Static energies from lattice QCD

Francesco Knechtli

QWG 2024, IISER Mohali, India, February 27, 2024

Funded by Deutsche Forschungsgemeinschaft German Research Foundation
FOR 5269
BERGISCHE UNIVERSITÄT WUPPERTAL

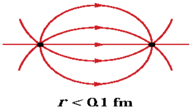
Plan of the talk

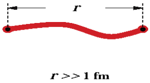
- The quark-mass dependence of the potential energy between static colour sources in the QCD vacuum with light and strange quarks [Bulava, Knechtli, Koch, Morningstar, Peardon, to appear soon]

The static potential - a probe of QCD at all scales

Potential energy levels (static energies) $V_n(r)\,, n = 0, 1, 2, \ldots$
of a quark-anti-quark static pair at distance r

Pure gauge theory:





asymptotic freedom,
perturbation theory

confinement, flux tube,
effective bosonic string theory

At large distances with dynamical (sea) quarks:

Formation of a pair of static-light mesons: string breaking

String breaking as a mixing phenomenon

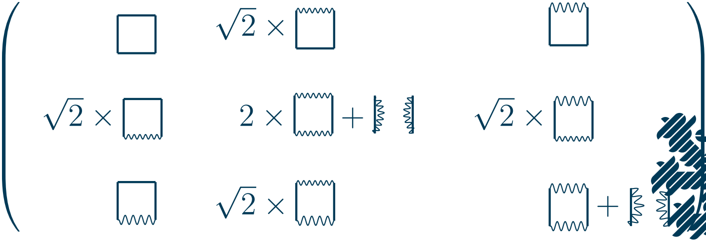
String breaking

- Describes the flattening of the potential $V_0(r)$ at large r
- *"The ground state potential $V_0(r)$ can therefore be called a static quark potential or a static meson potential"* [Sommer, Phys. Rept. 275(1) (1996)]
- Estimate $r_c \approx 1.5$ fm from $V_0(r_c) = 2E_{\bar{Q}l}$ [Alexandrou et al., Nucl.Phys. B414 (1994)]
- It has been observed as a mixing of "string-like" and two-meson operators [Drummond, 9805012; Philipsen and Wittig, 9807020; Knechtli and Sommer, 9807022, 0005021; Bali et al., 0505012, Bulava et al., 1902.04006]

String breaking of the ground state potential

- N_f=2+1 QCD: degenerate up, down (light) quarks and a strange quark
- Two avoided level crossings at $2E_{\bar{Q}l}$ (static-light) and $2E_{\bar{Q}s}$ (static-strange)
- First calculation in [Bulava et al., 1902.04006]
- Here: quark-mass dependence and extrapolation to the physical point

$C(\mathbf{r}, t) =$



CLS ensembles

id	N_{conf}	N_{conf}^{W}	t_0/a^2	N_s	N_t	m_π[MeV]	m_K[MeV]	$m_\pi L$
N203	94	752	5.1433(74)	48	128	340	440	5.4
N200	104	1664	5.1590(76)	48	128	280	460	4.4
D200	209	1117	5.1802(78)	64	128	200	480	4.2

- $N_\mathrm{f} = 2 + 1$ ensembles from CLS (Coordinated Lattice Simulations) [M. Bruno et al., 1411.3982; Bali et al. 1606.09039]
- Tree-level O(a^2) improved gauge action [Lüscher and Weisz, Comm. Math. Phys. 97 (1985)] and O(a) improved Wilson fermions with non-perterbative c_sw [Bulava and Schaefer, 1304.7093]
- Open b.c. in time and twisted-mass reweighting [Lüscher and Schaefer, 1206.2809]

CLS ensembles contd

- lattice spacing $a = 0.0633(4)(6)$ fm [Strassberger et al., 2112.06696]
- quark masses $m_{\mathrm{u}} = m_{\mathrm{d}} = m_l$ and m_{s} vary along $\sum_{f=\mathrm{u,d,s}} m_{\mathrm{bare},f}$ = constant ≈ physical value
- quark-mass parameter

$$\mu_l = \frac{3m_\pi^2}{m_\pi^2 + 2m_K^2} \approx \frac{3m_l}{2m_l + m_{\mathrm{s}}} \propto m_l$$

  $N_{\mathrm{f}} = 3$ symmetric point: $\mu_l = 1$
  physical point: $\mu_l = 0.1076$ ($m_\pi = 134.8$ MeV, $m_K = 494.2$ MeV isospin limit)
- physical units: we use the scale t_0 [Lüscher, 1006.4518] and $\sqrt{t_0} = 0.1443(7)(13)$ fm at the physical point [Strassberger et al., 2112.06696]

Techniques

Smearing

- Temporal gauge links: HYP2 static quark action [Hasenfratz and Knechtli, 0103029; Della Morte, Shindler, Sommer, 0506008; Donnellan et al, 1012.3037]
- Quarks: Distillation projection on a time-slice onto space spanned by lowest eigenmodes of the 3D gauge-covariant Laplace operator [Peardon et al, 0905.2160], stochastic estimation of quark propagators between distillation spaces [Morningstar et al, 1104.3870]

Techniques contd

Generalized Eigenvalue Problem (GEVP)

- For fixed inter-quark separation $\mathbf{r}$ solve the GEVP [Lüscher and Wolff, Nucl.Phys.B 339(1) (1990); Blossier et al., 0902.1265]

$$C(t)\, v_n(t,t_0) \;=\; \lambda_n(t,t_0)\, C(t_0)\, v_n(t,t_0)\,, n=0,1,2\,, t\geq t_0$$

We use $t_0=5$, solve GEVP at $t=10$ and project

$$\hat{C}_{ij} \;=\; (v_i(t_0,t_d), C(t) v_j(t_0,t_d))$$

We form the ratios

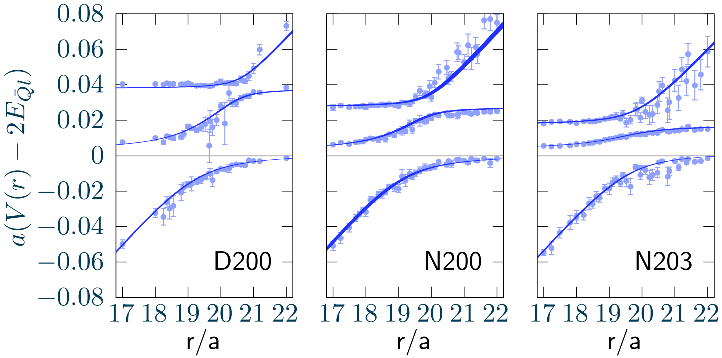
$$R_n(t) = \frac{\hat{C}_{nn}(t)}{C^2_{\bar{Q}l}(t)}\,, n=0,1,2$$

$C_{\bar{Q}l}$ is the correlator of a single static-light meson

- Correlated single-exponential fit to

$$R_n = \text{const.} \times \exp\left[-t(V_n - 2E_{\bar{Q}l})\right]$$

Potential levels



- Three lowest potential levels on each gauge ensembles
- Two avoided level crossings at twice the energy of static-light and static-strange mesons
- Off-axis distances to increase the resolution

Model

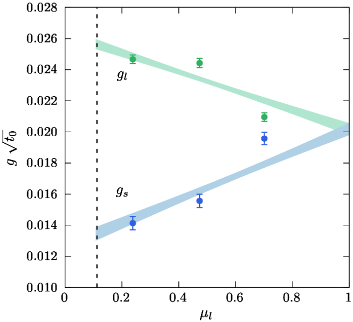
Model Hamiltonian

$$H(r) = \begin{pmatrix} \hat{V}(r) & \sqrt{2}g_l & g_s \\ \sqrt{2}g_l & \hat{E}_1 & 0 \\ g_s & 0 & \hat{E}_2 \end{pmatrix}, \hat{V}(r) = \hat{V}_0 + \sigma r + \gamma/r$$

- 7 fit parameters $\{\hat{E}_1, \hat{E}_2, g_l, g_s, \sigma, \hat{V}_0, \gamma\}$
- $\hat{V}(r)$, $\hat{E}_1$, $\hat{E}_2$ are the asymptotic energy levels for $r \to \infty$ up to O(r^{-1})
- Flavour factor $\sqrt{N_l = 2}$ in from of g_l, cf. matrix $C(\mathbf{r}, t)$
- Model parameters by minimization of *correlated* χ^2
- Fit ranges start at $4a$ (ground state), when gap $> 2m_\pi$ for excited states, maximal distance is $L/2$ (or below)

In the following: Mass extrapolations of fit parameters in units of t_0 to physical point assuming simple linear behaviour in μ_l

Mass extrapolations of the mixing coefficients

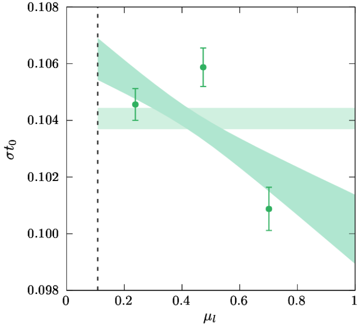


$$g_l\sqrt{t_0} = 0.0256(3), \qquad g_l = 35.0\,(5)_{\rm stat}(3)_{\rm sys}\,\mathrm{MeV}$$
$$g_s\sqrt{t_0} = 0.0134(4), \qquad g_s = 18.3\,(6)_{\rm stat}(2)_{\rm sys}\,\mathrm{MeV}$$

Mixing parameters g_l and g_s differ by $\approx$factor two

Some tension between data and model close to $\mu_l = 1$

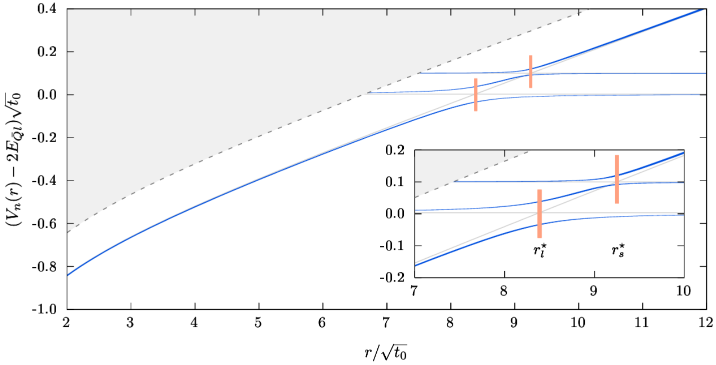
Mass extrapolations of the string tension



$$\sigma t_0 = 0.1061(7)(20), \quad \sqrt{\sigma} = 445(3)_{\text{stat}}(6)_{\text{sys}}\,\text{MeV}$$

String tension has only a mild quark mass dependence
Systematic error includes uncertainty from the difference between linear (central value) and constant fit

Potentials at the physical point



Gray region is ground state $+2m_\pi$

Crossings of $\hat{V}(r)$ with the asymptotic levels $\hat{E}_1$ and $\hat{E}_2$ resp.

$$r_l^\star = 8.39(3)\sqrt{t_0} = 1.211(7)_{\text{stat}}(11)_{\text{sys}}\text{ fm}$$

$$r_s^\star = 9.26(2)\sqrt{t_0} = 1.336(7)_{\text{stat}}(12)_{\text{sys}}\text{ fm}$$

Conclusions and Outlook

- Three lowest levels of the static potential up to $r \approx 1.7$ fm in QCD with up, down and strange quarks
- Simple model based on Cornell + mixing between string and static-meson states
- Parametrization of the physical potential
- Mixing parameter of light quarks $\approx 2\times$ of strange quark
- String tension at the physical point

  $$\sigma t_0 = 0.1061(7)(20), \quad \sqrt{\sigma} = 445(3)_{\rm stat}(6)_{\rm sys}\,\mathrm{MeV}$$

  For comparison: pure-gauge $\sigma t_0 \in [0.143, 0.159]$ from T_c/σ, cf. [Necco, 0309017]
  TUMQCD [Brambilla et al., 2206.03156], $N_{\rm f} = 2+1+1$: $\sqrt{\sigma} = 467(7)$, $482(7)$ MeV from Cornell fits at $r < 1$ fm
- Outlook: hybrid potentials using Laplacian trial states

  [Höllwieser et al., 2212.08485 and 2401.09453]