Memory effects in open quantum evolution of quarkonia (towards beyond NLO)

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In progress with Vyshakh B R

Quarkonia in the QGP: processes

- Various physical processes leading to suppression of quarkonia
  1. Gluo-dissociation, or absorption of energetic gluons *[Bhanot, Peskin (1979)]*
  2. Screening of the $Q\overline{Q}$ interaction *[Matsui, Satz (1986)]*
  3. Scattering with medium particles, or Landau damping (LD) *[Granchamp, Rapp (2001)]*. Can be seen as a complex potential*[Laine et. al. (2007)]*
- All these can be consistently incorporated in pNRQCD *[Brambilla et. al. 2008, 2011, 2013]*

pNRQCD

- Energy scales $M \gg \frac{1}{r} \gg E_b$ where r is the bound state size and E_b is the binding energy
- The lagrangian is

$$
\begin{aligned}
L_{\text{pNRQCD}} = \int d^3\mathbf{r}\ \mathrm{tr}\Big(&\mathcal{S}(\mathbf{r})^\dagger[i\partial_0 - h_s]\mathcal{S}(\mathbf{r}) \\
&+ \mathcal{O}(\mathbf{r})^\dagger[iD_0 - h_o]\mathcal{O}(\mathbf{r})\Big) \\
&+ \mathcal{O}^\dagger(\mathbf{r})\mathbf{r}\cdot g\mathbf{E}\mathcal{S}(\mathbf{r}) + \frac{1}{4}\{\mathcal{O}^\dagger(\mathbf{r})\{\mathbf{r}\cdot g\mathbf{E}, \mathcal{O}(\mathbf{r})\}\} + \ldots\Big)
\end{aligned}
$$

- $\mathbf{r}$ is the relative separation between $Q\overline{Q}$, $\mathcal{S}$ is the singlet wavefunction and $\mathcal{O}$ is the octet wavefunction
- $\mathbf{E}$ is the chromo-electric field, $h_{o,s} = -\frac{\nabla^2}{M} + V_{o,s}(\mathbf{r})$

Quarkonium in the QGP: a quantum system (S) in an environment (E)

- The dynamics of the bound state can be described using the theory of open quantum systems *[Akamatsu (2015)]*
- The $\bar{Q}Q$ system (S) and the QGP environment (E) interact with each other. Their combined evolution is unitary

$$i\frac{d\rho_{\mathrm{tot}}}{dt} = i[H_{\mathrm{tot}}, \rho_{\mathrm{tot}}]$$

- $H_{\mathrm{tot}} = H_{\mathrm{S}} + H_{\mathrm{E}} + V_{\mathrm{I}}$
- $Q\bar{Q}$ pair is in a mixed state described by a density matrix obtained by tracing out the E: $\rho_{\mathrm{S}} = \mathrm{tr}_{\mathrm{E}}(\rho_{\mathrm{tot}})$
- A master equation (in the interaction picture) up to $\mathcal{O}(V_{\mathrm{I}}^3)$

$$\frac{d\rho_{\mathrm{S}}}{dt} \approx -\int_0^t du\, [V_{\mathrm{I}}(t), [V_{\mathrm{I}}(u), \rho_{\mathrm{S}}(t)]]$$

[Breuer, Petruccione; Brambilla et. al. (2017)]

Weak-coupling Hamiltonian

- To start, consider $Q\bar{Q}$ is weakly coupled to each other

$$H_{\rm S} = \frac{p^2}{M}\mathbb{1} - \frac{C_F\alpha_s}{r}|s\rangle\langle s| + \frac{\alpha_s}{2N_c r}|o_a\rangle\langle o_a|$$

$$V_{\rm I} = -g\mathbf{r}\cdot\mathbf{E}^a\Big(\frac{1}{\sqrt{2N_c}}|s\rangle\langle o_a| + \frac{1}{\sqrt{2N_c}}|o_a\rangle\langle s| + \frac{1}{2}d_{abc}|o_b\rangle\langle o_c|\Big)$$

  Justified if $M \gg 1/r \gg E_b$, T
- $\rho_s(t) = \langle s|\rho_{\rm S}|s\rangle$, $\rho_o(t) = \langle o_a|\rho_{\rm S}|o_a\rangle$
- $\rho_{\rm S}$ is diagonal in s, o basis $\rho_{\rm S} = {\rm diag}\{\rho_s, \rho_o\}$. Similarly, $H_{\rm S} = {\rm diag}\{h_s, h_o\}$.
- Mixing between the two sectors comes from $V_{\rm I}$

The master equation

- The resulting master equation in Schrödinger picture is

$$i\frac{d\rho_{\mathrm{S}}}{dt} = [H_{\mathrm{S}}, \rho_{\mathrm{S}}] - i\int_0^t du\, \Gamma(u) \sum_{i=1,3}^{n=\pm,d} \Big(V_{ni}^{\dagger}(0) V_{ni}(u) \rho_{\mathrm{S}}(t) - V_{ni}(u) \rho_{\mathrm{S}}(t) V_{ni}^{\dagger}(0) + \mathrm{HC} \Big)$$

- $\Gamma(t) = g^2 \mathrm{tr}_{\mathrm{E}} \Big(E_i^a(t,0) E_i^a(0,0) \rho_{\mathrm{E}} \Big)$

The jump operators

- $\{V_{ni}(t)\}$ are time dependent jump operators corresponding to $s \to o$, $o \to s$, $o \to o$ transitions
- Explicitly,

$$V_{+i}(t) = e^{ih_s t} r_i e^{-ih_o t} \sqrt{C_F} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$V_{-i}(t) = e^{ih_o t} r_i e^{-ih_s t} \sqrt{\frac{1}{2N_c}} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$V_{di}(t) = e^{ih_o t} r_i e^{-ih_o t} \sqrt{\frac{N_c^2 - 4}{4N_c}} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} .$$

- Same structure as the NLO calculation of *[Brambilla et. al. (2022, 2023)]*. The key difference is the next step

Expansion in $\tau_{\mathrm{E}}/\tau_{Sys}$ (NLO)

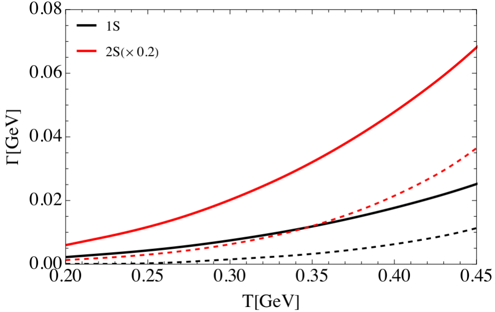
- $\Gamma(t)$ is substantial only for $t < \tau_{\mathrm{E}} \sim \frac{1}{T}$
- For example, in HTL, $\tau_{\mathrm{E}} \sim \frac{1}{gT}$
- $V_{ni}(t)$ evolves on a time scale of $\tau_{\mathrm{E}} \sim \frac{1}{E_b}$
- If $\tau_{\mathrm{E}} \ll \tau_{\mathrm{S}}$ then

$$V_{ni}(t) \sim e^{ih_\alpha t} r_i e^{-ih_\beta t} \approx r_i + it(h_\alpha r^i - r^i h_\beta) + \mathcal{O}\left[\left(\frac{\tau_{\mathrm{E}}}{\tau_{\mathrm{S}}}\right)^2\right]$$

- This gives the Lindblad equation (memoryless) which gives a very nice description of the LHC data for the suppression of Υ's *[Brambilla et. al. (2022, 2023)]*
- In Lindblad only $\Gamma(\omega = 0)$ required

Expansion in $\tau_{\mathrm{E}}/\tau_{Sys}$?

- However, $E_b \sim 500$MeV for $\Upsilon(1S)$, a little smaller for $\Upsilon(2S)$. On the other hand $T \lesssim 500$MeV
- For $\Upsilon(1S)$ in particular, it is worthwhile investigating whether further corrections in $\tau_{\mathrm{E}}/\tau_{\mathrm{S}}$ can have an effect on quantum dynamics
- Motivated by our results for finite frequency effects in the decay rates, *[Balbeer, Sharma (2023)]*



Angular momentum basis

- Assuming that the initial state is unpolarized, $\rho_{\mathrm{S}}(t)$ can be reduced into block diagonal form in the angular momentum basis just as in the Lindblad equation
- The blocks are given by

$$\rho_{\mathrm{S}}^{l} = \sum_{m} \langle l, m | \rho_{\mathrm{S}} | l, m \rangle$$

- Since V_{ni} are vector operators, use Wigner-Eckart to reduce the equation

The reduced equation

- $$i\frac{d\rho_{\mathrm{S}}^{l}}{dt} = [H_{\mathrm{S}}^{l}, \rho_l] - i\int_0^t du\, \Gamma(u) \sum^{n=\pm,d} \sum_{l'} \Big(T_n^{\dagger}(l \to l', 0)\, T_n(l \to l', u) \rho_s^l(t) - T_n(l' \to l, u) \rho_s^{l'} T_n^{\dagger}(l' \to l, 0) + \mathrm{HC} \Big)$$
- T_n's are reduced operators of the form

$$T_n(l \to l', t) = \sqrt{\frac{2l'+1}{2l+1}} \langle l' || V_{ni}(t) || l \rangle \qquad (1)$$

- T_n's can be thought of as transition operators that change l and also $s \to o$, $o \to s, o$
- They also change l by ± 1 as the interaction is dipolar
- Decomposition same as NLO but need to track the t dependence of V_{ni}

Numerical simulation

- $V_{ni}(t)$ can be evaluated by changing the basis

$$V_{ni}(u) \sim e^{ih_\alpha u} r_i e^{-ih_\beta u} = |\alpha_n\rangle\langle\beta_m|\langle\alpha_n|r^i|\beta_m\rangle e^{iu(\varepsilon_n^\alpha - \varepsilon_m^\beta)}$$

  Then need to integrate $\Gamma(u)V_{ni}(u)$ over u from $[0, t]$.
- Include continuum dynamics
- To evolve the density matrix, use the method of quantum trajectories. Evolve wavefunctions using a stochastic equation where the wavefunction evolves under H_{eff}, inter spaced with sudden jumps
- The wavefunction satisfies evolution equation with random noise. $d|\psi(t)\rangle = -iH_{\mathrm{eff}}|\psi(t)\rangle dt + F(\psi(t))dW(t)$

Numerical simulation

- The Non-Markovian density matrix equation can also be solved using quantum trajectories method
- Consider a general master equation

$$\frac{\partial \rho_{\mathrm{S}}}{\partial t} = A(t)\rho_{\mathrm{S}} + \rho_{\mathrm{S}} B^{\dagger}(t) + \sum_i C_i(t)\rho_{\mathrm{S}} D_i^{\dagger}(t)$$

- The main new feature is that $C_i \neq D_i$ because of the dependence on u
- The idea is to define a two component wavefunction:

$$|\psi(t)\rangle = (|\phi_1(t)\rangle \;\; |\phi_2(t)\rangle)$$

Breuer et. al. (1999)

- Then evolve the wavefunction using H_{eff} inter spaced with quantum jumps $J_i|\psi\rangle$

$$H_{\mathrm{eff}} = \mathrm{diag}\{A(t), \;\; B(t)\} \quad J_i = \mathrm{diag}\{C_i(t), \;\; D_i(t)\}$$

- Averaging $|\phi_1\rangle\langle\phi_2|$ over different jump realisations gives the density matrix

Parameters and model assumptions

- In the results I'll show, we have evolved $|\psi\rangle$ with $H_{\rm eff}$ and without incorporating quantum jumps
- Just $H_{\rm eff}$ evolution can give reasonable estimates for $\Upsilon(1S)$ *[Yao et. al. (2021), Brambilla et. al. (2022)]* but not the higher states
- For this preliminary study the correlation function used

$$\Gamma(t) = \frac{\kappa}{2\tau} e^{-|t|/\tau}$$

- Motivated by the HTL form of the gluon propagator. In the limit of $\tau \to 0$, goes to the LO Lindblad equation

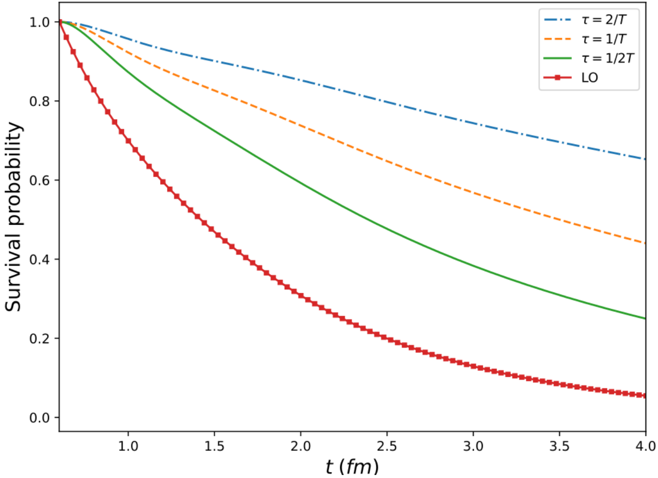
Parameters and medium

- We take the medium to be Bjorken expanding

$$T = T_0 \left(\frac{t_0}{t}\right)^{1/3}$$

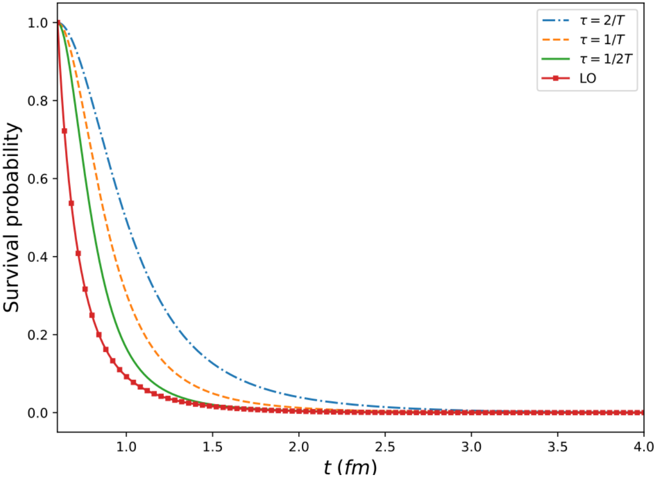
- We consider central collision geometry
- $T_0 = 400\text{MeV}$, $t_0 = 0.6\text{fm}/\text{c}$
- $\kappa = \frac{\hat{\kappa}}{T^3}$, $\hat{\kappa} = 2$
- $\hat{\gamma}$, which gives the imaginary part of the electric field correlator, is taken to be 0

Comparison of evolution



- $\Upsilon(1S)$ *[Vyshakh, Sharma (in progress)]*

Comparison of evolution



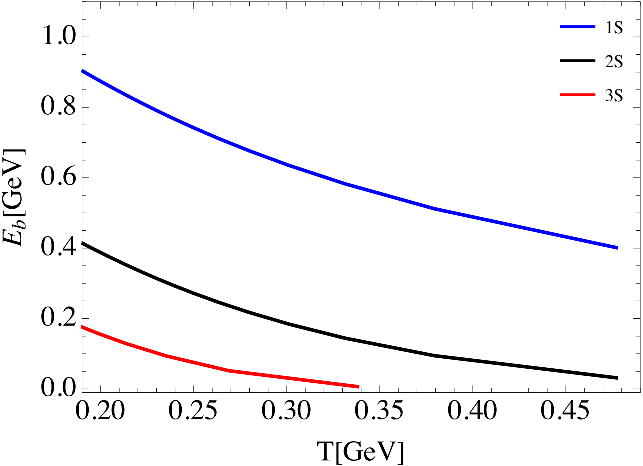
- $\Upsilon(2S)$ *[Vyshakh, Sharma (in progress)]*

Summary

- Forgoing the expansion in $\frac{\tau_{\mathrm{E}}}{\tau_{\mathrm{S}}}$ leads to a master equation with memory
- Leads to a weaker suppression than LO
- NLO captures the reduction in suppression but the extent is quite sensitive to $\tau_{\mathrm{S}} T$

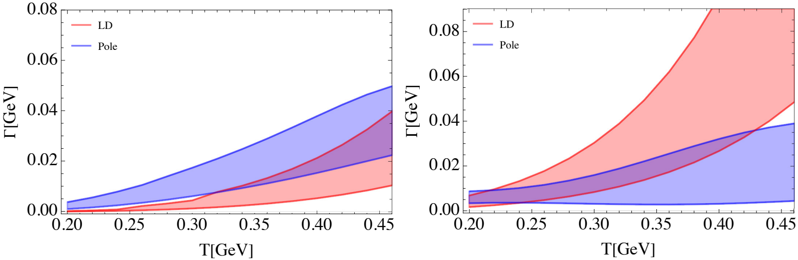
Backup slides

Binding energies



- Binding energy of the states with T
- Model dependence from the choice of V_s, V_o. We take V_s from *[Krouppa, Rothkopf, Strickland (2017); Lafferty, Rothkopf (2019)]*. Slightly weaker V_s in *[HOTQCD (2020, 2021)]*.

Comparison of all contributions



- Decay width with T *[Sharma, Singh (2023)]*