Thresholds effects on charmonium spectrum

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DFG Deutsche Forschungsgemeinschaft

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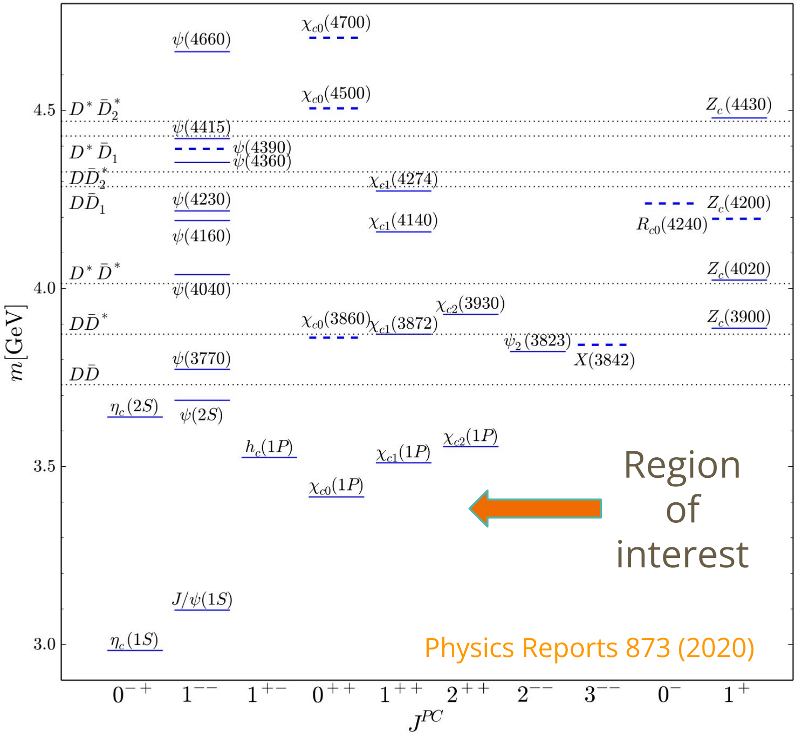
Outline

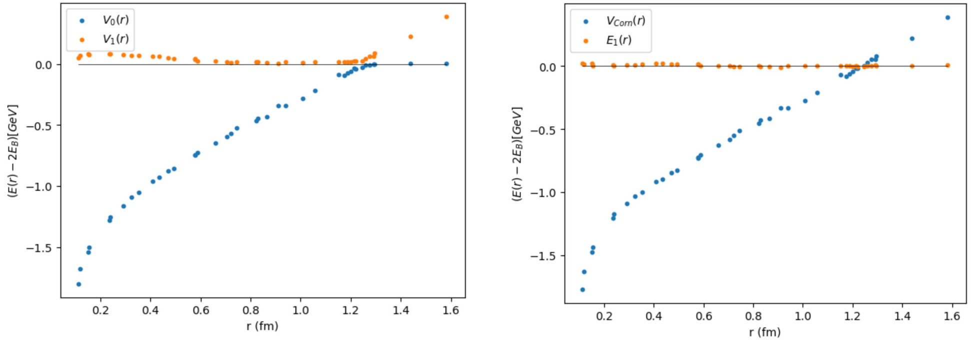
- Physical Picture vs. Lattice Picture
- BOEFT Lagrangian
- Coupled system approach
- Self-energy approach
- Comparison of the results
- Conclusions

Physical Picture vs. Lattice Picture

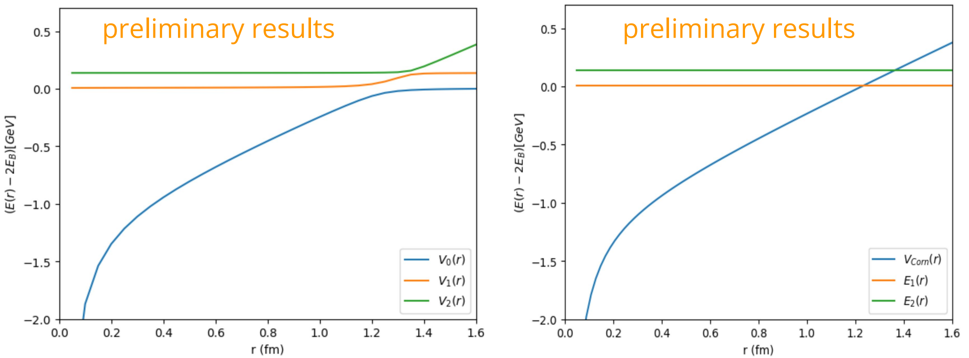


SESAM (2005)

lattice data taken from *Phys.Rev.D* 71 (2005)



J. Bulava, F. Knechtli, V. Koch, C. Morningstar, M. Peardon (2024)



BOEFT Lagrangian

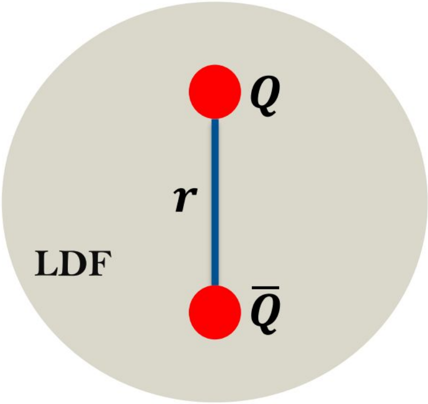
Irr. Reps. $D_{\infty h}$: Λ_η^σ (LDF)

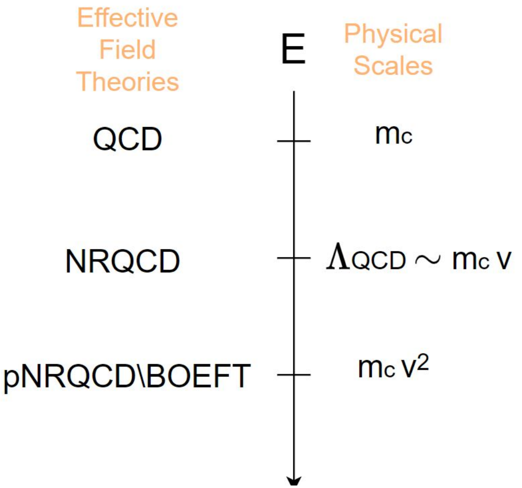
Quarkonium

Σ_g^+

Meson-Antimeson threshold

$K_{\bar q}^P \otimes K_q^P$	K^{PC}	Static energies $D_{\infty h}$
$(1/2)^- \otimes (1/2)^+$	0^{-+}	$\{\Sigma_u^-\}$
	1^{--}	$\{\Sigma_g^+, \Pi_g\}$
$(1/2)^- \otimes (1/2)^-$	0^{++}	$\{\Sigma_g^+\}$
	1^{+-}	$\{\Sigma_u^-, \Pi_u\}$
$(1/2)^- \otimes (3/2)^-$	1^{+-}	$\{\Sigma_u^-, \Pi_u\}$
	2^{++}	$\{\Sigma_g^+, \Pi_g, \Delta_g\}$
$(1/2)^+ \otimes (1/2)^-$	0^{-+}	$\{\Sigma_u^-\}$
	1^{--}	$\{\Sigma_g^+, \Pi_g\}$
$(1/2)^+ \otimes (3/2)^-$	1^{--}	$\{\Sigma_g^+, \Pi_g\}$
	2^{-+}	$\{\Sigma_u^-, \Pi_u, \Delta_u\}$
$(3/2)^+ \otimes (3/2)^-$	0^{-+}	$\{\Sigma_u^-\}$
	1^{--}	$\{\Sigma_g^+, \Pi_g\}$
	2^{-+}	$\{\Sigma_u^-, \Pi_u, \Delta_u\}$
	3^{--}	$\{\Sigma_g^+, \Pi_g, \Delta_g, \Phi_g\}$





$$\mathcal{L}_{tot} = \int d^3\boldsymbol{r} \left\{ \boldsymbol{\Psi}^\dagger(\boldsymbol{r},t) \left(i\partial_t + \frac{\nabla_r^2}{m_c} - V_\Psi \right) \boldsymbol{\Psi}(\boldsymbol{r},t) - \sum_{M=D,D_s} V_{\Psi M} \left(\boldsymbol{M}^\dagger(\boldsymbol{r},t)\, \boldsymbol{\Psi}(\boldsymbol{r},t) + h.c. \right) \right.$$
$$\left. + \sum_{M=D,D_s} \boldsymbol{M}^\dagger(\boldsymbol{r},t) \left(i\partial_t + \boldsymbol{P}_{10}^\dagger \frac{\nabla_r^2}{m_c} \boldsymbol{P}_{10} - V_M \right) \boldsymbol{M}(\boldsymbol{r},t) \right\}$$

Charmonium potential

$$V_\Psi = \frac{\gamma}{r} + \sigma r + 2m_c + \beta$$

Model parameters

$\sigma = 0.190\ GeV^2$ $\gamma = -0.432$ $m_c = 1.59\ GeV$

$V_D = 3.826\ GeV$ $V_{D_s} = 3.957\ GeV$ $\beta = -0.130\ GeV$

BOEFT Lagrangian

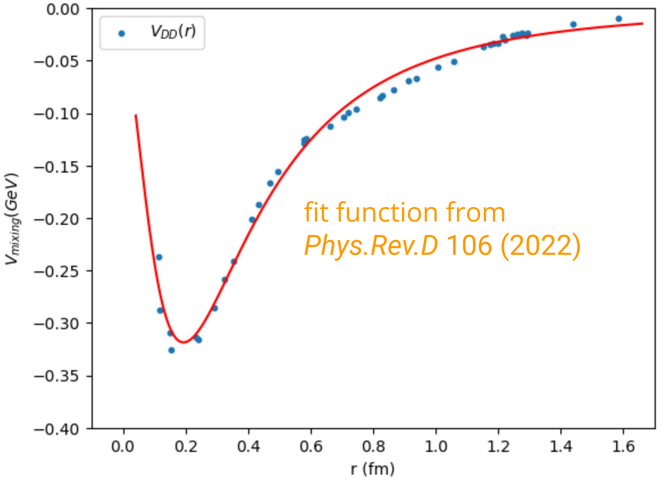
Charmonium spectrum

nL	charmonium $E_{exp}(MeV)$	$E_{th}(MeV)$
$1s$	3068	3095
$2s$	3674	3686
$3s$	4039	4128
$4s$	4421	4507
$5s$		4848
$1p$	3525	3502
$2p$	3927	3967
$3p$		4359
$4p$		4711
$5p$		5033
$1d$	3823	3794
$2d$		4202
$3d$		4565
$4d$		4896
$5d$		5204

Experimental values taken from R.L. Workman *et al.* (Particle Data Group)

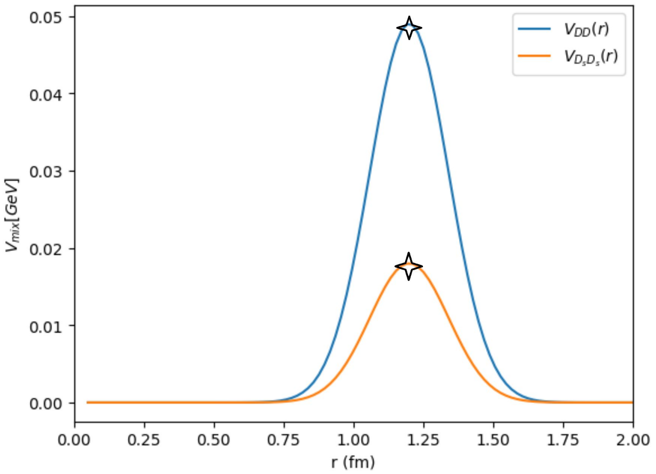
Mixing potentials

SESAM (2005)



Gaussian parametr.

parameters fixed from preliminary results of 'Bulava, F. Knechtli, V. Koch, C. Morningstar, M. Peardon' (2024)



BOEFT Lagrangian: mixing potentials

SESAM (2005)

$$V_{\Psi D}^{SESAM} = s(r)\left(c_1 r + c_2 r^3\right) + l(r)\ \frac{c_3}{r^3}$$

fit function from *Phys.Rev.D* 106 (2022)

$$s(r) = \left(\frac{r_0}{r+r_0}\right)^n \quad l(r) = \left(\frac{r}{r+r_0}\right)^n \quad c_1 = -0.723\ GeV^2 \quad c_2 = -15.251\ GeV^4 \quad c_3 = -13.991\ GeV^{-2}$$

$$r_0 = 0.25\ fm \quad n = 7$$

$$V_{\Psi D_s}^{SESAM} = \frac{1}{\sqrt{2}} V_{\Psi D}^{SESAM}$$

Gaussian

$$V_{\Psi D}^{gauss} = g_1\ \exp^{-\left(\frac{(r-r_c)^2}{\delta^2}\right)}$$

$$V_{\Psi D_s}^{gauss} = g_2\ \exp^{-\left(\frac{(r-r_c)^2}{\delta^2}\right)}$$

parameters fixed from preliminary results of 'Bulava, F. Knechtli, V. Koch, C. Morningstar, M. Peardon' (2024)

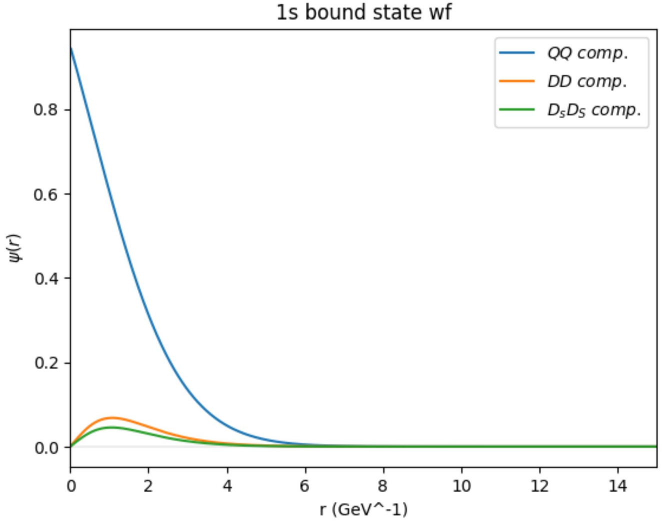
$$g_1 = 0.049\ GeV \quad g_2 = 0.018\ GeV \quad r_{min} = 1\ fm \quad r_{max} = 1.4\ fm$$

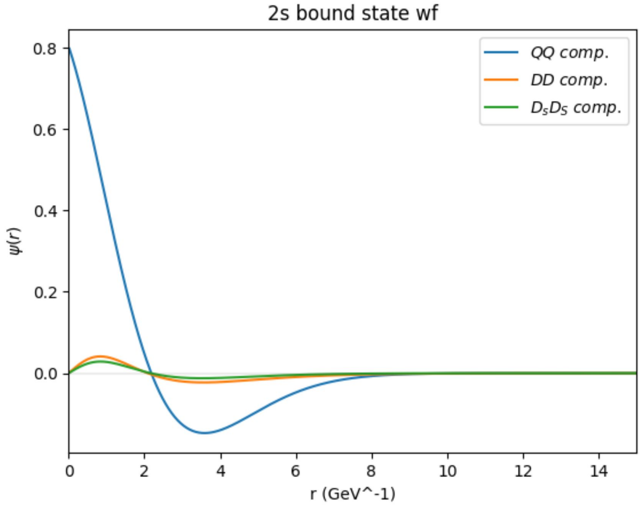
$$r_c = (r_{min} + r_{max})/2 \quad \delta = (r_{max} - r_{min})/2$$

Coupled system approach

Bound state (below threshold) e. o. m.

$$\begin{pmatrix} -\frac{\nabla^2}{m_Q} + V_\Psi & V_{\Psi D} & V_{\Psi D_s} \\ V_{\Psi D} & \vec{P}^*_{10}(-\frac{\nabla^2}{m_Q})\vec{P}_{10} + V_D & 0 \\ V_{\Psi D_s} & 0 & \vec{P}^*_{10}(-\frac{\nabla^2}{m_Q})\vec{P}_{10} + V_{D_s} \end{pmatrix} \begin{pmatrix} \psi_{Q\bar{Q}}(\boldsymbol{r}) \\ \psi_{D\bar{D}}(\boldsymbol{r}) \\ \psi_{D_s\bar{D}_s}(\boldsymbol{r}) \end{pmatrix} = E \begin{pmatrix} \psi_{Q\bar{Q}}(\boldsymbol{r}) \\ \psi_{D\bar{D}}(\boldsymbol{r}) \\ \psi_{D_s\bar{D}_s}(\boldsymbol{r}) \end{pmatrix}$$





Coupled system approach

Thresholds corrections in STA

nL	$\Delta E^{gauss}_{D\bar{D}}(MeV)$	$\Delta E^{flat}_{D\bar{D}}(MeV)$	$\Delta E^{SESAM}_{D\bar{D}}(MeV)$
$1s$		-2	-30
$2s$	-1	-6	-15
$1p$		-3	-22
$1d$	-1	-6	-15

nL	$\Delta E^{gauss}_{D_s\bar{D}_s}(MeV)$	$\Delta E^{flat}_{D_s\bar{D}_s}(MeV)$	$\Delta E^{SESAM}_{D_s\bar{D}_s}(MeV)$
$1s$			-14
$2s$		-1	-6
$1p$			-10
$1d$		-1	-6

Full thresholds corrections

nL	$\Delta E^{gauss}_{tot}(MeV)$	$\Delta E^{flat}_{tot}(MeV)$	$\Delta E^{SESAM}_{tot}(MeV)$
$1s$		-2	-43
$2s$	-1	-6	-20
$1p$		-3	-32
$1d$	-1	-6	-20

Self-energy approach

Exact charmonium propagator

$$\int dt e^{iEt} \langle 0|T\,\{\boldsymbol{\Psi}(t,\boldsymbol{r})\boldsymbol{\Psi}^\dagger(t,\boldsymbol{r}')\}|0\rangle_{exact} = \sum_{nl} i\,\frac{\psi_{nl}(\boldsymbol{r})\psi^*_{nl}(\boldsymbol{r}')}{E - E_{nl} - \Sigma_{nl}(E)}$$

similar methods in *Phys.Rev.D* 106 (2022)

Charmonium bound states

$$|c\bar{c}\rangle_{nl} = \int d^3\boldsymbol{r}\;\psi_{nl}(\boldsymbol{r})\;\boldsymbol{\Psi}^\dagger(\boldsymbol{r})\;|0\rangle$$

Meson-Antimeson states

$$|M\bar{M}\rangle_{kL} = \int d^3\boldsymbol{r} \sum_{\substack{l=|L-1|\\ m_l}}^{L+1} 4\pi i^{-l}\, j_l(kr)\, C^{Lm_L}_{lm_l1i}\, Y_l^{m_l}(\hat{r})\, P^i_{10}(\hat{r})\, \boldsymbol{M}^\dagger(\boldsymbol{r},t)|0\rangle$$

Diagrams resummation

Self-energy term

$$\Sigma_{nl}(E) = -\frac{2m_Q\,a}{\pi}\sum_{M=D,D_s}\int dk\,k^2\,\frac{\left(\sum_{l'=l-1}^{l+1} f^{l'}_{nl}(k)\right)^2}{m_Q(E-V_M)-k^2}$$

$$f^{l'}_{nl}(k) = C^{l0}_{l'010}\sqrt{\frac{2l'+1}{2l+1}}\int dr\,r^2\omega_{nl}(r)V_{\Psi M}(r)j_{l'}(kr)$$

Above threshold states

$$\Gamma^{M\bar{M}}_{nl} \simeq 2m_Q\,a\sqrt{m_Q\,(E_{nl}-V_M)}\left(\sum_{l'=l-1}^{l+1} f^{l'}_{nl}(k)\right)^2$$

$$M = D,\;D_s$$

Self-energy approach

Thresholds corrections in STA

nL	$\Delta E^{gauss}_{D\bar{D}}(MeV)$	$\Delta E^{flat}_{D\bar{D}}(MeV)$	$\Delta E^{SESAM}_{D\bar{D}}(MeV)$
$1s$		-3	-55
$2s$	-1	-6	-28
$1p$		-10	-83
$1d$	-2	-12	-40

nL	$\Delta E^{gauss}_{D_s\bar{D}_s}(MeV)$	$\Delta E^{flat}_{D_s\bar{D}_s}(MeV)$	$\Delta E^{SESAM}_{D_s\bar{D}_s}(MeV)$
$1s$			-26
$2s$		-1	-13
$1p$		-1	-40
$1d$		-1	-18

Full thresholds corrections

nL	$\Delta E^{gauss}_{tot}(MeV)$	$\Delta E^{flat}_{tot}(MeV)$	$\Delta E^{SESAM}_{tot}(MeV)$
$1s$		-3	-80
$2s$	-1	-6	-40
$1p$		-11	-120
$1d$	-2	-13	-57

Self-energy approach

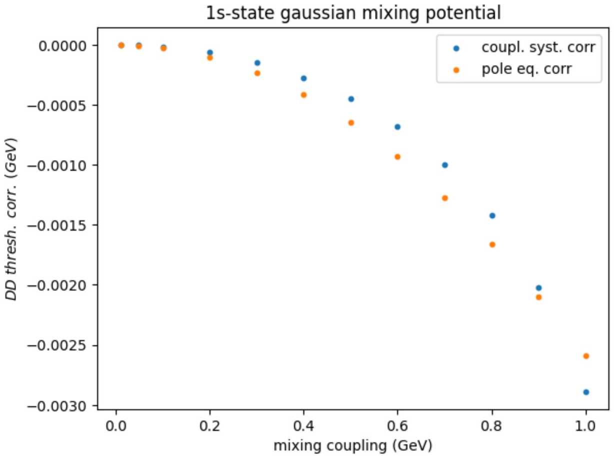
Decay rates for above threshold states

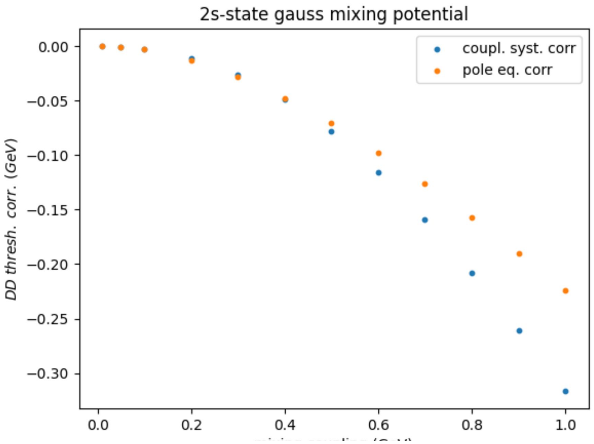
nL	$\Gamma^{gauss}_{D\bar{D}}(MeV)$	$\Gamma^{flat}_{D\bar{D}}(MeV)$	$\Gamma^{SESAM}_{D\bar{D}}(MeV)$
$3s$		6	11
$4s$		12	14
$5s$		10	12
$2p$	5	44	95
$3p$		62	103
$4p$	3	47	75
$5p$	1	35	55
$2d$	13	72	79
$3d$	2	47	62
$4d$	2	33	48
$5d$	2	25	37

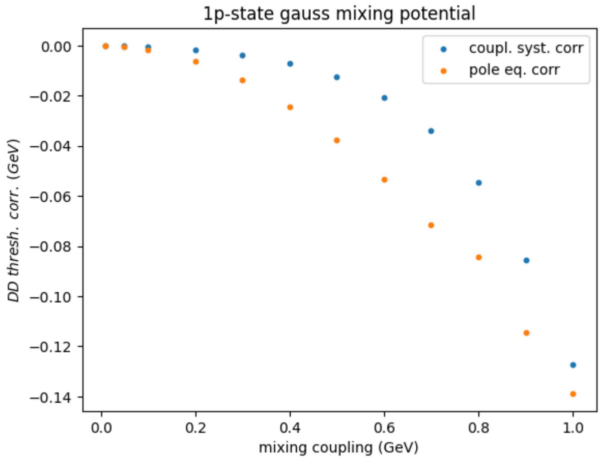
nL	$\Gamma^{gauss}_{D_s\bar{D}_s}(MeV)$	$\Gamma^{flat}_{D_s\bar{D}_s}(MeV)$	$\Gamma^{SESAM}_{D_s\bar{D}_s}(MeV)$
$3s$	1	1	
$4s$			4
$5s$		1	5
$2p$	1	5	
$3p$		3	33
$4p$		5	34
$5p$		5	28
$2d$		6	23
$3d$		7	28
$4d$		6	25
$5d$		5	21

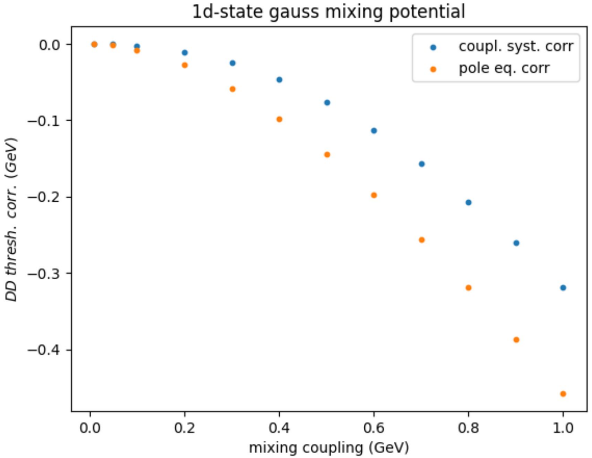
Comparison of the results

DD threshold



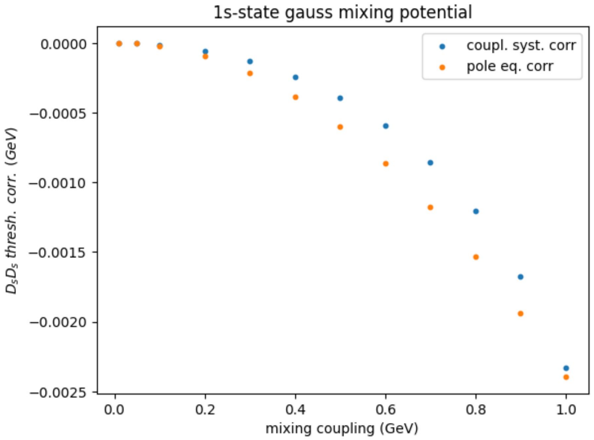


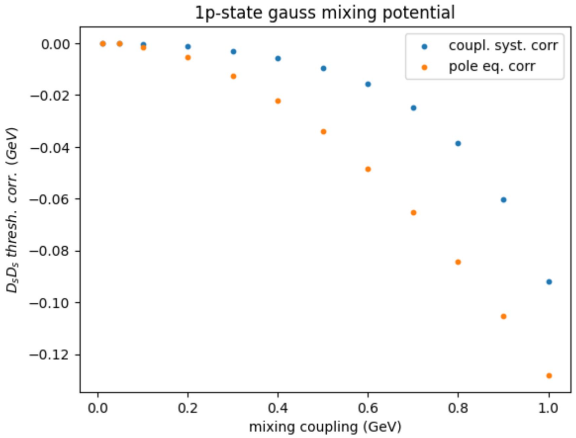


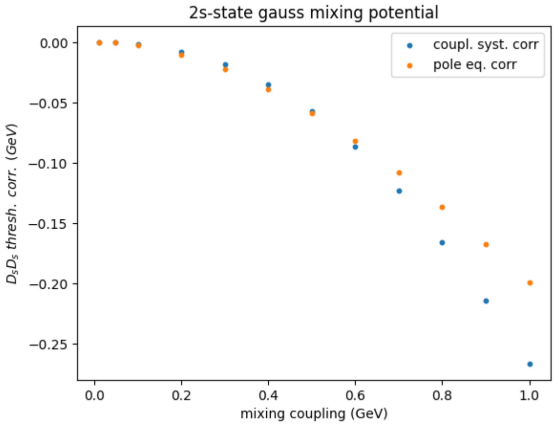


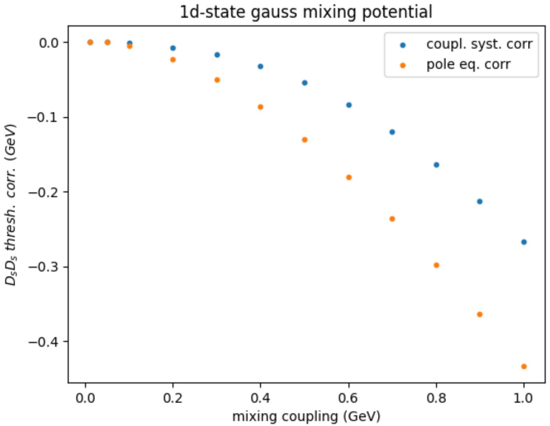
Comparison of the results

DsDs threshold









Conclusions

- Different approaches to study the problem
- LQCD inputs needed for the BOEFT lagrangian
- Different predictions for different mixing potentials, SESAM mixing potential predicts:
    - stronger corrections for lowest bound states ($\Delta E_{1s}^{SESAM} > \Delta E_{2s}^{SESAM}$) contrarily to the gaussian one;
    - stronger mixing with of thresholds with charmonium states w.r.t. gaussian one;
- Same predictions from the different methods (coupled system, self-energy) in the weak-coupling limit
- Decay rates key observable to identify the type of mixing potential