Transverse momentum dependent shape function for J/ψ production in SIDIS

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Outline

- Motivation

- Matching procedure
- Poles and the effective delta expansion
- The TMDShF tail
- Universality and process dependence

- Conclusions

Motivation

- TMDs incorporate the transverse momentum of quarks and gluons inside hadrons:
  "3D PDF"
- At small longitudinal momentum fraction x the gluons dominate (PDF), but hardly anything is known about the gluon TMDs experimentally.

- Heavy quarks are very sensitive to the gluon content of hadrons:
  - they are predominantly produced from gluons
  - not intrinsically present in hadrons at small momentum fractions.
- Furthermore, some quarkonium states, like the J/ψ, are relatively straightforward to detect and numerous events can be collected.

$\Longrightarrow$ Quarkonium, such as J/ψ production in SIDIS *Bacchetta et al. 2020*, can been considered as a main tool to extract gluon TMDs!

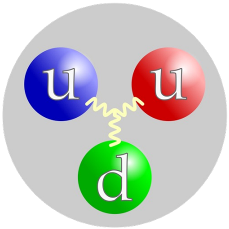
The Gluon TMDs

- Proton can be parameterized in terms of gluon TMDs, at LO (twist ~ 1/hard scale):

  Mulders and Rodrigues 2001

		Parent hadron polarization		
		Unpolarized	Longitudinal	Transverse
Parton polarization	U	f_1 (Number density)		$f_{1T}^{\perp}$ (Sivers)
	L		g_{1L} (Helicity)	g_{1T} (Worm-gear)
	T	$h_1^{\perp}$ (Boer-Mulders)	$h_{1L}^{\perp}$ (Worm-gear)	h_{1T} (Transversity) $h_{1T}^{\perp}$ (Pretzelosity)

- Unpolarized gluon distribution: f_1^g $\Longleftarrow$
- Linearly polarized gluon distribution: $h_1^{\perp g}$ $\Longleftarrow$



Models for quarkonium production

- Colour singlet model *Baier and Rückl 1983*

$$d\sigma[\mathcal{Q}] = \int dx_a\, dx_b\, f_a(x_a)\, f_b(x_b)\, d\hat{\sigma}_{a+b\to\mathcal{Q}+X}\, |R(0)|^2$$

- Nonrelativisitc QCD *Bodwin et al. 1997* $\Leftarrow$

$$d\sigma[\mathcal{Q}] = \sum_n \int dx_a\, dx_b\, f_a(x_a)\, f_b(x_b)\, d\hat{\sigma}_{a+b\to\mathcal{Q}[n]+X}\, \langle \mathcal{O}^{\mathcal{Q}}[n] \rangle$$

- $n = {}^{2S+1}L_J$

	$1,{}^1S_0$	$1,{}^3S_1$	$8,{}^1S_0$	$8,{}^3S_1$	$1,{}^1P_1$	$1,{}^3P_0$	$1,{}^3P_1$	$1,{}^3P_2$	$8,{}^1P_1$	$8,{}^3P_0$	$8,{}^3P_1$	$8,{}^3P_2$
J/ψ		1	v^3	v^4						v^4	v^4	v^4

- Colour evaporation model (improved by *Ma and Vogt 2016*)

$$d\sigma[\mathcal{Q}] = \mathcal{F}_{\mathcal{Q}} \int_{2m_Q}^{2m_h} dm_{qq}\, \frac{d\sigma_{qq}}{dm_{qq}}$$

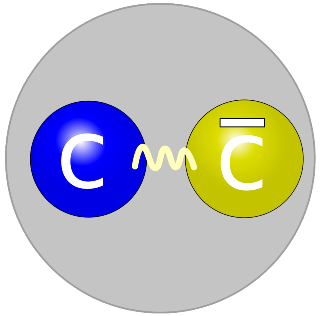
- J/ψ: $m_Q = m_c$; $m_h = m_D$

- Fragmentation functions *Kang et al. 2014*

$$d\sigma[\mathcal{Q}] = \int dx_a\, dx_b\, dz\, f_a(x_a)\, f_b(x_b)\, d\hat{\sigma}_{a+b\to c+X}\, D_{c\to\mathcal{Q}}(z) + \int dx_a\, dx_b\, dz\, f_a(x_a)\, f_b(x_b)\, d\hat{\sigma}_{a+b\to QQ+X}\, \boxed{D_{QQ\to\mathcal{Q}}(z)}$$

The shape function

- Binding of quarks described by NRQCD
- However, NRQCD does **not** incorporate:
  - Soft gluon exchange with other color sources that leads to final state smearing and TM dependence



- Therefore ones needs to include the TMDShF: $\Delta^{[n]}(z, \boldsymbol{k}_T^2)$

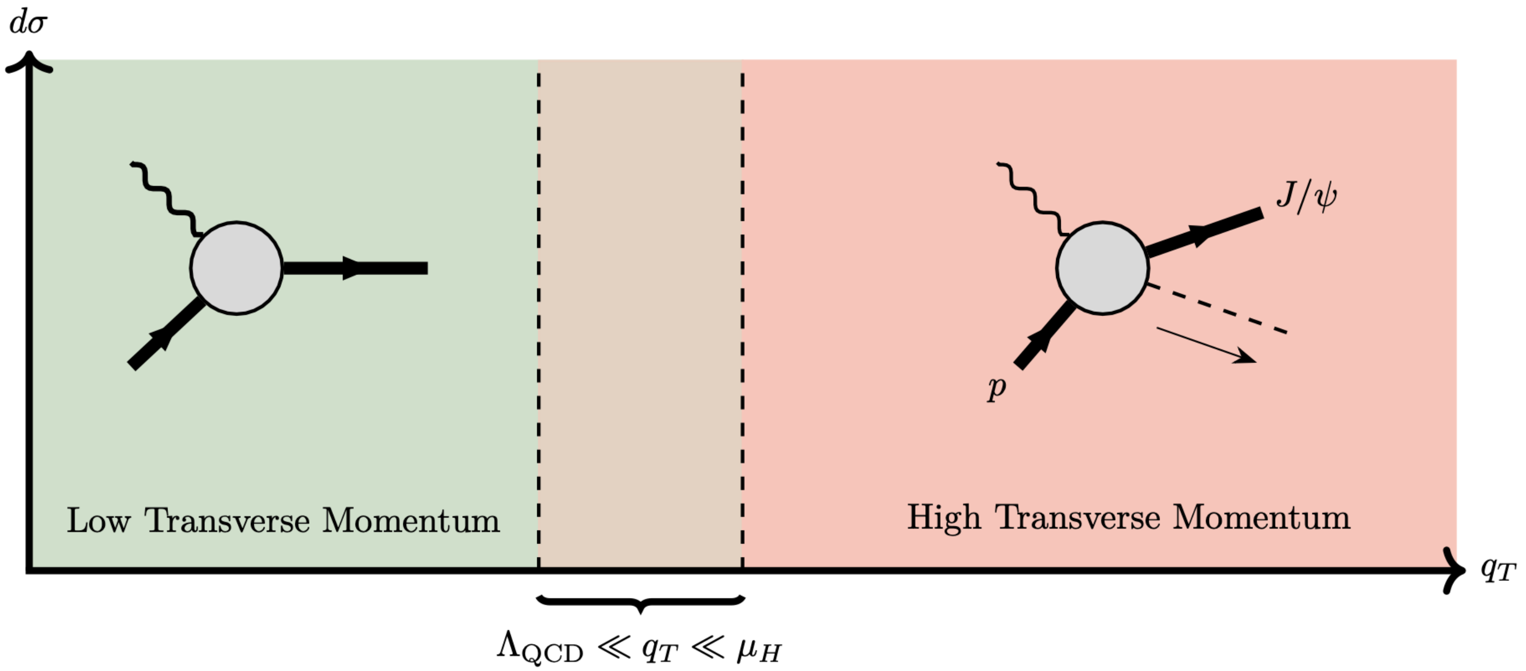
  Echevarria 2019 & Fleming et al. 2020

- *“Extension of the LMDEs in collinear factorization”*

- We assume that the TMDShF can be different when other TMDs are involved

Matching procedure

$$e(\ell) + p(P) \to e'(\ell') + \gamma^*(q) + p(P) \to e'(\ell') + J/\psi(P_\psi) + X$$



- TMD factorization

  $|\boldsymbol{q}_T| \ll \mu_H$

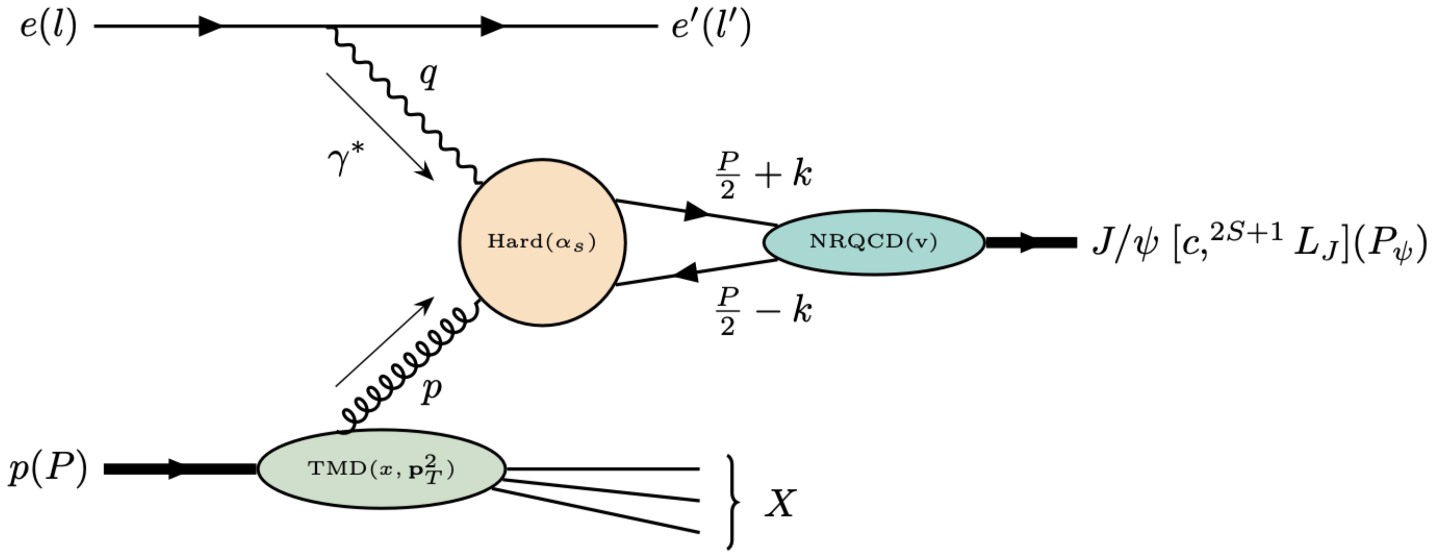
- Collinear factorization

  $|\boldsymbol{q}_T| \gg \Lambda_{\mathrm{QCD}}$

- In the overlap region the cross sections should match, which allows for extraction of the TMDShF *Boer et al. 2020*

$$|\boldsymbol{q}_T| = \frac{1}{z}|\boldsymbol{P}_{\psi\perp}|$$

Low Transverse Momentum (LTM)



- At LO 2 diagrams
- Solely CO production
- SIDIS variables:

$$x_B = \frac{Q^2}{2P\cdot q}, \quad y = \frac{P\cdot q}{P\cdot \ell}, \quad z = \frac{P\cdot P_\psi}{P\cdot q}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_B \mathrm{d}y \mathrm{d}z \mathrm{d}\boldsymbol{q}_T^2 \mathrm{d}\phi_\psi} = \frac{\alpha}{yQ^2}\left\{\left[1+(1-y)^2\right]\mathcal{F}_{UU,\perp} + 4(1-y)\mathcal{F}_{UU,\parallel} + 4(1-y)\cos 2\phi_\psi \mathcal{F}_{UU}^{\cos 2\phi_\psi}\right\}$$

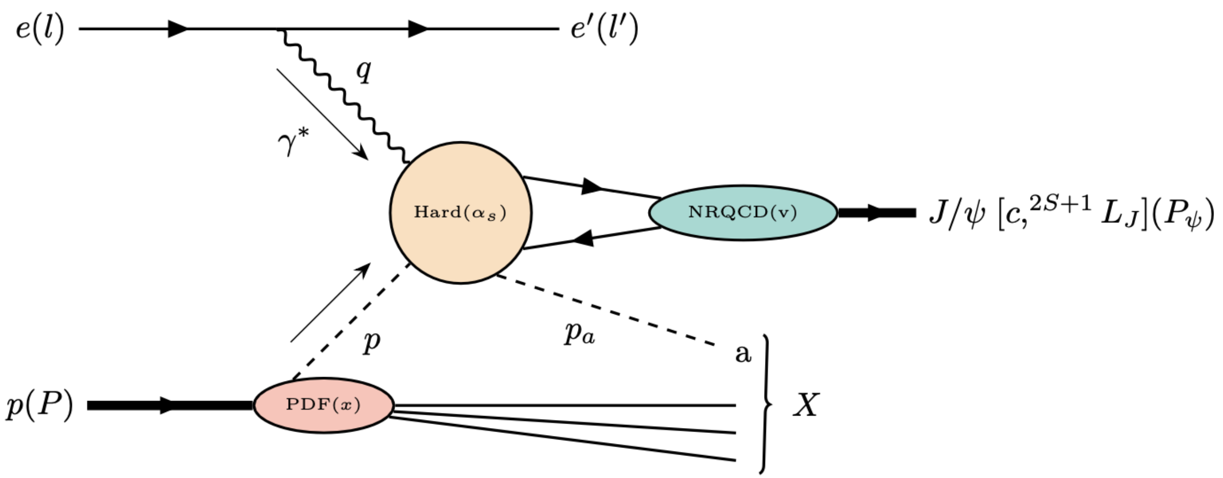
$$\mathcal{F}_{UU,\perp} = 2\pi^2 \frac{e_c^2\alpha\alpha_s}{M_\psi\left(M_\psi^2+Q^2\right)}\left(\mathcal{C}\left[f_1^g \Delta^{[{}^1S_0^{[8]}]}\right] + 4\frac{7M_\psi^4+2M_\psi^2Q^2+3Q^4}{M_\psi^2\left(M_\psi^2+Q^2\right)^2}\mathcal{C}\left[f_1^g\Delta^{[{}^3P_0^{[8]}]}\right]\right)$$

$$\mathcal{F}_{UU,\parallel} = 2\pi^2 \frac{e_c^2\alpha\alpha_s}{M_\psi\left(M_\psi^2+Q^2\right)}\left(16\frac{Q^2}{\left(M_\psi^2+Q^2\right)^2}\mathcal{C}\left[f_1^g\Delta^{[{}^3P_0^{[8]}]}\right]\right)$$

$$\mathcal{F}_{UU}^{\cos 2\phi_\psi} = \frac{\pi^2}{2}\frac{e_c^2\alpha\alpha_s}{M_\psi\left(M_\psi^2+Q^2\right)}\left(-\mathcal{C}\left[wh_1^{\perp g}\Delta_h^{[{}^1S_0^{[8]}]}\right] + 4\frac{3M_\psi^2-Q^2}{M_\psi^2\left(M_\psi^2+Q^2\right)}\mathcal{C}\left[wh_1^{\perp g}\Delta_h^{[{}^3P_0^{[8]}]}\right]\right)$$

- $\mathcal{F}_{UU}^{\cos\phi_\psi}$ is subleading power/twist

High Transverse Momentum (HTM)



- Large transverse momentum generated by extra outgoing particle
- At LO 12 diagrams
- CO & CS production
- Parton and outgoing particle are the same

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_B \mathrm{d}y \mathrm{d}z \mathrm{d}\boldsymbol{q}_T^2 \mathrm{d}\phi_\psi} = \frac{\alpha}{yQ^2}\Big\{\Big[1+(1-y)^2\Big]F_{UU,\perp} + 4(1-y)F_{UU,\parallel} + 2(2-y)\sqrt{1-y}\cos\phi_\psi F_{UU}^{\cos\phi_\psi} + 4(1-y)\cos 2\phi_\psi F_{UU}^{\cos 2\phi_\psi}\Big\}$$

$$\hat{x} = \frac{Q^2}{2p_a \cdot q} = \frac{x_B}{\xi}, \quad \hat{z} = \frac{p_a \cdot P_\psi}{p_a \cdot q} = z$$

$$\hat{x}_{\max} = \frac{Q^2}{M_\psi^2 + Q^2}$$

$$F_{UU,\mathcal{P}}^{\Phi} = \frac{z}{4}\sum_{n,a}\int_{x_B}^{\hat{x}_{\max}}\frac{d\hat{x}}{\hat{x}}\int_0^1\frac{d\hat{z}}{\hat{z}}\, f_1^a\left(\frac{x_B}{\hat{x}};\mu\right)\hat{H}_{\mathcal{P},\Phi}^{a[n]}(\hat{x},\hat{z})\,\langle\mathcal{O}^{J/\psi}(n)\rangle \times\delta\left(\frac{\mathbf{q}_T^2}{Q^2} + \frac{1-\hat{z}}{\hat{z}^2}\frac{M^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}}\right)\delta(z-\hat{z})\,,$$

Intermediate Transverse Momentum (ITM)

- The ITM can be obtained from the HTM by expanding the Dirac-delta at small q_T using **continuous** test functions *Boer et al. 2020*

$$\hat{x}' = \frac{\hat{x}}{\hat{x}_{\max}}$$

$$\delta\left(\frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}} - \frac{1-\hat{z}}{\hat{z}^2}\frac{M_\psi^2}{Q^2} - \frac{\boldsymbol{q}_T^2}{Q^2}\right) \sim \hat{x}_{\max}\left[\log\frac{M_\psi^2+Q^2}{\boldsymbol{q}_T^2}\,\delta(1-\hat{x}')\,\delta(1-\hat{z}) + \frac{\hat{x}'}{(1-\hat{x}')_+}\,\delta(1-\hat{z}) + \frac{M_\psi^2+Q^2}{M_\psi^2/\hat{z}+Q^2}\,\frac{\hat{z}}{(1-\hat{z})_+}\,\delta(1-\hat{x}')\right]$$

- Agrees with SIDIS *Meng et al. 1996*

$$\hat{x}_{\max} \to 1$$

- However, the hard factor contains **discontinuities**, made explicit by a decomposition into poles:

$$\hat{H}^{a[n]}_{\mathcal{P},\Phi}(\hat{x}',\hat{z}) = \hat{H}^{a[n];(0)}_{\mathcal{P},\Phi}(\hat{x}',\hat{z}) + \sum_{k=1}^{\infty}\left(\frac{1-\hat{z}}{1-\hat{x}'}\right)^k \hat{H}^{a[n];(k)}_{\mathcal{P},\Phi}(\hat{z})$$

- $a = g, \mathcal{P} = \perp, \parallel; k = 1,2$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_B\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}\boldsymbol{q}_T^2\,\mathrm{d}\phi_\psi} \equiv \mathrm{d}\sigma_A + \mathrm{d}\sigma_B + \mathrm{d}\sigma_C$$

The effective Dirac-delta expansion

$$\hat{H}^{a[n]}_{\mathcal{P},\Phi}(\hat{x}',\hat{z}) = \hat{H}^{a[n];(0)}_{\mathcal{P},\Phi}(\hat{x}',\hat{z}) + \sum_{k=1}^{\infty}\left(\frac{1-\hat{z}}{1-\hat{x}'}\right)^k \hat{H}^{a[n];(k)}_{\mathcal{P},\Phi}(\hat{z})$$

- Hard factors can be Taylor expanded (higher order terms solve the indeterminacy)

$$\hat{H}^{a[n];(k)}_{\mathcal{P}}(\hat{z}) = \hat{H}^{a[n];(k)}_{\mathcal{P}}(1) + \sum_m (1-\hat{z})^m \left.\frac{d^m \hat{H}^{a[n];(k)}_{\mathcal{P},\Phi}(\hat{z})}{d\hat{z}^m}\right|_{\hat{z}=1}$$

$$\hat{H}^{g[n];(1)}_{\mathcal{P}}(1) = -2\frac{M^2}{Q^2+M^2}\hat{H}^{g[n];(0)}_{\mathcal{P}}(1,1)\,,$$

$$\hat{H}^{g[n];(2)}_{\mathcal{P}}(1) = \left(\frac{M^2}{Q^2+M^2}\right)^2 \hat{H}^{g[n](0)}_{\mathcal{P}}(1,1)\,.$$

$$\delta\left(\frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}} - \frac{1-\hat{z}}{\hat{z}^2}\frac{M_\psi^2}{Q^2} - \frac{\boldsymbol{q}_T^2}{Q^2}\right) \sim \hat{x}_{\max}\left[\log\frac{M_\psi^2+Q^2}{\boldsymbol{q}_T^2}\delta(1-\hat{x}')\,\delta(1-\hat{z}) + \frac{\hat{x}'}{(1-\hat{x}')_+}\delta(1-\hat{z}) + \frac{M_\psi^2+Q^2}{M_\psi^2/\hat{z}+Q^2}\frac{\hat{z}}{(1-\hat{z})_+}\delta(1-\hat{x}')\right]$$

$$\delta_{\rm eff}(\hat{x}',\hat{z}) = \hat{x}_{\max}\left[\frac{1}{2}\left(\log\frac{M_\psi^2+Q^2}{\boldsymbol{q}_T^2} - 1 - \log\frac{M_\psi^2}{M_\psi^2+Q^2}\right)\delta(1-\hat{x}')\,\delta(1-\hat{z}) + \frac{\hat{x}'}{(1-\hat{x}')_+}\delta(1-\hat{z}) + \frac{M_\psi^2+Q^2}{M_\psi^2/\hat{z}+Q^2}\frac{\hat{z}}{(1-\hat{z})_+}\delta(1-\hat{x}')\right].$$

TMDShF tail

$$\mathcal{C}[f_1^g \Delta^{[n]}](x, \mathbf{q}_T^2) = \int d^2\mathbf{p}_T \int d^2\mathbf{k}_T \, \delta^2(\mathbf{p}_T + \mathbf{k}_T - \mathbf{q}_T) \, f_1^g(x, \mathbf{p}_T^2) \, \Delta^{[n]}(\mathbf{k}_T^2)$$

$$\mathcal{C}[w h_1^{\perp g} \Delta_h^{[n]}](x, \mathbf{q}_T^2) = \int d^2\mathbf{p}_T \int d^2\mathbf{k}_T \, \delta^2(\mathbf{p}_T + \mathbf{k}_T - \mathbf{q}_T) \, w(\mathbf{p}_T, \mathbf{k}_T) \, h_1^{\perp g}(x, \mathbf{p}_T^2) \, \Delta_h^{[n]}(\mathbf{k}_T^2)$$

$$w(\mathbf{p}_T, \mathbf{k}_T) = \frac{1}{2M_h^2 \mathbf{q}_T^2}[2(\mathbf{p}_T \cdot \mathbf{q}_T)^2 - \mathbf{p}_T^2 \mathbf{q}_T^2]$$

- Knowing the perturbative tail of the gluon TMDs (relate to PDFs), we obtain the LO TMDShF tail:

$$\widetilde{\Delta}^{[n]}(z, \boldsymbol{b}_T^2; \mu^2 = \widetilde{Q}^2) = \frac{1}{2\pi}\left[1 + \frac{\alpha_s}{2\pi} C_A \left(1 + \log\frac{M_\psi^2}{M_\psi^2 + Q^2}\right) \log\frac{\widetilde{Q}^2}{\mu_b^2}\right] \langle \mathcal{O}[n] \rangle \delta(1-z) + \mathcal{O}(\alpha_s^2) + \mathcal{O}(b_T \Lambda_{\mathrm{QCD}})$$

$$b_0 = 2e^{-\gamma_E} \approx 1.123 \qquad \mu_b = b_0 / |\boldsymbol{b}_T|$$

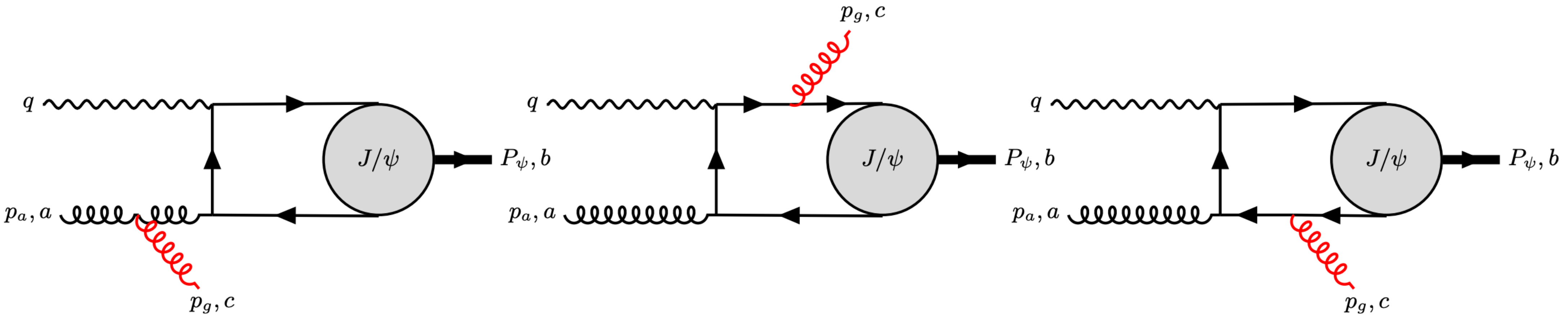
$$\Delta^{[n]}(z, \boldsymbol{k}_T^2; \widetilde{Q}^2) = -\frac{\alpha_s}{2\pi^2 \boldsymbol{k}_T^2} C_A \left(1 + \log\frac{M_\psi^2}{M_\psi^2 + Q^2}\right) \langle \mathcal{O}[n] \rangle \delta(1-z) + \mathcal{O}(\alpha_s^2)$$

$$|\boldsymbol{k}_T| \gg \Lambda_{\mathrm{QCD}}$$

- Particular scale: $\mu_H^2 \equiv \widetilde{Q}^2 = M_\psi^2 + Q^2$

- And the trivial TMDShF tail $\Delta_h^{[n]}(z, \boldsymbol{k}_T^2) = \delta(\boldsymbol{k}_T^2) \langle \mathcal{O}[n] \rangle \delta(1-z) + \mathcal{O}(\alpha_s)$
- Order $\alpha \alpha_s^3$ needed

Eikonal method: a validation



$$d\sigma_1 \propto \frac{g_s^2}{2(2\pi)^3} C_A \int_{\frac{p_{g\perp}^2}{M_\psi^2+Q^2}}^1 \frac{dx_g}{x_g}\left[2S_g(p_a,P_\psi) - S_g(P_\psi,P_\psi)\right]\overline{|M_0|}^2$$

$$= \frac{\alpha_s}{2\pi^2 \boldsymbol{p}_{g\perp}^2} C_A \left[\log\frac{M_\psi^2+Q^2}{\boldsymbol{p}_{g\perp}^2} + \log\frac{M_\psi^2+Q^2}{M_\psi^2} - 1\right]\overline{|M_0|}^2$$

$$x_g = \frac{p_g \cdot p_a}{q \cdot p_a}$$

$$S_g(v_1, v_2) = \frac{v_1 \cdot v_2}{(v_1 \cdot p_g)(v_2 \cdot p_g)}$$

- The TMDShF may also be seen as a fragmentation-like function of a $c\bar{c}$ pair into a J/ψ evaluated at ITM; inclusion of next order contributions might shine a light on their relation

Universality

- The q_T-divergent terms from the collinear limit are resummed in the Sudakov factor of the process, for a **general scale** μ_H:

$$S_A^{ep,\psi}(\boldsymbol{b}_T^2;\mu_H^2) = \frac{1}{2} S_A^g(\boldsymbol{b}_T^2;\mu_H^2) + B_{ep}(\mu_H^2)\log\frac{\mu_H^2}{\mu_b^2} \qquad B_{ep}(\mu_H^2) = -\frac{\alpha_s}{2\pi} C_A \left(1 + \log\frac{M_\psi^2 \mu_H^2}{(M_\psi^2+Q^2)^2}\right)$$

- In agreement with limit of open heavy quark- pair *Zhu et al. 2013*

- TMDShF should only depend on the scale M_ψ; Q may only enter through μ_H

$$\Delta_{ep}^{[n]}(\mu_H) = \Delta_{\rm ShF}^{[n]}(\mu_H) \times S_{ep}(\mu_H)$$

$$\widetilde{\Delta}_{\rm ShF}^{[n]}(z,\boldsymbol{b}_T^2;\mu_H^2) = \frac{1}{2\pi}\left[1 + \frac{\alpha_s}{2\pi} C_A \left(1 + \log\frac{M_\psi^2}{\mu_H^2}\right)\log\frac{\mu_H^2}{\mu_b^2}\right] \langle \mathcal{O}[n]\rangle \delta(1-z),$$

$$S_{ep}(\boldsymbol{b}_T^2;\mu_H^2) = 1 + \frac{\alpha_s}{2\pi} C_A \left(2\log\frac{\mu_H^2}{M_\psi^2+Q^2}\right)\log\frac{\mu_H^2}{\mu_b^2}\ .$$

$$pp \to J/\psi + X$$

Sun et al. 2013

$$S_{pp}(\mu_H^2) = 1 + \frac{\alpha_s}{2\pi} C_A \left(3\log\frac{\mu_H^2}{M_\psi^2}\right)\log\frac{\mu^2}{\mu_b^2}$$

Conclusions

- Revision of the procedure to derive the LO TMDShF perturbative tail for heavy quarkonium production: poles in the small q_T limit provide non-negligible finite terms in the expansion at small q_T; less divergent behavior as compared to the TMD fragmentation functions of light hadrons (what was found before).
- We expect that the presence of the poles is an intrinsic feature of any inclusive CO quarkonium production.
- Agreement with eikonal approximation and with the Sudakov factors obtained for open heavy-quark pair production in electron-proton and proton-proton collisions.
- Our TMDShF tails hold for every CO quarkonium state (unpolarised, L and T polarised) with the same quantum numbers as the J/ψ, e.g. Υ(nS) and ψ(2S).
- Only their magnitude is different by the LDMEs. This holds up to the precision considered, corresponding to the $\alpha\alpha_s^2$ and v^4 orders in the NRQCD double expansion.
- We split up into two terms: a process-independent quantity that we identify as the universal TMDShF, and an extra process-dependent soft factor.
- The universal TMDShFs are possible to extract at the EIC and LHC by appropriate choices of scales, which allows to relate different processes.