Introduction to Non-linear Longitudinal Beam Dynamics

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CERN

Introduction to Accelerator Physics

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The CERN Accelerator School

Outline

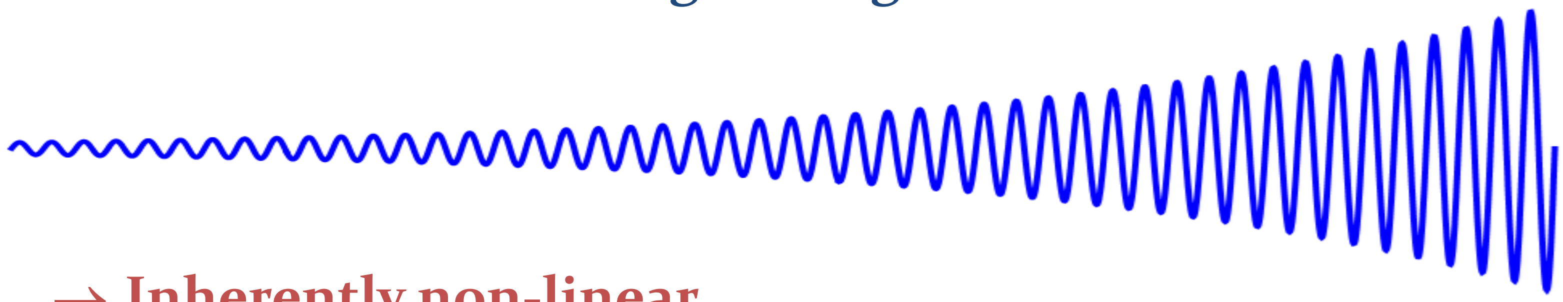
- **Introduction**
- **Linear and non-linear longitudinal dynamics**
  - Equations of motion, Hamiltonian, RF potential
- **Longitudinal manipulations**
  - Bunch length and distance control by multiple RF systems
  - Bunch rotation
- **Synchrotron frequency distribution**
  - Effect on longitudinal beam stability
- **Summary**

Introduction

Introduction

- **Signals generated by radio-frequency systems in particle accelerators are of the form** $V\,\sin(h\omega_{\rm rev}t)$

  $\rightarrow$ **Resonance effect: large voltage with little effort**



  $\rightarrow$ **Inherently non-linear**

  $\rightarrow$ **Linear longitudinal beam dynamics only an approximation**

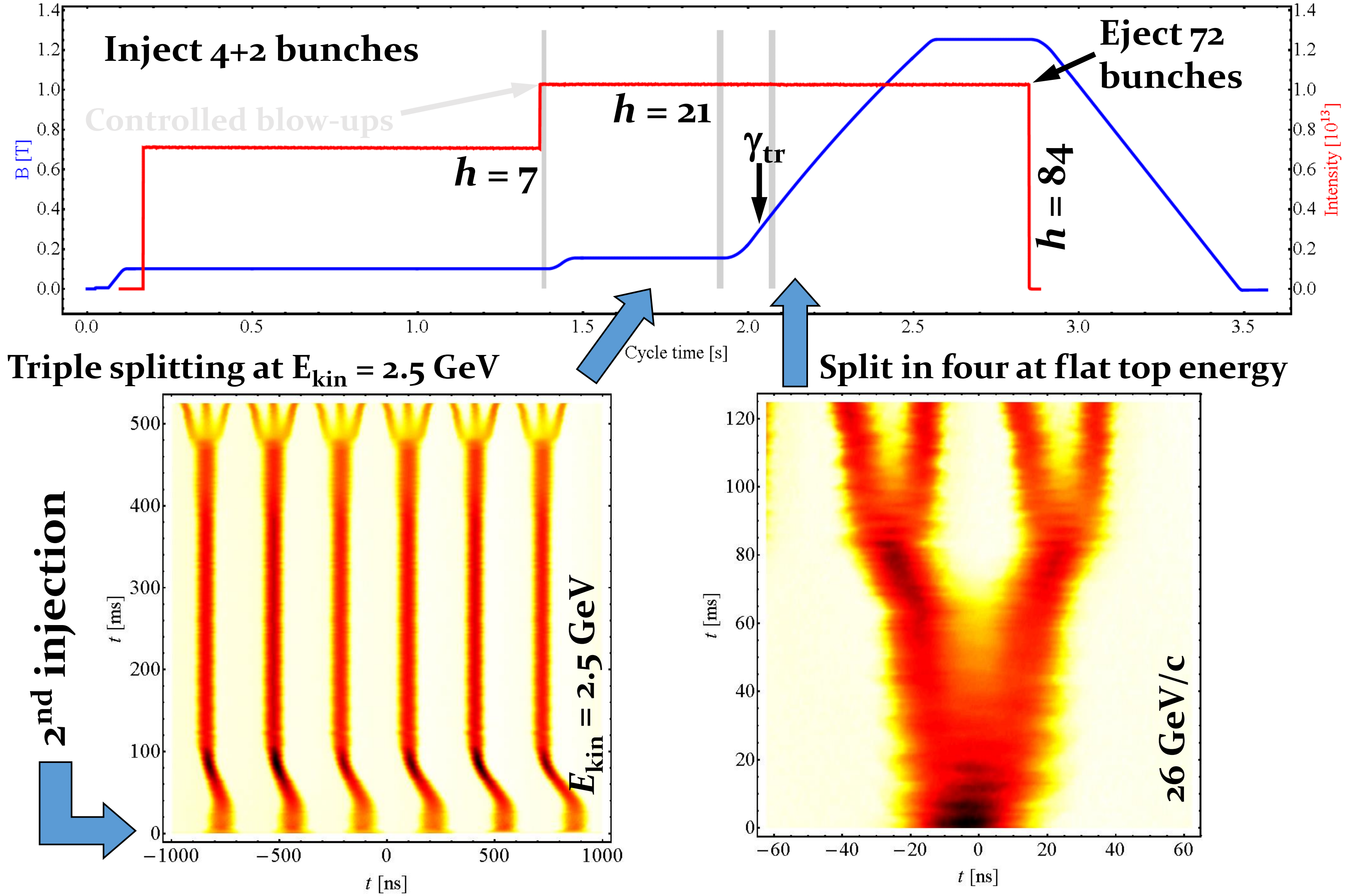
$\rightarrow$ **Effect of non-linearity on beam?**

$\rightarrow$ **Tools to describe and analyse non-linearity**

$\rightarrow$ **Use non-linearity to improve beam conditions**

Non-linear longitudinal dynamics

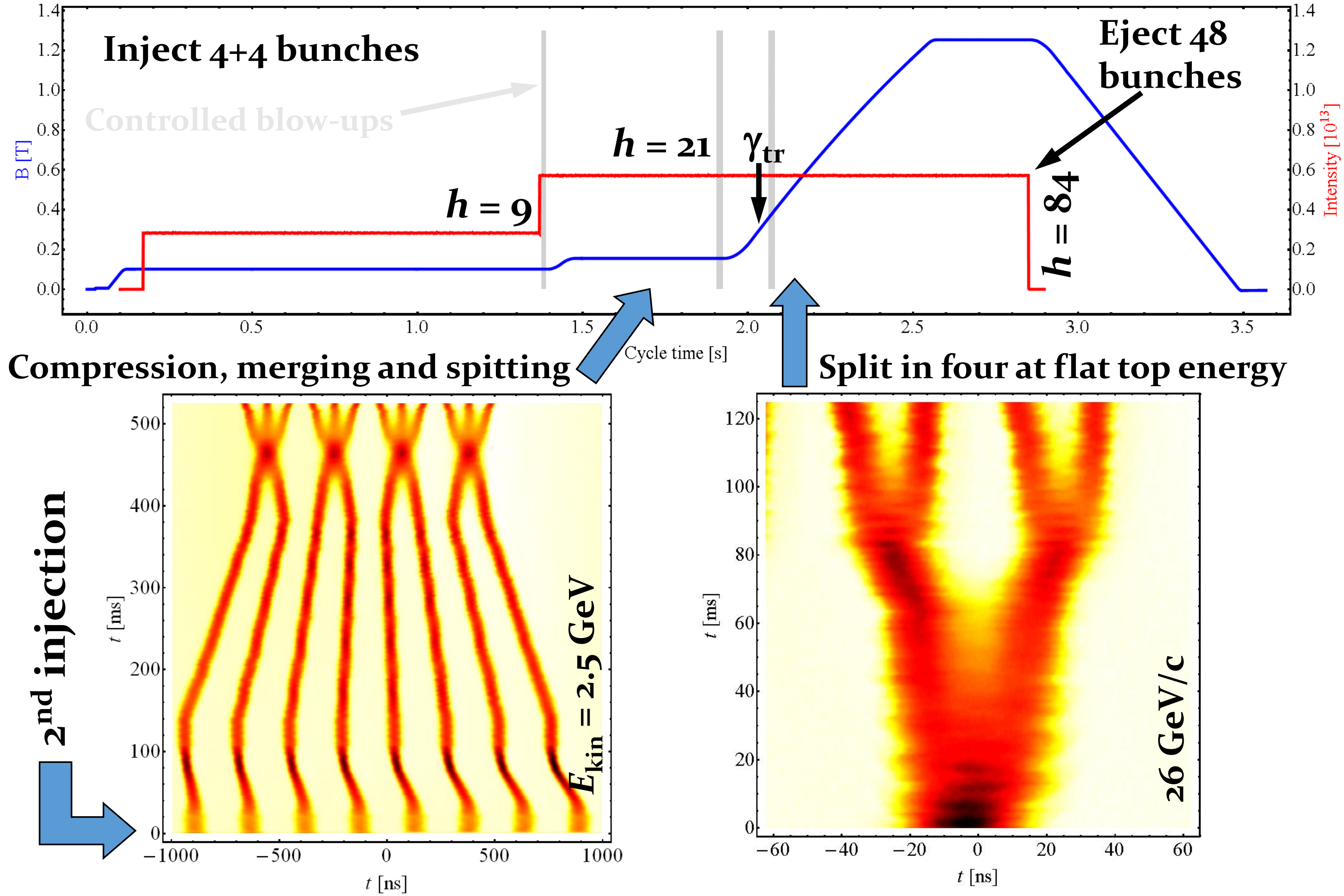
Example: LHC-type beam in the CERN PS



- **Non-linear RF allows to control all longitudinal parameters**

→ Number of bunches, bunch length and emittance, longitudinal stability

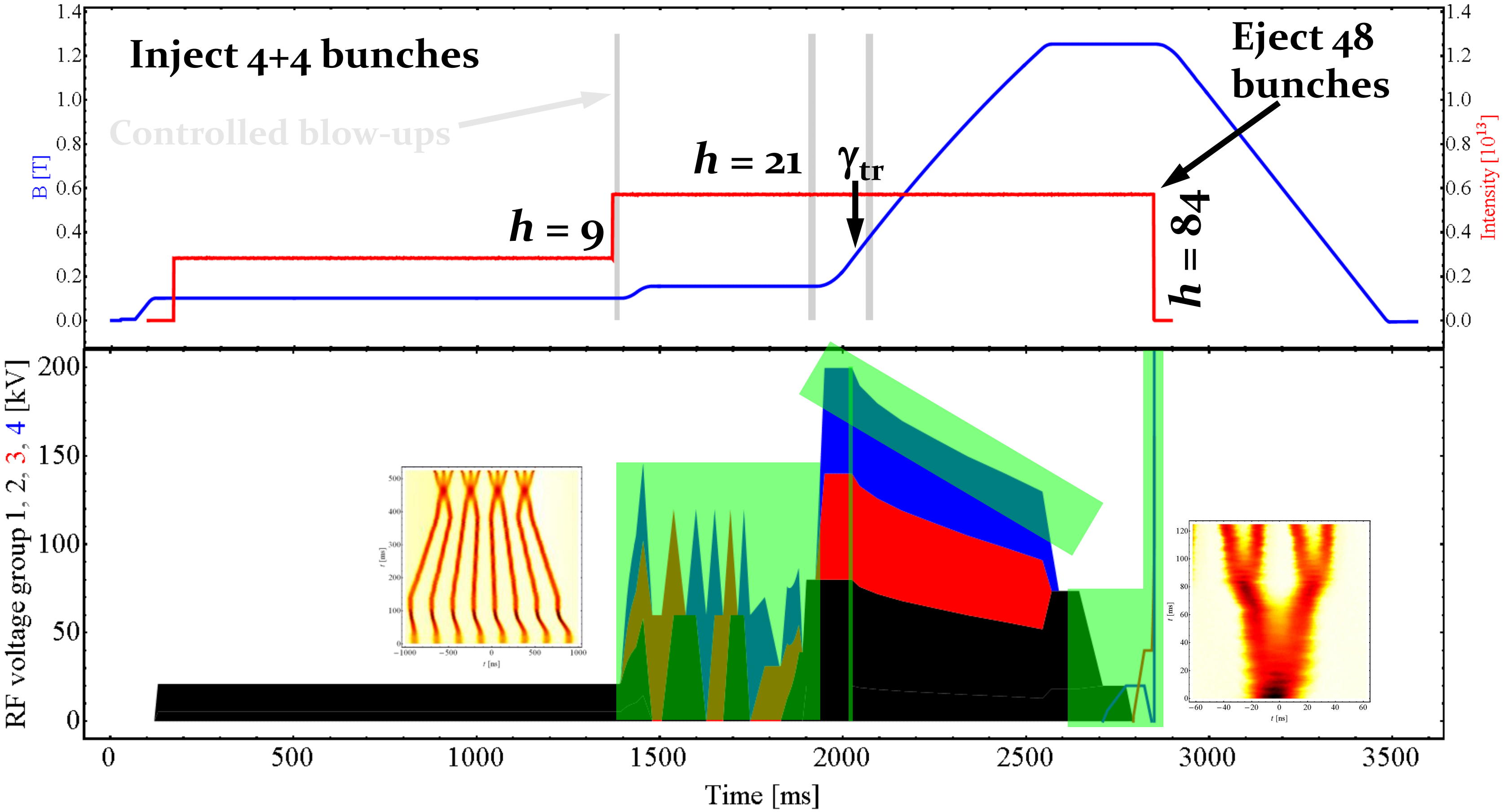
Example: LHC-type beam in the CERN PS



- **Non-linear RF allows to control all longitudinal parameters**

→ Number of bunches, bunch length and emittance, longitudinal stability

Where profit from non-linear RF?



→ RF manipulation from 8 bunches in h = 9 to 12 in h = 21

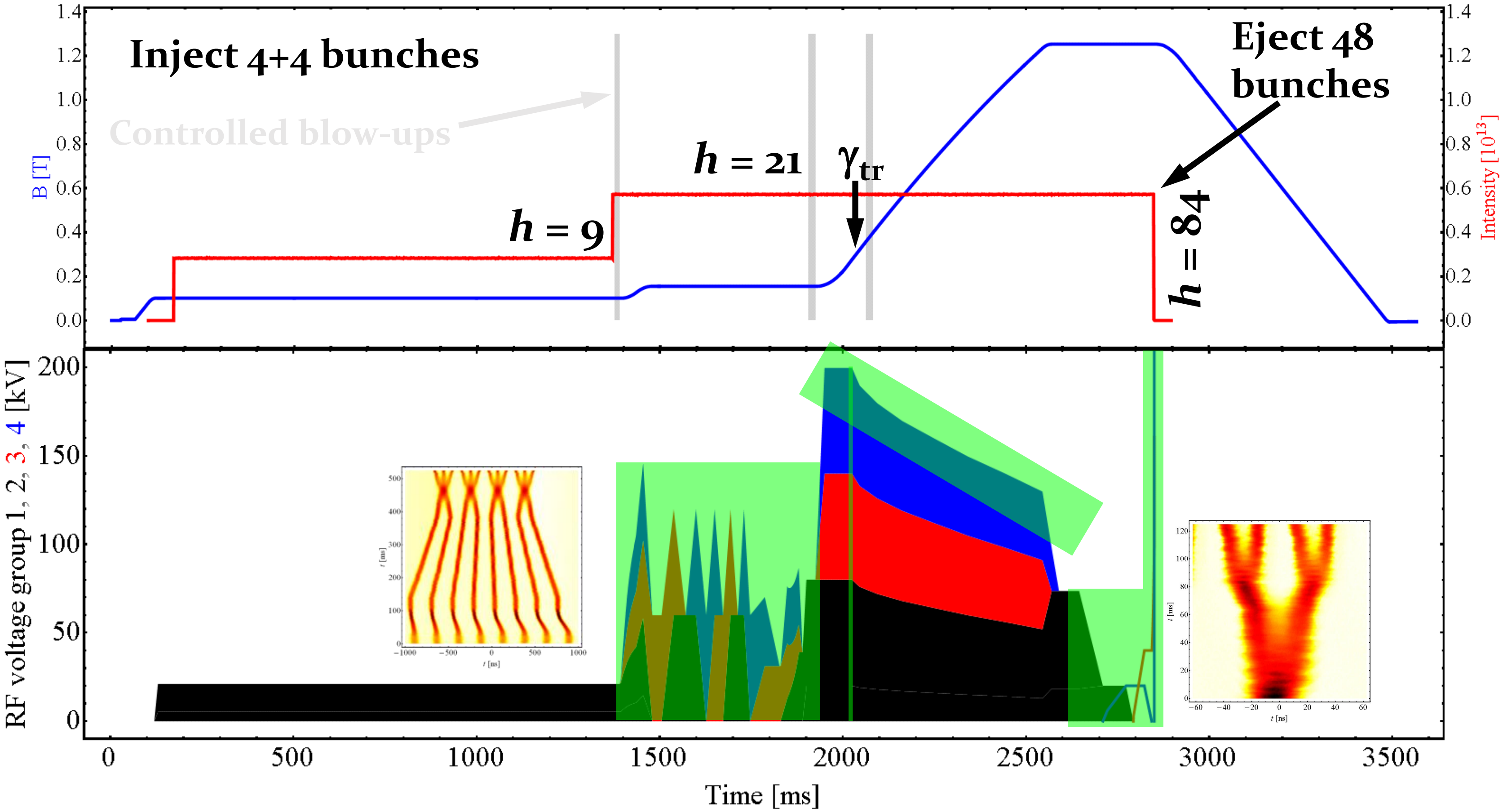
→ Transition crossing

→ RF voltage reduction during acceleration

→ Splitting at the flat-top

→ Bunch shortening (rotation) before extraction

Where profit from non-linear RF?



→ **RF manipulation from 8 bunches in *h* = 9 to 12 in *h* = 21**

→ **Transition crossing**

→ **RF voltage reduction during acceleration**

→ **Splitting at the flat-top**

→ **Bunch shortening (rotation) before extraction**

Applications

- **Introduce extra non-linearity**
  - **Bunch lengthening in double-harmonic RF system to reduce peak current (space charge)**

$$V_1 \sin(h_1 \omega_{\mathrm{rev}} t + \phi_1) + V_2 \sin(h_2 \omega_{\mathrm{rev}} t + \phi_2)$$

  - **Short and long bunches with multi-harmonic RF systems**

$$\sum_n V_n \sin(h_n \omega_{\mathrm{rev}} t + \phi_n)$$

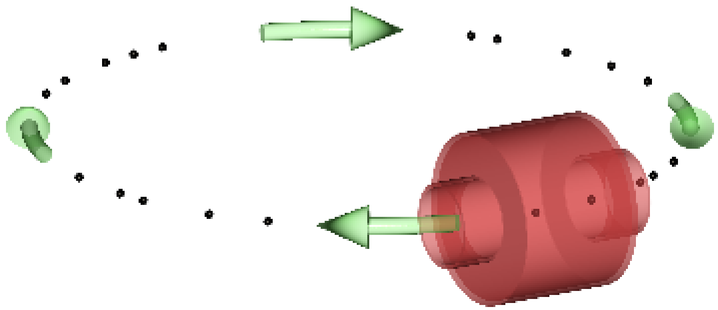
  - **Adapt bunch-to-bunch distance**

- **Profit from non-linearity for beam stabilization**
  - **Stabilize beam using higher-harmonic RF**
  - **Controlled longitudinal emittance blow-up**

Linear longitudinal beam dynamics

Interaction between particles and RF

Simple accelerator model:

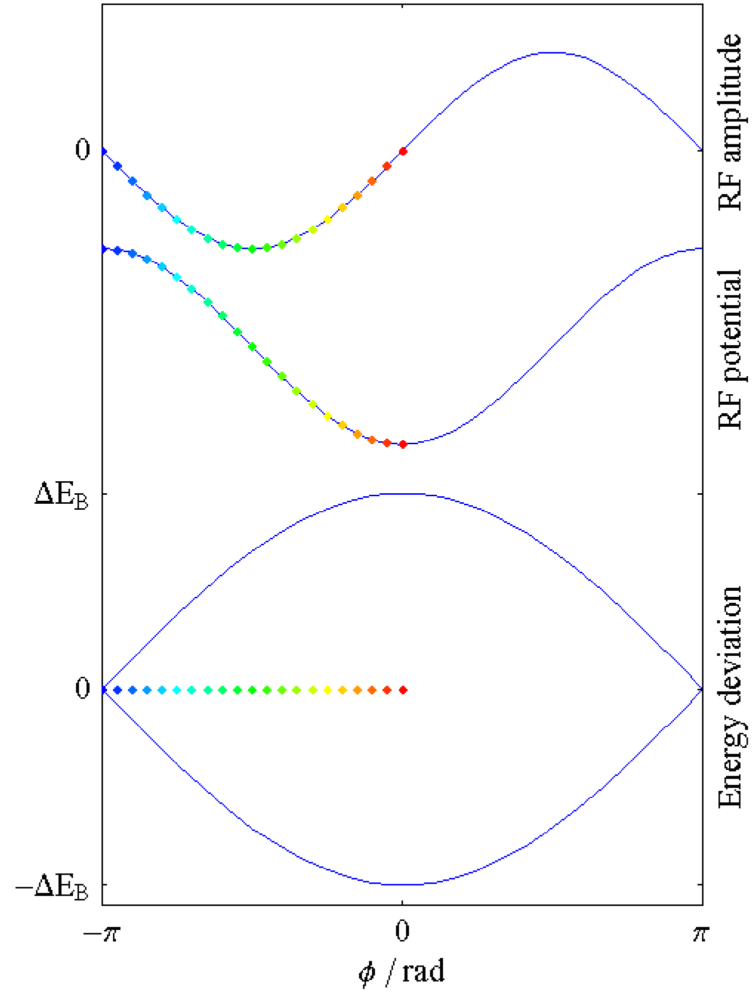


Energy dependent phase advance, ϕ:

$$\phi_{n+1} = \phi_n + 2\pi h \eta \frac{\Delta E_n}{\beta^2 E}, \; \eta = \frac{1}{\gamma_{\mathrm{tr}}^2} - \frac{1}{\gamma^2}$$

Phase dependent energy gain, ΔE:

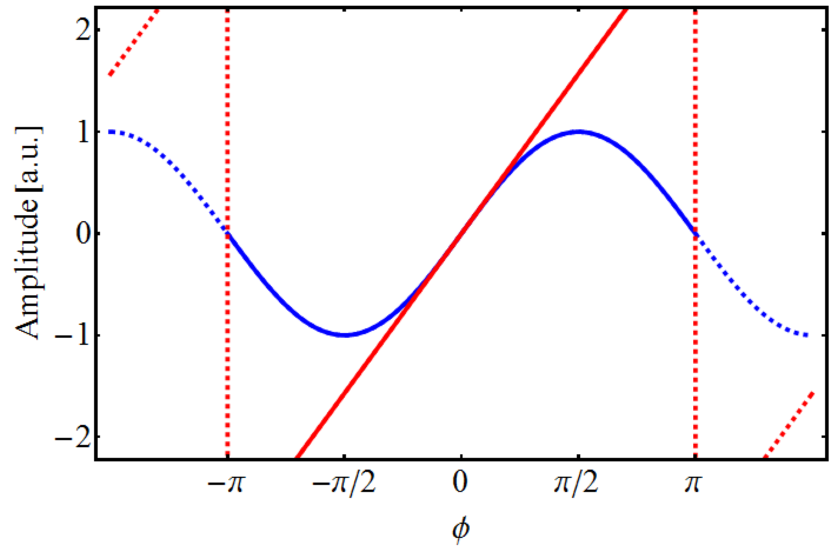
$$\Delta E_{n+1} = \Delta E_n + qVg(\phi_{n+1})$$



Works for arbitrary shape of acceleration amplitude $g(\phi)$

Linear longitudinal beam dynamics

- **Usual longitudinal beam dynamics already non-linear, since RF system usually provides sinusoidal amplitude**
- **Linear longitudinal beam dynamics?**



$$\frac{d}{dt}\phi = \frac{h\eta\omega_{\mathrm{rev}}}{pR}\left(\frac{\Delta E}{\omega_{\mathrm{rev}}}\right)$$

$$\frac{d}{dt}\left(\frac{\Delta E}{\omega_{\mathrm{rev}}}\right) = \frac{qV}{2\pi}\phi$$

same structure

$$\frac{dq}{dt} = \frac{\partial H}{\partial p}$$

$$\frac{dp}{dt} = -\frac{\partial H}{\partial q}$$

Linear longitudinal beam dynamics

- **Construct Hamiltonian from equations of motion**

$$\frac{d}{dt}\phi = \frac{h\eta\omega_{\mathrm{rev}}}{pR}\left(\frac{\Delta E}{\omega_{\mathrm{rev}}}\right)$$

$$\frac{d}{dt}\left(\frac{\Delta E}{\omega_{\mathrm{rev}}}\right) = \frac{qV}{2\pi}\phi$$

same structure

$$\frac{dq}{dt} = \frac{\partial H}{\partial p}$$

$$\frac{dp}{dt} = -\frac{\partial H}{\partial q}$$

$$q = \phi \qquad p = \frac{\Delta E}{\omega_{\mathrm{rev}}}$$

$$H(p,q) = T(p) + W(q)$$

- **Hamiltonian constant on trajectory**

→ **'Energy conservation'**

$$H(p,q) = H_{\mathrm{trajectory}}$$

Linear longitudinal beam dynamics

$$\frac{d}{dt}\phi = \frac{h\eta\omega_{\text{rev}}}{pR}\left(\frac{\Delta E}{\omega_{\text{rev}}}\right)$$

$$\frac{d}{dt}\left(\frac{\Delta E}{\omega_{\text{rev}}}\right) = \frac{qV}{2\pi}\phi$$

The Hamiltonian from the equations can be written as

$$H\left(\phi, \frac{\Delta E}{\omega_{\text{rev}}}\right) = \frac{1}{2}\frac{h\eta\omega_{\text{rev}}}{pR}\left(\frac{\Delta E}{\omega_{\text{rev}}}\right)^2 - \frac{1}{2}\frac{qV}{2\pi}\phi^2$$

$$= \frac{1}{2}\frac{pR}{h\eta\omega_{\text{rev}}}\dot{\phi}^2 - \frac{1}{2}\frac{qV}{2\pi}\phi^2$$

$$\eta = \frac{1}{\gamma_{\text{tr}}^2} - \frac{1}{\gamma^2}$$

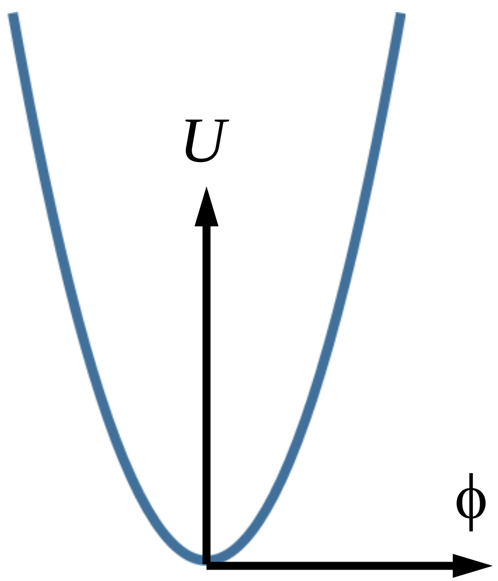
Linear longitudinal beam dynamics

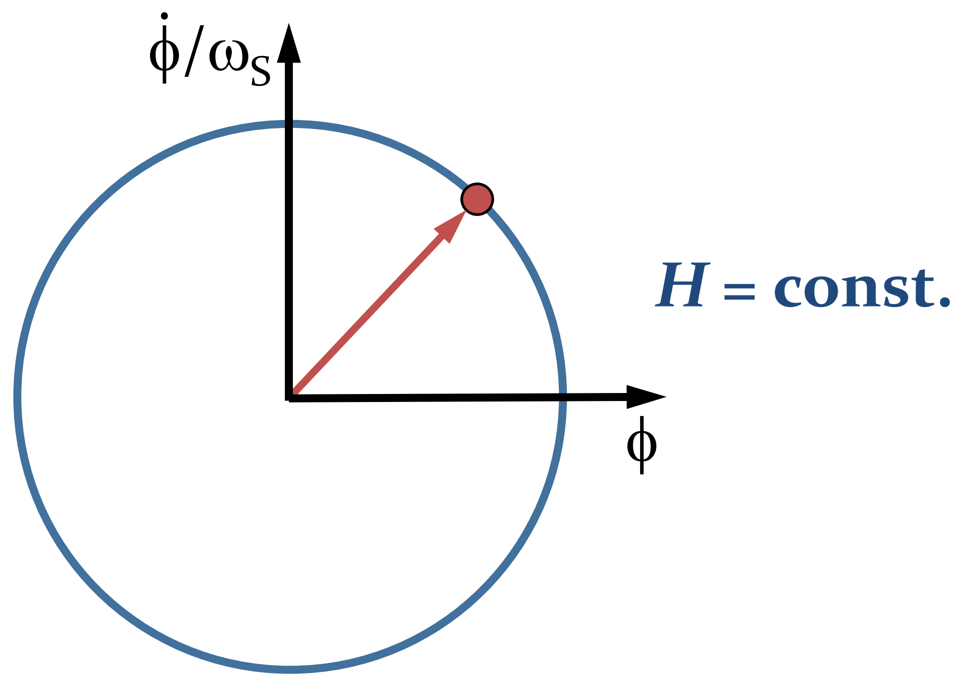
$$H\left(\phi, \frac{\dot{\phi}}{\omega_S}\right) = \frac{1}{2}\left(\frac{\dot{\phi}}{\omega_S}\right)^2 + \frac{1}{2}\phi^2 = T + W$$

→ **Particles move on circular trajectories in ϕ-$\dot{\phi}/\omega_S$ phase space**

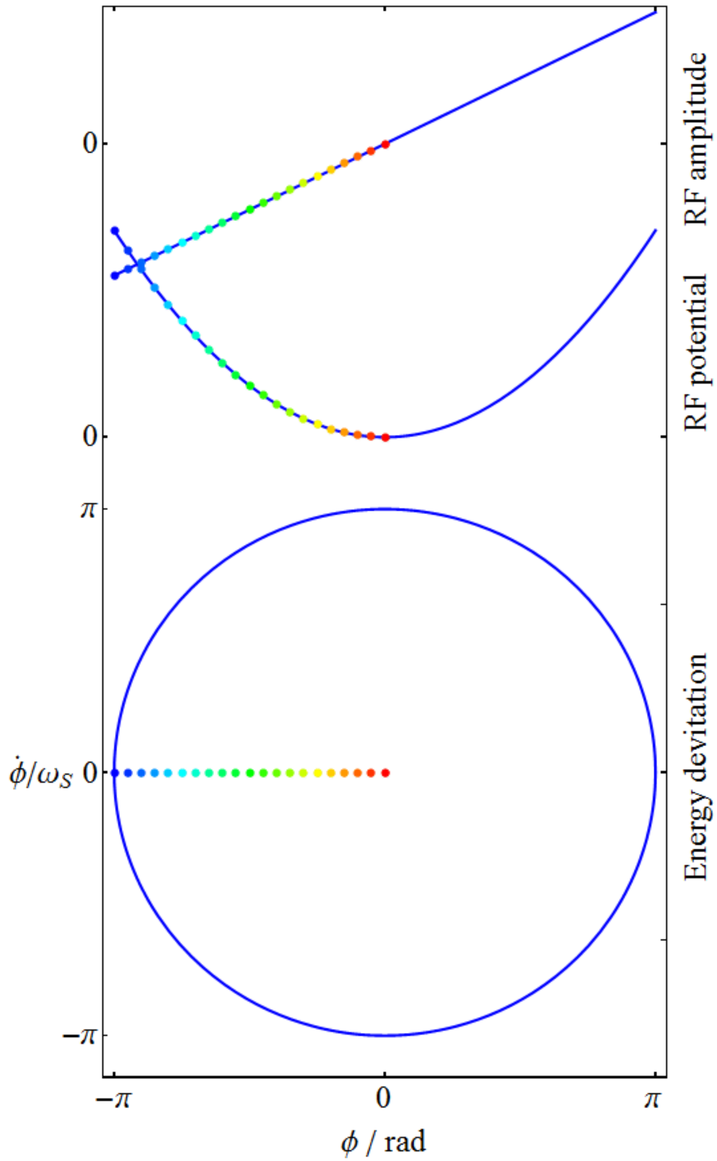
→ **RF potential is parabolic, $W(\phi) \sim \phi$**

→ **Hamiltonian is constant on these trajectories**





Linear longitudinal phase space



- **Simple model**
- **Circular trajectories**
- **All particles have same synchrotron frequency**
- **Normalized bucket area: $A_b = \pi r^2 = \pi^3$**

→ Harmonic oscillator

Non-linear longitudinal beam dynamics

Introduce most simple non-linearity

RF amplitude function $V\phi \rightarrow V\sin\phi$

$$\frac{d}{dt}\phi = \frac{h\eta\omega_{\mathrm{rev}}}{pR}\left(\frac{\Delta E}{\omega_{\mathrm{rev}}}\right)$$

$$\frac{d}{dt}\left(\frac{\Delta E}{\omega_{\mathrm{rev}}}\right) = \frac{qV}{2\pi}(\sin\phi - \sin\phi_{\mathrm{S}})$$

$$H\left(\phi, \frac{\Delta E}{\omega_{\mathrm{rev}}}\right) = \frac{1}{2}\frac{h\eta\omega_{\mathrm{rev}}}{pR}\left(\frac{\Delta E}{\omega_{\mathrm{rev}}}\right)^2 + \frac{qV}{2\pi}\left[\cos\phi - \cos\phi_{\mathrm{S}} + (\phi - \phi_{\mathrm{S}})\sin\phi_{\mathrm{S}}\right]$$

with $\phi = \phi_{\mathrm{S}} + \Delta\phi$ **this becomes**

$$H\left(\Delta\phi, \frac{\Delta E}{\omega_{\mathrm{rev}}}\right) = \frac{1}{2}\frac{h\eta\omega_{\mathrm{rev}}}{pR}\left(\frac{\Delta E}{\omega_{\mathrm{rev}}}\right)^2 + \frac{qV}{2\pi}\left[\cos(\phi_{\mathrm{S}} + \Delta\phi) - \cos\phi_{\mathrm{S}} + \Delta\phi\sin\phi_{\mathrm{S}}\right]$$

→ Standard longitudinal beam dynamics → Lectures F. Tecker

Introduce most simple non-linearity

$$H\left(\Delta\phi, \frac{\Delta E}{\omega_{\mathrm{rev}}}\right) = \frac{1}{2}\frac{h\eta\omega_{\mathrm{rev}}}{pR}\left(\frac{\Delta E}{\omega_{\mathrm{rev}}}\right)^2 + \frac{qV}{2\pi}\left[\cos(\phi_{\mathrm{S}} + \Delta\phi) - \cos\phi_{\mathrm{S}} + \Delta\phi\sin\phi_{\mathrm{S}}\right]$$

using

$$\begin{aligned}\cos(\phi_{\mathrm{S}} + \Delta\phi) &= \cos\phi_{\mathrm{S}}\cos\Delta\phi - \sin\phi_{\mathrm{S}}\sin\Delta\phi \\ &\simeq \cos\phi_{\mathrm{S}}\left(1 - \frac{1}{2}\Delta\phi^2\right) - \sin\phi_{\mathrm{S}}\Delta\phi\end{aligned}$$

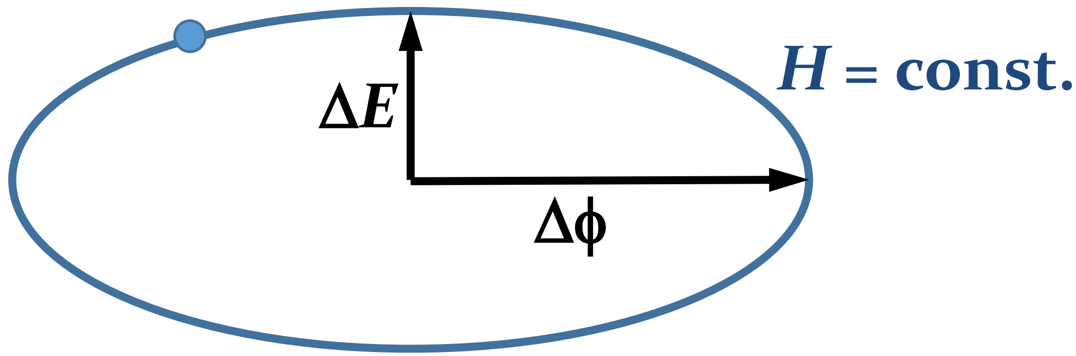
this Hamiltonian simplifies to

$$H\left(\Delta\phi, \frac{\Delta E}{\omega_{\mathrm{rev}}}\right) \simeq \frac{1}{2}\frac{h\eta\omega_{\mathrm{rev}}}{pR}\left(\frac{\Delta E}{\omega_{\mathrm{rev}}}\right)^2 - \frac{1}{2}\frac{qV}{2\pi}\cos\phi_{\mathrm{S}}\Delta\phi^2$$

Linear part of non-linear bucket

$$H\left(\Delta\phi, \frac{\Delta E}{\omega_{\mathrm{rev}}}\right) \simeq \frac{1}{2}\frac{h\eta\omega_{\mathrm{rev}}}{pR}\left(\frac{\Delta E}{\omega_{\mathrm{rev}}}\right)^2 - \frac{1}{2}\frac{qV}{2\pi}\cos\phi_S \Delta\phi^2$$

- **In the centre of the bucket, particles move on elliptical trajectories in Δϕ-Δ*E* phase space**
- **Hamiltonian is constant on these trajectories**

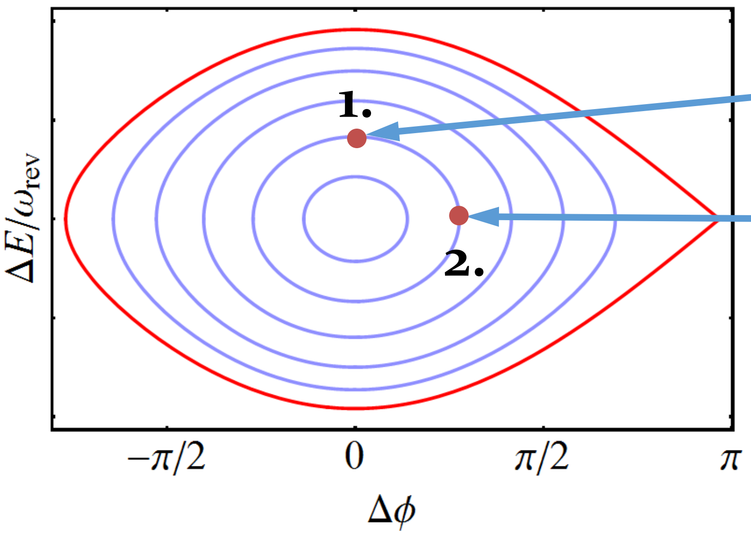


- **In the bucket centre, particles oscillate with the synchrotron frequency, $\omega_S = 2\pi f_S$**

$$\omega_S^2 = -\frac{h\eta\omega_{\mathrm{rev}} qV\cos\phi_S}{2\pi pR} \qquad \eta = \frac{1}{\gamma_{\mathrm{tr}}^2} - \frac{1}{\gamma^2}$$

Longitudinal emittance

- **Compare two particles on the same trajectory**
  1. **No phase deviation**
  2. **No energy deviation**



$$H\left(\Delta\phi = 0, \frac{\Delta E}{\omega_{\rm rev}}\right) = \frac{1}{2}\frac{h\eta\omega_{\rm rev}}{pR}\left(\frac{\Delta E}{\omega_{\rm rev}}\right)^2$$

$$H\left(\Delta\phi, \frac{\Delta E}{\omega_{\rm rev}} = 0\right) = -\frac{1}{2}\frac{qV}{2\pi}\cos\phi_S \Delta\,\phi^2$$

Longitudinal emittance

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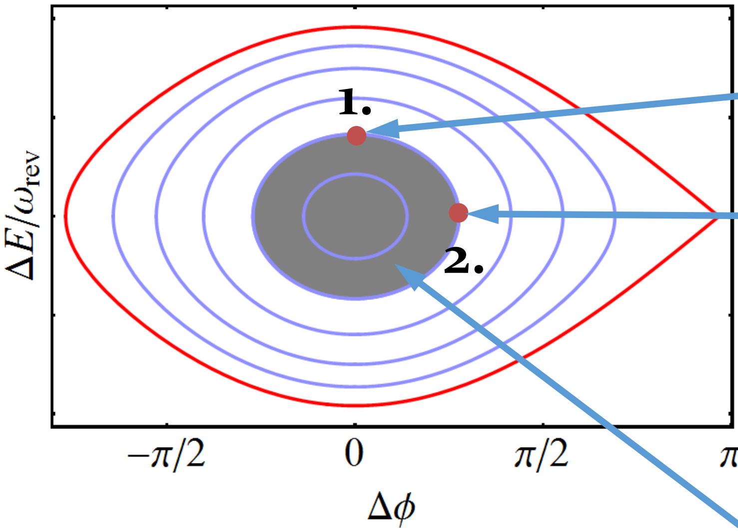

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Longitudinal emittance, ε_l

~ **Surface occupied by particles in longitudinal phase space**

→ **Preserved in physical [$\pi\Delta\tau\,\Delta E$] = eVs**

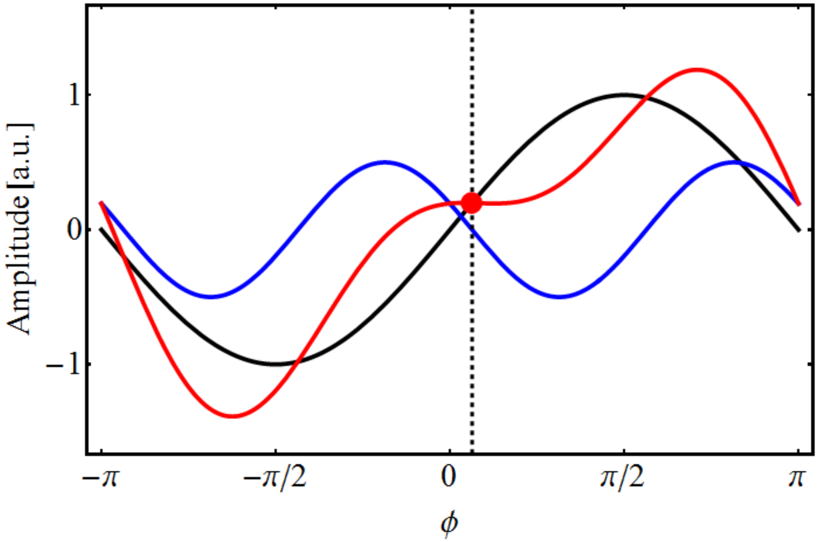
$$\varepsilon_l = \frac{2}{h\omega_{\rm rev}}\int_{\Delta\phi_i}^{\Delta\phi_f} \Delta E(\Delta\phi)\, d(\Delta\phi)$$

More non-linearity: multi-harmonic RF

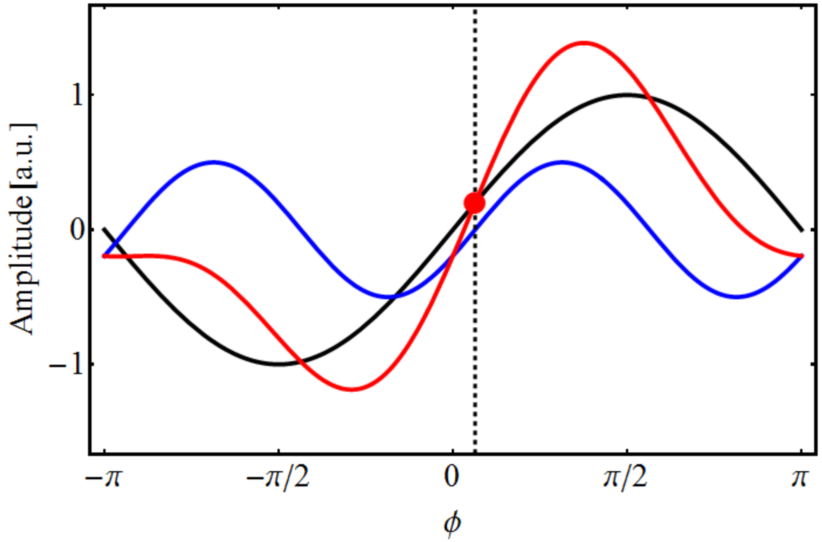
RF amplitude $$V \sin\phi \rightarrow V\left[\sin\phi + r\sin(n\phi + \phi_1)\right]$$

- **Example case $n = 2$ and $r = 0.5$**

Both RF systems in counter-phase



Both RF systems in phase at bunch



→ **Local voltage gradient decreased**

→ **Bunch is stretched**

→ **Lower peak current**

→ **Local voltage gradient increased**

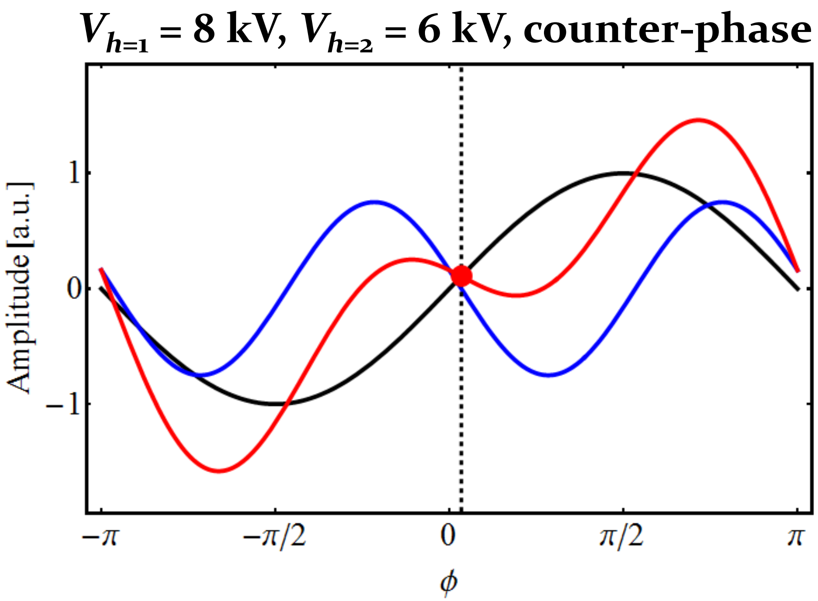
→ **Bunch is compressed**

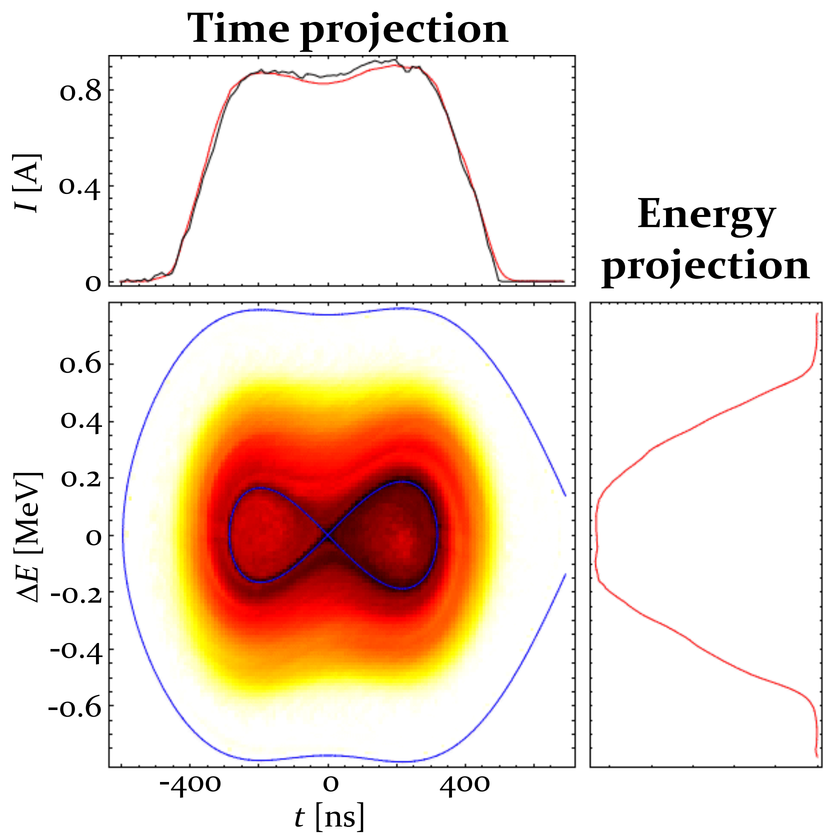
→ **Higher peak current**

Example application: space charge in PSB

RF amplitude $V \sin\phi \rightarrow V[\sin\phi + r\sin(n\phi + \phi_1)]$

→ Space charge ∝ instantaneous current





- **Inverted gradient at bucket centre**
- **Flattened bunch with reduced peak current** → Space charge reduction at low energy

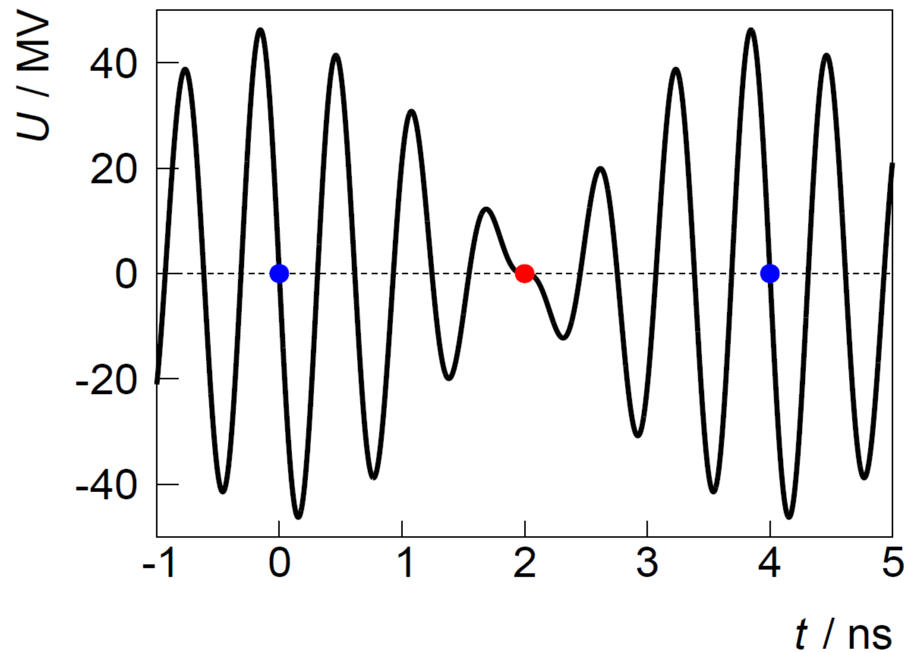
Long and short bunches simultaneously

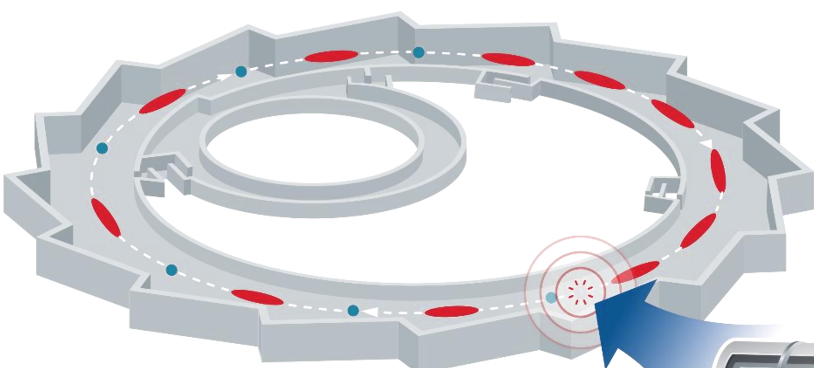
Markus Ries et al.

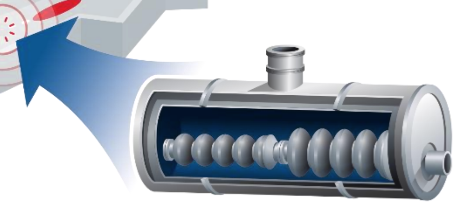
- Example BESSY VSR

→ Depending on user of synchrotron radiation: need long or short bunches

☞ Do long and short bunches simultaneously!







- 4 × 0.5 GHz NC (existing)
- 4 × 1.5 GHz supercond.
- 4 × 1.75 GHz supercond.

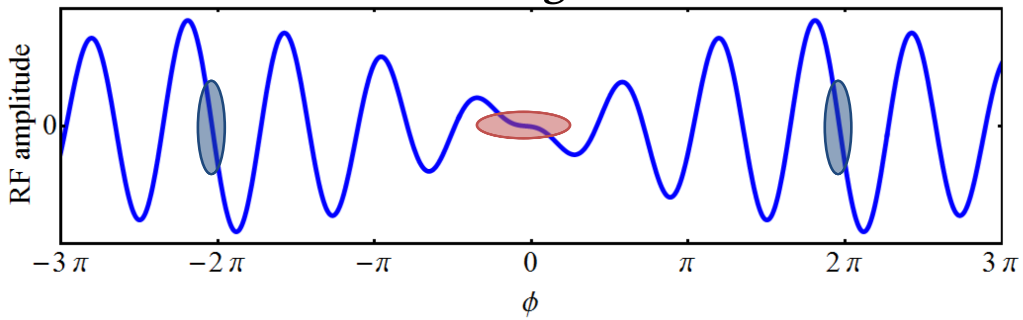
HZB Helmholtz Zentrum Berlin BESSY VSR

Bunch length modulation

Markus Ries et al.

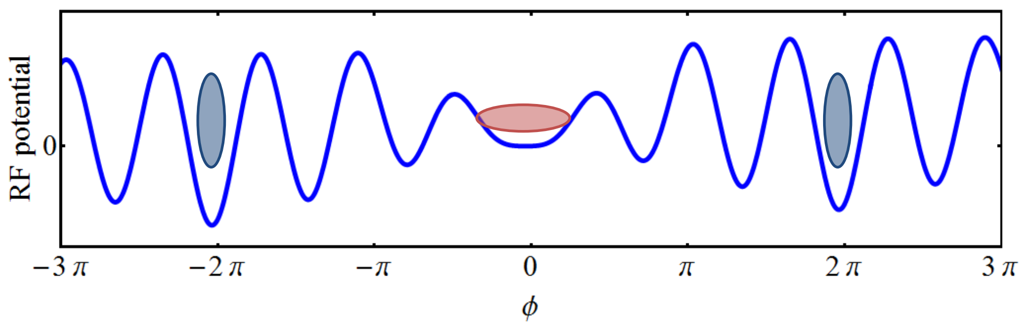
- **Future 3-harmonic RF system for BESSY VSR**

RF voltage sum



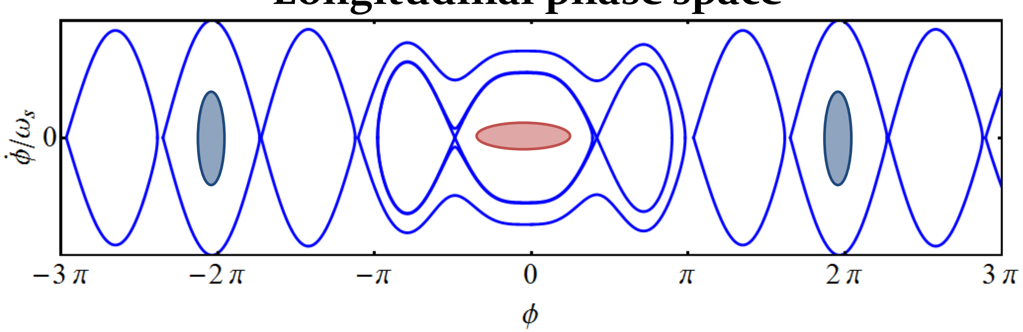
RF voltages sum up or cancel

RF Potential



Short deep or bathtub potential well

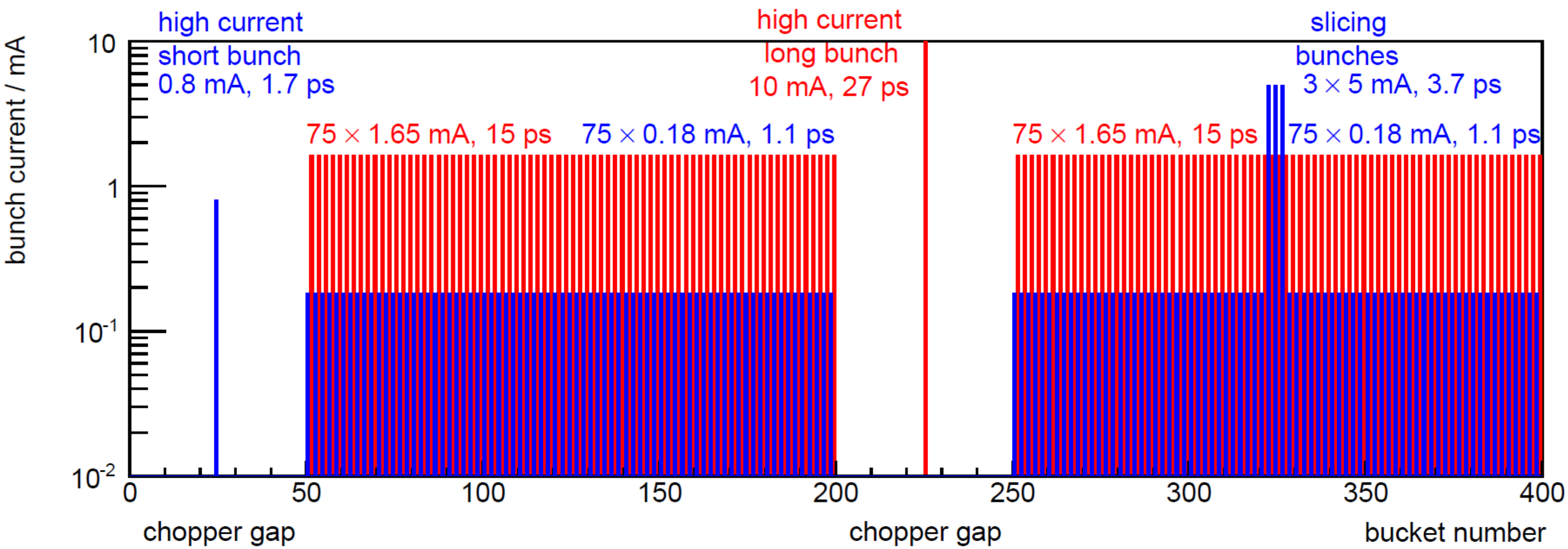
Longitudinal phase space



Short or stretched bucket

Filling pattern

Markus Ries et al.



- 300 mA average current

→ High-current single bunches

→ short (0.8 mA) & long (10 mA)

→ Special high-current density bunches

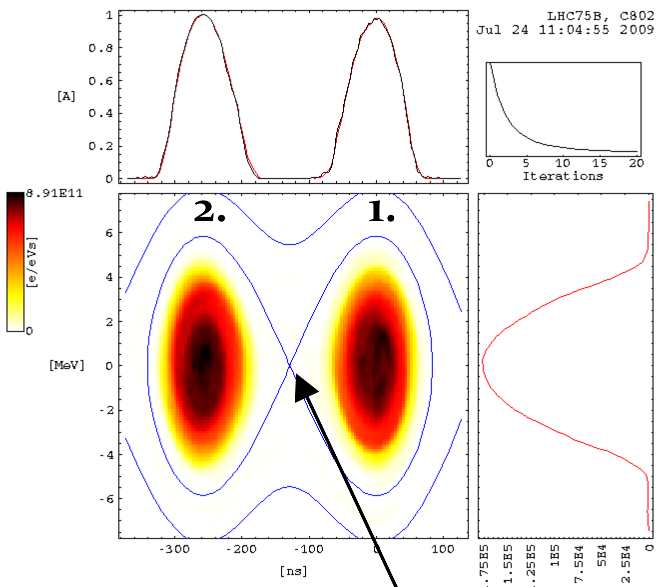
♨ Two electron storage ring in one

♨ Thanks to longitudinal beam dynamics trick

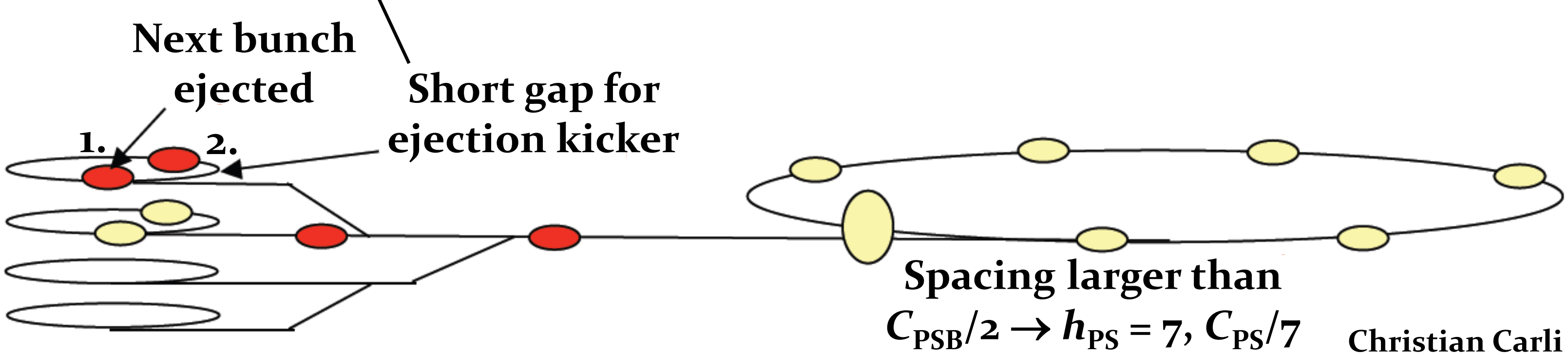
Example: adjust bunch spacing

- Was used at CERN PSB-to-PS to transfer 2 bunches at once
- Circumference ratio $C_{PS}/C_{PSB} = 4$

→ Ratio virtually moved to 2/7: use $h_{RF} = 2 + 1$



1. Add $h1$ component such that bunches approach to 245 ns (small spacing) → big spacing becomes 327 ns
2. Synchronize on $h1$ to the PS
3. Trigger extraction kicker in-between the small spacing
4. Eject two bunches per ring at a distance of 327 ns



Christian Carli

Introduce general non-linearity

Replace $V \sin\phi \rightarrow V g(\phi)$ $\rightarrow$ **arbitrary amplitude**

Equations of motion

$$\frac{d}{dt}\phi = \frac{h\eta\omega_{\mathrm{rev}}}{pR}\left(\frac{\Delta E}{\omega_{\mathrm{rev}}}\right)$$

$$\frac{d}{dt}\left(\frac{\Delta E}{\omega_{\mathrm{rev}}}\right) = \frac{qV}{2\pi}\left[g(\phi) - g(\phi_{\mathrm{S}})\right]$$

same structure ⟷

$$\frac{dq}{dt} = \frac{\partial H}{\partial p}$$

$$\frac{dp}{dt} = -\frac{\partial H}{\partial q}$$

The Hamiltonian describing the system becomes

$$H\left(\phi, \frac{\Delta E}{\omega_{\mathrm{rev}}}\right) = \frac{1}{2}\frac{h\eta\omega_{\mathrm{rev}}}{pR}\left(\frac{\Delta E}{\omega_{\mathrm{rev}}}\right)^2 - \frac{qV}{2\pi}\left[\int g(\phi)d\phi - g(\phi_{\mathrm{S}})\phi\right]$$

$$\eta = \frac{1}{\gamma_{\mathrm{tr}}^2} - \frac{1}{\gamma^2}$$

Arbitrary RF waveform

$$H\left(\phi, \frac{\Delta E}{\omega_{\mathrm{rev}}}\right) = \frac{1}{2}\frac{h\eta\omega_{\mathrm{rev}}}{pR}\left(\frac{\Delta E}{\omega_{\mathrm{rev}}}\right)^2 - \frac{qV}{2\pi}\left[\int g(\phi)d\phi - g(\phi_{\mathrm{S}})\phi\right]$$

Using $$\dot{\phi} = \frac{h\eta\omega_{\mathrm{rev}}}{pR}\left(\frac{\Delta E}{\omega_{\mathrm{rev}}}\right)$$

The Hamiltonian can be rewritten, with RF potential *W*(ϕ)

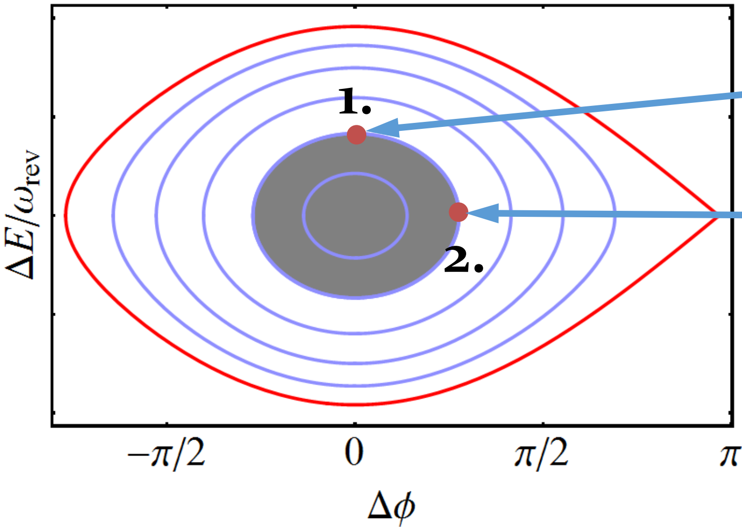
$$H(\phi, \dot{\phi}) = \frac{1}{2}\left(\frac{\dot{\phi}}{\omega_S}\right)^2 + W(\phi)$$

$$W(\phi) = \frac{1}{\cos\phi_{\mathrm{S}}}\left[\int g(\phi)\,d\phi - g(\phi_{\mathrm{S}})\phi\right]$$

Longitudinal beam manipulations using non-linearity

Change RF voltage to change bunch length?

→ **Calculate aspect ratio of bucket trajectories**



$$H\left(\Delta\phi = 0, \frac{\Delta E}{\omega_{\mathrm{rev}}}\right) = \frac{1}{2}\frac{h\eta\omega_{\mathrm{rev}}}{pR}\left(\frac{\Delta E}{\omega_{\mathrm{rev}}}\right)^2$$

$$H\left(\Delta\phi, \frac{\Delta E}{\omega_{\mathrm{rev}}} = 0\right) = -\frac{1}{2}\frac{qV}{2\pi}\cos\phi_S \Delta\phi^2$$

Equating both sides gives

$$\left(\frac{\Delta E}{\Delta\tau}\right)^2 = -\frac{qV}{2\pi}E\beta^2 h\omega_{\mathrm{rev}}^2\frac{\cos\phi_S}{\eta}$$

with emittance as $\varepsilon_l = \pi\Delta\tau\Delta E = \mathrm{const.}$ $\rightarrow$ $\Delta\tau \propto \frac{1}{\sqrt[4]{V}}$

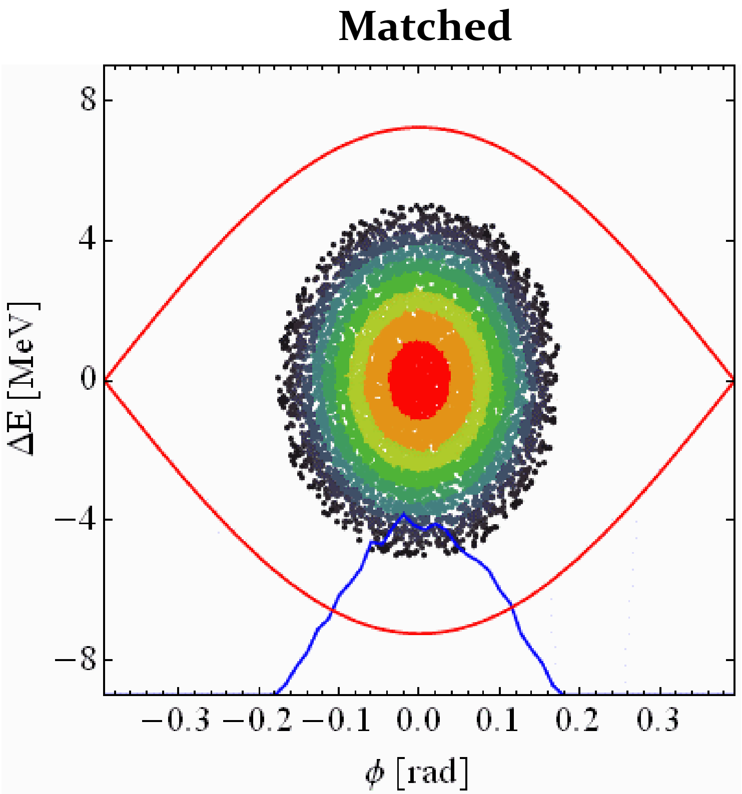
→ **Not efficient at all**

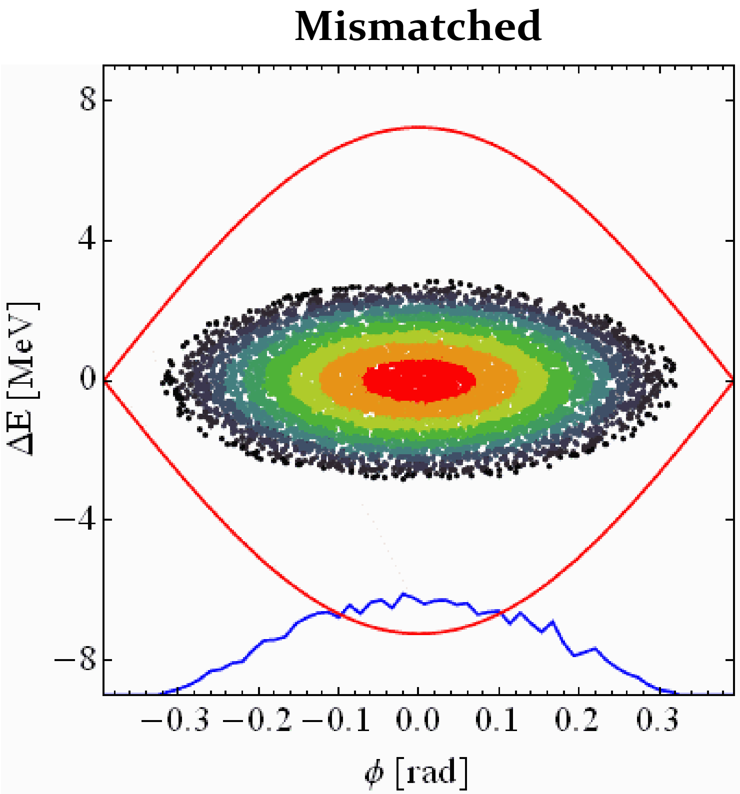
→ **16 times more RF voltage needed to cut bunch length in half**

Abrupt change of RF voltage

→ **Individual particles in matched bunch oscillate but no macroscopic motion**

→ **Abruptly changing the RF voltage flips particles to new trajectories**





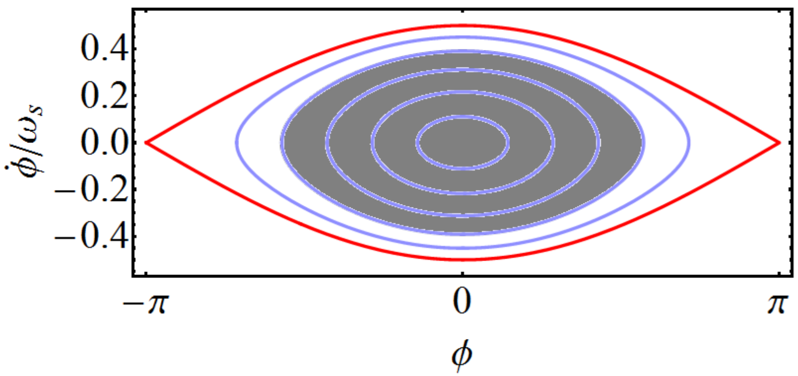
→ **The bunch distribution seems to rotate**

→ **Exchange of bunch length and momentum spread**

Introduce sudden change: bunch rotation

→ **Quickly exchange longitudinal phase space behind bunch**

→ **Increase RF voltage much faster than period of f_S**



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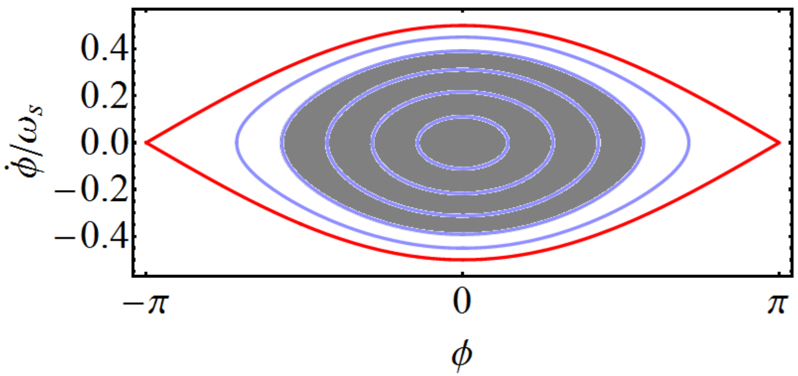
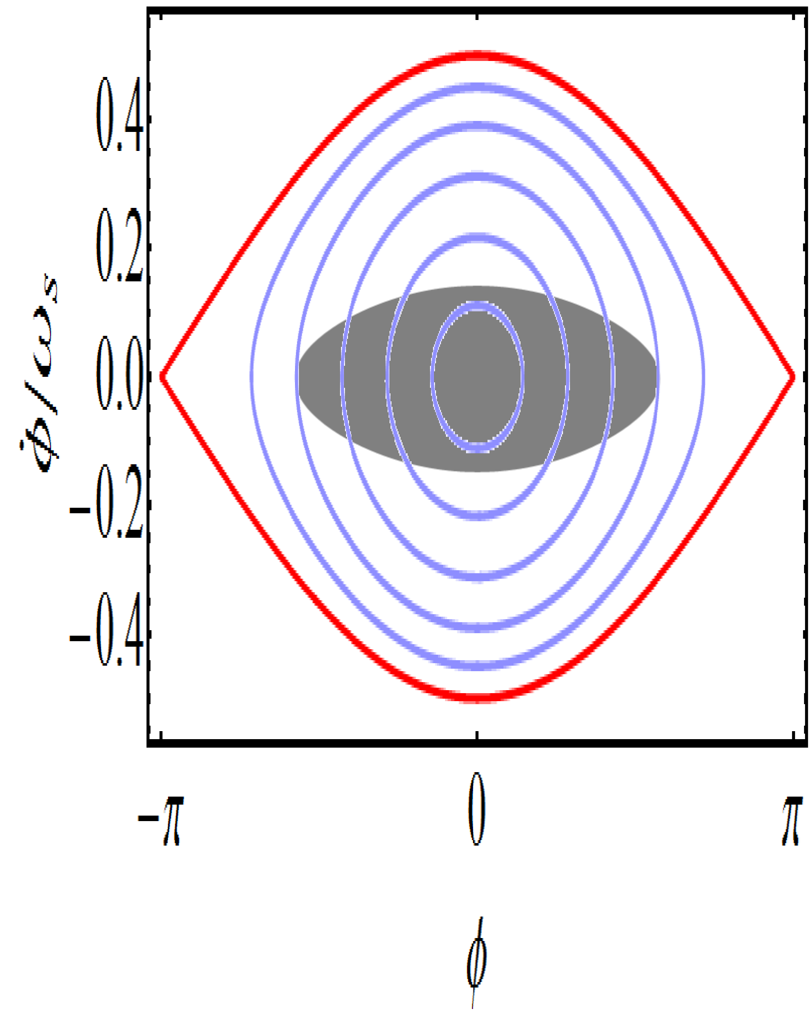
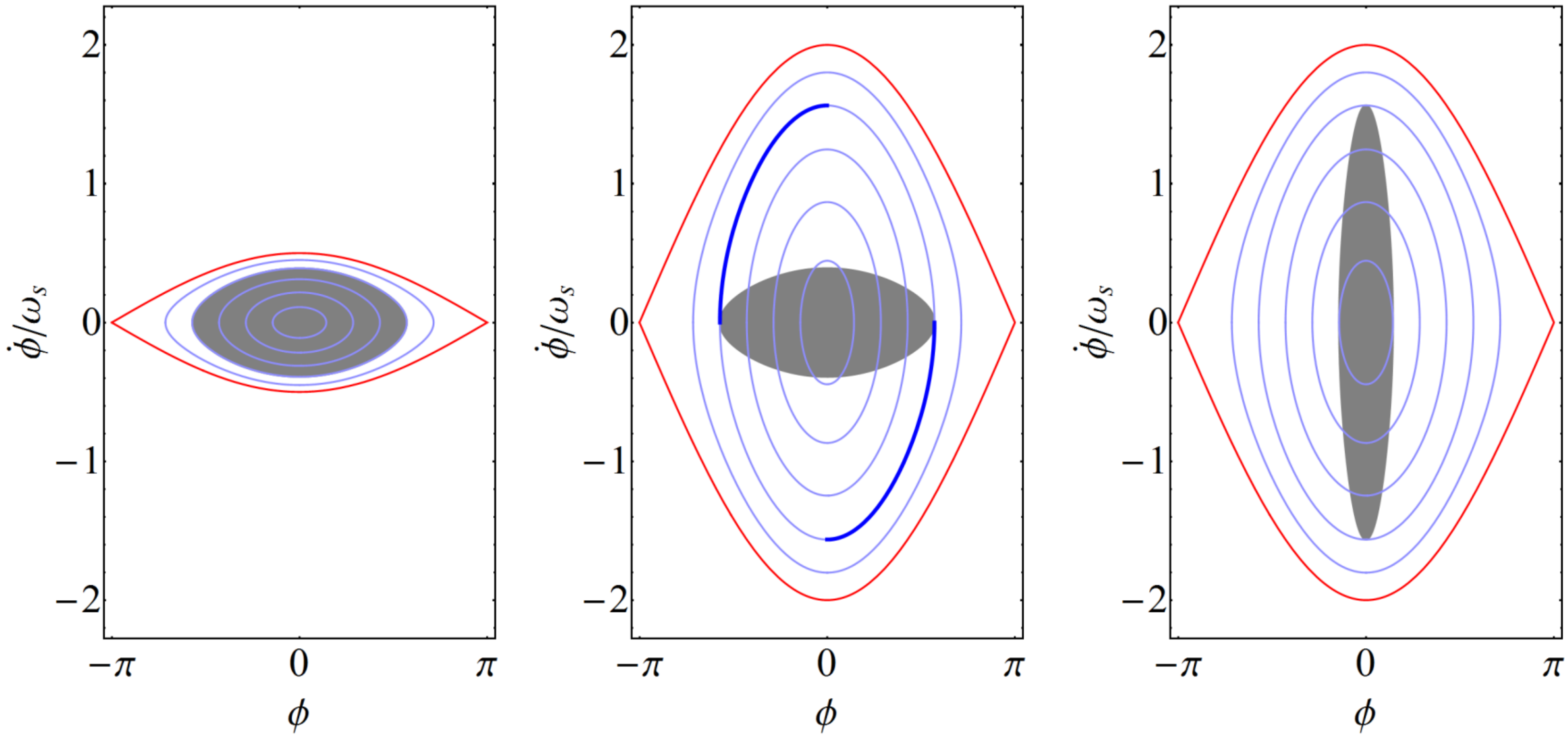




Introduce sudden change: bunch rotation

→ **Switch RF voltage much faster than period of f_s**

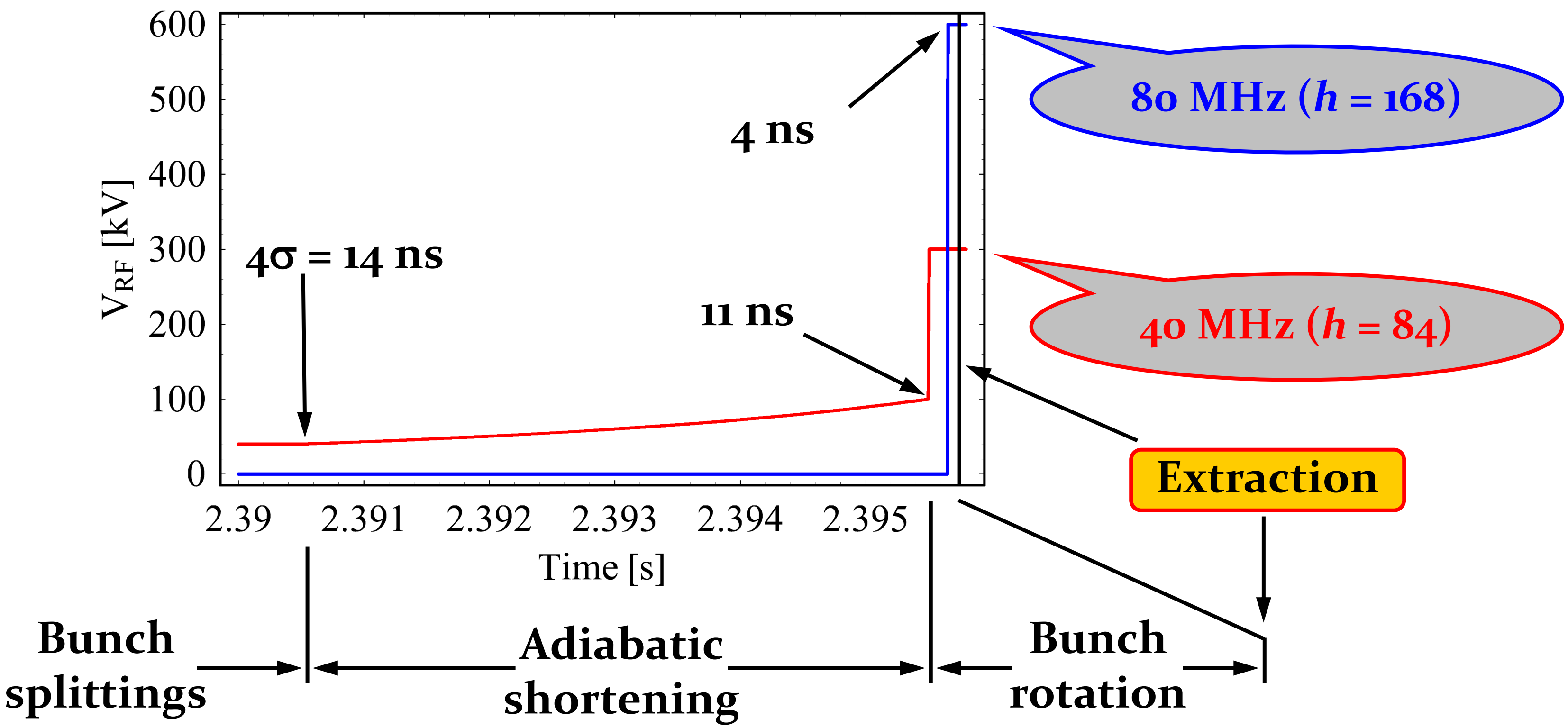


$$V_i \propto \left(\frac{\Delta E_i}{\Delta \tau_i}\right)^2 \qquad V_f \propto \left(\frac{\Delta E_f}{\Delta \tau_i}\right)^2 \qquad \frac{\Delta \tau_f}{\Delta \tau_i} = \frac{\Delta E_i}{\Delta E_f} = \sqrt{\frac{V_i}{V_f}}$$

Example: PS to SPS transfer at CERN

- **Fit 14 ns long bunches into 5 ns long buckets in the SPS**

→ Double-step bunch rotation



Example: rotation at PS-SPS transfer

→ Bunch length now proportional to $1/\sqrt{V}$ and not $1/\sqrt[4]{V}$

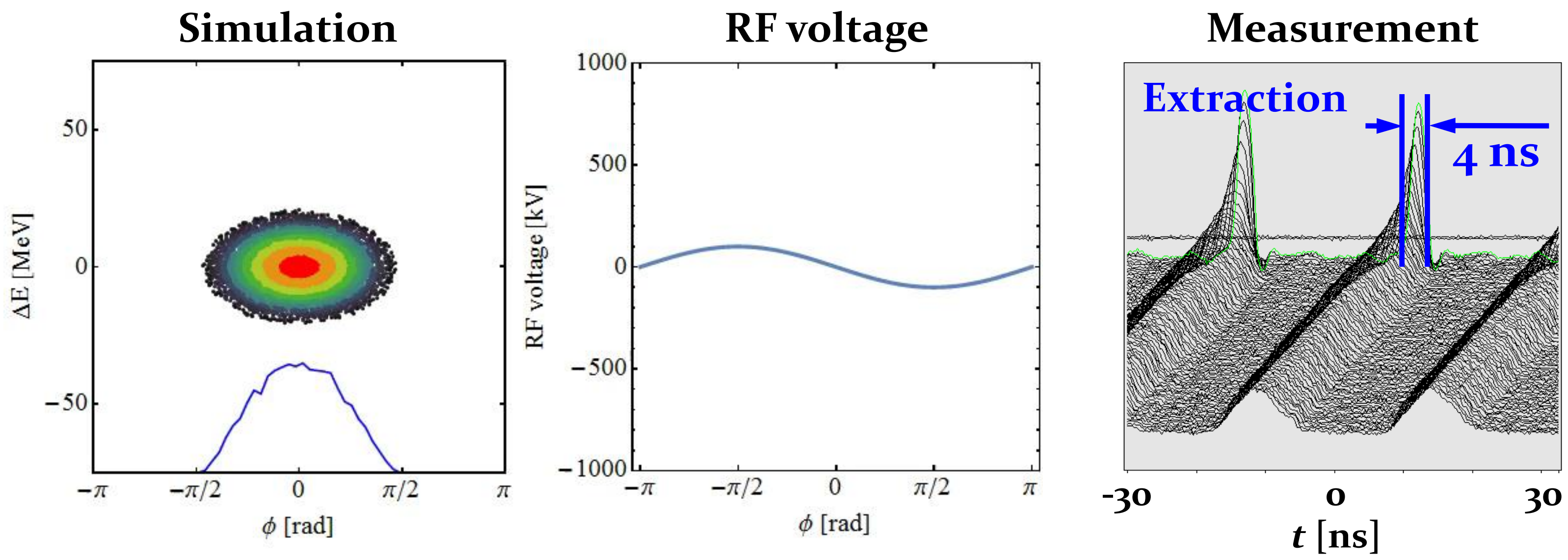
→ Can save enormous RF voltage

→ Bunch shortening from 14 ns to 4 ns (ratio ~3.5)

→ Starting from 100 kV at 40 MHz

→ Slow shortening would require $100\ \text{kV} \cdot 3.5^4 \sim 15\ \text{MV}$

→ Installed RF voltage is only about 1.2 MV

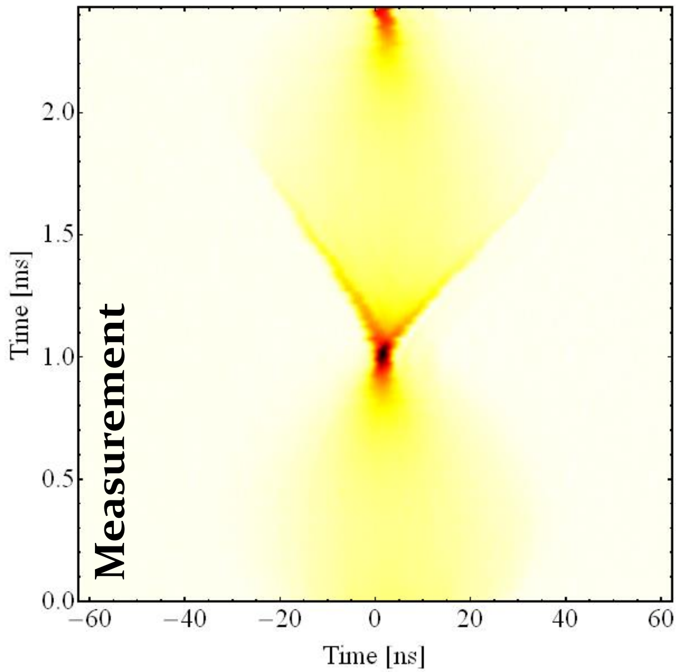


Profiting from the non-linear rotation

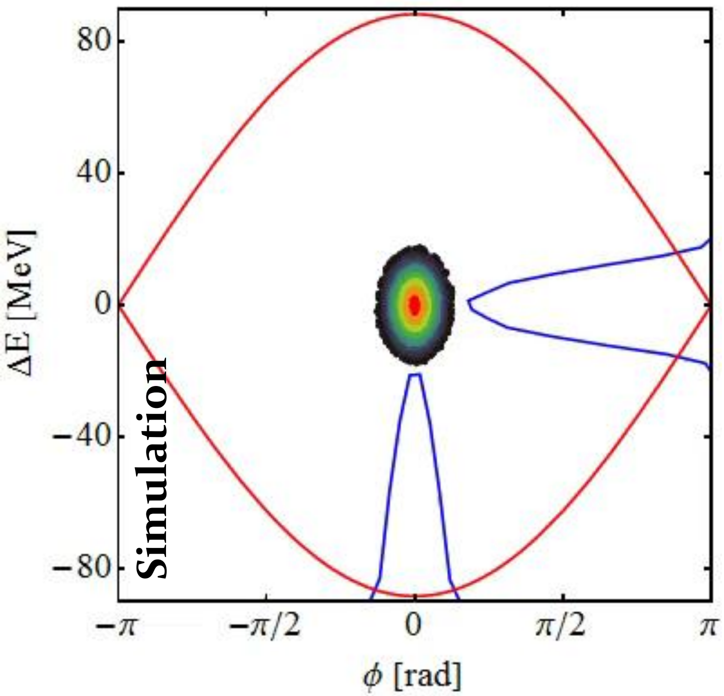
Need large momentum spread for slow extraction

1. **Jump RF phase such that bunch at unstable fixed point**
2. **Jump back**
3. **Let bunch rotate, switch RF off at large momentum spread**

Bunch profile evolution



Longitudinal phase space

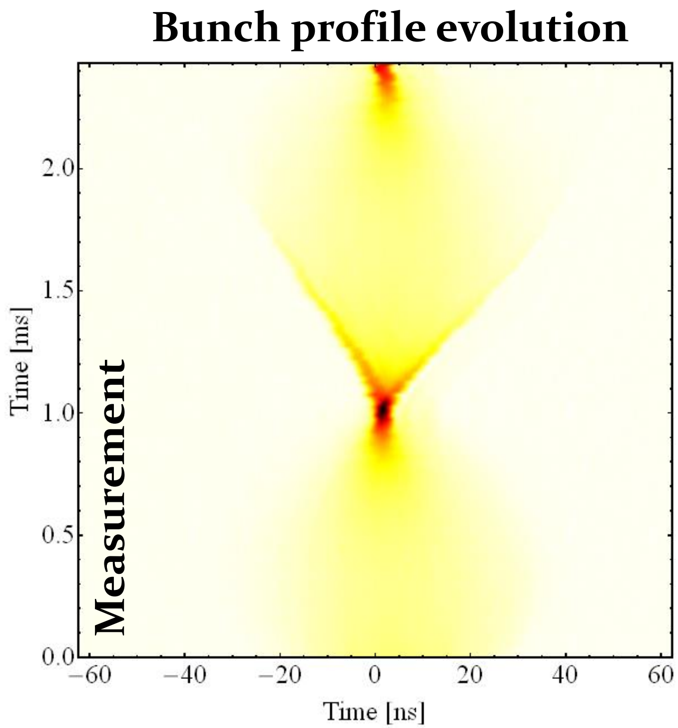


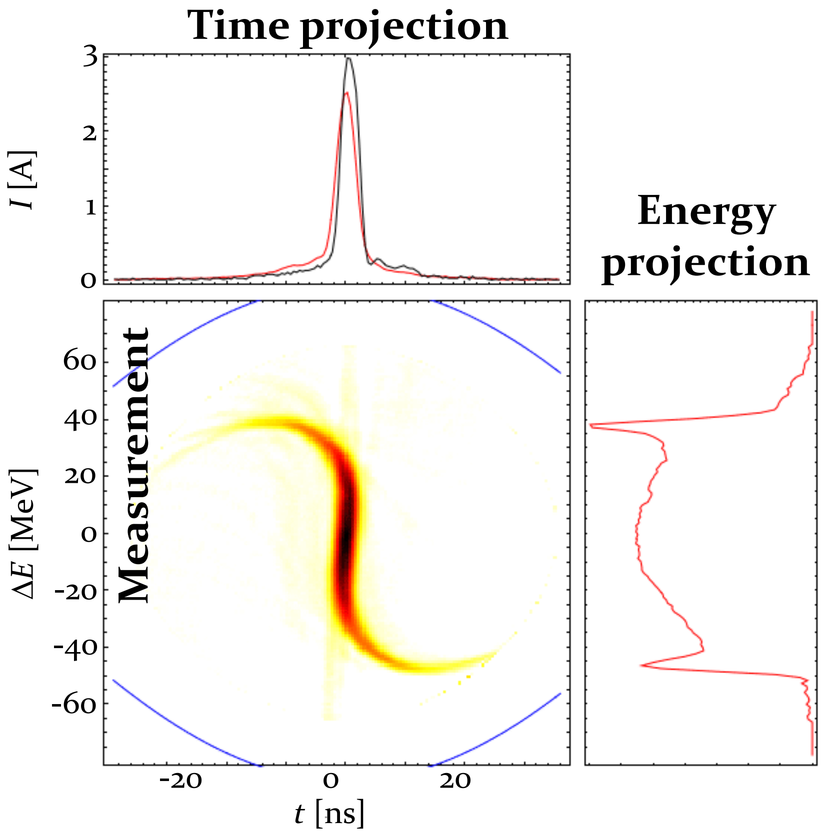
→ **Non-linearly of bunch rotation helps**

Example: using the non-linearity

Need large momentum spread for slow extraction

1. **Jump RF phase such that bunch at unstable fixed point**
2. **Jump back**
3. **Let bunch rotate, switch RF off at large momentum spread**





→ Almost constant momentum distribution after rotation

Synchrotron frequency distribution

General synchrotron frequency

- **Synchrotron frequency depends on trajectory**

→ Calculate average velocity for given trajectories in longitudinal phase space → Action angle, *J*

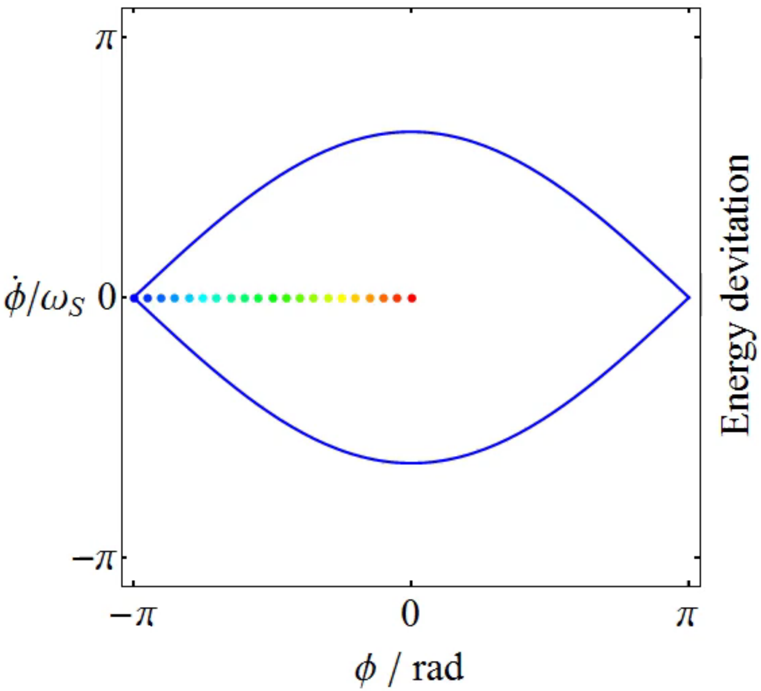
$$J(H) = \frac{1}{2\pi\omega_S} \oint \dot{\phi}(\phi)\, d\phi$$

The angular frequency becomes

$$\omega(H) = \frac{d}{dJ} H$$

General expression for ω_S

$$\frac{\omega(H)}{\omega_S} = \frac{\sqrt{2}\pi}{\int_{\phi_l}^{\phi_u} \frac{1}{\sqrt{H/\omega_S^2 - W(\phi)}}\, d\phi}$$



(for bucket boundaries $\phi_l \rightarrow \phi_u$)

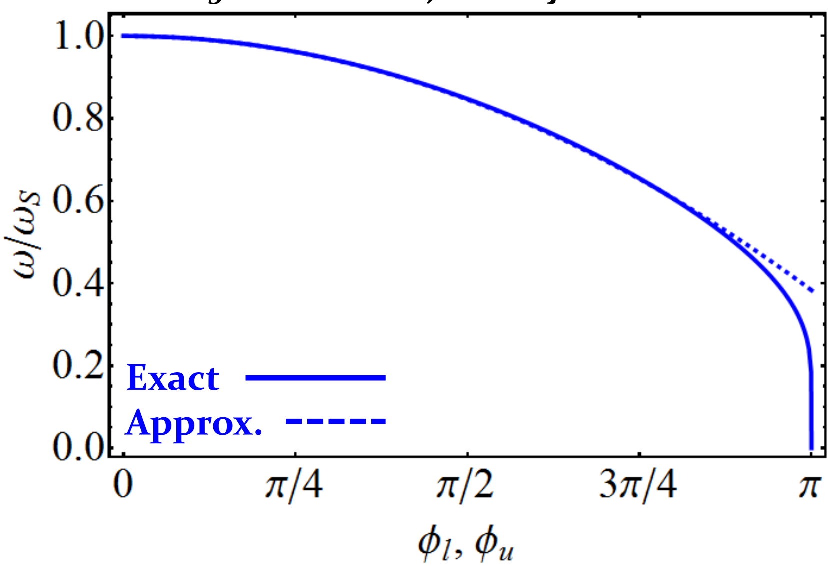
Distribution for stationary bucket

- **Single-harmonic RF in stationary bucket**

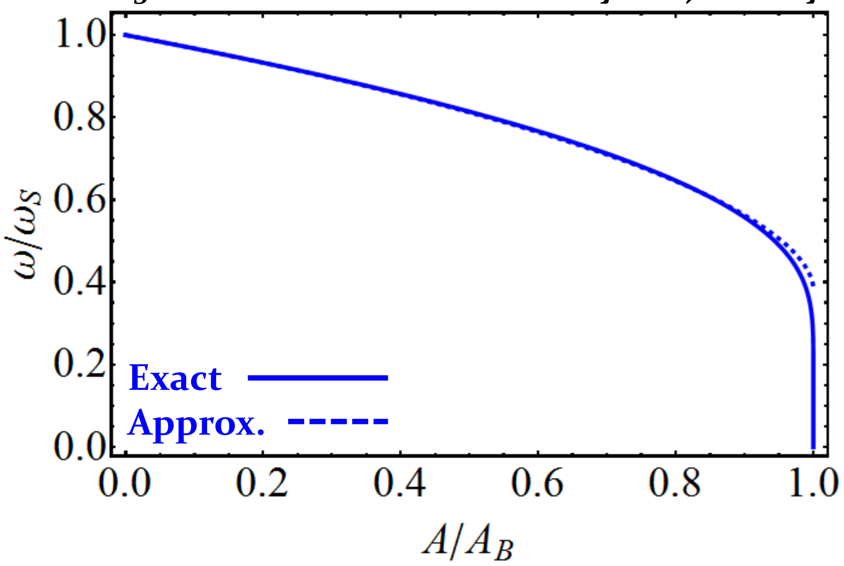
$$\frac{\omega(\Delta\phi_u)}{\omega_S} = \frac{\pi}{2K[\sin(\phi_u/2)]} \simeq 1 - \frac{\phi_u^2}{16}$$

***K*(*x*): 1st kind elliptical integral function**

ω_S versus trajectory duration



ω_S vs. surface encircled by trajectory



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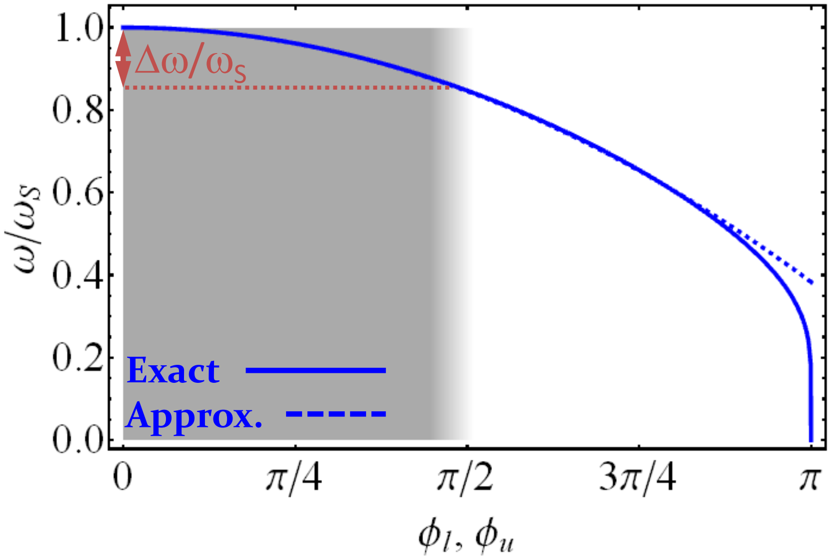

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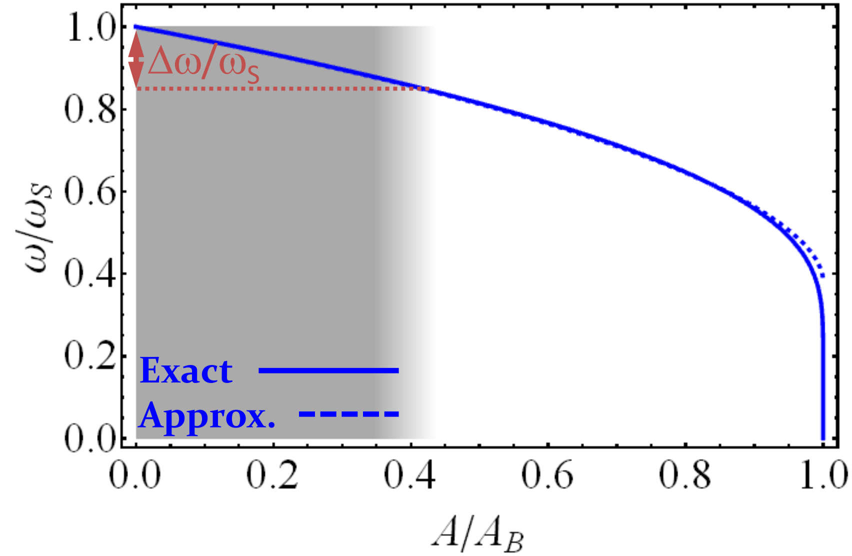


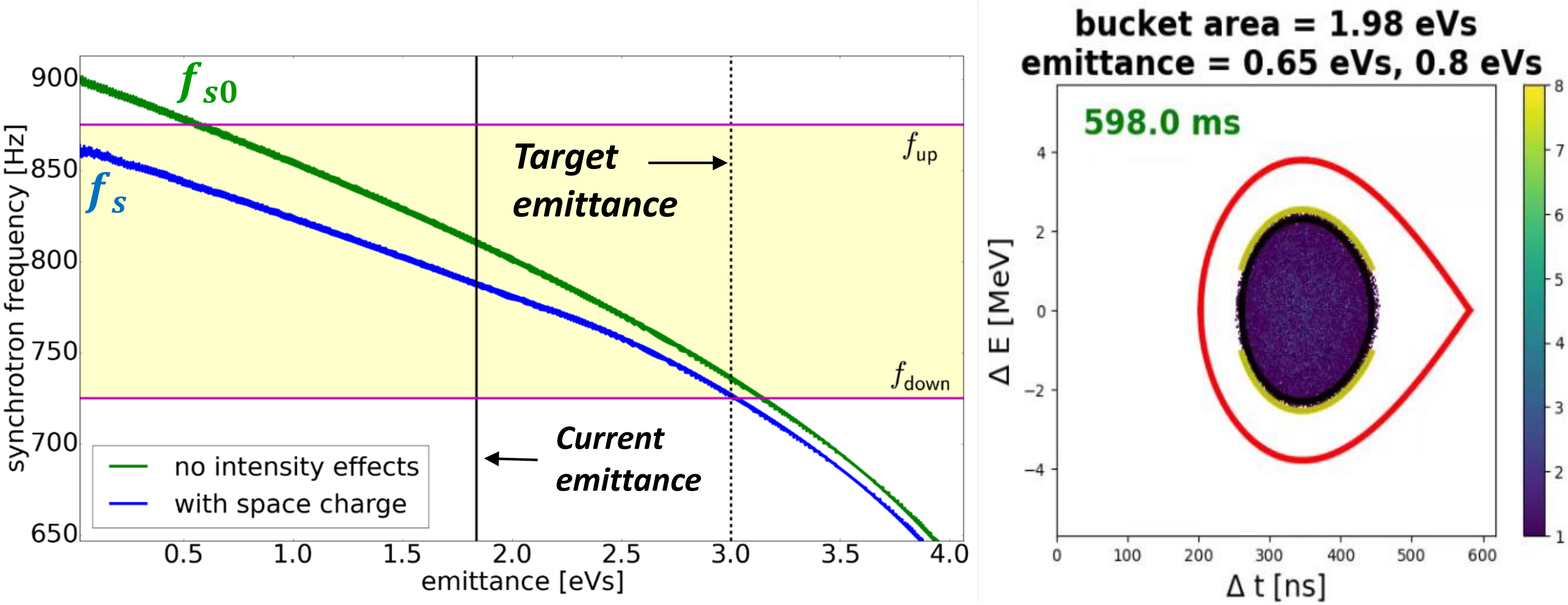
→ **Different synchrotron frequencies of particles in bunch**

→ **Total spread $\Delta\omega/\omega_S$ depends on filling factor of bucket**

Example: Emittance control with noise

- **Noise excitation of bunch by band-width limited noise**

→ **Controlled longitudinal blow-up in the PSB**

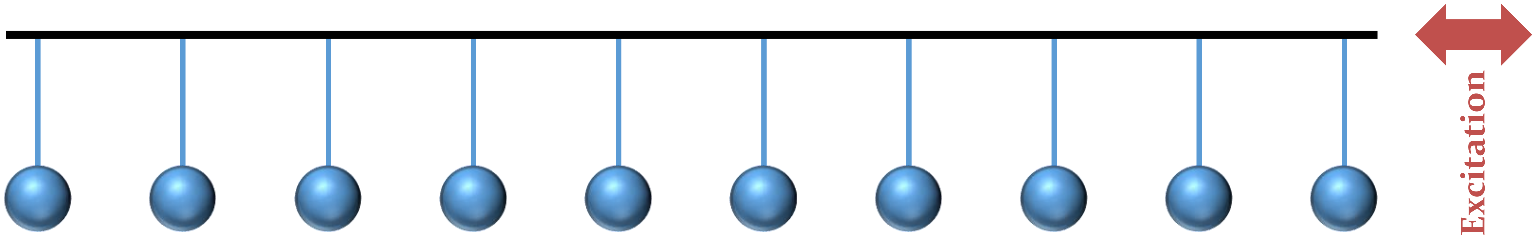


1. **Choose upper frequency to cover synchrotron frequency at bunch centre**
2. **Choose lower frequency to match target emittance**
3. **Excite**

D. Quartullo

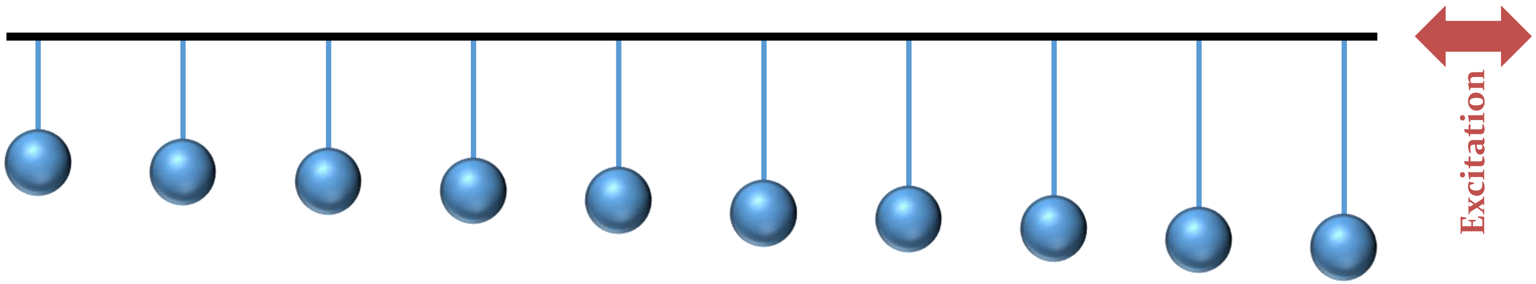
Analogy: pendulums mounted on a bar

- **All particles have the same resonance frequency**



→ **Easy to excite macroscopic oscillation**

- **Resonance frequencies of individual particles varies**

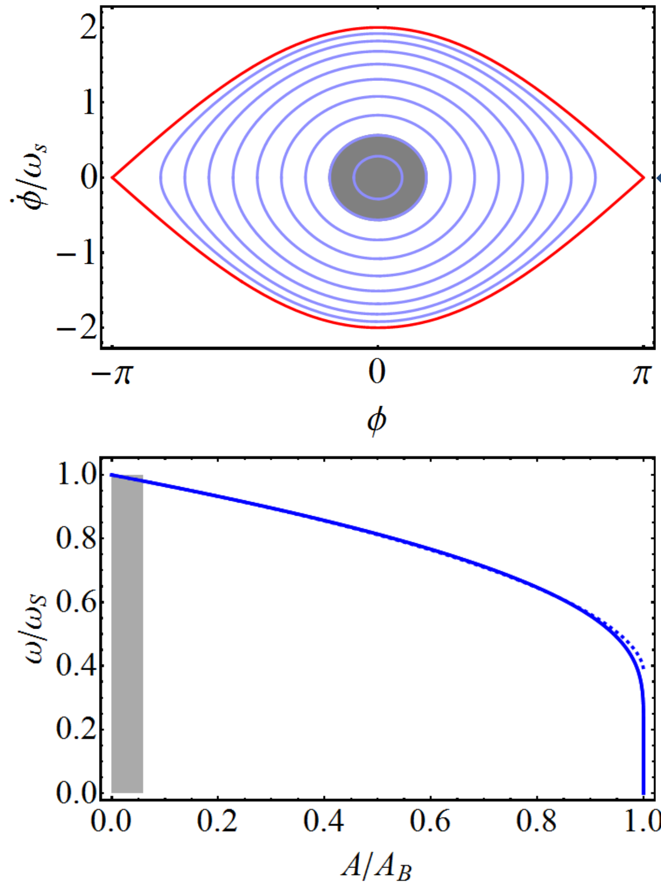


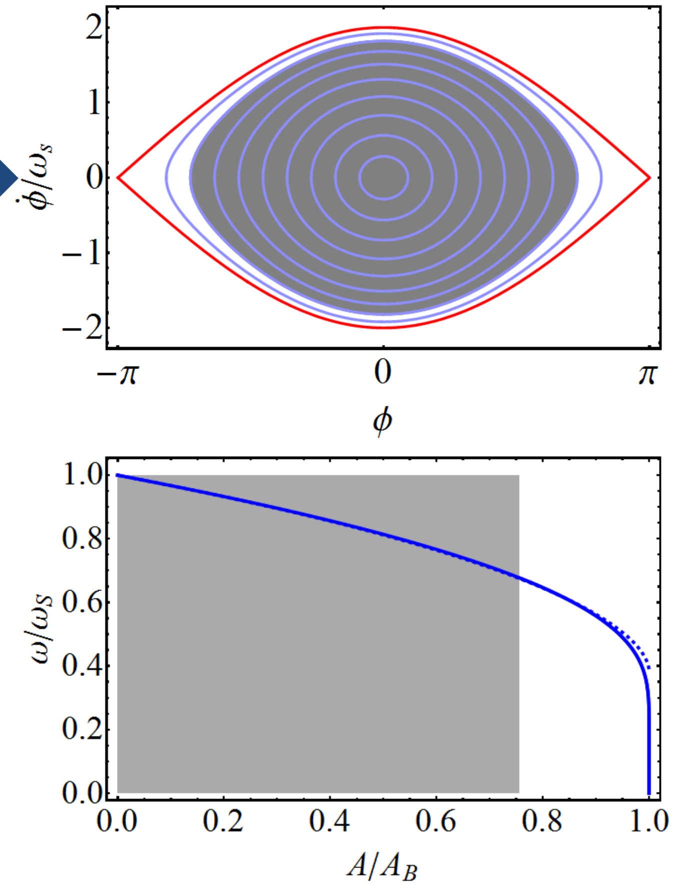
→ **Difficult to excite macroscopic oscillation**

→ **Large synchrotron frequency spread increases stability**

Bucket filling ratio

Smaller or larger bunch or bucket? What is more stable?





- **Easy to excite**

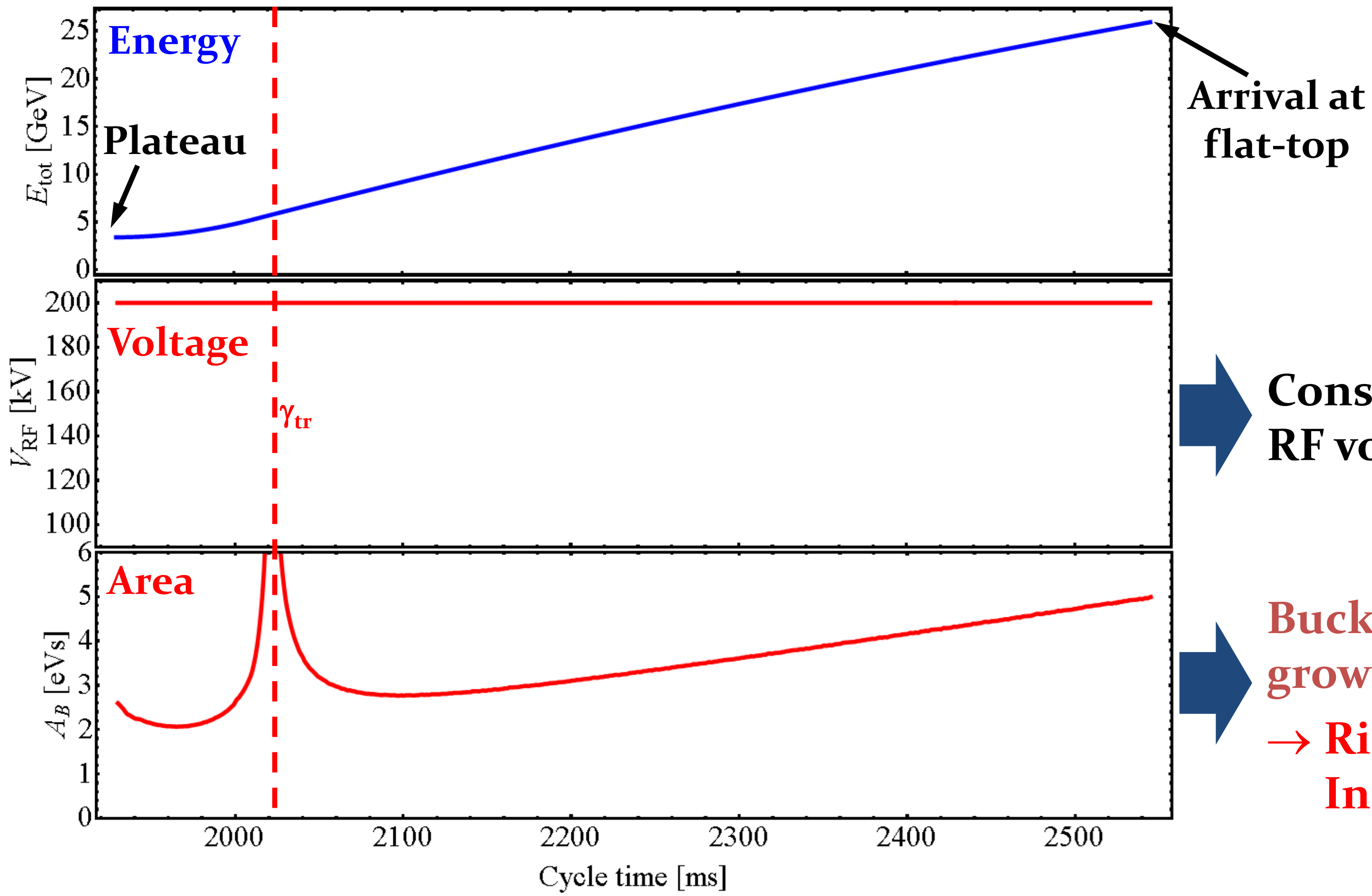
→ Prone to instability

- **Large f_s spread**

→ More stable

Example: stabilization with lower voltage

→ **Acceleration of protons in the CERN PS** (E_{total} = 3.4 → 26 GeV)



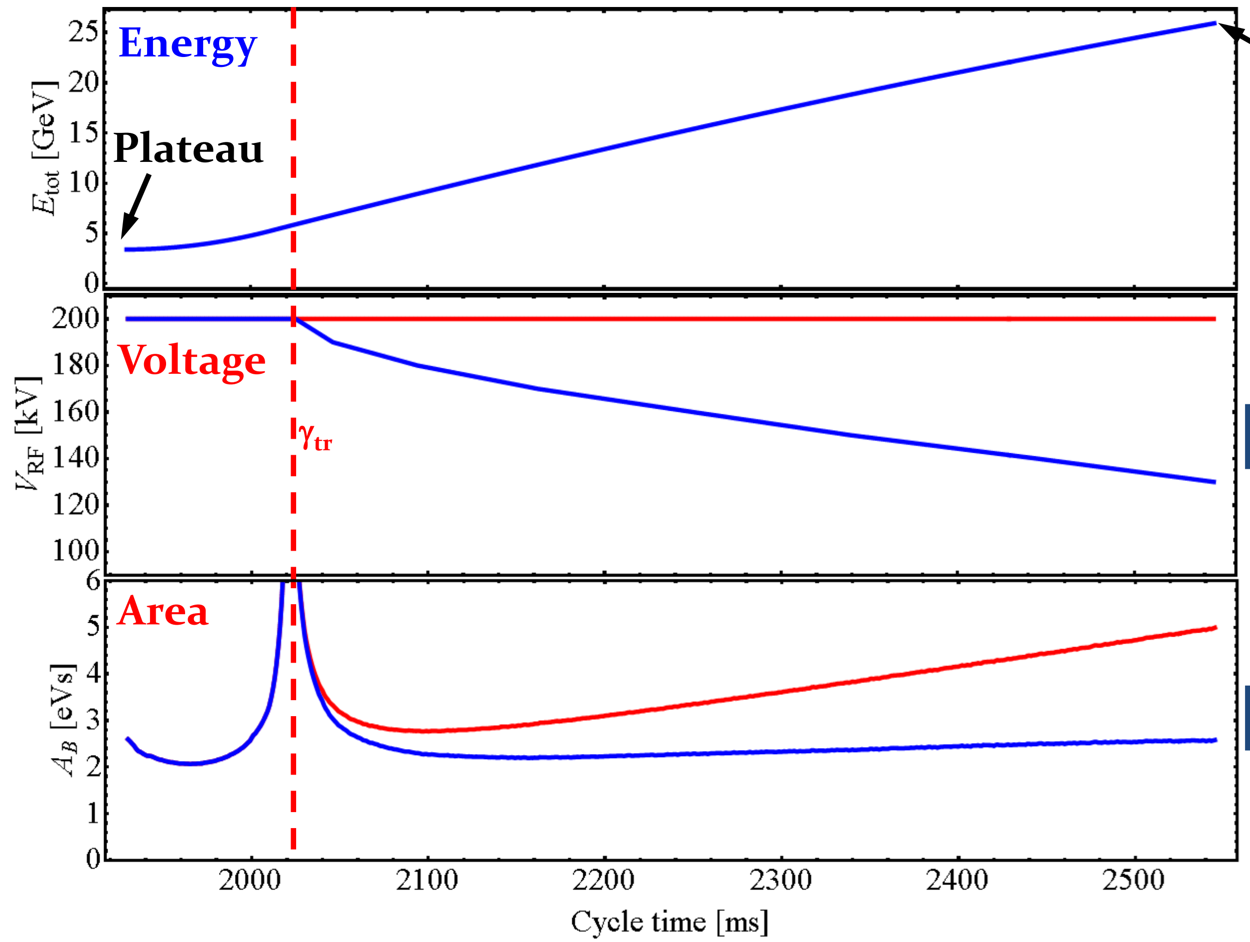
Constant RF voltage

Bucket area grows

→ Risk of Instability

Example: stabilization with lower voltage

→ **Acceleration of protons in the CERN PS (3.4 → 26 GeV total)**



Constant RF voltage

Bucket area constant → Instability

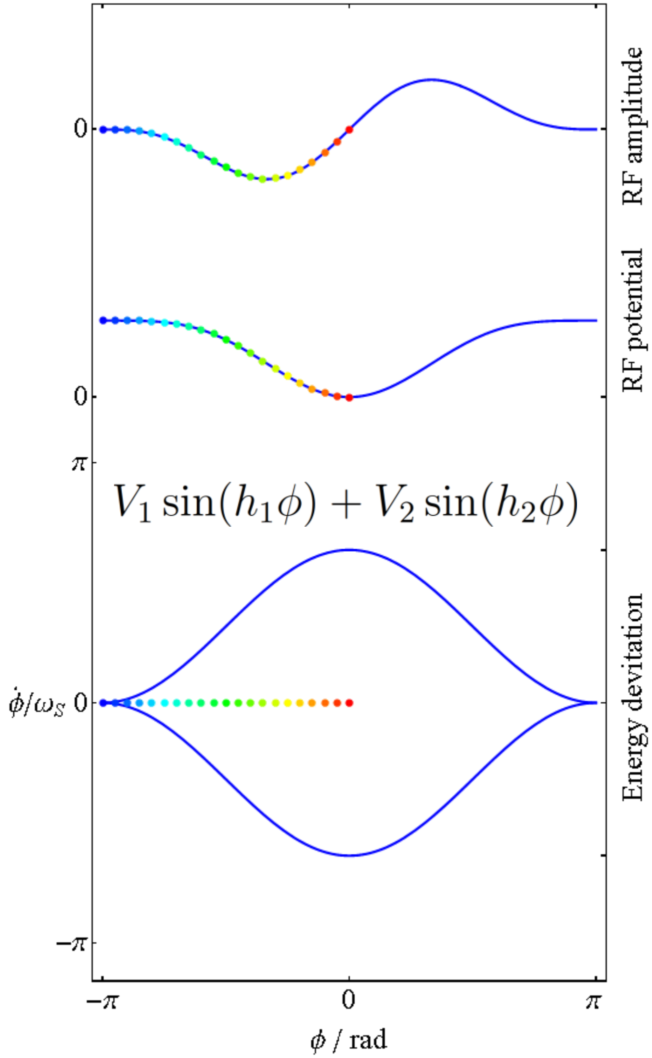
- **Same principle also applied in SPS and LHC**

→ **Prevent bucket filling to decrease**

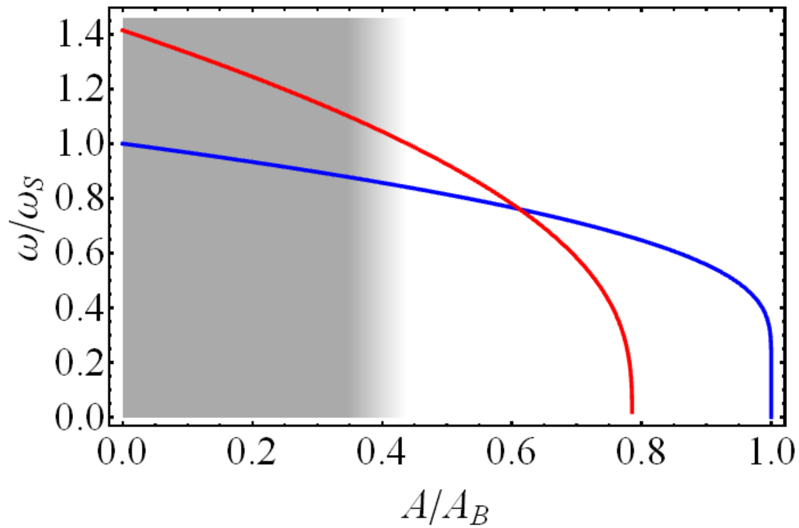
Additional non-linearity by double RF

→ RF system at twice the main frequency and at half amplitude

$V_2/V_1 = 1/2$, $h_2/h_1 = 2$



Synchrotron frequency



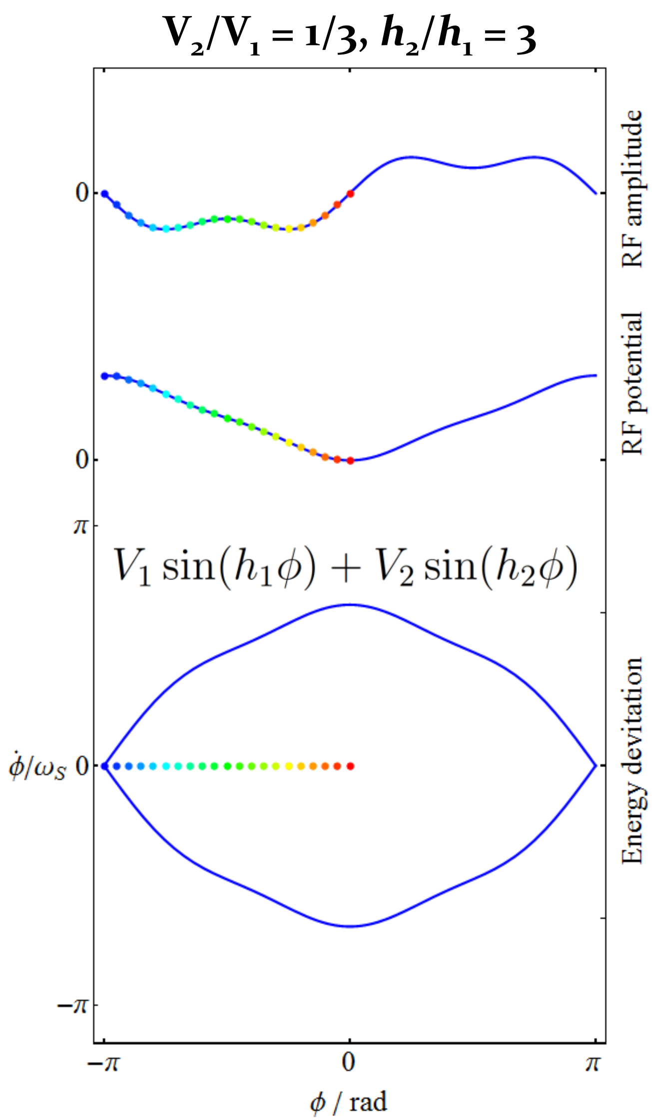
- **Both RF systems in phase**

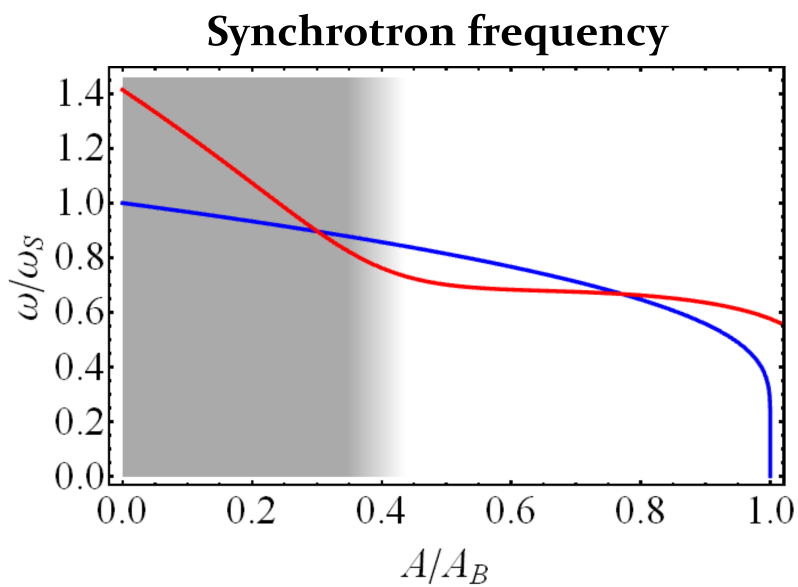
→ **Important increase in synchrotron frequency spread**

→ **Improves stability**

Additional non-linearity by double RF

→ RF system at twice the main frequency and at half amplitude





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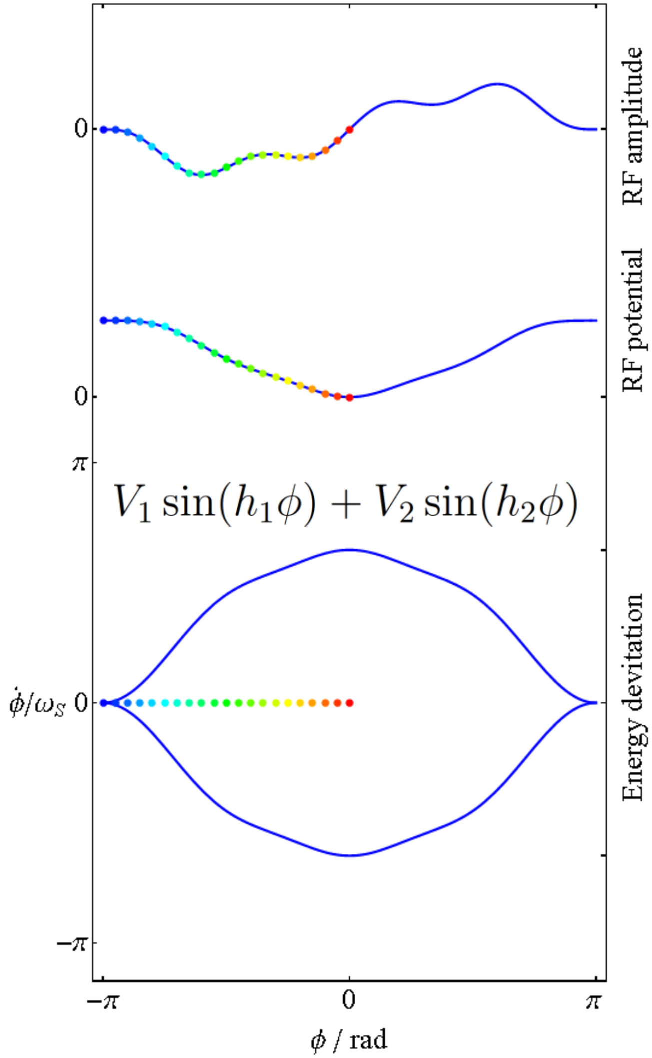
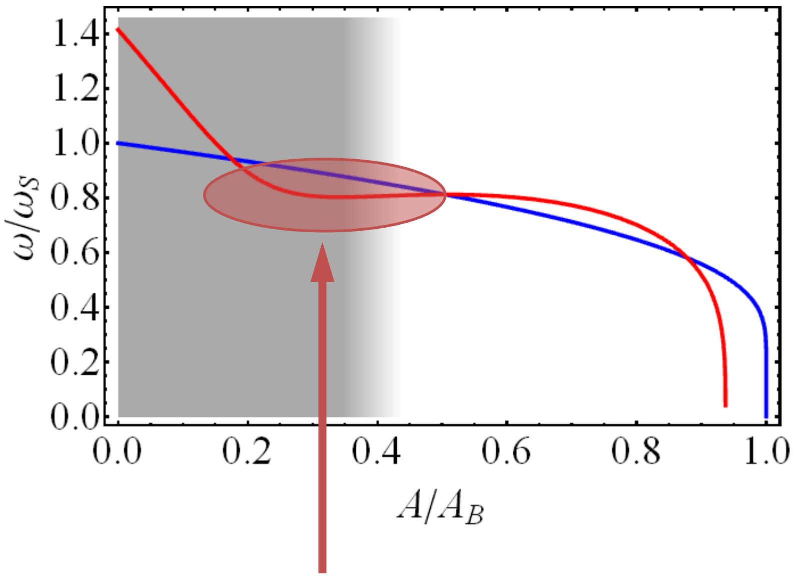




- **Local regions of bunch with no f_S gradient**

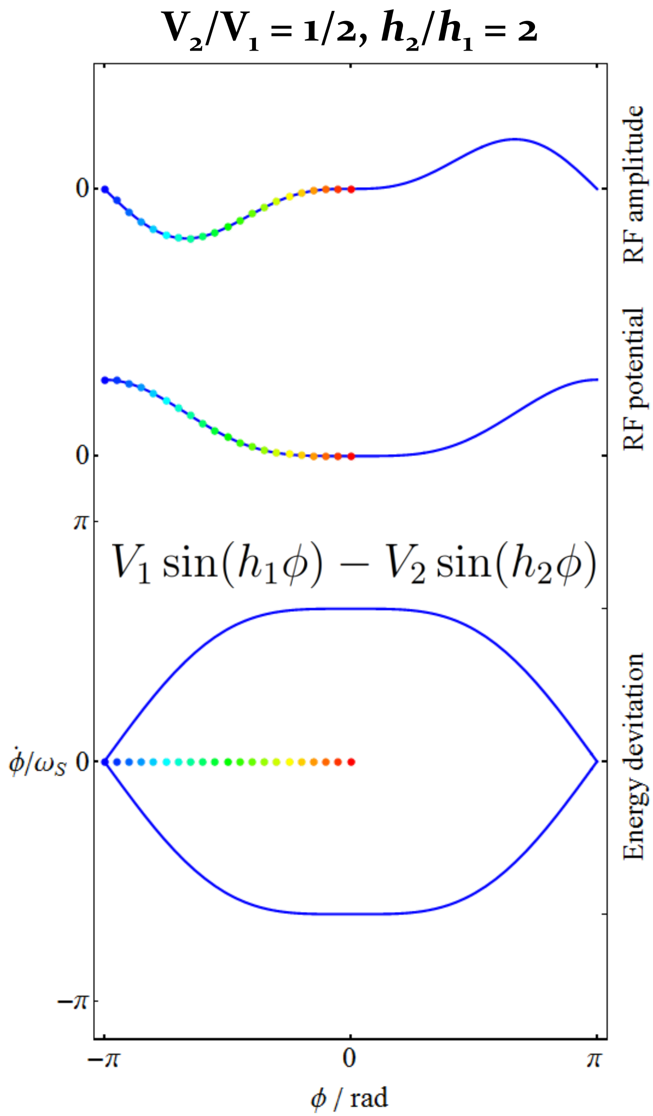
→ **Again prone to instability**

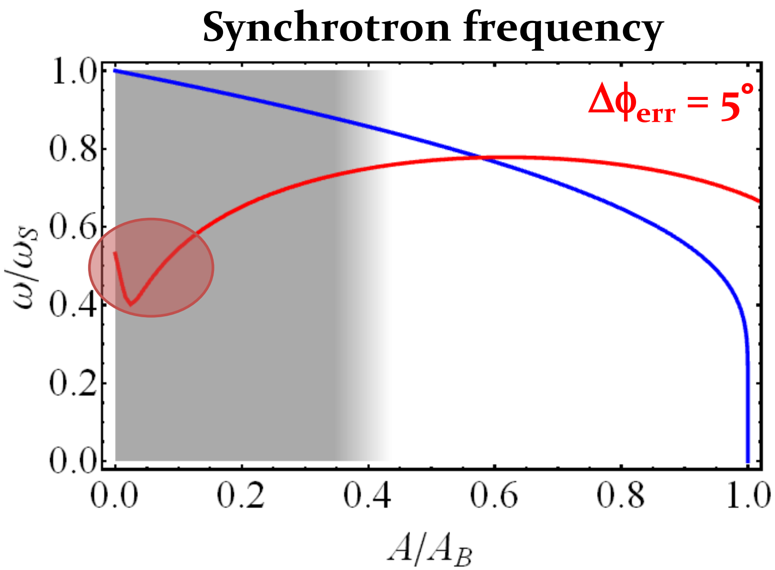
→ **Reduce voltage of 2nd harmonic RF system**

→ **Improving stability depends on appropriate voltage ratio**

Two RF systems in counter-phase?

→ 2^{nd} RF twice frequency, half amplitude in counter-phase





- Large frequency spread at bunch centre with perfectly adjusted phases

→ Minor phase offset causes locally unstable regions

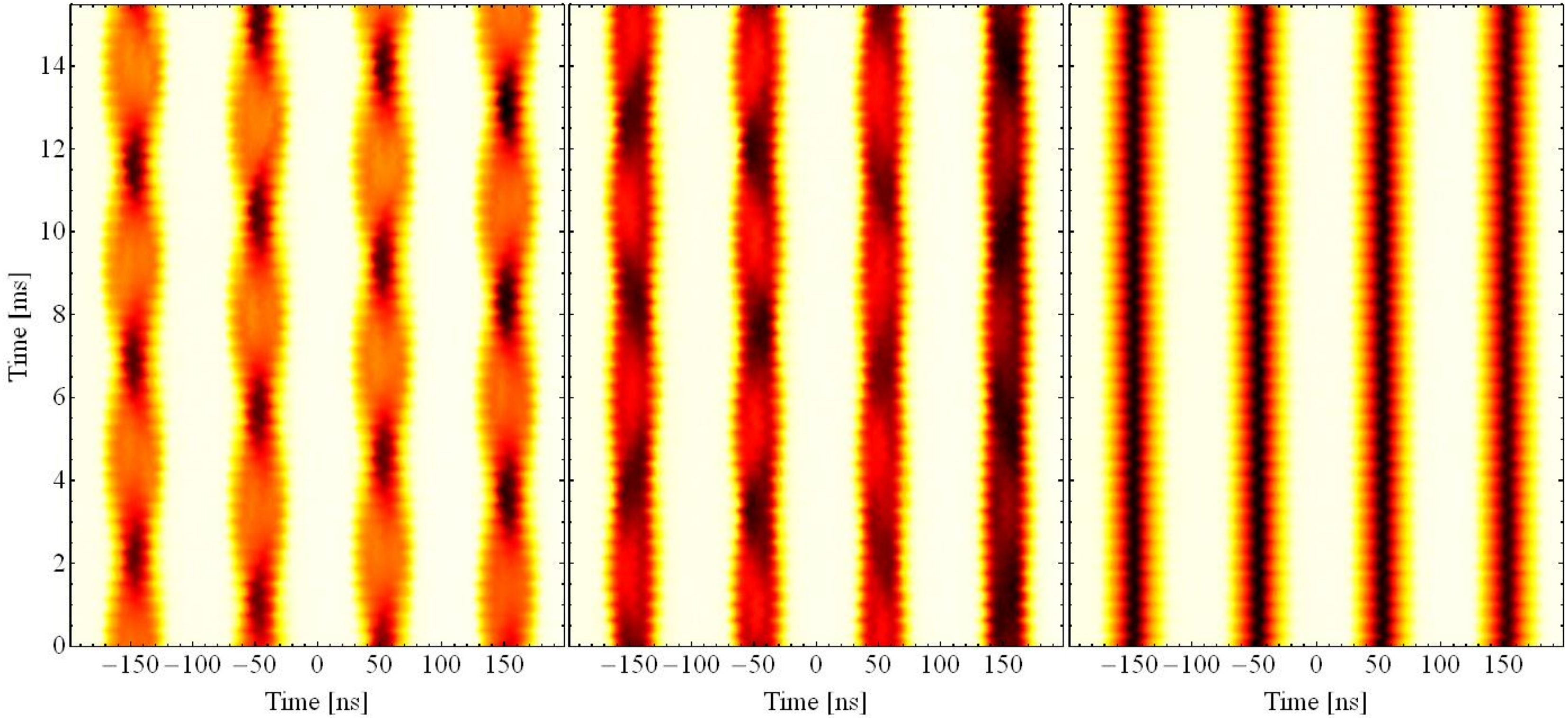
→ Works only for very short bunches

→ Electron accelerators

Example: damping observations in the PS

- **Quadrupolar coupled-bunch oscillations at flat-top**
- **Main RF system:** $h_1 = 21$, 10 MHz, 4 out of 18 bunches
- **Higher-harmonic RF system:** $h_2 = 84$, 40 MHz

Single-harmonic RF | **(Imperfect) counter-phase** | **Double RF, in phase**



Both RF systems in phase:

→ Highest peak current, but most stable

Summary

- **Longitudinal beam dynamics**

  → Everything non-linear

- **Longitudinal manipulations**

  → Tricks to adjust length and distance of bunches

  → Do more with less RF

- **Synchrotron frequency spread**

  → More RF voltage may result in less stability

  → Higher peak density may be more stable

  → Improve stability and control emittance

A big Thank You

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**Wolfgang Höfle, Andreas Jankowiak, Erk Jensen,
Danilo Quartullo, Markus Ries, Elena Shaposhnikova,
Frank Tecker**

Thank you very much for your attention!

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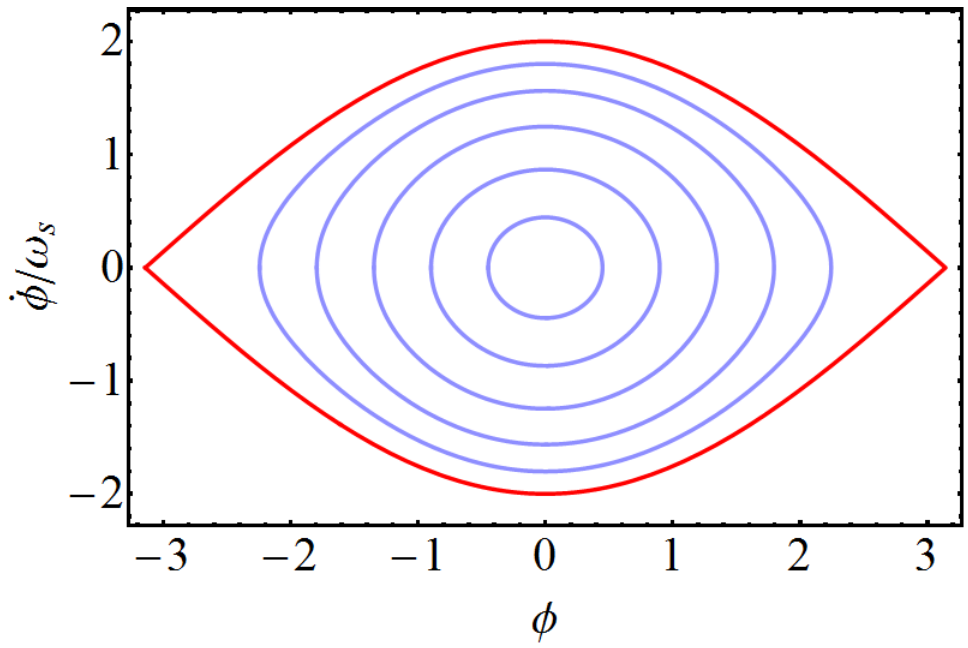
Spare slides

Stationary bucket in normalized coordinates

→ RF bucket properties become independent from accelerator parameters

→ Significant simplification of equations, easy to use

Example of stationary bucket



→ **Bucket height**

$$\frac{\dot{\phi}_B}{\omega_S} = 2\,\mathrm{rad}$$

→ **Bucket area**

$$\frac{A_B}{\omega_S} = 16\,\mathrm{rad}^2$$

→ **Exception:** conservation of longitudinal phase space