Quark mass dependence of T_{cc} using lattice QCD and lhc

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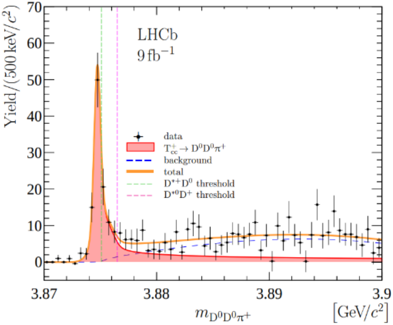
with **S. Collins**, **A. Nefediev** and **S. Prelovsek**.
Based on article **arXiv:2402.14715**

Doubly heavy tetraquarks: T_{cc}^{+}



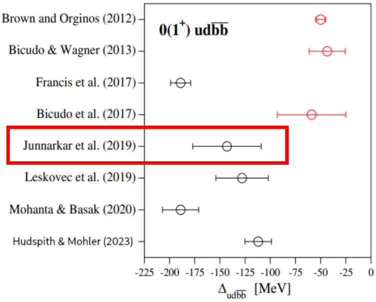
LHCb: 2109.01038, 2109.01056

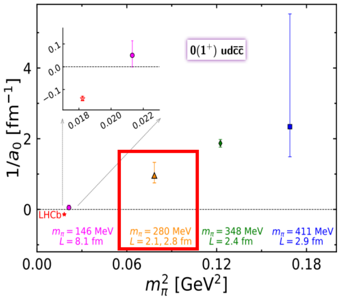
$$\delta m \equiv m_{T_{cc}^{+}} - (m_{D^{*+}} + m_{D^0})$$

$$\delta m_{\text{pole}} = -360 \pm 40^{+4}_{-0} \ \text{keV}/c^2 \,,$$
$$\Gamma_{\text{pole}} = 48 \pm 2^{+0}_{-14} \ \text{keV} \,.$$

- The doubly charmed tetraquark T_{cc}^{+}, $I = 0$ and favours $J^P = 1^+$. Nature Phys., Nature Comm. 2022
  Striking similarities with the longest known heavy exotic, X(3872).
- No features observed in $D^0 D^+ \pi^+$: possibly not $I = 1$.
- Many more exotic tetraquark candidates discovered recently, T_{cs}, $T_{c\bar{s}}$, $X(6900)$.
  Prospects also for T_{bc} in the near future. See talk by Ivan Polyakov at Hadron 2023
- Doubly heavy tetraquarks: theory proposals date back to 1980s.
  c.f. Ader&Richard PRD25(1982)2370

Doubly heavy tetraquarks using lattice QCD, T_{bb} and T_{cc}: $I(J^P) = 0(1^+)$





- Deeper binding in doubly bottom tetraquarks $\mathcal{O}(100 MeV)$. Fig: Hudspith&Mohler 2023
  Red box: ILGTI work on QQ tetraquarks: Junnarkar, Mathur, MP PRD 2019
- Shallow bound state in doubly charm tetraquarks $\mathcal{O}(100 keV)$. Fig: Lyu *et al.*PRL 2023
  Red box: T_{cc} (RQCD) [PRL 2022] and its quark mass dependence [2402.14715].
- Several recent calculations in the bottom-charm tetraquark sector.
  A summary of different lattice investigations $\rightarrow$ see review by Pedro Bicudo, 2212.07793
  see also talk by Archana Radhakrishnan earlier today.

Finite volume spectrum and infinite volume physics

- On a finite volume Euclidean lattice : Discrete energy spectrum
  Cannot constrain infinite volume scattering amplitude.

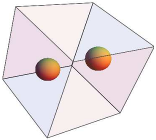
Maiani-Testa 1990

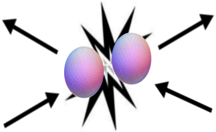
- Non-interacting two-hadron levels are given by
  $$E(L) = \sqrt{m_1^2 + \boldsymbol{p}_1^2} + \sqrt{m_2^2 + \boldsymbol{p}_2^2} \text{ where } \boldsymbol{p}_{1,2} = \frac{2\pi}{L}(n_x, n_y, n_z).$$

- Switching on the interaction: $\boldsymbol{p}_{1,2} \neq \frac{2\pi}{L}(n_x, n_y, n_z)$. *e.g.* in 1D $\boldsymbol{p}_{1,2} = \frac{2\pi}{L}n + \frac{2}{L}\delta(k)$.

- Lüscher's formalsim: **finite volume level shifts $\Leftrightarrow$ infinite volume phase shifts.**

Lüscher 1991





- Generalizations of Lüscher's formalism: *c.f.* Briceño 2014
  Quite complex problem: inelastic resonances $(R \to H_1H_2, H_3H_4)$

Scattering amplitude parametrization

- Scattering amplitude: $S = 1 + i\frac{4p}{E_{cm}}T$

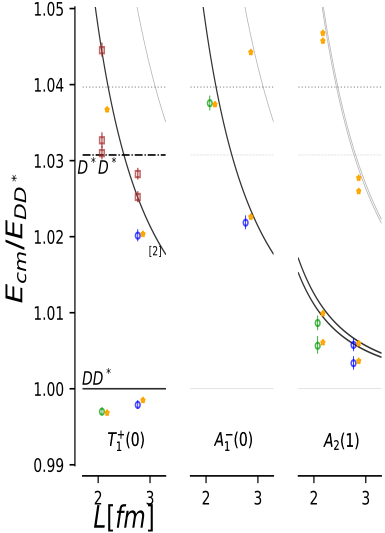
- For an elastic scattering, and assuming only S-wave,

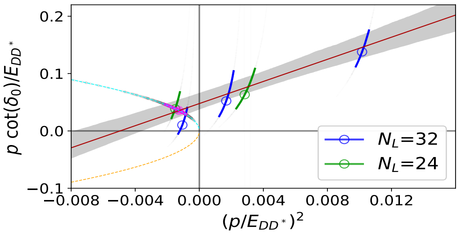
$$T^{-1} = \frac{2\tilde{K}^{-1}}{E_{cm}} - i\frac{2p}{E_{cm}}, \quad \text{with} \quad \tilde{K}^{-1} = p.cot\delta(p)$$

  (virtual/bound) state constraint below threshold: $p.cot\delta(p) = (+/-)\sqrt{-p^2}$

- Lüscher's prescription: $p.cot\delta(p) = \mathcal{F}(p)$, where $\mathcal{F}(p)$ is a known mathematical function. p^2 is determined from each extracted finite volume energy splittings.

- Parametrize $p.cot\delta(p)$ as different functions of p.
  Effective Range Expansion (ERE): $p.cot\delta(p) = a_0^{-1} + 0.5r_0p^2 + \beta_i p^{2i+4}$.
  The best fits and fit estimates determined to represent the energy dependence of the amplitude.





MP, Prelovsek PRL 2022

$m_\pi \sim 280$ MeV

- Fit quality:
  $\chi^2/d.o.f. = 3.7/5.$
- Fit parameters:
  $a_0^{(1)} = 1.04(0.29)$ fm & $r_0^{(1)} = 0.96(^{+0.18}_{-0.20})$ fm
  $a_1^{(0)} = 0.076(^{+0.008}_{-0.009})$ fm^3 & $r_1^{(0)} = 6.9(2.1)$ fm^{-1}
- Binding energy:
  $\delta m_{T_{cc}} = -9.9(^{+3.6}_{-7.2})$ MeV.
- First evaluation of the DD^* amplitude in T_{cc} channel.

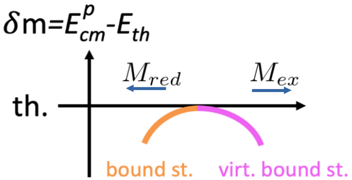
+/g refers to positive parity, -/u refers to negative parity.

Results and inferences with ERE approach

- A shallow virtual bound state pole in s-wave related to T_{cc}.

	m_D [MeV]	$\delta m_{T_{cc}}$ [MeV]	T_{cc}
lat. ($m_\pi \simeq 280$ MeV, $m_c^{(h)}$)	1927(1)	$-9.9^{+3.6}_{-7.2}$	virtual bound st.
lat. ($m_\pi \simeq 280$ MeV, $m_c^{(l)}$)	1762(1)	$-15.0(^{+4.6}_{-9.3})$	virtual bound st.
exp.	1864.85(5)	$-0.36(4)$	bound st.

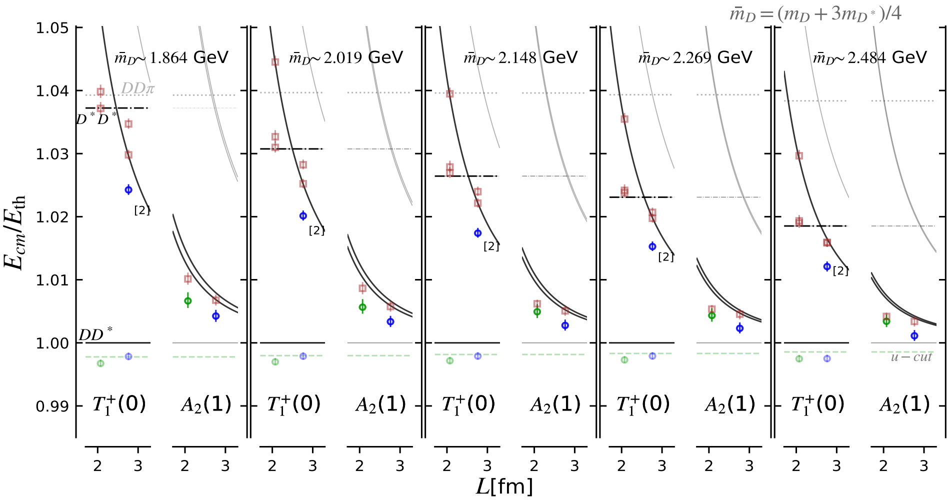
- Observations in line with the expected behaviour of a near-threshold molecular bound state pole in simple Quantum Mechanical potentials.



MP, Prelovsek PRL 2022. See a video demonstration at the end.

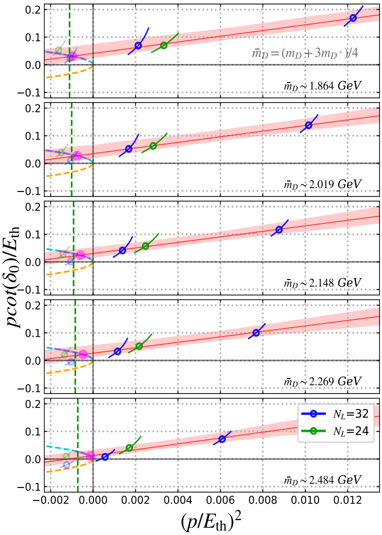
- $M_{red}(\propto m_c)$ is the reduced mass of the DD^* system.
- The mass of the particle exchanged during the interaction $M_{ex}(\propto m_{u/d})$.

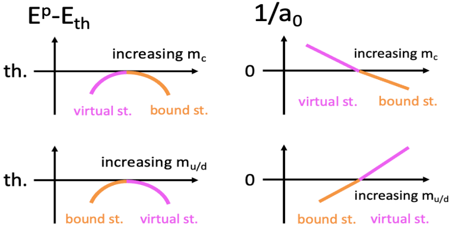
DD^* finite volume spectrum at five different m_c



Heavy quark mass dependence of the interacting DD^* spectrum in the finite volume.

m_c dependence of the T_{cc} pole [ERE]





Collins, Nefediev, MP, Prelovsek 2402:14715

$m_\pi \sim 280$ MeV

- Virtual bound poles at all values of m_c.
- m_c dependence: Purely attractive, roughly m_Q independent DD^* potential.
- Critical mass: $$\bar{m}_D^{\rm crit}({\rm ERE}) = 2.71(^{+34}_{-26})\ {\rm GeV}$$
- ERE: Questionable [OPE interactions and lhc]

Pion exchange interactions/left-hand cut: ERE and QC

- A two fold problem: (Unphysical pion masses used in lattice)

  $m_\pi > m_{D^*} - m_D \quad \Rightarrow \quad D^* \to D\pi$ is kinematicaly forbidden.

  $2 \to 2$ Generalized LQC: does not subthreshold lhc effects.

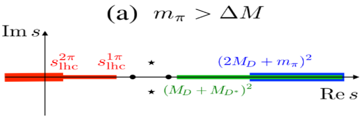
  Raposo&Hansen 2311.18793, Dawid *et al* 2303.04394, Hansen *et al* 2401.06609

  ERE convergence fails at the nearest singularity.

  Left-hand cut in the DD^* system close below the DD^* threshold.

  Du *et al* 2303.09441[PRL]

- Unphysical pion masses ($m_\pi > \Delta M = M_{D^*} - M_D$, stable D^* meson):



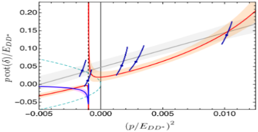


Figure taken from Du *et al* 2303.09441[PRL]

Long range pion exchange interactions: the origin of left-hand singularity and cut.
Fits with a potential that incorporates the one pion exchange:

Virtual bound states $\Rightarrow$ Virtual resonances

One-pion exchange interaction/left-hand cut

- OPE from the lowest order NR Lagrangian

$$\mathcal{L} = \frac{g_c}{2f_\pi} \boldsymbol{D}^{*\dagger} \cdot \boldsymbol{\nabla}\pi^a \tau^a D + h.c. \quad \Rightarrow \quad V_\pi(\boldsymbol{p}, \boldsymbol{p}') = 3\left(\frac{g_c}{2f_\pi}\right)^2 \frac{(\boldsymbol{\epsilon}\cdot\boldsymbol{q})(\boldsymbol{q}\cdot\boldsymbol{\epsilon}'^*)}{u - m_\pi^2}$$

Fleming *et al.* hep-ph/0703168, Hu&Mehen hep-ph/0511321

- Upon S-wave projection, we have

$$V_\pi^S(p,p) = \frac{g_c^2}{4f_\pi^2}\left[\frac{m_\pi^2 - q_0^2}{4p^2} \ln\left(1 + \frac{4p^2}{m_\pi^2 - q_0^2}\right) - 1\right]$$

Logarithmic function branch cut → infinite set of Riemann sheets

- With the finite branch point at

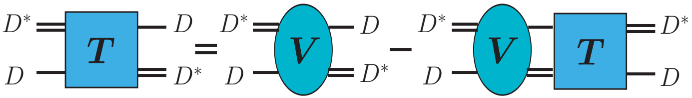
$$\boldsymbol{p_{\mathrm{lhc}}^2 = \frac{1}{4}(q_0^2 - m_\pi^2) < 0} \text{ for all lattice setups.}$$

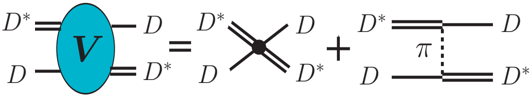
with $q_0 \simeq m_{D^*} - m_D$, where the $D^{(*)}$-meson recoil terms are ignored.

Du *et al.* 2303.09441[PRL]

- Consequences:
  - Complex phase shifts below the lhc.
  - Modified near-threshold energy dependence.

Solving Lippmann-Schwinger Equation for the DD^* amplitude





- The potential: a sum of short range and long range interactions

$$V(\boldsymbol{p}, \boldsymbol{p}') = V_{\mathrm{CT}}(p, p') + V_\pi^S(p, p') \quad \text{with} \quad V_{\mathrm{CT}}(p, p') = 2c_0 + 2c_2(p^2 + p'^2) + \mathcal{O}(p^4, p'^4)$$

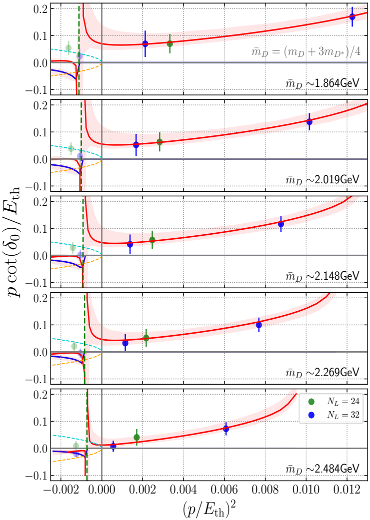
- The scattering amplitude $\qquad T^{-1} \propto p \cot \delta_0 - ip$

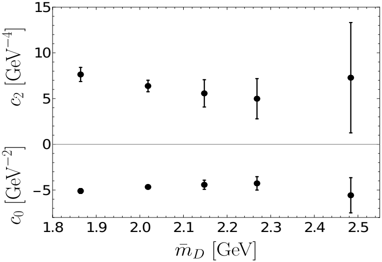
- The pion decay constant f_π and $DD^*\pi$ coupling g_c
  at $m_\pi \sim 280$ MeV following the 1-loop χPT.

Du *et al* 2303.09441[PRL]

Collins, Nefediev, MP, Prelovsek 2402:14715

m_c dependence of the T_{cc} pole [EFT]



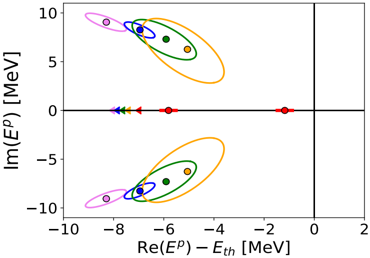


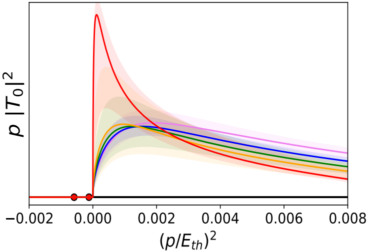
- Resonance poles below threshold at all values of m_c except the heaviest.
- At the heaviest m_c: virtual bound poles
- Counter terms: Short range attraction
  Weak long range repulsion from OPE.
- Weak m_c dependence in $V(\boldsymbol{p}, \boldsymbol{p}')$.

Collins, Nefediev, MP, Prelovsek 2402:14715

$m_\pi \sim 280$ MeV

Pole positions and scattering rate [EFT]

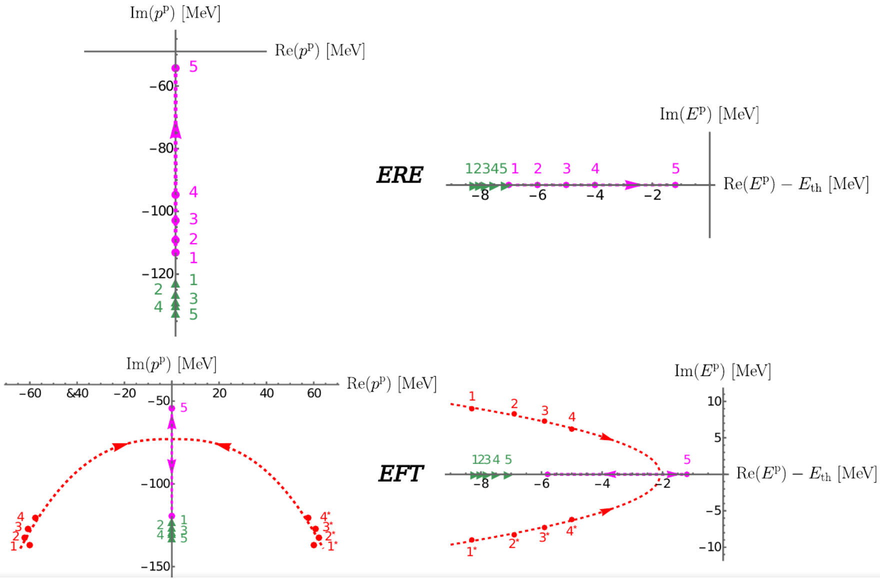




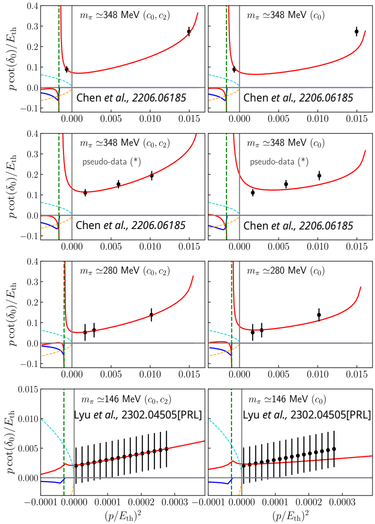
- Subthreshold resonance pole pair moving towards the real axis with increasing m_c.
- Collide on the real axis below threshold and turn back-to-back.
  At the heaviest m_c: virtual bound poles [in Red]
- With increasing m_c, subthreshold resonance poles evolves to become
  a pair of virtual bound poles.
- Enhancement in the DD^* scattering rate $(p|T_0|^2)$.

Collins, Nefediev, MP, Prelovsek 2402:14715

Pole trajectory of T_{cc}^{+}: ERE Vs EFT



m_π dependence of the T_{cc} pole [EFT]



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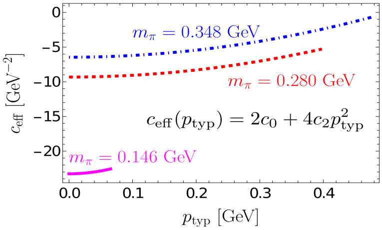
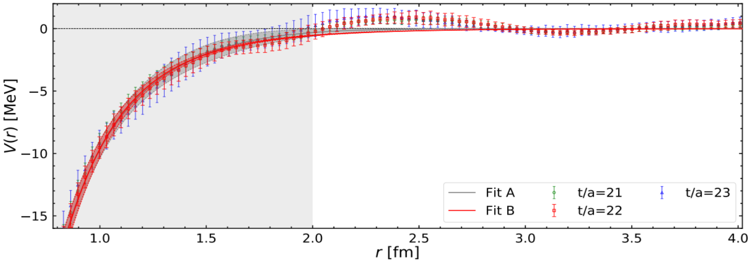
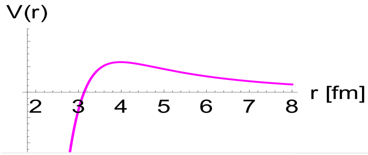


- Qualitative study of m_π dependence using $V_{\rm CT}(p,p') = 2c_0 + 2c_2(p^2 + p'^2)$
- Two parameter fit (c_0, c_2) [left] and a single parameter fit (c_0, with $c_2 = 0$) [right].
- Resonance poles at m_π ∼348 and ∼280 MeV. Shallow virtual bound poles at m_π =146 MeV.
- Stronger attraction for lighter m_π. [$c_{\rm eff}$] stronger binding in T_{cc} for lighter pions.
- m_π =146 MeV: HALQCD procedure.

The HALQCD DD^* potential and EFT fits





Regularized potential: *c.f.*

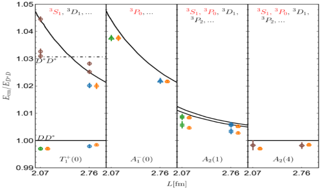
Törnqvist hep-ph/9310247,

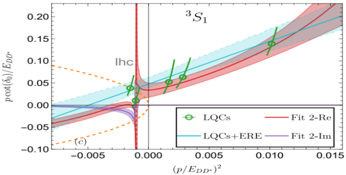
Liu *et al.*0801.3540,

Thomas&Close 0805.3653

- The DD^* potential at $m_\pi \sim 146$ MeV:
  HALQCD procedure Lyu *et al.* 2303.04505[PRL]
- EFT suggests virtual bound poles:
  Consistent with HALQCD results.
- Potential consistent with a short range attraction.
  A slight long range repulsion from OPE.
- Caution: Qualitative comparison with HALQCD results.
  DD^* phase shifts from lattice-extracted DD^* potential in coordinate space.

- An effective field theory incorporating OPE with a plane wave basis expansion.





Lu Meng *et al* arXiv:2312.01930

Virtual bound states ⇒ Virtual resonances [$m_\pi \sim$280 MeV]

- Modified 3-particle (Lüscher) Quantization Condition:

Hansen, Romero-Lopez, Sharpe, 2401.06609, Raposo, Hansen, 2311.18793

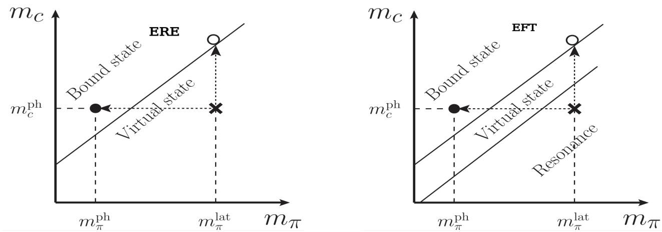
See a recent talk by Romero-Lopez here

A rigorous procedure, but demands multiple lattice inputs.
- $D\pi$ finite volume spectrum up to the $D\pi\pi$ threshold.
- Isovector DD finite volume spectrum up to the $DD\pi$ threshold.
- Isoscalar $DD\pi$ finite volume spectrum up to the $DD\pi\pi$ threshold.

Summary

- T_{cc} on the lattice, long range pion exchange interactions and left-hand cuts

- Analysis using an Effective Range Expansion and an Effective Field Theory.
  Trajectory of the T_{cc} pole.



- Either parametrizations of DD^* interactions indicate
  a possibly bound system for heavier m_c and lighter m_π.

- The binding of T_{cc}^+ observed in experiments:
  Possibly a delicate interplay between m_c and m_π.

Thank you

Quark mass dependence: a QuanMech understanding

$$R \propto M_{red} \propto 1/M_{ex}$$