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Higher order perturbative calculation of EE and κ and γ

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Contents

Motivation: Evolution of Quarkonium in QGP.

- Brief overview: Open Quantum System and pNRQCD,

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- Transport coefficients depend on correlators,

↓

- Introduce correlator of interest,

↓

- Correlators in different representations.

Quarkonia in QGP:
The underlying theory I

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Quarkonium treated in pNRQCD

$$M \gg 1/a0 \gg T \gg E$$

Evolution of quarkonium in QGP ⇨ A **Lindblad** equation

$$\frac{d\rho(t)}{dt} = -i[\mathcal{H}, \rho] + \sum_n \left(C_n \rho C_n^\dagger - \frac{1}{2}\{C_n^\dagger C_n, \rho\} \right)$$

Environment shifts hamiltonian proportional to γ,

$$\delta M = 3a_0^2\gamma$$

Collapse operators proportional to **κ**,

$$\Gamma = 3a_0^2\kappa$$

Brambilla et al. (2019)

Quarkonia in QGP: The underlying theory II

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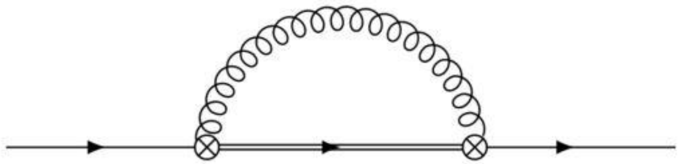
An important quantity: Singlet self-energy,



Given the hierarchy of scales T>>E transport coefficients,

$$\kappa = \frac{g^2}{6N_c} Re \int_{-\infty}^{\infty} dt \langle \mathcal{T} E_i^a(t) W^{ab}(t,0) E_i^b(0) \rangle$$

$$\gamma = \frac{g^2}{6N_c} Im \int_{-\infty}^{\infty} dt \langle \mathcal{T} E_i^a(t) W^{ab}(t,0) E_i^b(0) \rangle$$

$$= -2\alpha_s^2 T^3 \zeta(3) C_F (\frac{4}{3} N_c + N_f)$$

Brambilla et al. (2018)

Correlator of Interest

Correlator

$$\langle E_i^a(t) W^{ab}(t,0) E_i^b(0) \rangle_T$$

Wilson line in adjoint representation

Note: Correlator defined in real-time

We work in **(p)QCD** in Euclidean or Imaginary-time formalism

Analytic continuation of correlator is highly non-trivial

Motivation: Determination of κ from a euclidean correlator

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Imaginary-time Approach

Work in Euclidean or **Imaginary-time** Formalism.

Why? ⇨ Another approach to thermal QCD

⇨ Make contact with Lattice QCD!

In practice, correlators @ LO ⇨ 1-Loop calculation and

@NLO ⇨2-Loop

Imaginary-time Loop Integrals

Perturbative evaluation of correlators require loop integrals

$$\int_{-\infty}^{\infty} \frac{dp_0}{2\pi} \int_{-\infty}^{\infty} \frac{d^d\mathbf{p}}{(2\pi)^d} f(p_0, \mathbf{p})$$

where we use dimensional regularization d = 3-2ε to handle divergences

In Imaginary-time, temporal integral becomes a sum

$$\int_{-\infty}^{\infty} \frac{dp_0}{2\pi} \int_{-\infty}^{\infty} \frac{d^d\mathbf{p}}{(2\pi)^d} f(p_0, \mathbf{p}) \to T \sum_{\omega_n} \int_{-\infty}^{\infty} \frac{d^d\mathbf{p}}{(2\pi)^d} f(\omega_n, \mathbf{p})$$

where for bosons $\omega_n = 2\pi T n$ and for fermions $\omega_n = 2\pi T(n+1)$

Imaginary-time approach: Fundamental Correlator I

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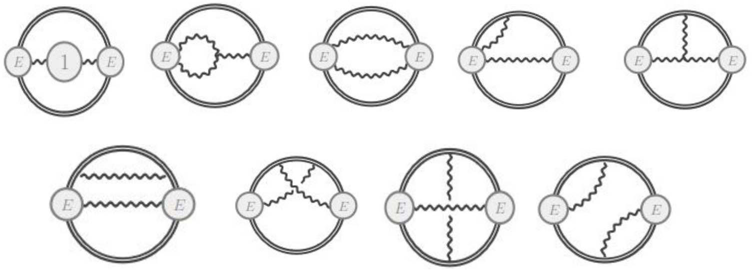
Correlator derived in Caron-Huot et al. (2009),

$$G^{HQ}(\tau) = -\frac{1}{3}\frac{\langle ReTr\,[U(\beta,\tau)gE(\tau,0)U(\tau,0)gE(0,0)]\rangle}{\langle ReTr\,[U(\beta,0)]\rangle}$$

with $U = \mathcal{P}exp\{igT^a \int_0^\beta d\tau' A_0^a(\tau')\}$

κ @ NLO has been evaluated from correlator. Diagrammatic contribution,



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Imaginary-time approach: Fundamental Correlator II

A general way of estimating **κ** from Euclidean correlator is through

$$G(\omega_n) = \int_0^\beta e^{i\omega_n \tau} G(\tau)$$

$$\rho(\omega) = Im[G(\omega_n \to -i(\omega + i\epsilon)]$$

$$\kappa = lim_{\omega \to 0} \frac{2T\rho(\omega)}{\omega}$$

Caron-Huot et al. (2009), Burnier et al. (2010), Petreczky and Teany (2006)

The euclidean evaluation of gamma was given in

$$\gamma = -\int_0^\beta d\tau G(\tau) \quad (*)$$

γ @NLO of fundamental correlator,

$$\gamma_{fund}^{LO} = -2\alpha_s^2 T^3 \zeta(3) C_F N_F$$

Eller et al. (2019)

What about κ?

Both the spectral function $\rho(\omega)$ and κ have been evaluated for,

- Adjoint correlator in real-time

Brambilla et al. (2019), Binder et al. (2022)

- Fundamental correlator in imaginary-time **and** real-time

Moore and Teany (2005), Casalderrey-Solana and Teany (2006), Caron-Huot and Moore (2008), Caron-Huot et al (2009), Burnier et al. (2010)

Conclusion:

$$\kappa^{adj} = \kappa^{fund}$$

Question: Can we derive a κ^{adj} from a euclidean correlator?

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Imaginary-time approach: One step closer

Interested in (for now) NLO adjoint correlators ⇨ 2-loop problem

Method:
Tackle higher-order loops by scalarization and reduce the complexity of integrals ⇨ IBP relations
Schröder et al. (2024), Davydychev and Schröder (2022)

Advantage: Evaluation of correlator and transport coefficients is analytically tractable
Disadvantage: Need to manually find IBPs

For the first time, evaluating EE directly (and almost analytically)

Imaginary-time approach: Symmetric Correlator

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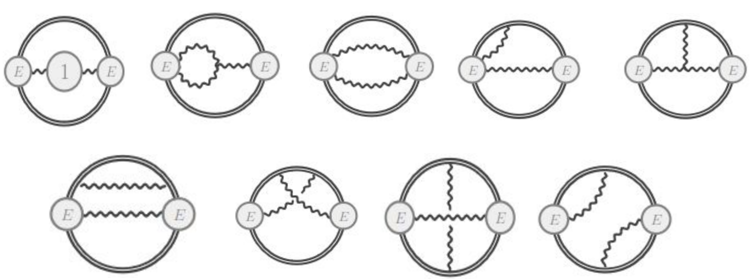
An attempt at a symmetric adjoint correlator in imaginary time

$$\langle (EW)^{ac}(EW)^{ca}\rangle = \langle f^{abl} f^{cdk} E_i^l(0) W^{bc}(0,\tau) E_i^k(\tau) W^{da}(\tau,\beta)\rangle$$

with $E^{ab} = f^{abc} E^c$

Wilson lines are in adjoint representation. @NLO



Imaginary-time approach: Further Work

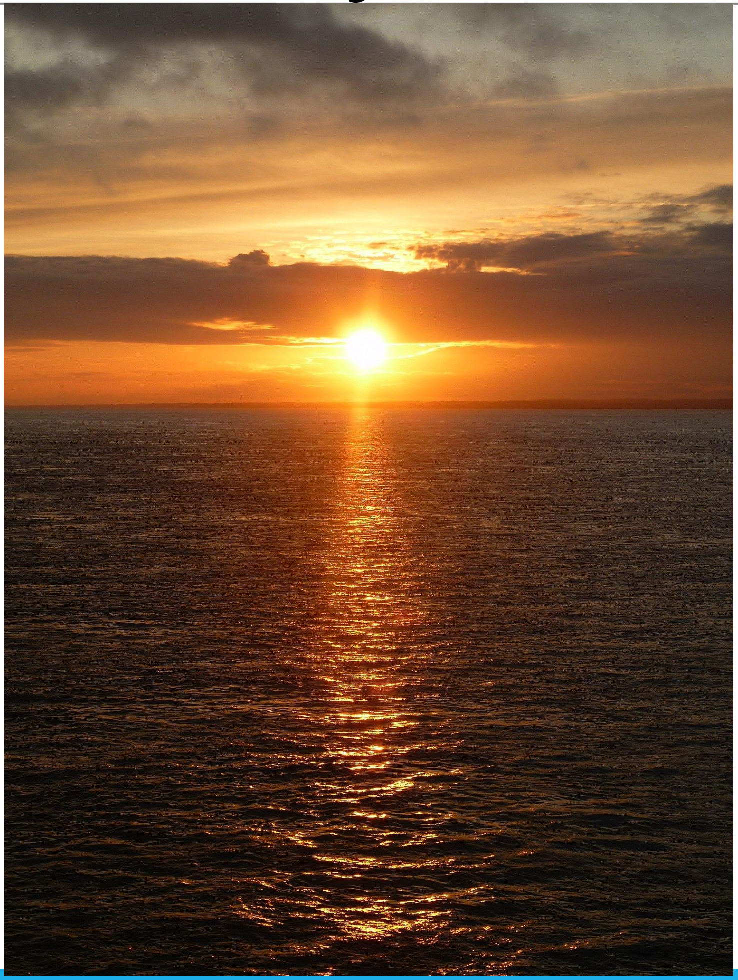
- Method provides techniques to calculate Euclidean correlators and transport coefficients
- N^2LO and beyond with easily generalisable methods?
- Compare to other works, [Scheihing-Hitschfeld and Yao (2023)]

$$\kappa_{\text{adj}} = \lim_{\omega \to 0} \frac{T}{2\omega} \left[\rho^{++}_{\text{adj}}(\omega) - \rho^{++}_{\text{adj}}(-\omega) \right]$$

$$\gamma_{\text{adj}} = -\int_0^\beta \mathrm{d}\tau \, G_{\text{adj}}(\tau) - \frac{1}{2\pi} \int_{-\infty}^{+\infty} \mathrm{d}\omega \frac{1 + 2n_B(|\omega|)}{|\omega|} \rho^{++}_{\text{adj}}(\omega)$$

- Make contact with lattice

Supplementary Material



An Example

Practically the sum integral of bosons,

$$T\int_{\mathbf{p}}\sum_{p_0=-\infty}^{\infty} = T\int_{\mathbf{p}}\sum_{p_0=-\infty}^{-1} + T\int_{\mathbf{p}}\sum_{p_0=1}^{\infty} + T\int_{\mathbf{p}}$$

Fermions have **no** zero mode(ie set p0 to zero). Given,

$$T^2\sum_{p_0}\int_{\mathbf{p}}\sum_{q_0}\int_{\mathbf{q}}\frac{e^{ip_0t}}{(p^2+p_0^2)(q^2+q_0^2)}$$

Evaluating in DimReg d = 3-2eps spatial integrals first and then sums (zero modes are zero in DimReg)

$$\frac{T^4}{8\pi}\Gamma(1-\frac{d}{2})^2 Li_{2-d}(1)(Li_{2-d}(e^{-2i\pi tT}) + Li_{2-d}(e^{+2i\pi tT}))$$

with $Li_s(z) = \sum_{n=1}^{\infty}\frac{z^n}{n^s}$

Its γ…

Must integrate over thermal circle as well.

$$T^2 \int_0^\beta dt \sum_{p_0} \int_{\mathbf{p}} \sum_{q_0} \int_{\mathbf{q}} \frac{e^{ip_0 t}}{(p^2 + p_0^2)(q^2 + q_0^2)}$$

Accounting for zero modes as well, we perform first the time-integral → spatial integral → sums(if available). We see that,

$$T^2 \int_0^\beta dt \sum_{p_0} \int_{\mathbf{p}} \sum_{q_0} \int_{\mathbf{q}} \frac{e^{ip_0 t}}{(p^2 + p_0^2)(q^2 + q_0^2)} = 0$$

Its κ…

We follow Burnier et al. (2010), start by performing Matsubara sums

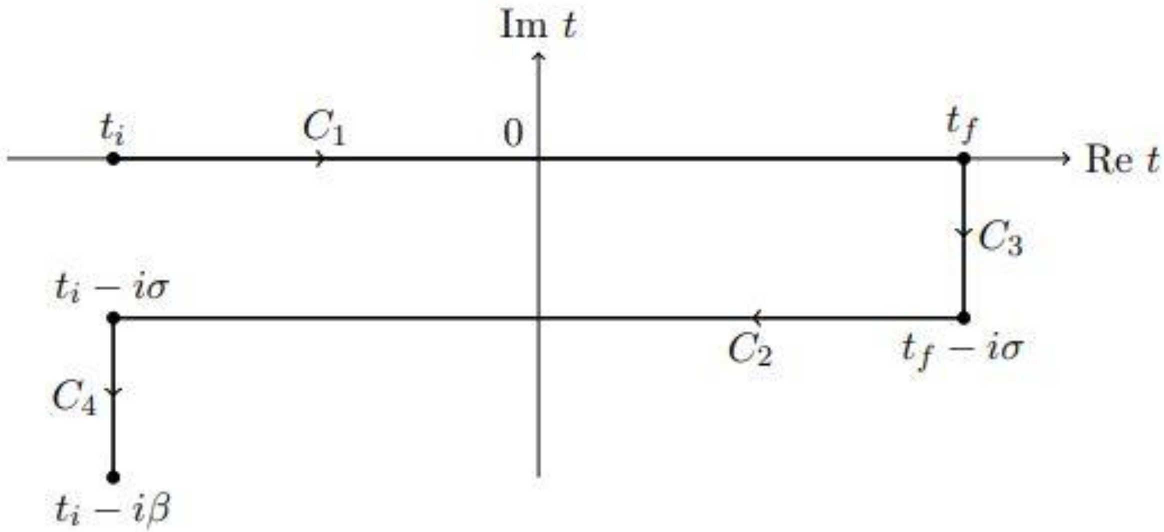
$$T\sum_{p_0} e^{ip_0\tau}\frac{1}{p_0^2+p^2} = \frac{\eta_B(p)}{2p}(e^{(\beta-\tau)p}+e^{\tau p})$$

We then Fourier transform such that $G(\omega_n) = \int_0^\beta e^{i\omega_n\tau}G(\tau)$, proceed to analytically continue discrete frequency to real-time continues frequency while only taking Imaginary part. Use

$$\kappa = lim_{\omega\to 0}\frac{2T\rho(\omega)}{\omega}$$

Finally we get a **κ**-contribution of $-\frac{T^3}{96\pi}$ which has the correct scaling of g^4T^3

Real-time Formalism



Real-time path integral, figure taken from PhD thesis of Peter Vander Griend, adapted from M. L. Bellac, Thermal Field Theory,