Complex potential at *T>0* and heavy quark diffusion coefficient in 2+1 flavor QCD

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- Introduction: spectral functions at *T>0*
- Complex potential at *T>0*
  Balla et al (HotQCD), PRD 105 (2022) 054513
  Bazavov et al (HotQCD), arXiv:2308.16587
- Heavy quark diffusion coefficient from lattice QCD

Altenkort, Kaczmarek, Larsen, Mukherjee, PP, Shu, Stendebach (HotQCD), PRL130 (2023) 231902
Altenkort, de la Cruz Kaczmarek, Larsen, Mukherjee, PP, Shu, Stendebach (HotQCD), PRL 132 (2024) 051902

QWG 2024 :The 16th International Workshop on Heavy Quarkonium, 26 February 2024 to 1 March 2024, IISER Mohali

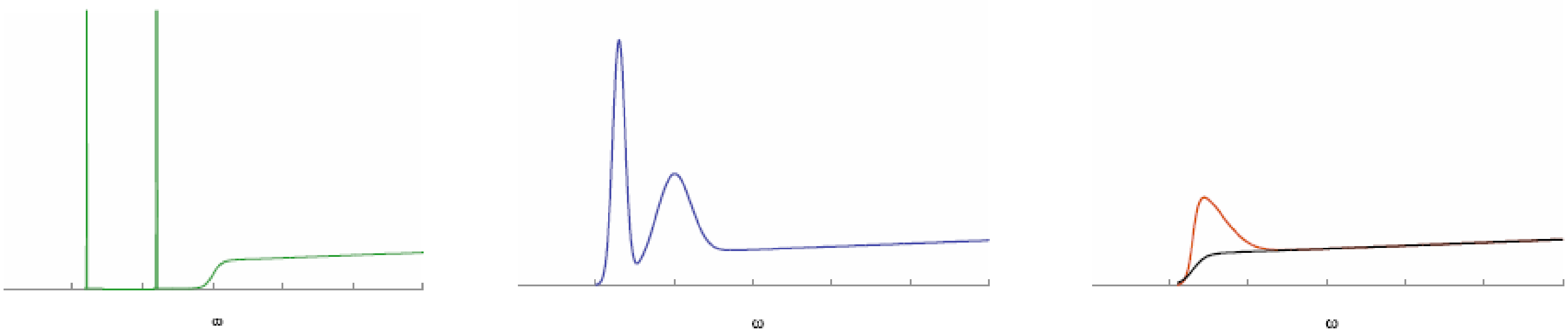
Meson correlators and spectral functions

Vacuum and in-medium properties as well as dissolution of mesons are encoded in the spectral functions:

$$\rho(\omega, p, T) = \frac{1}{2\pi}\mathrm{Im}\int_{-\infty}^{\infty} dt e^{i\omega t}\int d^3x e^{ipx}\langle[O(x,t), O(0,0)]\rangle_T, \quad O(x,t) \sim \bar{Q}(x,t)\Gamma Q(x,t)$$

Melting is seen as progressive broadening and disappearance of the bound state peaks

Modifications of quarkonium yields in heavy ion collisions   Matsui and Satz, PLB 178 (1986) 416



$$C(\tau, T) = \sum_x \langle O(x,\tau) O(0,0)\rangle_T \quad \longleftrightarrow \quad C(\tau, T) = \int_0^{\infty} d\omega \rho(\omega, T)\frac{\cosh(\omega\cdot(\tau - 1/(2T)))}{\sinh(\omega/(2T))}$$

Consider large τ behavior of $C(\tau, T=0)$:

$$C(\tau, T) \sim \sum_n |\langle 0|O|n\rangle|^2 e^{-M_n\tau} \simeq f_1 e^{-M_1\tau} + f_2 e^{-M_2\tau} + \ldots$$

$T > 0: \quad \tau < 1/T \Rightarrow \text{reconstruct } \rho(\omega, T)$

Quark anti-quark potential at T>0

Conjecture, Matsui and Satz, PLB 178 (86) 416 $-\frac{4}{3}\frac{\alpha_s}{r}+\sigma r \to -\frac{4}{3}\frac{\alpha_s}{r}e^{-m_D r}, T > T_c$

Extending pNRQCD to *T>0*: the potential is complex, the real part can have thermal correction but is not necessarily screened, except when $r \sim 1/m_D \gg 1/T$

Weak coupling calculation: Laine, Philipsen, Romatschke, Tassler, JHEP 03 (06) 054
Brambilla, Ghiglieri, PP, Vairo, PRD 78 (08) 014017

Quarkonium melting is caused by imaginary part of the potential not by screening !
Escobedo, Soto, PRA 78 (2008) 032520, Laine, NPA 820 (2009) 25C

Calculate the potential non-perturbatively on the lattice by considering Wilson loops of size *r x τ* at *T>0*

$$W(r,\tau,T) = \int_{-\infty}^{\infty} \rho_r(\omega,T) e^{-\omega\tau}$$

If potential at $T > 0$ exists the $\rho_r(\omega,T)$ should have a well define peak at $\omega \simeq \mathrm{Re}V(r,T)$, and the width of the peak is $\mathrm{Im}V(r,T)$

Rothkopf, Hatsuda, Sasaki, PRL 108 (2012) 162001

Challenge: reconstruct $\rho_r(\omega,T) \Rightarrow$ subtract the high ω part of the spectral function using $T = 0$ lattice results and model the ground state peak by a Gaussian form

Details of the lattice calculations

HISQ action, $T = 153 - 352$ MeV

$$a = 0.028 \text{ fm}, m_l = m_s/5, (m_\pi = 320 \text{ MeV}),\ 96^3 \times N_\tau, N_\tau = 56,\ 36,\ 32,\ 28,\ 24,\ 20$$

$$a = 0.040 \text{ fm}, m_l = m_s/20, (m_\pi = 160 \text{ MeV}),\ 64^3 \times N_\tau, N_\tau = 64,\ 32,\ 30,\ 28,\ 26,\ 24,\ 22,\ 20,\ 18,\ 16$$

$$a = 0.049 \text{ fm}, m_l = m_s/20, (m_\pi = 160 \text{ MeV}),\ 64^3 \times N_\tau, N_\tau = 64,\ 26,\ 24,\ 22,\ 20,\ 18,\ 16$$

Calculate correlation functions of temporal Wilson line instead of Wilson loops (better signal)

Gradient (Zeuthen) flow for noise reduction:

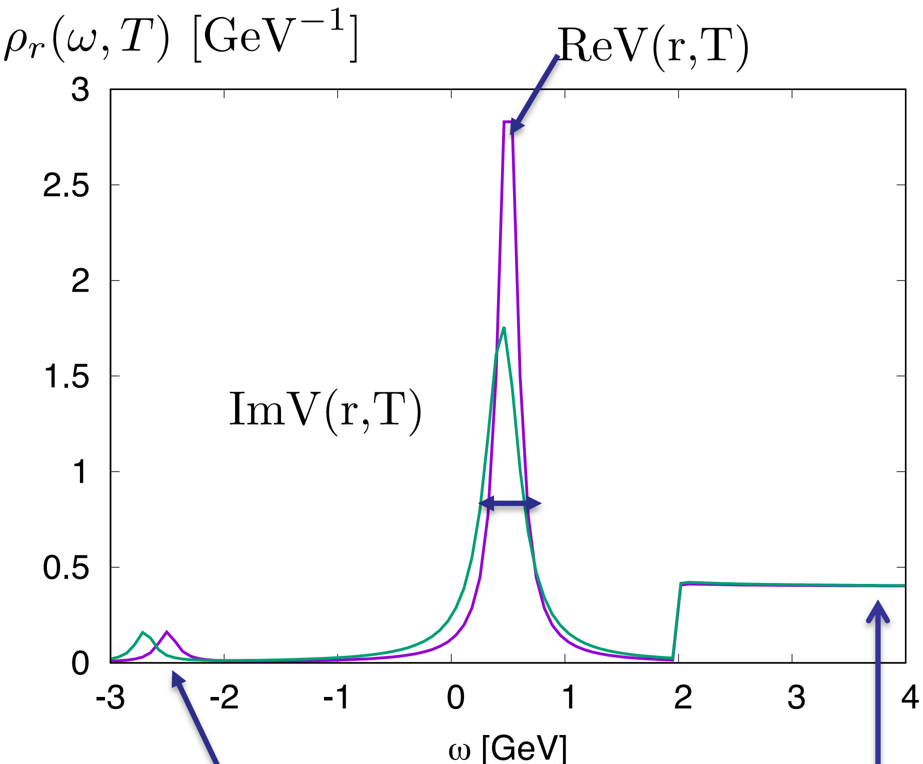
$$A_\mu(x) \to B_\mu(\tau_F, x) \qquad B_\mu(0, x) = A_\mu(x) \qquad \partial_{\tau_F} B_\mu(\tau_F, x) = -g_0^2 \frac{\delta S_{\text{YM}}[B]}{\delta B_\mu(\tau_F, x)}$$

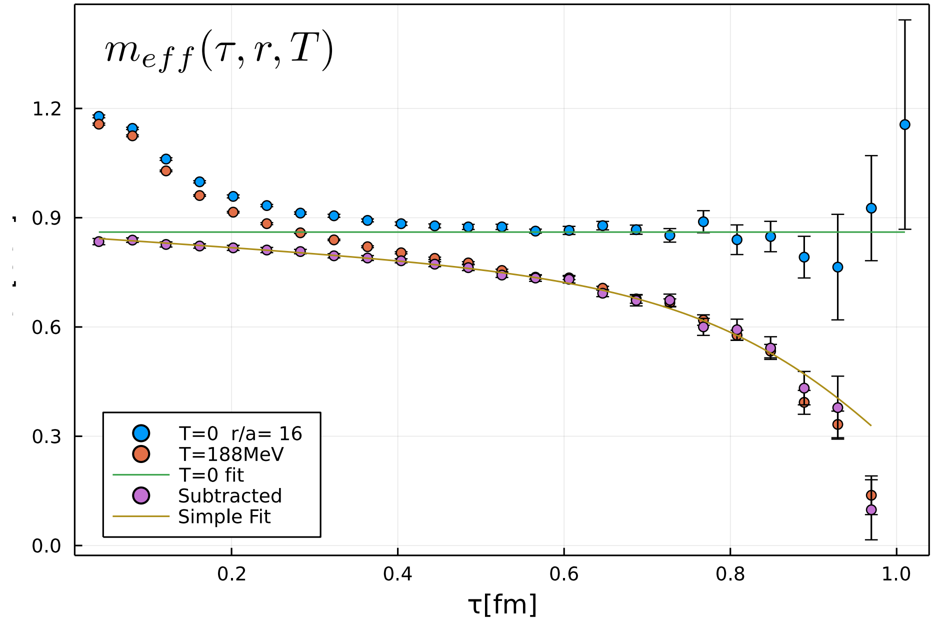
Gauge fields are smeared in the radius $\sqrt{8\tau_F}$

$$\sqrt{8\tau_F}T = 0.04 - 0.05$$

$$m_{eff}(r, \tau, T) = -\partial_\tau \log(W(\tau, r, T)) \simeq \frac{1}{a} \ln \frac{W(r, \tau, T)}{W(r, \tau + a, T)}$$

Spectral function and effective masses





$$\rho_r(\omega,T) = \rho_r^{tail}(\omega,T) + \rho_r^{peak}(\omega,T) + \rho_r^{high}(\omega)$$

See, Bala et al (HotQCD), PRD 105 (2022) 054513

$$W^{high}(r,\tau) = \int_{-\infty}^{\infty} d\omega \rho_r^{high}(\omega) e^{-\omega\tau}$$

On the lattice: $W^{high}(r,\tau) =$
$$W(r,\tau,T=0) - A_0 \exp(-V(r)\tau)$$

No plateau at $T>0$ in m_{eff} at $T>0$

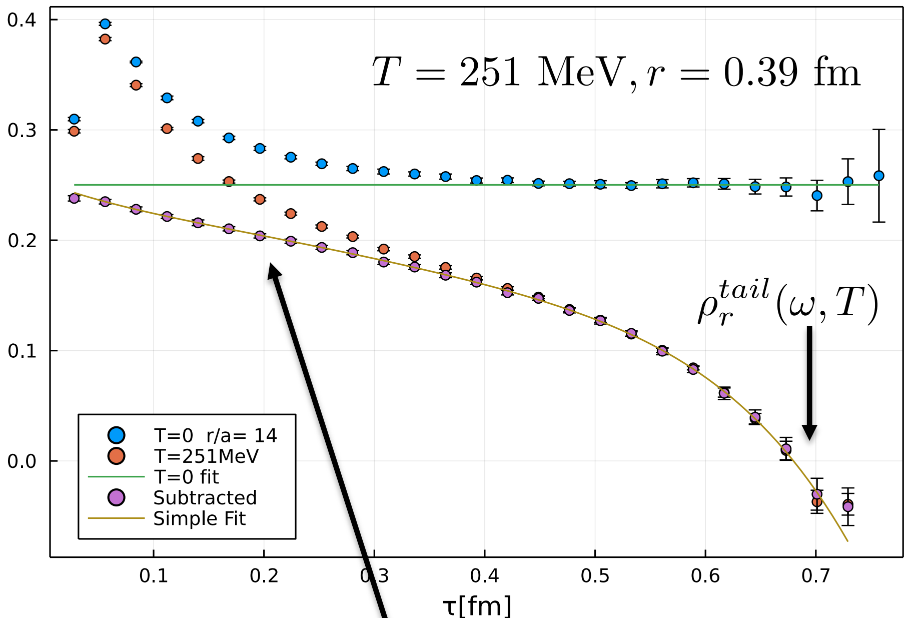
Only tiny T-dependence for small τ

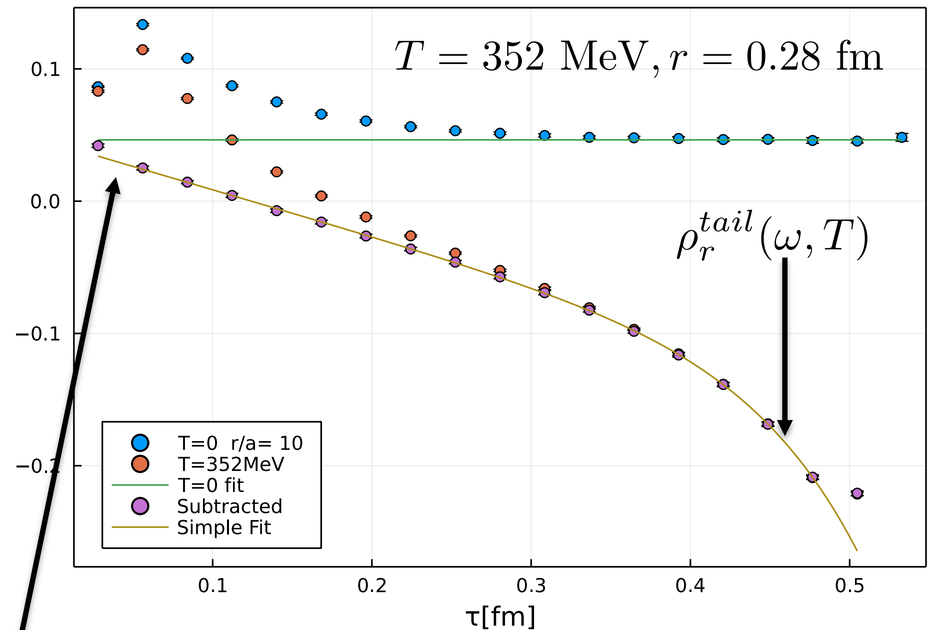
Distortions at small τ due to flow

$$W^{sub}(r,\tau,T) = W(r,\tau,T) - W^{high}(r,\tau)$$

Spectral function and effective masses

- No plateau in m_{eff} at $T > 0$ and only small T-dependence for small τ
- Distortions for small τ are largely removed by subtraction





m_{eff} for the subtracted correlator has milder τ-dependence, which is approximately linear

Thermal width

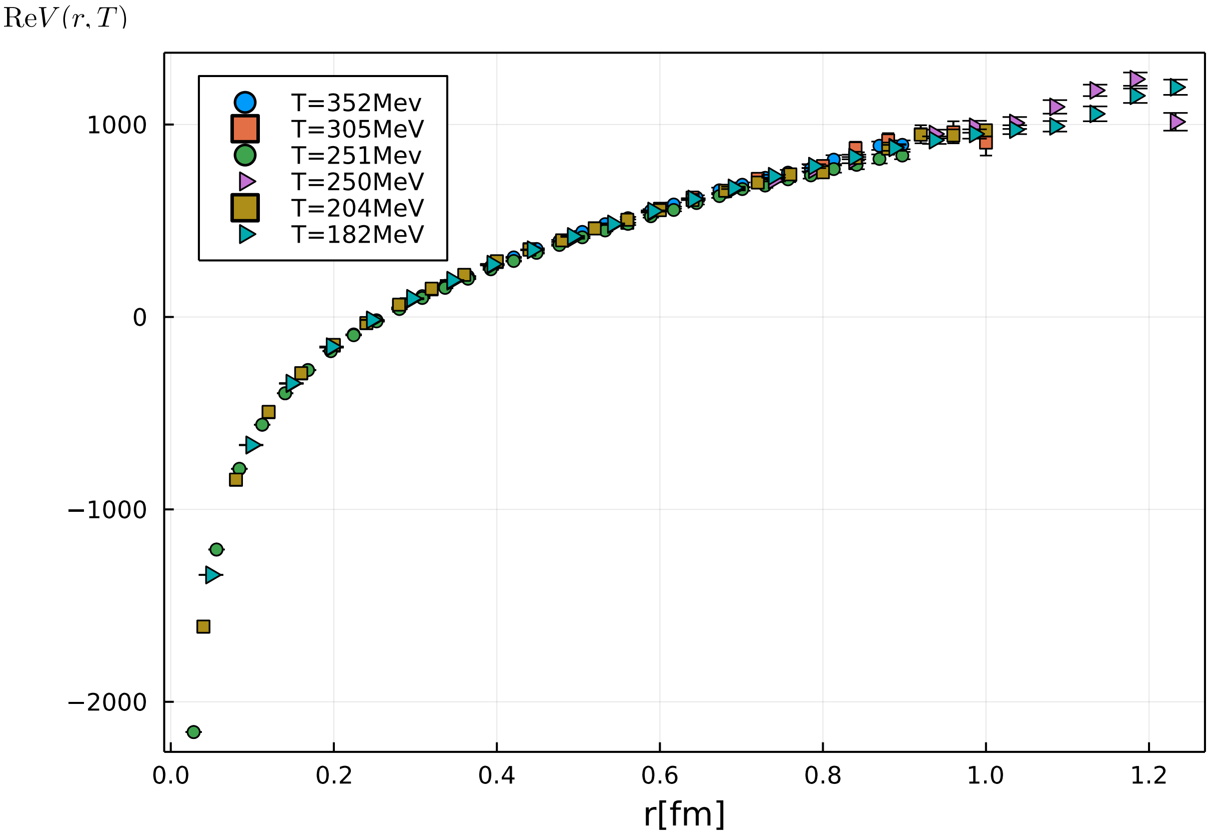
If $\rho_r(\omega, T)$ is Gaussian m_{eff} is linear in τ

$$m_{eff}(\tau \simeq 0, r, T) \simeq V(r, T = 0)$$

Model spectral function and the complex potential

$$\rho_r^{peak}(\omega,T)=\frac{A}{\pi}\frac{\Gamma(\omega,r,T)}{(\omega-\mathrm{Re}V(r,T))^2+\Gamma^2(\omega,r,T)}.\qquad \rho_r^{tail}(\omega,T)=A^{tail}\delta(\omega-E^{tail})$$

$$\Gamma(\omega,r,T)=\begin{cases}\Gamma_0(r,T) & -2\Gamma_0<\omega<2\Gamma_0\\ 0 & n \text{ otherwise}\end{cases}$$

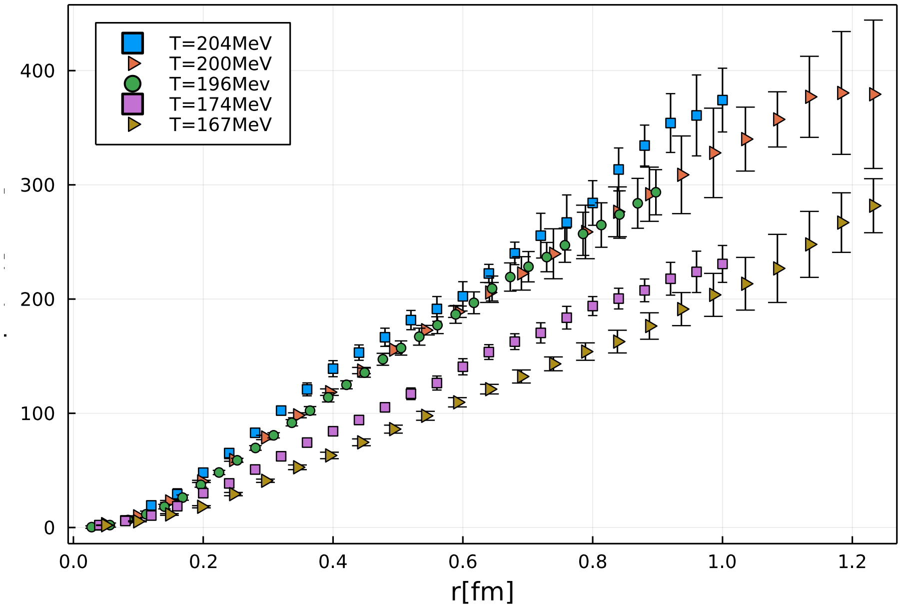


$\mathrm{Re}V(r,T)$ shows only tiny temperature dependence and no hint of screening !

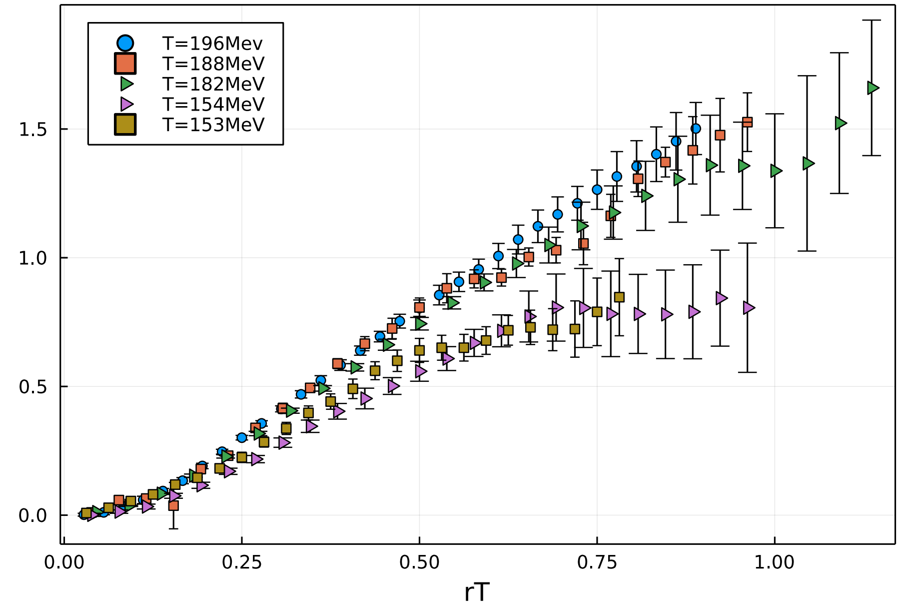
The same result if $\rho_r^{peak}(\omega,T)\sim\exp(-(\omega-\mathrm{Re}V(r,T))^2/(\mathrm{Im}V(r,T))^2)$

Imaginary part of the potential

$\mathrm{Im}V(r,T)$ [GeV]



$\mathrm{Im}V(r,T)/T$



circles: $a = 0.028$ fm, squares: $a = 0.040$ fm, triangles: $a = 0.049$ fm

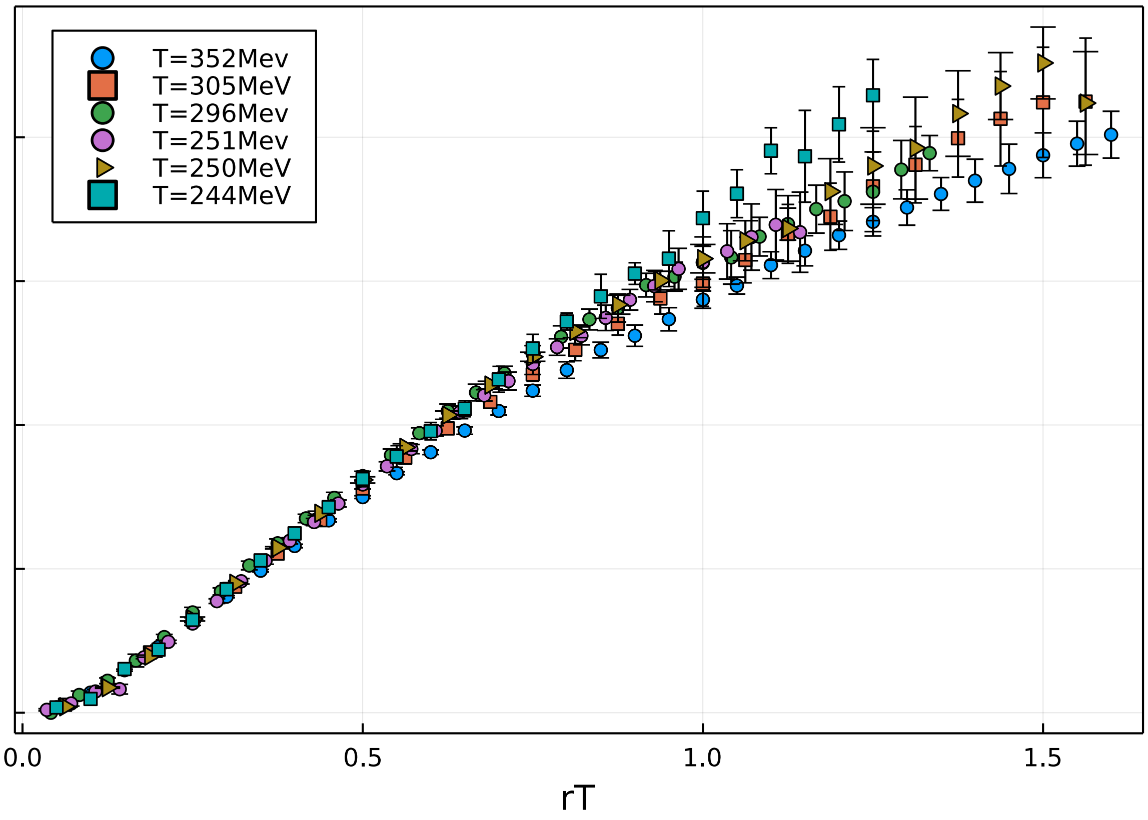
$\mathrm{Im}V(r,T)$ increases with increasing temperature and distance

No apparent quark mass effects for $T > 196$ MeV

No apparent cutoff effects

Imaginary part of the potential (cont'd)

$\mathrm{Im}V(r,T)/T$



For 244 MeV $< T <$ 352 MeV $\mathrm{Im}V(r,T)/T$ approximately scales with rT as one would expect based on weak coupling calculations

Current-current correlators and heavy quark diffusion coefficient

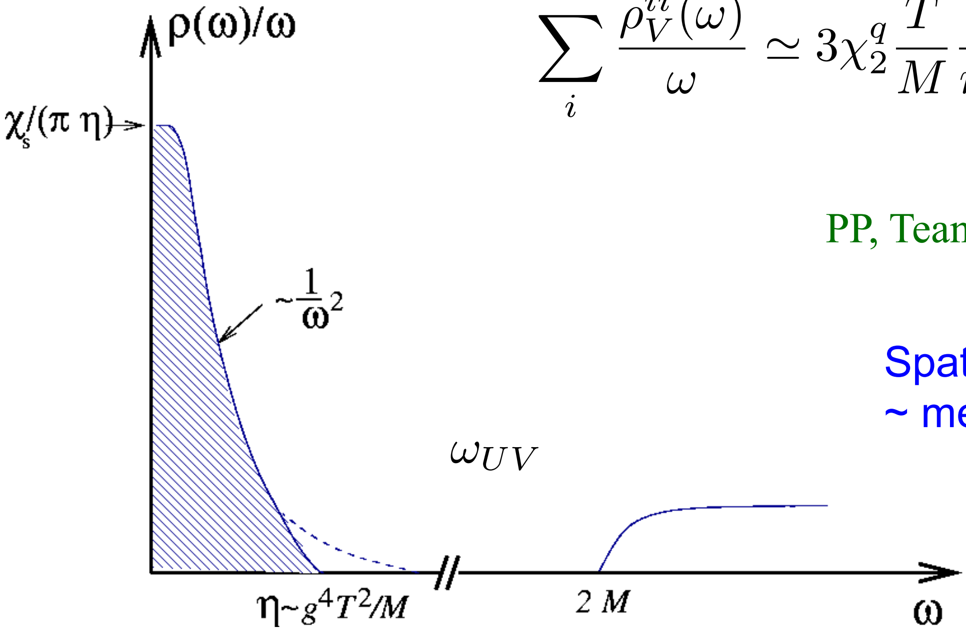
$$\rho_V^{\mu\nu}(\omega) \equiv \int_{-\infty}^{\infty} dt e^{i\omega t} \int d^3x \left\langle [\hat{J}^\mu(t,\vec{x}), \hat{J}^\nu(0,\vec{0})] \right\rangle$$

$$\partial_t p_i = -\eta p_i + f_i(t),$$

$$\langle f_i(t) f_j(t') \rangle = \kappa \delta_{ij} \delta(t - t')$$

$$\kappa = 2MT\eta = 2T^2/D_s$$

Momentum diffusion coefficient



$$\sum_i \frac{\rho_V^{ii}(\omega)}{\omega} \simeq 3\chi_2^q \frac{T}{M} \frac{\eta}{\eta^2 + \omega^2}, \quad \omega < \omega_{UV}, \quad \eta = \frac{T}{M}\frac{1}{D_s}$$

drag constant

PP, Teaney, PRD 72 (2006) 014508

Spatial diffusion constant
~ mean free path (weak coupling)

$$D_s \sim \frac{1}{g^4 T}$$

area under the peak ~ $\chi_2^q \frac{T}{M}$

heavy quark coefficient ~ width of the peak

For large quark mass the transport peak is very narrow even for strong coupling and its difficult to reconstruct it accurately from Euclidean correlator calculated on the lattice

Heavy quark diffusion and lattice QCD

Obtain the momentum heavy quark transport coefficient through the force correlator

$$\langle f_i(t) f_j(t)\rangle = \langle E_i(t) E_j(t')\rangle + \frac{1}{3}\langle \mathbf{v}^2\rangle \langle \delta_{ij} B_k(t) B_k(t') - B_i(t') B_j(t)\rangle \qquad \langle \mathbf{v}^2\rangle = \frac{3T}{M}$$

$$t \to i\tau$$

Can be rigorously derived in Heavy Quark Effective Theory

Casalderrey-Solana, Teaney, PRD 74 (2006) 085012; Caron-Huot, Laine, Moore, JHEP 0904 ('09) 053

Bouttefeux, Laine, JHEP 12 (2020) 150

$$G_E(\tau) = -\frac{1}{3}\sum_{i=1}^{3} \frac{\left\langle \mathrm{ReTr}\left[U(\beta,\tau)\, gE_i(\tau,\vec{0})\, U(\tau,0)\, gE_i(0,\vec{0})\right]\right\rangle}{\left\langle \mathrm{ReTr}[U(\beta,0)]\right\rangle} \qquad \kappa_E = \lim_{\omega\to 0}\frac{2T}{\omega}\rho_E(\omega)$$

$$G_B(\tau) = -\frac{1}{3}\sum_{i=1}^{3} \frac{\left\langle \mathrm{ReTr}\left[U(\beta,\tau)\, gB_i(\tau,\vec{0})\, U(\tau,0)\, gB_i(0,\vec{0})\right]\right\rangle}{\left\langle \mathrm{ReTr}[U(\beta,0)]\right\rangle} \qquad \kappa_B = \lim_{\omega\to 0}\frac{2T}{\omega}\rho_B(\omega)$$

$$G_{E,B}(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi}\rho_{E,B}(\omega)\frac{\cosh\left(\tau - \frac{1}{2T}\right)\omega}{\sinh\frac{\omega}{2T}}$$

$$\kappa = \kappa_E + \frac{2}{3}\langle \mathbf{v}^2\rangle \kappa_B$$

Extracting momentum diffusion coefficient from the lattice

2+1 flavor QCD with $m_l = m_s/5$ ($m_\pi = 320$ MeV), $T = 195 - 354$ MeV, $96^3 \times N_\tau$ lattices with $N_\tau = 36,\ 32,\ 28,\ 24,\ 20$; additional $64^3 \times N_\tau$ lattices with $N_\tau = 20,\ 22,\ 24 \Rightarrow 3$ lattice spacings at each T; Gradient flow for noise reduction

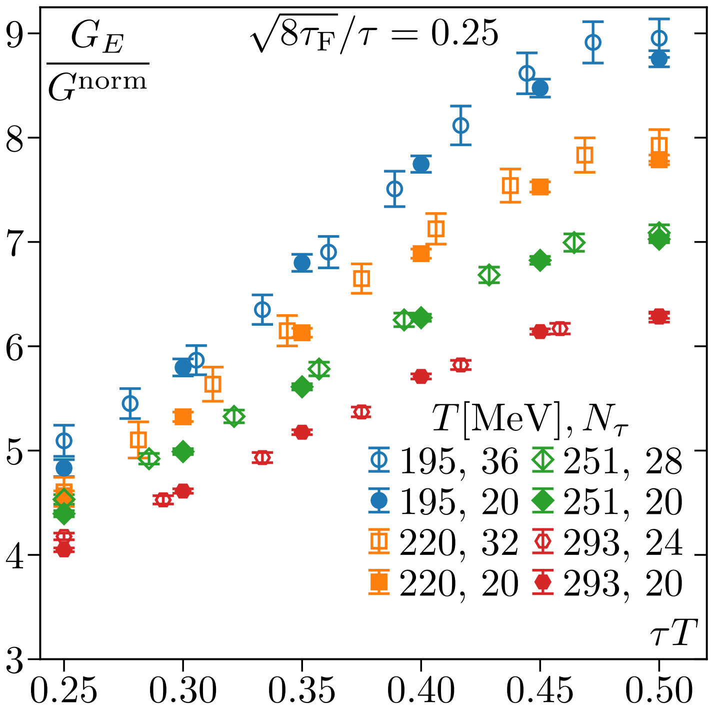
$$A_\mu(x) \to B_\mu(\tau_F, x) \qquad \partial_{\tau_F} B_\mu(\tau_F, x) = -g_0^2 \frac{\delta S_{\rm YM}[B]}{\delta B_\mu(\tau_F, x)}$$

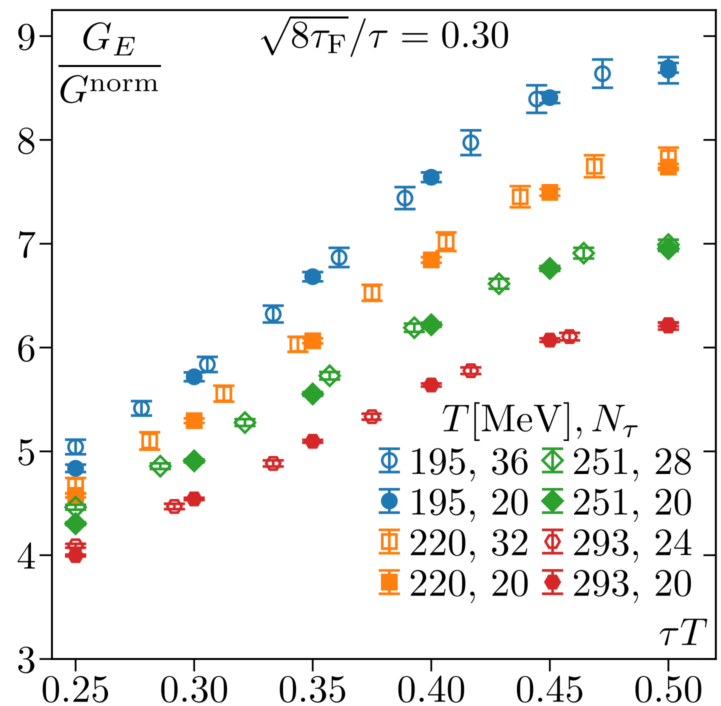
$$B_\mu(0, x) = A_\mu(x)$$

Symanzik gauge action and Zeuthen flow

Gauge fields are smeared in the radius $\sqrt{8\tau_F}$

$$a < \sqrt{8\tau_F} < \tau/3$$





We see small cutoff effects thanks improved actions

Analysis and modeling the chromo-electric correlator

Analysis of the chromo-electric correlator:

- Extrapolate the lattice results on the chromo-electric correlator to the continuum limit
- Perform the zero flow time extrapolation

Fits to model spectral function:

$$\rho^{low}(\omega, T) = \frac{\kappa\omega}{2T} \qquad \rho^{high}(\omega) = \rho^{LO,NLO}(\omega)$$

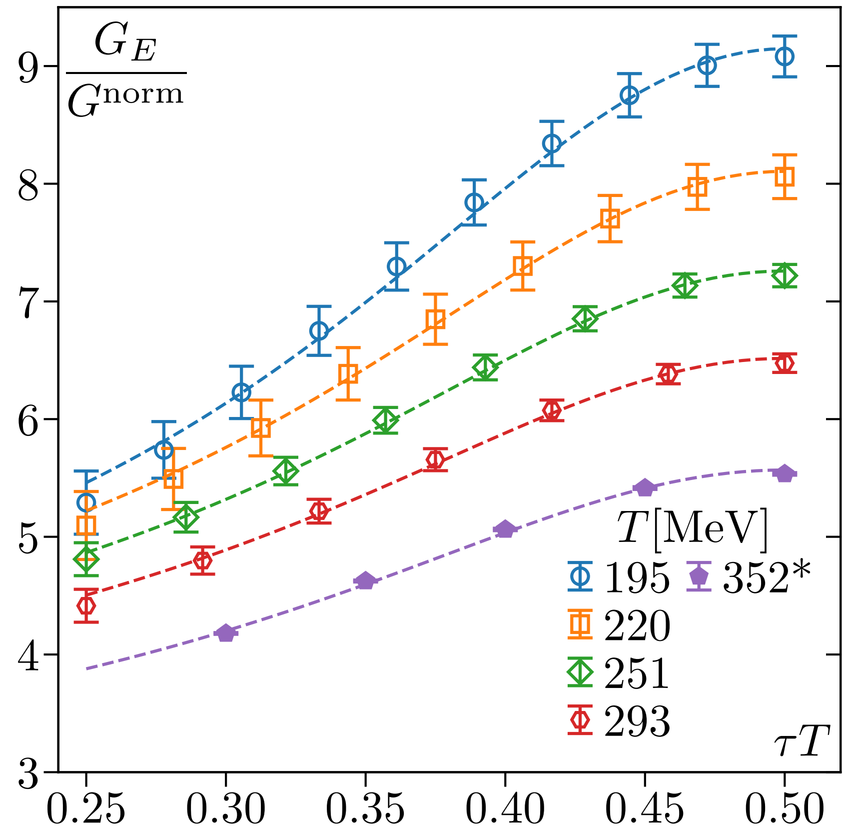
$$\rho^{max}(\omega, T) = max(\rho^{low}(\omega, T), \rho^{high}(\omega))$$

$$\rho^{smax}(\omega, T) = \sqrt{(\rho^{low})^2 + (\rho^{high})^2}$$

$$\rho^{pow}(\omega, T) = \rho^{low}(\omega, T), \ \omega \le \omega_{IR}$$

$$\rho^{pow}(\omega, T) = A\omega^{\alpha}, \ \omega_{IR} < \omega < \omega_{UV}$$

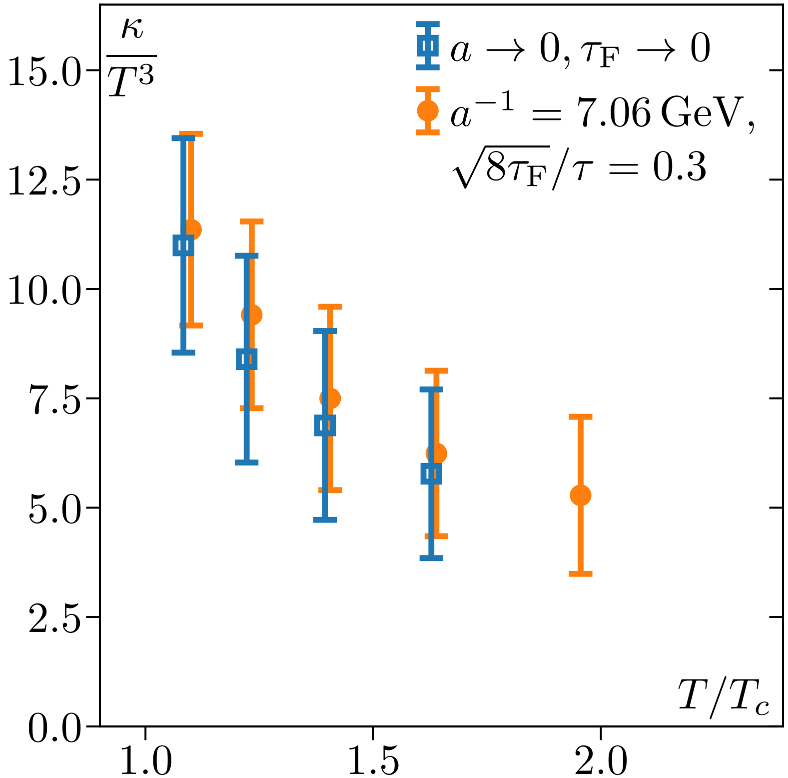
$$\rho^{pow}(\omega) = \rho^{high}(\omega), \ \omega \ge \omega_{UV}$$

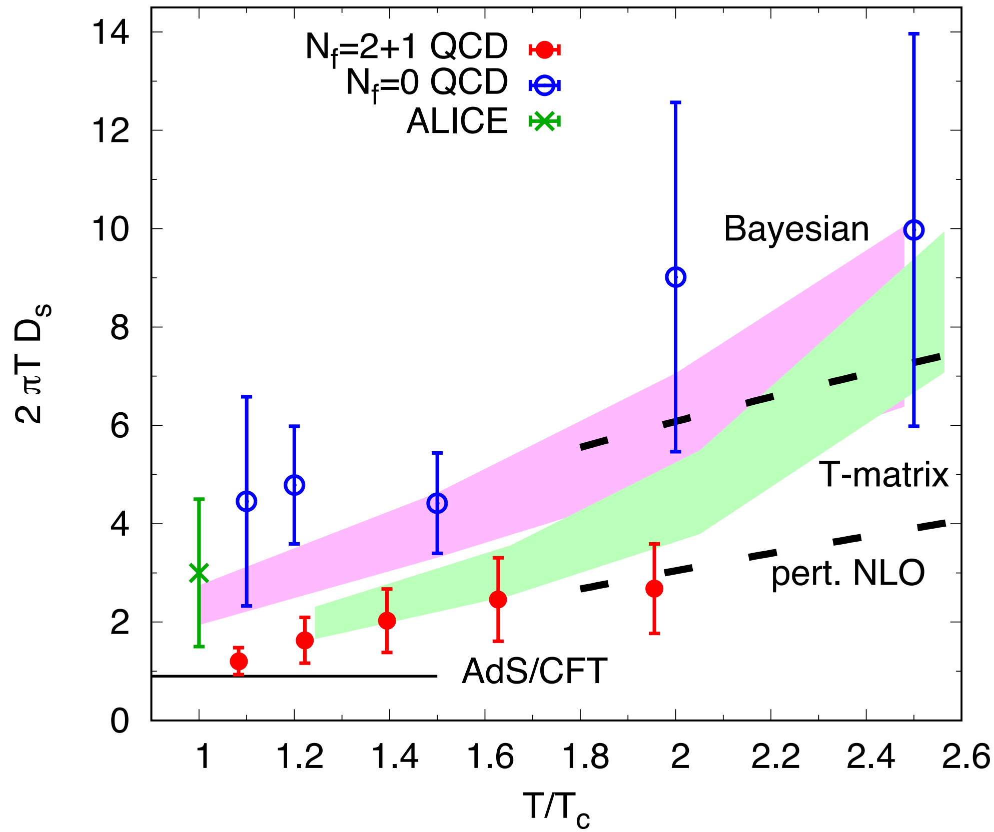


Heavy quark diffusion coefficient in QCD

- κ/T^3 has significant temperature dependence
- D_s is significantly smaller in 2+1 flavor QCD than in quenched QCD and is close to the AdS/CFT limit

$$D_s = \frac{2T^2}{\kappa}$$





Altenkort et al (HotQCD), PRL130 (2023) 231902

Lattice calculations of the chromo-magnetic correlator

The chromo-magnetic correlator has non-trivial anomalous dimension and needs renormalization even in the continuum limit

$$G_B^{\text{phys.}}(\tau,T)=\lim_{\tau_F\to 0} Z_{\text{match}}(\bar{\mu}_T,\bar{\mu}_{\tau_F},\mu_F)\,G_B(\tau,T,\tau_F).$$

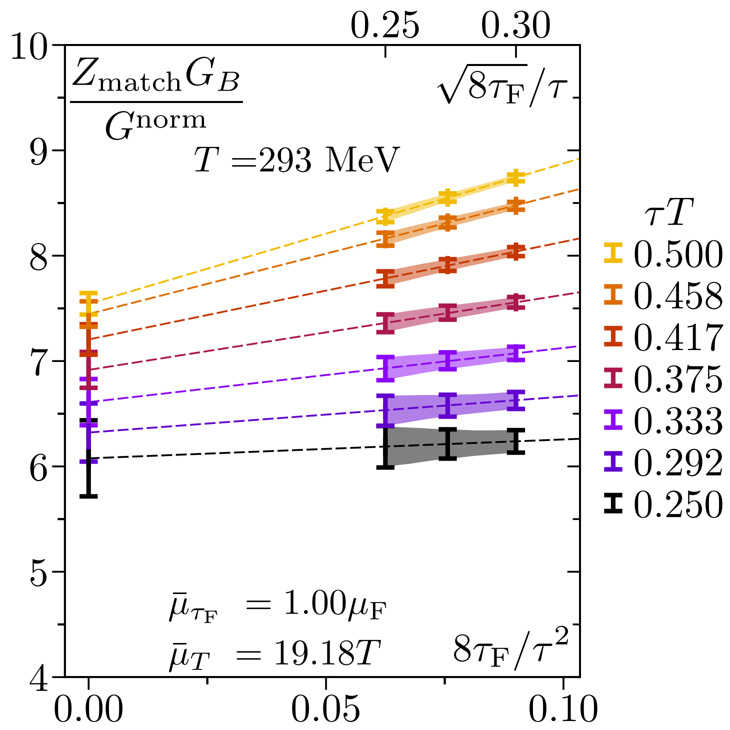
$$\ln Z_{\text{match}}=\int_{\bar{\mu}_T^2}^{\bar{\mu}_{\tau_F}^2}\gamma_0 g^2_{\overline{\text{MS}}}(\bar{\mu})\frac{d\bar{\mu}^2}{\bar{\mu}^2}+\gamma_0 g^2_{\overline{\text{MS}}}(\bar{\mu}_T)\left[\ln\frac{\bar{\mu}_T^2}{(4\pi T)^2}-2+2\gamma_E\right]-\gamma_0 g^2_{\overline{\text{MS}}}(\bar{\mu}_{\tau_F})\left[\ln\frac{\bar{\mu}_{\tau_F}^2}{4\,\mu_F^2}+\gamma_E\right]$$

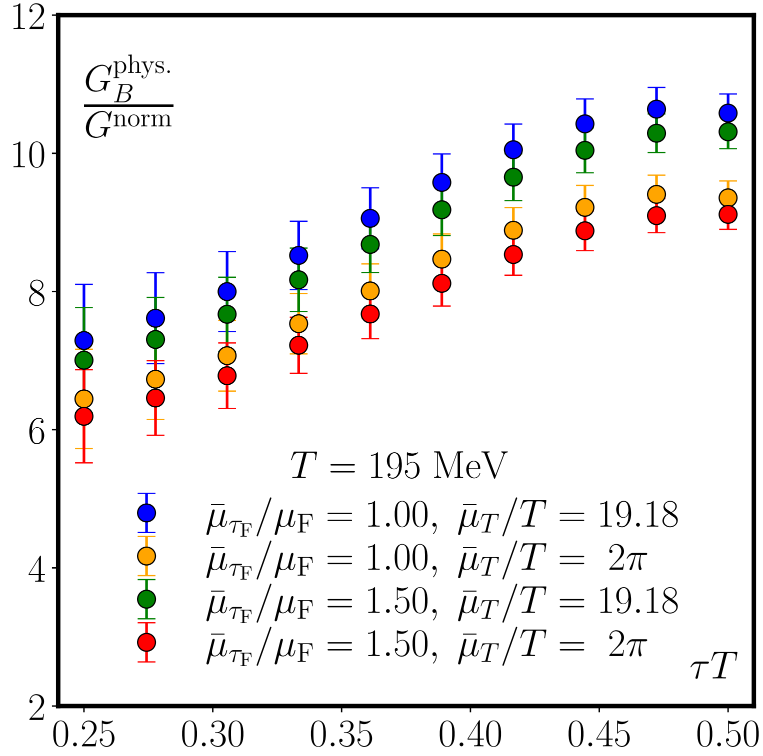
Evolution from $\bar{\mu}_{\tau_F}$ to $\bar{\mu}_T$

Matching to physical correlator

Matching between the gradient flow scheme and $\overline{MS}$ scheme at $\bar{\mu}_{\tau_F}$

Altenkort, de la Cruz Kaczmarek, Larsen, Mukherjee, PP, Shu, Stendebach (HotQCD), PRL 132 (2024) 051902





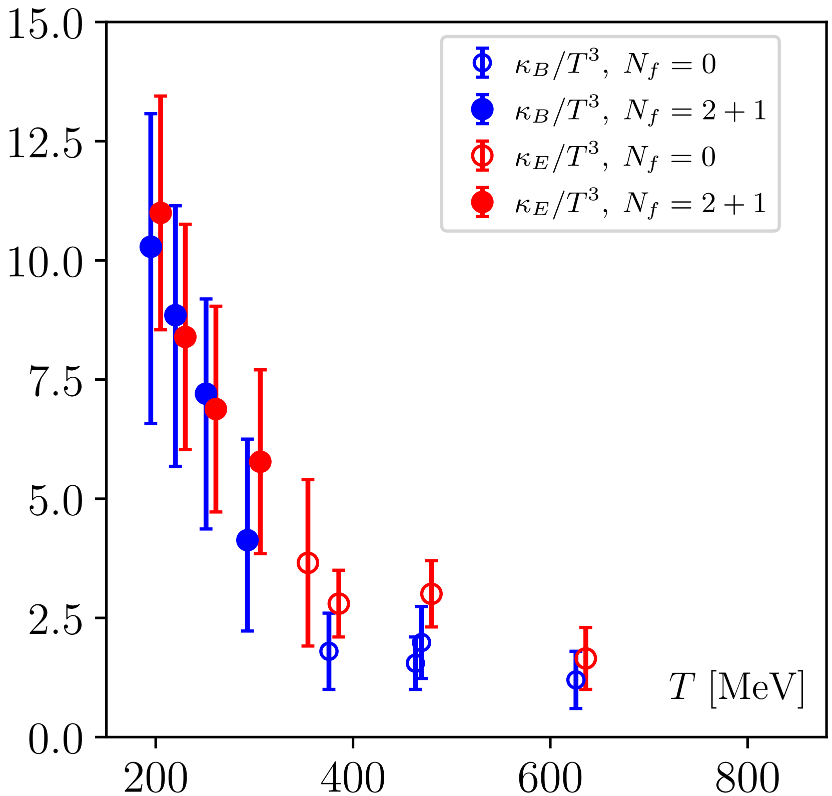
The M-dependence of the heavy quark diffusion coefficient

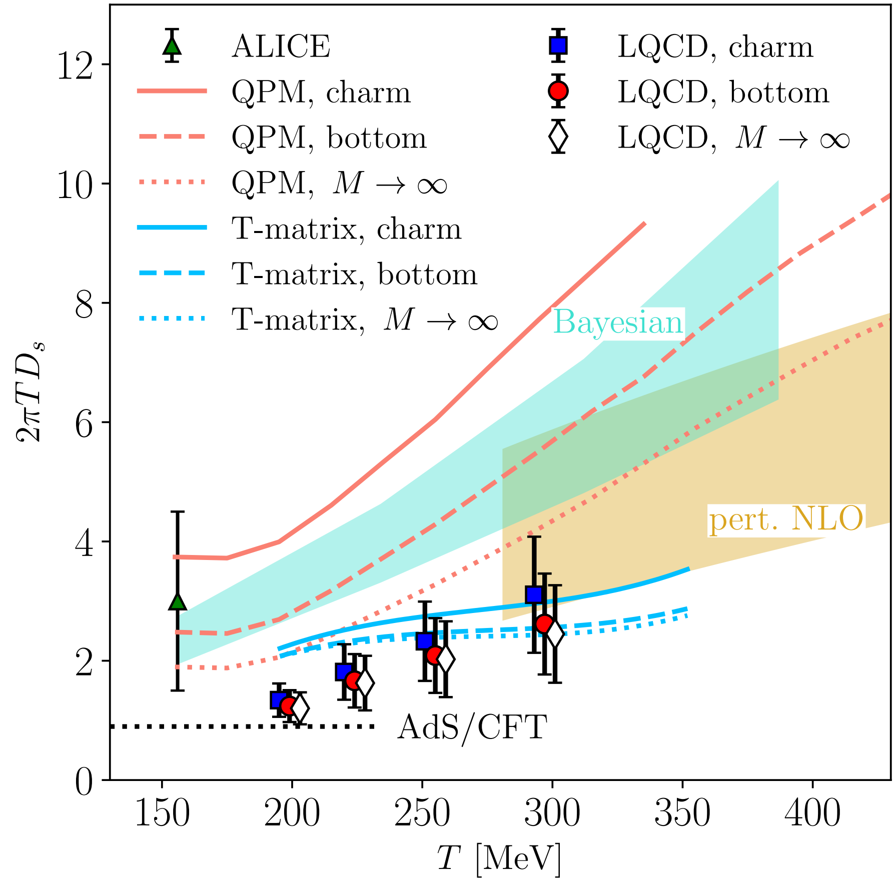
$$\rho_B^{\mathrm{uv,phys.}}(\omega) = K c_B^{\mathrm{uv,LO,NLO}}(\mu, \bar{\mu}_T) \rho_B^{\mathrm{uv,LO,NLO}}(\omega, \mu)$$

$$c_B^{\mathrm{NLO}}(\mu, \bar{\mu}_T) = \exp\left(\int_{\bar{\mu}_T^2}^{\mu^2} \gamma_0 g^2_{\overline{\mathrm{MS}}}(\bar{\mu}) \frac{d\bar{\mu}^2}{\bar{\mu}^2} \right), c_B^{\mathrm{LO}} = 1$$

$$D_s = \frac{2T^2}{\kappa_E + 2\kappa_B \langle \mathbf{v}^2 \rangle / 3} \cdot \frac{\langle p^2 \rangle}{3MT}$$

M, $\langle v^2 \rangle$, $\langle p^2 \rangle$ from a quasi-particle model





Altenkort et al (HotQCD), PRL 132 (2024) 051902

Summary

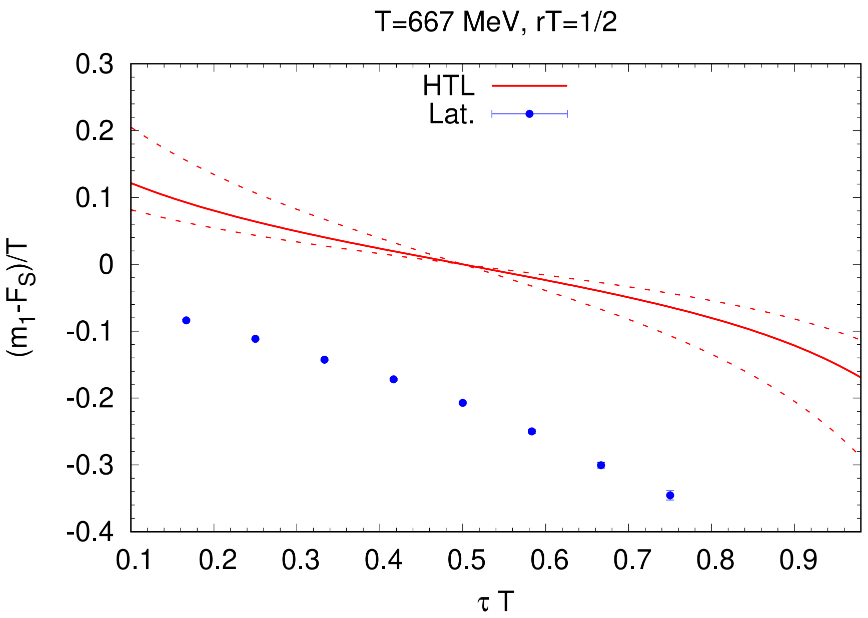
- First full QCD calculation of the heavy quark diffusion coefficient become available now and indicate that D_s is smaller than un quenched QCD and close to the AdS/CFT bound; the heavy quark mass dependence of D_s is small
- Lattice calculations confirm the existence of the imaginary part of the potential; There is no evidence for the screening of the real part of the potential $\Rightarrow$ quarkonium melting is not related to color screening. This not unexpected from the point of view of EFT approach
- The imaginary part of the potential increases with the temperature and distance, and for 244 MeV $< T <$ 352 MeV scales with the temperature reaching values $\sim 2T$

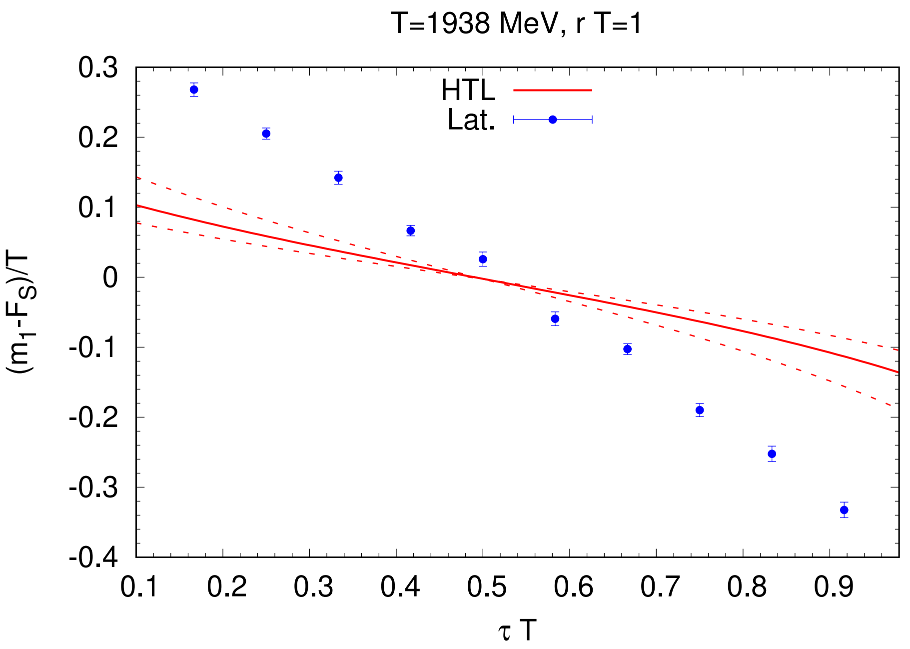
Back-up: Comparison with HTL perturbation theory

In HTL perturbation theory ReV is screened, but HTL approximation is valid only for $r \sim 1/m_D$ and assumes $m_D \ll T$ (problematic in realistic setup)

Lattice results on $W(r,\tau,T)$ can be compared with the HTL calculations

Burnier, Rothkopf, PRD 87 (2013) 114019

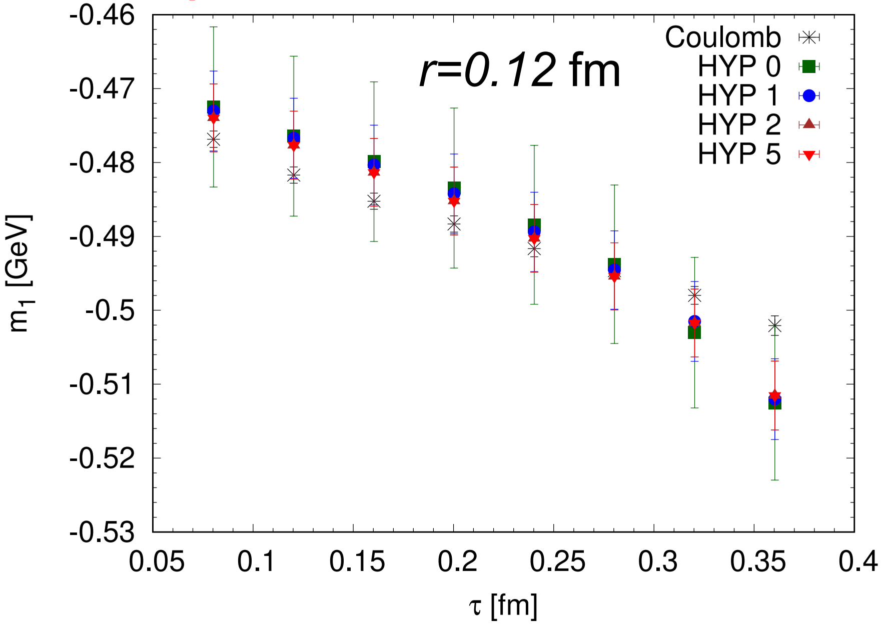


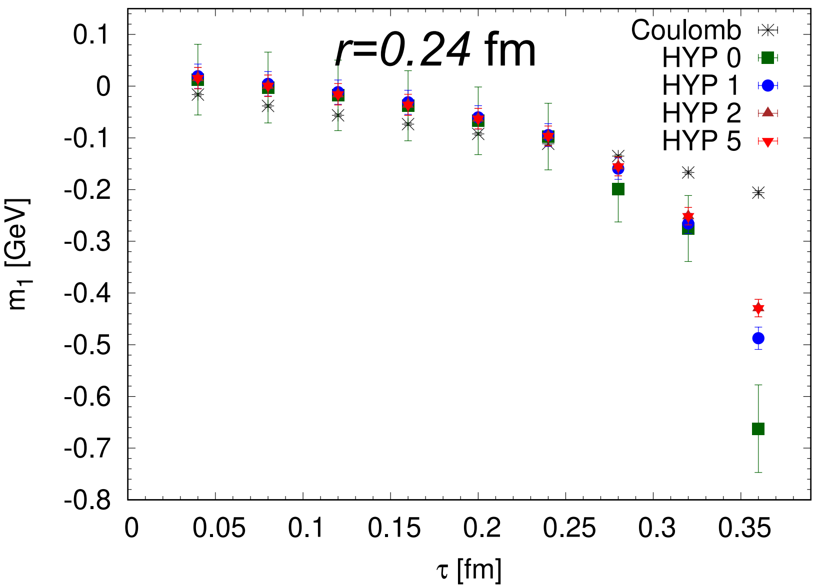


In HTL approximation Re$V(r,T) = F_S(r,T)$

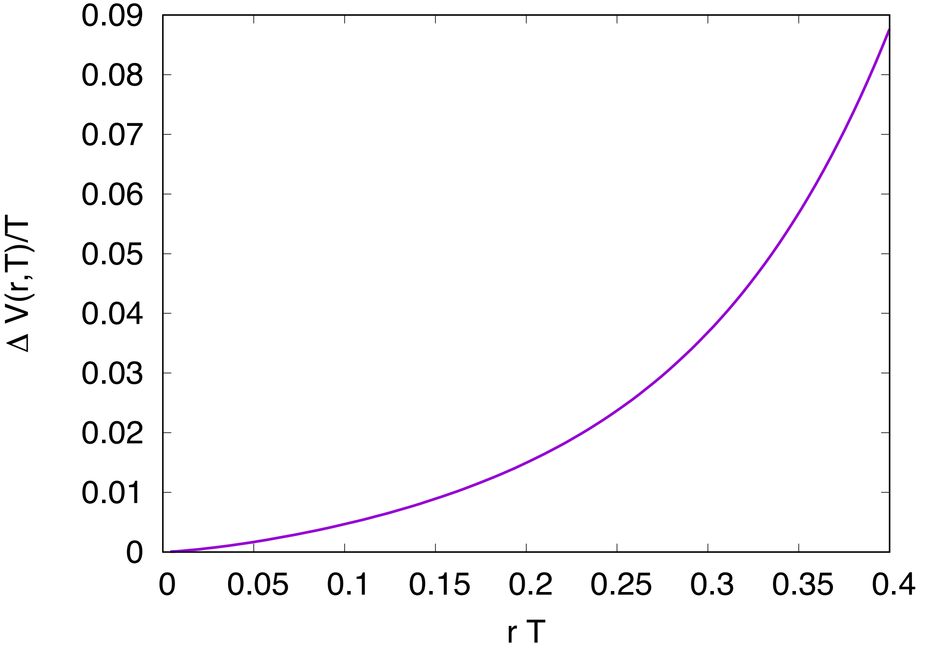
No agreement between the lattice and the HTL results even at the highest T

Back-up : subtracted effective masses for the Wilson loops

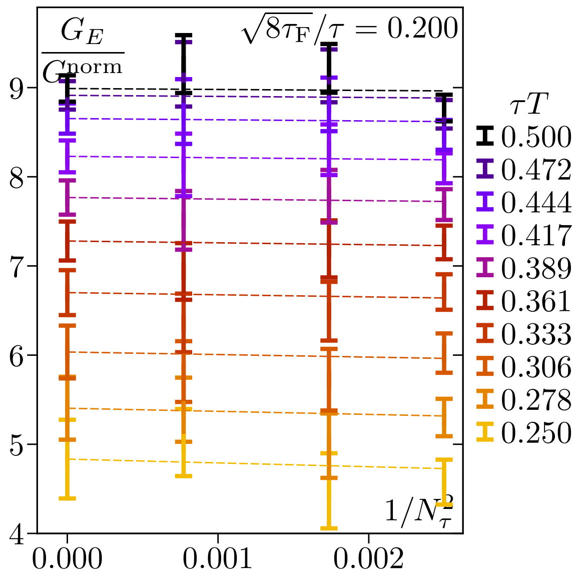


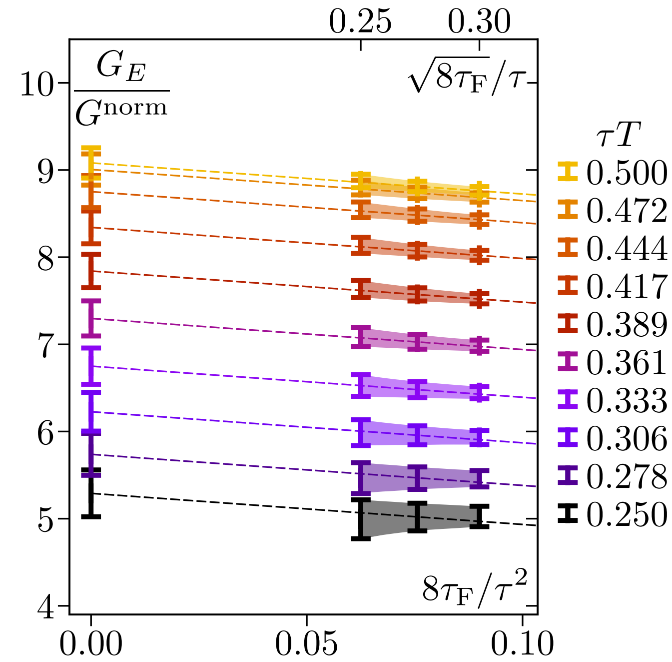


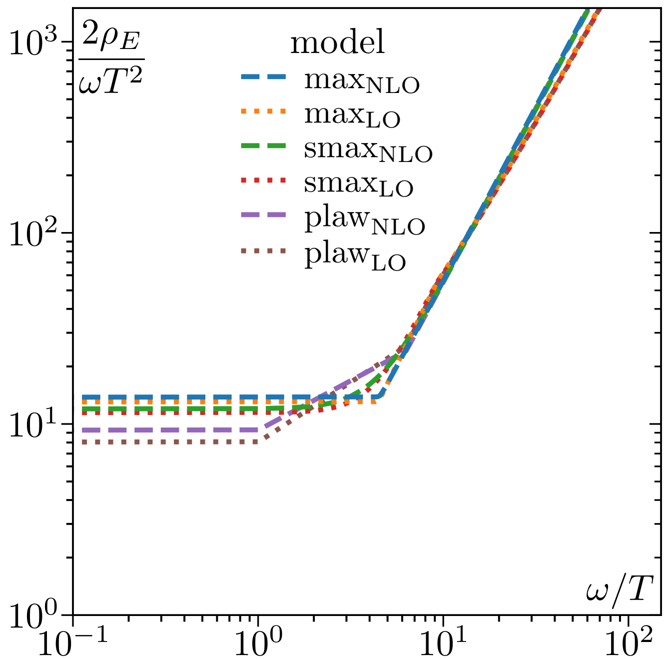
Thermal correction to the real part of the potential in weakly coupled pNRQCD:

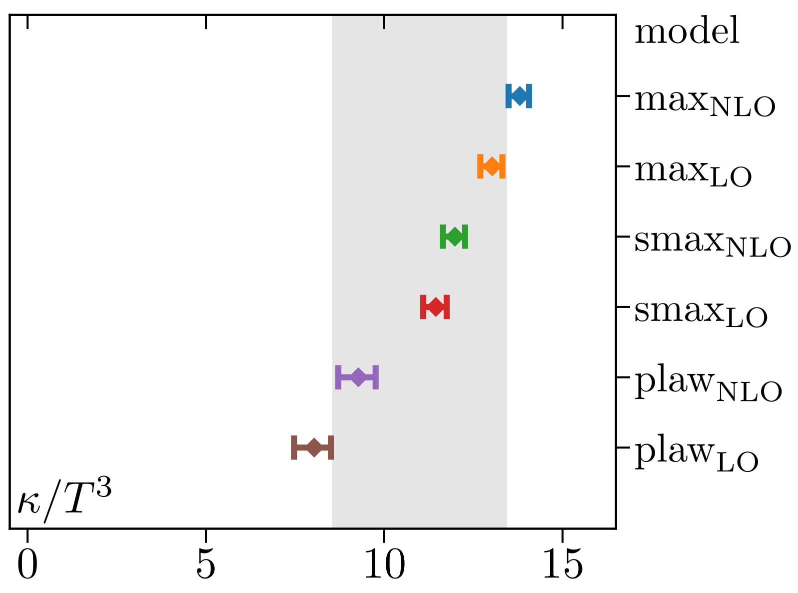


Back-up: extrapolations at T=195 MeV:

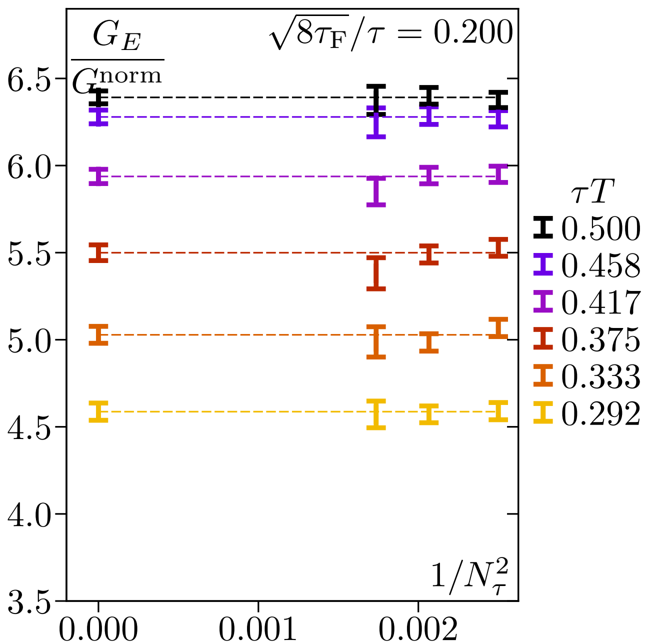


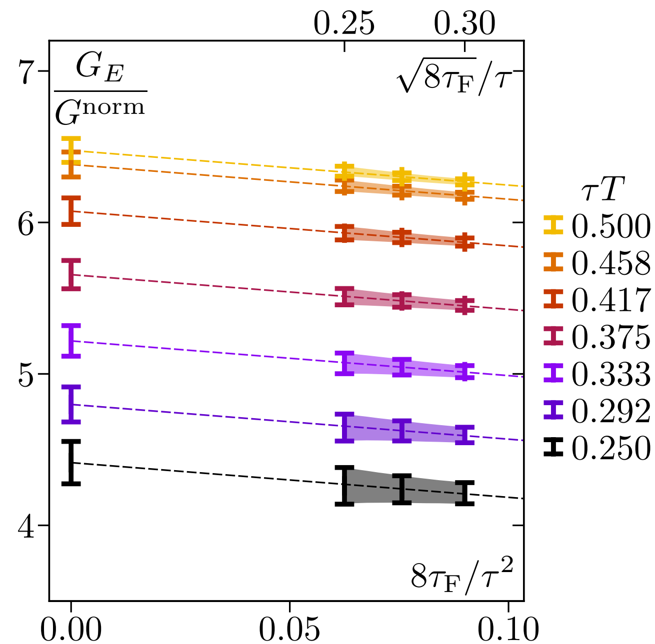


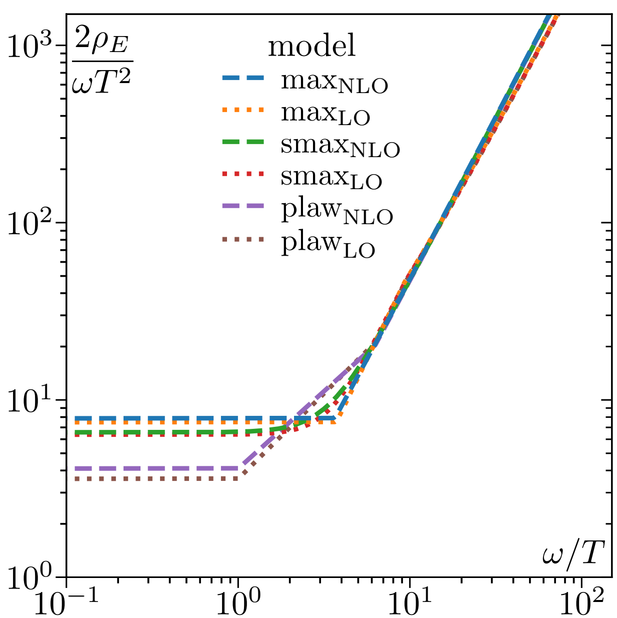


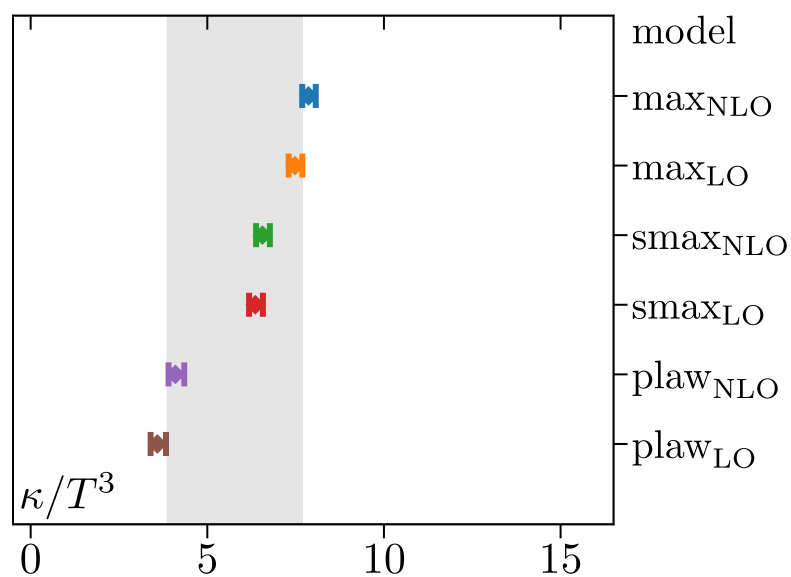


Backup: extrapolations at T=293 MeV:

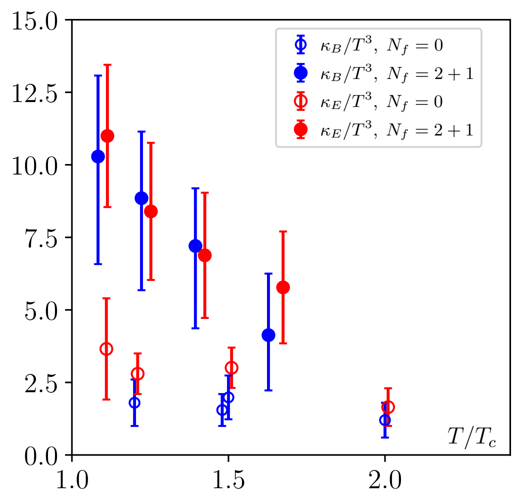


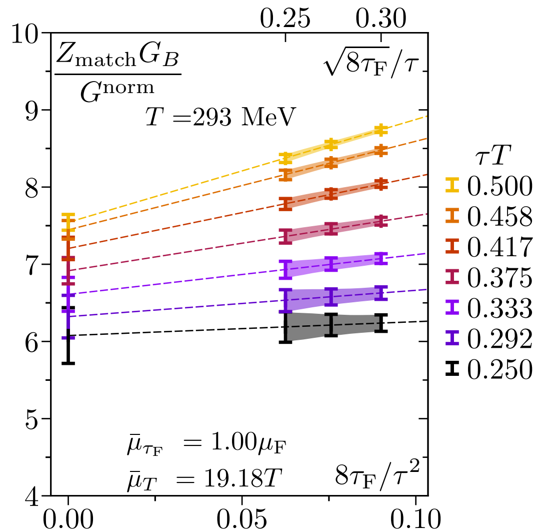


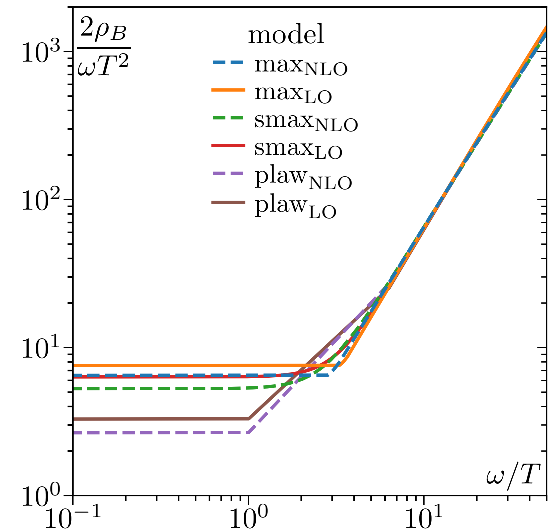




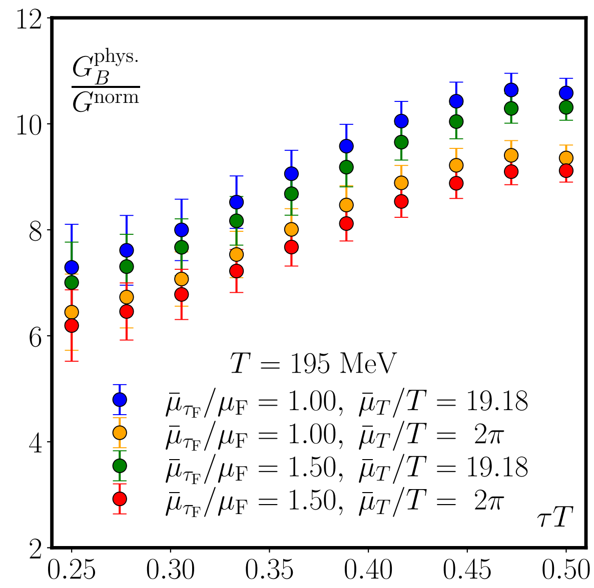
Back-up : calculations of the chromo-magnetic correlator

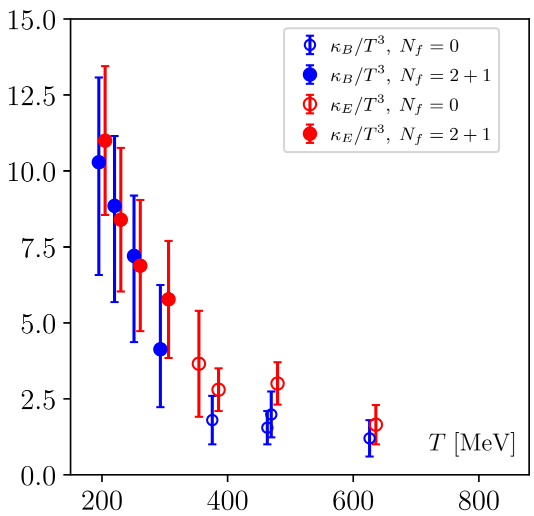




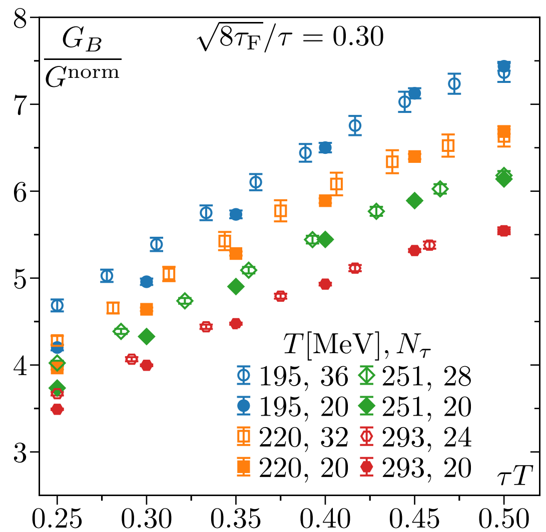


$$B_i(\mathbf{x}) \equiv \frac{\epsilon_{ijk}}{2}\Big(U_j(\mathbf{x})U_k(\mathbf{x}+\hat{j}) - U_k(\mathbf{x})U_j(\mathbf{x}+\hat{k})\Big)\,.$$

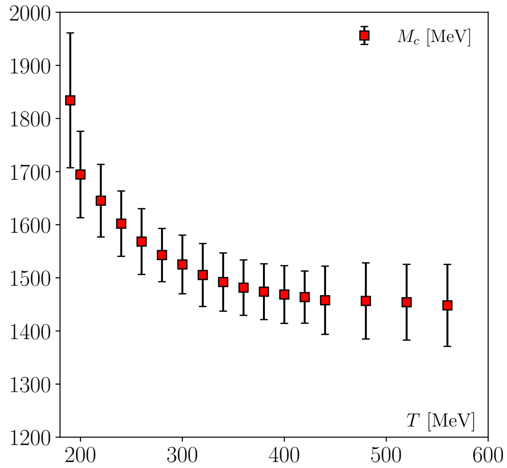




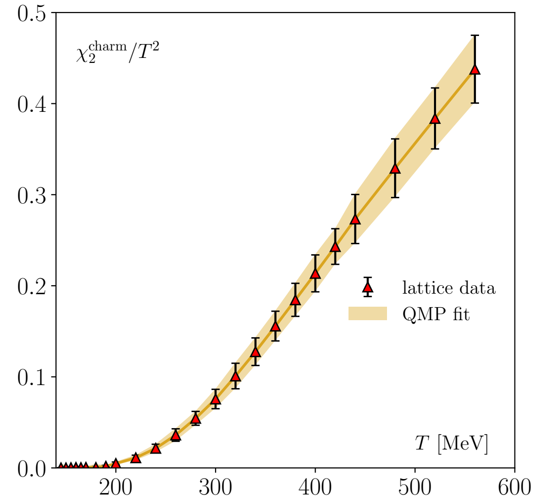
$\kappa_B/T^3,\ N_f=0$
$\kappa_B/T^3,\ N_f=2+1$
$\kappa_E/T^3,\ N_f=0$
$\kappa_E/T^3,\ N_f=2+1$
T [MeV]



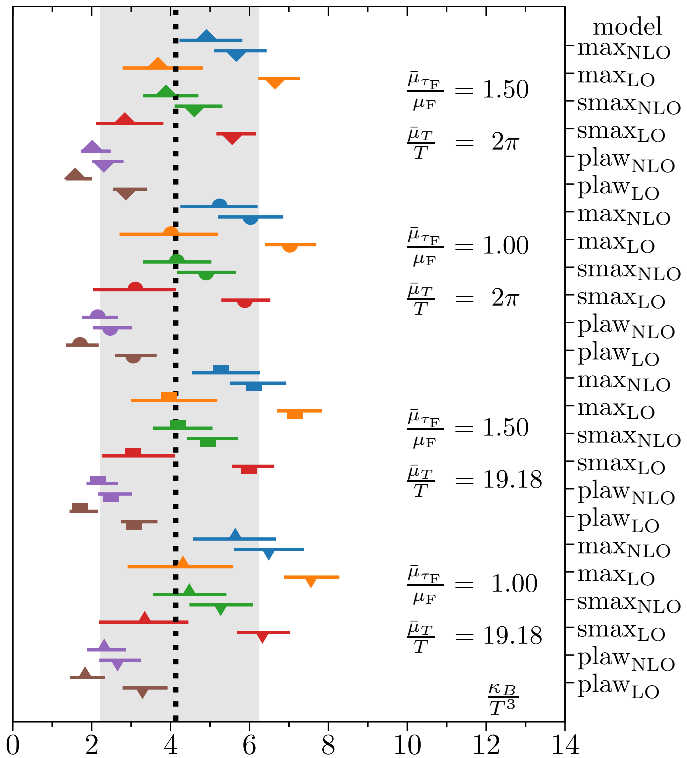
$\frac{G_B}{G^{\mathrm{norm}}}$
$\sqrt{8\tau_F}/\tau = 0.30$
T[MeV], N_τ
195, 36
195, 20
220, 32
220, 20
251, 28
251, 20
293, 24
293, 20
τT



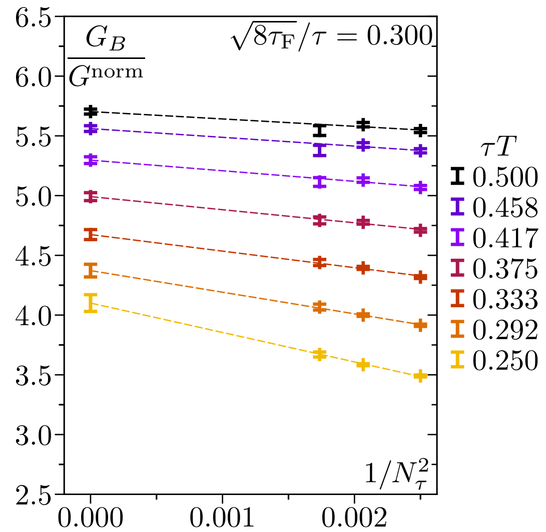
M_c [MeV]
T [MeV]



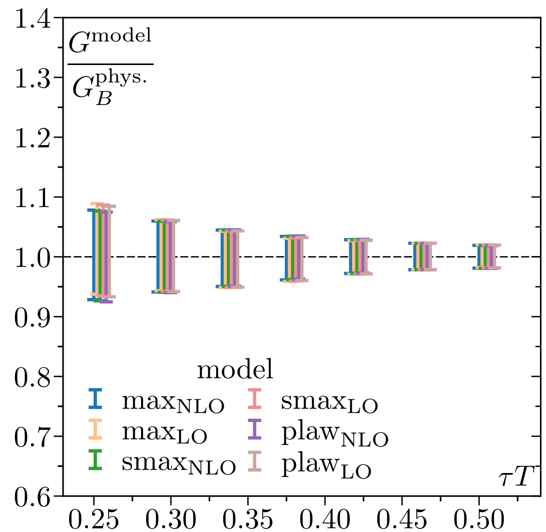
$\chi_2^{\mathrm{charm}}/T^2$
lattice data
QMP fit
T [MeV]



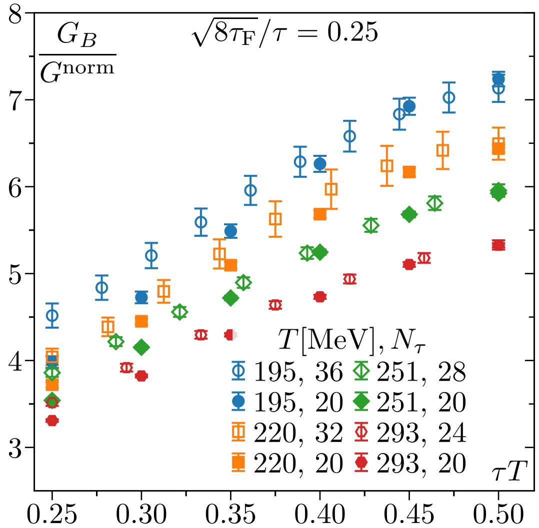
model
$\max_{\mathrm{NLO}}$
$\max_{\mathrm{LO}}$
$\mathrm{smax}_{\mathrm{NLO}}$
$\mathrm{smax}_{\mathrm{LO}}$
$\mathrm{plaw}_{\mathrm{NLO}}$
$\mathrm{plaw}_{\mathrm{LO}}$
$\frac{\bar{\mu}_{\tau_F}}{\mu_F} = 1.50$
$\frac{\bar{\mu}_{T}}{T} = 2\pi$
$\frac{\bar{\mu}_{\tau_F}}{\mu_F} = 1.00$
$\frac{\bar{\mu}_{T}}{T} = 2\pi$
$\frac{\bar{\mu}_{\tau_F}}{\mu_F} = 1.50$
$\frac{\bar{\mu}_{T}}{T} = 19.18$
$\frac{\bar{\mu}_{\tau_F}}{\mu_F} = 1.00$
$\frac{\bar{\mu}_{T}}{T} = 19.18$
$\frac{\kappa_B}{T^3}$
0 2 4 6 8 10 12 14

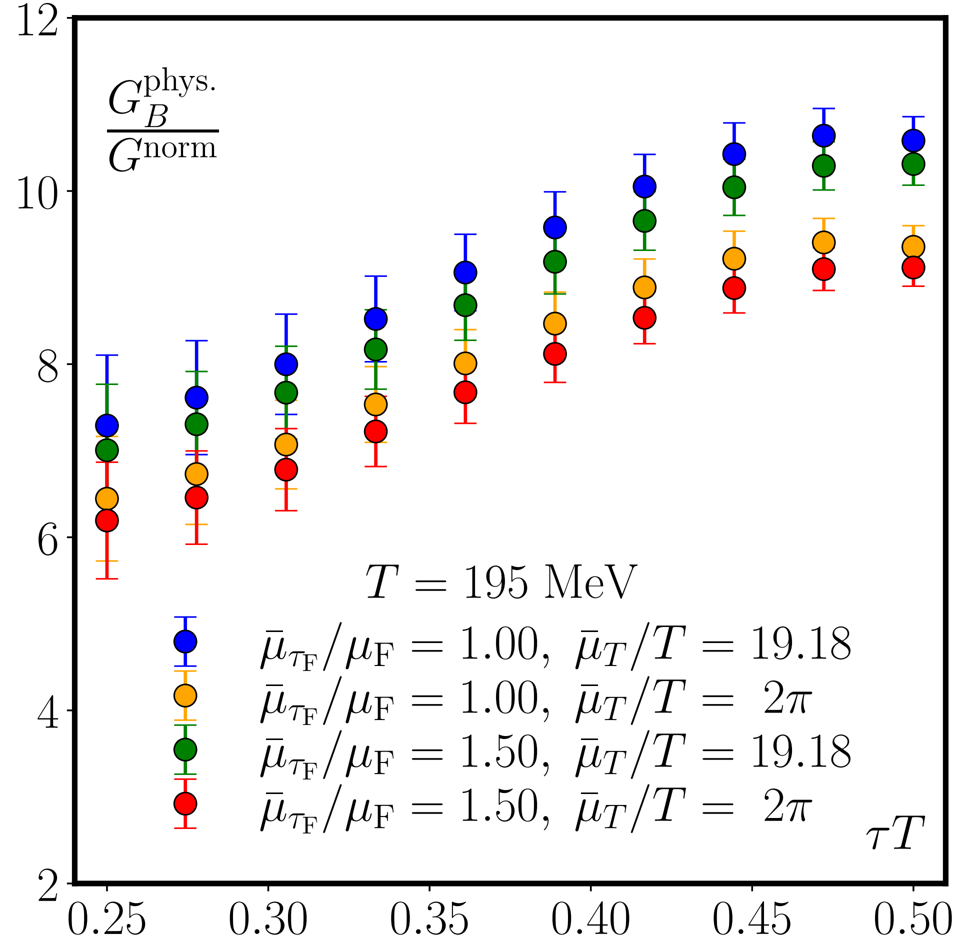


$\frac{G_B}{G^{\mathrm{norm}}}$
$\sqrt{8\tau_F}/\tau = 0.300$
τT
0.500
0.458
0.417
0.375
0.333
0.292
0.250
$1/N_\tau^2$
6.5 6.0 5.5 5.0 4.5 4.0 3.5 3.0 2.5
0.000 0.001 0.002



$\frac{G^{\mathrm{model}}}{G_B^{\mathrm{phys.}}}$
model
$\max_{\mathrm{NLO}}$
$\max_{\mathrm{LO}}$
$\mathrm{smax}_{\mathrm{NLO}}$
$\mathrm{smax}_{\mathrm{LO}}$
$\mathrm{plaw}_{\mathrm{NLO}}$
$\mathrm{plaw}_{\mathrm{LO}}$
τT
1.4 1.3 1.2 1.1 1.0 0.9 0.8 0.7 0.6
0.25 0.30 0.35 0.40 0.45 0.50


$\frac{G_B}{G^{\mathrm{norm}}}$
$\sqrt{8\tau_F}/\tau = 0.25$
T[MeV], N_τ
195, 36
195, 20
220, 32
220, 20
251, 28
251, 20
293, 24
293, 20
τT


$\frac{G_B^{\mathrm{phys.}}}{G^{\mathrm{norm}}}$
$T = 195$ MeV
$\bar{\mu}_{\tau_F}/\mu_F = 1.00,\ \bar{\mu}_T/T = 19.18$
$\bar{\mu}_{\tau_F}/\mu_F = 1.00,\ \bar{\mu}_T/T = 2\pi$
$\bar{\mu}_{\tau_F}/\mu_F = 1.50,\ \bar{\mu}_T/T = 19.18$
$\bar{\mu}_{\tau_F}/\mu_F = 1.50,\ \bar{\mu}_T/T = 2\pi$
τT