Effect of magnetic field on heavy Quarkonia immersed in a hot and dense QCD medium

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Magnetic fields in noncentral events in uRHIC collisions

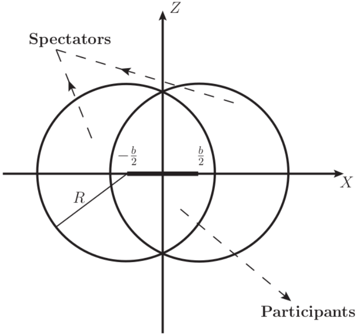


Figure: *

Schematic of a non-central collision event.[Bethke, Prog. Part. Nucl. Phys. 2007].

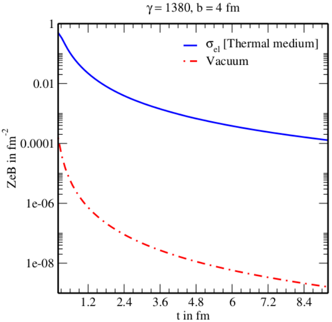


Figure: *

Variation of Magnetic field with time.[Shubhalaxmi, BP JHEP 2019)].

- Magnetic field produced by fast moving spectator quarks.
- Magnetic fields of the order of 10^{17}-10^{19} Gauss[Kharzeev *et al*, Nucl.Phys. A **803**, 227 (2008)].

Decay of magnetic field strength suppressed in a conducting medium

- Motivates the properties of heavy quarkonia in the presence of magnetic field.

Motivation

The properties of heavy quarkonia at finite temperature and chemical potential have been established up to some extent, **Matsui and Satz**

- Magnetic field is produced in non-central events of collider experiments. However, magnetic field decays very fast in vacuum but it could be strong or weak, depending on:
- i) The time scale of heavy quark pairs formed in the plasma frame
- ii) the electrical properties of medium.

- Since magnetic field breaks the rotational symmetry, thus one need to revisit the covariant structure of gluon self-energy tensor.
- To be specific, how the studies of quarkonia already done in finite temperature get altered in weak and strong magnetic field.

Outline

- Resummed Gluon Propagator:
  $(T \neq 0, \mu_q \neq 0)$; $(T \neq 0, \mu_q \neq 0, \text{Strong } B \neq 0)$; $(T \neq 0, \text{Weak } B \neq 0)$
- Heavy Quarkonia in $T \neq 0, \mu_q \neq 0$
- Heavy Quarkonia in $T \neq 0, \text{Strong } B \neq 0$
- Heavy Quarkonia in $T \neq 0, \mu_q \neq 0, \text{Strong } B \neq 0$
- Heavy Quarkonia in $T \neq 0, \text{Weak } B \neq 0$
- Resummed Quark Propagator:
  $(T \neq 0, \mu_q \neq 0)$; $(T \neq 0, \mu_q \neq 0, \text{Strong } B \neq 0)$; $(T \neq 0, \mu_q \neq 0, \text{Weak } B \neq 0)$

Quarkonium Spectroscopy

- Since $M_{Q\bar{Q}} \gg \Lambda_{\rm QCD}$ so the non-relativistic potential theory is a useful method for studying quarkonium bound states and describes the spectroscopy of quarkonium well.
- Schrödinger equation: Properties of Quarkonia

$$\boxed{\left[2m_Q - \frac{1}{m_Q}\nabla^2 + V(r)\right]\phi_i(r) = M_i\phi_i(r)}$$

$\Rightarrow M_i$: Mass of i-th bound state
$\Rightarrow \phi_i(r)$: Corresponding eigenfunctions
$\Rightarrow < r_i^2 >= \int d^3r\, r^2|\phi_i(r)|^2$: Size

State	J/ψ	χ_c	ψ'	Υ	χ_b	Υ'	χ_b'	Υ''
mass(GeV)	3.10	3.53	3.68	9.46	9.99	10.02	10.26	10.36
B.E. (GeV)	0.64	0.20	0.05	1.10	0.67	0.54	0.31	0.20
r_0 (fm)	0.50	0.72	0.90	0.28	0.44	0.56	0.68	0.78

Table: Masses, Binding energies etc. for different quarkonium states.

Layout of our calculation

- Thermalization of quark and gluon propagators.
- Calculation of the re-summed retarded/advanced and symmetric propagators by the real and imaginary part of retarded/advanced self-energies.
- The real and imaginary component of dielectric permittivities are obtained by the resummed retarded and symmetric propagators, repespectively.
- Inverse Fourier transform gives the real and imaginary parts of $Q\bar{Q}$ potential.
- Schrödinger equation is solved with the real-part of the potential to obtain both the energy eigenvalues and eigenfunctions to calculate the size and binding energies of quarkonia.
- The imaginary part is used to estimate the medium-induced thermal width of the resonances, *Landau damping*.

Propagators in Real-time Formalism

- Propagator at finite temperature develops into a matrix structure

$$D_{ab}(K) = U_{ac}(K) \begin{pmatrix} D_0(K) & 0 \\ 0 & (D_0(K))^* \end{pmatrix}_{cd} U_{db}(K)$$

$$U(K) = \begin{pmatrix} \sqrt{1 \pm n(k_0)} & \pm n(k_0) \\ n(k_0) & \sqrt{1 \pm n(k_0)} \end{pmatrix}$$

- Keldysh representation:

$$D_R^0 = D_{11}^0 - D_{12}^0, \; D_A^0 = D_{11}^0 - D_{21}^0, \; D_S^0 = D_{11}^0 + D_{22}^0,$$
$$\Pi_R = \Pi_{11} + \Pi_{12}, \; \Pi_A = \Pi_{11} + \Pi_{21}, \; \Pi_S = \Pi_{11} + \Pi_{22}$$

- Re-summation by Dyson-Schwinger equation:

$$D_{R,A}^L = D_{R,A}^{L(0)} + D_{R,A}^{L(0)} \Pi_{R,A}^L D_{R,A}^L$$

$$D_S^L = D_S^{L(0)} + D_R^{L(0)} \Pi_R^L D_{S(0)}^L + D_S^0 \Pi_A D_A + D_R^0 \Pi_S D_A$$

- Expressed explicitly by the self-energies

$$D_{R,A}^L(k) = \frac{1}{\mathbf{k}^2 - \mathrm{Re}\Pi_R^L(k) \mp \mathrm{i}\mathrm{Im}\Pi_R^L(k)},$$
$$D_S^L(k) = (1 + 2n_B(k_0))\, \mathrm{sgn}(k_0) \left(D_R^L(k) - D_A^L(k) \right)$$

Gluon Self-energy at $T \neq 0$

Heat bath defines a local rest frame, u_μ, so the Lorentz invariance is broken. So larger tensor basis is needed, constructed by the available four-vectors, k_μ, u_μ and tensor $g_{\mu\nu}$ by the two orthogonal tensors, compatible with the physical degree of freedom:

$$\Pi^{\mu\nu}(p_0, \mathbf{p}) = \Pi_T(p_0, \mathbf{p}) P_T^{\mu\nu} + \Pi_L(p_0, \mathbf{p}) P_L^{\mu\nu}$$

where

$$P_T^{\mu\nu} = g^{\mu\nu} - \frac{P^\mu P^\nu}{P^2} - P_L^{\mu\nu}$$

$$P_L^{\mu\nu} = -\frac{P^2}{p^2}\left(u^\mu - \frac{p^0 P^\mu}{P^2}\right)\left(u^\nu - \frac{p^0 P^\nu}{P^2}\right)$$

$u^\mu = (1, 0, 0, 0)$

$P_\mu P_T^{\mu\nu} = P_\mu P_L^{\mu\nu} = 0$

- Longitudinal and transverse parts of self-energy:

$$\Pi_L(p_0, p) = -\frac{(p_0^2 - p^2)}{p^2} m_D^2 \left(1 - \frac{p_0}{2p} \ln \frac{p_0 + p}{p_0 - p}\right), \quad (1)$$

$$\Pi_T(p_0, p) = \frac{m_D^2}{2} \frac{p_0^2}{p^2} \left(1 - \frac{(p_0^2 - p^2)}{2p_0 p} \ln \frac{p_0 + p}{p_0 - p}\right) \quad (2)$$

$$\text{Debye Mass} \quad m_D^2 = g^2 T^2 \left\{\frac{N_c}{3} + \frac{N_f}{6}\right\} \quad (3)$$

Gluon Self-energy at $T \neq 0$ continued..

- Resummed gluon propagator:

$$D^{\mu\nu} = \frac{P_T^{\mu\nu}}{P^2 + \Pi_T} + \frac{P_L^{\mu\nu}}{P^2 + \Pi_L} \tag{4}$$

- Real and imaginary part (in static limit):

$$\mathrm{Re}\ D^{00}(p_0 = 0, p) = -\frac{1}{p^2 + m_D^2}, \qquad \mathrm{Im}\ D^{00}(p_0 = 0, p) = \frac{\pi T m_D^2}{p(p^2 + m_D^2)^2} \tag{5}$$

- Nonperturbative contribution to the $Q\bar{Q}$ potential: A nonperturbative term induced by dimension two gluon condensate is added to the HTL full gluon propagator in order to incorporate the long distance behaviour of the potential.
- Real-part of the resummed propagator:

$$\mathrm{Re}\ D^{00}(p_0 = 0, p) = -\frac{1}{p^2 + m_D^2} - \underbrace{\frac{m_G^2}{(p^2 + m_D^2)^2}}_{m_G^2 = \frac{2\sigma}{\alpha}} \tag{6}$$

Gluon Self-energy at $T \neq 0$: $Q\bar{Q}$ Potential at $T \neq 0$ Vineet, BP PRD 2010, EPJC 2011

- Imaginary part of the resummed propagator:

$$\mathrm{Im}\, D^{00}(p_0 = 0, p) = \frac{\pi T m_D^2}{p(p^2+m_D^2)^2} + \underbrace{\frac{2\pi T m_D^2 m_G^2}{p(p^2+m_D^2)^3}} \qquad (7)$$

- Real-part of the potential ($\hat{r} = r m_D$):

$$\mathrm{Re}\, V(r;T) = C_F\, g^2 \int \frac{d^3p}{(2\pi)^3}(e^{ip.r}-1)\,\mathrm{Re}\, D^{00}(p_0=0,p) \qquad (8)$$

$$= -\frac{4}{3}\alpha_s\left(\frac{e^{-\hat{r}}}{r}+m_D\right)+\frac{4}{3}\frac{\sigma}{m_D}\left(1-e^{-\hat{r}}\right) \qquad (9)$$

- Schrödinger Equation $\Rightarrow$ Binding Energy

$$\boxed{\left[-\frac{1}{m_Q}\nabla^2 + V(r;T)\right]\phi^i{}_n(r) = E^i{}_n\phi^i{}_n(r)}$$

Imaginary-part of $Q\bar{Q}$ potential at $T \neq 0$

$$
\begin{aligned}
\mathrm{Im}\; V(r;T) &= C_F\, g^2 \int \frac{d^3p}{(2\pi)^3}(e^{ip.r}-1)\;\mathrm{Im}\; D^{00}(p_0=0,p) && (10)\\
&= -\frac{4}{3}\alpha_s T\psi_1(\hat{r}) - \frac{16\sigma T}{3m_D^2}\psi_2(\hat{r})\\
\psi_1(\hat{r}) &\approx -\frac{1}{9}\hat{r}^2\,(3\ln\hat{r}-4+3\gamma_E)\\
\psi_2(\hat{r}) &\approx \frac{\hat{r}^2}{12}+\frac{\hat{r}^4}{900}\,(15\ln\hat{r}-23+15\gamma_E) && (11)
\end{aligned}
$$

• Thermal Width: $\boxed{\Gamma_i = -2\int_0^\infty \mathrm{Im}\; V(r;B,T)|\Phi_i(r)|^2 d\tau}$

• $Q\bar{Q}$ potential at $T \neq 0$, $\mu \neq 0$:
Potential will remain the same, except the Debye mass will now be modified as:

$$
m_D^2 = g^2T^2\left\{\frac{N_c}{3}+\frac{N_f}{6}\left(1+\frac{3\mu^2}{\pi^2T^2}\right)\right\} \qquad (12)
$$

The dissociation chemical potentials of J/ψ at T = 40 and 50 MeV are found to be 1611 and 1560 MeV, respectively.

Gluon Self-energy at $T \neq 0$, Strong B, Mujeeb, Bhaswar, BP EPJC 2017, Mujeeb, Bhaswar, BP, Partha NPB 2019

Magnetic field breaks the rotational symmetry in the system because it introduces an anisotropy in space. Thus, one can define a four vector b_μ.

•Tensor basis: In the rest frame of the heat bath, $u^\mu = (1,0,0,0)$, b^μ can be defined uniquely as projection of $\tilde{F}_{\mu\nu}$ along u^ν:

$$b_\mu = \frac{1}{2B}\epsilon_{\mu\nu\rho\lambda}u^\nu F^{\rho\lambda} = \frac{1}{B}u^\nu \tilde{F}_{\mu\nu} = (0,0,0,1). \tag{13}$$

•Therefore, in addition to the P_L and P_T, we have,

$$\begin{aligned}
P_\parallel^{\mu\nu}(k) &= -\frac{k^0 k^z}{k_\parallel^2}\left(b^\mu u^\nu + u^\mu b^\nu\right) + \frac{1}{k_\parallel^2}\left[\left(k^0\right)^2 b^\mu b^\nu + \left(k^z\right)^2 u^\mu u^\nu\right], \\
&= -\left(g_\parallel^{\mu\nu} - \frac{k_\parallel^\mu k_\parallel^\nu}{k_\parallel^2}\right), \\
P_\perp^{\mu\nu}(k) &= \frac{1}{k_\perp^2}\Big[-k_\perp^2 g^{\mu\nu} + k^0\left(k^\mu u^\nu + u^\mu k^\nu\right) - k^z\left(k^\mu b^\nu + b^\mu k^\nu\right) + \\
&\quad k^0 k^z\left(b^\mu u^\nu + u^\mu b^\nu\right) - k^\mu k^\nu + \left(k_\perp^2 - \left(k^0\right)^2\right) u^\mu u^\nu - \mathbf{k}^2 b^\mu b^\nu\Big], \\
&= -\left(g_\perp^{\mu\nu} - \frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2}\right)
\end{aligned} \tag{14}$$

$g_\parallel^{\mu\nu} = \mathrm{diag}(1,0,0-1)$, $g_\perp^{\mu\nu} = \mathrm{diag}(0,-1,-1,0)$, $k_\parallel^2 = (k_0)^2 - (k_z)^2$, $k_\perp^2 =$

Gluon-self energy at finite T, strong B *continued*

$$\Pi^{\mu\nu}(k) = P_L^{\mu\nu}\Pi^{g,L}(k) + P_T^{\mu\nu}\Pi^{g,T}(k) + P_\parallel^{\mu\nu}\Pi^{q,\parallel}(k) + P_\perp^{\mu\nu}\Pi^{q,\perp}(k), \qquad (15)$$

- In strong magnetic field, $\Pi^{q,\perp} \approx 0$, therefore, longitudinal-component of gluon self-energy ($P_T^{00}, P_\perp^{00} = 0$)

$$\Pi(k) = \Pi^{q,\parallel}(k) + \Pi^{g,L}(k), \qquad (16)$$

- The 11-component of quark propagator

$$iS_{11}(p) = \left[\frac{1}{p_\parallel^2 - m_f^2 + i\epsilon} + 2\pi i n(p_0)\,\delta\left(p_\parallel^2 - m_f^2\right)\right] \left(1 + \gamma^0\gamma^3\gamma^5\right)\left(\gamma^0 p_0 - \gamma^3 p_z + m_f\right) e^{\frac{-p_\perp^2}{|q_f B|}}. \qquad (17)$$

- Hence the "11"-component of the self-energy matrix

$$\Pi^{q,\mu\nu}(k) = \frac{ig^2}{2}\sum_f \int \frac{d^4p}{(2\pi)^4}\,\mathrm{Tr}\left[\gamma^\mu S_{11}(p)\gamma^\nu S_{11}(q)\right]$$

Gluon-self energy at finite T, strong B *continued*

- The trace over the γ matrices is calculated as

$$L^{\mu\nu} = 8\left[p_{\parallel}^{\mu} q_{\parallel}^{\nu} + p_{\parallel}^{\nu} q_{\parallel}^{\mu} - g_{\parallel}^{\mu\nu}\left((p\cdot q)_{\parallel} - m_f^2\right)\right].$$

- Running coupling, g is

$$\alpha_s^{\parallel}(eB) = \frac{g^2}{4\pi} = \frac{1}{\alpha_s^0(\mu_0)^{-1} + \frac{11N_c}{12\pi}\ln\left(\frac{k_z^2+M_B^2}{\mu_0^2}\right) + \frac{1}{3\pi}\sum_f \frac{|q_f B|}{\sigma}},$$

$$\alpha_s^0(\mu_0) = \frac{12\pi}{11N_c \ln\left(\frac{(\mu_0^2+M_B^2)}{\Lambda_V^2}\right)}.$$

where $k_z = 0.1\sqrt{eB}$, $\mu_0 = 1.1\mathrm{GeV}$, $\Lambda_V = 0.385\mathrm{GeV}$, $M_B \sim 1\mathrm{GeV}$, and $\sigma = 0.18\mathrm{GeV}^2$.

- $\Pi^{q,\mu\nu}(k)$ becomes factorizable:

$$\Pi^{q,\mu\nu}(k) = \sum_f \Pi^{q,\mu\nu}(k_{\parallel})\, A(k_{\perp}),\; A(k_{\perp}) = \frac{\pi\,|q_f|\,B}{2}.$$

- Decomposing into vacuum and medium components:

$$\Pi^{q,\mu\nu}(k_{\parallel}) \equiv \Pi_V^{q,\mu\nu}(k_{\parallel}) + \Pi_n^{q,\mu\nu}(k_{\parallel}) + \Pi_{n^2}^{q,\mu\nu}(k_{\parallel})$$

Gluon-self energy at finite T, strong B *continued*

$$\Pi_V^{q,\mu\nu}\left(k_\parallel\right) = \frac{ig^2}{2(2\pi)^4}\int dp_0 dp_z L^{\mu\nu}\left\{\frac{1}{\left(q_\parallel^2 - m_f^2 + i\epsilon\right)}\frac{1}{\left(p_\parallel^2 - m_f^2 + i\epsilon\right)}\right\},$$

$$\Pi_n^{q,\mu\nu}\left(k_\parallel\right) = \frac{ig^2(2\pi i)}{2(2\pi)^4}\int dp_0 dp_z L^{\mu\nu}\left\{n\left(p_0\right)\frac{\delta\left(p_\parallel^2 - m_f^2\right)}{\left(q_\parallel^2 - m_f^2 + i\epsilon\right)} + n\left(q_0\right)\frac{\delta\left(q_\parallel^2 - m_f^2\right)}{\left(p_\parallel^2 - m_f^2 + i\epsilon\right)}\right\},$$

$$\Pi_{n^2}^{q,\mu\nu}\left(k_\parallel\right) = \frac{ig^2}{2(2\pi)^4}\int dp_0 dp_z L^{\mu\nu}\left\{\left(-4\pi^2\right) n\left(p_0\right) n\left(q_0\right)\delta\left(p_\parallel^2 - m_f^2\right)\delta\left(q_\parallel^2 - m_f^2\right)\right\}.$$

- Real- and imaginary- part of $\Pi_V^{q,\mu\nu}\left(k_\parallel\right)$ for massless case

$$\boxed{\mathrm{Re}\,\Pi_V{}^{q,\parallel}\left(k_0,k_z\right) = \frac{g^2}{4\pi^2}\sum_f |q_f B|\,\frac{k_z^2}{k_\parallel^2};\quad \mathrm{Im}\Pi_V^{q,\parallel}\left(k_0,k_z\right) = 0}$$

- Real- and imaginary- part of $\Pi_n^{q,\mu\nu}\left(k_\parallel\right)$

$$\begin{aligned}
\mathrm{Re}\Pi_n^{q,\parallel}\left(k_\parallel\right) &= -\frac{g^2}{2(2\pi)^3}\int dp_0 dp_z L^{00}\Bigg[n\left(p_0\right)\frac{\left\{\delta\left(p_0-\omega_p\right)+\delta\left(p_0+\omega_p\right)\right\}}{2\omega_p\left(q_0^2-q_z^2-m_f^2\right)} \\
&+ n\left(q_0\right)\frac{\left\{\delta\left(q_0-\omega_q\right)+\delta\left(q_0+\omega_q\right)\right\}}{2\omega_q\left(p_0^2-p_z^2-m_f^2\right)}\Bigg]
\end{aligned}$$

Gluon-self energy at finite T, strong B *continued*

$$L^{00} = 8\left[p_0 q_0 + p_z q_z + m_f^2\right]\ ;\ \omega_p \quad = \sqrt{p_z^2 + m_f^2};\ \omega_q = \sqrt{q_z^2 + m_f^2} = \sqrt{(p_z - k_z)^2 + m_f^2}.$$

$$\mathrm{Re}\ \Pi_n^{q,\|}(k_\|) = -\frac{4g^2}{(2\pi)^3}\frac{k_z T}{k_0^2 - k_z^2}\left(\int_{-\infty}^{\infty} dt\,\frac{1}{e^t + 1} - \int_{-\infty}^{\infty} dt'\,\frac{1}{e^{t'} + 1}\right) = 0$$

⇒ demonstrates the results of (1+1) dimensional massless QED, Schwinger model, where the medium does not permeate to the vaccum.

- $\mathrm{Im}\ \Pi_n^{q,\|}\left(k_\|\right) = 0.$
- $\mathrm{Re}\Pi_{n^2}^{q,\|}\left(k_0, k_z\right) = 0.$
- Imaginary part is also obtained from the retarded current-current correlator as

$$\mathrm{Im}\Pi_{n^2}^{q,\|}(k_0, k_z) = -\frac{g^2\sum_f |q_f|B}{8\pi}k_0[\delta(k_0 - k_z) + \delta(k_0 + k_z)],$$

•$|q_f B|/8\pi$ is the transverse density of states for the LLL states.

- Real-part of quark-loop contribution by the vacuum contribution only:

$$\mathrm{Re}\Pi^{q,\|}(k_0, k_z) = \mathrm{Re}\Pi_V^{q,\|}(k_0, k_z) = \frac{g^2}{4\pi^2}\sum_f |q_f|B\frac{k_z^2}{k_\|^2}\ ,$$

Gluon-self energy at finite T, strong B *continued*, Mujeeb, Bhaswar, BP,EPJC 2017

- Imaginary-part of the quark-loop contribution:

$$\mathrm{Im}\Pi^{q,\parallel}(k_0,k_z) = \mathrm{Im}\Pi^{q,\parallel}_{n^2}(k_0,k_z) = \frac{-g^2\sum_f |q_f|B}{8\pi} k_0[\delta(k_0-k_z)+\delta(k_0+k_z)]$$

- **Gluon-loop:** The real- and imaginary-parts are

$$\mathrm{Re}\Pi^{g,L}(k_0,\mathbf{k}) = g'^2T^2\left(\frac{k_0}{2|\mathbf{k}|}\ln\left|\frac{k_0+|\mathbf{k}|}{k_0-|\mathbf{k}|}\right|-1\right),$$

$$\mathrm{Im}\Pi^{g,L}(k_0,\mathbf{k}) = -g'^2T^2\frac{\pi k_0}{2|\mathbf{k}|};\quad \alpha'(T)=\frac{6\pi}{(33-2N_f)\ln(\frac{2\pi T}{\Lambda_{QCD}})},$$

- Gluon self-energy in $T\neq 0$ and strong B:

$$\begin{aligned}
\mathrm{Re}\Pi^{L'}(k_0,\mathbf{k}) &= \mathrm{Re}\,\Pi^{g,L}(k_0,\mathbf{k})+\mathrm{Re}\,\Pi^{q,\parallel}(k_0,k_z),\\
&= g'^2T^2\left(\frac{k_0}{2|\mathbf{k}|}\ln\left|\frac{k_0+|\mathbf{k}|}{k_0-|\mathbf{k}|}\right|-1\right)+\frac{g^2}{4\pi^2}\sum_f |q_f|B\frac{k_z^2}{k_\parallel^2},\\
\mathrm{Im}\Pi^{L'}(k_0,\mathbf{k}) &= \mathrm{Im}\Pi^{g,L}(k_0,\mathbf{k})+\mathrm{Im}\Pi^{q,\parallel}(k_0,k_z),\\
&= -g'^2T^2\frac{\pi k_0}{2|\mathbf{k}|}-\frac{g^2\sum_f|q_f|B}{8\pi}k_0[\delta(k_0-k_z)+\delta(k_0+k_z)] \quad (18)
\end{aligned}$$

•**For massles quarks** $m_D^2 = g'^2 T^2 + \frac{g^2}{4\pi^2} \sum_f |q_f B|$

•**For physical quark masses** $m_D^2 = g'^2 T^2 + \frac{g^2}{2\pi^2 T} \sum_f |q_f B| \int_0^\infty dp_z \frac{e^{\beta\sqrt{p_z^2+m_f^2}}}{\left(1+e^{\beta\sqrt{p_z^2+m_f^2}}\right)^2}$

•Real-part of resummed gluon propagator:

$$\text{Re } D^{00}(p_0 = 0, p) = -\frac{1}{p^2 + m_D^2} - \frac{m_G^2}{(p^2 + m_D^2)^2} \tag{19}$$

- Imaginary-part of resummed gluon propagator:

$$\text{Im } D^{00}(p_0 = 0, p) = \sum_f \frac{g^2 |q_f B| m_f^2}{4\pi} \frac{1}{p_3^2 (p^2 + m_D^2)^2} + \frac{\pi T m_g^2}{p(p^2 + m_D^2)^2} + \frac{2\pi T m_g^2 m_G^2}{p(p^2 + m_D^2)^3} \tag{20}$$

- Real-part of the potential:

$$\boxed{\text{Re } V(r; T, B) = -\frac{4}{3}\alpha_s \left(\frac{e^{-\hat{r}}}{r} + m_D(T, B)\right) + \frac{4}{3} \frac{\sigma}{m_D(T, B)} \left(1 - e^{-\hat{r}}\right)}$$

•The long range part of the quarkonium potential is affected much more by magnetic field as compared to the short range part.

Quarkonia at $T \neq 0$, Strong B, Mujeeb, BP, Bhaswar, Partha NPA 2020

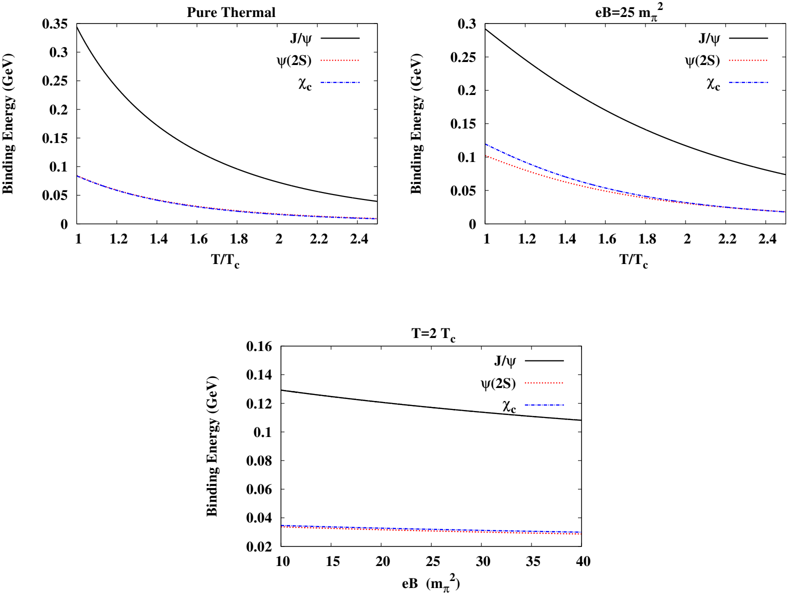
•Imaginary-part of the $Q\bar{Q}$-potential:

$$\mathrm{Im}\, V(r;T,B) = \sum_f \alpha_s g^2 m_f \frac{|q_f B|}{3\pi^2}\left[\frac{\pi}{2m_D^3} - \frac{\pi e^{-\hat{r}}}{2m_D^3} - \frac{\pi \hat{r} e^{-\hat{r}}}{2m_D^3} - \frac{2\hat{r}}{m_D}\int_0^\infty \frac{pdp}{(p^2+m_D^2)^2} \int_0^{pr} \frac{\sin t}{t} dt\right] - \frac{4}{3}\frac{\alpha_s T m_g^2}{m_D^2}\psi_1(\hat{r}) - \frac{16\sigma T m_g^2}{3m_D^4}\psi_2(\hat{r}) \quad (21)$$

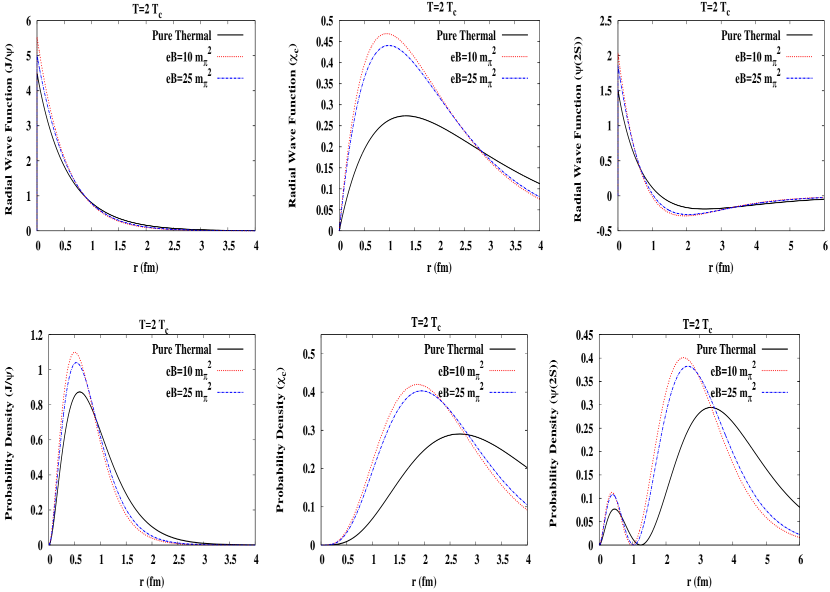
- With $eB = 6m_\pi^2$ the J/ψ is dissociated at $2T_c$, and with $eB = 4m_\pi^2$ the χ_c is dissociated at $1.1T_c$ in comparison, with eB = 0 the J/ψ is dissociated at $T = 1.6T_c$ and χ_c is dissociated at $T = 0.8T_c$.
- However, with the further increase of magnetic field the dissociation temperatures decrease.

State	Pure Thermal (eB=0) T_D (in T_c)	In presence of magnetic field T_D (in T_c) (eB (m_π^2))
J/ψ	1.60	2.0 (6.50) 1.8 (27.0) 1.5 (68.0)
χ_c	0.80	1.1 (3.7) 1.0 (12)
$\psi(2S)$	0.70	$< 1 (< m_\pi^2)$

Temperature dependence of binding energy

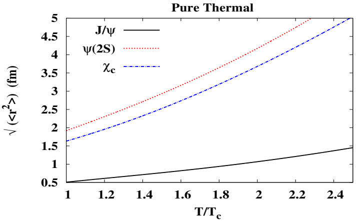


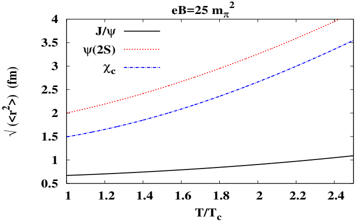
Wavefunction and Probability density



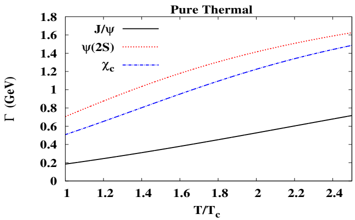
Continue..

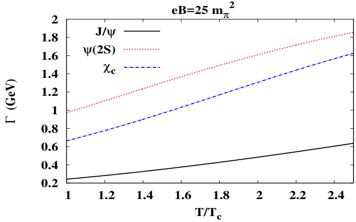
- **Average Size:** $\sqrt{r_i^2} = \left(\int d\tau\ r^2 \mid \Phi_i(r) \mid^2\right)^{1/2}$





- **Thermal Width:** $\Gamma_i = -2\int_0^\infty \mathrm{Im}\ V(r;B,T)|\Phi_i(r)|^2 d\tau$





$Q\bar{Q}$ potential at $T \neq 0$, $\mu \neq 0$ and strong B

- Real part of resummed gluon propagator:

$$\mathrm{Re}\ D^{00}(p_0 = 0, p) = -\frac{1}{p^2 + m_D^2(T, B, \mu)} - \frac{m_G^2}{(p^2 + m_D^2(T, B, \mu))^2} \qquad (22)$$

- Imaginary part of resummed gluon propagator:

$$\mathrm{Im}\ D^{00}(p_0 = 0, p) = \sum_f \frac{g^2 |q_f B| m_f^2}{4\pi} \frac{1}{p_3^2 (p^2 + m_D^2(T, B, \mu))^2} + \frac{\pi T m_g^2}{p(p^2 + m_D^2(T, B, \mu))^2} + \frac{2\pi T m_g^2 m_G^2}{p(p^2 + m_D^2(T, B, \mu))^3} \qquad (23)$$

- Real part of the potential:

$$\mathrm{Re}\ V(r; T, B, \mu) = -\frac{4}{3}\alpha_s \left(\frac{e^{-\hat{r}}}{r} + m_D(T, B, \mu) \right) + \frac{4}{3} \frac{\sigma}{m_D(T, B, \mu)} \left(1 - e^{-\hat{r}} \right) \qquad (24)$$

- Imaginary part of the potential:

$$\mathrm{Im}\ V(r; T, B, \mu) = \sum_f \alpha_s g^2 m_f \frac{|q_f B|}{3\pi^2} \left[\frac{\pi}{2m_D^3} - \frac{\pi e^{-\hat{r}}}{2m_D^3} - \frac{\pi \hat{r} e^{-\hat{r}}}{2m_D^3} - \frac{2\hat{r}}{m_D} \int_0^\infty \frac{p\,dp}{(p^2 + m_D^2)^2} \int_0^{pr} \frac{\sin t}{t} dt \right] - \frac{4}{3} \frac{\alpha_s T m_g^2}{m_D^2} \psi_1(\hat{r}) - \frac{16\sigma T m_g^2}{3 m_D^4} \psi_2(\hat{r}) \qquad (25)$$

Dissociation temperature of $Q\bar{Q}$ bound states, Salman, BP NPB 2023

$$m_{\mathrm{D}}^2 = \sum_f g^2 \frac{|q_f B|}{4\pi^2 T} \int_0^\infty dk_3 \; \{n^+(E_1)(1 - n^+(E_1)) + n^-(E_1)(1 - n^-(E_1))\} + g^2 T^2 \left(\frac{N_c}{3}\right)$$

- Dissociation temperature:

	T_D (in terms of $T_c = 155$ MeV), $eB = 15\ m_\pi^2$	
State	J/ψ	Υ
μ =0	1.64	1.95
$\mu = 60$	1.69	1.97
$\mu = 100$	1.75	2.00

- Conclusions:
- Baryon asymmetry makes the real-part of potential slightly more attractive and weakens the imaginary-part
- Dissciation temperature gets enhanced at finite μ

Form Factor Approach of resummed gluon propagator at $T \neq 0$, Strong B

- Gluon self-energy in magnetic field

$$\Pi^{\mu\nu}(P) = b(P)B^{\mu\nu}(P) + c(P)R^{\mu\nu}(P) + d(P)M^{\mu\nu}(P) + a(P)N^{\mu\nu}(P) \tag{26}$$

- The projection tensors

$$B^{\mu\nu}(P) = \frac{\bar{u}^\mu \bar{u}^\nu}{\bar{u}^2} \qquad R^{\mu\nu}(P) = g_\perp^{\mu\nu} - \frac{P_\perp^\mu P_\perp^\nu}{P_\perp^2}$$

$$M^{\mu\nu}(P) = \frac{\bar{n}^\mu \bar{n}^\nu}{\bar{n}^2} \qquad N^{\mu\nu}(P) = \frac{\bar{u}^\mu \bar{n}^\nu + \bar{u}^\nu \bar{n}^\mu}{\sqrt{\bar{u}^2}\sqrt{\bar{n}^2}}$$

$$\bar{u}^\mu = \left(g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2}\right) u_\nu \quad \bar{n}^\mu = \left(\tilde{g}^{\mu\nu} - \frac{\tilde{Q}^\mu \tilde{Q}^\nu}{\tilde{Q}^2}\right) n_\nu$$

$\Longrightarrow \tilde{g}^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$ and $\tilde{Q}^\mu = Q^\mu - (Q.u)u^\mu$

- The form factors

$$b(P) = B^{\mu\nu}(P)\,\Pi_{\mu\nu}(P) \qquad c(P) = R^{\mu\nu}(P)\,\Pi_{\mu\nu}(P)$$

$$d(P) = M^{\mu\nu}(P)\,\Pi_{\mu\nu}(P) \qquad a(P) = \frac{1}{2}N^{\mu\nu}(P)\,\Pi_{\mu\nu}(P)$$

Form factor Approach of resummed Gluon propagator at $T \neq 0$, Strong B continued

$$D^{\mu\nu}(P) = \frac{(P^2 - d(P))B^{\mu\nu}}{(P^2 - b(P))(P^2 - d(P)) - a^2(P)} + \frac{R^{\mu\nu}}{P^2 - c(P)} + \frac{(P^2 - b(P))M^{\mu\nu}}{(P^2 - b(P))(P^2 - d(P)) - a^2(P)} + \frac{a(P)N^{\mu\nu}}{(P^2 - b(P))(P^2 - d(P)) - a^2(P)} \quad (27)$$

- $D^{00}(P)$ in the static limit becomes ($R^{00} = M^{00} = N^{00} = 0$):

$$D^{00}(p_0 = 0, p) = -\frac{1}{(p^2 + b(p_0 = 0, p))} \quad (28)$$

form factor $a(p_0 = 0, p)$ vanishes

- Real and imaginary parts (in static limit):

$$\mathrm{Re}\, D^{00}(p_0 = 0, p) = -\frac{1}{p^2 + m_D^2} \quad (29)$$

$$\mathrm{Im}\, D^{00}(p_0 = 0, p) = \sum_f \frac{g^2 |q_f B| m_f^2}{4\pi} \frac{1}{p_3^2 (p^2 + m_D^2)^2} + \frac{\pi T m_g^2}{p(p^2 + m_D^2)^2} \quad (30)$$

where

$$m_{\mathrm{D}}^2 = \sum_f g^2 \frac{|q_f B|}{2\pi^2 T} \int_0^\infty dk_3 \; \{n(E_1)(1 - n(E_1))\} + g^2 T^2 \left(\frac{N_c}{3}\right)$$

Gluon self-energy in $T \neq 0$, weak B, Mujeeb, BP PRd 2021

$$\text{Quark}-\text{loop}: i\Pi^{\mu\nu}(Q) = \sum_f \frac{g^2}{2}\int \frac{d^4K}{(2\pi)^4} Tr\left[\gamma^\nu iS(K)\gamma^\mu iS(P)\right];\ (P = K - Q)$$

•Quark Propagator in weak B up to $O(q_fB)^2$:

$$\begin{aligned}
iS(K) &= i\frac{(\not{K}+m_f)}{K^2-m_f^2} - q_fB\frac{\gamma_1\gamma_2(\not{K}_\parallel+m_f)}{(K^2-m_f^2)^2} - 2i(q_fB)^2\frac{[K_\perp^2(\not{K}_\parallel+m_f)+\not{K}_\perp(m_f^2-K_\parallel^2)]}{(K^2-m_f^2)^4} \\
&\equiv S_0(K)+S_1(K)+S_2(K) \\
\Pi^{\mu\nu}(Q) &= \Pi^{\mu\nu}_{(0,0)}(Q)+\Pi^{\mu\nu}_{(1,1)}(Q)+2\Pi^{\mu\nu}_{(2,0)}(Q)+O[(q_fB)^3],\ \text{where} \\
\Pi^{\mu\nu}_{(0,0)}(Q) &= \sum_f \frac{ig^2}{2}\int\frac{d^4K}{(2\pi)^4}\frac{Tr[\gamma^\nu(\not{K}+m_f)\gamma^\mu(\not{P}+m_f)]}{(K^2-m_f^2)(P^2-m_f^2)} \\
\Pi^{\mu\nu}_{(1,1)}(Q) &= -\sum_f \frac{ig^2(q_fB)^2}{2}\int\frac{d^4K}{(2\pi)^4}\frac{Tr[\gamma^\nu\gamma_1\gamma_2(\not{K}_\parallel+m_f)\gamma^\mu\gamma_1\gamma_2(\not{P}_\parallel+m_f)]}{(K^2-m_f^2)^2(P^2-m_f^2)^2} \\
\Pi^{\mu\nu}_{(2,0)}(Q) &= -\sum_f \frac{2ig^2(q_fB)^2}{2}\int\frac{d^4K}{(2\pi)^4}\frac{M^{\mu\nu}}{(K^2-m_f^2)^4(P^2-m_f^2)^4} \\
M^{\mu\nu} &= Tr\left[\gamma^\nu\{K_\perp^2(\not{K}_\parallel+m_f)+\not{K}_\perp(m_f^2-K_\parallel^2)\}\gamma^\mu(\not{P}+m_f)\right]
\end{aligned}$$

$$\Rightarrow \boxed{\Pi^{\mu\nu}(Q) = \Pi_0^{\mu\nu}(Q)(O\sim(q_fB)^0)+\Pi_2^{\mu\nu}(Q)(O\sim(q_fB)^2)}$$

Gluon self energy in weak B Mujeeb, BP PRD 2021

$$\text{General Form}: \Pi^{\mu\nu}(Q) = bB^{\mu\nu} + cR^{\mu\nu} + dM^{\mu\nu} + aN^{\mu\nu}$$

$$B^{\mu\nu} = \frac{\bar{u}^\mu \bar{u}^\nu}{\bar{u}^2},\; R^{\mu\nu} = g_\perp^{\mu\nu} - \frac{Q_\perp^\mu Q_\perp^\nu}{Q_\perp^2},\; M^{\mu\nu} = \frac{\bar{n}^\mu \bar{n}^\nu}{\bar{n}^2},\; N^{\mu\nu} = \frac{\bar{u}^\mu \bar{n}^\nu + \bar{u}^\nu \bar{n}^\mu}{\sqrt{\bar{u}^2}\sqrt{\bar{n}^2}}$$

$$\bar{u}^\mu = \left(g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2}\right) u_\nu \quad \bar{n}^\mu = \left(\tilde{g}^{\mu\nu} - \frac{\tilde{Q}^\mu \tilde{Q}^\nu}{\tilde{Q}^2}\right) n_\nu$$

- $\tilde{g}^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$ and $\tilde{Q}^\mu = Q^\mu - (Q.u)u^\mu$

- **Form Factors**

$$b = B^{\mu\nu}\Pi_{\mu\nu}; \quad c = R^{\mu\nu}\Pi_{\mu\nu}; \quad d = M^{\mu\nu}\Pi_{\mu\nu}; \quad a = \frac{1}{2}N^{\mu\nu}\Pi_{\mu\nu}$$

General form resummed gluon propagator in Landau gauge:

$$D^{\mu\nu}(Q) = \frac{(Q^2-d)B^{\mu\nu}}{(Q^2-b)(Q^2-d)-a^2} + \frac{R^{\mu\nu}}{Q^2-c} + \frac{(Q^2-b)M^{\mu\nu}}{(Q^2-b)(Q^2-d)-a^2} + \frac{aN^{\mu\nu}}{(Q^2-b)(Q^2-d)-a^2}$$

$\Rightarrow$ Only the "00"-component is required for deriving the heavy quark potential ($R^{00} = M^{00} = N^{00} = 0$)

$$D^{00}(Q) = \frac{(Q^2-d)\bar{u}^2}{(Q^2-b)(Q^2-d)-a^2},$$

Form factor a: $a = a_0 + a_2 \Rightarrow a_0 = 0$ and $a_2 : O \sim (q_f B)^2$

$$D^{00}(Q) = \frac{\bar{u}^2}{(Q^2-b)}$$

Form factor b**:** $b = b_0 + b_2$; **Form factor** b_0 **(**$O \sim [(q_f B)^0]$**)**

$$\mathrm{Re}\; b_0(q_0=0) = g^2 T^2 \frac{N_f}{6}; \quad \left[\frac{\mathrm{Im}\; b_0(q_0,q)}{q_0}\right]_{q_0=0} = \frac{g^2 T^2 N_f}{6}\frac{\pi}{2q}$$

Continue..

- Adding the gluonic contribution

$$\mathrm{Re}\ b_0(q_0=0) = g^2T^2\left(\frac{N_c}{3}+\frac{N_f}{6}\right)$$

$$\left[\frac{\mathrm{Im}\ b_0(q_0,q)}{q_0}\right]_{q_0=0} = g^2T^2\left(\frac{N_c}{3}+\frac{N_f}{6}\right)\frac{\pi}{2q}$$

Form factor b_2 ($O \sim [(q_fB)^2]$)

$$\mathrm{Re}\ b_2(q_0=0) = \sum_f \frac{g^2}{12\pi^2T^2}(q_fB)^2\sum_{l=1}^{\infty}(-1)^{l+1}l^2K_0(\frac{m_fl}{T})$$

$$\left[\frac{\mathrm{Im}\ b_2(q_0,q)}{q_0}\right]_{q_0=0} = \frac{1}{q}\left[\sum_f \frac{g^2(q_fB)^2}{16\pi T^2}\sum_{l=1}^{\infty}(-1)^{l+1}l^2K_0\left(\frac{m_fl}{T}\right) - \sum_f \frac{g^2(q_fB)^2}{96\pi T^2}\sum_{l=1}^{\infty}(-1)^{l+1}l^2K_2\left(\frac{m_fl}{T}\right) + \sum_f \frac{g^2(q_fB)^2}{768\pi}\frac{(8T-7\pi m_f)}{m_f^2T}\right]$$

$\Longrightarrow$ K_0 and K_2 are the modified Bessel functions of second kind

Continue..

- The total contribution of form factor b

$$
\begin{aligned}
\mathrm{Re}\, b(q_0 = 0) &= g^2 T^2 \left(\frac{N_c}{3} + \frac{N_f}{6}\right) \\
&+ \sum_f \frac{g^2}{12\pi^2 T^2} (q_f B)^2 \sum_{l=1}^{\infty} (-1)^{l+1} l^2 K_0(\frac{m_f l}{T}) \\
\left[\frac{\mathrm{Im}\, b(q_0, q)}{q_0}\right]_{q_0=0} &= g^2 T^2 \left(\frac{N_c}{3} + \frac{N_f}{6}\right) \frac{\pi}{2q} \\
&+ \frac{1}{q} \left[\sum_f \frac{g^2 (q_f B)^2}{16\pi T^2} \sum_{l=1}^{\infty} (-1)^{l+1} l^2 K_0 \left(\frac{m_f l}{T}\right)\right. \\
&- \sum_f \frac{g^2 (q_f B)^2}{96\pi T^2} \sum_{l=1}^{\infty} (-1)^{l+1} l^2 K_2 \left(\frac{m_f l}{T}\right) \\
&\left. + \sum_f \frac{g^2 (q_f B)^2}{768\pi} \frac{(8T - 7\pi m_f)}{m_f^2 T}\right]
\end{aligned}
$$

$\Rightarrow$ Debye screering mass: $M_D^2 = \mathrm{Re}\, b(q_0 = 0)$

$Q\bar{Q}$ potential in Weak B at $T \neq 0$

$$V(r;T,B) = \int \frac{d^3\mathbf{q}}{(2\pi)^{3/2}} (e^{i\mathbf{k}.\mathbf{r}} - 1) \frac{V(\mathbf{q})}{\epsilon(\mathbf{q})}$$

- Dielectric permittivity: $\frac{1}{\epsilon(\mathbf{q})} = -\lim_{q_0 \to 0} \mathbf{q}^2 D^{00}(q_0 \mathbf{q})$

$$\mathrm{Re}\ D^{00}(q_0 = 0) = -\frac{1}{q^2 + M_D^2} - \frac{m_G^2}{(q^2 + M_D^2)^2}$$

$$\mathrm{Im}\ D^{00}(q_0 = 0, q) = \frac{\pi T M_B^2}{q(q^2 + M_D^2)} + \frac{2\pi T M_B^2 m_G^2}{q(q^2 + M_D^2)^3}$$

- Real and imaginary-parts of the dielectric permittivities

$$\frac{1}{\mathrm{Re}\ \epsilon(\mathbf{q})} = \frac{\mathbf{q}^2}{\mathbf{q}^2 + M_D^2} + \frac{\mathbf{q}^2 m_G^2}{(\mathbf{q}^2 + M_D^2)^2}$$

$$\frac{1}{\mathrm{Im}\ \epsilon(\mathbf{q})} = -\frac{\mathbf{q}\pi T M_B^2}{(\mathbf{q}^2 + M_D^2)^2} - \frac{2\mathbf{q}\pi T M_B^2 m_G^2}{(\mathbf{q}^2 + M_D^2)^3}$$

$Q\bar{Q}$ potential in weak B, $T \neq 0$

$$\mathrm{ReV(r;T,B)} = -\frac{4}{3}\alpha_s\left(\frac{e^{-\hat{r}}}{r} + M_D\right) + \frac{4}{3}\frac{\sigma}{M_D}\left(1 - e^{-\hat{r}}\right), \quad \sigma = \frac{\alpha m_G^2}{2}$$

$$\mathrm{ImV_C(r;T,B)} = -\frac{4}{3}\frac{\alpha_s T M^2_{(T,B)}}{M_D^2}\phi_2(\hat{r}), \quad \mathrm{ImV_S(r;T,B)} = -\frac{4\sigma T M^2_{(T,B)}}{M_D^4}\phi_3(\hat{r})$$

$$\phi_2(\hat{r}) = 2\int_0^\infty \frac{z\,dz}{(z^2+1)^2}\left[1 - \frac{\sin z\hat{r}}{z\hat{r}}\right], \quad (31)$$

$$\phi_3(\hat{r}) = 2\int_0^\infty \frac{z\,dz}{(z^2+1)^3}\left[1 - \frac{\sin z\hat{r}}{z\hat{r}}\right], \quad (32)$$

• In the small $\hat{r}$ limit ($\hat{r} \ll 1$),

$\phi_2(\hat{r}) \approx -\frac{1}{9}\hat{r}^2\left(3\ln\hat{r} - 4 + 3\gamma_E\right)$,

$\phi_3(\hat{r}) \approx \frac{\hat{r}^2}{12} + \frac{\hat{r}^4}{900}\left(15\ln\hat{r} - 23 + 15\gamma_E\right)$

	Dissociation Temperatures T_d in T_c	
State	J/ψ	Υ
Pure Thermal (eB=0)	1.80	3.50
$eB = 0.5 m_\pi^2$	1.74	3.43

Effect of magnetic field on Heavy quark transport coefficients

- Momentum and spatial diffusion coefficnets, energy loss etc. have been calculated by the cutting rules to Heavy quark self-energy in weak magnetic field.**Debarshi, BP, PRD 2024**
- One has to extend these calculations in an expanding medium.

Thank you

Quark Propagator at $T \neq 0$, $\mu \neq 0$

General Formof Quark Self – energy : $\Sigma(P) = -\mathcal{A}(p_0, |\mathbf{p}|)\not{P} - \mathcal{B}(p_0, |\mathbf{p}|)\not{u}$, (33)

- **Form Factors:**

$$\mathcal{A}(p_0, |\mathbf{p}|) = \frac{m_{th}^2}{|\mathbf{p}|^2} Q_1\left(\frac{p_0}{|\mathbf{p}|}\right)$$

$$\mathcal{B}(p_0, |\mathbf{p}|) = -\frac{m_{th}^2}{|\mathbf{p}|}\left[\frac{p_0}{|\mathbf{p}|} Q_1\left(\frac{p_0}{|\mathbf{p}|}\right) - Q_0\left(\frac{p_0}{|\mathbf{p}|}\right)\right]$$

- Resummed quark propagator in the chiral basis:

$$S^{*-1}(P) = S_0^{-1} - \Sigma(P)$$

$$= \frac{1}{2}\left[P_L \frac{\not{L}}{L^2/2} P_R + P_R \frac{\not{R}}{R^2/2} P_L\right]; \qquad P_L = \frac{(1+\gamma^5)}{2}, P_R = \frac{(1-\gamma^5)}{2}$$

$$L^2 = (1+\mathcal{A})^2 P^2 + 2(1+\mathcal{A})\mathcal{B} p_0 + \mathcal{B}^2$$

$$R^2 = (1+\mathcal{A})^2 P^2 + 2(1+\mathcal{A})\mathcal{B} p_0 + \mathcal{B}^2$$

$$\frac{L^2}{2}\Big|_{p_0=0, |\mathbf{p}|\to 0} = \frac{R^2}{2}\Big|_{p_0=0, |\mathbf{p}|\to 0} \to \boxed{m_{th}^2 = \frac{g^2 T^2}{6}\left(1 + \frac{\mu^2}{\pi^2 T^2}\right)}$$

Quark Propagator at $T \neq 0$, $\mathrm{S}trong\ B$, $\mu \neq 0$

- Quark propagator in a magnetic field

$$iS_n(k) = \sum_n \frac{-id_n(\alpha)D + d_n'(\alpha)\bar{D}}{k_\parallel^2 - m_f^2 + 2n|q_f B|} + i\frac{\gamma \cdot k_\perp}{k_\perp^2} \tag{34}$$

where Landau Levels, $n = 0, 1, 2 \cdots$ and $\alpha = k_\perp^2/|q_f B|$

$$\begin{aligned}
D &= (m_f + \gamma \cdot k_\parallel) + \gamma \cdot k_\perp \frac{m_f^2 - k_\parallel^2}{k_\parallel^2}, \\
\bar{D} &= \gamma_1\gamma_2(m_f + \gamma \cdot k_\parallel) \\
d_n(\alpha) &= (-1)^n e^{-\alpha} C_n(2\alpha), \\
d_n'(\alpha) &= \frac{\partial d_n}{\partial \alpha}, \\
C_n(2\alpha) &= L_n(2\alpha) - L_{n-1}(2\alpha)
\end{aligned}$$

- In strong magnetic field ($|q_i B| >> T^2 >> m_i^2$) ($n = 0$ lowest Landau level)

$$S_0(k) = ie^{-\frac{k_\perp^2}{|q_i B|}} \frac{(\gamma^0 k_0 - \gamma^3 k_z + m_i)}{k_\parallel^2 - m_i^2}\left(1 - \gamma^0\gamma^3\gamma^5\right) \tag{35}$$

Strong Magnetic field continued....

•General form of Quark self-energy in strong B

$$\Sigma(p_{\|}) = A_1\gamma^\mu u_\mu + A_2\gamma^\mu b_\mu + A_3\gamma^5\gamma^\mu u_\mu + A_4\gamma^5\gamma^\mu b_\mu \tag{36}$$

where

$$A_1 = \frac{1}{4}\mathrm{Tr}[\Sigma\gamma^\mu u_\mu] = \frac{g^2|q_iB|}{3\pi^2}\left[\frac{\pi T}{2m_{i,0}} - \ln(2) + \frac{7\mu^2\zeta(3)}{8\pi^2T^2} - \frac{31\mu^4\zeta(5)}{32\pi^4T^4}\right]\frac{p_0}{p_{\|}^2}$$

$$A_2 = -\frac{1}{4}\mathrm{Tr}[\Sigma\gamma^\mu b_\mu] = \frac{g^2|q_iB|}{3\pi^2}\left[\frac{\pi T}{2m_{i,0}} - \ln(2) + \frac{7\mu^2\zeta(3)}{8\pi^2T^2} - \frac{31\mu^4\zeta(5)}{32\pi^4T^4}\right]\frac{p_z}{p_{\|}^2}$$

$$A_3 = \frac{1}{4}\mathrm{Tr}[\gamma^5\Sigma\gamma^\mu u_\mu] = -\frac{g^2|q_iB|}{3\pi^2}\left[\frac{\pi T}{2m_{i,0}} - \ln(2) + \frac{7\mu^2\zeta(3)}{8\pi^2T^2} - \frac{31\mu^4\zeta(5)}{32\pi^4T^4}\right]\frac{p_z}{p_{\|}^2}$$

$$A_4 = -\frac{1}{4}\mathrm{Tr}[\gamma^5\Sigma\gamma^\mu b_\mu] = -\frac{g^2|q_iB|}{3\pi^2}\left[\frac{\pi T}{2m_{i,0}} - \ln(2) + \frac{7\mu^2\zeta(3)}{8\pi^2T^2} - \frac{31\mu^4\zeta(5)}{32\pi^4T^4}\right]\frac{p_0}{p_{\|}^2}$$

Strong Magnetic field continued....

- Resummed quark propagator in chiral basis:

$$S(p_{\|}) = \frac{1}{2}\left[P_L \frac{\gamma^\mu X_\mu}{X^2/2} P_R + P_R \frac{\gamma^\mu Y_\mu}{Y^2/2} P_L\right] \tag{37}$$

where

$$\frac{X^2}{2} = X_1^2 = \frac{1}{2}\left[p_0 - (A_1 - A_2)\right]^2 - \frac{1}{2}\left[p_z + (A_2 - A_1)\right]^2 , \tag{38}$$

$$\frac{Y^2}{2} = Y_1^2 = \frac{1}{2}\left[p_0 - (A_1 + A_2)\right]^2 - \frac{1}{2}\left[p_z + (A_2 + A_1)\right]^2 \ . \tag{39}$$

- Effective mass of i-th flavour:

$$m_{i,B}^2 = \frac{g^2|q_iB|}{3\pi^2}\left[\frac{\pi T}{2m_{i,0}} - \ln(2) + \frac{7\mu^2\zeta(3)}{8\pi^2T^2} - \frac{31\mu^4\zeta(5)}{32\pi^4T^4}\right]$$

Both left and right-handed chiral modes have equal mass.

Quark Propagator at $T \neq 0$, $\mathrm{W}eak\ B$, $\mu \neq 0$

- In weak magnetic field ($T^2 >> |q_i B| >> m_i^2$):

$$S(k) = \frac{\not{k} + m_i}{k^2 - m_i^2} - \frac{\gamma_1\gamma_2(\gamma.k_{||} + m_i)}{(k^2 - m_i^2)^2}(q_i B) \qquad (40)$$

- Quark self-energy

$$\Sigma(p_0, |\mathbf{p}|) = -a_1(p_0, |\mathbf{p}|)\not{P} - a_2(p_0, |\mathbf{p}|)\not{u} - a_3(p_0, |\mathbf{p}|)\gamma_5\not{u} - a_4(p_0, |\mathbf{p}|)\gamma_5\not{b}, \quad (41)$$

  - Form Factors:

$$\begin{aligned}
a_1(p_0, |\mathbf{p}|) &= \frac{m_{th}^2}{|\mathbf{p}|^2} Q_1\left(\frac{p_0}{|\mathbf{p}|}\right), \\
a_2(p_0, |\mathbf{p}|) &= -\frac{m_{th}^2}{|\mathbf{p}|}\left[\frac{p_0}{|\mathbf{p}|} Q_1\left(\frac{p_0}{|\mathbf{p}|}\right) - Q_0\left(\frac{p_0}{|\mathbf{p}|}\right)\right], \\
a_3(p_0, |\mathbf{p}|) &= -4g^2 C_F M^2 \frac{p_z}{|\mathbf{p}|^2} Q_1\left(\frac{p_0}{|\mathbf{p}|}\right), \\
a_4(p_0, |\mathbf{p}|) &= -4g^2 C_F M^2 \frac{1}{|\mathbf{p}|} Q_0\left(\frac{p_0}{|\mathbf{p}|}\right),
\end{aligned}$$

Weak Magnetic field continued....

where

$$M^2(T,\mu,B) = \frac{|q_i B|}{16\pi^2}\left(\frac{\pi T}{2m_{i0}} - \ln 2 + \frac{7\mu^2\zeta(3)}{8\pi^2 T^2}\right),$$

- Resummed quark propagator in chiral basis

$$S^*(P) = \frac{1}{2}\left[P_L \frac{\not{L}}{L^2/2} P_R + P_R \frac{\not{R}}{R^2/2} P_L\right], \qquad (42)$$

where

$$L^2 = (1+a_1)^2 P^2 + 2(1+a_1)(a_2+a_3)p_0 - 2a_4(1+a_1)p_z + (a_2+a_3)^2 - a_4^2,$$
$$R^2 = (1+a_1)^2 P^2 + 2(1+a_1)(a_2-a_3)p_0 + 2a_4(1+a_1)p_z + (a_2-a_3)^2 - a_4^2.$$

- Effective mass for L- and R-modes:

$$m_L^2 = m_{th}^2 + 4g^2 C_F M^2$$
$$m_R^2 = m_{th}^2 - 4g^2 C_F M^2$$

Different mass for left-and right-handed chiral modes

Effect of magnetic field on Heavy quark transport coefficients

Debarshi, BP, PRD 2024

•Momentum and spatial diffusion coefficnets, energy loss etc. have been calculated by the cutting rules to Heavy quark self-energy in weak magnetic field.

•One has to extend these calculations in an expanding medium.

Thank you