Temperature and quark mass dependence of the heavy quark diffusion coefficient

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Heavy-light mesons in deconfined plasma

- For moderate momenta heavy quarks in a plasma, with $M \gg T$, a Langevin framework can be used.

$$\frac{dp_i}{dt} = \xi_i(t) - \eta_D(p) p_i, \qquad \langle \xi_i(t)\, \xi_j(t') \rangle = \kappa_{ij}\, \delta(t - t')$$

Svetitsky '88; Mustafa et al., '97; Moore & Teaney '05; Rapp & van Hees '05

- Leads to the Fokker-Planck equation

$$\frac{\partial f_Q(p,t)}{\partial t} = -\frac{\partial}{\partial p_i} \left[p_i\, \eta_D(p)\, f_Q(p,t) \right] + \frac{\partial^2}{\partial p_i\, \partial p_j} \left[\kappa_{ij}(p)\, f_Q(p,t) \right]$$

- For low momenta, just one coefficient κ. Standard nonrelativistic relations:

$$\eta_D = \frac{\kappa}{2\, M\, T}, \qquad \langle x^2(t) \rangle = 6 D_s t, \qquad D_s = \frac{2T^2}{\kappa}$$

- A field theoretic definition of κ can be given from the force-force correlator:

$$3\kappa = \frac{1}{\chi} \lim_{\omega \to 0} \int_{-\infty}^{\infty} dt\, e^{i\omega t} \int d^3x \left\langle \frac{1}{2} \left\{ F_i(t,x)\, ,\, F_i(0,0) \right\} \right\rangle$$

Calculation of κ

- Expanding the force term in a series in $1/M$:

$$F^i = M\frac{dJ^i}{dt} = \phi^\dagger \left\{ -gE^i + \frac{[D^i, D^2 + c_b g\sigma\cdot B]}{2M} + ... \right\} \phi$$

- In leading order in $1/M$ one gets only the gE force.

$$\kappa = \frac{1}{3\chi} \lim_{\omega\to 0} \int_{-\infty}^{\infty} dt\, e^{i\omega t} \, \langle \mathrm{Tr}\, W(t,-\infty)^\dagger \, gE_i(t)\, W(t,0)\, gE_i(0)\, W(0,-\infty) \rangle$$

J. Casalderrey-Solana & D. Teaney, PRD 74 (2006) 085012;
S. Caron-Huot, M. Laine & G. Moore, JHEP 04 (2009) 53;
A. Bouttefeux & M. Laine, JHEP 12 (2020) 150

- κ_E has been calculated to NLO in HTL PT.

$$\frac{\kappa_E}{T^3} = \frac{2\,g^4}{27\,\pi}\left[N_c\left(\ln\frac{2T}{m_D} + \xi\right) + \frac{N_f}{2}\left(\ln\frac{4T}{m_D} + \xi\right) + \frac{N_c\, m_D}{T} C + \mathcal{O}(g^2)\right]$$

- Series in g rather than in α. NLO corrections very large.

Caron-Huot & Moore, PRL 100 (2008) 052301

Calculation of κ_E

- $$\kappa_E = \frac{1}{3\chi} \lim_{\omega \to 0} \int_{-\infty}^{\infty} dt\, e^{i\omega t} \, \langle \mathrm{Tr}\, W(t,-\infty)^\dagger \, gE_i(t)\, W(t,0)\, gE_i(0)\, W(0,-\infty) \rangle$$
- Nonperturbative evaluation: calculate the *EE* Matsubara correlator on the lattice.
- Connection to the real correlator through the spectral function.
$$G_{EE}(\tau) = \int_0^\infty \frac{d\omega}{\pi} \, \rho_{EE}(\omega) \, \frac{\cosh \omega(\tau - 1/2T)}{\sinh \omega/2T}$$
- κ_E can be extracted from the infrared behavior of $\rho_{EE}(\omega)$:
$$\rho_{IR} \underset{\omega \to 0}{\approx} \frac{\kappa_E\, \omega}{2T}$$
- The *EE* correlator has been investigated for a gluonic plasma by multiple groups, and κ_E extracted in the temperature range $T_c < T \lesssim 3.5\, T_c$.

κ_E from lattice

- Finding continuum extrapolated $G_{EE}(\tau)$ straightforward as the correlator is ultraviolet finite.
- Only a finite renormalization factor $Z_E(a)$, known to NLO, or its boosted form, in conjunction with standard signal enhancement methods.

Banerjee, Datta, Gavai, Majumdar, PRD (2012), NP A1038(2023)122721
Francis et al. PRD (2015); Brambilla, et al. (TUMQCD), PRD (2020)

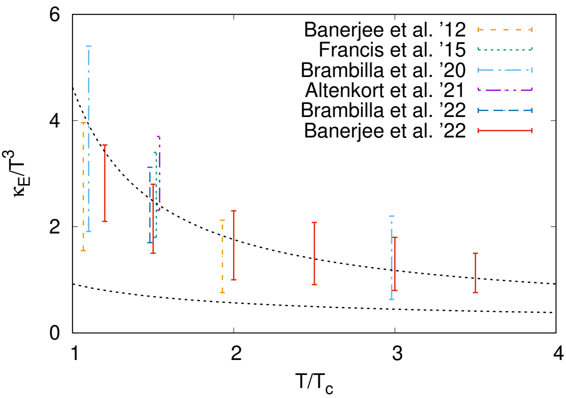
- Besides, Gradient flow has been used in some calculations: both operator renormalization and signal enhancement.

Altenkort et al.,PRD103(2021)014511; Brambilla et al,PRD107(2023)054508

- Extraction of κ_E from $G_{EE}(\tau)$ not straightforward. We try modelling $\rho_{EE}(\omega)$ in various ways, consistent with the limiting behaviours

$$\rho_{EE}^{IR}(\omega) \equiv \frac{\kappa\,\omega}{2T}, \qquad \rho_{UV}(\omega) \equiv \frac{g^2(\mu) C_f \omega^3}{6\pi}$$

κ_E from lattice

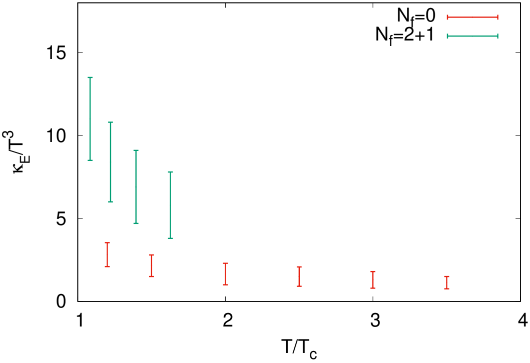


- Results of different groups agree very well.
- The error is dominated by uncertainty in the extraction of $\rho_{EE}(\omega)$.
- Matches well with NLO pert. theory.

κ_E for QCD with $N_f = 2+1$

Remarkably, a calculation with 2+1 flavors of thermal quarks, with $m_\pi \approx 320$ MeV, $T_c \approx 180$ MeV, has been carried out.

HotQCD (Altenkort, et al.), PRL 130 (2023) 231902



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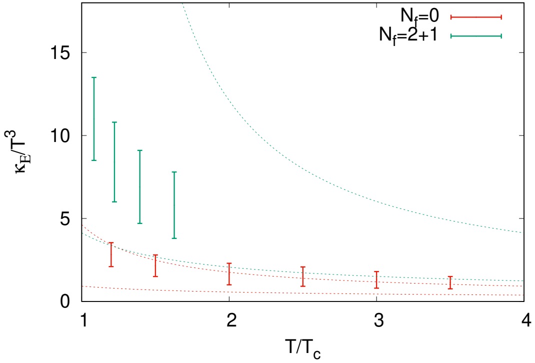

- The results are consistent with NLO pert. theory.

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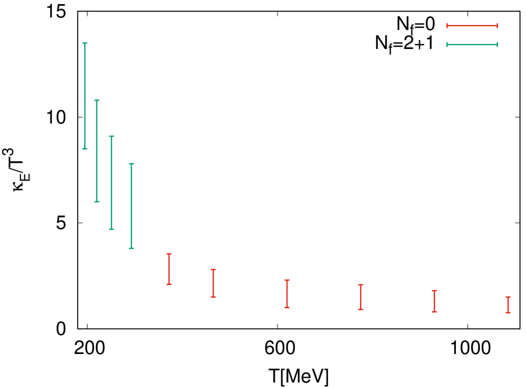
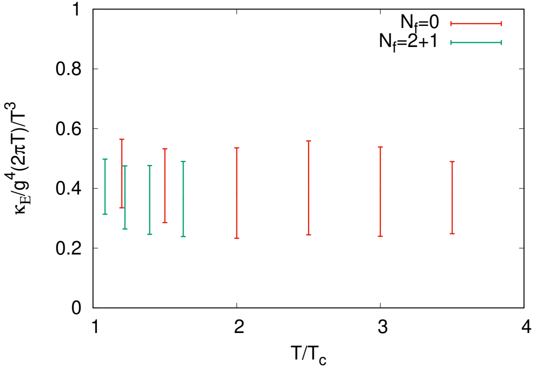


- Much of the N_f dependence can be attributed to the difference in T_c ($T_c^{N_f=0} \approx 310$ MeV).

Parametrization of κ_E

An interesting observation: (Altenkort et al., 2402.09337)
results for κ_E for T near T_c can be parametrized as $c\, g^4(2\pi T)\, T^3$, with similar c for both the gluonic and the 2+1 flavor theory.



- This is not perturbative: coefficient N_f dependent in PT.
- Also in this temperature range g^5 term dominates in NLO PT.

$1/m_Q$ correction

- At $\mathcal{O}\left(\frac{1}{M}\right)$ the magnetic field force needs to be included.
- Under certain assumptions, (Bouttefeux & Laine, JHEP 12 ('20) 150)

$$\kappa_Q \;\approx\; \kappa_E \;+\; \frac{2}{3}\,\langle v^2\rangle\,\kappa_B\,, \qquad \langle \gamma v^2\rangle \;=\; \frac{3T}{M_{\rm kin}}$$

- κ_B is similar to κ_E, for the $B-B$ correlator.
  The B operator has an anomalous dimension.
  We need to calulate

$$c_B(\mu)\,G_{BB}(\mu,T;\tau)$$

  where G_{BB} is the correlator renormalized at scale μ, and c_B is a Wilson coefficient. The μ dependence cancels in the product.
- This becomes equivalent to evaluating the $\overline{MS}$ correlator at scale $\mu_T \approx 19.2T$.

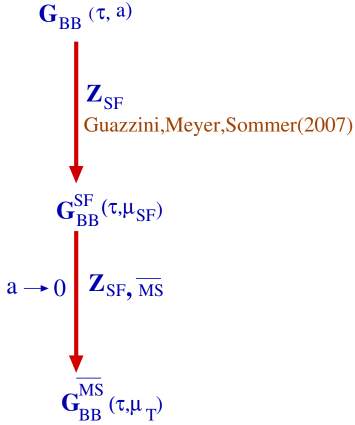
  M. Laine, JHEP 06 ('21) 139
- Renormalize the latice discretized operators, take continuum limit, and calculate $G_{BB}(\mu_T,\tau)$.

κ_B for gluonic plasma

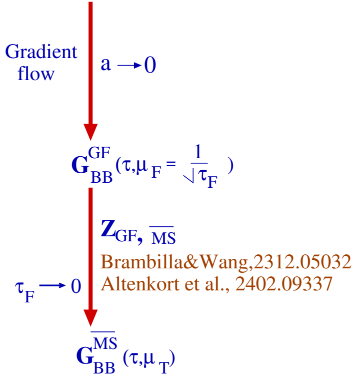
Results for κ_B for gluonic plasma in the temperature range 1-3 T_c.

Banerjee, Datta & Laine, JHEP 08 (2022) 128
Brambilla et al., PRD107(2023)074503, Altenkort et al., 2402.09337

Our calculation: nonperturbative renormalization of ALPHA.

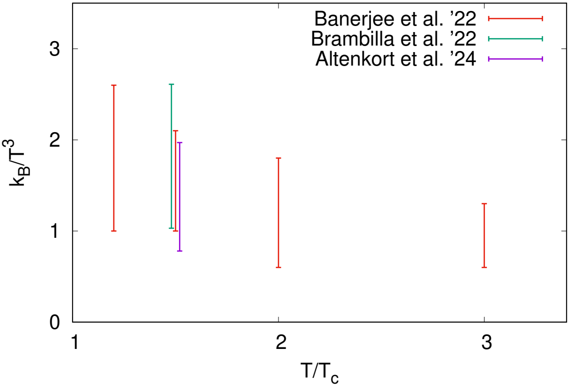


Calculations using gradient flow.



κ_B for gluonic plasma from BB correlator

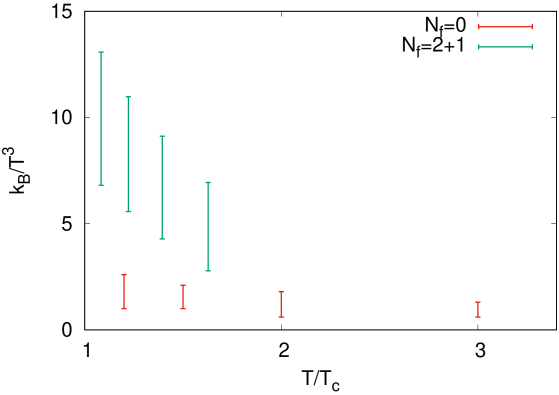
The different calculations give consistent results.



Effect of thermal quarks

The results for the theory with dynamical quarks (with m_π=320 MeV) have come out recently. The calculation uses gradient flow.

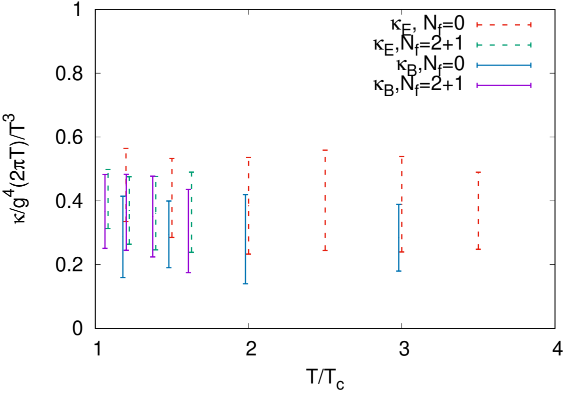
Altenkort et al. (HotQCD), PRL 132 (2024) 051902



The discrepancy is largely due to the difference in T_c.

κ_E, κ_B

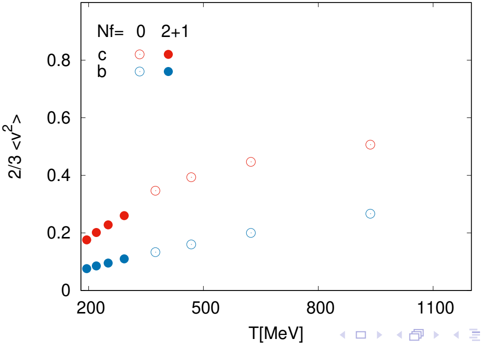
κ_E and κ_B are comparable in magnitude, and both scale approximately like $g^4(2\pi T)T^3$ in the 1-3 T_c range.



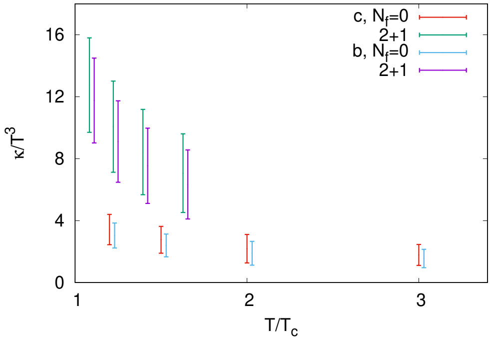
κ_Q to $\mathcal{O}\left(\frac{1}{M}\right)$

- To $\mathcal{O}\left(\frac{1}{M}\right)$ $\kappa_Q \approx \kappa_E + \frac{2}{3}\langle v^2\rangle\, \kappa_B$
- With $\kappa_B \lesssim \kappa_E$, $\langle v^2\rangle$ gives the size of $\mathcal{O}\left(\frac{1}{M}\right)$ correction.
- We obtained $\langle v^2\rangle$ from the constant part of $\frac{\langle J^i\, J^i\rangle}{\langle J^0\, J^0\rangle}$.

  Burnier & Laine, JHEP 11 ('12) 086
- Calculated from the susceptibility in Altenkort et al.



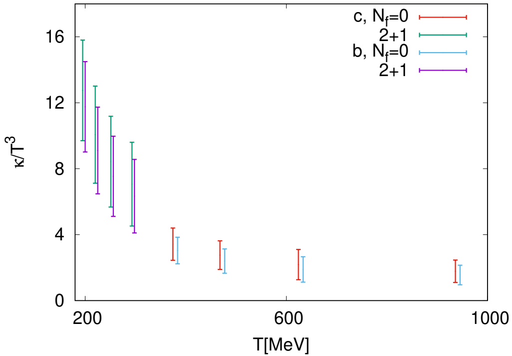
κ_c, κ_b



• Correction to the static limit is $\lesssim$ 10% for bottom near T_c for the gluonic theory, rising to $\approx$ 15% at 2 T_c. For the three flavor theory it is $<$ 10% for $T < 1.6T_c$.

• Even for charm the $1/M$ correction is $\approx$ 25% near T_c for the quenched and $<$ 20% for the 3-flavor theory.

κ_c, κ_b



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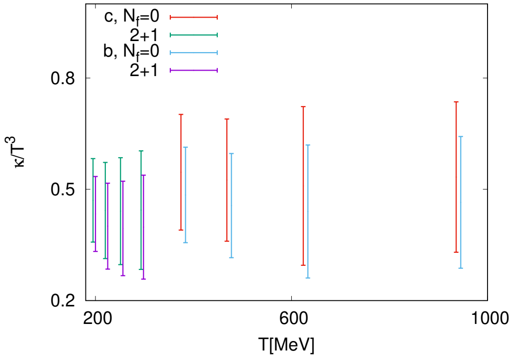
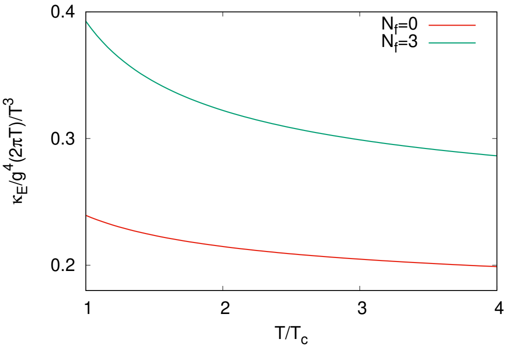
Summary

- The heavy quark diffusion coefficient can be calculated on the lattice using a nonrelativistic expansion.
- The first two terms in the expansion have been calculated.
- For the quenched theory (only thermal gluons), results of different groups agree.
  The different calculations differ in the renormalization procedure, but the analysis is similar.
- Results for $N_f = 2 + 1$ thermal quarks have been recently available.
- Much of the difference of the quenched and $N_f = 2 + 1$ results can be attributed to the difference in T_c.
- The temperature dependence of both κ_E and κ_B can be parametrized as $c\, g^4(2\pi T)\, T^3$, with $c \sim 0.3 - 0.4$.

EXTRA SLIDE: Behavior of $\kappa_{c,b}/g^4T^3$ in NLO

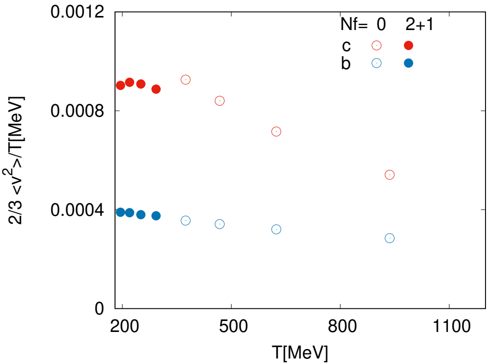


EXTRA SLIDE: Behavior of κ_E/g^4T^3 in NLO



- For the $N_f = 3$ curve we have taken T_c=180 MeV.
- At these temperatures the NLO term $\sim g^5$ dominates.

EXTRA SLIDE: $\langle v^2 \rangle / T$



EXTRA SLIDE: $\rho(\omega)$ and κ_E

- $\rho^{IR}_{EE}(\omega) \equiv \frac{\kappa\,\omega}{2T}$, $\quad \rho_{UV}(\omega) \equiv \frac{g^2(\mu) C_f \omega^3}{6\pi}$, $\quad \mu = \max(\omega, \pi T)$
- Model $\rho(\omega)$ as

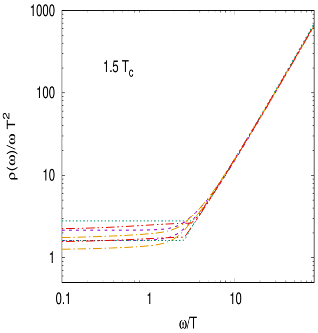
  $\rho_1(\omega) \equiv \max\left(\rho^{IR}_{EE}(\omega),\ c\rho_{UV}(\omega)\right)$
- Physically better motivated: Francis et al. '15

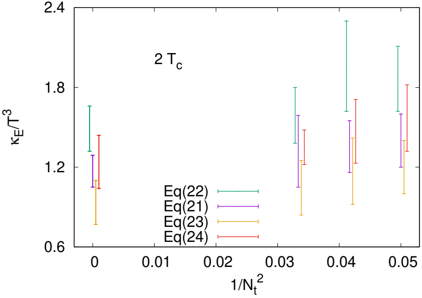
  $\rho_2(\omega) \equiv \sqrt{(\rho^{IR}_{EE}(\omega))^2 + (c\rho_{UV}(\omega))^2}$
- Also tried adding a Fourier mode (Francis et al. '15)

$$\rho_{1f,2f}(\omega) \equiv (1 + d \sin \pi y)\ \rho^{c=1}_{1,2}(\omega), \qquad y = \frac{\log\left(1 + \frac{\omega}{\pi T}\right)}{1 + \log\left(1 + \frac{\omega}{\pi T}\right)}$$

- At each temperature, calculate the correlator at three lattice spacings and take continuum limit.
- took a superset of the values obtained from different forms.
- Also looked at the fits for individual lattices.

κ_E


1000
100
10
1
0.1
1
10
1.5 T_c
ρ(ω)/ω T^2
ω/T


2.4
1.8
1.2
0.6
0
0.01
0.02
0.03
0.04
0.05
2 T_c
κ_E/T^3
1/N_t^2
Eq(22)
Eq(21)
Eq(23)
Eq(24)

EXTRA SLIDE: Analysis of $G_{BB}(\tau)$

- $G_{BB}(\tau)$ has very different shape from $G_{EE}(\tau)$, in particular at short times.
- Analysis similar to the *EE* correlator, but subtle differences arise because of the anomalous dimension.

$$\rho^{IR}_{EE}(\omega) \equiv \frac{\kappa\,\omega}{2T}, \qquad \rho_{UV}(\omega) \equiv \frac{g^2(\mu) C_f \omega^3}{6\pi}, \qquad \mu = \max\left(\omega^{1-\frac{\gamma_0}{b_0}} (\pi T)^{\gamma_0/b_0},\ \pi T\right)$$