

# Resolving negative cross section of quarkonium hadroproduction using soft gluon factorization



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IISER Mohali

# Outline

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**I. Introduction**

**II. Soft gluon factorization**

**III. Phenomenological studies**

**IV. Summary**



## I. Introduction

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# Introduction



## ➤ NRQCD factorization Bodwin, Braaten, Lepage, PRD, 1995

$$(2\pi)^3 2P_H^0 \frac{d\sigma_H}{d^3P_H} = \sum_n d\hat{\sigma}_n(P_H) \langle \mathcal{O}_n^H \rangle$$

$d\hat{\sigma}_n$  : production of a heavy quark pair in state  $n({}^{2S+1}L_J^{[c]})$ .

$\langle \mathcal{O}_n^H \rangle$  : the hadronization of  $Q\bar{Q}(n)$  to  $H$ ;

can be ordered in powers of  $v$ ;

universality.

## ➤ A glory history

- Solved IR divergences in P-wave quarkonium decay
- Explained  $\psi'$  surplus
- Explained  $\chi_{c2}/\chi_{c1}$  production ratio
- ....

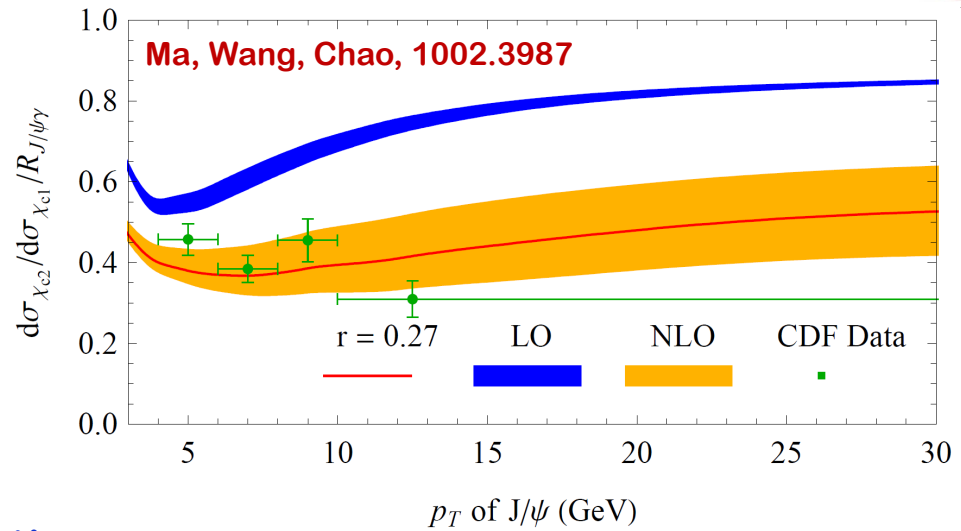
## ➤ Difficulty

- Polarization puzzle
- Universality problem

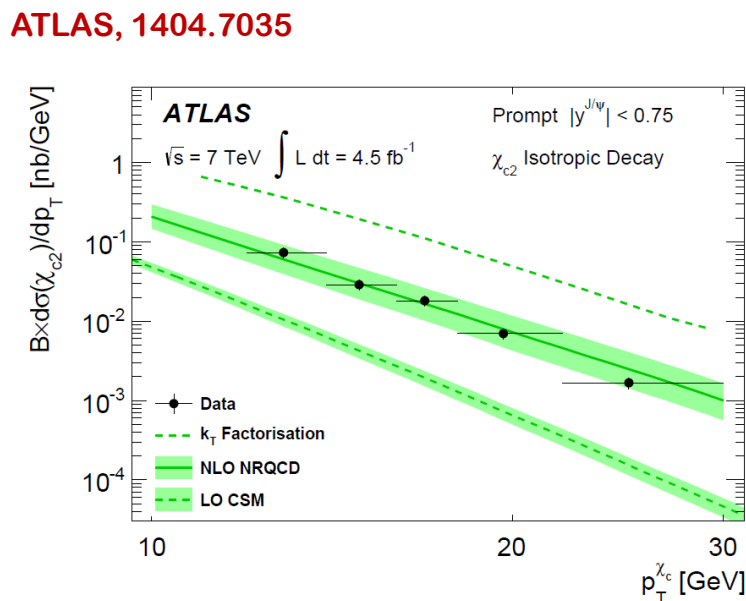
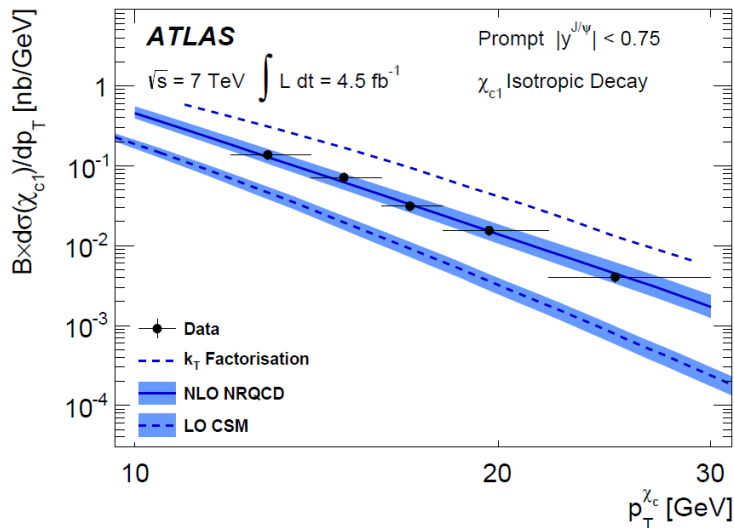
# ➤ Difficulty : negative cross sections

□ Explain  $\chi_{cJ}$  production

- The ratio  $R_{\chi_c} = \sigma_{\chi_{c2}}/\sigma_{\chi_{c1}}$   
LO NRQCD:  $R_{\chi_c} = 5/3$



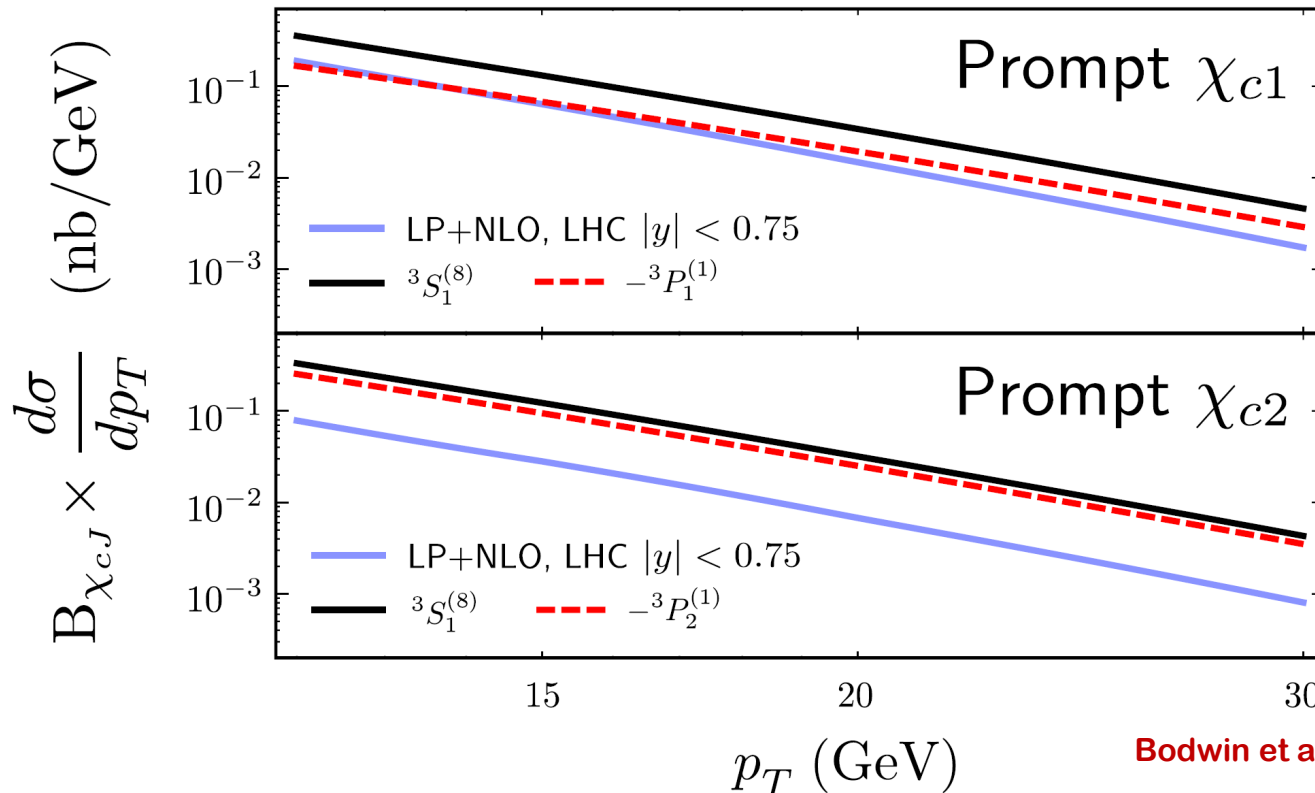
- The differential cross sections **ATLAS, 1404.7035**



□ There are substantial cancellations between  ${}^3S_1^{[8]}$  and  ${}^3P_J^{[1]}$

$$d\sigma(\chi_{cJ}) = (2J + 1)d\hat{\sigma}[{}^3S_1^{[8]}]\langle\mathcal{O}^{\chi_{c0}}({}^3S_1^{[8]})\rangle + (2J + 1)d\hat{\sigma}[{}^3P_J^{[1]}]\frac{\langle\mathcal{O}^{\chi_{c0}}({}^3P_0^{[1]})\rangle}{m_c^2}$$

Positive Negative

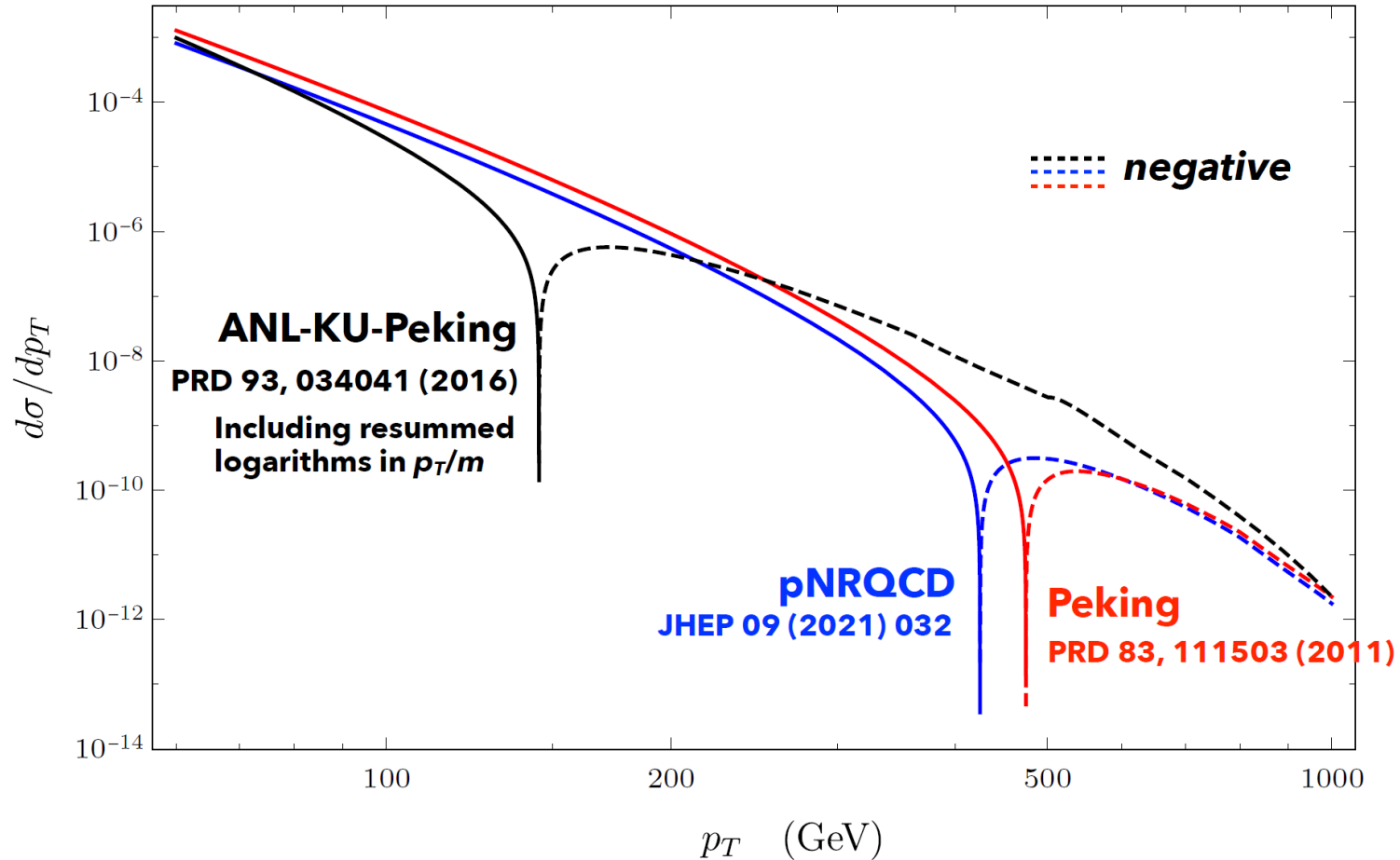


Bodwin et al., 1509.07904

- Perturbation unstable

- Cross sections turn negative at large  $p_T$

$$pp \rightarrow \chi_c + X \quad y = 2.0 \quad \sqrt{s} = 13 \text{ TeV}$$



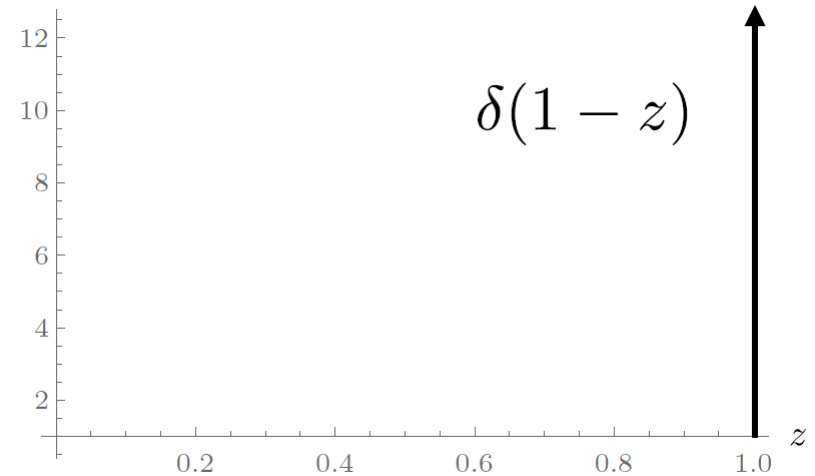
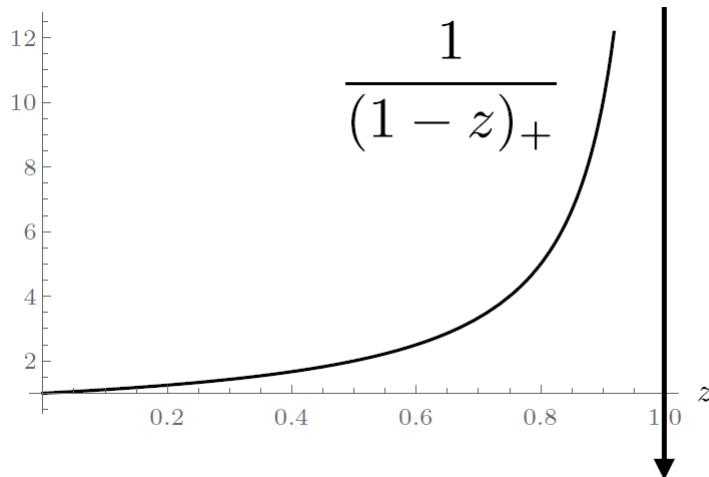
Hee Sok Chung, talk at The 15th International Workshop on Heavy Quarkonium

## □ Why?

$$d\sigma(\chi_{cJ}) = (2J + 1)d\hat{\sigma}[{}^3S_1^{[8]}]\langle\mathcal{O}^{\chi_{c0}}({}^3S_1^{[8]})\rangle + (2J + 1)d\hat{\sigma}[{}^3P_J^{[1]}]\frac{\langle\mathcal{O}^{\chi_{c0}}({}^3P_0^{[1]})\rangle}{m_c^2}$$

$$d\hat{\sigma}[{}^3P_J^{[1]}] = d\hat{\sigma}_g \otimes \left\{ 0 \times \alpha_s + \frac{2\alpha_s^2}{27N_c m_c^5} \left[ \left( \frac{Q_J}{2J+1} - \log \frac{\Lambda}{2m_c} \right) \delta(1-z) + \frac{z}{(1-z)_+} + \frac{P_J(z)}{2J+1} \right] \right\}$$

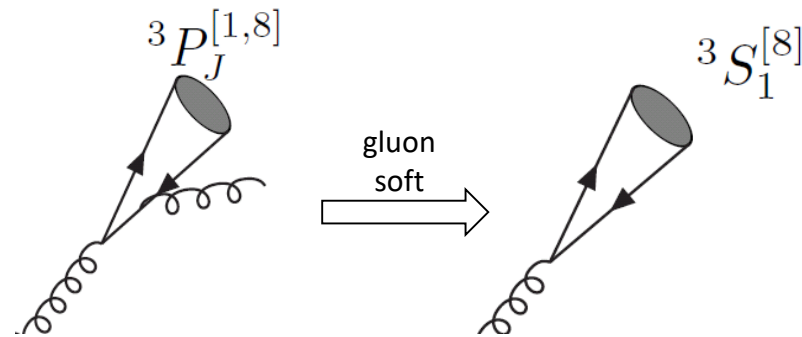
$$d\hat{\sigma}[{}^3S_1^{[8]}] = d\hat{\sigma}_g \otimes \frac{\pi\alpha_s}{24m_c^3} \delta(1-z)$$



Hee Sok Chung, talk at The 15th International Workshop on Heavy Quarkonium

- Cross section at very large  $p_T$  will depend strongly on  $z \rightarrow 1$  behavior of FFs





- **Soft gluon in P-wave: factorized to S-wave matrix element**
  - **Plus functions: remnants of the infrared subtraction in matching the  ${}^3P_J^{[1]}$  SDCs**
  - **Subtraction scheme: at zero momentum, which contributes the largest production rate. Over subtracted!**
  - **Solution:** soft gluon momentum should be kept during subtraction process, or resum kinematic effects to all powers in  $v$ .
- **Soft gluon factorization:** resum a dominant series of power corrections (kinematic effects) and log corrections **Ma, Chao, 1703.08402;**  
**Chen, Ma, 2005.08786.**



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# Soft gluon factorization(SGF)



## ➤ Factorization

Ma, Chao, 1703.08402;  
Chen, Jin, Ma, Meng 2103.15121.

$$(2\pi)^3 2P_H^0 \frac{d\sigma_H}{d^3 P_H} = \sum_{n,n'} \int \frac{dz}{z^2} d\hat{\sigma}_{[nn']} (P_H/z, m_Q, \mu_f) F_{[nn'] \rightarrow H}(z, M_H, m_Q, \mu_f),$$

- $d\hat{\sigma}_{[nn']}$  : perturbatively calculable hard parts
- $F_{[nn'] \rightarrow H}$  : nonperturbative soft gluon distributions (SGDs)
- $n = {}^{2S+1}L_J^{[c]}$
- $P_H$  : momentum of quarkonium
- $M_H$  : mass of quarkonium
- $z = P_H^+ / P^+$  : the longitudinal momentum fraction with  $P$  denoting the total momentum of the intermediate  $Q\bar{Q}$  pair

# ➤ Soft gluon distributions (SGDs)

## □ Operator definition

- Expectation values of bilocal operators in QCD vacuum

$$F_{[nn'] \rightarrow H}(z, M_H, m_Q, \mu_f) = P_H^+ \int \frac{db^-}{2\pi} e^{-iP_H^+ b^- / z} \langle 0 | [\bar{\Psi} \mathcal{K}_n \Psi]^\dagger(0) [a_H^\dagger a_H] [\bar{\Psi} \mathcal{K}_{n'} \Psi](b^-) | 0 \rangle_S,$$

with

$$a_H^\dagger a_H = \sum_X \sum_{J_z^H} |H + X\rangle \langle H + X|$$

$$\mathcal{K}_n(rb) = \frac{\sqrt{M_H}}{M_H + 2m} \frac{M_H + \not{P}_H}{2M_H} \Gamma_n \frac{M_H - \not{P}_H}{2M_H} \mathcal{C}^{[c]}$$

Spin project operators:

$$\Gamma_n = \sum_{L_z, S_z} \langle L, L_z; S, S_z | J, J_z \rangle \Gamma_{LL_z}^o \Gamma_{SS_z}^s$$

Color project operators:

$$\mathcal{C}^{[1]} = \frac{\mathbf{1}_c}{\sqrt{N_c}} \quad \mathcal{C}^{[8]} = \sqrt{2} t^{\bar{a}} \Phi_{a\bar{a}}^{(A)}(rb)$$

□ Gauge link

$$\Phi_l(rb^-) = \mathcal{P} \exp \left[ -ig_s \int_0^\infty d\xi l \cdot A(rb^- + \xi l) \right],$$

□ Evaluated in small region

- Subscript “S”: evaluate the matrix element in the region where off-shellness of all particles is much smaller than heavy quark mass

➤ FFs in SGF

- $D_{f \rightarrow H}$ : single parton FFs
- $\mathcal{D}_{[Q\bar{Q}(\kappa)] \rightarrow H}$ : double parton FFs
- $\hat{z} = z/x$

$$\begin{aligned}
 & D_{f \rightarrow H}(z, \mu_0) \\
 &= \sum_{n, n'} \int \frac{dx}{x} \hat{D}_{f \rightarrow Q\bar{Q}[nn']}(\hat{z}; M_H/x, m_Q, \mu_0, \mu_\Lambda) \\
 &\quad \times F_{[nn'] \rightarrow H}(x, M_H, m_Q, \mu_\Lambda), \tag{2a}
 \end{aligned}$$

$$\begin{aligned}
 & \mathcal{D}_{[Q\bar{Q}(\kappa)] \rightarrow H}(z, \zeta, \zeta', \mu_0) \\
 &= \sum_{n, n'} \int \frac{dx}{x} \hat{D}_{[Q\bar{Q}(\kappa)] \rightarrow Q\bar{Q}[nn']}(\hat{z}, \zeta, \zeta'; M_H/x, m_Q, \mu_0, \mu_\Lambda) \\
 &\quad \times F_{[nn'] \rightarrow H}(x, M_H, m_Q, \mu_\Lambda), \tag{2b}
 \end{aligned}$$

## □ Short distance hard parts at LO

$$\hat{D}_{g \rightarrow Q\bar{Q}[{}^3S_{1,T}^{[8]}]}^{LO,(0)}(z, M_H, \mu_0, \mu_\Lambda) = \frac{\pi\alpha_s}{(N_c^2 - 1)} \frac{8}{M_H^3} \delta(1 - z), \quad (9a)$$

$$\begin{aligned} \hat{D}_{g \rightarrow Q\bar{Q}[{}^1S_0^{[8]}]}^{LO,(0)}(z, M_H, \mu_0, \mu_\Lambda) \\ = \frac{8\alpha_s^2}{M_H^3} \frac{N_c^2 - 4}{2N_c(N_c^2 - 1)} \left[ (1 - z) \ln[1 - z] - z^2 + \frac{3}{2}z \right], \quad (9b) \end{aligned}$$

$$\begin{aligned} \hat{D}_{g \rightarrow Q\bar{Q}[{}^3P_0^{[1]}]}^{LO,(0)}(z; M_H, \mu_0, \mu_\Lambda) \\ = \frac{32\alpha_s^2}{M_H^5 N_c} \frac{2}{9} \left[ \frac{1}{36} z(837 - 162z + 72z^2 + 40z^3 + 8z^4) \right. \\ \left. + \frac{9}{2} (5 - 3z) \ln(1 - z) \right], \quad (9c) \end{aligned}$$

- The P-wave short distance hard parts do not include terms proportional to plus distributions

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# Phenomenological studies



## ➤ Collinear factorization

### □ Heavy quarkonium production at large $p_T$

$$d\sigma_{A+B \rightarrow H+X}(p) \approx \sum_{i,j} f_{i/A}(x_1, \mu_F) f_{j/B}(x_2, \mu_F) \left\{ \sum_f D_{f \rightarrow H}(z, \mu_F) \otimes d\hat{\sigma}_{i+j \rightarrow f+X}(\hat{P}/z, \mu_F) \right. \\ \left. + \sum_{\kappa} \mathcal{D}_{[Q\bar{Q}(\kappa)] \rightarrow H}(z, \zeta, \zeta', \mu_F) \otimes d\hat{\sigma}_{i+j \rightarrow [Q\bar{Q}(\kappa)]+X}(\hat{P}(1 \pm \zeta)/2z, \hat{P}(1 \pm \zeta')/2z, \mu_F) \right\},$$

Kang, Ma, Qiu, Sterman, 1401.0923

### □ Factorization of FFs

- SGF
- NRQCD factorization

### □ Nonperturbative model for SGDs

$$F^{\text{mod}}(x) = \frac{N^H \Gamma(M_H b / \bar{\Lambda}) (1-x)^{b-1} x^{M_H b / \bar{\Lambda} - b - 1}}{\Gamma(M_H b / \bar{\Lambda} - b) \Gamma(b)},$$

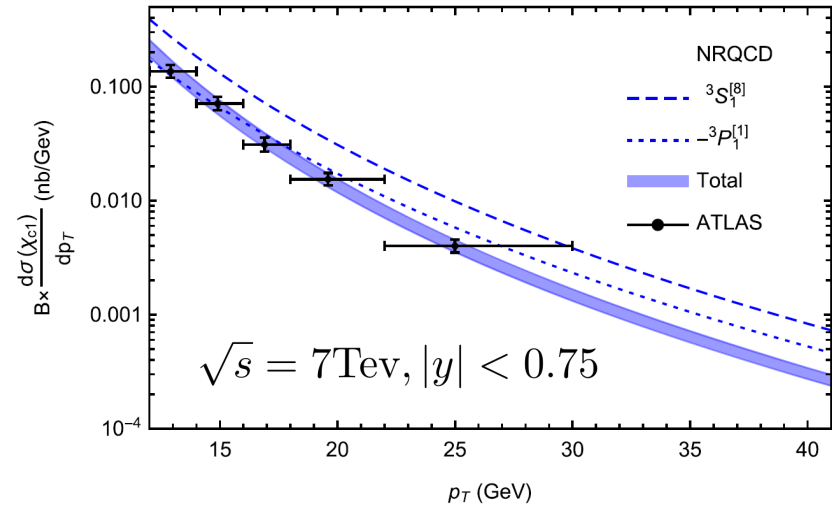
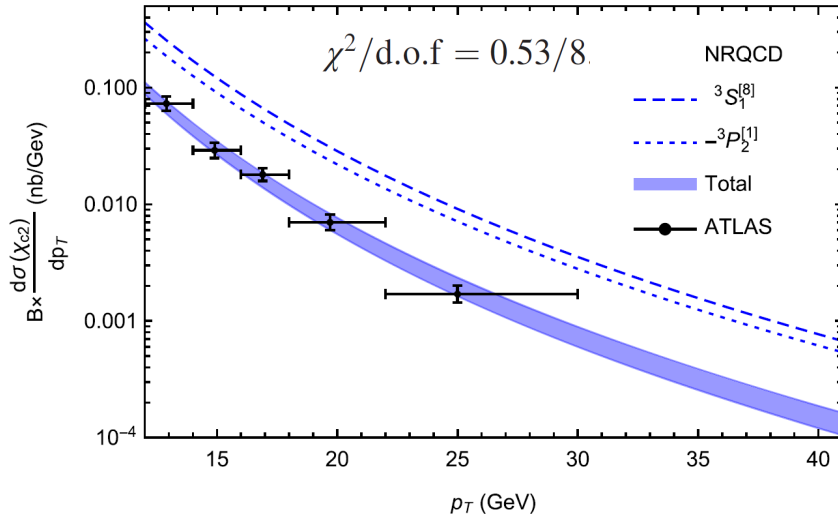
- $N^H$ : the normalization,  $N^H[n] \approx \langle \mathcal{O}^H(n) \rangle$
- $\bar{\Lambda}$ : the average radiated momentum in the hadronization process
- $b$ : related to the second moment of model function



# ➤ Production of $\chi_{cJ}$

## □ NRQCD factorization

- The fitted cross sections compared with ATLAS data



- Define the ratio

$$r(\chi_{c0}) \equiv \frac{\langle \mathcal{O}_{\chi_{c0}}(3S_1^{[8]}) \rangle}{\langle \mathcal{O}_{\chi_{c0}}(3P_0^{[1]}) \rangle / m_c^2},$$

- The cross sections

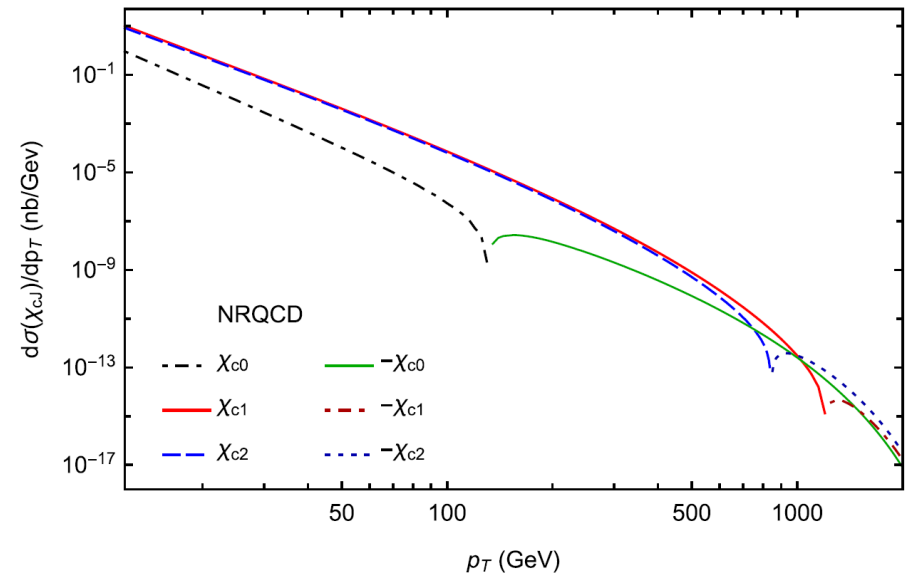
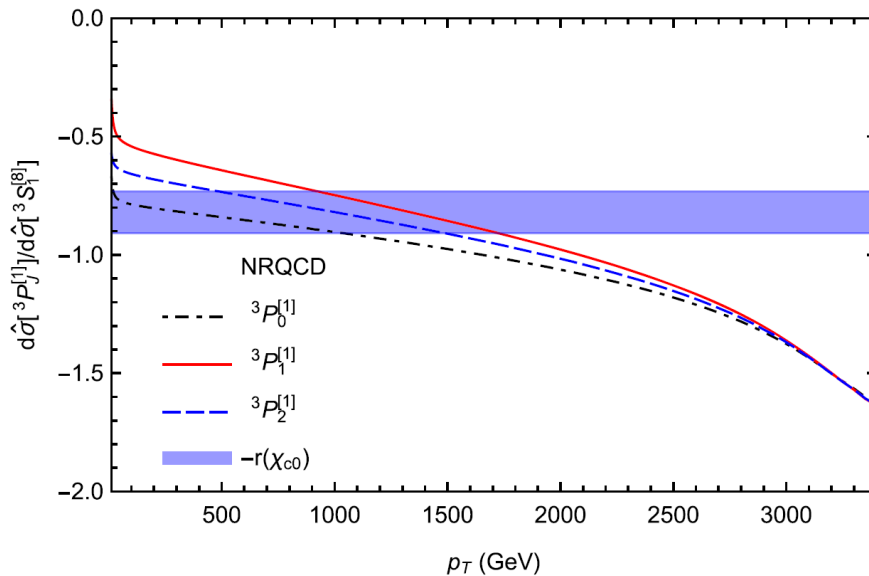
$$d\sigma(\chi_{cJ}) = (2J + 1) d\hat{\sigma}[3S_1^{[8]}] \frac{\langle \mathcal{O}_{\chi_{c0}}(3P_0^{[1]}) \rangle}{m_c^2} \left[ r(\chi_{c0}) + \frac{d\hat{\sigma}[3P_J^{[1]}]}{d\hat{\sigma}[3S_1^{[8]}]} \right].$$

- To achieve a positive cross section, it is necessary to have

$$\frac{d\hat{\sigma}[{}^3P_J^{[1]}]}{d\hat{\sigma}[{}^3S_1^{[8]}]} > -r(\chi_{c0}).$$

- Left: comparison between the ratios and  $-r(\chi_{c0})$

Right: the  $p_T$  distributions when the LDMEs take the central values



- The ratios fall below the lower bound of  $-r(\chi_{c0})$  at very large  $p_T$

## □ SGF

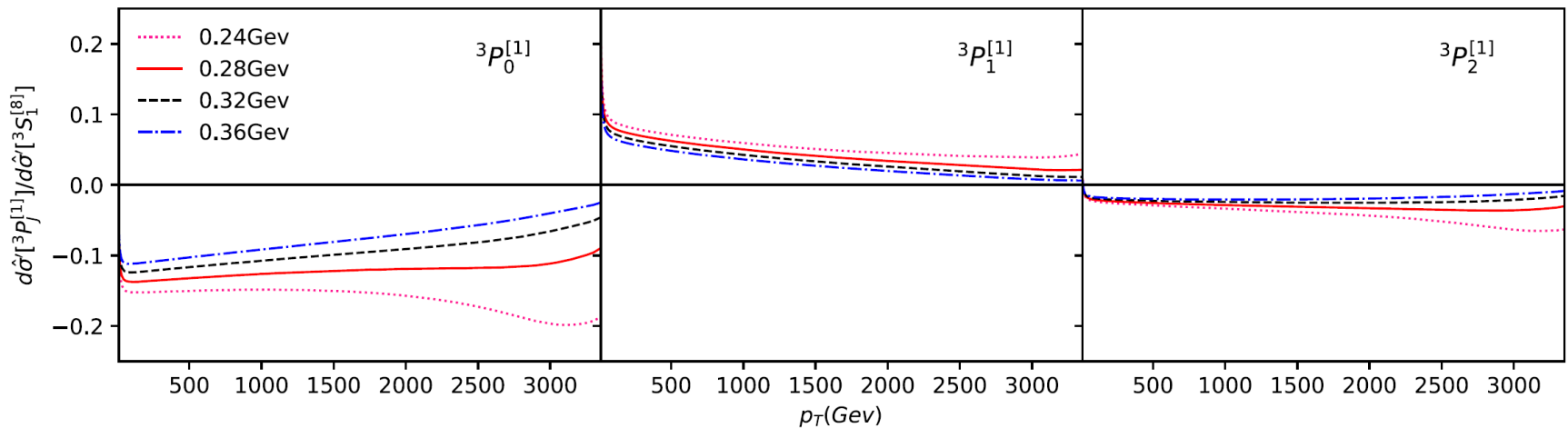
- The cross sections

$$d\sigma(\chi_{cJ}) = (2J + 1)d\hat{\sigma}'[{}^3S_1^{[8]}] \frac{N^{\chi_{c0}}[{}^3P_0^{[1]}]}{m_c^2} \left[ r'(\chi_{c0}) + \frac{d\hat{\sigma}'[{}^3P_J^{[1]}]}{d\hat{\sigma}'[{}^3S_1^{[8]}]} \right].$$

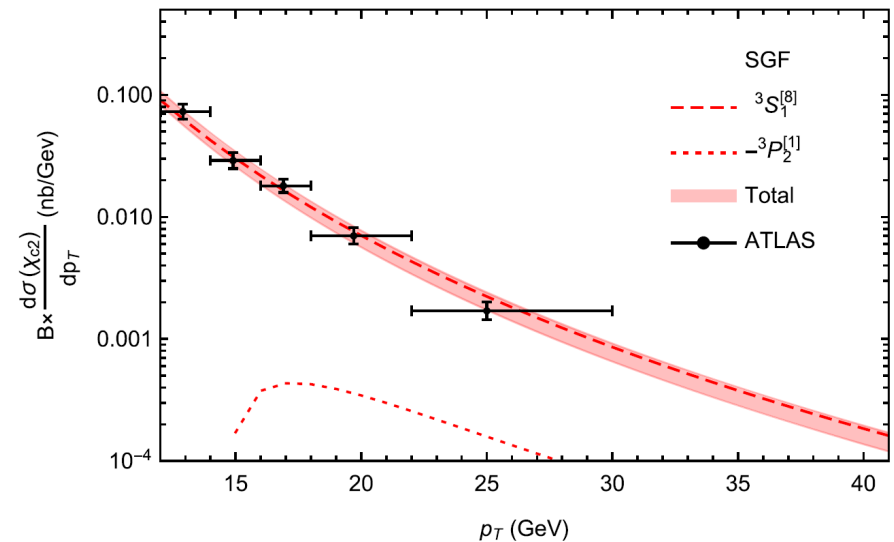
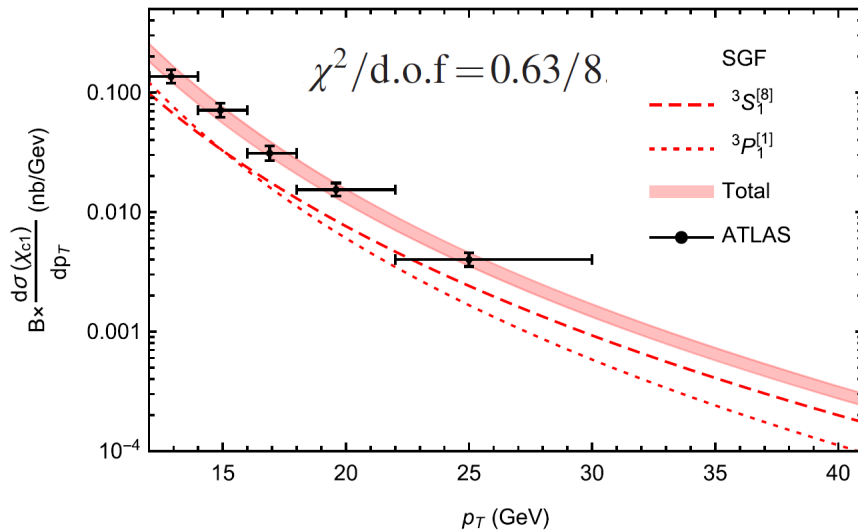
with

$$r'(\chi_{c0}) \equiv \frac{N^{\chi_{c0}}[{}^3S_1^{[8]}]}{N^{\chi_{c0}}[{}^3P_0^{[1]}]/m_c^2}.$$

- $d\hat{\sigma}'[{}^3P_J^{[1]}]/d\hat{\sigma}'[{}^3S_1^{[8]}]$  is sensitive to the parameters  $\bar{\Lambda}$
- Fix  $\bar{\Lambda}[{}^3S_1^{[8]}] = 0.4\text{Gev}$  and vary  $\bar{\Lambda}[{}^3P_J^{[1]}] = 0.36, 0.32, 0.28, 0.24\text{Gev}$



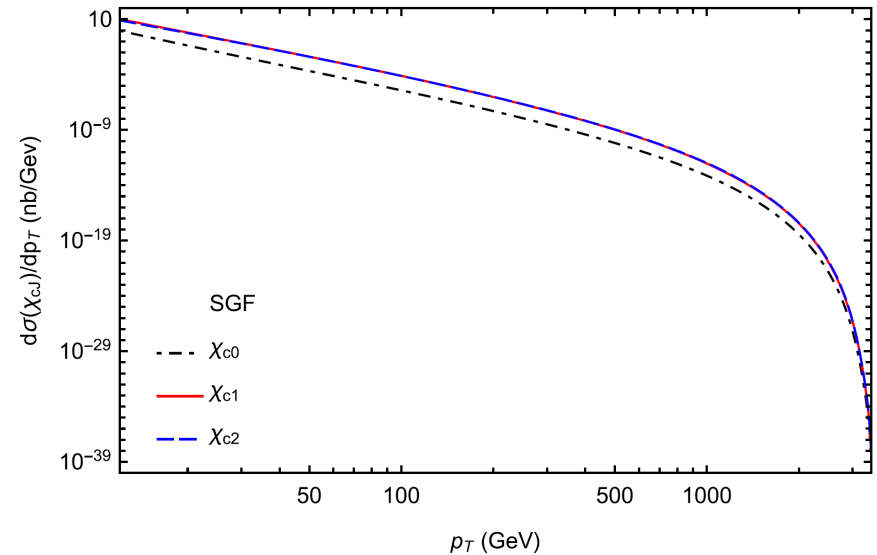
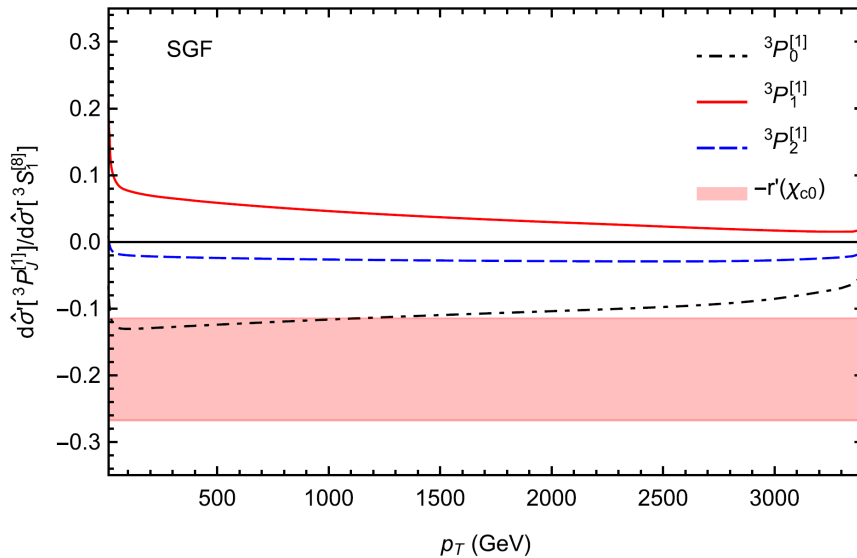
- A constraint relation is suggested:  $\bar{\Lambda}[{}^3P_J^{[1]}] \geq 0.7\bar{\Lambda}[{}^3S_1^{[8]}]$
- We set  $\bar{\Lambda}[{}^3S_1^{[8]}] = 0.4\text{Gev}$  and  $\bar{\Lambda}[{}^3P_J^{[1]}] = 0.3\text{Gev}$
- The fitted cross sections compared with ATLAS data



- The fit to experimental data is as good as that in NRQCD factorization

- Left: comparison between the ratios and  $-r'(\chi_{c0})$

Right: the  $p_T$  distributions when the parameters take the central values



- There is a wide range of  $r'(\chi_{c0})$  in which the ratios is larger than  $-r'(\chi_{c0})$
- The negative cross section problem is resolved in SGF

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# Summary



- We studied the hadroproduction of  $\chi_{cJ}$  using the SGF and NRQCD factorization;
- We confirm that the NRQCD predictions for  $\chi_{cJ}$  production rates at the LHC turn negative at sufficiently large  $p_T$  ;
- Our results show that the fit to experimental data in SGF is as good as that in NRQCD factorization;
- Our results show that the negative cross section problem in NRQCD can be resolved in SGF;
- **It will be very useful to apply SGF to study the polarizations of  $\psi(ns)$  production at LHC in the future.**

***Thank you!***