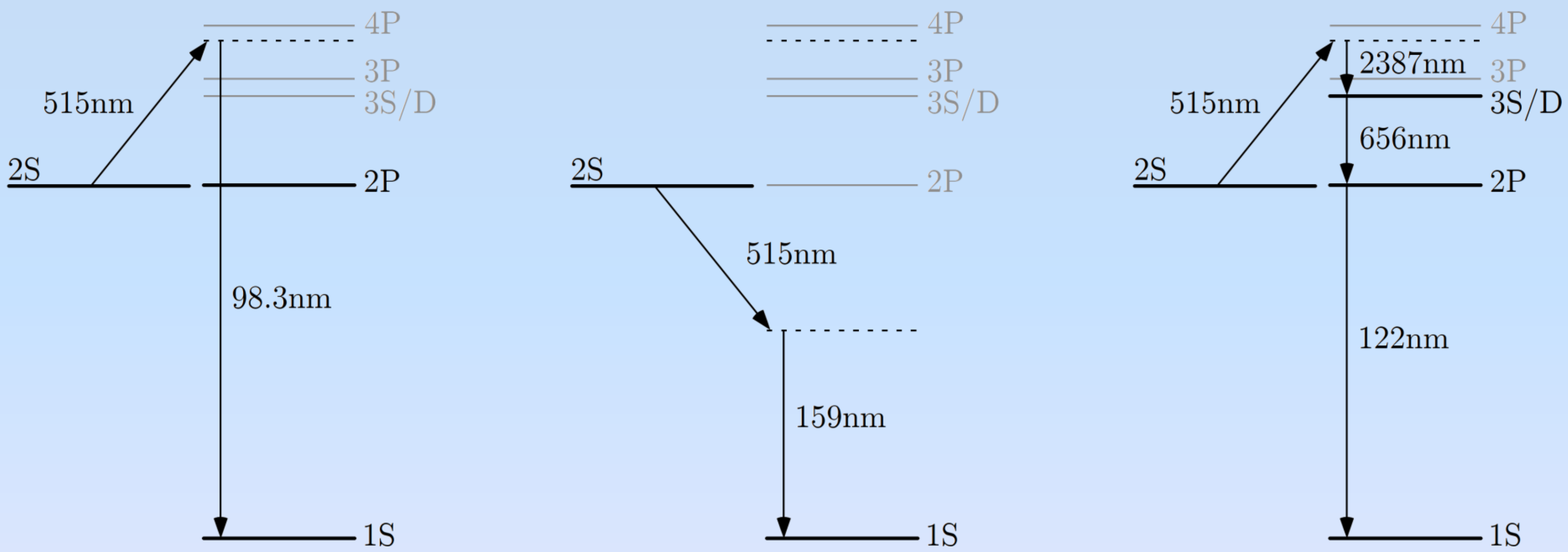
Atomic Properties

Natural width / kHz

n	S	P	D
2	$1.3\ 10^{-3}$	100000	-
3	1000	30000	10000
4	680	13000	4400
5	430	6600	2300
6	280	3900	1300

	H	Rb
Mass	1 u	87 u
Wavelength	121 nm	780 nm
Linewidth	100 MHz	6 MHz
Doppler Temp.	2.3 mK	145 μK
Recoil Temp.	1.2 mK	360 nK
Recoil velocity	3.2 m/s	5.6 mm/s
$\frac{\text{Linewidth}}{\text{Doppler shift}}$	3.7	800

Quenching Decay Paths



Laser Sources

515 nm, 6 kW (power enhanced), $w_0 = 80\ \mu$m

ECDL 1030 nm

Fiber Amplifier

SHG PP-Mg:SLT

243 nm, 5 W (power enhanced), $w_0 = 500\ \mu$m

ECDL 972 nm

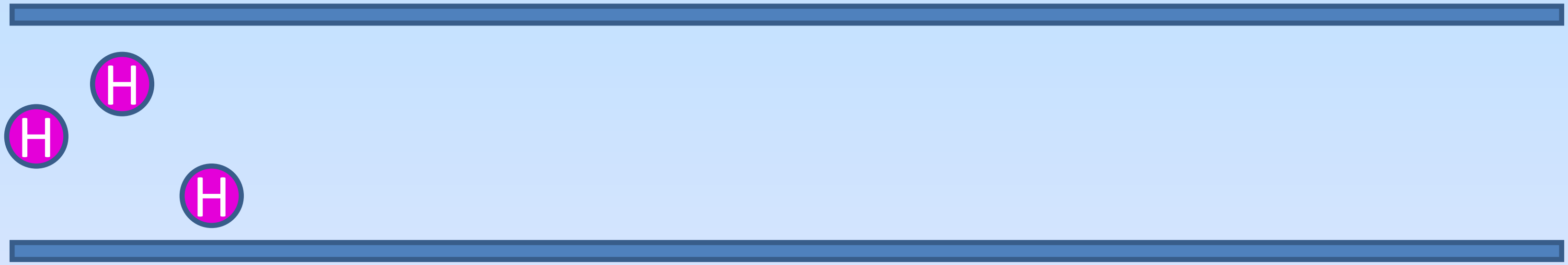
TA

SHG PP-Mg:SLT

Resonant SHG

„Zacharias“ Effect

- Slow atoms are being pushed out of the atomic beam by fast moving atoms.



Thank you for your Attention