Charmonium spectroscopy with optimal distillation profiles

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QWG 2024, IISER Mohali, India, February 26 to March 1, 2024

FOR 5269

BERGISCHE UNIVERSITÄT WUPPERTAL

FOR5269: Future methods for studying confined gluons in QCD

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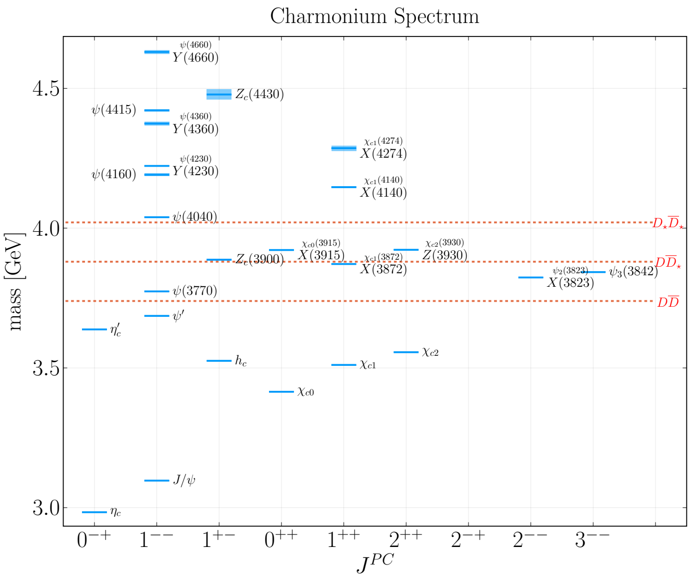
Collaboration between **physics** and **applied math** at BUW, DESY Zeuthen and Trinity College Dublin.

Main goals:

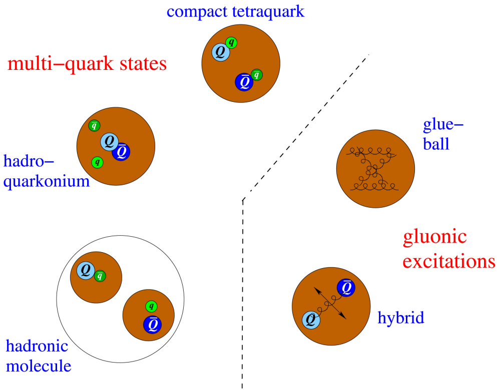
- Disconnected contributions in **charmonium**.
- Glueballs and mixing in dynamical QCD.
- String breaking in hybrid potentials.
- New schemes for molecular dynamics.
- Distillation + Multigrid framework.
- ...

https://confluence.desy.de/display/for5269



Plot by T. Korzec

Some observed states are not compatible with a $\bar{q}q$ composition. Alternatives [Brambilla *et al.* (2019)]:



Lattice QCD

Simulate QCD via Monte-Carlo methods in a Euclidean space-time lattice.

- Discretization introduces lattice spacing a.
- Quarks ψ live in lattice sites, gluons U live in links between sites.
- Lattice Dirac operator $\mathbf{D}$ is a **large** but sparse matrix $(10^7 \times 10^7)$
- Action $S[\bar{\psi}, \psi, U]$ recovers correct $a \to 0$ limit.

Measure expected values of observables $\mathcal{A}[\bar{\psi}, \psi, U]$:

- $\langle \mathcal{A} \rangle$ gives physical information, e.g energies.
- Sample gluon configurations distributed as $\propto e^{-S}$.
- Statistical errors $\propto \dfrac{1}{\sqrt{\# \text{ of measurements}}}$ + other effects

Hadron spectroscopy in lattice QCD

Nature: What is the mass of a $J^{PC} = 0^{-+}$ $\bar{c}c$ state, e.g η_c?

- SO(3) reduces to cubic group $\mathbb{O}$: $(0^{\pm\pm}, 1^{\pm\pm}, 2^{\pm\pm}, ...) \to (A_1^{\pm\pm}, A_2^{\pm\pm}, E^{\pm\pm}, T_1^{\pm\pm}, T_2^{\pm\pm})$.
- Flavor-singlet channels are **blind** to quark content.

Lattice: What is the mass of a A_1^{-+} with dominant $\bar{c}c$ content?

1. Define operator $\mathcal{O}[\bar{\psi}, \psi, U]$ with fixed quantum numbers.
2. Calculate two-point temporal correlation function

$$\langle \mathcal{O}(t)\bar{\mathcal{O}}(0) \rangle = \frac{1}{Z} \int d\psi d\bar{\psi} dU \mathcal{O}(t)\bar{\mathcal{O}}(0) e^{-S}$$

$$\approx \frac{1}{N} \sum_i (\ldots) \to \text{Monte Carlo for} \int dU$$

$$= \sum_n | \langle n | \hat{O}^\dagger | \Omega \rangle |^2 e^{-E_n t} \overset{t\to\infty}{\approx} | \langle 0 | \hat{O}^\dagger | \Omega \rangle |^2 e^{-E_0 t}$$

Hadron spectroscopy in lattice QCD

Build correlation matrix between different operators with equal quantum numbers

$$C_{ij}(t) = \left\langle \mathcal{O}_i(t)\bar{\mathcal{O}}_j(0) \right\rangle$$

and solve a generalized eigenvalue problem (GEVP)

$$C(t)w_n(t,t_G) = \rho_n(t,t_G)C(t_G)w_n(t,t_G)$$

to get

$$\rho_n(t,t_G) \overset{t\to\infty}{\approx} c_n e^{-E_n t} \rightarrow \text{Energies of states}$$

$$\tilde{\mathcal{O}}_n = \sum_k w_n^{(k)}(t_1,t_G)\mathcal{O}_k \rightarrow \text{Operator closest to } |n\rangle$$

[Lüscher and Wolff (1999), Blossier et al. (2009)]

Charmonium on the lattice

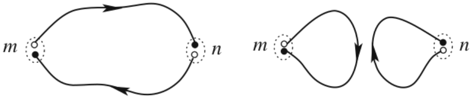
Mesonic operators

$$\mathcal{O}(t) = \bar{c}(t)\Gamma c(t), \ \Gamma = \{\gamma_5, \gamma_i, \gamma_5\gamma_i, \nabla_i, ...\}$$

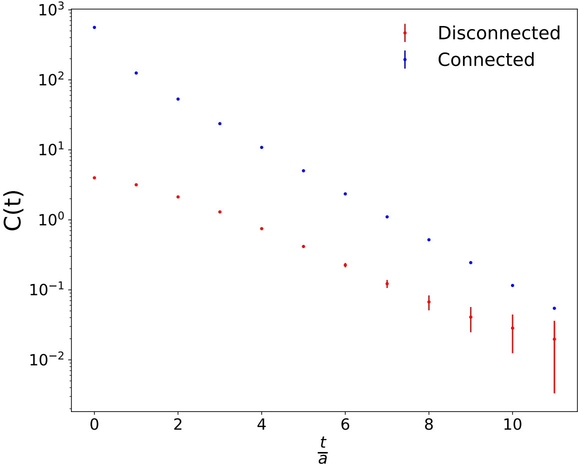
with correlation function

$$C(t) = -\left\langle \text{Tr}\left(\Gamma D^{-1}[t,0]\Gamma D^{-1}[0,t]\right)\right\rangle_{\text{gauge}} \quad \text{Connected}$$

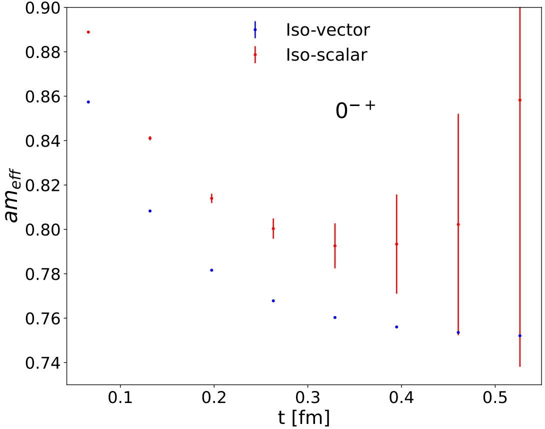
$$+\left\langle \text{Tr}\left(\Gamma D^{-1}[t,t]\right) \text{Tr}\left(\Gamma D^{-1}[0,0]\right)\right\rangle_{\text{gauge}} \quad \text{Disconnected}$$



- Inversions D^{-1} are the main computational cost.
- Disconnected contribution is the most **expensive** and **noisy**, being often **neglected** (OZI suppression).



Severe signal-to-noise problem in disconnected piece.
Why not just work at small times?



Excited-state contamination is significant at small times!

$$C(t) = |\langle 0|\hat{O}^\dagger|\Omega\rangle|^2 e^{-E_0 t} + \sum_{n>0} |\langle n|\hat{O}^\dagger|\Omega\rangle|^2 e^{-E_n t}$$

Optimal distillation profiles

Distillation [Peardon *et al.* (2009)]

- Project quark fields onto low-dimensional subspace of smooth, gauge-covariant fields → **Smearing**.
- $\psi(t) \to V[t]V[t]^\dagger\psi(t)$, with $V[t]$ the low modes of the 3D gauge-covariant Laplacian operator.
- Perambulators: $\tau[t_1,t_2] = V[t_1]^\dagger D^{-1} V[t_2]$
- Elementals: $\Phi[t] = V[t]^\dagger \Gamma V[t]$

$$C(t) = -\left\langle \mathrm{Tr}\left(\Phi[t]\tau[t,0]\Phi[0]\tau[0,t]\right)\right\rangle_{\mathrm{gauge}} + \left\langle \mathrm{Tr}\left(\Phi[t]\tau[t,t]\right)\mathrm{Tr}\left(\Phi[0]\tau[0,0]\right)\right\rangle_{\mathrm{gauge}}$$

High inversion cost but matrices have manageable sizes and perambulators are recycled for any choice of Γ.

Improved Distillation Phys. Rev. D 106, 034501 (2022)

- Exploit further freedom: $V[t]V[t]^\dagger \to V[t]J[t]V[t]^\dagger$ with quark distillation profile $J_k[t]_{ij} = \delta_{ij} g_k\left(\lambda_i[t]\right)$.
- Build optimal meson profiles solving a GEVP with different quark distillation profiles.

$$\mathcal{O}^{(\Gamma,n)}(t) = \sum_k a_k^{(\Gamma,n)} \bar{\psi}(t) V[t] J_k[t]^\dagger V[t]^\dagger \Gamma V[t] J_k[t] V[t]^\dagger \psi(t)$$

$$f^{(\Gamma,n)}\left(\lambda_i[t], \lambda_j[t]\right) = \sum_k a_k^{(\Gamma,n)} g_k^*\left(\lambda_i[t]\right) g_k\left(\lambda_j[t]\right)$$

$$\Phi[t]^{(\Gamma,n)}_{\substack{ij\\ \alpha\beta}} = f^{(\Gamma,n)}\left(\lambda_i[t], \lambda_j[t]\right) \Phi[t]_{\substack{ij\\ \alpha\beta}}$$

One optimal profile per Γ and energy level n.

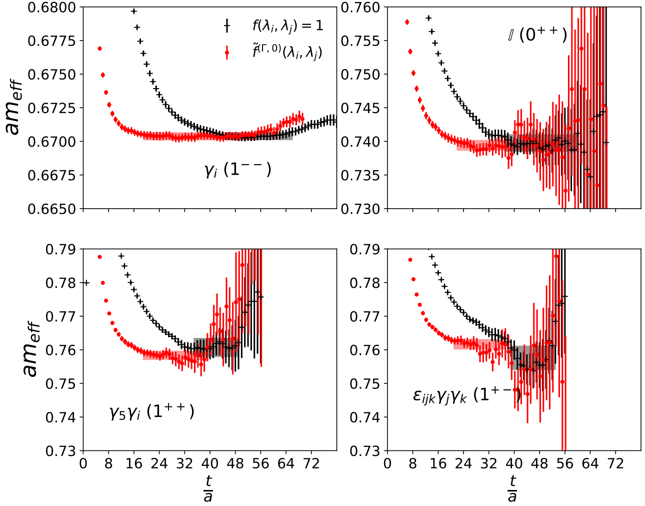
A close-to-physical setup

- $N_f = 3+1$ Clover-improved Wilson fermions with mass-dependent improvement for charm quark, $\beta = 3.43$, $\kappa_l = 0.13599$, $\kappa_c = 0.13088$ in a 144×48^3 lattice. [P. Fritzsch *et al.* (2018), R. Höllwieser *et al.* (2020)]
- $a = 0.04292(52)$ fm.
- SU(3) light flavor symmetric point + physical charm mass ($\eta_c \approx 3$ GeV).

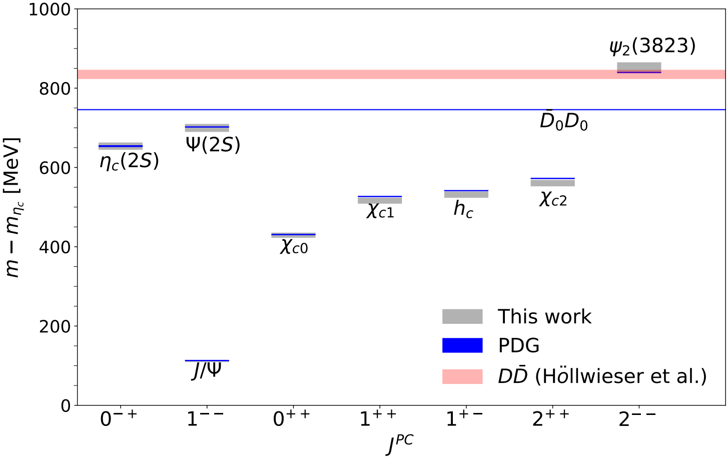
Advantages:

- Charm physics close to physical point and small lattice spacing.
- Presence of light quarks introduces decay channels.
- At SU(3) flavor symmetric point:

$$m_{\bar{c}c}^{\text{phys.}} = m_{\bar{c}c}^{\text{sym.}} + \sum_{i=u,d,s} \left.\frac{\partial m_{\bar{c}c}}{\partial m_i}\right|_{\text{sym.}} \left(m_i^{\text{sym.}} - m_i^{\text{phys.}}\right) + \mathcal{O}\left(\mathbf{\Delta m^2}\right)$$



Optimals profiles suppress excited-state contamination.
Earlier and longer plateau regions.



Good agreement with nature *despite* omission of disconnected contributions.

Hyperfine splitting $\Delta m_{\text{HF}} = m_{\text{J}/\Psi} - m_{\eta_c}$

- Experiment: $113.0(5)$ MeV. [R. L. Workman *et al.*, (2022)]
- Lattice: $118.6(1.1),\ 116.2(1.1)$ MeV [C. DeTar *et al.* (2019), D. Hatton *et al.* (2020)]
- This work: $111.8(1.4)$MeV.

Other mass splittings [C. DeTar *et al.* (2019)]

Splitting	This work	DeTar *et al.*	Experiment
$\overline{1P} - \overline{1S}$	$447.3(5.5)$	$462.2(4.5)$	$456.64(14)$
Spin-Orbit	$43.93(\mathbf{87})$	$46.6(3.0)$	$46.60(8)$
Tensor	$14.43(\mathbf{41})$	$17.0(2.3)$	$16.27(7)$
$1P$ HF	$-0.2(\mathbf{1.6})$	$-6.1(4.2)$	$-0.09(14)$
$2S$ HF	$45.9(1.8)$		$48(1)$

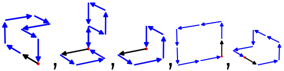
$$\Delta m_{\text{SO}} = \frac{1}{9}\left(5m_{2^{++}} - 3m_{1^{++}} - 2m_{0^{++}}\right),$$
$$\Delta m_{\text{tensor}} = \frac{1}{9}\left(3m_{1^{++}} - m_{2^{++}} - 2m_{0^{++}}\right)$$
$$\Delta m_{\text{1P HF}} = m_{\overline{1P}} - m_{1^{+-}}$$

Glueballs on the lattice

Bound states of only gluons arising from their self-interaction. Experimental detection is **difficult** due to decays and mixing with mesons.

On the lattice:

- Correlations are heavily affected by **signal-to-noise problem**, needing **large** statistics.
- Mixing with mesons makes **identification difficult**.

Popular operators:  $\rightarrow$ 3D Wilson loops.

Quenched lattice QCD [C. Morningstar and M. Peardon, (1999)]:

$0^{++} : 1730 \pm 80$ MeV $\rightarrow f_0(1710)$?

$2^{++} : 2400 \pm 120$ MeV, $0^{-+} : 2590 \pm 130$ MeV

Glueballs are **unstable** in full dynamical QCD!

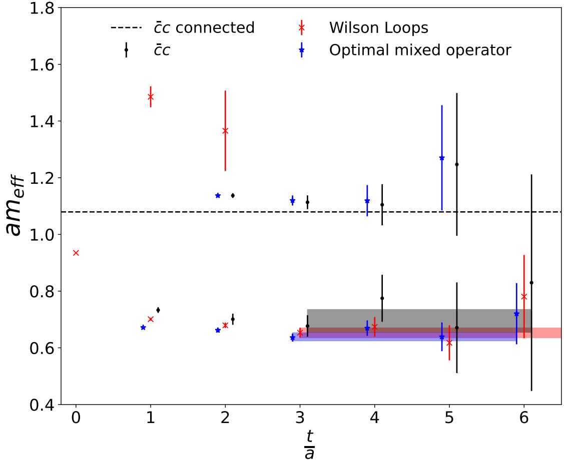
Start with simplified setup:

- $N_f = 2$ QCD, degenerate charm quarks at half the physical charm quark mass.
- Absence of light quarks restricts mixing to only charmonium.
- No decays into light hadrons.

Correlation matrix involves mesonic and gluonic operators:

$$C(t) = \begin{pmatrix} \langle \bar{c}(t)c(t) \cdot \bar{c}(0)c(0) \rangle & \langle \bar{c}(t)c(t) \cdot G(0) \rangle \\ \langle G(t) \cdot \bar{c}(0)c(0) \rangle & \langle G(t) \cdot G(0) \rangle \end{pmatrix}$$

$\langle \bar{c}(t)c(t) \cdot G(0) \rangle \neq 0$ implies **mixing**.
$\rightarrow$ Optimal operator includes mesonic and gluonic components.



We include multiple profiles and gluonic operators. Ground state is mostly gluonic, first excitation is mostly mesonic (χ_{c0}).

Conclusions and Outlook

Charmonium spectroscopy with optimal distillation profiles:

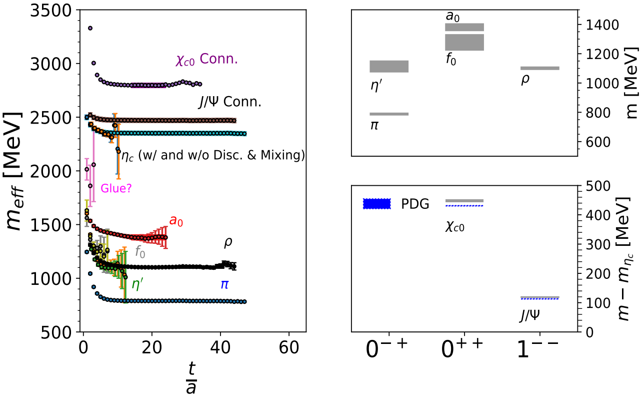
- ✓ **Good agreement** with nature.
- ✓ Significant **improvement** over distillation and other stochastic methods: excited-state contamination, precision, etc...
- ✓ **Facilitates** including disconnected contributions, i.e possible light decays/mixing.
- ✓ **Facilitates** a first study of charmonium-glueball mixing in a simplified setup.

Work in progress within FOR5269:

- ? Better operators for $\bar{c}c$, glueballs, **multi-particle** states, **static-light** mesons, **static** potentials, ...
- ? Better methods to tackle SNR problem, e.g multi-level updates for quenched QCD in 2312.11372.
- ? Better methods to solve $Dx = b$.

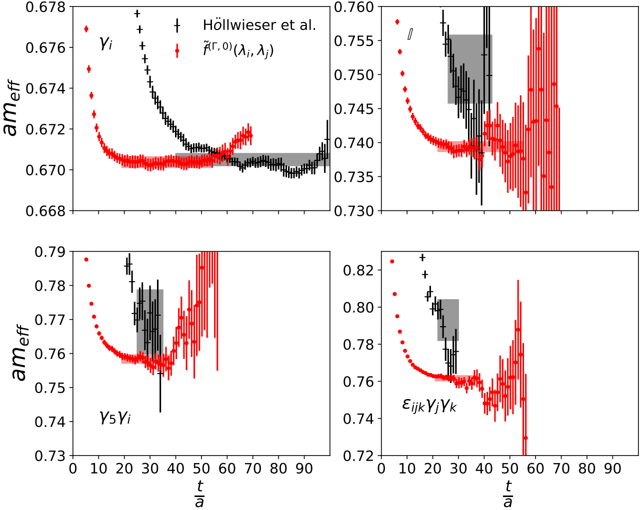
Outlook: Including the light quarks

$N_f = 3 + 1$ at SU(3) flavor-symmetric point + 800 MeV pions



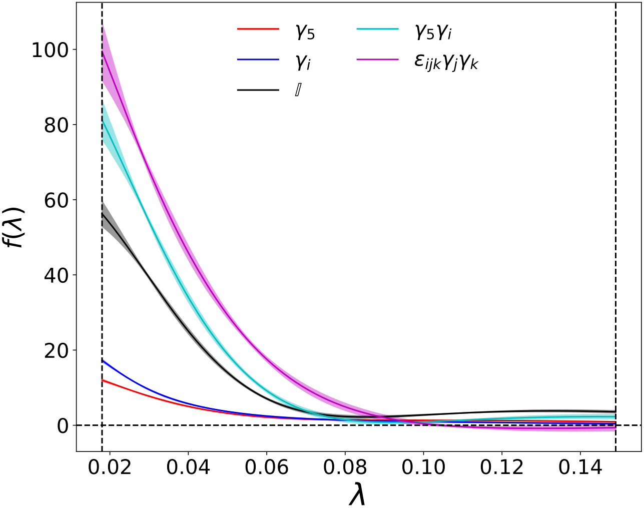
2 (heavy) pion threshold for scalar glueball ≈ 1.6 GeV.
Need 2-particle operators! (See 2312.16740 for other details.)

Thank you for your attention!

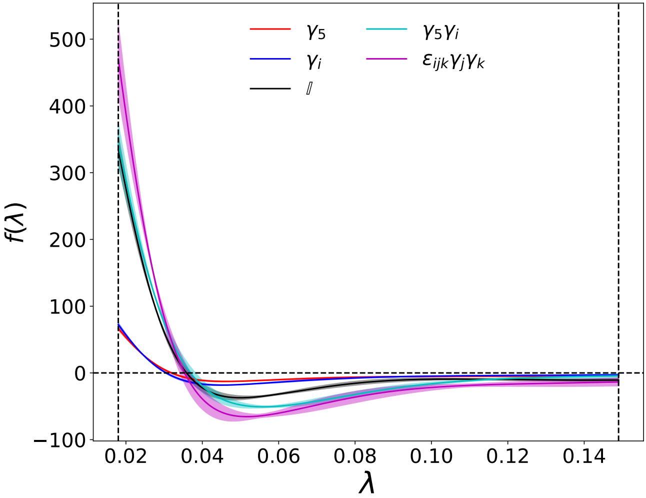


Significant improvement on previous study of the same setup.

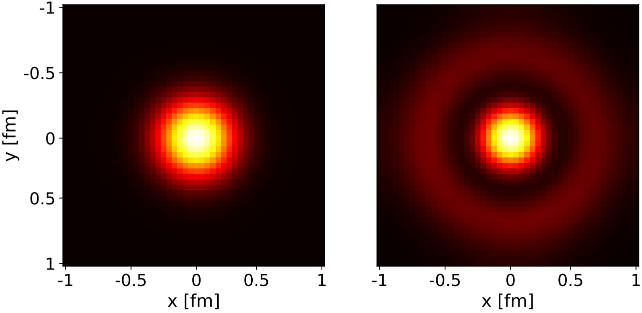
[R. Höllwieser *et al.* (2020)]



Optimal meson distillation profiles of ground state of local Γ operators.



Optimal meson distillation profiles of first excitation of local Γ operators.



Spatial profile for $\Gamma = \gamma_5\ (0^{-+})$

- S-wave behavior.
- Node-like structure in first excitation.
- Lattice size provides high resolution.
- Finite-volume effects under control.