# Why Quarkonium Hybrid Coupling to Two S-Wave Heavy-Light Mesons is Not Suppressed

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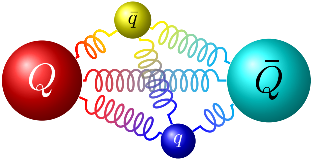
# Born-Oppenheimer Approximation for QCD

K.J. Juge, J. Kuti and C.J. Morningstar 1999

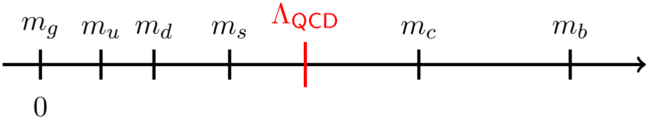



### Heavy degrees of freedom

- heavy quarks

### Light-QCD fields

- gluons
- light quarks



# The Born-Oppenheimer Hamiltonian

## Expansion in powers of $1/m$

$$H_{\text{BO}}(\vec{r},\vec{p}) = H_{\text{static}}(\vec{r}) + \frac{p^2}{m} + \ldots$$

## Leading order ($m \to \infty$): the static limit

$$H_{\text{static}}(\vec{r}) = \sum_n |\zeta_n(\vec{r})\rangle\, V_n(r)\, \langle\zeta_n(\vec{r})|$$

$n$ Born-Oppenheimer quantum numbers

$V_n(r)$ energy levels of light QCD with static $Q, \bar{Q}$ at distance $r$

$|\zeta_n(\vec{r})\rangle$ eigenstates of light QCD with static $Q, \bar{Q}$ at $+\vec{r}/2, -\vec{r}/2$

# Matching with Lattice QCD

**Correlation matrix in light QCD with static $Q, \bar{Q}$ at $+\vec{r}/2, -\vec{r}/2$**

$$C_{ij}(r,\tau,\tau_0) = \langle 0|\mathcal{O}_i(\vec{r},\tau)U(\tau,\tau_0)\mathcal{O}_j^\dagger(\vec{r},\tau_0)|0\rangle$$

The correlation matrix $\mathbf{C}$ can be calculated using lattice QCD.

| QCD | quantity that is determined | B-O |
|---|---|---|
| $\mathbf{C}$ eigenvalues at large $\tau$ | static energy levels | $V_n(r)$ |
| $\mathbf{C}$ eigenvectors at large $\tau$ | mixing angles | $\vert\zeta_n(\vec{r})\rangle$ |

**Truncation to $N$ channels**

$N$ eigenvalues and eigenvectors $\rightarrow$ truncated B-O approximation

# Diabatic Born-Oppenheimer Approximation

W. Lichten 1963; F.T. Smith 1969

## Adiabatic Schrödinger equation

$$-\frac{1}{m}\left(\vec{\nabla} + \vec{\mathbf{\Pi}}(\vec{r})\right)^2 \mathbf{\Psi}(\vec{r}) + \mathbf{V}_{\text{diag}}(r)\mathbf{\Psi}(\vec{r}) = E\mathbf{\Psi}(\vec{r})$$

transitions proceed through nonadiabatic coupling matrix $\vec{\mathbf{\Pi}}(\vec{r})$

unitary transformation

## Diabatic Schrödinger equation

$$-\frac{\nabla^2}{m}\mathbf{\Psi}(\vec{r}) + \mathbf{V}(\vec{r})\mathbf{\Psi}(\vec{r}) = E\mathbf{\Psi}(\vec{r})$$

transitions proceed through diabatic potential matrix $\mathbf{V}(\vec{r})$

# From Static Quarks to Quarkonium Hybrids

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- The static energy levels with Born-Oppenheimer quantum numbers $\eta, \lambda$ are the eigenvalues of a matrix $\mathbf{G}^{\eta,\lambda}(r)$ that solely depends on the distance $r$ between $Q$ and $\bar{Q}$.

- The diabatic potential matrix that depends on the relative position $\vec{r}$ of $Q$ and $\bar{Q}$ is a linear combination of the matrices $\mathbf{G}^{\eta,\lambda}(r)$ for different values of $\lambda$,

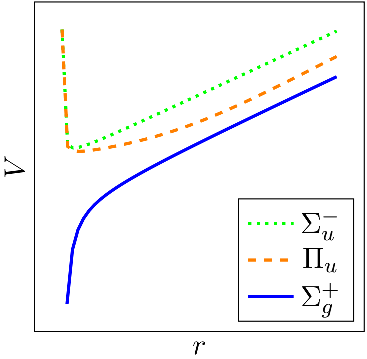
$$V^{\eta}_{i,\sigma;i',\sigma'}(\vec{r}) = \sum_{\lambda} D^{j_i}_{\sigma,\lambda}(\varphi,\theta,\psi) D^{j_{i'}}_{\sigma',\lambda}(\varphi,\theta,\psi)^* G^{\eta,\lambda}_{i,i'}(r),$$

where the angular dependence is governed by Wigner $D$-matrix elements.

# Static Energy Levels of Pure $SU(3)$ Gauge Theory

K.J. Juge, J. Kuti and C.J. Morningstar 1999

S. Capitani, O. Philipsen, C. Reisinger, C. Riehl and M. Wagner 2019




**$\Pi_u, \Sigma_u^-$: hybrid potentials**

- $r \to 0$: $1^{+-}$ gluelump
- $r \to \infty$: $N = 1, 3$ string

**$\Sigma_g^+$: quarkonium potential**

- $r \to 0$: $0^{++}$ vacuum
- $r \to \infty$: $N = 0$ string

# Diabatic Schrödinger Equation for Quarkonium Hybrids

- The quarkonium-hybrid spectrum forms degenerate multiplets of heavy-quark spin symmetry.
- The potential matrix for each hybrid multiplet is given by:
  - angular-momentum coefficients,
  - functions of $r$ calculable using lattice QCD.

**Potential matrix for $H_1$ multiplet: $J^{PC} = 1^{--}, (0,1,2)^{-+}$**

$$\begin{pmatrix} \frac{1}{3}\left[2V_{\Pi_u}(r)+V_{\Sigma_u^-}(r)\right] & \frac{\sqrt{2}}{3}\left[V_{\Pi_u}(r)-V_{\Sigma_u^-}(r)\right] & \sqrt{\frac{1}{3}}g(r) \\ \frac{\sqrt{2}}{3}\left[V_{\Pi_u}(r)-V_{\Sigma_u^-}(r)\right] & \frac{1}{3}\left[V_{\Pi_u}(r)+2V_{\Sigma_u^-}(r)\right] & -\sqrt{\frac{2}{3}}g(r) \\ \sqrt{\frac{1}{3}}g(r) & -\sqrt{\frac{2}{3}}g(r) & V_{B^{(*)}\bar{B}^{(*)}}(r) \end{pmatrix}$$

# Spectrum and Decays of Quarkonium Hybrids

## Quarkonium Hybrids

Are associated with poles of the S-matrix for heavy-meson pairs.

## S-matrix

Can be calculated nonperturbatively by solving the Schrödinger equation for coupled $Q\bar{Q}$ and heavy-meson-pair channels.

## Decay selection rules

Can be determined using Born-Oppenheimer symmetries.

# Born-Oppenheimer Selection Rules for Hybrids

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1. The Born-Oppenheimer quantum numbers are:
   - $\Pi_u$ and $\Sigma_u^-$ for quarkonium hybrids,
   - $\Sigma_g^+$, $\Pi_g$, and $\Sigma_u^-$ for pairs of S-wave heavy mesons.
2. The Born-Oppenheimer quantum numbers are conserved.
3. Decays into pairs of S-wave heavy mesons are:
   - allowed for pure $\Sigma_u^-$ or mixed $\Pi_u/\Sigma_u^-$ quarkonium hybrids,
   - forbidden for pure $\Pi_u$ quarkonium hybrids.

# Decays of Lowest Hybrids into Two S-Wave Heavy Mesons

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| | Multiplet | $J^{PC}$ | | Potential |
|---|---|---|---|---|
| allowed | $H_1$ | $1^{--}$ | $(0,1,2)^{-+}$ | $\Pi_u/\Sigma_u^-$ |
| forbidden | $H_2$ | $1^{++}$ | $(0,1,2)^{+-}$ | $\Pi_u$ |
| allowed | $H_3$ | $0^{++}$ | $1^{+-}$ | $\Sigma_u^-$ |
| | $H_4$ | $2^{++}$ | $(1,2,3)^{+-}$ | $\Pi_u/\Sigma_u^-$ |
| forbidden | $H_5$ | $2^{--}$ | $(1,2,3)^{-+}$ | $\Pi_u$ |

See talk by Chunjiang Shi on Friday about the decays of the lowest $1^{-+}$ charmoniumlike hybrid in lattice QCD.

- Quarkonium hybrids can be studied *ab initio* using the Born-Oppenheimer approximation for QCD.
- One can derive model-independent selection rules for decays into pairs of heavy mesons.
- The Born-Oppenheimer selection rules allow decays of many quarkonium hybrids into pairs of S-wave heavy mesons.
- This finding contradicts the conventional wisdom of the last 40 years from constituent models that hybrid mesons are forbidden to decay into pairs of S-wave heavy mesons.

## Born-Oppenheimer Symmetries

The static $Q\bar{Q}$ break

- rotations,
- parity,
- charge-conjugation,

down to

- cylindrical symmetries,
- combined $CP$ symmetry.

The quantum numbers are not

- $J$ angular momentum,
- $P$ parity,
- $C$ charge-conjugation,

but rather

- $\lambda$ angular momentum projection on the $Q\bar{Q}$ axis,
- $\eta$ ($g$ or $u$) $CP = +$ or $-$.

**Heavy-quark spin symmetry**

Static energy levels are independent of the heavy-quark spins.