Quarkonium potentials, hybrid spin-dependent potentials and quarkonium-hybrid mixing

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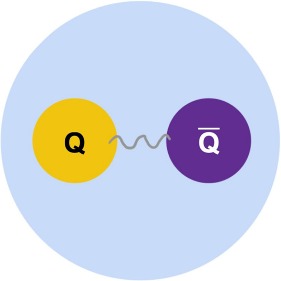
Potential for ordinary quarkonium

$$V_{Q\bar{Q}}(r) = V^{(0)}_{Q\bar{Q}}(r) + \frac{1}{m_Q} V^{(1)}_{Q\bar{Q}}(r) + \frac{1}{m_Q^2}\left(V^{(2)}_{Q\bar{Q},\mathrm{SD}}(r) + V^{(2)}_{Q\bar{Q},\mathrm{SI}}(r)\right) + \mathcal{O}(m_Q^{-3})$$

[Eichten, Feinberg, 1981], [Barchielli et al. 1988], [Pineda, Vairo, 2001], [Brambilla, 2022]

- Corrections known up to N^3LO in perturbation theory, but computations in lattice QCD face difficulties. [Buchmüller, Ng, Tye, 1981], [Peset, Pineda, Stahlhofen, 2016], [Bali, Schilling, Wachter, 1997], [Koma, Koma, 2007]
- **Difficulties** UV-noise, renormalization and matching can be adressed with gradient flow. [Lüscher, 2010], [Brambilla, Leino, Mayer-Steudte, Vairo, 2023], [Brambilla, Wang, 2023]
- **Analogous** expressions recently derived for spin-dependent hybrid potentials. [Brambilla, Lai, Segovia, Castellà, 2020], [Soto, Castellà, 2021], [Soto, Valls, 2023]



Potentials in terms of generalized Wilson loops

Potentials are computed using correlators $\langle\langle\ldots\rangle\rangle = \langle\ldots\rangle_W / \langle 1\rangle_W$.

$$V_{p^2} = \frac{1}{2}\left\{\boldsymbol{p}^2, \left(\mathcal{I}_2(E_z(t,0)E_z(0,0)) + \mathcal{I}_2(E_z(t,r)E_z(0,0))\right)\right\},$$

$$V_{LS} = \epsilon_{ijz}\frac{c_F(\mu)}{2r}\left(2\mathcal{I}_1(B_i(t,0)E_j(0,0)) + \mathcal{I}_1(B_i(t,r)E_j(0,0))\right)\boldsymbol{LS},$$

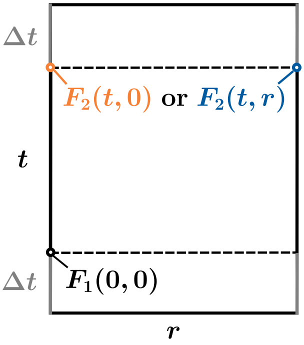
$$V_{S^2} = \frac{2c_F^2(\mu)}{3}\sum_i \left(\mathcal{I}_0(B_i(t,r)B_i(0,0))\right)(\boldsymbol{S}_1\boldsymbol{S}_2), \ \ldots$$

where

$$\mathcal{I}_n(F_2(t,r_2)F_1(0,r_1)) = \lim_{T\to\infty}\int_0^T dt\ t^n\ \langle\langle g^2 F_2(t,r_2)F_1(0,r_1)\rangle\rangle_{(c)}$$

$\boldsymbol{r} = (0,0,r)$; $c_F(\mu)$: Matching coefficient for B-insertions;
$n = 0, 1, 2.$

Generalized Wilson loop $\langle F_2(t)F_1(0)\rangle_W$:



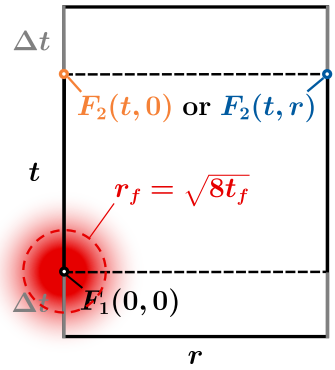
Gradient Flow

- Lattice-$F_{\mu\nu}$ requires renormalization, e.g. clover definition $F_{\mu\nu} = (\Pi_{\mu\nu} - \Pi^\dagger_{\mu\nu})/2$ via $\bar{F}_{\mu\nu} = (\Pi_{\mu\nu} + \Pi^\dagger_{\mu\nu})/2$. [Huntley, Michael, 1986]
- Flow equation:

$$\dot{B}_\mu = D_\nu G_{\mu\nu}, \qquad B_\mu|_{t=0} = A_\mu,$$
$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu], \quad D_\mu = \partial_\mu + [B_\mu, \cdot]$$

- Fields at flow time $t_f > 0$ are **smooth** and **renormalized**. [Lüscher, 2010], [Suzuki, 2013], [Shindler, 2023]
- Related flow radius $r_f = \sqrt{8t_f}$ acts as regulator, makes **continuum extrapolation** possible. [Brambilla, Wang, 2023]
- $\sqrt{8t_f}$ must not be too large, to prevent unpredictable effects.

[Eller, Moore, 2018], [Brambilla, Leino, Mayer-Steudte, Vairo, 2023]



Lattice setup

Ensembles:

Ensemble name	β	TL^3/a^4	a [fm]	$N_{\mathrm{Conf.}}$
A	6.284	$48 \cdot 24^3$	0.06	10000
B	6.451	$60 \cdot 30^3$	0.048	4400
C	6.594	$72 \cdot 36^3$	0.04	2200

- Generation of ensembles: `CL2QCD` [Sciarra et al., 2021]
- Error propagation and fitting: `pyerrors` [Joswig, Kuberski, Kuhlmann, Neuendorf, 2024]
- Monte Carlo integration of lattice propagator: `vegas` [Lepage, 2023]
- Flow times were set to $\sqrt{8t_f}/a = 0.6, 0.8, 0.96, 1.12$ for Ensemble C and held fixed in physical units for Ensembles A and B.

Static potential

Michael Eichberg

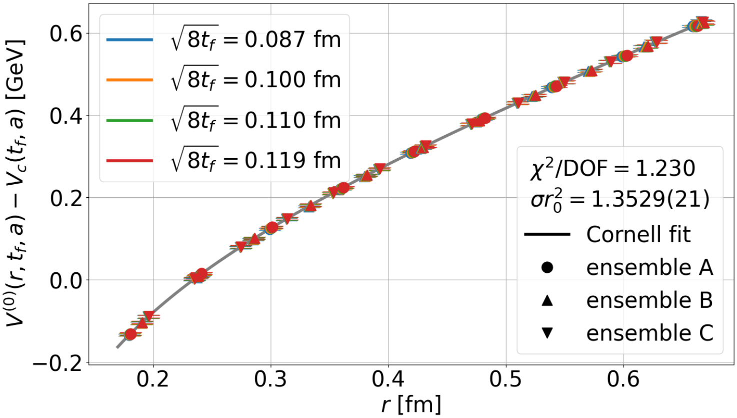


Figure: Static potential from all ensembles. Mass shifts V_c, as well as a- and t_f-dependence at tree level, have been removed. Physical scale is set using $r_0 = 0.5$ fm.

Relativistic corrections

Michael Eichberg

[Bali, Schilling, Wachter, 1997], [Brambilla, Groher, Martinez, Vairo, 2014]

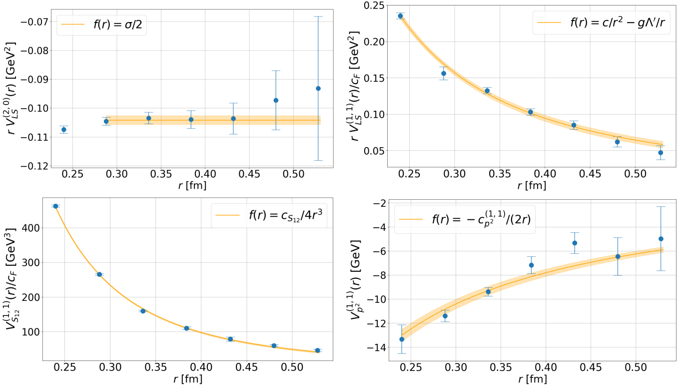


Figure: $V_{LS}^{(2,0)}$, $V_{LS}^{(1,1)}$, $V_{S_{12}}^{(1,1)}$ and $V_{p^2}^{(1,1)}$ from ensemble B at $\sqrt{8t_f} \approx 0.1$ fm.

Required steps:

- Compute correlators from lattice ensembles.
- Fit ansatz $\langle\langle F_2(t) F_1(0) \rangle\rangle \sim \sum_k D_{12}^{(k)} e^{-(E_k - E_{\Sigma_g^+})t}$ for every set of (a, t_f, r) and determine t-integral analytically.
- Determine suitable fit range, such that $\sigma_{\mathrm{stat.}}$ is small, but flow effects are not too large.
- Correlators at e.g. $t \lesssim 2\sqrt{8t_f}$ are the most precise, but have to be omitted!

Gromes and BBMP relations

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[Gromes, 1984], [Barchielli, Montaldi, Prosperi, 1986]

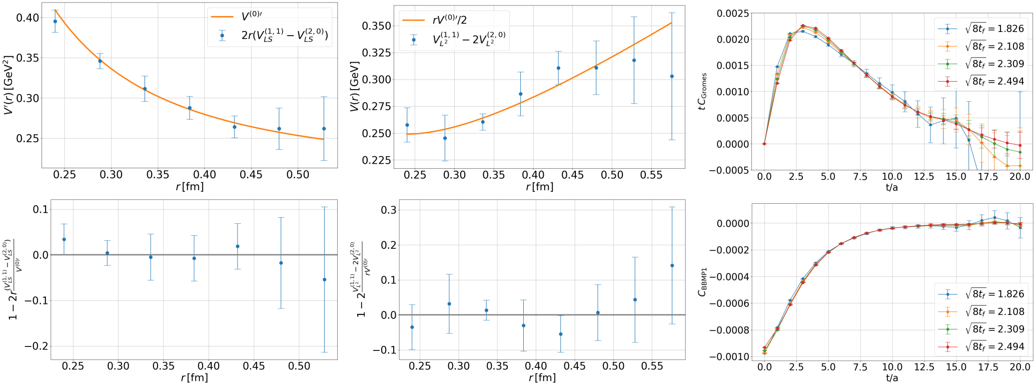


Figure: Gromes (left) and first BBMP relation (center) at $\sqrt{8t_f} \approx 0.1$ fm, as well as respective correlators (right), all from ensemble B ($a \approx 0.048$ fm).

Conclusions I

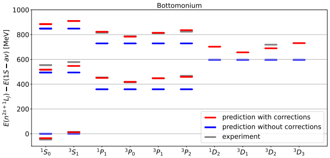
Summary

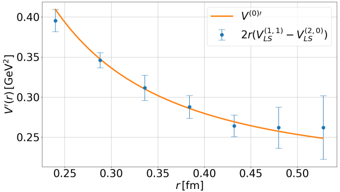
- $\mathcal{O}(1/m_Q^2)$-corrections to the static $Q\bar{Q}$-potential have been computed at various t_fs for distances up to ~ 0.5 fm.
- Gradient flow provides noise reduction and renormalization.
- Results in agreement with Gromes and BBMP relations.

Outlook

- Perform continuum limit for all potentials and extrapolate $t_f \to 0$ or match between GF and $\overline{ms}$.
- Determine quarkonium masses using potential results in a Schrödinger equation.

$$\left(-\frac{\Delta}{m_Q} + V_{Q\bar{Q}}\right)\psi(r) = E\psi(r)$$





NRQCD-BOEFT for hybrid mesons

Carolin Schlosser

$$V_{\kappa^P} = V^{(0)}_{\kappa^P} + \frac{1}{m_Q}\left(V^{(1)\mathrm{SI}}_{\kappa^P} + V^{(1)\mathrm{SD}}_{\kappa^P}\right) + \mathcal{O}(m_Q^{-2}),$$

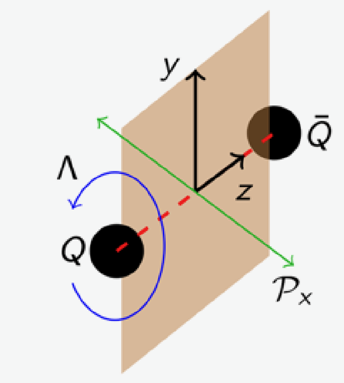
$$V^{(1)\mathrm{SD}}_{\kappa^P} = \sum_{\Lambda,\Lambda'} \mathcal{P}_{\kappa\Lambda}\left(V^{sa}_{\kappa^P\,\Lambda\Lambda'}(r)\boldsymbol{S}_{Q\bar{Q}}(\mathcal{P}_{10}\boldsymbol{S}_\kappa) + V^{sb}_{\kappa^P\,\Lambda\Lambda'}(r)\boldsymbol{S}_{Q\bar{Q}}(\mathcal{P}_{11}\boldsymbol{S}_\kappa)\right)\mathcal{P}_{\kappa\Lambda'}.$$

[Brambilla, Lai, Segovia, Castellà, 2019], [Soto, Castellà, 2020], [Soto, Valls, 2023]

- κ^P: Glueball spin and parity; Here, **lowest hybrid meson** considered ($\kappa^P = 1^+$).
- Static pot. given by linear combination $V^{(0)}_{\kappa^P} = \sum_\Lambda V_{\Lambda^\epsilon_\eta}\mathcal{P}_{\kappa\Lambda^\epsilon_\eta}$.
  [Hasenfratz, Horge, Kuti, Richard, 1980], [Perantonis, Michael, 1990]
- **Mixing** between ordinary $Q\bar{Q}$ and hybrids given by

$$V^{i,j}_{\mathrm{mix}}(\boldsymbol{r}) = \delta_{ij}V^{\mathrm{mix}}_{\Pi}(r) - (\boldsymbol{e}_r)^i(\boldsymbol{e}_r)^j\left(V^{\mathrm{mix}}_{\Sigma}(r) - V^{\mathrm{mix}}_{\Pi}(r)\right).$$

[Oncala, Soto, 2017]



1^+ spin-dependent hybrid potentials

Carolin Schlosser

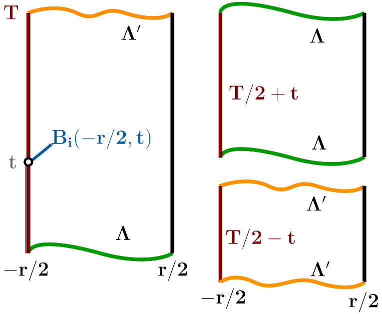
- For $T \to \infty$ spin-potenials reduce to:

$$V_{11}^{sa}(r) = -\frac{c_F}{2} \langle 0, \Pi_u^+ | B_z(-\boldsymbol{r}/2) | 0, \Pi_u^- \rangle (r),$$

$$V_{10}^{sb}(r) = -\frac{c_F}{4} \Big(\langle 0, \Pi_u^- | B_x(-\boldsymbol{r}/2) | 0, \Sigma_u^- \rangle (r) + \langle 0, \Pi_u^+ | B_y(-\boldsymbol{r}/2) | 0, \Sigma_u^- \rangle (r) \Big),$$

$$V_{\Pi_u}^{\mathrm{mix}}(r) = -\frac{c_F}{8} \Big(\langle 0, \Pi_u^+ | B_x(-\boldsymbol{r}/2) | 0, \Sigma_g^+ \rangle (r) - \langle 0, \Pi_u^- | B_y(-\boldsymbol{r}/2) | 0, \Sigma_g^+ \rangle (r) \Big),$$

$$V_{\Sigma_u^-}^{\mathrm{mix}}(r) = -\frac{c_F}{4} \langle 0, \Sigma_u^- | B_z(-\boldsymbol{r}/2) | 0, \Sigma_g^+ \rangle (r).$$



- Matrix elements in terms of generalized Wilson loops:

$$\langle 0, \lambda | B_i(-\boldsymbol{r}/2) | 0, \lambda' \rangle = \lim_{T \to \infty} \frac{W_{\lambda\lambda'}^{B_i}(t, T, r)}{\sqrt{W_\lambda(T, r) W_{\lambda'}(T, r)}} \sqrt{\frac{W_{\lambda'}(T/2 - t, r) W_\lambda(T/2 + t, r)}{W_\lambda(T/2 - t, r) W_{\lambda'}(T/2 + t, r)}}$$

Hybrid B-field matrix elements

Carolin Schlosser, Preliminary

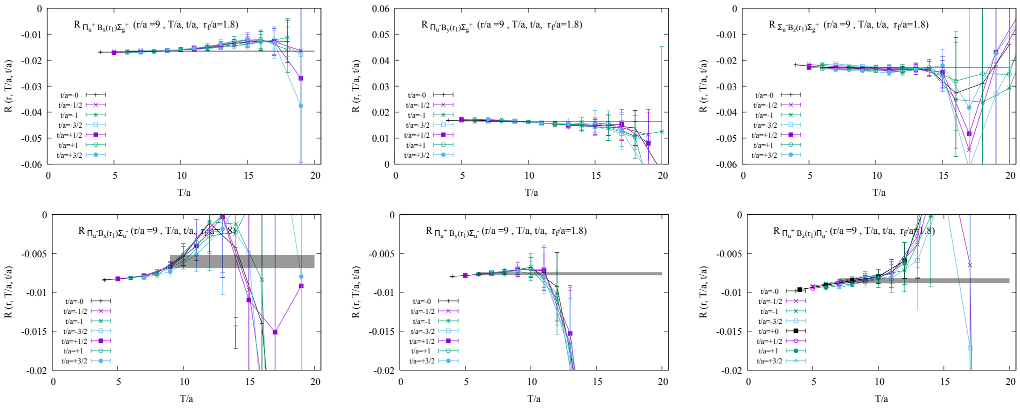


Figure: Plateau fits for hybrid B-field matrix elements at finite lattice spacing and finite flow time for various positions t of the insertion.

Spin-dependent hybrid potentials

Carolin Schlosser, Preliminary

$$V_{11}^{sa}(r) = \mp \frac{2c_F \pi^2 |g\Lambda'''|}{\kappa} \frac{1}{r^3} \quad V_{10}^{sb}(r) = \mp \frac{c_F g \Lambda' \pi^2}{\sqrt{\pi\kappa}} \frac{1}{r^2} \quad V_{\Pi_u}^{\text{mix}}(r) = \frac{\pi^{3/2} g \Lambda' c_F}{2\sqrt{\kappa}} \frac{1}{r^2} \quad V_{\Sigma_u^-}^{\text{mix}}(r) = \frac{\mp \pi^2 |g\Lambda'''| c_F}{\kappa} \frac{1}{r^3}$$

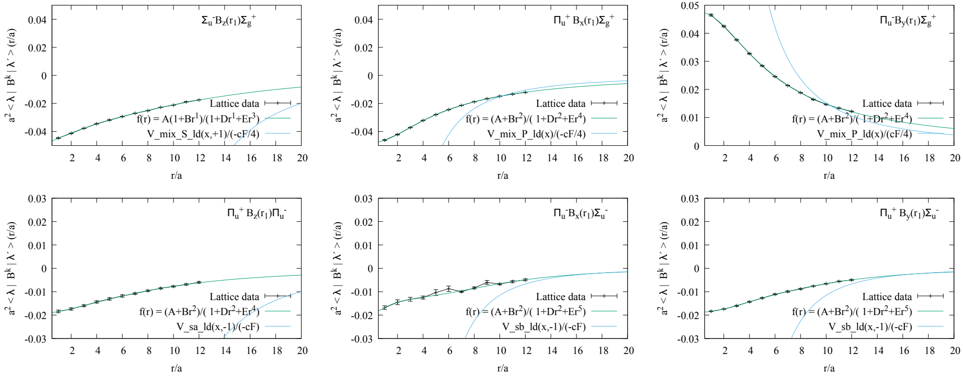


Figure: Lattice results for hybrid spin-potentials from ensemble A ($a \approx 0.06$ fm) at $\sqrt{8t_f} \approx 0.108$ fm with analytical long distance predictions from [Oncala, Soto, 2017].

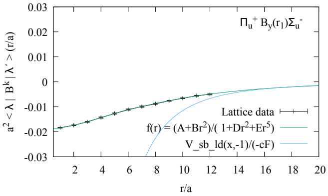
Conclusions II

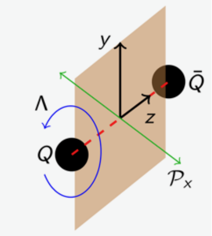
Summary

- $\mathcal{O}(1/m_Q)$ hybrid spin dependent and mixing corrections potentials have been computed at finite lattice spacing and t_f for distances up to ~ 0.7 fm.
- Gradient flow provides noise reduction and renormalization.
- Analytical prediction for asymptotic long distance behavior in coarse agreement with first preliminary lattice results.

Outlook

- Compute potentials at various lattice spacings and flow times and extrapolate $a \to 0$, $t_f \to 0$.
- Find suitable parametrizations for potentials.
- Once finalized, use results to determine hybrid meson spectra in the BO-approximation.





Thank you for your attention!

Tree level improvement of $V^{(0)}$

Fit ansatz: [Schlosser, Wagner, 2021]

$$V^{(0)}_{\mathrm{fit}}(r,t_f,a) = -\frac{c}{r}\mathrm{erf}\left(\frac{r}{\sqrt{8t_f}}\right) + \sigma r + V_c(a,t_f) + \tilde{c}\left(\frac{\mathrm{erf}(r/\sqrt{8t_f})}{r} - 4\pi G(\boldsymbol{r},t_f)\right),$$

where $V_c(a,t_f)$ is a constant shift and $G(\boldsymbol{r},t_f)$ the lattice propagator in gradient flow:

[Brambilla, Leino, Mayer-Steudte, Vairo, 2023]

$$G(\boldsymbol{r},t_f) = \int_{[-\pi,\pi]^3} \frac{\mathrm{d}^3 p}{(2\pi)^3} e^{i\boldsymbol{pr}} \frac{\exp(-2t_f \sum_i (2\sin(p_i/2))^2)}{\sum_i (2\sin(p_i/2))^2}.$$

What is plotted is the Cornell potential and corrected data points:

$$V^{(0)}(r,t_f=0) = -\frac{c}{r} + \sigma r,$$

$$V^{(0)}_{\mathrm{lat,corr.}}(r,t_f,a) = V^{(0)}_{\mathrm{lat,meas.}}(r,t_f,a) - V_c(a,t_f) - \tilde{c}\left(\frac{\mathrm{erf}\left(\frac{r}{\sqrt{8t_f}}\right)}{r} - 4\pi G(\boldsymbol{r},t_f)\right) - c\frac{1-\mathrm{erf}\left(\frac{r}{\sqrt{8t_f}}\right)}{r}.$$

Correlators

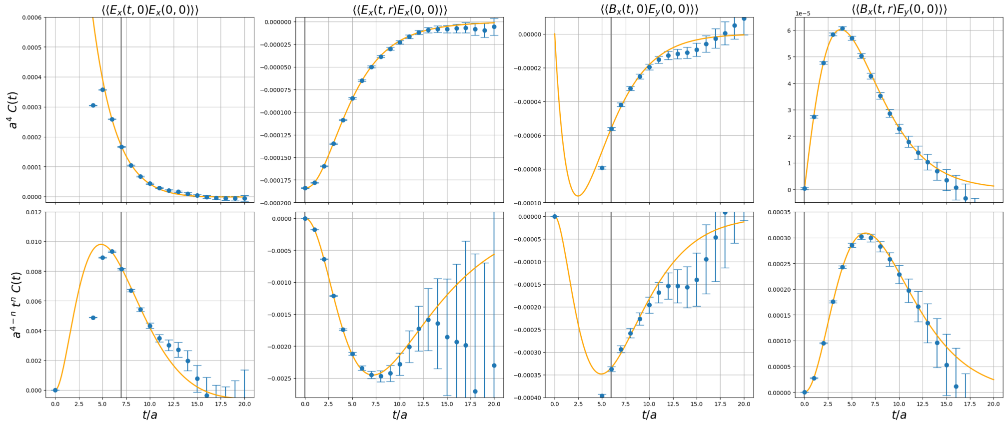


Figure: Correlators from ensemble B ($a \approx 0.048$ fm) where integral is weighted with t or t^2 for $\sqrt{8t_f} \approx 0.119$ fm. Grey lines indicate where separation between insertions surpasses $2\sqrt{8t_f}$.

Noise reduction

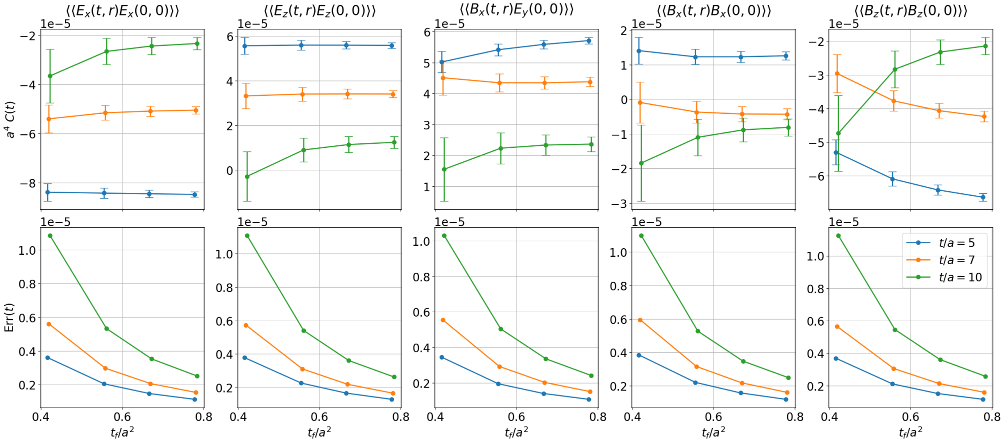


Figure: Mean values and errors for different $\langle\langle F_2(t)F_1(0)\rangle\rangle$ from ensemble B ($a \approx 0.048$ fm) as function of t_f/a^2 for $r/a = 8$ and various t/a.