Recap 3rd Lecture

Optical functions / Twiss parameters: $\sigma_u = \sqrt{\varepsilon_u \beta_u}, \quad \sigma_{u'} = \sqrt{\varepsilon_u \gamma_u}, \quad \overline{uu'} = -\varepsilon_u \alpha_u$

Ellipse equation: $\varepsilon_u = \gamma_u u^2 + 2\alpha_u uu' + \beta_u u'^2$

General solution of equation of motion: $u(s) = A\sqrt{\beta_u(s)}\cos\left(\mu_u(s) + \varphi\right)$

Courant-Snyder invariant A: πA^2 = area of single particle ellipse

Optical functions ↔ 4 Twiss parameters are linked via $\frac{\mathrm{d}}{\mathrm{d}s}\mu(s) = \frac{1}{\beta(s)}, \quad \frac{\mathrm{d}}{\mathrm{d}s}\beta(s) = -2\alpha(s), \quad \gamma(s) = \frac{1+\alpha^2(s)}{\beta(s)}$

Separation of beam's internal properties and impact of magnets optics!

Transformation of Twiss parameters using the beta matrix $\mathbf{B} = \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}, \qquad \varepsilon \cdot \mathbf{B} = \mathbf{\Sigma}_{\text{beam}} = \begin{pmatrix} \sigma_x^2 & \overline{xx'} \\ \overline{xx'} & \sigma_{x'}^2 \end{pmatrix}$

Transformation: $\mathbf{B}(s) = \mathbf{M}(s_0, s) \cdot \mathbf{B}(s_0) \cdot {}^T\mathbf{M}(s_0, s)$

(Particle beam ↔ TEM_{00} light beam: $4\pi\varepsilon = \lambda$ **)**

Twiss matrix: **M** only dependent on $\alpha, \alpha_0, \beta, \beta_0, \gamma, \gamma_0$ and μ

Weak focusing: $\beta > \rho$, **strong focusing stability criterion**: $|\mathrm{Tr}\{\mathbf{M}\}| \leq 2$