Inclusive Production of Fully Charmed Tetraquarks at the LHC and EIC

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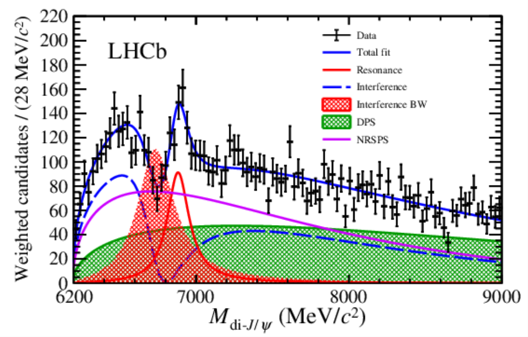
Motivation

Discovery of $X(6900)$



Invariant mass spectrum of J/ψ-pair candidates (LHCb, 2020)

- First fullly-charm tetraquark candidate
- Strong decay to two J/ψ, $C = +$
- Confirmed by both the ATLAS and CMS collaborations

Theoretical Interpretation of $X(6900)$

- Radial excitation of 0^{++} tetraquark: ***Z.G.Wang, 2020; Lü et al., 2020; Giron and Lebed 2020; Karliner and Rosner 2020; J.Zhao et al., 2020; R.Zhu, 2020; B.-C.Yang et al., 2020; Z. Zhao et al., 2020; Ke et al., 2021***
- P-wave tetraquark: ***Liu et al., 2020; H.-X.Chen, et al., 2020; R.Zhu, 2020***
- ground-state S-wave tetraquark: ***Gordillo et al., 2020***

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- ground-state S-wave tetraquark: ***Gordillo et al., 2020***
- 0^{++} hybird: ***Wan and Qiao, 2020***
- Resonance of charmonium scattering: ***G.Yang et al., 2020; Jin et al., 2020;***
- Kinematic cusp of final-state interaction: ***J.-Z. Wang et al., 2020; X.-K.Dong et al.2021; Guo and Oller, 2021; Gong et al., 2020***
- BSM explanation: ***J.-W.Zhu et al., 2020; Dosch et al., 2020***

Theoretical Studies of Fully-heavy Tetraquark

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- Search for the fully-bottom tetraquark on Lattice NRQCD: found no indication of any states below $2\eta_b$ threshold in the $0^{++}, 1^{+-}$ and 2^{++} channels. ***Hughes et al., 2018***

Production

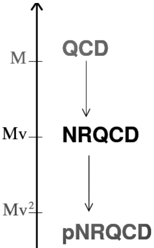
- Duality relations: ***Berezhnoy et al., 2011, 2012; Kaliner et al., 2017***
- Color evaporation model: ***Carvalho et al., 2016; Maciuła et al., 2020***
- $\gamma\gamma$ interaction: ***Gonçalves, Moreira, 2021***

Production

- Duality relations: ***Berezhnoy et al., 2011, 2012; Kaliner et al., 2017***
- Color evaporation model: ***Carvalho et al., 2016; Maciuła et al., 2020***
- NRQCD-inspired: ***Ma, Zhang, 2020; Feng et al., 2020,2021; Zhu, 2020***
- $\gamma\gamma$ interaction: ***Gonçalves, Moreira, 2021***

NRQCD Factorization

NRQCD Factorization *Bodwin, Braaten, Lepage, 1995*

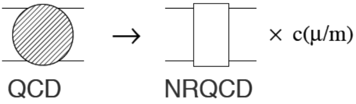


Vairo, Hadron 2011

- Quarkonium energy scale (Braaten, 1997)

	$c\bar{c}$	$b\bar{b}$	$t\bar{t}$
M	$1.5\,\mathrm{GeV}$	$4.7\,\mathrm{GeV}$	$180\,\mathrm{GeV}$
Mv	$0.9\,\mathrm{GeV}$	$1.5\,\mathrm{GeV}$	$16\,\mathrm{GeV}$
Mv^2	$0.5\,\mathrm{GeV}$	$0.5\,\mathrm{GeV}$	$1.5\,\mathrm{GeV}$

- Integrate out the heavy($\sim M$) degrees of freedom



Qiu, 2011

NRQCD Factorization

- To produce T_{4Q}, one needs to produce two charm quarks and two anti-charm quarks at short distances $\sim 1/m_Q$ before the hadronization.

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- NRQCD factorization formula

$$\sigma(T_{4Q}) = \sum_n \frac{F_n(\mu_\Lambda)}{m_Q^{d_n-4}} \left\langle 0 \left| \mathcal{O}_n^{T_{4Q}}(\mu_\Lambda) \right| 0 \right\rangle ,$$

- NRQCD production operators

$$\mathcal{O}_n^{T_{4Q}} = O_n \left(\sum_X \sum_{m_J} |T_{4Q}+X\rangle \langle T_{4Q}+X| \right) O_{n'}^\dagger .$$

NRQCD Operators

We construct all the NRQCD local operators at leading order of velocity expansion for the S-wave tetraquark with $J^{PC}=0^{++},1^{+-},2^{++}$

$$O^{(0)}_{\bar{\mathbf{3}}\otimes\mathbf{3}} = -\frac{1}{\sqrt{3}}[\psi_a^t(\mathrm{i}\sigma^2)\sigma^i\psi_b][\chi_c^\dagger\sigma^i(\mathrm{i}\sigma^2)\chi_d^*]\mathcal{C}^{ab;cd}_{\bar{\mathbf{3}}\otimes\mathbf{3}}$$

$$O^{(0)}_{\mathbf{6}\otimes\bar{\mathbf{6}}} = [\psi_a^t(\mathrm{i}\sigma^2)\psi_b][\chi_c^\dagger(\mathrm{i}\sigma^2)\chi_d^*]\,\mathcal{C}^{ab;cd}_{\mathbf{6}\otimes\bar{\mathbf{6}}},$$

$$O^{i;(1)}_{\bar{\mathbf{3}}\otimes\mathbf{3}} = \frac{\mathrm{i}}{\sqrt{2}}\epsilon^{ijk}\left(\psi_a^\dagger\sigma^j\mathrm{i}\sigma^2\psi_b^*\right)\left(\chi_c^t\mathrm{i}\sigma^2\sigma^k\chi_d\right)\mathcal{C}^{ab;cd}_{\bar{\mathbf{3}}\otimes\mathbf{3}}$$

$$O^{\alpha\beta;(2)}_{\bar{\mathbf{3}}\otimes\mathbf{3}} = [\psi_a^t(\mathrm{i}\sigma^2)\sigma^m\psi_b][\chi_c^\dagger\sigma^n(\mathrm{i}\sigma^2)\chi_d^*]\;\Gamma^{\alpha\beta;mn}\;\mathcal{C}^{ab;cd}_{\bar{\mathbf{3}}\otimes\mathbf{3}},$$

$$\mathcal{C}^{ab;cd}_{\bar{\mathbf{3}}\otimes\mathbf{3}} := \frac{1}{2\sqrt{3}}(\delta^{ac}\delta^{bd}-\delta^{ad}\delta^{bc}),\quad \mathcal{C}^{ab;cd}_{\mathbf{6}\otimes\bar{\mathbf{6}}} := \frac{1}{2\sqrt{6}}(\delta^{ac}\delta^{bd}+\delta^{ad}\delta^{bc})$$

$$\Gamma^{kl;mn} := \frac{1}{2}(\delta^{km}\delta^{ln}+\delta^{kn}\delta^{lm}-\frac{2}{3}\delta^{kl}\delta^{mn})$$

NRQCD Operators

- The operators manifest the correct C/P-parity under the charge conjugation/parity transformations

$$\psi \to \mathrm{i}\left(\chi^\dagger \sigma^2\right)^t, \quad \chi \to -\mathrm{i}\left(\psi^\dagger \sigma^2\right)^t$$

$$\psi(t, \mathbf{r}) \to \psi(t, -\mathbf{r}), \quad \chi(t, \mathbf{r}) \to -\chi(t, -\mathbf{r})$$

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- We use the basis in which the diquark and anti-diquark in the color-triplet and color-sexet, respectively. The operators can also be constructed from quark-antiquark pairs in the color-singlet and color-octet.

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- These NRQCD operators can also be inferred by performing the Foldy-Wouthuysen-Tani transformation from the QCD interpolating currents in QCD sum rules. ***H.-X. Chen et al.,2020***

$pp \to T_{4c} + X$

Inclusive Production of T_{4c} on LHC

- QCD factorization:

$$\mathrm{d}\sigma\left(p+p\to T_{4c}+X\right)=\sum_{i,j=q,g}\int_0^1 \mathrm{d}x_1\mathrm{d}x_2\,f_{i/p}\left(x_1,\mu\right)f_{j/p}\left(x_2,\mu\right)$$
$$\times\,\mathrm{d}\hat{\sigma}_{ij\to T_{4c}+X}(x_1x_2s,\mu),$$

Inclusive Production of T_{4c} on LHC

- QCD factorization:

$$\mathrm{d}\sigma\left(p+p \to T_{4c}+X\right) = \sum_{i,j=q,g} \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2\, f_{i/p}\left(x_1,\mu\right) f_{j/p}\left(x_2,\mu\right) \times \mathrm{d}\hat{\sigma}_{ij\to T_{4c}+X}(x_1 x_2 s, \mu),$$

- NRQCD factorization formula

$$\frac{\mathrm{d}\hat{\sigma}_{T_{4c}^n+X}}{\mathrm{d}\hat{t}} = \frac{2M_{T_{4c}}}{m_c^{14}}\left[F_{3,3}^n \left\langle \mathcal{O}_{3,3}^n \right\rangle + F_{3,6}^n\, 2\mathrm{Re}\left\langle \mathcal{O}_{3,6}^n \right\rangle + F_{6,6}^n \left\langle \mathcal{O}_{6,6}^n \right\rangle\right]$$

$\hat{s}$, $\hat{t}$: Mandelstam variables of parton scattering;
F_n: SDCs;
$\left\langle \mathcal{O}_n^{T_{4c}} \right\rangle$: vacuum matrix elements of NRQCD production operators

Perturbative Matching

Since the SDCs are insensitive to the long-distance physics, one can use the perturbative matching procedure to determine the SDCs.

- Replace the physical tetraquark state T_{4c}^{J} with a free 4-quark state
- Calculate both sides of factorization formula in perturbative QCD and perturbative NRQCD
- Solving the factorization formula to determine the SDCs.

E.g.,

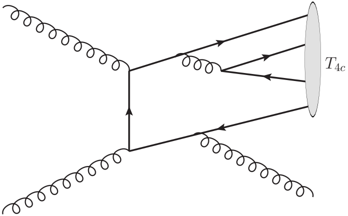
$$\left|\mathcal{T}^{J,m_j}_{\bar{\mathbf{3}}\otimes\mathbf{3}}(Q)\right\rangle = \frac{1}{2}\sum_{s_*,\lambda_*}\left\langle\frac{1}{2}\lambda_1\frac{1}{2}\lambda_2\middle|1s_1\right\rangle\left\langle\frac{1}{2}\lambda_3\frac{1}{2}\lambda_4\middle|1s_2\right\rangle\langle 1s_1 1s_2|Jm_j\rangle$$

$$\mathcal{C}^{ab;cd}_{\bar{\mathbf{3}}\otimes\mathbf{3}}\left|c_a^{\lambda_1}(q_1)c_b^{\lambda_2}(P-q_1)\bar{c}_c^{\lambda_3}(q_2)\bar{c}_d^{\lambda_4}(Q-P-q_2)\right\rangle$$

$$\Rightarrow\left\langle\mathcal{T}^{J,m_j}_{\bar{\mathbf{3}}\otimes\mathbf{3}}(Q)\middle|\varepsilon(m_j)\cdot\mathcal{O}^{(J)\dagger}_{\bar{\mathbf{3}}\otimes\mathbf{3}}\middle|0\right\rangle = 4$$

Feynman Diagrams

- There are about 600 Feynman diagrams for the partonic process $gg \to T_{4c} + g$.



SDCs

- The full expression of SDCs is too lengthy, here we only present the asymptotic behavior of SDCs at large transverse momentum.

$$F_{3,3}^{0^{++}} = -\frac{8836\pi^4 m_c^6 \alpha_s^5 \left(\hat{s}\hat{t}+\hat{s}^2+\hat{t}^2\right)^4}{243\hat{s}^5\hat{t}^3\left(\hat{s}+\hat{t}\right)^3} + \mathcal{O}\left(\frac{m_c^7}{p_T^7}\right),$$

$$F_{3,6}^{0^{++}} = -\frac{4418\sqrt{\frac{2}{3}}\pi^4 m_c^6 \alpha_s^5 \left(\hat{s}\hat{t}+\hat{s}^2+\hat{t}^2\right)^4}{81\hat{s}^5\hat{t}^3\left(\hat{s}+\hat{t}\right)^3} + \mathcal{O}\left(\frac{m_c^7}{p_T^7}\right),$$

$$F_{6,6}^{0^{++}} = -\frac{4418\pi^4 m_c^6 \alpha_s^5 \left(\hat{s}\hat{t}+\hat{s}^2+\hat{t}^2\right)^4}{81\hat{s}^5\hat{t}^3\left(\hat{s}+\hat{t}\right)^3} + \mathcal{O}\left(\frac{m_c^7}{p_T^7}\right),$$

$$F_{3,3}^{1^{+-}} = \frac{960400\pi^4 m_c^8 \alpha_s^5 \left(\hat{s}\hat{t}+\hat{s}^2+\hat{t}^2\right)^2}{2187\hat{s}^4\hat{t}^2\left(\hat{s}+\hat{t}\right)^2} + \mathcal{O}\left(\frac{m_c^9}{p_T^9}\right),$$

$$F_{3,3}^{2^{++}} = -\frac{140936\pi^4 m_c^6 \alpha_s^5 \left(\hat{s}\hat{t}+\hat{s}^2+\hat{t}^2\right)^4}{1215\hat{s}^5\hat{t}^3\left(\hat{s}+\hat{t}\right)^3} + \mathcal{O}\left(\frac{m_c^7}{p_T^7}\right).$$

LDMEs

- Four-body potential models are adopted to estimate the LDMEs. The results are proportional to the wave functions at the origin, where the color structure labels C_1 and C_2 indicate the color configurations $\bar{\mathbf{3}} \otimes \mathbf{3}$ or $\mathbf{6} \otimes \bar{\mathbf{6}}$.

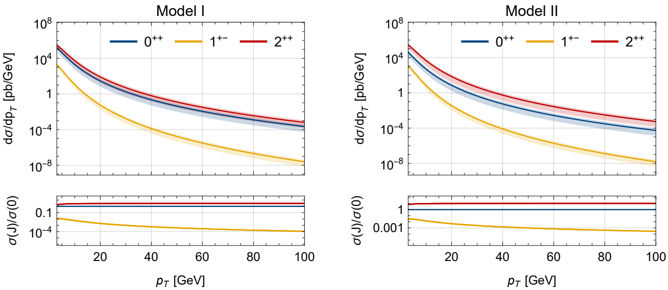
$$\left\langle O^{(0)}_{C_1,C_2} \right\rangle \approx 16\psi_{C_1}(\mathbf{0})\psi^*_{C_2}(\mathbf{0}), \quad \left\langle O^{(2)}_{C_1,C_2} \right\rangle \approx 80\psi_{C_1}(\mathbf{0})\psi^*_{C_2}(\mathbf{0}).$$

- Numerical results: (GeV^9)
  Model I : ***Lü, Chen, Dong, EPJC2020***
  Model II : ***M.-S. Liu, F.-X. Liu, et al., 2020***

	0^{++}			1^{+-}	2^{++}
	$\left\langle O^{(0)}_{3,3} \right\rangle$	$\left\langle O^{(0)}_{3,6} \right\rangle$	$\left\langle O^{(0)}_{6,6} \right\rangle$	$\left\langle O^{(1)}_{3,3} \right\rangle$	$\left\langle O^{(2)}_{3,3} \right\rangle$
Model I	0.0347	0.0211	0.0128	0.0780	0.072
Model II	0.0187	-0.0161	0.0139	0.0480	0.0628

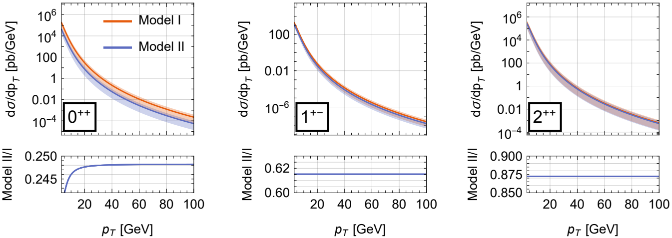
Phenomenology - p_{T} Spectrum

- $\sqrt{s} = 13\,\mathrm{TeV}$;
  `CTEQ14` PDF sets;
  $\mu \in [m_T/2, 2m_T]$, $\quad m_T = \sqrt{m_{T_{4c}}^2 + p_T^2}$;
  $m_c = 1.5\,\mathrm{GeV}$;
  $p_T \in [20, 60]\,\mathrm{GeV}$;
  rapidity cut $-5 \le y \le 5$
- The p_{T} spectra of the S-wave T_{4c} at the LHC predicted from two potential models.

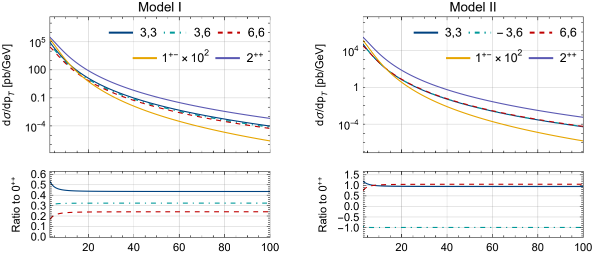


Phenomenology - Model Comparison

- Comparison of the p_{T} distributions of the S-wave T_{4c} between two phenomenological potential models.

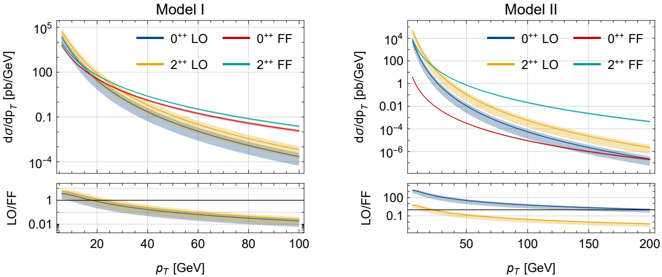


- Contributions from different color configurations



Phenomenology - vs Fragmentation

- Comparison of the p_{T} distributions of the T_{4c} between this work and from the fragmentation mechanism. *Feng,et al., PRD2022*



Phenomenology - σ & Event Numbers

- The integrated production rates for various S-wave T_{4c} states ($6\,\mathrm{GeV} \leq p_T \leq 100\,\mathrm{GeV}$) and the estimated event yields.
- Luminosity: $3000\,\mathrm{fb}^{-1}$.

	Model I		Model II	
	σ [nb]	$N_{\mathrm{events}}/10^9$	σ [nb]	$N_{\mathrm{events}}/10^9$
0^{++}	67 ± 47	200 ± 140	16 ± 11	49 ± 34
1^{+-}	0.52 ± 0.29	1.6 ± 0.9	0.32 ± 0.18	1.0 ± 0.5
2^{++}	168 ± 118	504 ± 354	147 ± 103	440 ± 309

$$\gamma p \to T_{4c} + X$$

Factorization Formula

Utilizing the EPA and QCD factorization theorem, the inclusive production cross section of a hadron T_{4c} at the EIC can be written as

$$\frac{d\sigma}{dzdp_T} = \sum_i \int_{x_\gamma^{\min}}^1 dx_\gamma \frac{2x_i p_T}{z(1-z)} f_{\gamma/e}\left(x_\gamma\right) f_{i/p}\left(x_i\right) \\ \times \frac{\mathrm{d}\hat{\sigma}(\gamma + i \to T_{4c} + X, \mu)}{\mathrm{d}\hat{t}},$$

$z := P_{T_{4c}} \cdot P_p / P_\gamma \cdot P_p$: elasticity parameter
x_i: momentum fraction of parton i,
x_γ: momentum fraction carried by the photon relative to the electron
$x_\gamma^{\min} = \frac{M_T^2 - m_H^2 z}{s\, z(1-z)}$
$f_{i/p}$: proton PDF
$f_{\gamma/e}$: photon flux under EPA

NRQCD Factorization

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$$\frac{\mathrm{d}\hat{\sigma}(\gamma g \to T_{4c}^{(1)} + X)}{\mathrm{d}\hat{t}} = \frac{2M_{T_{4c}}}{m_c^{14}} F_{3,3}^{(1)}(\hat{s},\hat{t}) \left\langle O_{3,3}^{(1)} \right\rangle$$

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- LO partonic channel: $\gamma + g \to T_{4c} + g$
- C-parity conservation ⇝ vector tetraquark state 1^{+-}

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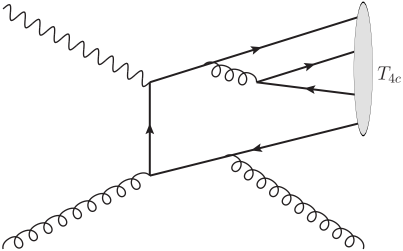
- more than 300 tree-level Feynman diagrams



SDC

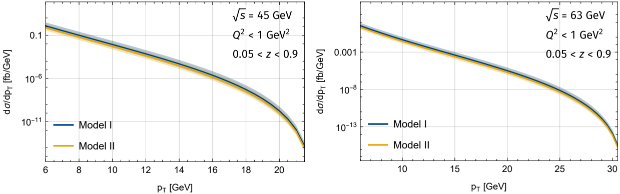
$$
\begin{aligned}
&F_{3,3}^{(1)}(\hat{s},\hat{t}) = \pi^3 e_Q^2 \alpha_s^4 r_s^2 \left[72 r_t^8 \left(5445 - 5298 r_t + 1462 r_t^2 - 184 r_t^3 + 49 r_t^4\right) - 432 r_t^7 \left(-5445 + 9879 r_t - 5524 r_t^2\right.\right.\\
&\left.+1030 r_t^3 - 112 r_t^4 + 42 r_t^5\right) r_s + 2 r_t^6 \left(3332340 - 8427078 r_t + 8454303 r_t^2 - 4101132 r_t^3 + 1115650 r_t^4\right.\\
&\left.-253810 r_t^5 + 43627 r_t^6\right) r_s^2 + 2 r_t^5 \left(5880600 - 17892198 r_t + 25180533 r_t^2 - 22035111 r_t^3 + 12807704 r_t^4\right.\\
&\left.-4945126 r_t^5 + 1195291 r_t^6 - 138533 r_t^7\right) r_s^3 + r_t^4 \left(14113440 - 49523400 r_t + 83600442 r_t^2 - 101112318 r_t^3\right.\\
&\left.+94409051 r_t^4 - 60657225 r_t^5 + 24055510 r_t^6 - 5305354 r_t^7 + 505879 r_t^8\right) r_s^4 + r_t^3 \left(11761200 - 49523400 r_t\right.\\
&+ 95733756 r_t^2 - 135804348 r_t^3 + 164472260 r_t^4 - 151209848 r_t^5 + 91395217 r_t^6 - 33278237 r_t^7 + 6611864 r_t^8\\
&\left.-555538 r_t^9\right) r_s^5 + r_t^2 \left(6664680 - 35784396 r_t + 83600442 r_t^2 - 135804348 r_t^3 + 186897370 r_t^4 - 206629419 r_t^5\right.\\
&\left.+164091573 r_t^6 - 86266517 r_t^7 + 27956171 r_t^8 - 5016861 r_t^9 + 381715 r_t^{10}\right) r_s^6 + r_t \left(2352240 - 16854156 r_t\right.\\
&+ 50361066 r_t^2 - 101112318 r_t^3 + 164472260 r_t^4 - 206629419 r_t^5 + 187216756 r_t^6 - 119518674 r_t^7 + 52323094 r_t^8\\
&\left.-14762980 r_t^9 + 2381419 r_t^{10} - 165406 r_t^{11}\right) r_s^7 + \left(392040 - 4267728 r_t + 16908606 r_t^2 - 44070222 r_t^3\right.\\
&+ 94409051 r_t^4 - 151209848 r_t^5 + 164091573 r_t^6 - 119518674 r_t^7 + 59925804 r_t^8 - 20969265 r_t^9 + 4946107 r_t^{10}\\
&\left.-698919 r_t^{11} + 43850 r_t^{12}\right) r_s^8 + \left(-381456 + 2386368 r_t - 8202264 r_t^2 + 25615408 r_t^3 - 60657225 r_t^4 + 91395217 r_t^5\right.\\
&\left.-86266517 r_t^6 + 52323094 r_t^7 - 20969265 r_t^8 + 5682942 r_t^9 - 1042547 r_t^{10} + 119941 r_t^{11} - 6480 r_t^{12}\right) r_s^9\\
&+ \left(105264 - 444960 r_t + 2231300 r_t^2 - 9890252 r_t^3 + 24055510 r_t^4 - 33278237 r_t^5 + 27956171 r_t^6 - 14762980 r_t^7\right.\\
&\left.+ 4946107 r_t^8 - 1042547 r_t^9 + 135646 r_t^{10} - 10512 r_t^{11} + 408 r_t^{12}\right) r_s^{10} + \left(-13248 + 48384 r_t - 507620 r_t^2 + 2390582 r_t^3\right.\\
&\left.-5305354 r_t^4 + 6611864 r_t^5 - 5016861 r_t^6 + 2381419 r_t^7 - 698919 r_t^8 + 119941 r_t^9 - 10512 r_t^{10} + 324 r_t^{11}\right) r_s^{11}\\
&\left.+ \left(2 - 3 r_t + r_t^2\right)^2 \left(882 - 1890 r_t + 13277 r_t^2 - 21970 r_t^3 + 14354 r_t^4 - 4032 r_t^5 + 408 r_t^6\right) r_s^{12}\right]\\
&\times \left\{331776 \left(3 - r_s\right)^2 \left(2 - r_s\right)^2 \left(1 - r_s\right)^2 \left(r_s \left(2 - r_t\right) - 2 r_t\right)^2 \left(3 - r_t\right)^2 \left(2 - r_t\right)^2 \left(1 - r_t\right)^2\right.\\
&\left.\times \left(r_s + r_t\right)^2 \left(r_s \left(3 - 2 r_t\right) - 3 r_t\right)^2\right\}^{-1}, \quad r_s := 16 m_c^2/\hat{s},\ r_t := 16 m_c^2/\hat{t}
\end{aligned}
$$

Asymptotic form of the SDC in large p_T :

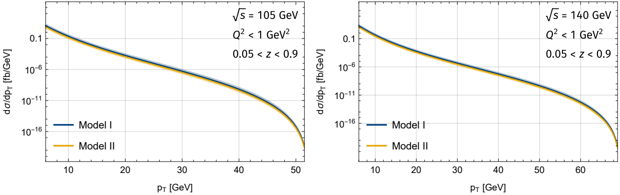
$$
F_{3,3}^{(1)}(\hat{s},\hat{t}) = \frac{1210 \pi^3 \alpha_s^4 e_c^2 m_c^8 \left(\hat{s}^2 + \hat{s}\hat{t} + \hat{t}^2\right)^2}{729 \hat{s}^4 \hat{t}^2 (\hat{s} + \hat{t})^2} + \mathcal{O}\left(\frac{m_c^9}{p_T^9}\right).
$$

Phenomenology

- Comparison of p_T distributions with LDMEs estimated from two phenomenological models.



(a) $\sqrt{s} = 45$ GeV **(b)** $\sqrt{s} = 63$ GeV



(c) $\sqrt{s} = 105$ GeV **(d)** $\sqrt{s} = 140$ GeV

Phenomenology

- The p_T-integrated cross section for T_{4c} inclusive production.
- Integrated luminosity:
  $100\ \mathrm{fb}^{-1}$ /yr@EIC;
  $50.5\ \mathrm{fb}^{-1}$/yr@EicC;
  $468\ \mathrm{pb}^{-1}$@HERA.
- p_T range:
  $6-20\,\mathrm{GeV}$@EIC&HERA;
  $6-9\,\mathrm{GeV}$@EicC.

	$\sqrt{s}$ [GeV]	Model I		Model II	
		σ [fb]	N	σ [fb]	N
EIC	44.7	1.0	96	0.59	59
	63.2	3.0	300	1.9	190
	104.9	11	1100	6.7	670
	140.7	20	2000	12	1200
HERA	319	67	31	41	19
EicC	20	0.00066	0.033	0.00041	0.020

Summary

- We propose a model-independent approach to study the production of fully heavy tetraquark, based on NRQCD factorization.
- The production rates of T_{4c} appears to be significant on the LHC due to the huge luminosity.
- EIC is the most promising ep collider for detecting the vector T_{4c} events.
- Model-independent estimates on the NRQCD matrix elements are required to make more reliable phenomenological predictions.

Thanks!