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Imperfections and Correction

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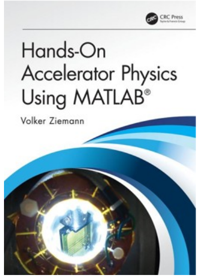
Background material (Proceedings)
https://arxiv.org/abs/2006.11016
even more (+example code in MATLAB)
https://www.crcpress.com/9781138589940



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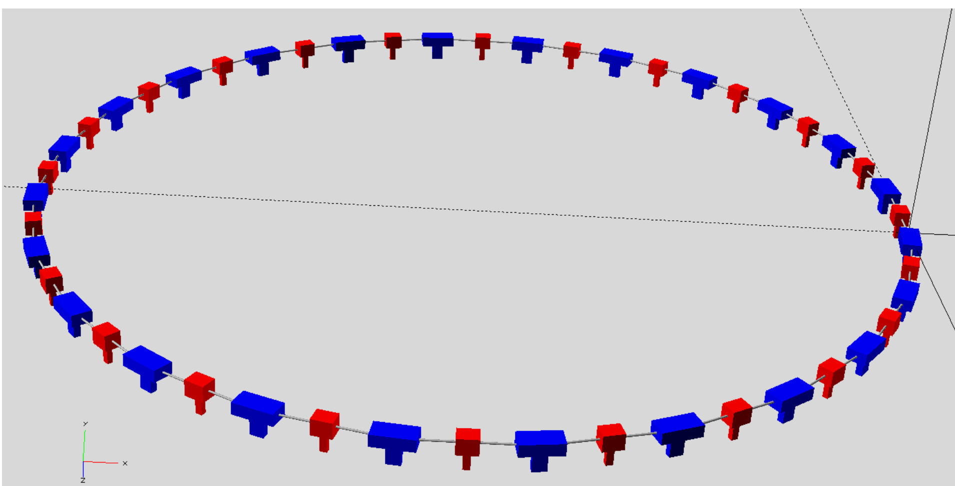
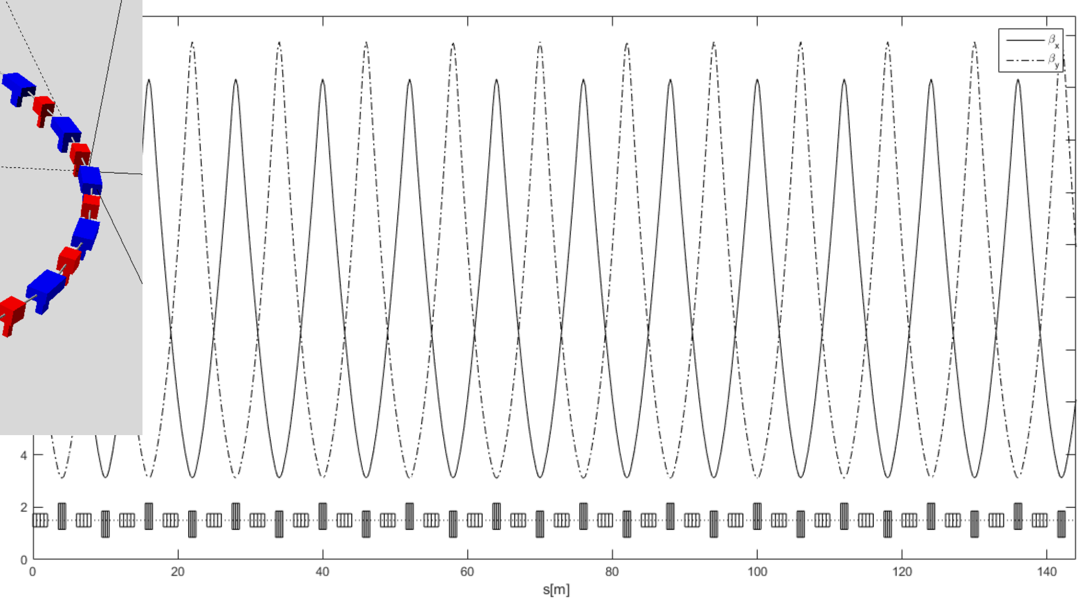
What is this talk about?

- First, you come up with lattice and design optics
  - nice and shiny beta functions
  - high periodicity → systematic errors cancel





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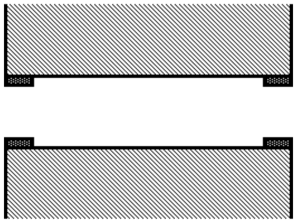
But then...

- ...the accelerator is built, and..
  - the magnets are not quite where they should be;
  - power supplies have calibration errors;
  - magnets have incorrect shims;
  - the diagnostics might have imperfections, too
    - Beam position monitors
    - Screens



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Therefore...

- I talk about
  - things that can go wrong (courtesy of Mrs Murphy...)
    → Imperfections
  - how to figure out what is wrong
    → Diagnostics to use
  - and fix it
    → Corrections

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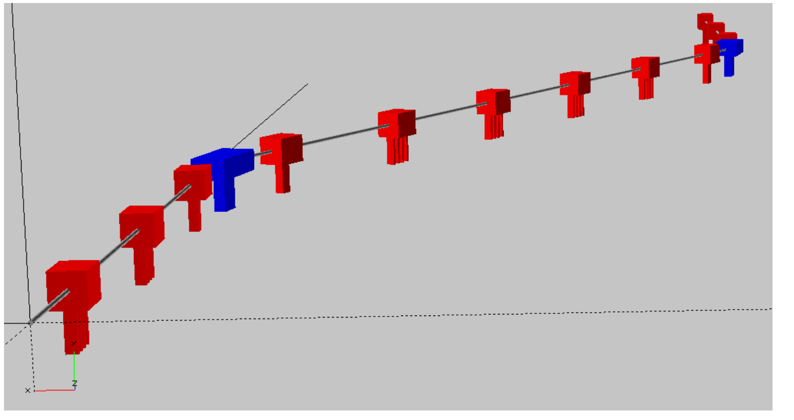
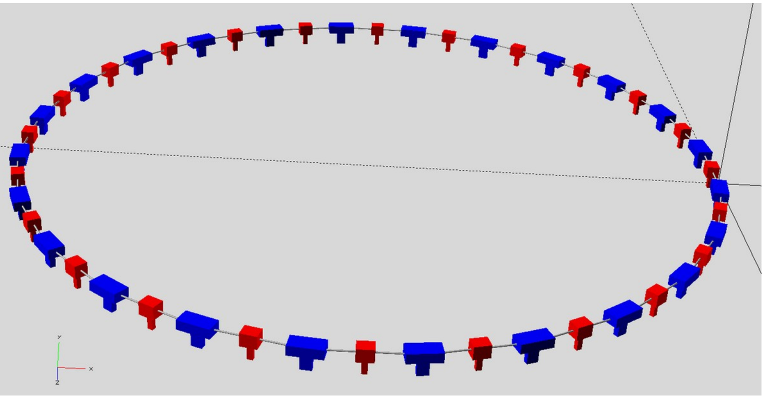
Outline

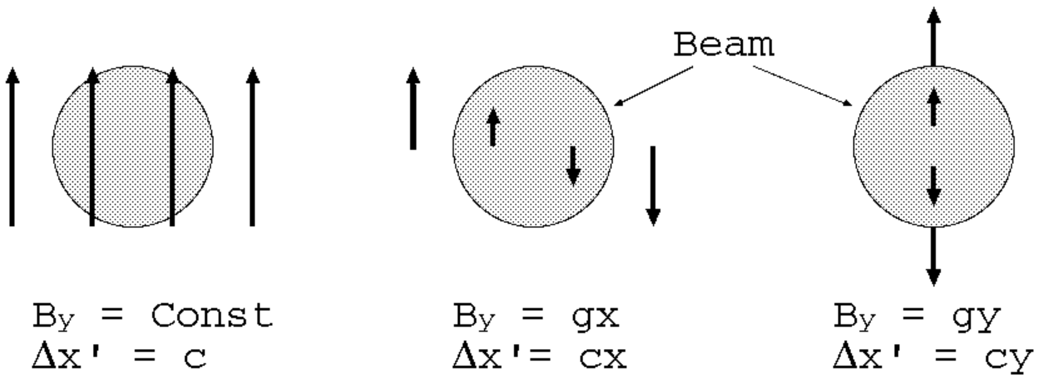
- Imperfections
- Straight systems
  - Beam lines and Linac
  - Imperfections and their corrections
- Rings
  - Imperfections and their corrections

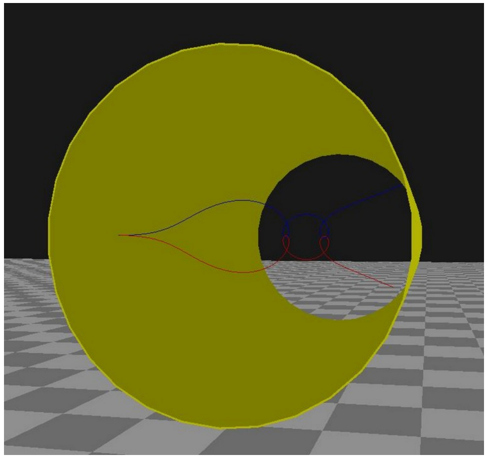




Part 1: Linear Imperfections

- Spoil the 'nice&shiny™' periodic magnet lattice
  - due to unwanted magnetic fields in the wrong place
- that's where the beam is
  - constant: dipole kick
  - gradient: focusing
  - skew gradient: coupling
- Solenoid fields
  - detector
  - electron cooler





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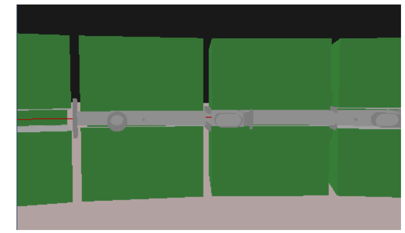
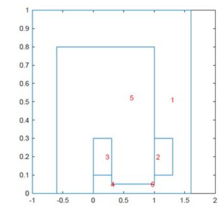
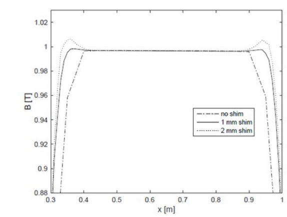
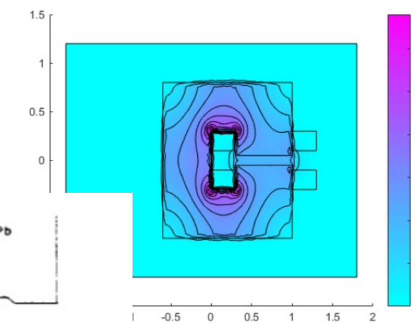
Sources of Imperfections

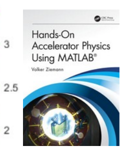
- Anything that is not in the design lattice
- Fringe fields and cross talk between magnets
- Saturation of magnets
- Power supply calibration and read-back errors
- Wrong shims
- Earth magnetic field in low-energy beam lines
- Nickel layers in the wrong place
- Solenoids in detectors or coolers
- Weak focusing from wigglers
- Tilt and roll angles of magnets
- Misaligned magnets (or beams)











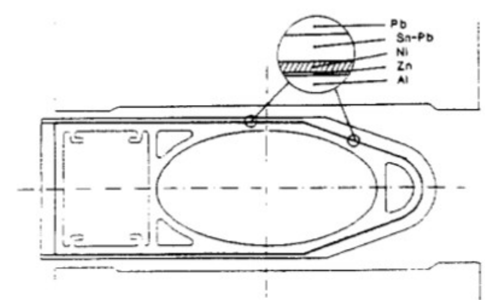


Figure 1: *The LEP dipole chamber and its nickel layer*

J. Billan et al., PAC 1993

Alignment

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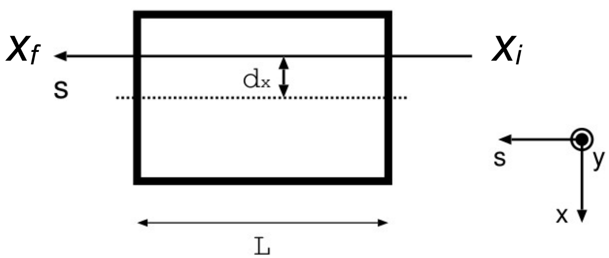
- How do you do it?
  - Magnets on tables
  - Fiducialization to pods
  - Triangulation
- How well can you do it?
  - 0.2-0.3 mm OK
  - <0.1 mm increasingly more difficult
  - more difficult in large installations
- Sub-micron for linear colliders → beam-based

Photo: R. Ruber, CTF3-TBTS

Transversely displaced elements

- Misalignment of linear elements



$$\begin{pmatrix} x_f \\ x'_f \end{pmatrix} = \begin{pmatrix} -d_x \\ 0 \end{pmatrix} + \tilde{R}\left[\begin{pmatrix} d_x \\ 0 \end{pmatrix} + \begin{pmatrix} x_i \\ x'_i \end{pmatrix}\right]$$

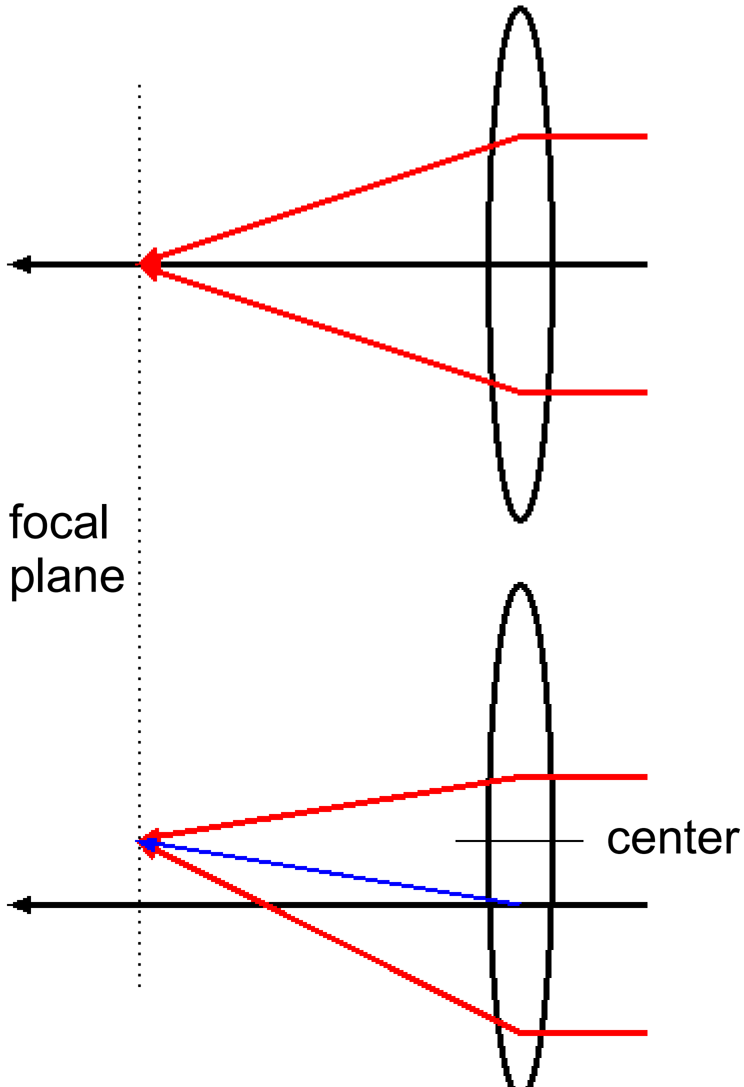
$$= \left[\tilde{R} - 1\right]\begin{pmatrix} d_x \\ 0 \end{pmatrix} + \tilde{R}\begin{pmatrix} x_i \\ x'_i \end{pmatrix} = \vec{q} + \tilde{R}\begin{pmatrix} x_i \\ x'_i \end{pmatrix}$$

- and for a thin quadrupole...

$$\vec{q} = \left[\tilde{R} - 1\right]\begin{pmatrix} d_x \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ -\frac{1}{f} & 0 \end{pmatrix}\begin{pmatrix} d_x \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{d_x}{f} \end{pmatrix}$$

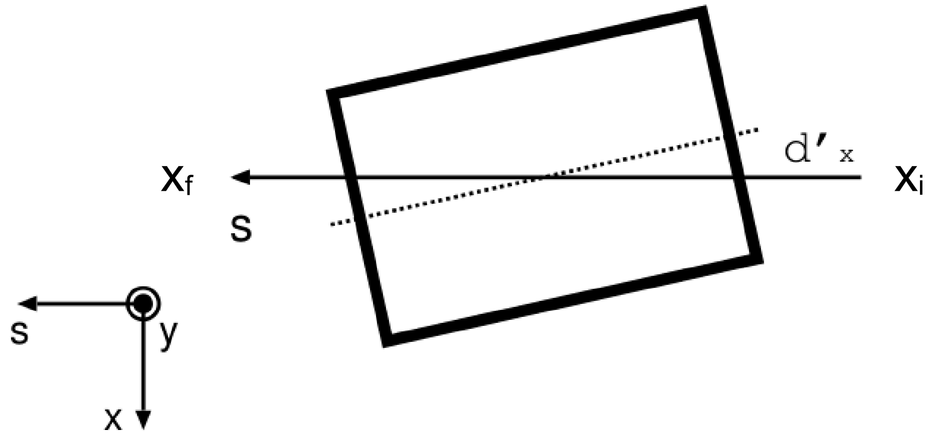
- An additional dipolar kick appears → **feed-down**

Misaligned quadupoles focus just as good as centered ones



- Same focal length despite misalignment.
- Lower ray is further away from the quad center and is bent more.
- Upper ray is closer to axis and is bent less.
- But they kick the centroid of the beam.

Tilted elements



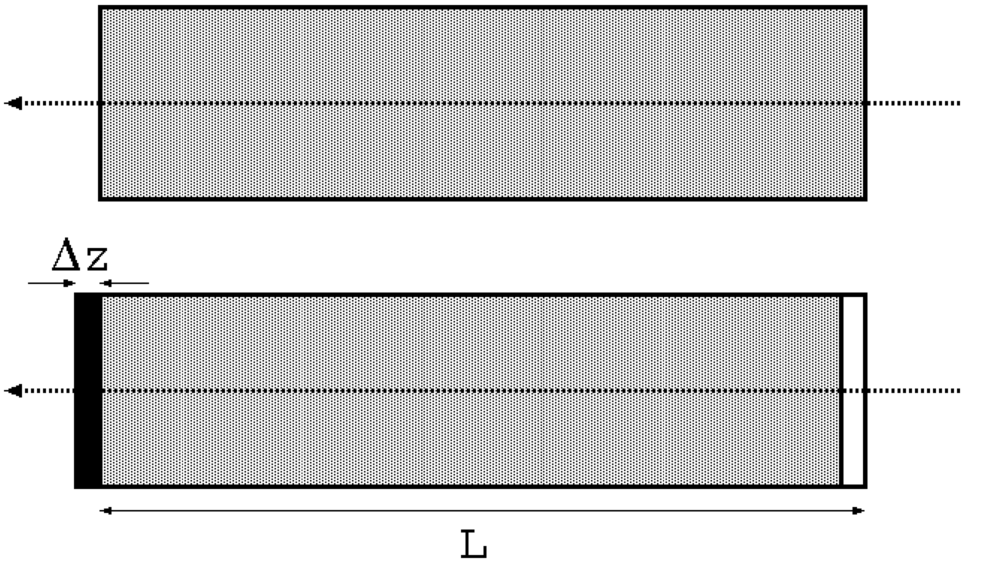
- come in, step right and point left, go through, step right again and point right

$$
\begin{aligned}
\begin{pmatrix} x_f \\ x'_f \end{pmatrix} &= \begin{pmatrix} -d'_x L/2 \\ -d'_x \end{pmatrix} + \hat{R}\left[\begin{pmatrix} -d'_x L/2 \\ d'_x \end{pmatrix} + \begin{pmatrix} x_i \\ x'_i \end{pmatrix}\right] \\
&= \left[\hat{R} + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right]\begin{pmatrix} -d'_x L/2 \\ d'_x \end{pmatrix} + \hat{R}\begin{pmatrix} x_i \\ x'_i \end{pmatrix} \quad = \vec{q} + \hat{R}\begin{pmatrix} x_i \\ x'_i \end{pmatrix}
\end{aligned}
$$

- Again,normal transport and a constant vector

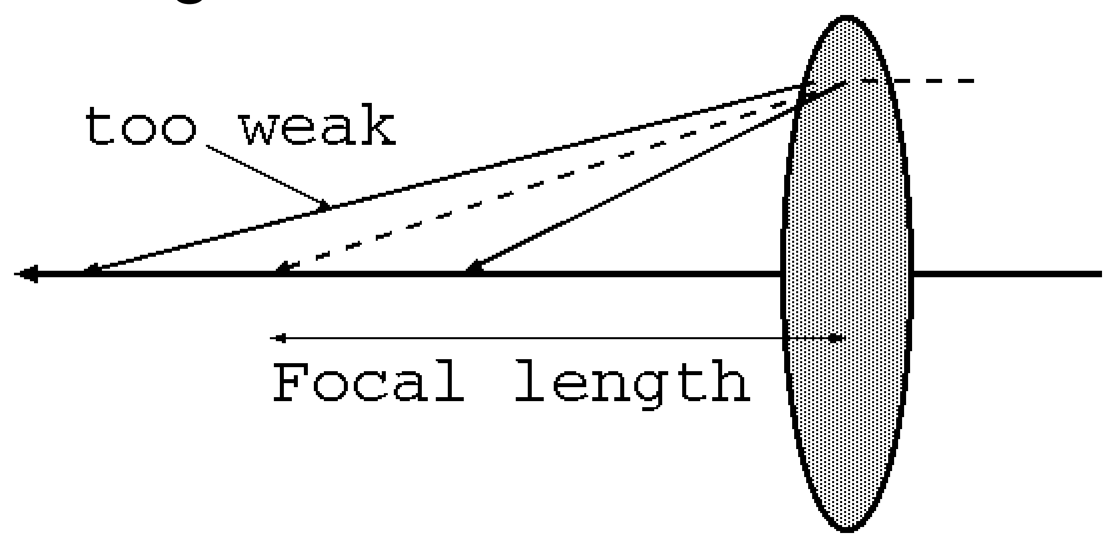
Longitudinally Shifted Elements

- Add a short positive element on one side and the negative on the other.
- Dipole
  - kick on either side
- Quadrupoles
  - thin quadrupoles



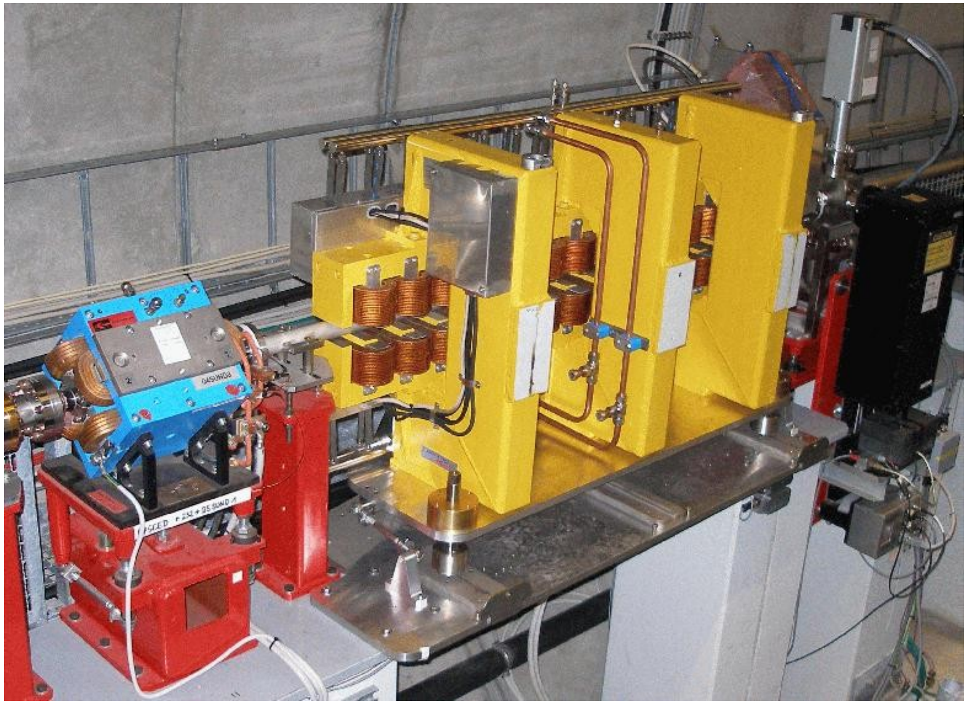
Incorrectly powered quadrupoles

- Focal length changes
  - beam matrix differs from the expected
  - beta functions change
  - in rings, the tune changes



Undulators and Wigglers

- $B_y \sim \cos(2\pi s/\lambda_u) \rightarrow$ horizontal oscillations
- $\partial B_y/\partial s = \partial B_s/\partial y \rightarrow$ vertically changing B_s
- Focus vertically (only)
- Many Rbends
- weak effect $(l/\rho)^2$, but
- changing excitation
  - affects orbit;
  - affects tune.



"Hilda"

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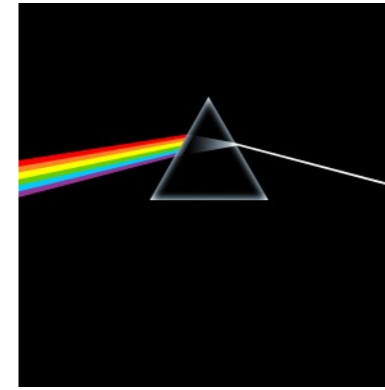
Dispersion

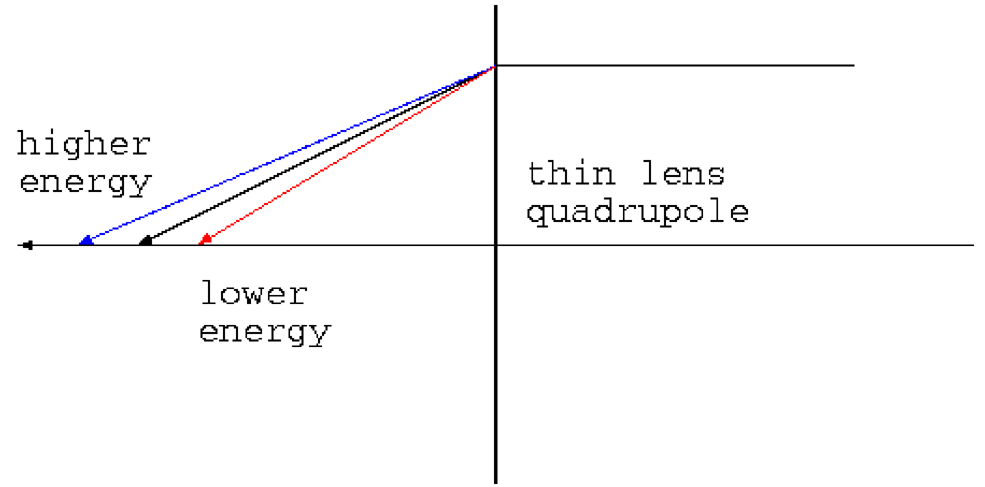


- Effect of magnetic fields on the beam (~*B/p*) with $p=p_0(1+\delta)$ is reduced by $1+\delta$
- Every dipole behaves as a spectrometer
  - separates the particles according to their momentum
  - even dipole correctors contribute
- In planar systems the vertical dispersion is by design zero
  - but rolled dipoles (and quadrupoles) make it non-zero.

...revisit Wolfgang's slides, section 5

Chromaticity

- Also quadrupolar fields are reduced by $1+\delta$
  - longitudinal location of the focal plane depends on momentum and enlarges the beam sizes at the IP
  - chromaticity $Q'=dQ/d\delta$
  - tune spread



…again, revisit Wolfgang's Section 5.

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Measuring Dispersion and Chromaticity

- Change the beam energy in rings by changing the RF frequency
  - and look at orbit changes on BPMs → dispersion
  - and measure the tune → chromaticity
- In transfer lines or linacs change the energy of the injected beam.
- Optionally, may scale all magnets with the same factor
  - all beam observables are proportional to B/p.

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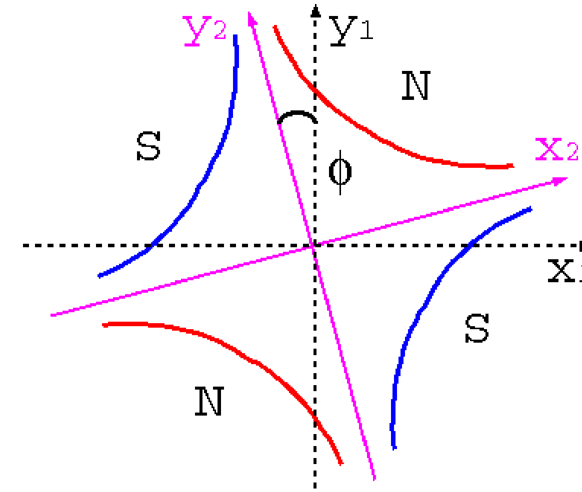
Rolled elements



- Coordinate rotation

$$\begin{pmatrix} x_2 \\ x_2' \\ y_2 \\ y_2' \end{pmatrix} = \begin{pmatrix} \cos\phi & 0 & \sin\phi & 0 \\ 0 & \cos\phi & 0 & \sin\phi \\ -\sin\phi & 0 & \cos\phi & 0 \\ 0 & -\sin\phi & 0 & \cos\phi \end{pmatrix} \begin{pmatrix} x_1 \\ x_1' \\ y_1 \\ y_1' \end{pmatrix}$$

- Sandwich roll-left before the element and then roll-right after the element
- Example: quad to skew-quad (example, thin quad)

$$Q_s = R(-\pi/4) \begin{pmatrix} Q_f & 0_2 \\ 0_2 & Q_d \end{pmatrix} R(\pi/4) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1/f & 0 \\ 0 & 0 & 1 & 0 \\ 1/f & 0 & 0 & 1 \end{pmatrix}$$

verify this on paper

- Mixes the transverse planes $\rightarrow$ betatron coupling

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Reminder: Multipoles

- Magnet builder's view (b_m: upright, a_m: skew)

$$B_y + iB_x = B_0 \sum_{m=1}^{\infty} (b_m + ia_m) \left(\frac{x+iy}{R_0} \right)^{m-1}$$

m=1 is dipole

- How the beam "sees" the fields

modulo a sign due to the particle type

$$\Delta x' - i\Delta y' = \frac{(B_y + iB_x)L}{B\rho} = \sum_{n=0}^{\infty} \frac{k_n L}{n!} (x+iy)^n$$

n=0 is dipole

- Multipole coefficients
  - real part: upright
  - imaginary part: skew

$$\frac{k_n L}{n!} = \frac{(B_0/R_0^n)L}{B\rho} (b_{n+1} + ia_{n+1})$$

$$k_n L = \frac{d^n B/dx^n}{B\rho} L$$

Feed-down from displaced multipoles

- Kick from thin multipole $\Delta x' - i\Delta y' = \frac{k_n L}{n!}(x+iy)^n$

- and from a horizontally displaced multipole

$$\Delta x' - i\Delta y' = \frac{k_n L}{n!}(x + d_x + iy)^n$$
$$= \frac{k_n L}{n!}(x+iy)^n + \frac{k_n L}{n!}\sum_{k=0}^{n-1}\binom{n}{k} d_x^{n-k}(x+iy)^k$$

  - binomial expansion, such as $(z+d)^2=z^2+2zd+d^2$ $z=x+iy$

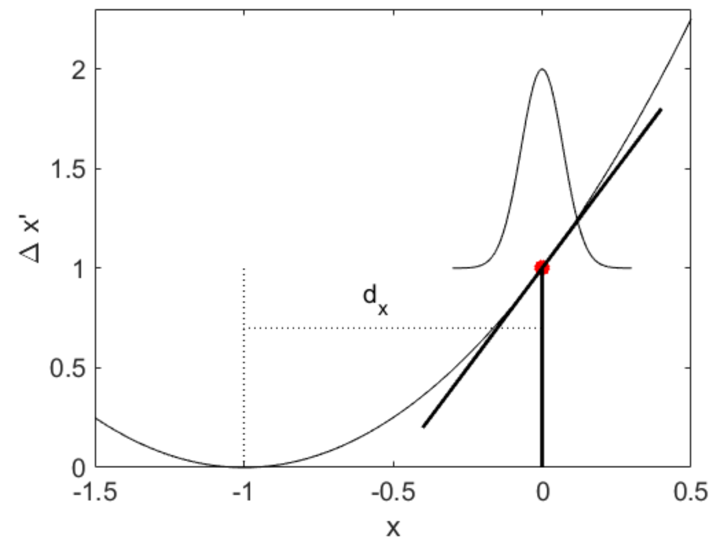
- Displaced multipole still works as intended, but also generates **all** lower multipoles.

Feed-down from sextupoles

- Horizontally displaced by d_x

$$\Delta x' - i\Delta y' = \frac{k_2 L}{2}\left[(x+iy)^2 + 2d_x(x+iy) + d_x^2)\right]$$

  - additional quadrupolar and dipolar kicks.



- Vertically displaced by d_y

$$\Delta x' - i\Delta y' = \frac{k_2 L}{2}(x+iy+id_y)^2 = \frac{k_2 L}{2}\left[(x+iy)^2 + 2id_y(x+iy) - d_y^2)\right]$$

  - Additional skew-quadrupolar and dipole kicks.
  - Vertically displaced sextupoles cause coupling.

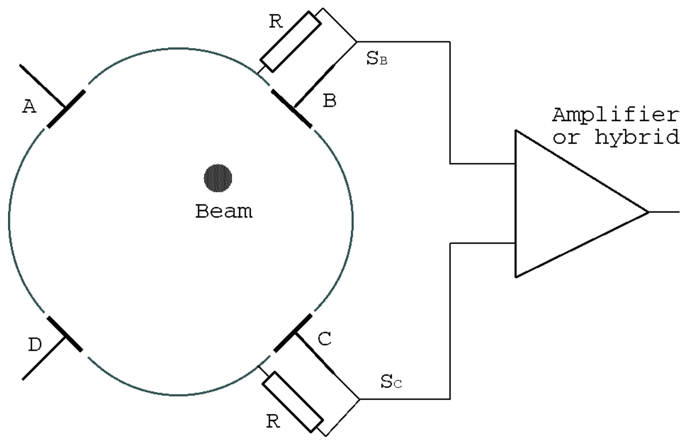
Detrimental effects

- Dipole fields cause beam to be in wrong place
  - losses, bad if you have a multi-MJ beam;
  - Background in the experiments.
- Gradients change the beam size, this spoils
  - Luminosity, if you work on a collider;
  - Coherence, if you work on a light source.
- Breaks the symmetry of the optics of a ring
  - more resonances;
  - reduces dynamic aperture.
- Need observations to figure out what's wrong.

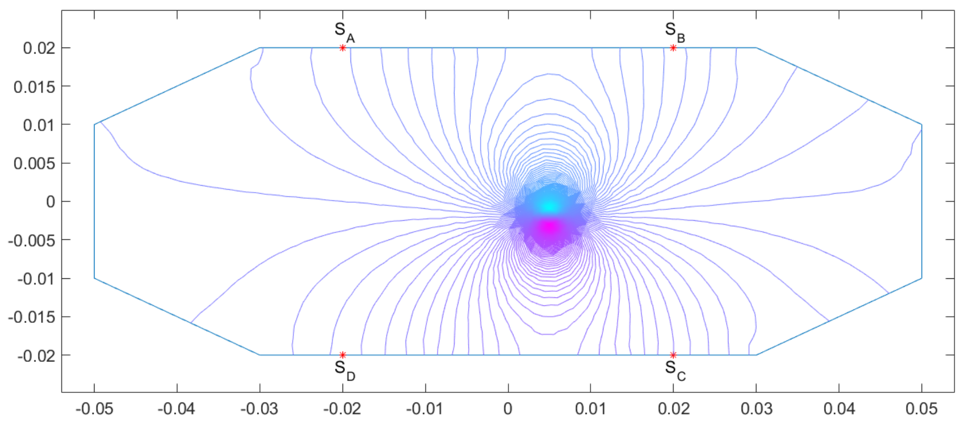
Beam Position Monitors and their Imperfections

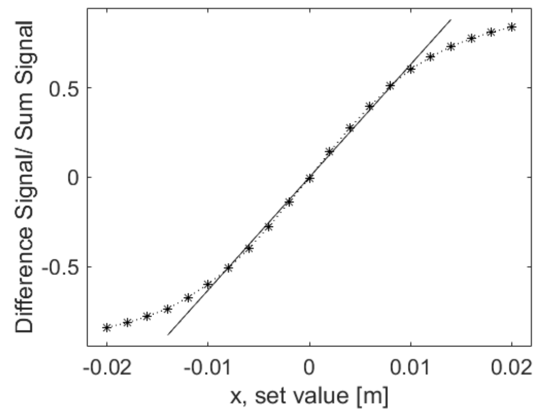
Details in Peter's talks

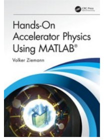
- Transverse offset
- (Longitudinal position)
- Electrical offset
- Scale error



$$x = k_x \frac{(S_A + S_D) - (S_B + S_C)}{S_A + S_B + S_C + S_D}$$







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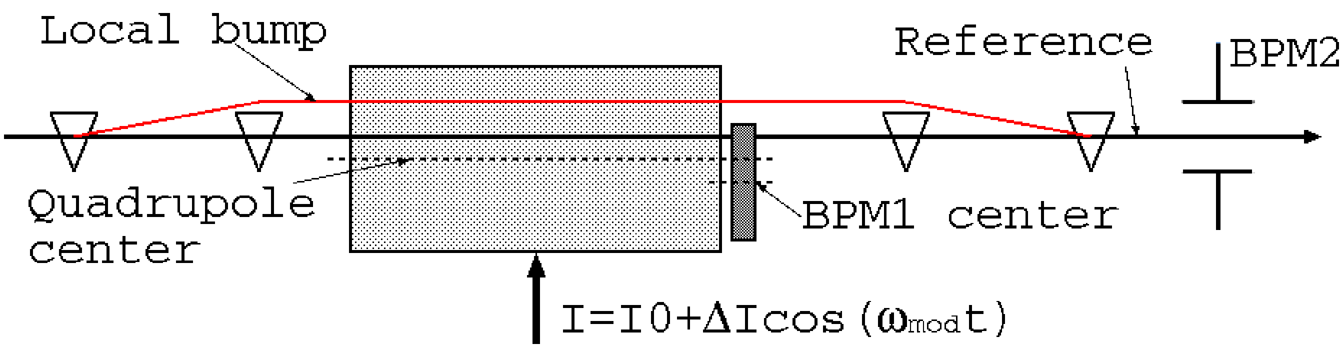
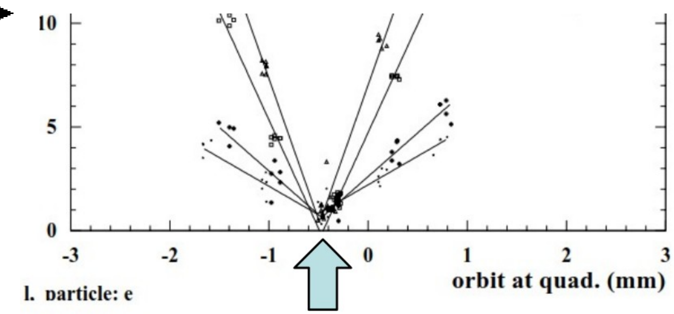
Find offsets with K-modulation

- BPM+Quadrupole are often rigidly mounted next to each other on the same girder



J. Wenninger
CAS 2018



- Modulate gradient of quadrupole
  - Deflection from quadrupole *x' = x'(ω)* is also modulated.
  - Observe on BPM2 and minimize signal by moving beam with a bump → quadrupole center.
  - Reading of BPM1 gives BPM1 offset relative to quad.

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Screens et al. and their Bugs

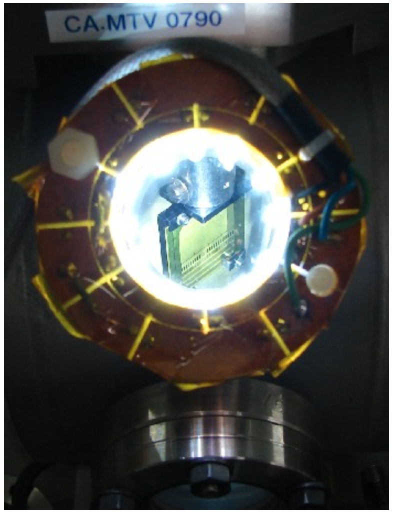

Photo taken by M. Jacewicz

- Transverse position
- Scale errors from the optical system
  - place fiducial marks on the screen
- Looking at an angle
- Depth of focus limitations, especially at large magnification levels
- Burnt-out spots on fluorescent screens
- Non-linear response of screen and saturation

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That's all for now, folks

- Take-home messages
  - Imperfections are characterized by the multipolarity of an equivalent magnet in the wrong place.
  - Describe misalignment by coordinate transformations.
  - Diagnostics can be in the wrong place, show scale errors, or non-linear response.

- Next lecture
  - Beamlines and linacs.
  - What can go wrong and how to fix it.

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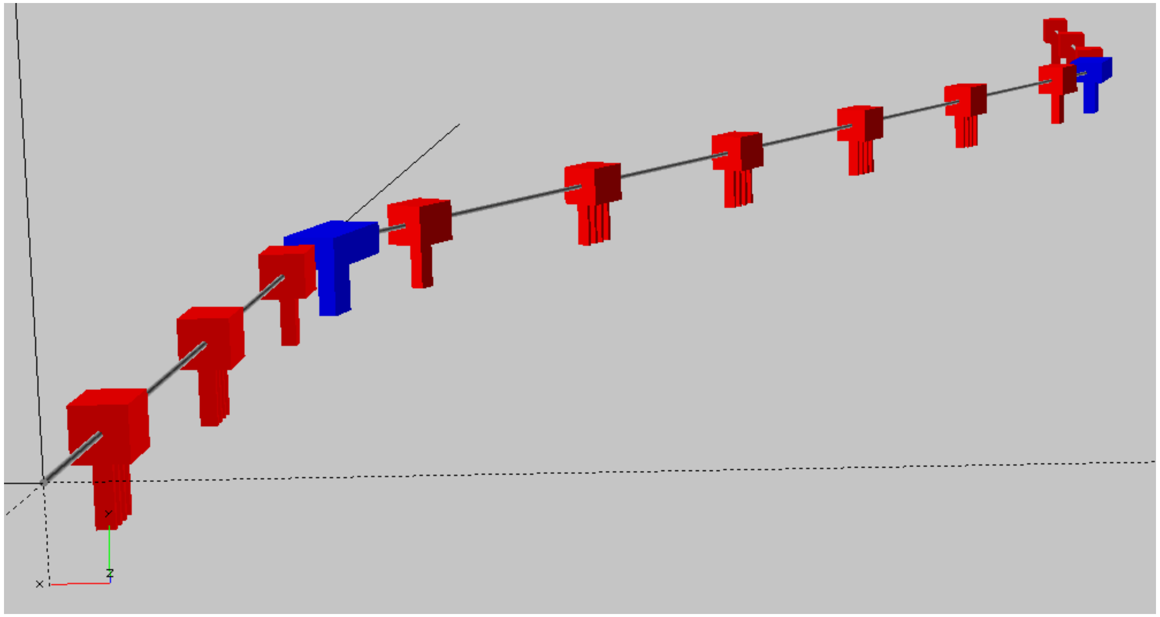
Things to think about...

- Construct the transfer matrix of a longitudinally displaced (along the beam line) thin quad.
- Does a vertically displaced octupole cause linear coupling?
- When is a magnet “short” and the thin-lens approximation justified?

Imperfections and their Correction in Beam Lines or Linacs

- Dipole errors
- Gradient errors
- Skew-gradient errors
- Filamentation



Transfer matrices—or what do all those numbers really mean?

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- Consider two-dimensional example

$$\begin{pmatrix} x_{look} \\ x'_{look} \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} \begin{pmatrix} x_{change} \\ x'_{change} \end{pmatrix}$$

1st: look -index 2nd: change index

  - R_{11}: by how much does x_{look} change when changing x_{change}?
  - **R_{12}**: by how much does x_{look} change when changing x'_{change}?
  - R_{21}: by how much does x'_{look} change when changing x_{change}?
  - R_{22}: by how much does x'_{look} change when changing x'_{change}?
- Generalizes to 4D and 6D

Transfer matrices in linacs

- Just keep this in mind...
- The beam energy at the location for the kick (change) and the observation (look) point may be different.
- Adiabatic damping
  - transverse momentum p_x is constant
  - longitudinal momentum p_s increases (acceleration!)
  - $x'=p_x/p_s$ scales with $p_s = \beta\gamma\, mc$
- R_{12} then scales with $(\beta\gamma)_{kick}/(\beta\gamma)_{look}$

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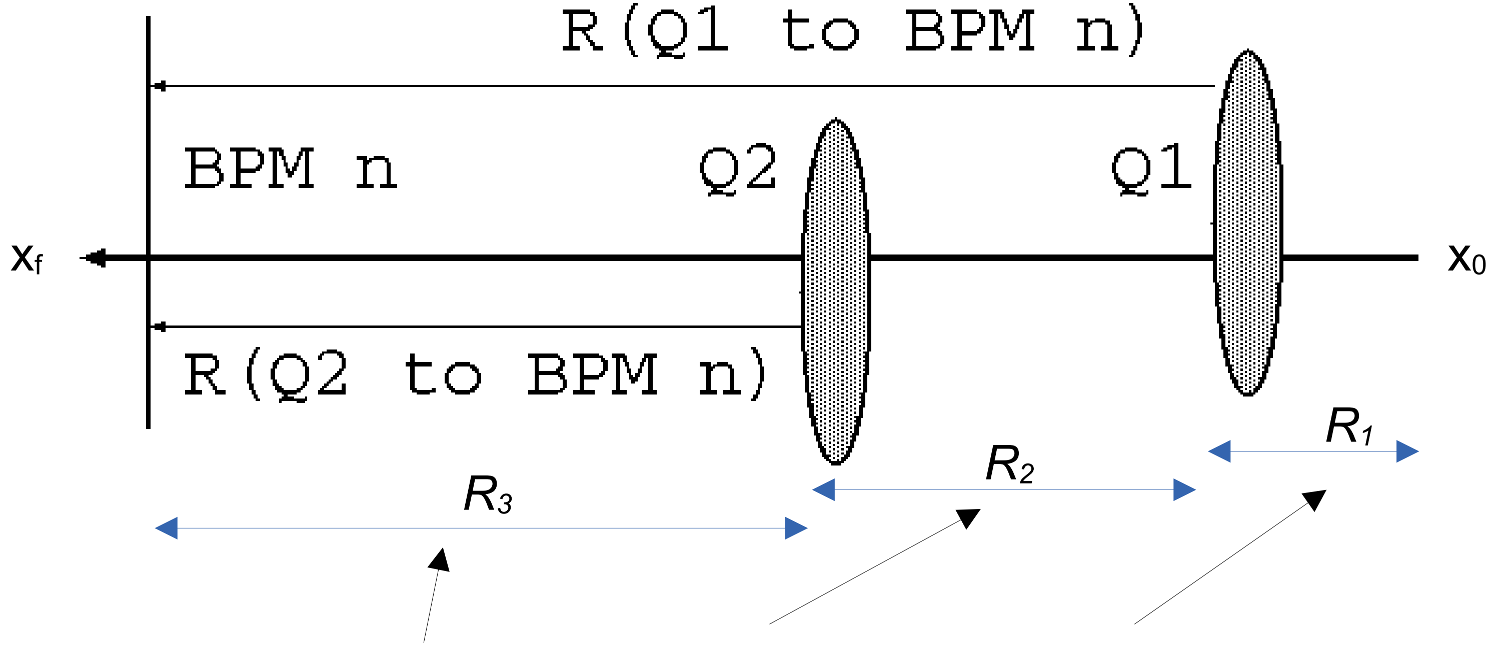
Two displaced quads

Linear and independent superposition of the perturbations



$$\vec{x}_f = R_3 (\vec{q}_2 + R_2)(\vec{q}_1 + R_1)\vec{x}_0$$

$$= R_3\vec{q}_2 + R_3 R_2 \vec{q}_1 + R_3 R_2 R_1 \vec{x}_0$$

Perturbation of quad2 through the rest of the beam line

Perturbation of quad1 through the rest of the beam line

Unperturbed transport of incoming particle all the way to the end.

Many, many dipole errors

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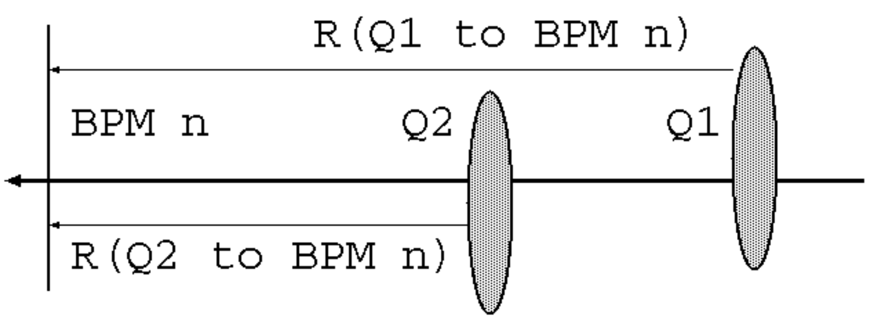
- Each misaligned element with label *k* may add a misalignment dipole-kick $\vec{q}_k$

$$\begin{aligned}\vec{x}_n &= R_n \cdots (\vec{q}_{k+1} + R_{k+1})(\vec{q}_k + R_k) \cdots (\vec{q}_1 + R_1)\vec{x}_0 \\ &= R_n \cdots R_1 \vec{x}_0 + \sum_{j=1}^{n-1} (R_n \cdots R_{j+1}) \vec{q}_j\end{aligned}$$

- Simple interpretation
  - at the look-point (BPM) *n* all perturbing kicks are added with the transfer matrix from kick to end



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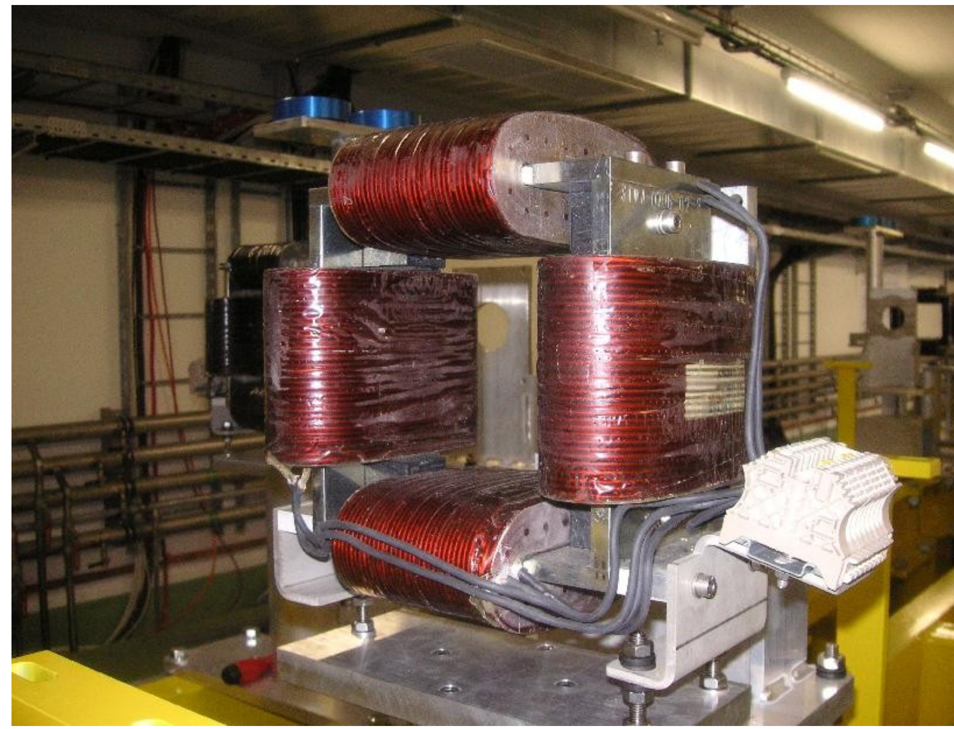
Correct with orbit correctors

- small dipole magnet, here for both planes (steerer for CTF3-TBTS)
- affects the beam like any other error



$$\begin{pmatrix} x_1 \\ x_1' \end{pmatrix} = \begin{pmatrix} 0 \\ \theta \end{pmatrix} + \begin{pmatrix} x_0 \\ x_0' \end{pmatrix}$$

$$\vec{x}_1 = \vec{q} + \tilde{R}\vec{x}_0$$

- treat just as additional misalignment

Local trajectory Bumps

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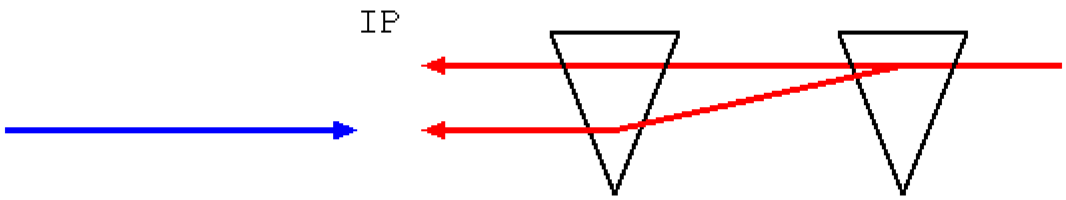
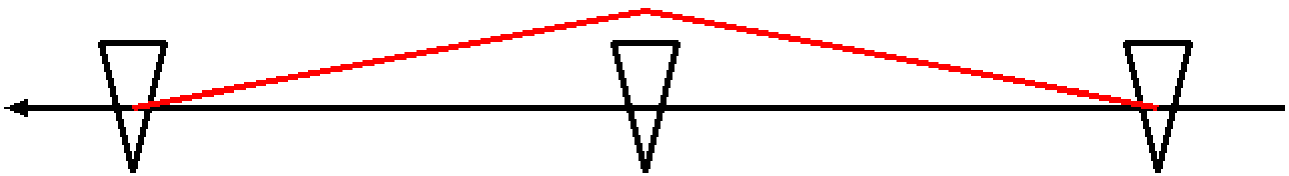
- Occasionally a particular displacement or angle of the orbit at a given point might be required
- Displace orbit at IP to bring beams into collision



- or a slight excursion (3-bump)



- Differential changes ('by' not 'to')

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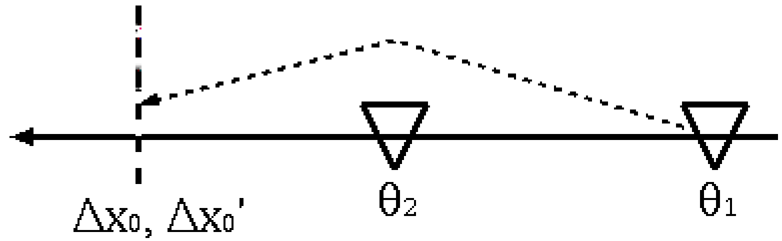
Trajectory knob

- Change position and angle at reference point



- Remember that kicks add up with TM from source to observation or reference point

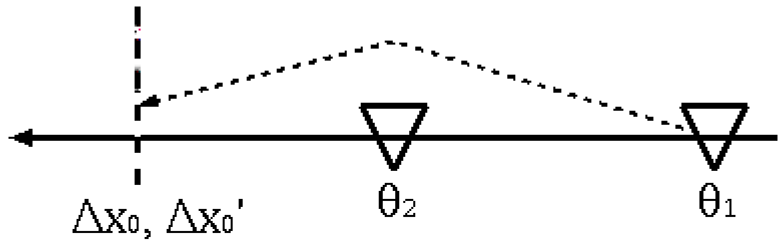
$$\begin{pmatrix} \Delta x_0 \\ \Delta x_0' \end{pmatrix} = \begin{pmatrix} R_{12}^{01} & R_{12}^{02} \\ R_{22}^{01} & R_{22}^{02} \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}$$

- and the **columns of the inverse matrix** are the knobs

$$\begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} = \begin{pmatrix} R_{12}^{01} & R_{12}^{02} \\ R_{22}^{01} & R_{22}^{02} \end{pmatrix}^{-1} \begin{pmatrix} \Delta x_0 \\ \Delta x_0' \end{pmatrix}$$

A trivial example

- Two steering magnets with drift between them



$$R^{02} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \qquad R^{01} = \begin{pmatrix} 1 & 2L \\ 0 & 1 \end{pmatrix}$$

- Response matrix

$$\begin{pmatrix} \Delta x_0 \\ \Delta x_0' \end{pmatrix} = \begin{pmatrix} R^{01}_{12} & R^{02}_{12} \\ R^{01}_{22} & R^{02}_{22} \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} = \begin{pmatrix} 2L & L \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}$$

- Knobs

$$\begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} = \frac{1}{L} \begin{pmatrix} 1 & -L \\ -1 & 2L \end{pmatrix} \begin{pmatrix} \Delta x_0 \\ \Delta x_0' \end{pmatrix} \longrightarrow \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} = \frac{1}{L} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \Delta x_0$$

Almost common sense!

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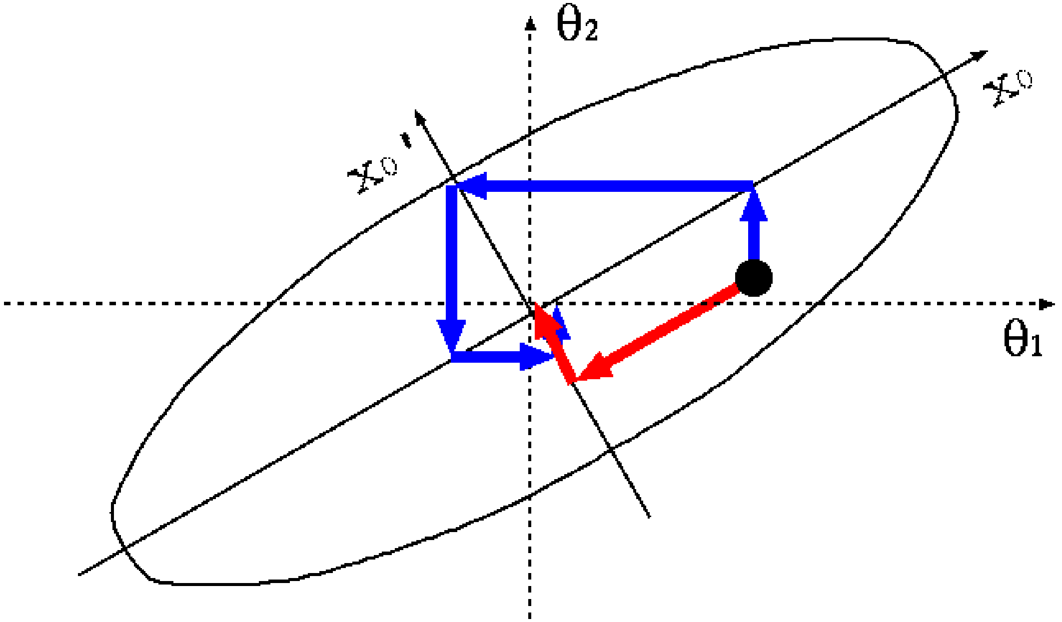
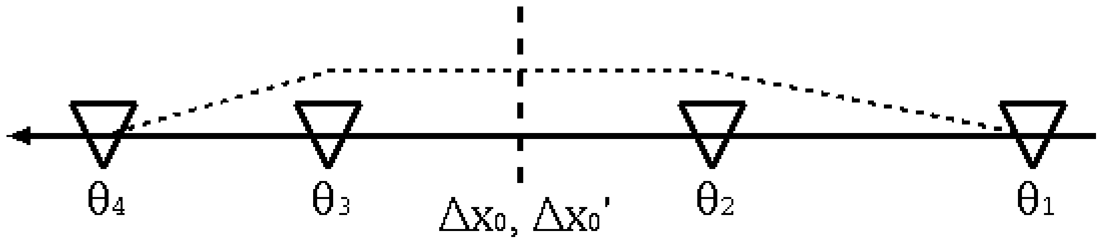
Remark about Orthogonality

- Knobs are orthogonal
- Optimize one parameter without screwing up the other(s).
  - Faster convergence
  - Enables heuristic optimization
  - Deterministic
- Use physics rather than hardware parameters



4-Bump



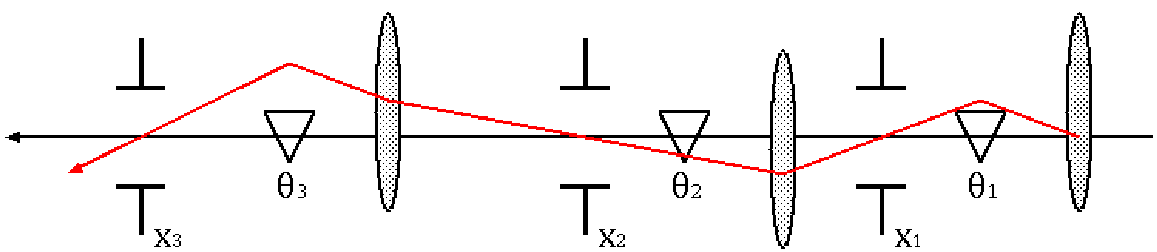
- Use four steerers to adjust angle and position at a center point and then flatten orbit downstream of the last steerer.

$$\begin{pmatrix} \Delta x_0 \\ \Delta x_0' \\ x_f = 0 \\ x_f' = 0 \end{pmatrix} = \begin{pmatrix} R_{12}^{01} & R_{12}^{02} & 0 & 0 \\ R_{22}^{01} & R_{22}^{02} & 0 & 0 \\ R_{12}^{f1} & R_{12}^{f2} & R_{12}^{f3} & R_{12}^{f4} \\ R_{22}^{f1} & R_{22}^{f2} & R_{22}^{f3} & R_{22}^{f4} \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{pmatrix}$$

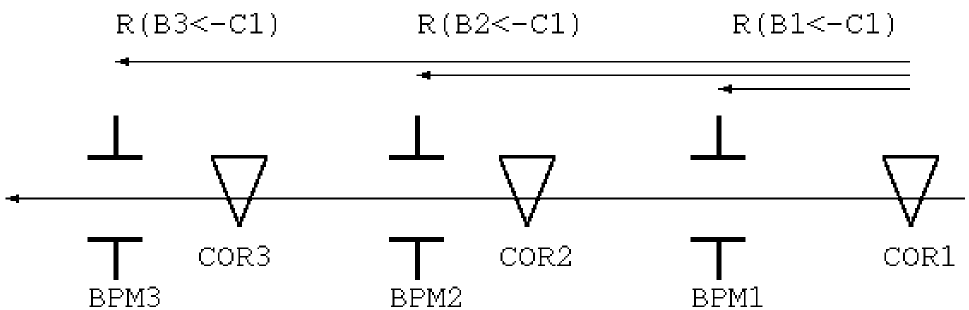
- Invert matrix and express thetas as a function of the constraints x_0 and x_0'
- Gives the required steering excitations θ_j as a function of x_0 and x_0' $\rightarrow$ Multiknob

Orbit Correction in Beamline #1

- Observe the orbit on beam-position monitors
- and correct with steering dipoles
- How much do we have to change the steering magnets in order to compensate the observed orbit either to zero or some other 'golden orbit'.
- In a beam line the effect of a corrector on the downstream orbit is given by transfer matrix element R_{12}
- One-to-one steering



Orbit correction in a Beamline #2



$$\begin{pmatrix} -x_1 \\ -x_2 \\ -x_3 \end{pmatrix} = \begin{pmatrix} R_{12}^{11} & 0 & 0 \\ R_{12}^{21} & R_{12}^{22} & 0 \\ R_{12}^{31} & R_{12}^{32} & R_{12}^{33} \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{pmatrix}$$

- Observed beam positions x_1, x_2, and x_3
- Only downstream BPM can be affected
- Linear algebra problem to **invert matrix** and find required corrector excitations θ_j to produce negative of observed x_i
- Include BPM errors by left-multiplying the equation with

$$\bar{\Lambda} = \mathrm{diag}\left(\frac{1}{\sigma_1}, \ldots, \frac{1}{\sigma_n}\right)$$

This weights each BPM measurement by its inverse error. Good BPMs are trusted more!

How to get the response matrix?

- With the computer (MADX or any other code)
  - tables of transfer matrix elements
  - but it is based on a model and somewhat idealized
  - no BPM or COR scale errors known
- Experimentally by measuring difference orbits
  - record reference orbit $\vec{x}_0$
  - change steering magnet $\Delta\theta_j$
  - record changed orbit $\vec{x}_j$
  - Build response matrix one column at a time

$$A = \left(\frac{\vec{x}_1 - \vec{x}_0}{\Delta\theta_1} , \quad \frac{\vec{x}_2 - \vec{x}_0}{\Delta\theta_2} , \quad \cdots \right)$$

Solving $-x = A\theta$

- A is an $n \times m$ matrix, n BPM and m correctors
- $n=m$ and matrix A is non-degenerate:

$$\vec{\theta} = -A^{-1}\vec{x}$$

- $m<n$: too few correctors, least squares $\chi^2 = |-\vec{x} - A\vec{\theta}|^2$

$$\vec{\theta} = -(A^t A)^{-1} A^t \vec{x}$$

- MICADO: pick the most effective, fix orbit, the next effective, fix residual orbit, the next...
  - good for large rings with many BPM and COR
- $m>n$ or degenerate: singular-value dec. (SVD)

Digression on SVD

- Singular Value Decomposition $A = O\Lambda U^t$
  - may need to zero-pad
  - U is orthogonal, a coordinate rotation
  - Λ is diagonal, it stretches the coordinates by λ_i
  - O is orthogonal and rotates, but differently
- If A is symmetric → eigenvalue decomposition
- Inversion is trivial $\text{"}A^{-1}\text{"} = U\Lambda^{-1}O^t$
  - invert only in sub-space where you can if $\lambda \neq 0$
  - and set projection onto degenerate subspace to zero "1/0 = 0" (see *Numerical Recipes* for a discussion)

Comment on Matrix Inversion

- Many correction problems can be brought into a generic form, if you
  - pretend you know the excitation of all controllers (think correctors, θ)
  - determine the response matrix (expt. or numerically)

    $C_{ij} = \partial \text{Observable}_i / \partial \text{Controller}_j$
  - to predict the changes of the observable y (think BPM)

$\pm y = C\theta$

- Then invert the response matrix C to determine the controller values required to change the observable by some value.

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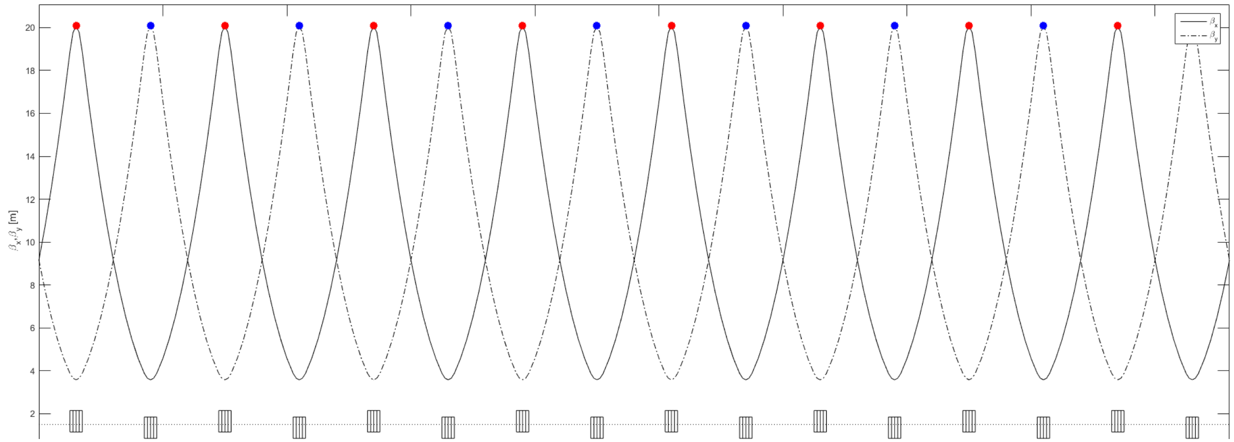
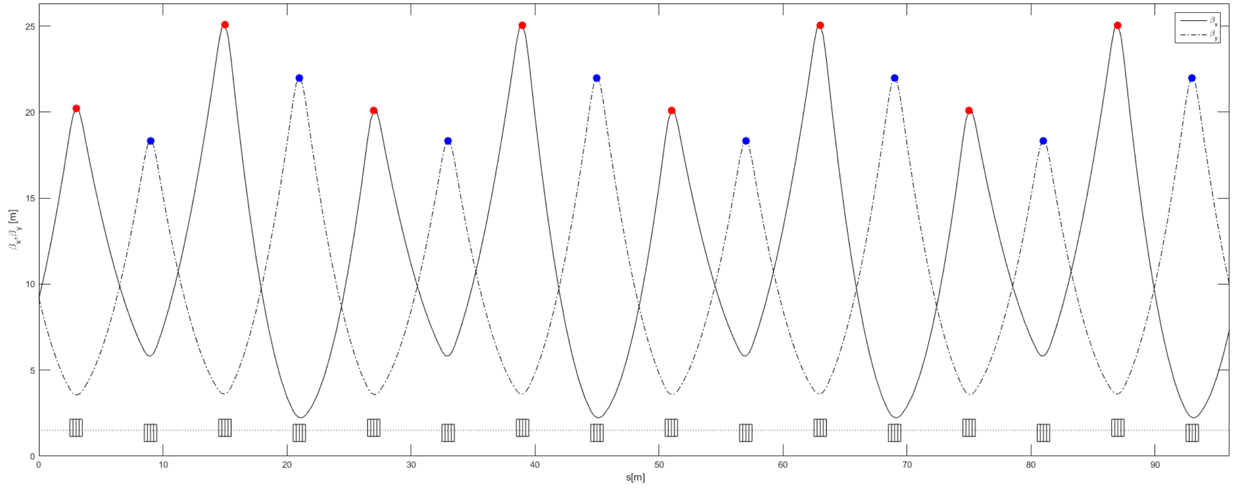
Effect of gradient errors

Eight 90° FODO cells, first quad 10% too low



Unperturbed lattice

Nice and repetitive beta functions



Repeats after 2 cells or 2 x 90°

Beta-function "beats"

Injection into following beam line or ring is compromised

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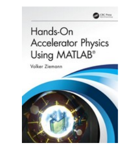
Beam lines: Gradient errors

- Gradient errors cause the beam matrix or beta functions β to differ from their design values $\hat{\beta}$
- Downstream beam size

lengthy derivation



$$\bar{\sigma}_x^2 = \varepsilon\bar{\beta}\left[B_{mag} + \sqrt{B_{mag}^2 - 1}\cos(2\mu - \varphi)\right]$$

  – enlarged effective emittance, beta-beat oscillations with twice the betatron phase advance μ

- This is called mismatch and is quantified by

$$B_{mag} = \frac{1}{2}\left[\left(\frac{\hat{\beta}}{\beta} + \frac{\beta}{\hat{\beta}}\right) + \beta\hat{\beta}\left(\frac{\alpha}{\beta} - \frac{\hat{\alpha}}{\hat{\beta}}\right)^2\right]$$

- For a single thin quad we have

$$B_{mag} = 1 + \frac{\hat{\beta}^2}{2f^2}$$

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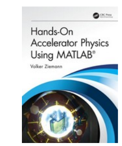
Filamentation #1

- What happens when we inject a mismatched beam into a ring with chromaticity Q' ?

$$\sigma_n^2 = \varepsilon\bar{\beta}\left[B_{mag} + \sqrt{B_{mag}^2 - 1}\ \cos(4\pi n(Q + Q'\delta) - \varphi)\right]$$

lengthy derivation



  - with momentum distribution $$\psi(\delta) = \frac{1}{\sqrt{2\pi}\sigma_\delta} e^{-\delta^2/2\sigma_\delta^2}$$

- Averaging over δ gives

$$\sigma_n^2 = \varepsilon\bar{\beta}\left[B_{mag} + e^{-2(2\pi Q'\sigma_\delta)^2 n^2}\sqrt{B_{mag}^2 - 1}\ \cos(4\pi n Q - \varphi)\right]$$

- Oscillates with *2 x Q*, 'damps' with *exp(-n²)*, and leaves an increased beam size (by B_{mag}).

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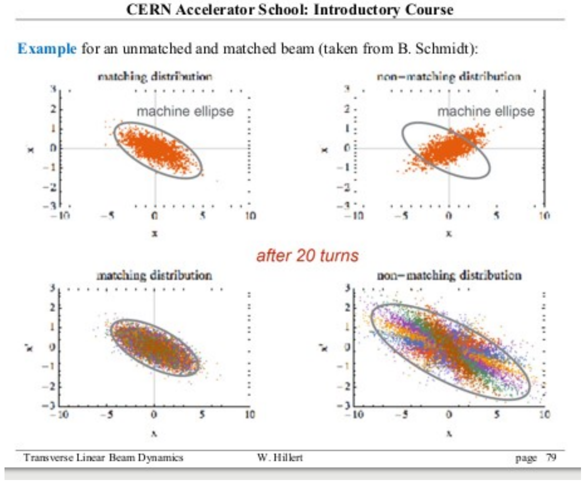
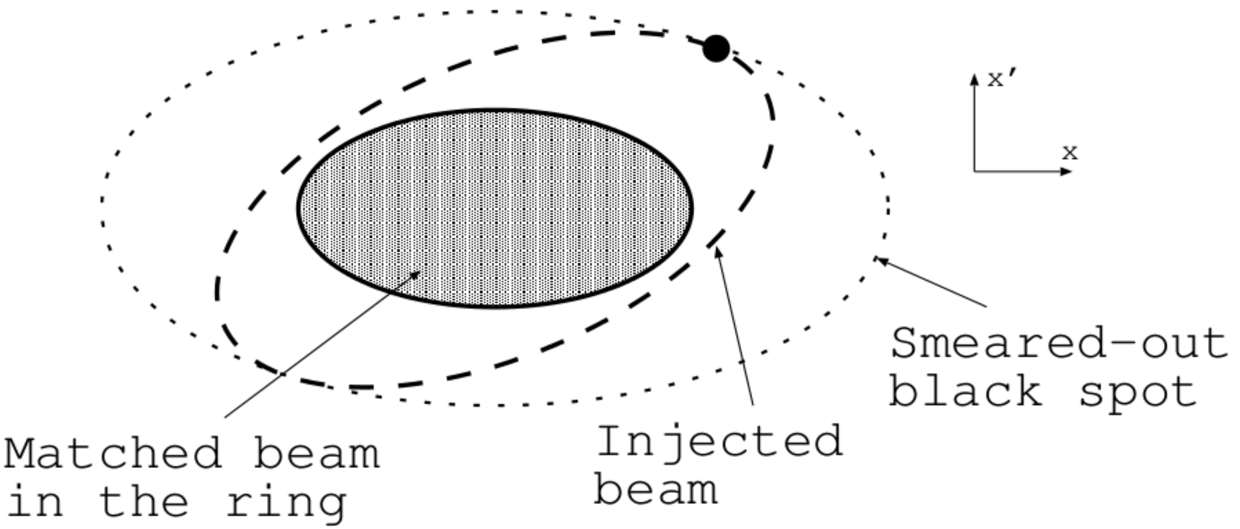
Filamentation #2

You've seen it before...


CERN Accelerator School: Introductory Course

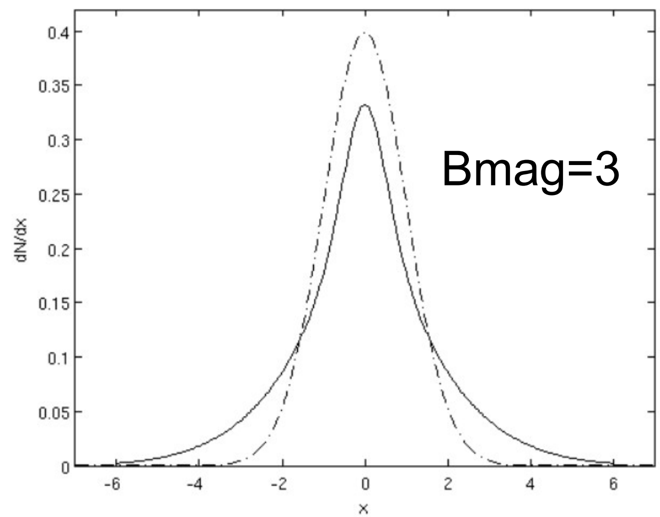
Example for an unmatched and matched beam (taken from B. Schmidt):

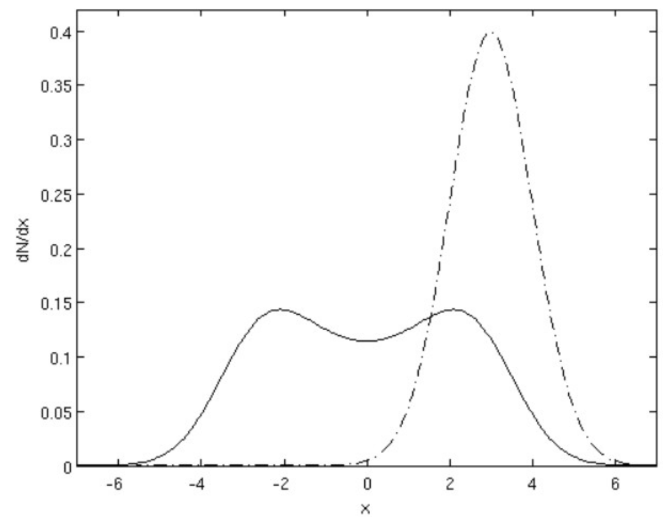
Transverse Linear Beam Dynamics W. Hillert page 79



Injecting with transverse offset also leads to filamentation

Final distribution is not Gaussian





Measuring Beam Matrices

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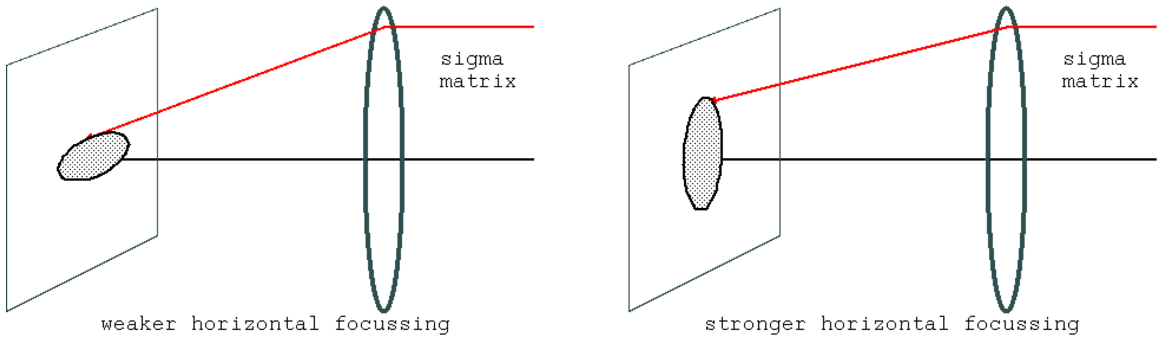

$$\bar{\sigma} = R(f)\ \sigma\ R(f)^t$$

Vary quadrupole and observe changes on a screen, usually one plane at a time

- Beam size on screen depends on quad setting

$$\bar{\sigma}_x^2 = \bar{\sigma}_{11} = R_{11}^2\sigma_{11} + 2R_{11}R_{12}\sigma_{12} + R_{12}^2\sigma_{22}$$

- where *R=R(f)*, use several measurement and solve for the three sigma matrix elements

$$\varepsilon_x^2 = \sigma_{11}\sigma_{22} - \sigma_{12}^2 \qquad \beta_x = \sigma_{11}/\varepsilon_x \qquad \alpha_x = -\sigma_{12}/\varepsilon_x$$

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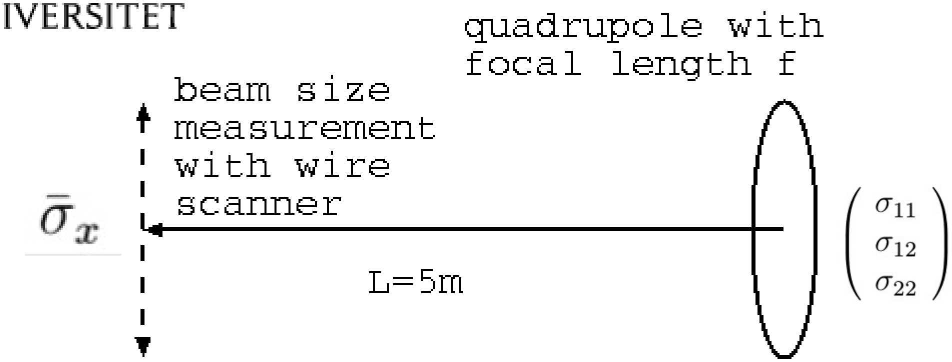
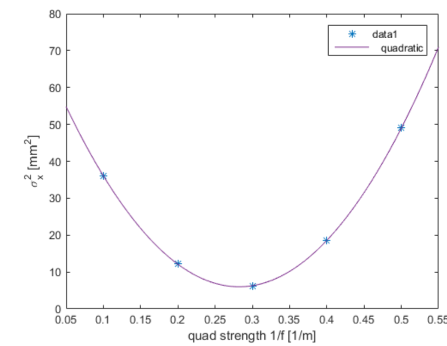
A worked example: Quad scan



$1/f$ [1/m]	$\bar{\sigma}_x$ [mm]
0.1	6.0
0.2	3.5
0.3	2.5
0.4	4.3
0.5	7.0



- Transfer matrix $R = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} = \begin{pmatrix} 1-l/f & l \\ -1/f & 1 \end{pmatrix}$
- Relate unknown beam matrix to measurements

$$\begin{aligned} \bar{\sigma}_x^2 &= R_{11}^2\sigma_{11} + 2R_{11}R_{12}\sigma_{12} + R_{12}^2\sigma_{22} \\ &= (1-l/f)^2\sigma_{11} + 2l(1-l/f)\sigma_{12} + l^2\sigma_{22} \\ &= \left(\frac{l}{f}\right)^2\sigma_{11} - \left(\frac{l}{f}\right)(2\sigma_{11} + 2l\sigma_{12}) + (\sigma_{11} + 2l\sigma_{12} + l^2\sigma_{22}) \end{aligned}$$

- Indeed a parabola in *l/f*

Quad scan #2

- Build matrix of the type *y=Ax*
  - and with error bars $\Sigma_k=2\sigma_k\Delta\sigma_k$

$$\begin{pmatrix} \bar{\sigma}_{x,1}^2 \\ \bar{\sigma}_{x,2}^2 \\ \bar{\sigma}_{x,3}^2 \\ \bar{\sigma}_{x,4}^2 \\ \bar{\sigma}_{x,5}^2 \end{pmatrix} = \begin{pmatrix} (1-l/f_1)^2 & 2l(1-l/f_1) & l^2 \\ (1-l/f_2)^2 & 2l(1-l/f_2) & l^2 \\ (1-l/f_3)^2 & 2l(1-l/f_3) & l^2 \\ (1-l/f_4)^2 & 2l(1-l/f_4) & l^2 \\ (1-l/f_5)^2 & 2l(1-l/f_5) & l^2 \end{pmatrix} \begin{pmatrix} \sigma_{11} \\ \sigma_{12} \\ \sigma_{22} \end{pmatrix}$$

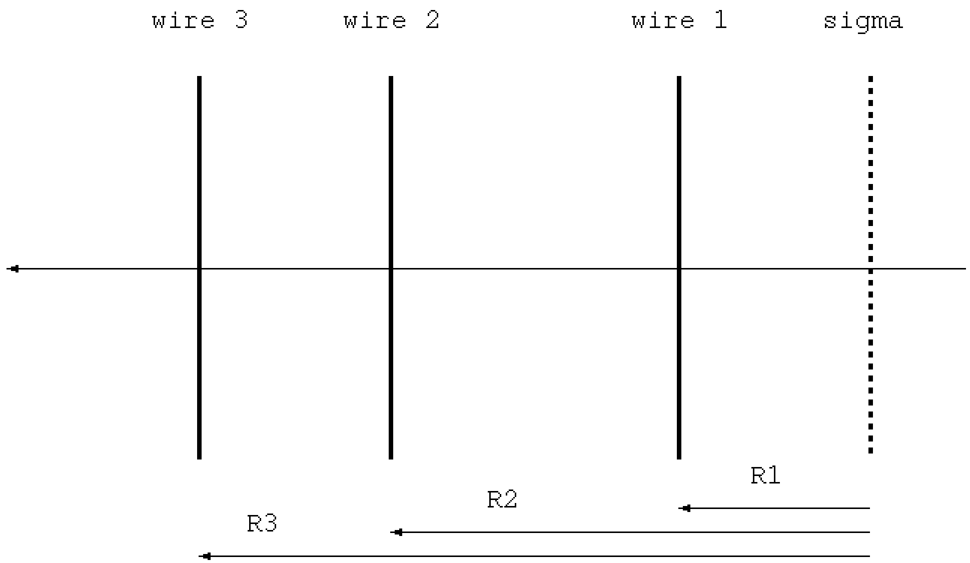
$$\begin{pmatrix} \frac{\bar{\sigma}_{x,1}^2}{\Sigma_1} \\ \frac{\bar{\sigma}_{x,2}^2}{\Sigma_2} \\ \frac{\bar{\sigma}_{x,3}^2}{\Sigma_3} \\ \frac{\bar{\sigma}_{x,4}^2}{\Sigma_4} \\ \frac{\bar{\sigma}_{x,5}^2}{\Sigma_5} \end{pmatrix} = \begin{pmatrix} \frac{(1-l/f_1)^2}{\Sigma_1} & \frac{2l(1-l/f_1)}{\Sigma_1} & \frac{l^2}{\Sigma_1} \\ \frac{(1-l/f_2)^2}{\Sigma_2} & \frac{2l(1-l/f_1)}{\Sigma_2} & \frac{l^2}{\Sigma_2} \\ \frac{(1-l/f_3)^2}{\Sigma_3} & \frac{2l(1-l/f_1)}{\Sigma_3} & \frac{l^2}{\Sigma_3} \\ \frac{(1-l/f_4)^2}{\Sigma_4} & \frac{2l(1-l/f_1)}{\Sigma_4} & \frac{l^2}{\Sigma_4} \\ \frac{(1-l/f_5)^2}{\Sigma_5} & \frac{2l(1-l/f_1)}{\Sigma_5} & \frac{l^2}{\Sigma_5} \end{pmatrix} \begin{pmatrix} \sigma_{11} \\ \sigma_{12} \\ \sigma_{22} \end{pmatrix}$$

- Solve by least-squares pseudo-inverse

$$x=(A^tA)^{-1}A^ty$$

- with the covariance matrix $Cov=(A^tA)^{-1}$
  - diagonal elements are square of error bars of fit parameter x

Or use several wire scanners



$$\begin{aligned}
\bar{\sigma}_1^2 &= (R^1)^2_{11}\sigma_{11} + 2R^1_{11}R^1_{12}\sigma_{12} + (R^1)^2_{12}\sigma_{22} \\
\bar{\sigma}_2^2 &= (R^2)^2_{11}\sigma_{11} + 2R^2_{11}R^2_{12}\sigma_{12} + (R^2)^2_{12}\sigma_{22} \\
\bar{\sigma}_3^2 &= (R^3)^2_{11}\sigma_{11} + 2R^3_{11}R^3_{12}\sigma_{12} + (R^3)^2_{12}\sigma_{22}
\end{aligned}$$

$$\begin{pmatrix} \bar{\sigma}_1^2 \\ \bar{\sigma}_2^2 \\ \bar{\sigma}_3^2 \end{pmatrix} = \begin{pmatrix} (R^1)^2_{11} & 2R^1_{11}R^1_{12} & (R^1)^2_{12} \\ (R^2)^2_{11} & 2R^2_{11}R^2_{12} & (R^2)^2_{12} \\ (R^3)^2_{11} & 2R^3_{11}R^3_{12} & (R^3)^2_{12} \end{pmatrix} \begin{pmatrix} \sigma_{11} \\ \sigma_{12} \\ \sigma_{22} \end{pmatrix}$$

- $(A^t A)^{-1} A^t$ - gymnastics with error bar estimates
- Derive emittance and betas after σ_{ij} is found by inversion

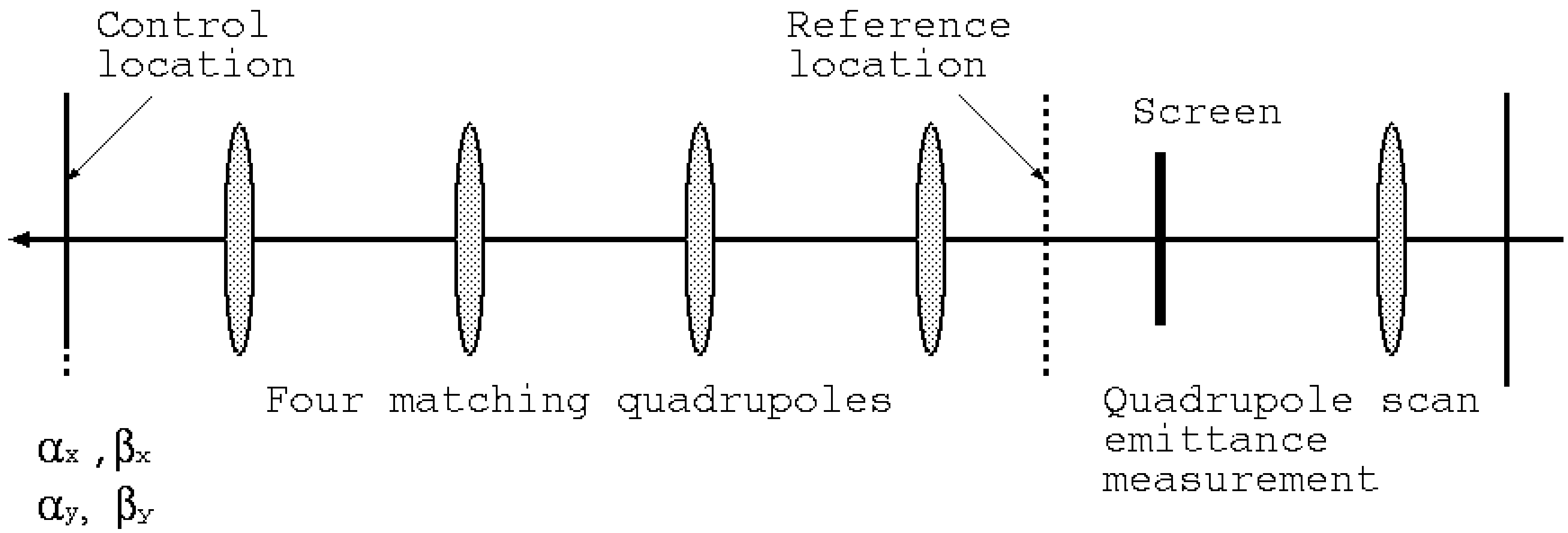
$$\varepsilon_x^2 = \sigma_{11}\sigma_{22} - \sigma_{12}^2 \qquad \beta_x = \sigma_{11}/\varepsilon_x \qquad \alpha_x = -\sigma_{12}/\varepsilon_x$$

- Can use several more wire scanners which allows χ^2 calculation for goodness-of-fit estimate

Fix beam matrix a.k.a. Beta match

$$\gamma_x = \frac{1+\alpha_x^2}{\beta_x}$$

- Uncoupled beam matrix $\varepsilon_x \begin{pmatrix} \beta_x & -\alpha_x \\ -\alpha_x & \gamma_x \end{pmatrix}$
  - need four quadrupoles to adjust $\alpha_x, \beta_x, \alpha_y,$ and β_y
  - non-linear optimizer (MADX matching module)



Waist knob

- Finding quad-excitations to match beta functions (or sigma matrix) is a non-linear problem
- and depends on the incoming beam matrix.
- Tricky, but one sometimes can build knobs, based on the design optics, to correct some observable
  - conceptually: linearizing around a working point
- Example:
  - IP-waist knob
  - $d\alpha_x/dQuad_{1,2}$ and $d\alpha_y/dQuad_{1,2}$

Beam lines: Skew-gradient errors

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- Transfer matrix for a skew-quadrupole

$$S = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1/f & 0 \\ 0 & 0 & 1 & 0 \\ 1/f & 0 & 0 & 1 \end{pmatrix}$$

- Vertical part of the sigma-matrix after skew quad

$$\begin{pmatrix} \hat{\sigma}_{33} & \hat{\sigma}_{34} \\ \hat{\sigma}_{34} & \hat{\sigma}_{44} \end{pmatrix} = \begin{pmatrix} \sigma_{33} & \sigma_{34} \\ \sigma_{34} & \sigma_{44} + \sigma_{11}/f^2 \end{pmatrix}$$

verify this on paper!

- *Projected emittance* after skew quadrupole

$$\hat{\varepsilon}_y^2 = \varepsilon_y^2 + \frac{\sigma_{11}\sigma_{33}}{f^2} = \varepsilon_y^2 \left(1 + \frac{\varepsilon_x}{\varepsilon_y} \frac{\beta_x \beta_y}{f^2} \right)$$

  - Problem with flat beams. Increases with ratio $\varepsilon_x/\varepsilon_y$ and is proportional to both beta functions.

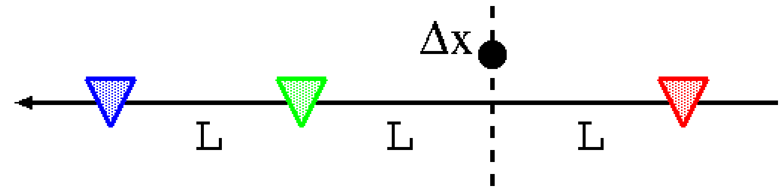
- Problem in Final-Focus Systems with flat beams. Solenoid fields need compensation.

That’s all for now, folks

- Take-home messages
  - Linear superposition of dipole-like errors.
  - Gradient errors mess up beam sizes.
    - Beta beat and B_{mag}
  - Skew gradients cause problems with flat beams.
  - Correction often involves to invert a “response matrix”
- Next time
  - same thing as today, but in rings, where the beam bites its own tail.

Things to think about...

- Can you determine the relative excitation of the three steering magnets without doing matrix algebra?



- You've carefully checked the optics of your linac before powering the RF and found it to be perfect, but then nothing works when you power the accelerating structures. Any ideas why?
- How many steerers and quads do you need to adjust the vertical position and angle and, additionally, the horizontal Twiss parameters?

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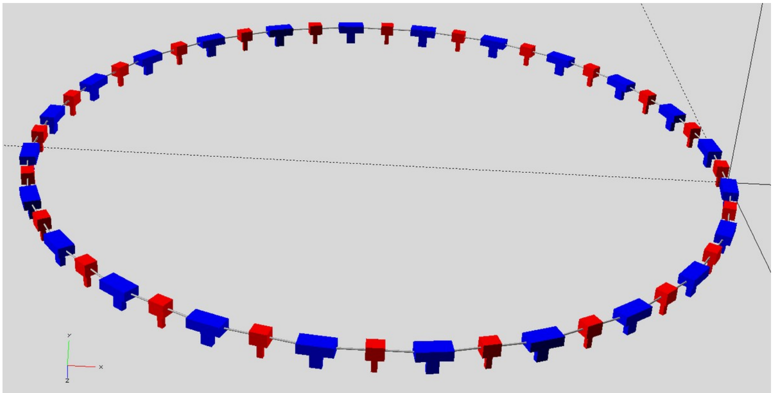
Imperfections in a Ring

- Effect of a localized kick on orbit
- Effect of a localized gradient error
- Effect of a skew gradient error
- Stop-bands and resonances



Dipole errors in a Ring

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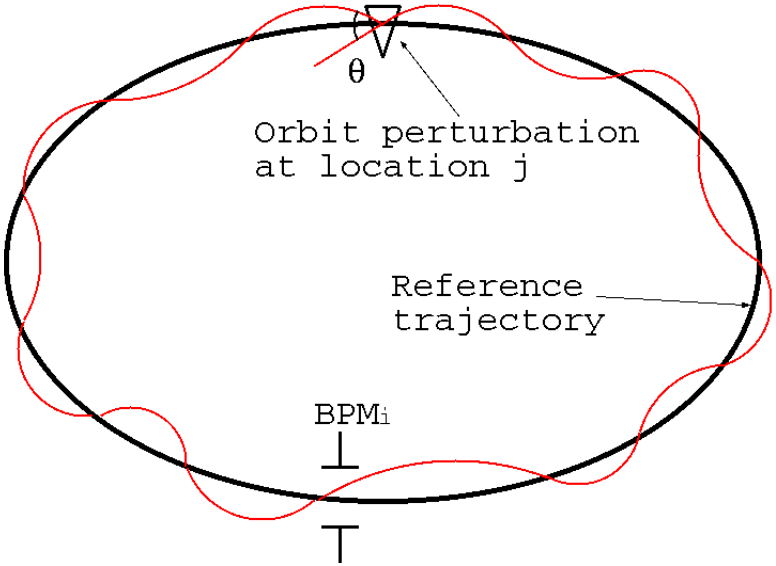
- Beam bites its tail → periodic boundary conditions → closed orbit
- Orbit after perturbation at *j*



$$\vec{x}_j = R^{jj}\vec{x}_j + \vec{q}_j$$

$$\vec{x}_j = (1 - R^{jj})^{-1}\vec{q}_j$$

- Propagate to BPM i

$$\vec{x}_i = R^{ij}\vec{x}_j = R^{ij}(1 - R^{jj})^{-1}\vec{q}_j = C^{ij}\vec{q}_j$$

- Response coefficients $C^{ij} = R^{ij}(1 - R^{jj})^{-1}$
  - just like transfer matrix in beam line, but with built-in closed-orbit constraint.

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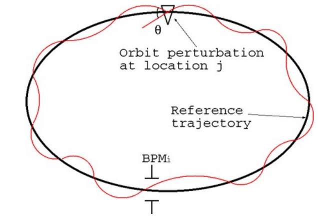
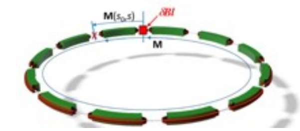
Response coefficients with beta functions





- Express transfer-matrices through beta functions

$$\begin{pmatrix} x \\ x' \end{pmatrix}_j = \begin{pmatrix} \cos(2\pi Q) & \beta_j \sin(2\pi Q) \\ -\sin(2\pi Q)/\beta_j & \cos(2\pi Q) \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_j + \begin{pmatrix} 0 \\ \theta \end{pmatrix}$$

- Solve for closed orbit

$$\begin{pmatrix} x \\ x' \end{pmatrix}_j = \frac{\theta}{2} \begin{pmatrix} \beta_j \cot(\pi Q) \\ 1 \end{pmatrix}$$

- Transfer matrix to BPM i

$$R^{ij} = \begin{pmatrix} \sqrt{\beta_i} & 0 \\ -\alpha_i/\sqrt{\beta_i} & 1/\sqrt{\beta_i} \end{pmatrix} \begin{pmatrix} \cos\mu_{ij} & \sin\mu_{ij} \\ -\sin\mu_{ij} & \cos\mu_{ij} \end{pmatrix} \begin{pmatrix} 1/\sqrt{\beta_j} & 0 \\ 0 & \sqrt{\beta_j} \end{pmatrix}$$

- Response coefficient

$$x_i = \left[\frac{\sqrt{\beta_i \beta_j}}{2\sin(\pi Q)} \cos(\mu_{ij} - \pi Q) \right] \theta$$

Divergences at integer tunes

$$C_{12}^{ij} = \frac{\partial BPM_i(x)}{\partial COR_j(x')}$$

You’ve seen it before, check out Wolfgang’s slides

Quadrupole alignment amplification factor

- Consider randomly displaced quadrupoles

$$\theta_j = d_j/f \qquad \langle d_j \rangle = 0 \qquad \langle d_j d_k \rangle = \sigma_d^2 \delta_{jk}$$

- Incoherently (RMS) add all contributions

$$\begin{aligned}\langle x_i^2 \rangle &= \langle \left[\sum_j \frac{\sqrt{\beta_i \beta_j}}{2 \sin \pi Q} \cos(\mu_{ij} - \pi Q) \frac{d_j}{f_j} \right] \left[\sum_k \frac{\sqrt{\beta_i \beta_k}}{2 \sin \pi Q} \cos(\mu_{ik} - \pi Q) \frac{d_k}{f_k} \right] \rangle \\ &= \sum_j \frac{\beta_i \beta_j}{(2 \sin \pi Q)^2} \cos^2(\mu_{ij} - \pi Q) \frac{\sigma_d^2}{f_j^2}\end{aligned}$$

- Misalignment amplification factor $\sqrt{\langle x_i^2 \rangle} \approx \sqrt{N_q} \frac{\bar{\beta}/\bar{f}}{2\sqrt{2} \sin \pi Q} \sigma_d$
  - large rings with large N_q are sensitive,
  - such as LHC and FCC.

Response Coefficients with RF

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- Radio-frequency system constrains the revolution time

$C = vT$

$$\frac{\Delta T}{T} = \frac{\Delta C}{C} - \frac{\Delta v}{v} = \left(\alpha - \frac{1}{\gamma^2}\right)\delta$$

- but a horizontal kick causes a horizontal closed orbit distortion which causes the circumference to change by $\Delta C = D_x \theta_x$ (6x6 TM is symplectic, and if uncoupled: $R_{16} = R_{52}$)
- Since RF fixes the revolution frequency the momentum of the particle has to adjust to $\delta = -D_j \theta / \eta C$
- ...and the particle moves on a dispersion trajectory.
- Complete response coefficient between BPM_i and dipole error or COR_j

$$C_{12}^{ij} = \left[\frac{\sqrt{\beta_i \beta_j}}{2\sin(\pi Q)}\cos(\mu_{ij} - \pi Q) - \frac{D_i D_j}{\eta C}\right]$$

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Orbit Correction in a Ring

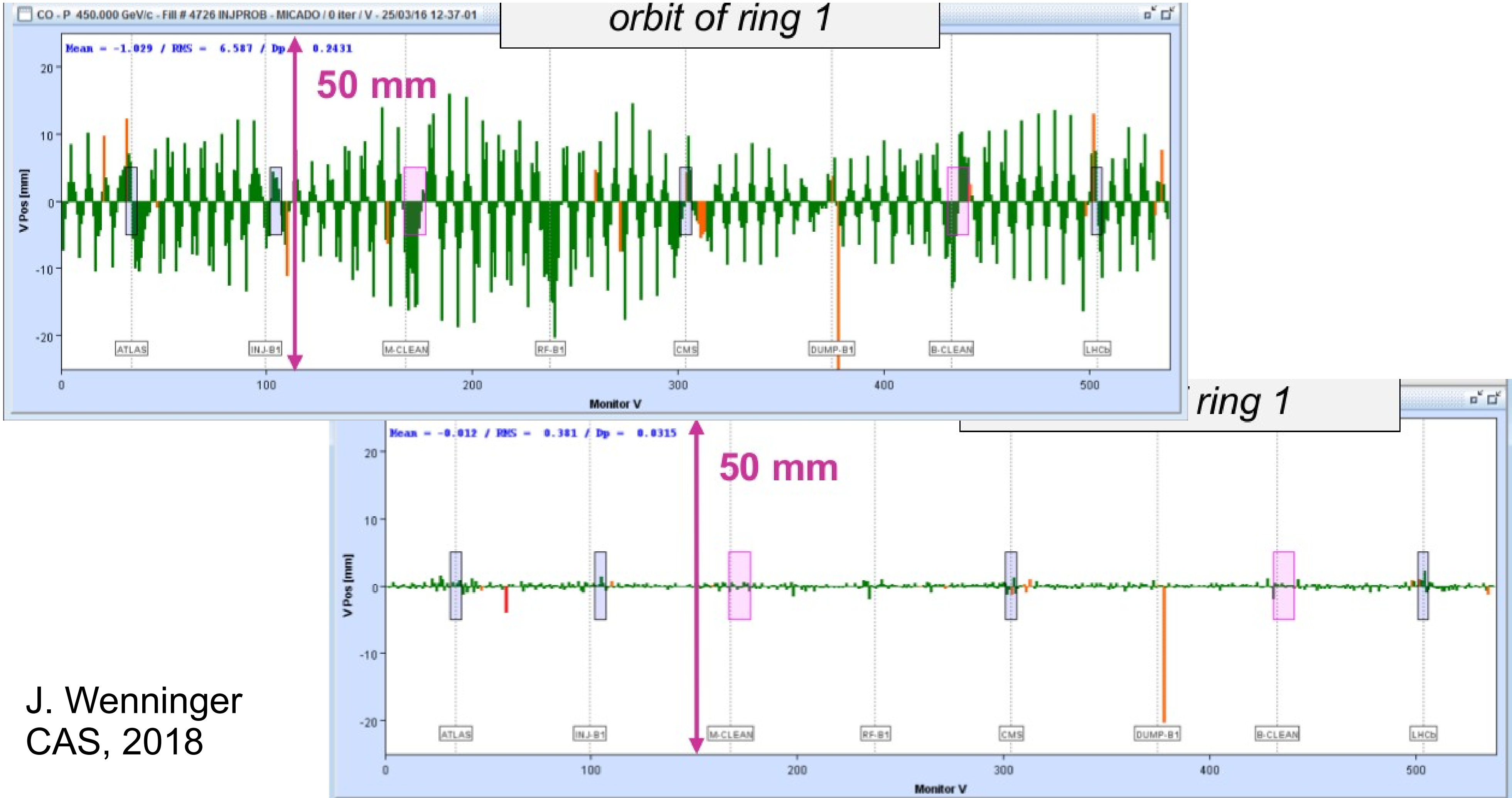
- Every steering magnet affects every BPM
  - orbit response coefficients and matrix $C^{ij} = R^{ij}(1 - R^{jj})^{-1}$
- Compensate measured positions x_i by inverting

$$\begin{pmatrix} -x_1 \\ -x_2 \\ \vdots \\ -x_m \end{pmatrix} = \begin{pmatrix} C_{12}^{11} & C_{12}^{12} & \dots & C_{12}^{1n} \\ C_{12}^{21} & C_{12}^{22} & \dots & C_{12}^{2n} \\ \vdots & \vdots & \ddots & \vdots \\ C_{12}^{m1} & C_{12}^{m2} & \dots & C_{12}^{mn} \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{pmatrix}$$

- and also in the vertical plane
- left-multiply with diagonal BPM error matrix $\bar{\Lambda} = \mathrm{diag}\left(\frac{1}{\sigma_1}, \dots, \frac{1}{\sigma_n}\right)$
- use either calculated or measured response matrix
- inversion with pseudo-inverse, MICADO, or SVD

Example: orbit correction

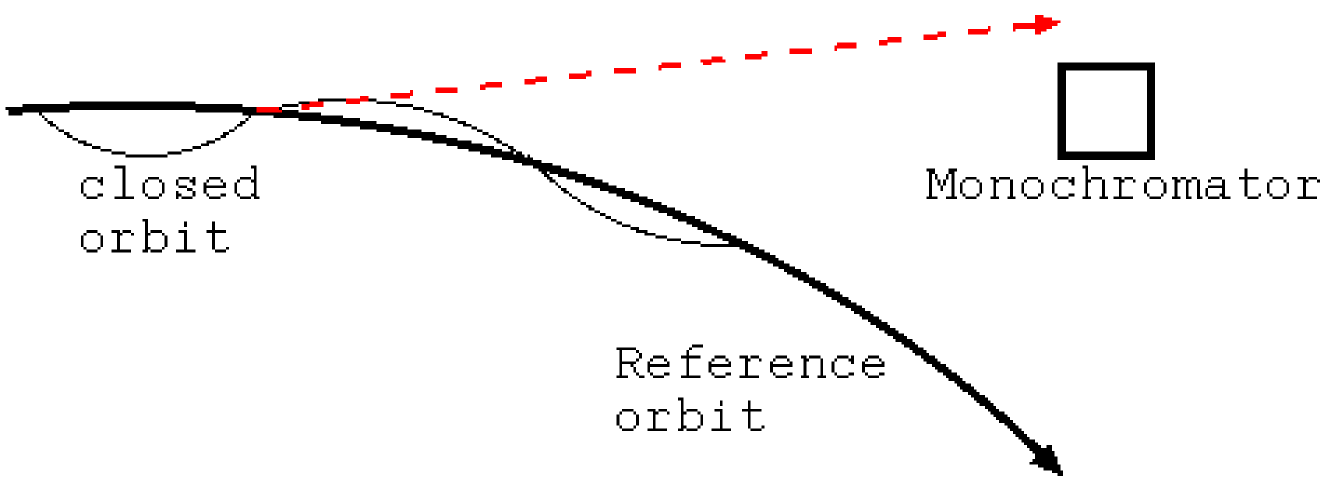
Vertical orbit in LHC, before and after correction



J. Wenninger
CAS, 2018

Steering synchrotron beam lines

- steer synchrotron light beam onto experiment
- fix angle at source point
- incorporate in orbit correction by +L,vBPM,-L



Dispersion-free steering

- Steering magnets are small dipoles and also affect the dispersion (in ring and linac) besides the orbit.
- Take into account with dispersion response matrix $S_{ij}=dD_i/d\theta_j=d^2x_i/d\delta d\theta_j$ $\quad$ $(D_i=dx_i/d\delta)$
  - Either numerically or from measurements
- Simultaneously correct orbit and dispersion
  - weight with Σs
  - more constraints
  - same number of correctors

$$\begin{pmatrix} \vdots \\ x_i/\Sigma_i \\ \vdots \\ D_i/\hat{\Sigma}_i \\ \vdots \end{pmatrix} = \begin{pmatrix} C_{ij}/\Sigma_i \\ S_{ij}/\hat{\Sigma}_i \end{pmatrix} \begin{pmatrix} \vdots \\ \theta_j \\ \vdots \end{pmatrix}$$

Gradient Errors in a Ring

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- Add a gradient error (modeled as a thin quad) to a ring with $\mu=2\pi Q$

$$R_Q R = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \begin{pmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ -\frac{1+\alpha^2}{\beta}\sin\mu & \cos\mu - \alpha\sin\mu \end{pmatrix}$$

$$= \begin{pmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ -(\cos\mu + \alpha\sin\mu)/f + \gamma\sin\mu & \cos\mu - \alpha\sin\mu - (\beta/f)\sin\mu \end{pmatrix}$$

- Trace gives the perturbed tune $\bar{Q} = Q + \Delta Q$

$$2\cos(2\pi(Q+\Delta Q)) = 2\cos(2\pi Q) - \frac{\beta}{f}\sin(2\pi Q)$$

- and if β/f is small, the tune-shift is $\Delta Q \approx \frac{\beta}{4\pi f}$
- Gradient errors change the tune!

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Changes of the beta function and stop bands

- From R_{12} get the change in the beta function

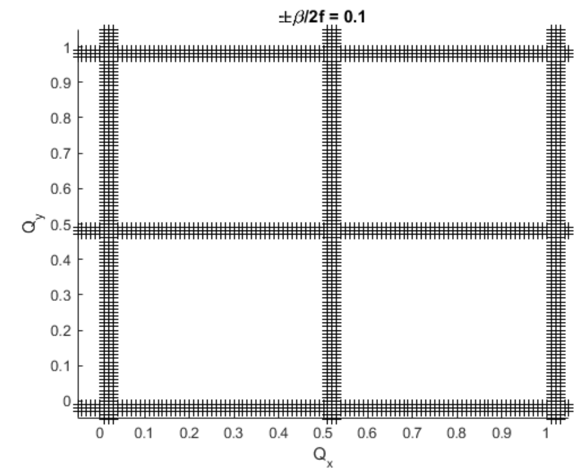
$$\bar{\beta} = \frac{\beta \sin(2\pi Q)}{\sin(2\pi(Q+\Delta Q))} \approx \beta\left[1 + 2\pi\Delta Q \cot(2\pi Q)\right]$$

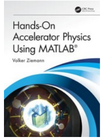
$$\frac{\Delta\beta}{\beta} = 2\pi\Delta Q \cot(2\pi Q) \approx \frac{\beta}{2f}\cot(2\pi Q)$$

- Divergences at half-integer values of the tune
- Stability requires

$$\left|\cos(2\pi Q) - \frac{\beta}{2f}\sin(2\pi Q)\right| \le 1$$

  - stop-band width





You've seen it before, check out Wolfgang's slides

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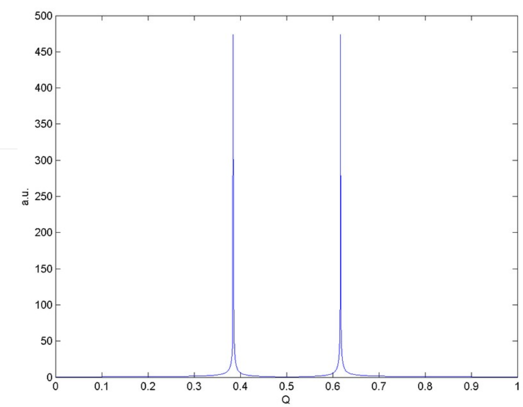
Measuring the Tune

- Kick beam and look at BPM difference-signal on spectrum analyzer
  - and dividing the observed frequency by the revolution frequency gives the fractional part of the tune
- Turn by turn BPM recordings and FFT
  - is it Q or 1-Q?
  - change QF and see which way the tune moves

```
Qx=0.616;
n=1:1024;
x=sin(2*pi*Qx*n);
plot(n/1024,abs(fft(x)));
xlabel('Q_x'); ylabel('|fft(x)|')
```



- PLL in LHC: Beam is band-pass, tickle it, and detect synchronously

Tune Correction

- Use a variable quadrupole with $1/f = \Delta k_1 l$
- Changes both Q_x and Q_y $\quad \Delta Q_x = \frac{\beta_{1x}}{4\pi f_1} \quad \text{and} \quad \Delta Q_y = -\frac{\beta_{1y}}{4\pi f_1}$
- Use two independent quadrupoles

$$\begin{aligned} \Delta Q_x &= \frac{\beta_{1x}}{4\pi f_1} + \frac{\beta_{2x}}{4\pi f_2} \\ \Delta Q_y &= -\frac{\beta_{1y}}{4\pi f_1} - \frac{\beta_{2y}}{4\pi f_2} \end{aligned} \qquad \begin{pmatrix} \Delta Q_x \\ \Delta Q_y \end{pmatrix} = \frac{1}{4\pi} \begin{pmatrix} \beta_{1x} & \beta_{2x} \\ -\beta_{1y} & -\beta_{2y} \end{pmatrix} \begin{pmatrix} 1/f_1 \\ 1/f_2 \end{pmatrix}$$

- Solve by inversion

$$\begin{pmatrix} 1/f_1 \\ 1/f_2 \end{pmatrix} = \frac{-4\pi}{\beta_{1x}\beta_{2y} - \beta_{2x}\beta_{1y}} \begin{pmatrix} -\beta_{2y} & -\beta_{2x} \\ \beta_{1y} & \beta_{1x} \end{pmatrix} \begin{pmatrix} \Delta Q_x \\ \Delta Q_y \end{pmatrix}$$

- Quads on same power supply → sum of betas

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Measuring beta functions

- Change quadrupole and observe tune variation

$$\Delta Q_x = \frac{\beta_{1x}}{4\pi f_1} \quad \text{and} \quad \Delta Q_y = -\frac{\beta_{1y}}{4\pi f_1}$$

- Need independent power supplies
  - or piggy-back boost supply
  - or a shunt resistor
- May get sums of betas in quads-on-the-same-power-supply.

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Model Calibration #1

- Compare measured $\hat{C}^{ij}$ orbit response matrix to computer model C^{ij}
  - enormous amount of data $2 \times N_{bpm} \times N_{cor}$
- and blame the difference on quad gradients g_k or other parameters p_l
  - much fewer fit-parameters N_{quad} and N_{para}

$$\hat{C}^{ij} - C^{ij} = \sum_k \frac{\partial C^{ij}}{\partial g_k} \Delta g_k + \sum_l \frac{\partial C^{ij}}{\partial p_l} \Delta p_l$$

- First used in SPEAR and later perfected in NSLS → LOCO

Model Calibration #2

- Normally the parameters p_l are BPM and corrector scale errors
  - fit for N_{quad} gradients and $2\ x\ (N_{bpm}+N_{cor})$ scales

$$\hat{C}^{ij} - C^{ij} = \sum_k \frac{\partial C^{ij}}{\partial g_k}\Delta g_k + C^{ij}\Delta x^i - C^{ij}\Delta y^j$$

- Determine derivatives $\partial C^{ij}/\partial g_k$ numerically by changing a gradient and re-calculating all response coefficients, then taking differences
- BPM-cor degeneracy → SVD needed to invert
- Converges, if χ^2/DOF is close to unity

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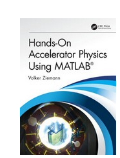
micro-LOCO

- 2 Quads, 2 BPM, 2 COR, only horizontal "C_{12}"
  - ill-defined, but useful to see the structure of matrix
  - gradient errors Δg, BPM scales Δx, corrector scales Δy
- Blame difference on Δg,Δx,Δy

$$C^{ij} = R^{ij}(1 - R^{jj})^{-1}$$

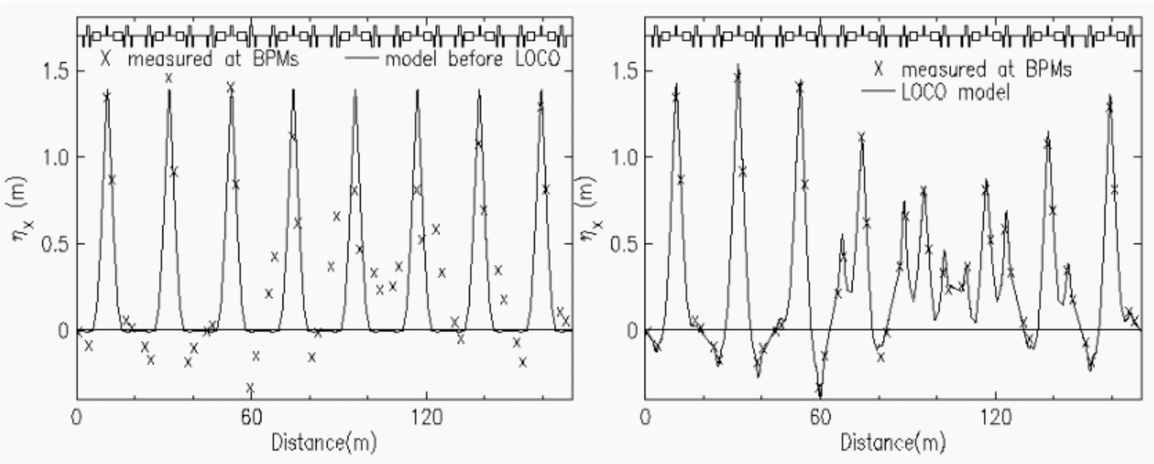
$$\begin{pmatrix} \hat{C}^{11} - C^{11} \\ \hat{C}^{21} - C^{21} \\ \hat{C}^{12} - C^{12} \\ \hat{C}^{22} - C^{22} \end{pmatrix} = \begin{pmatrix} \frac{\partial C^{11}}{\partial g_1} & \frac{\partial C^{11}}{\partial g_2} & C^{11} & 0 & -C^{11} & 0 \\ \frac{\partial C^{21}}{\partial g_1} & \frac{\partial C^{21}}{\partial g_2} & 0 & C^{21} & -C^{21} & 0 \\ \frac{\partial C^{12}}{\partial g_1} & \frac{\partial C^{12}}{\partial g_2} & C^{12} & 0 & 0 & -C^{12} \\ \frac{\partial C^{22}}{\partial g_1} & \frac{\partial C^{22}}{\partial g_2} & 0 & C^{22} & 0 & -C^{22} \end{pmatrix} \begin{pmatrix} \Delta g_1 \\ \Delta g_2 \\ \Delta x_1 \\ \Delta x_2 \\ \Delta y_1 \\ \Delta y_2 \end{pmatrix}$$

See example code in Matlab



Experience

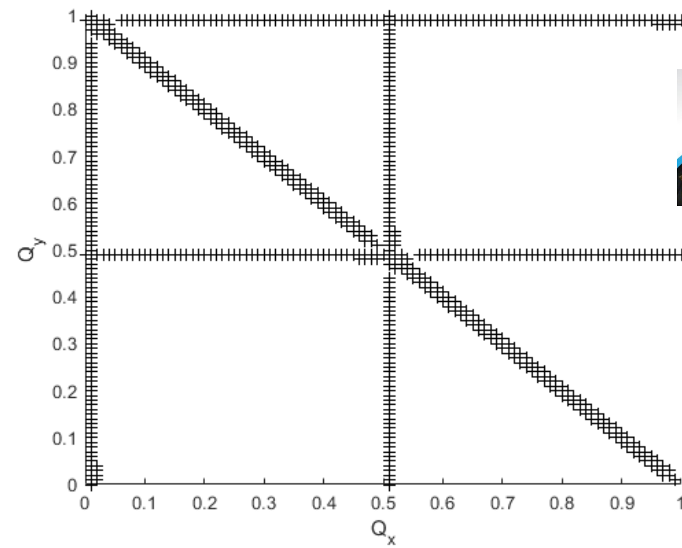
- SPEAR: could explain measured tunes to within 4×10^{-3} from quadrupole settings which had percent errors (J. Corbett, M. Lee, VZ, PAC93).
- NSLS: LOCO, $\Delta\beta/\beta = 10^{-3}$, dispersion fixed, emittance factor 2 improved (J. Safranek, NIMA 388, 1997)

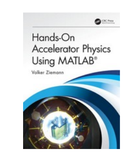


- and practically every light source since then uses it.

Skew-gradient stop bands

- Why are skew-gradient errors bad?
  - they also add stop bands along the diagonals
- Ring with single skew
  - with strength $\sqrt{\beta_x \beta_y}/f = 0.2$
- Calculate the eigentunes
  - Edwards-Teng algorithm
- for each pair Q_x, Q_y
- make cross if unstable
  - complex or NAN in Matlab





Measuring Coupling

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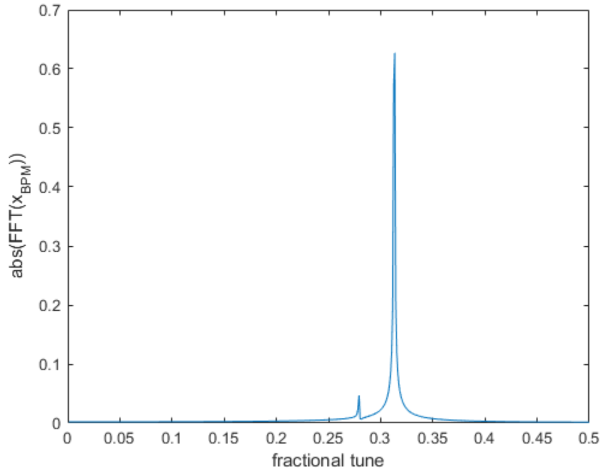
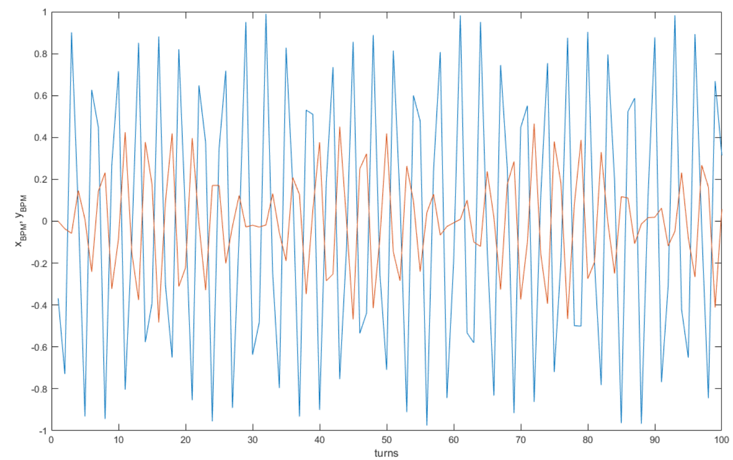
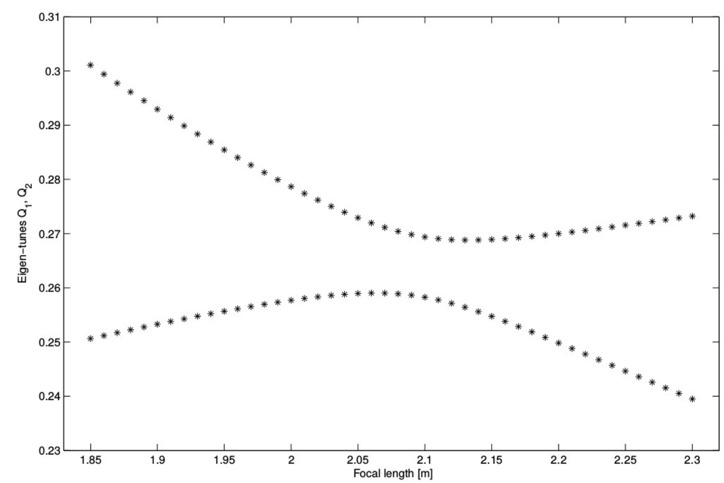
- BPM turn-by-turn data cross talk, beating





- Closest tune
  - try to make the tunes equal with an upright quad
  - measure tunes
  - coupling 'repels' the tunes



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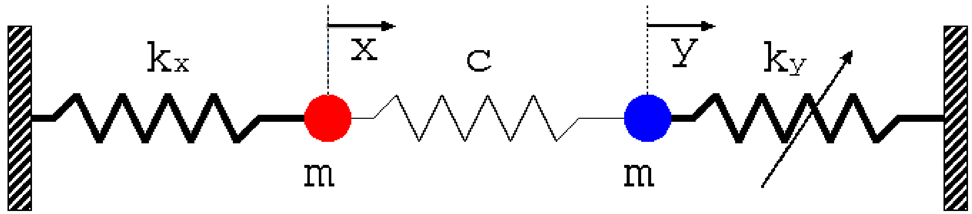
Coupling: mechanical analogy



- Two weakly coupled oscillators: simple to find the equations of motion

$$
\begin{aligned}
0 &= m\ddot{x} + (k_x + c)x - cy \\
0 &= m\ddot{y} + (k_y + c)y - cx
\end{aligned}
$$

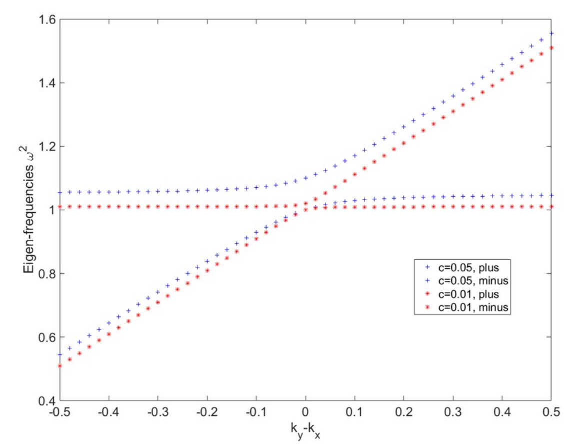
- and eigen-frequencies

$$\omega^2 = \frac{k_x + k_y + 2c}{2m} \pm \sqrt{\left(\frac{k_x - k_y}{2m}\right)^2 + \frac{c^2}{m^2}}$$

- Minimum tune separation
- Excite one mass, get beating

Translation for accelerator physicists:

x → horiz. betatr. osc.
y → vert. betatr. osc.
$k_x/m \rightarrow Q_x^2$
$k_y/m \rightarrow Q_y^2$ (adj.)
c/m → coupling source



Coupling correction

- Use a single skew-quad if that is all you have to minimize the closest tune.
- Otherwise build knobs for the four resonance-driving terms with normalized skew gradients

$$\begin{pmatrix} \mathrm{Re}(F_-) \\ \mathrm{Im}(F_-) \\ \mathrm{Re}(F_+) \\ \mathrm{Im}(F_+) \end{pmatrix} = \begin{pmatrix} \cos(\mu_{x1}-\mu_{y1}) & \dots & \cos(\mu_{x4}-\mu_{y4}) \\ \sin(\mu_{x1}-\mu_{y1}) & \dots & \sin(\mu_{x4}-\mu_{y4}) \\ \cos(\mu_{x1}+\mu_{y1}) & \dots & \cos(\mu_{x4}+\mu_{y4}) \\ \sin(\mu_{x1}+\mu_{y1}) & \dots & \sin(\mu_{x4}+\mu_{y4}) \end{pmatrix} \begin{pmatrix} \kappa_1 \\ \kappa_2 \\ \kappa_3 \\ \kappa_4 \end{pmatrix}$$

$$F_\pm = \sum_j \frac{\beta_{x,j}\beta_{y,j}}{2f_j} e^{i(\mu_{x,j}\pm\mu_{y,j})}$$

$$\kappa_i = \sqrt{\beta_{xi}\beta_{yi}}/2f_i$$

- and empirically minimize each RDT,
  - often F_- (if tunes are close) is sufficient
- Choose phases *μ* to make the condition number of the matrix as close to unity as possible.

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Measuring Chromaticity Q'

- Reminder: chromaticity is the momentum-dependence of the tunes: $Q = Q_0+Q'\delta$
- Force the momentum to change by changing the RF frequency. The beam follows, because synchrotron oscillations are stable.

$$-\frac{\Delta f_{rf}}{f_{rf}} = \frac{\Delta T}{T} = \eta\delta = \left(\alpha - \frac{1}{\gamma^2}\right)\delta \qquad \rightarrow \qquad \delta = -\frac{1}{\eta}\frac{\Delta f_{rf}}{f_{rf}}$$

- Plot tune change ΔQ versus $\Delta f_{rf}/f_{rf}$. The slope is proportional to (1/chromaticity Q') [can also use PLL]

$$Q' = \frac{\Delta Q}{\delta} = -\eta\frac{\Delta Q}{\Delta f_{rf}/f_{rf}}$$

Chromaticity correction

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- Need **controllable and momentum-dependent quadrupole** to compensate or at least change the natural chromaticity Q'=dQ/dδ.
- Momentum dependent feed-down: Use sextupole with dispersion, replace d_x by $D_x\delta$

$$\Delta x' - i\Delta y' = \frac{k_2 L}{2}\left[(x+iy)^2 + 2D_x\delta(x+iy) + D_x^2\delta^2)\right]$$

- Linear (quadrupolar) term with effective focal length that is momentum dependent

$$\frac{1}{f_\delta} = k_2 L D_x \delta$$

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Chromaticity correction #2

- Momentum-dependent tune shifts

$$\Delta Q_x = \frac{k_2 L D_x \beta_x}{4\pi}\delta \qquad \Delta Q_y = -\frac{k_2 L D_x \beta_y}{4\pi}\delta$$

- Build correction matrix in the same way as for the tune correction for ΔQ'=ΔQ/δ

$$\begin{pmatrix} \Delta Q'_x \\ \Delta Q'_y \end{pmatrix} = \frac{1}{4\pi}\begin{pmatrix} D_{1x}\beta_{1x} & D_{2x}\beta_{2x} \\ -D_{1x}\beta_{1y} & -D_{2x}\beta_{2y} \end{pmatrix}\begin{pmatrix} (k_2 L)_1 \\ (k_2 L)_2 \end{pmatrix}$$

- Invert to find sextupole excitations k_2L that add chromaticities to partially compensate the natural

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Winding down

- We looked at the sources of all evil, the imperfections,
- and how they affect
  - the orbit
  - the optics (beta functions, etc)
- and figured out how to fix it.
- Lots of things to think about, for example...

Things to think about...

- In your 3 GeV electron ring ($B\rho \approx 10$ Tm) you have 0.5 m long quads with a gradient of $dB_y/dx=5$ T/m. What is their approximate focal length?
- The beta function at the quad is about 8.5 m. By what percentage do you have to change the quad excitation in order to change the tune by 3×10^{-3}?
- Find out what's wrong in your accelerator at home and fix it.

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Bloopers

- LEP vacuum pipe soldering
- Beer bottle in LEP
- Stand-up metal-piece in magnet
- Shielding in SLC damping ring

- These non-standard 'imperfections' are very difficult to identify, but it is good to keep in mind that even such odd-balls occur.