

All-heavy Tetraquarks in QCD

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QWG 2024 (IISER Mohali) - Feb 29, 2024



Hadrons and more

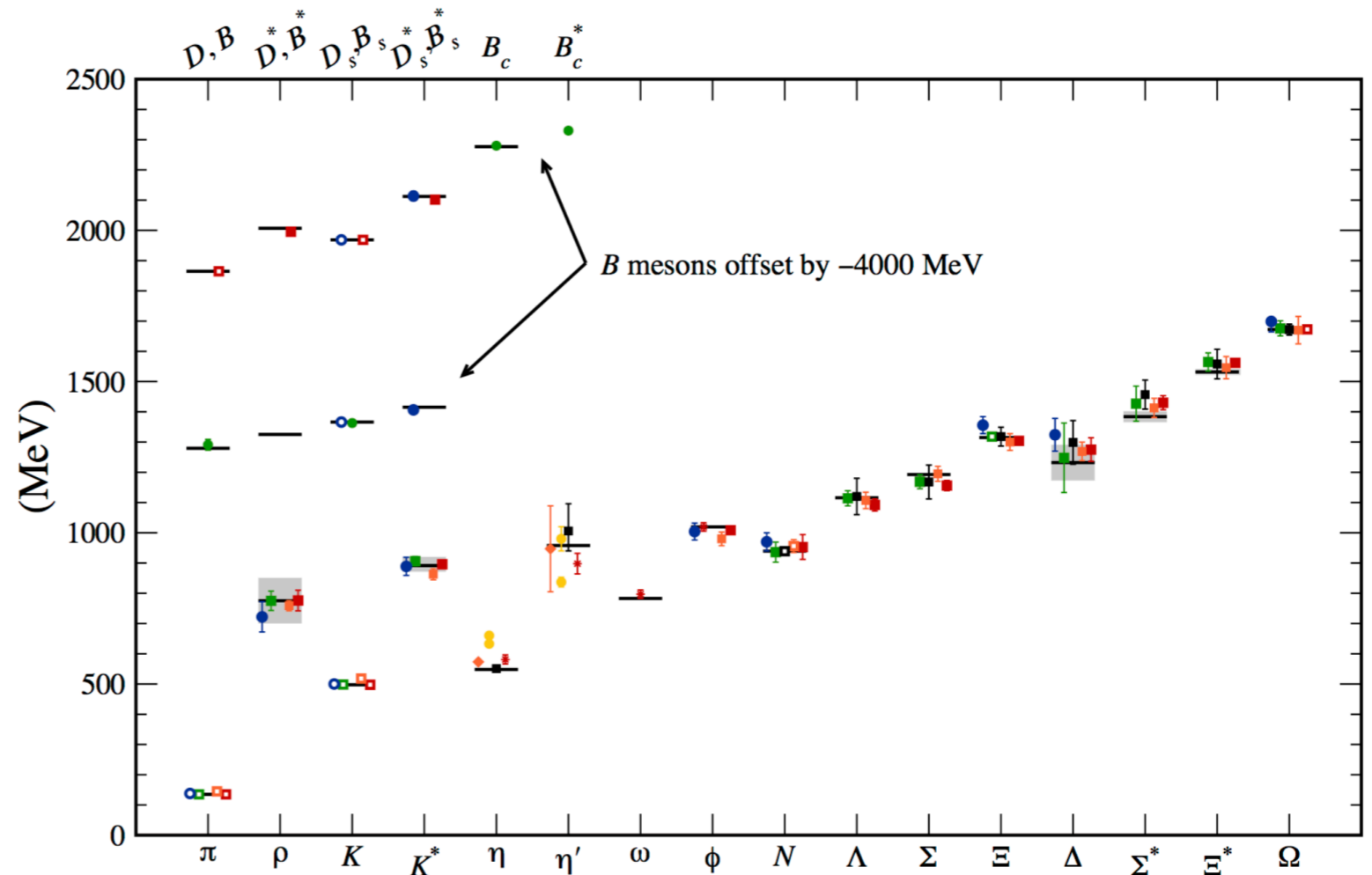
LQCD hadron spectrum: Kronfeld '12

LQCD: spectroscopy complete for most common hadrons

Exotic hadrons: recent experiments, Belle, LHCb etc. E.g. tetra/pentaquarks, dibaryons etc.

Exotics hard in LQCD, dense excited state spectrum, computationally inefficient

[Kronfeld et al. 2207.07641](#)



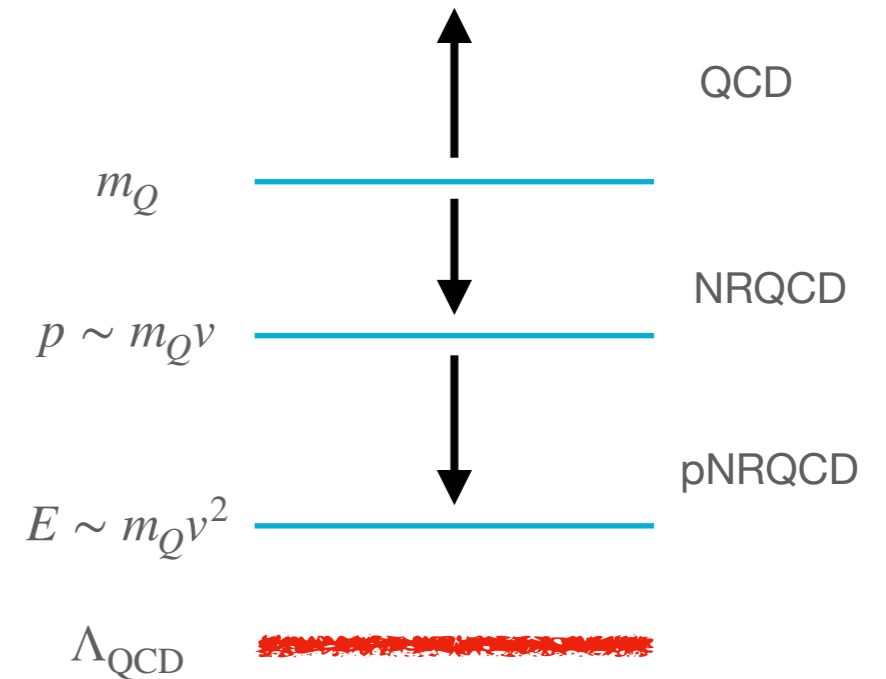
Can EFT help us understand all-heavy exotics and nuclear interactions at unphysical quark masses?

BA and Wagman, PRD 108

HQET and (p)NRQCD

Heavy quarks and their bound states call for **multi-scale** treatment

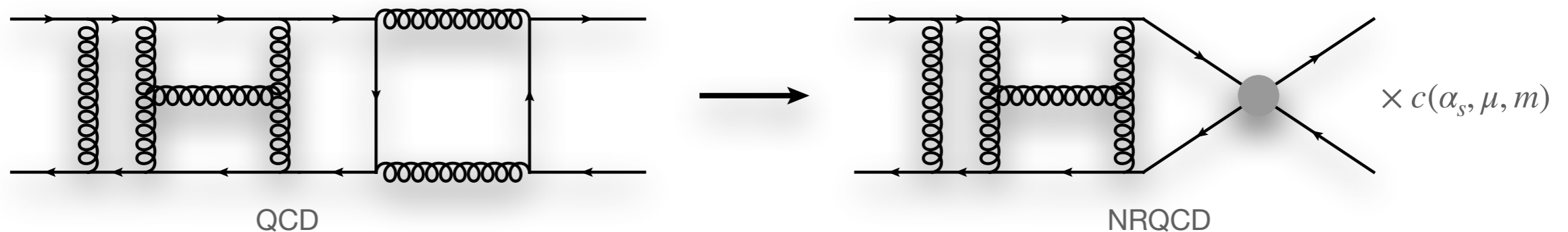
Expansion at the Lagrangian level in $m_Q \gg v \sim \alpha_s \Rightarrow$ **EFT description**



HQET & NRQCD: Operators and matching known to $D = 8$, NNLO and above for specific applications [Gerlach et al. '19](#); [Gunawardana et al. '17](#); [BA, Kniehl, Soto '20](#)

pNRQCD: Static $1/r$ potentials known to $N^3\text{LO}$ and finite mass + spin-dependent terms to $m^2\text{NNLO}$ [Review: Pineda '11](#)

Applications: $t, b, c, B, \eta_{b,c}, \Upsilon(1S), J/\psi, \dots$ mass, decay, splitting, cross-sections etc.



pNRQCD for (multi-)hadrons

Extending the pNRQCD Lagrangian \Rightarrow **new operators**

Quarkonium operator:

$$L_{\psi\chi}^{\text{pot}} = - \int d^3\mathbf{r}_1 d^3\mathbf{r}_2 \psi_i^\dagger(t, \mathbf{r}_1) \chi_j(t, \mathbf{r}_2) \chi_k^\dagger(t, \mathbf{r}_2) \psi_l(t, \mathbf{r}_1) \left[\delta_{ij} \delta_{kl} V_1^{\psi\chi}(\mathbf{r}, \mathbf{p}_1, \mathbf{p}_2, \mathbf{S}_1, \mathbf{S}_2) + T_{ij}^a T_{kl}^a V_8^{\psi\chi}(\mathbf{r}, \mathbf{p}_1, \mathbf{p}_2, \mathbf{S}_1, \mathbf{S}_2) \right]$$

Di-quark operator:

$$L_{\psi\psi}^{\text{pot}} = - \int d^3\mathbf{r}_1 d^3\mathbf{r}_2 \psi_i^\dagger(t, \mathbf{r}_1) \psi_j^\dagger(t, \mathbf{r}_2) \psi_k(t, \mathbf{r}_2) \psi_l(t, \mathbf{r}_1) \left[\epsilon_{ijm} \epsilon_{klm} V_3^{\psi\psi}(\mathbf{r}, \mathbf{p}_1, \mathbf{p}_2, \mathbf{S}_1, \mathbf{S}_2) + \left(\delta_{il} \delta_{jk} + \delta_{jl} \delta_{ik} \right) V_6^{\psi\psi}(\mathbf{r}, \mathbf{p}_1, \mathbf{p}_2, \mathbf{S}_1, \mathbf{S}_2) \right]$$

Tri-quark operator:

$$L_{3\psi}^{\text{pot}} = - \int d^3\mathbf{r}_1 d^3\mathbf{r}_2 d^3\mathbf{r}_3 \psi_i^\dagger(t, \mathbf{r}_1) \psi_j^\dagger(t, \mathbf{r}_2) \psi_k^\dagger(t, \mathbf{r}_3) \psi_l(t, \mathbf{r}_3) \psi_m(t, \mathbf{r}_2) \psi_n(t, \mathbf{r}_1) V_{ijk}^{3\psi}(\mathbf{r}, \mathbf{p}_1, \mathbf{p}_2, \mathbf{S}_1, \mathbf{S}_2)$$

$$V_{ijk}^{3\psi} = \epsilon_{ijk} \epsilon_{lmn} V_1^{3\psi} + \left(\epsilon_{ijp} T_{pk}^a \epsilon_{lmo} T_{on}^a \right) V_{8A}^{3\psi} + \left(\epsilon_{ikp} T_{jp}^a + \epsilon_{jkp} T_{ip}^a \right) \left(\epsilon_{lno} T_{mo}^a + \epsilon_{mno} T_{lo}^a \right) V_{8S}^{3\psi} + \left(\delta_{il} \delta_{jm} \delta_{kn} + \text{perms} \right) V_{10}^{3\psi}$$

For $N_q > 3$ N -body operators contribute at NNLO+

Two-body potentials

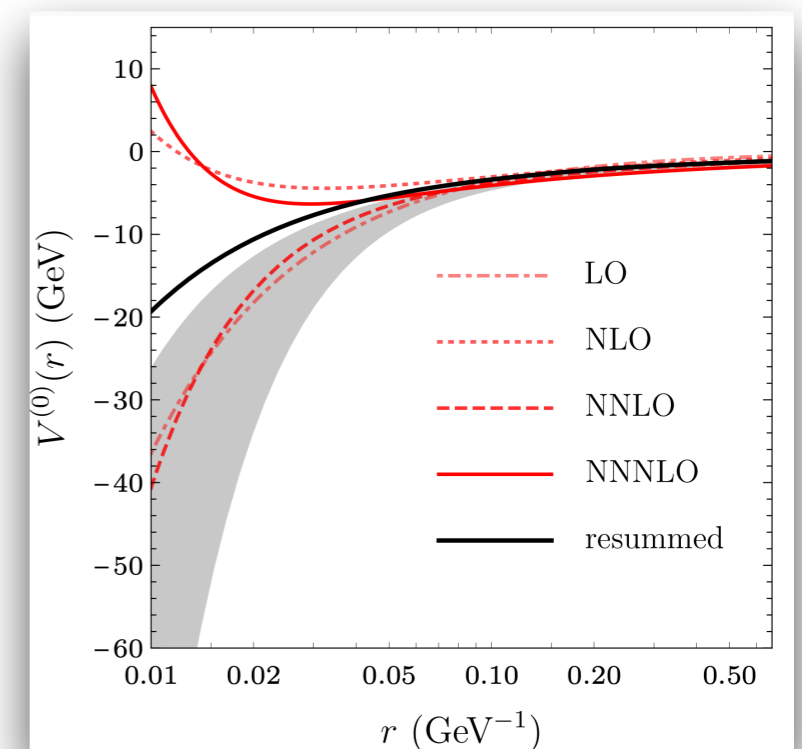
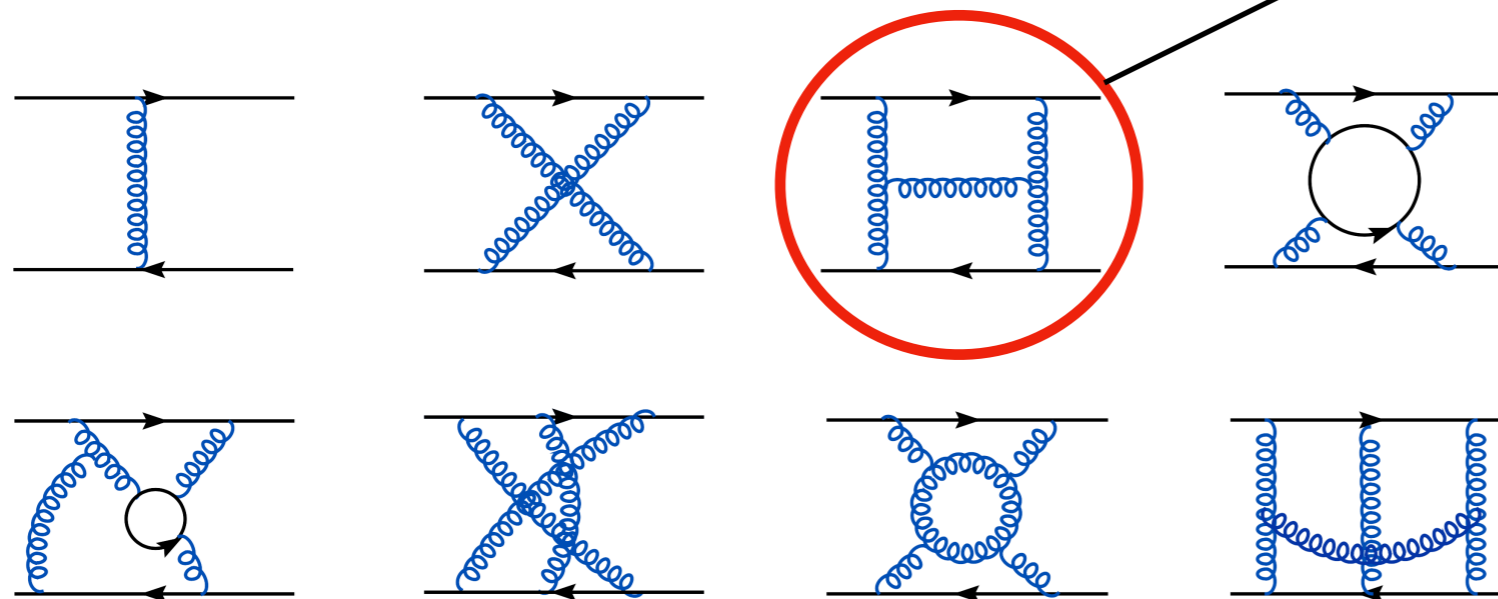
Potential of two heavy fermions under $SU(N_c)$: $V_c(r) = \sum_{n,k} \frac{1}{m^n} V_c^{(n,k)}$

Known to $\mathcal{O}(1/m^2)$, NLO $\forall c$, N³LO for $c = \mathbf{1, 8}$ Steinhauser et al. '98 - '18

Spin-dependent pieces arise in $V_c^{2,k}(r)$ at m^2 LO Review: Pineda '11

Color relation (NNLO): $V_\rho(r) = -\mathcal{C}_\rho^{\text{tree}} \left(\frac{1}{C_F} V_1^{\psi\chi}(r) - \frac{\alpha_s^3}{(4\pi)^2} \frac{\delta a_\rho}{r} \right)$

H.S. Chung '20



N-body potential

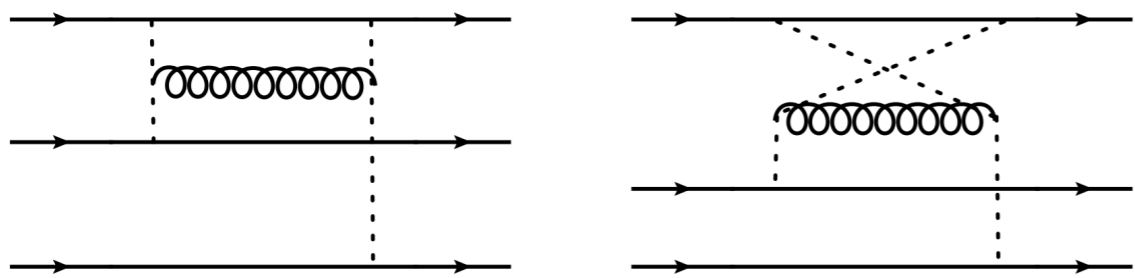
Three-body Lagrangian for general $N_c \mapsto$ Hamiltonian straightforward

$$L_{3\psi}^{\text{pot}} = - \int d^3\mathbf{r}_1 d^3\mathbf{r}_2 d^3\mathbf{r}_3 \psi_i^\dagger(t, \mathbf{r}_1) \psi_j^\dagger(t, \mathbf{r}_2) \psi_k^\dagger(t, \mathbf{r}_3) \psi_l(t, \mathbf{r}_3) \psi_m(t, \mathbf{r}_2) \psi_n(t, \mathbf{r}_1) \\ \times \left[\mathcal{F}_{ijk}^A \mathcal{F}_{lmn}^A V_A^{3\psi} + \mathcal{F}_{ijk}^S \mathcal{F}_{lmn}^S V_S^{3\psi} + \sum_{r,s} \mathcal{F}_{ijkq_1\dots q_{N_c-3}}^{M_{r,s}} V_{M_{r,s}}^{3\psi} \mathcal{F}_{lmnq_1\dots q_{N_c-3}}^{M_{r,s}} \right]$$

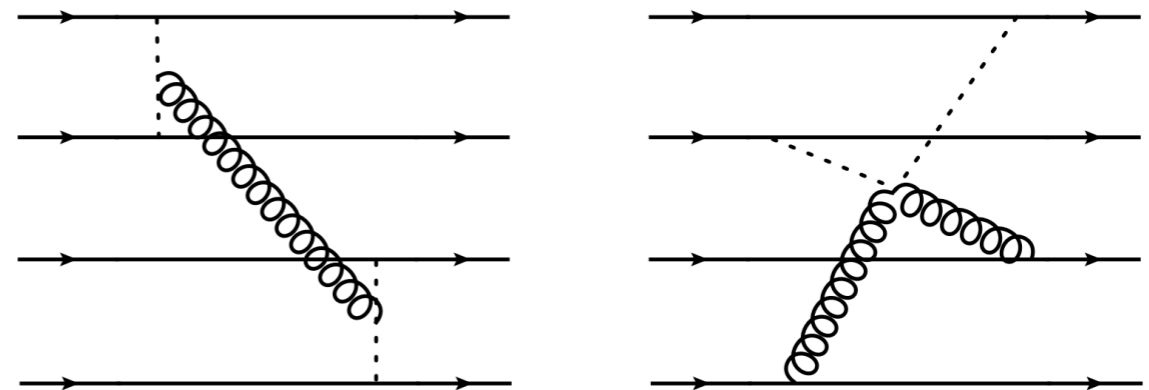
Three-body potential (NNLO)

$$V_\rho^{3\psi} = \alpha \left(\frac{\alpha}{4\pi} \right)^2 \left[\mathcal{C}_{\rho uv}^{3\psi,1} v_3(\mathbf{r}_{12}, \mathbf{r}_{13}) + \mathcal{C}_{\rho uv}^{3\psi,2} v_3(\mathbf{r}_{12}, \mathbf{r}_{23}) + \mathcal{C}_{\rho uv}^{3\psi,3} v_3(\mathbf{r}_{13}, \mathbf{r}_{23}) \right]$$

with v_3 from 3-body diagrams (no spectators + transverse g connector)



Non-exponentiating and non-zero 3-body diagram (NNLO)



NNLO 4-body diagrams (vanishing for baryons)

Variational methods

Modern nuclear physics has many ways to solve Schrödinger equations

Problem: No a priori knowledge of eigenstates of system \Rightarrow **MC methods**

\Rightarrow **Hamiltonian** of N -quark system and **trial wavefunction:**

$$\Psi_T(\mathbf{r}, \boldsymbol{\alpha}) = \sum_{i=1}^M \alpha_i \Phi_i(\mathbf{r}) \quad \text{with } \mathbf{r} \equiv \mathbf{r}_{1\dots N} \text{ and } \textit{variational parameters } \boldsymbol{\alpha}$$

\Rightarrow **Expectation value:**

$$E_0 \leq \langle \psi | H | \psi \rangle = \int d\mathbf{R} \psi^*(\mathbf{R}) \left[\sum_i \frac{\nabla_i^2}{2m_i} + \sum_{i < j} V(\mathbf{r}_i - \mathbf{r}_j) \right] \psi(\mathbf{R})$$

Idea: Vary $\boldsymbol{\alpha}$ with differential program that performs minimisation iteratively

Green's function Monte-Carlo

We now have Ψ_T and E_{\min} but what if \exists *lower-lying states*?

Limited in VMC by choice and size of basis \Rightarrow need new tool

GFMC: Given Ψ_T from VMC diffuse in time and reach **true ground-state energy**

$$|\Psi_T\rangle = \lim_{\tau \rightarrow \infty} e^{-\hat{H}\tau} |\psi_T\rangle$$

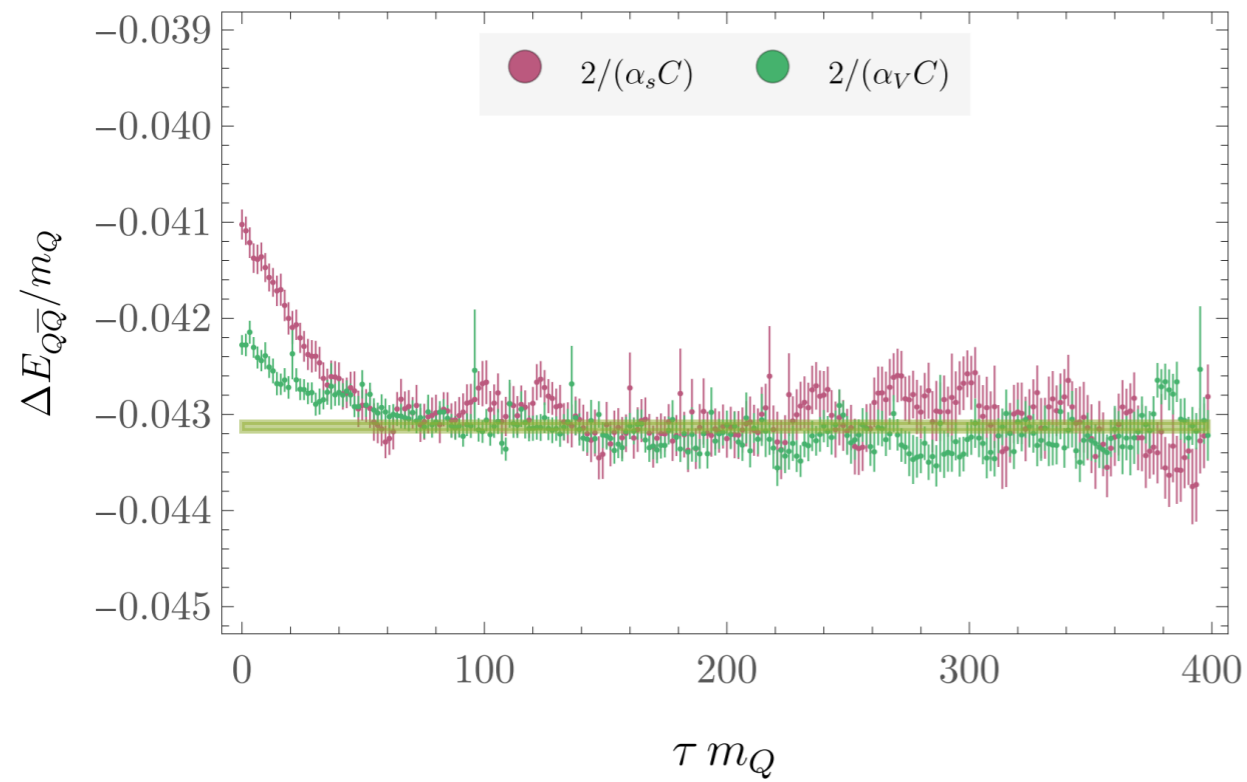
In practice: Take small time $\delta\tau = \tau/L$ for $L \gg 1$ and perform Trotter-Suzuki expansion

$$|0\rangle = \lim_{\tau \rightarrow \infty} e^{-H\tau} |\psi\rangle = \lim_{\tau \rightarrow \infty} \lim_{\delta\tau \rightarrow 0} \left(e^{-V\delta\tau/2} e^{-K\delta\tau} e^{-V\delta\tau/2} \right)^{\tau/\delta\tau} |\psi\rangle$$

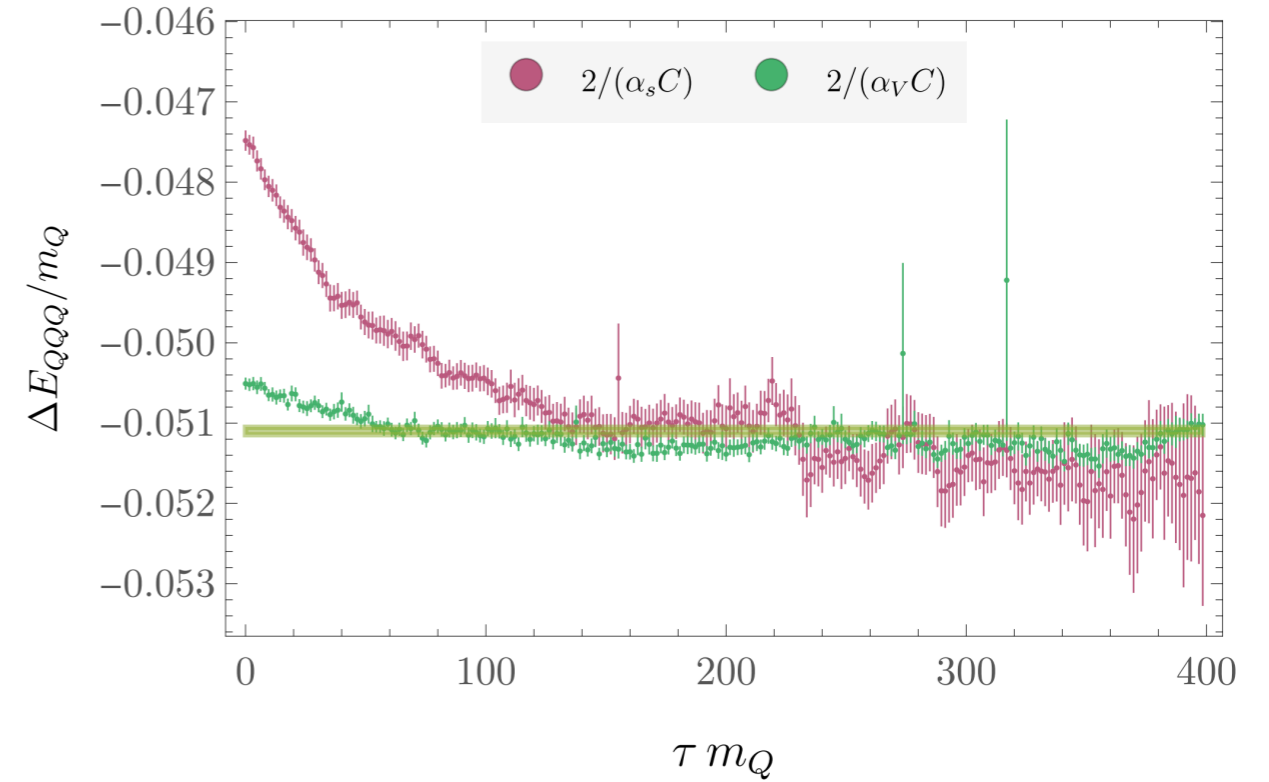
VMC vs. GFMC

$\alpha_s = 0.2$

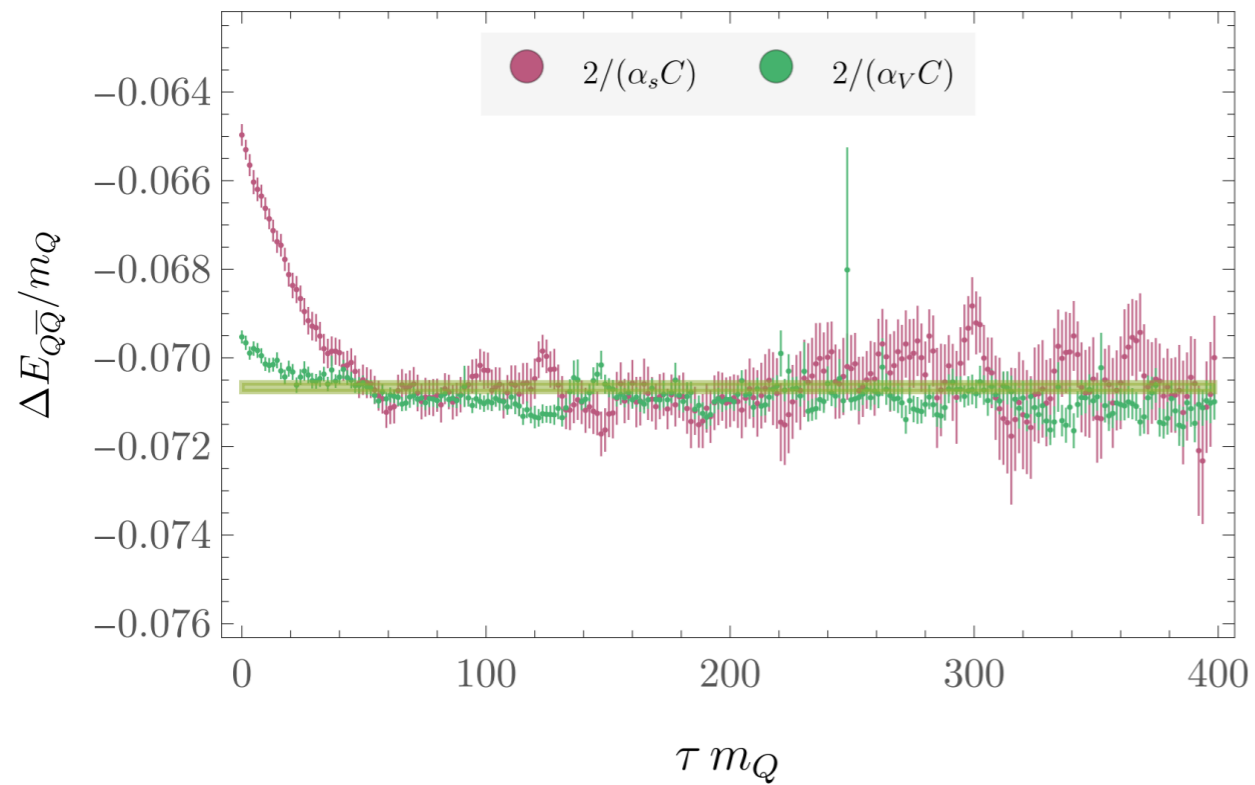
NLO



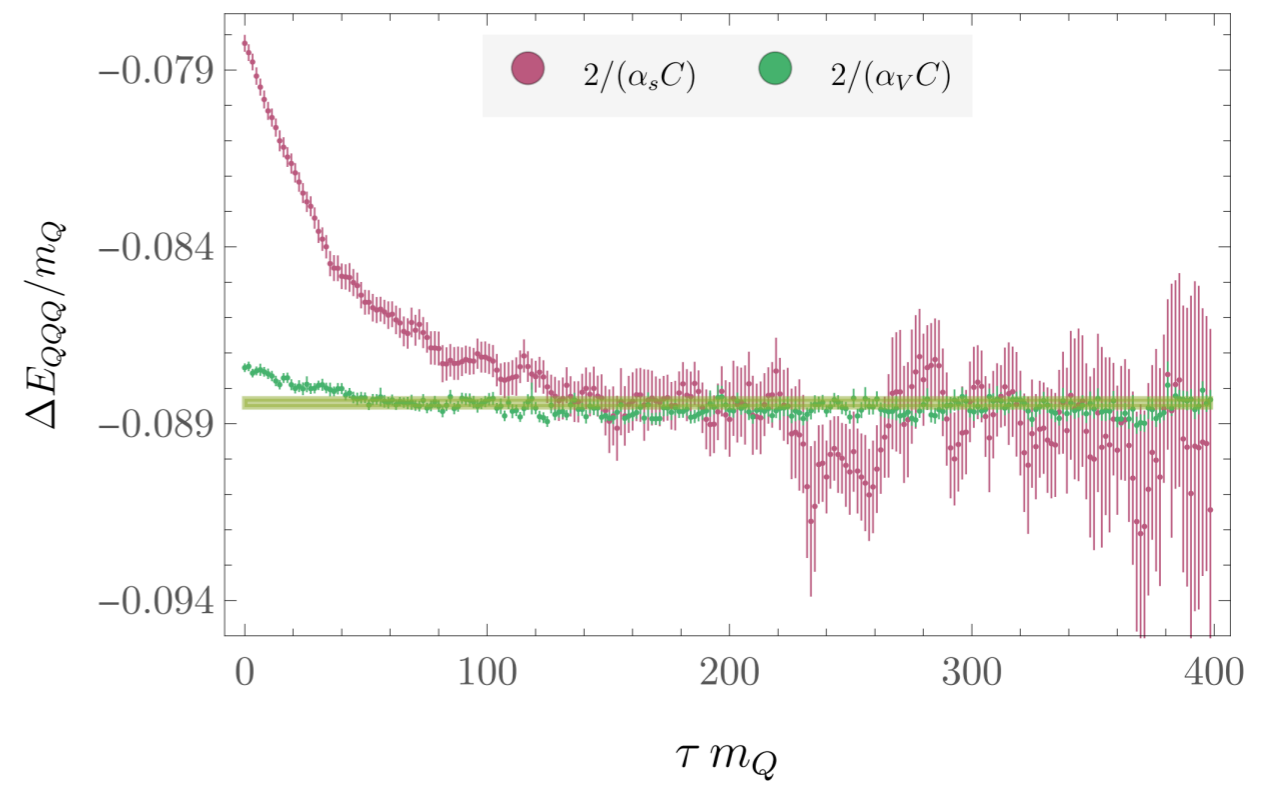
NLO



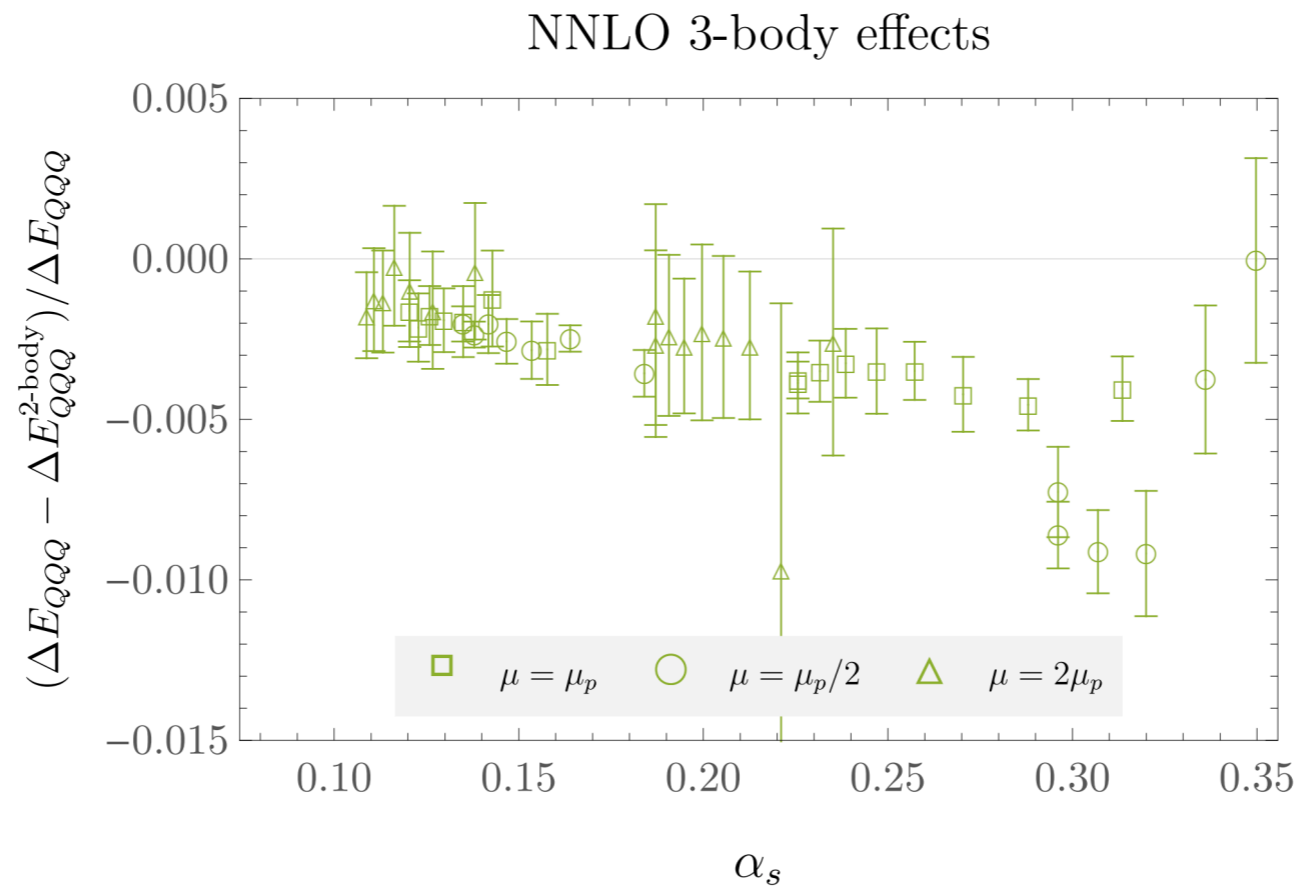
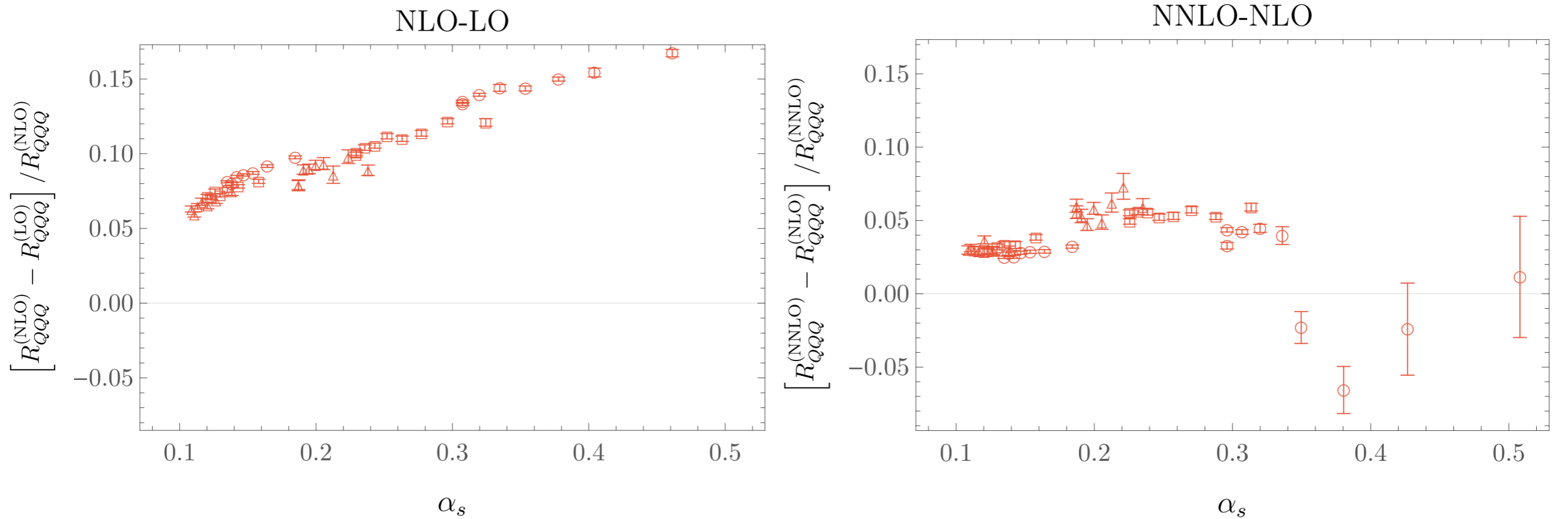
NNLO



NNLO



Order comparison



Baryons in pNRQCD

BA and Wagman, PRD 108

For $m_Q \gg \Lambda_{\text{QCD}}$, potential can be computed as power series in $\alpha_s(\mu)$ where $\mu \propto m_Q$

Multiple color structure arise, e.g.

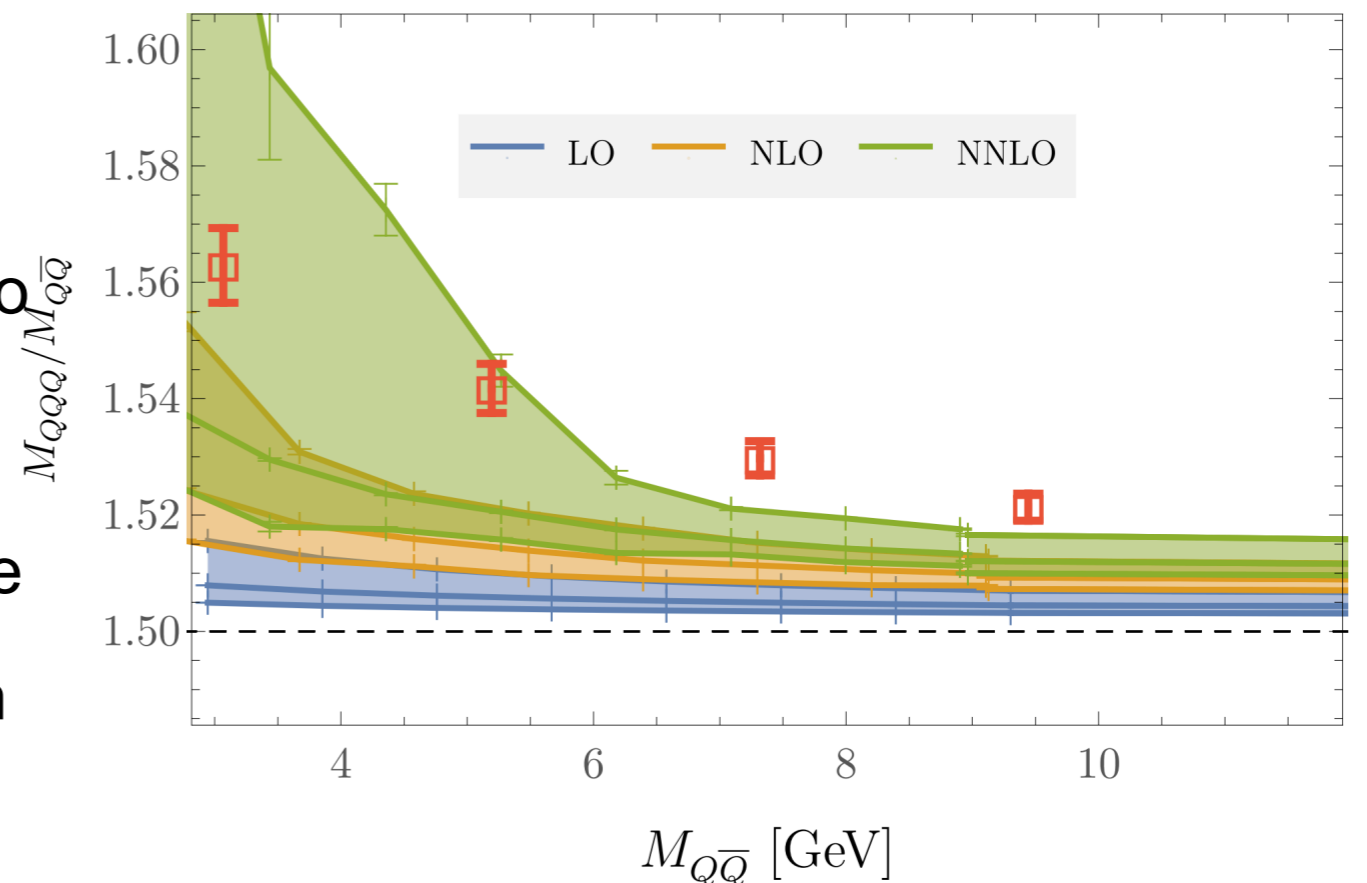
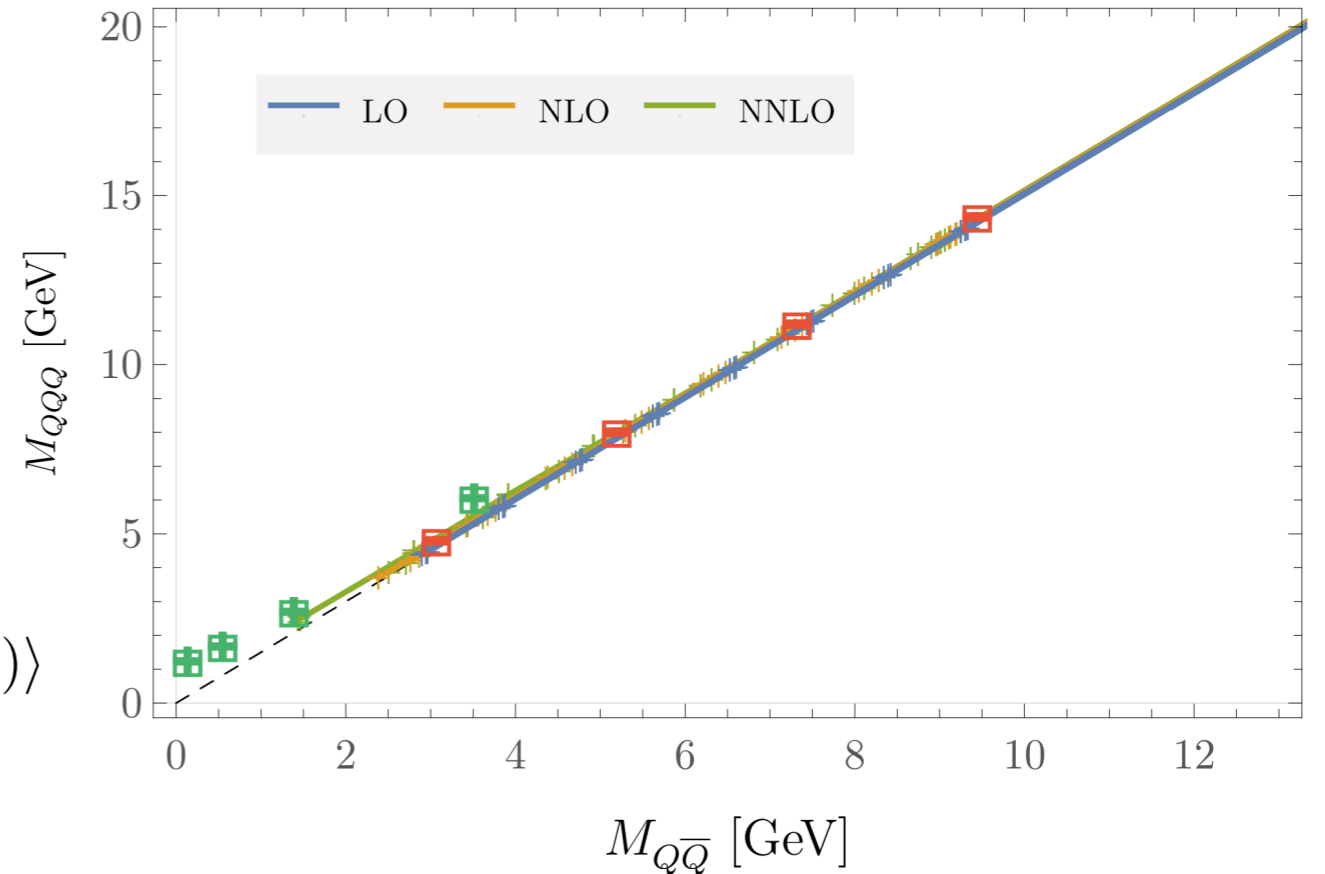
$$V |Q_i(r)Q_j(0)\rangle = \frac{1}{2} [(\delta_{il}\delta_{jk} + \delta_{jl}\delta_{ik})V_S(r) + \epsilon_{ijm}\epsilon_{klm}V_A(r)] |Q_k(r)Q_l(0) + Q_l(0)Q_k(r)\rangle$$

Triply-heavy baryon binding due to

$$V_A(r) = -\frac{2}{3} \left[\frac{\alpha_s(\mu)}{r} + O(\alpha_s(\mu)^2) \right]$$

Pole masses can be fixed by matching to observed quarkonium masses

Potential + masses completely determine pNRQCD Hamiltonian, enables predictions of e.g. triply-heavy baryon masses



QCD mass spectra

Renormalise m_Q with respect to corresponding spin-averaged $1S$ mass: $M_{Q\bar{Q}} = (3M_{S=1} + M_{S=0})/4$

| $1S$ mesons | Order | $\alpha_s(\mu)$ | m_Q | χ^2/dof | $M_{Q\bar{Q}}$ | Measured $M_{Q\bar{Q}}$ |
|----------------------|------------|-----------------|---------|---------------------|----------------|-------------------------|
| (Υ, η_b) | LO (exact) | 0.214850 | 4.77050 | - | 9.44295 | 9.44295(90) |
| (Υ, η_b) | NLO | 0.227318 | 4.86886 | 1.09 | 9.44255(46) | 9.44295(90) |
| (Υ, η_b) | NNLO | 0.224750 | 4.96983 | 1.15 | 9.44386(40) | 9.44295(90) |
| $(J/\psi, \eta_c)$ | LO (exact) | 0.282678 | 1.56206 | - | 3.06865 | 3.06865(10) |
| $(J/\psi, \eta_c)$ | NLO | 0.313613 | 1.65234 | 1.12 | 3.06870(33) | 3.06865(10) |
| $(J/\psi, \eta_c)$ | NNLO | 0.297100 | 1.77092 | 0.59 | 3.06824(45) | 3.06865(10) |

Using $m_Q^{\text{ren}} \leftrightarrow \alpha_s$ to obtain BE \Rightarrow triply-heavy baryon mass spectrum: $M_{Q_1 Q_2 Q_3} = \Delta E_{Q_1 Q_2 Q_3} + \sum_{i=1}^3 m_{Q_i}$

| Baryon | This work: M_{QQQ} | This work: χ^2/dof | Variational Methods | Lattice QCD |
|----------------|---|----------------------------------|---------------------------------------|---------------|
| Ω_{ccc} | LO: 4.626723(8) NLO: 4.66561(42) NNLO: 4.69981(29) | LO: 1.0 NLO: 1.4 NNLO: 1.1 | LO: 4.76(6) NNLO+mNLO: 4.97(20) | 4.796(8)(18) |
| Ω_{ccb} | LO: 7.815284(14) NLO: 7.86032(34) NNLO: 7.91297(28) | LO: 1.3 NLO: 1.3 NNLO: 1.0 | LO: 7.98(7) NNLO+mNLO: 8.20(15) | 8.007(9)(20) |
| Ω_{cbb} | LO: 11.03611(1) NLO: 11.09206(28) NNLO: 11.110463(46) | LO: 1.5 NLO: 1.1 NNLO: 1.1 | LO: 11.48(12) NNLO+mNLO: 11.34(26) | 11.195(8)(20) |
| Ω_{bbb} | LO: 14.12990(3) NLO: 14.25397(4) NNLO: 14.25903(54) | LO: 1.2 NLO: 1.4 NNLO: 1.3 | LO: 14.76(18) NNLO+mNLO: 14.57(25) | 14.371(4)(12) |

Color scaling

Attained **precise** scaling relations for N_c mesons and baryons. E.g. for NLO barons:

$$\frac{\Delta E_{B_d}^{(\text{NLO})}}{m_d \alpha_d^2 N_c^4} \approx -A_{B_d}^{(\text{LO},0)} - \alpha_d A_{B_d}^{(\text{NLO},1)} - \alpha_d^2 A_{B_d}^{(\text{NLO},2)}$$

$$A_{B_d}^{(\text{LO},0)} \approx \frac{0.013281(40)}{N_c} + \frac{0.02077(34)}{N_c^2} - \frac{0.0231(7)}{N_c^3}$$

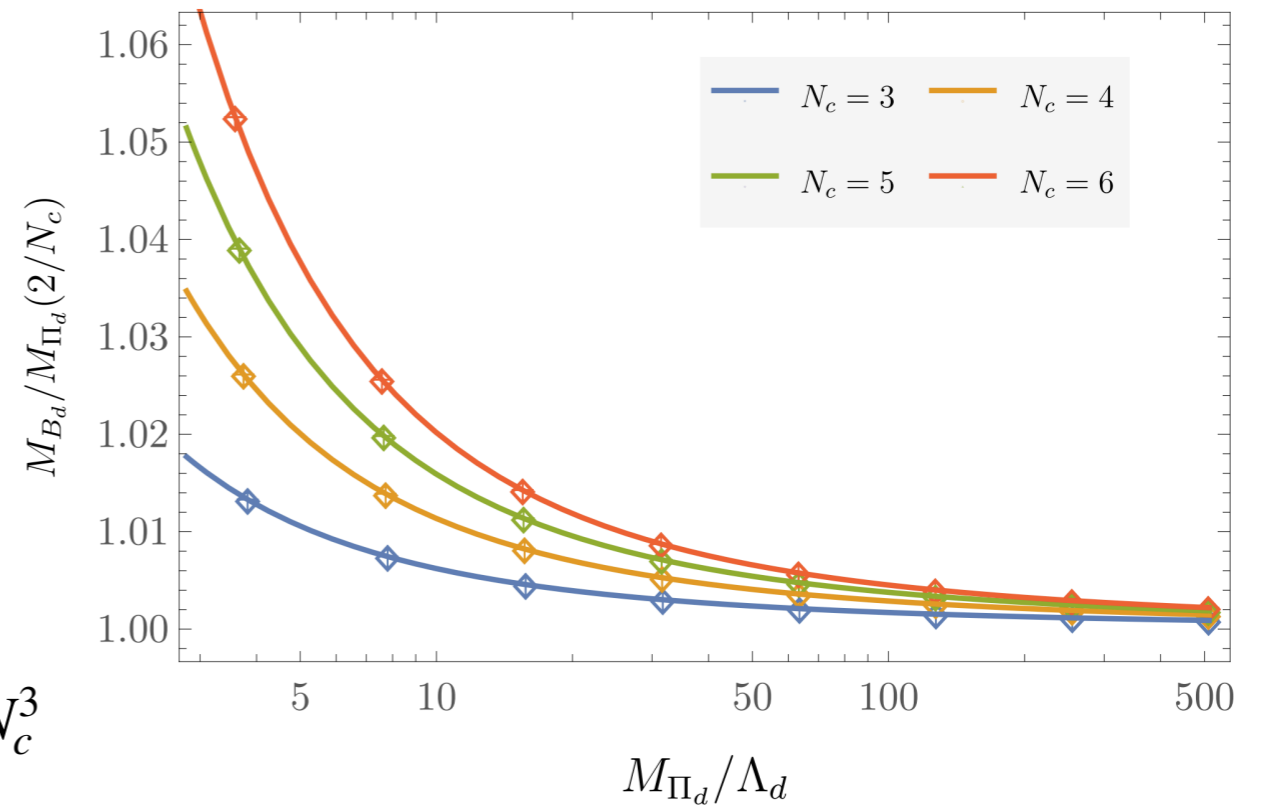
$$A_{B_d}^{(\text{NLO},1)} \approx 0.0224(20) + \frac{0.179(18)}{N_c} - \frac{0.183(37)}{N_c^2}$$

$$A_{B_d}^{(\text{NLO},2)} \approx 0.0459(9)$$

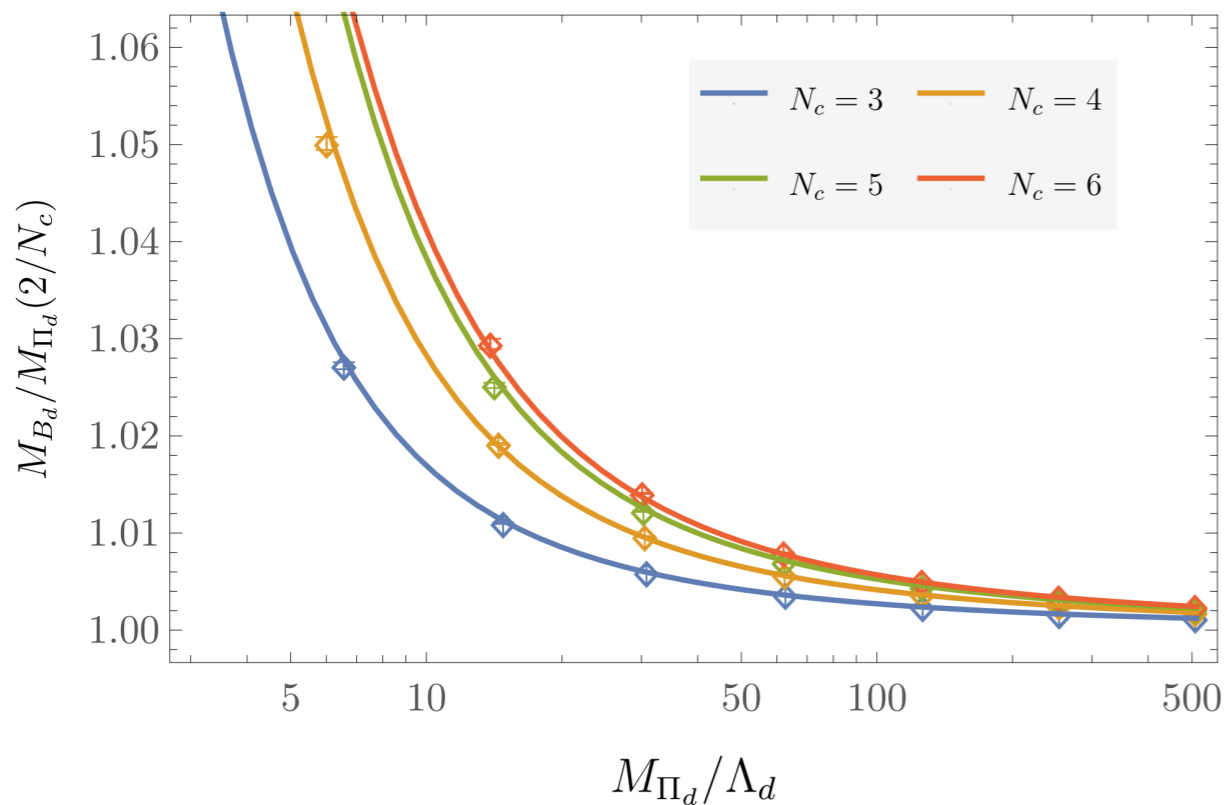
Consistent with large N_c arguments $\Delta E_{B_d}/m_d \sim \alpha_d^2 N_c^3$

Witten '79

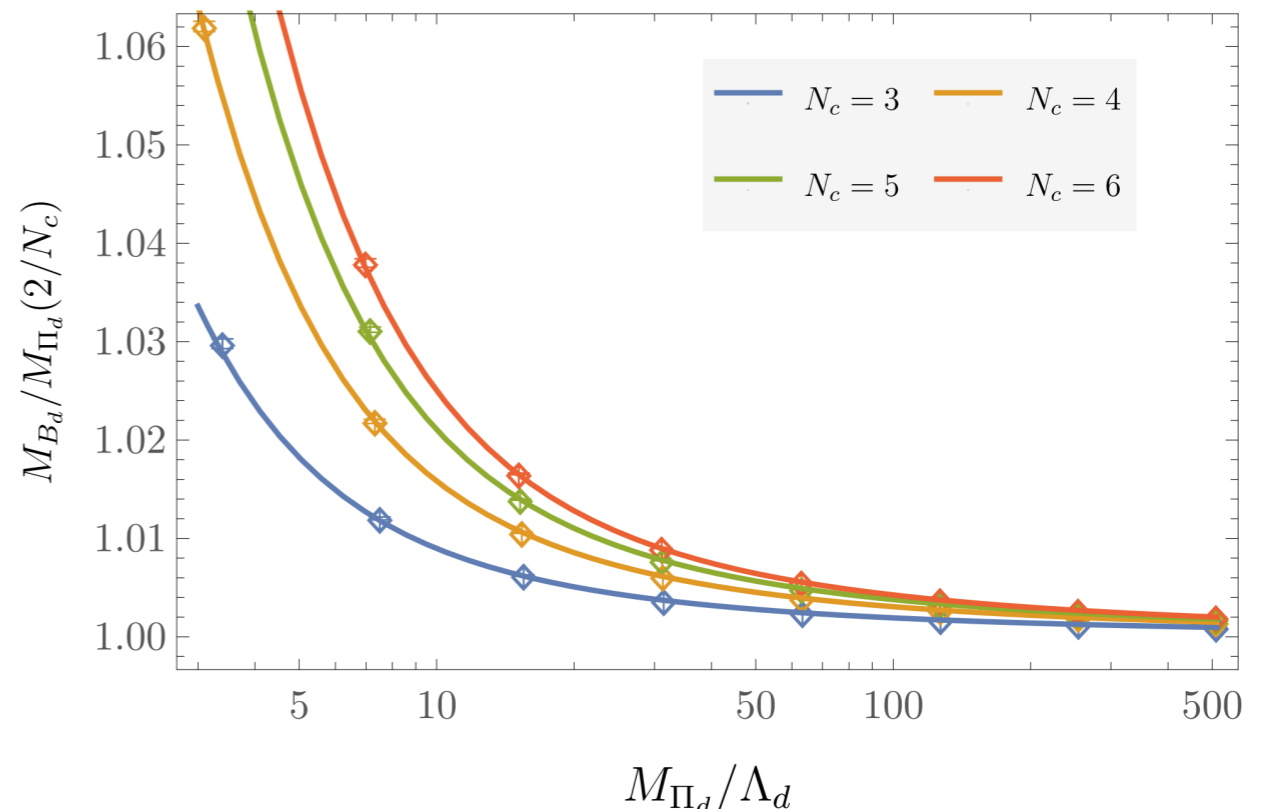
LO



NNLO



NLO

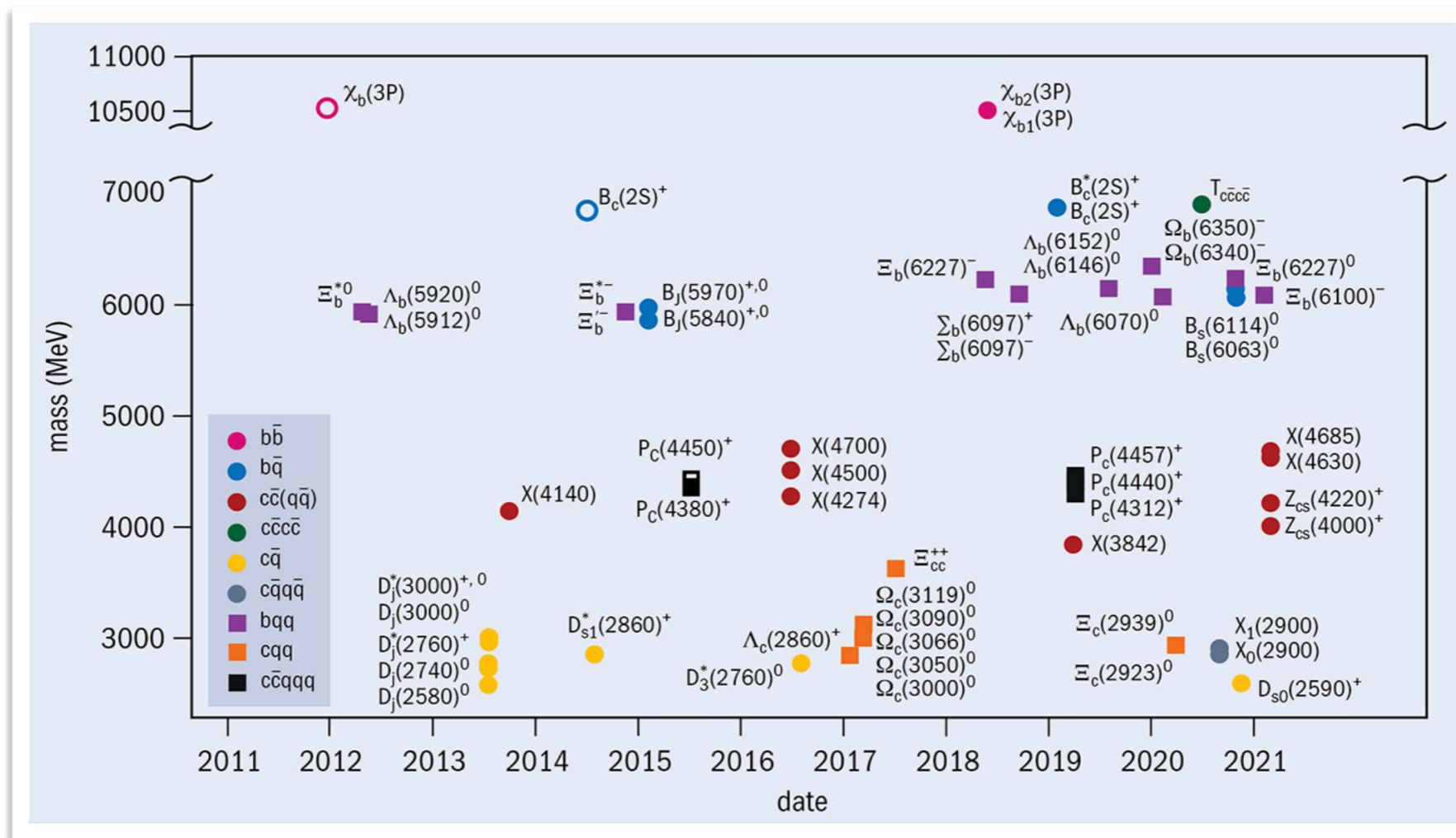


Tetraquarks

New exotics being observed — many are tetraquark candidates and first all-heavy candidate X(6900) observed...?

Heavy-light tetraquarks hard to study \Rightarrow instead clean all-heavy system with our formalism

Aim: Tackle all-heavy constituent exotics in QCD and beyond starting with tetraquarks



Positronium molecules in pNRQED

Same methods can be used to study “molecular” states in pNRQED such as Ps_2 the positronium molecule

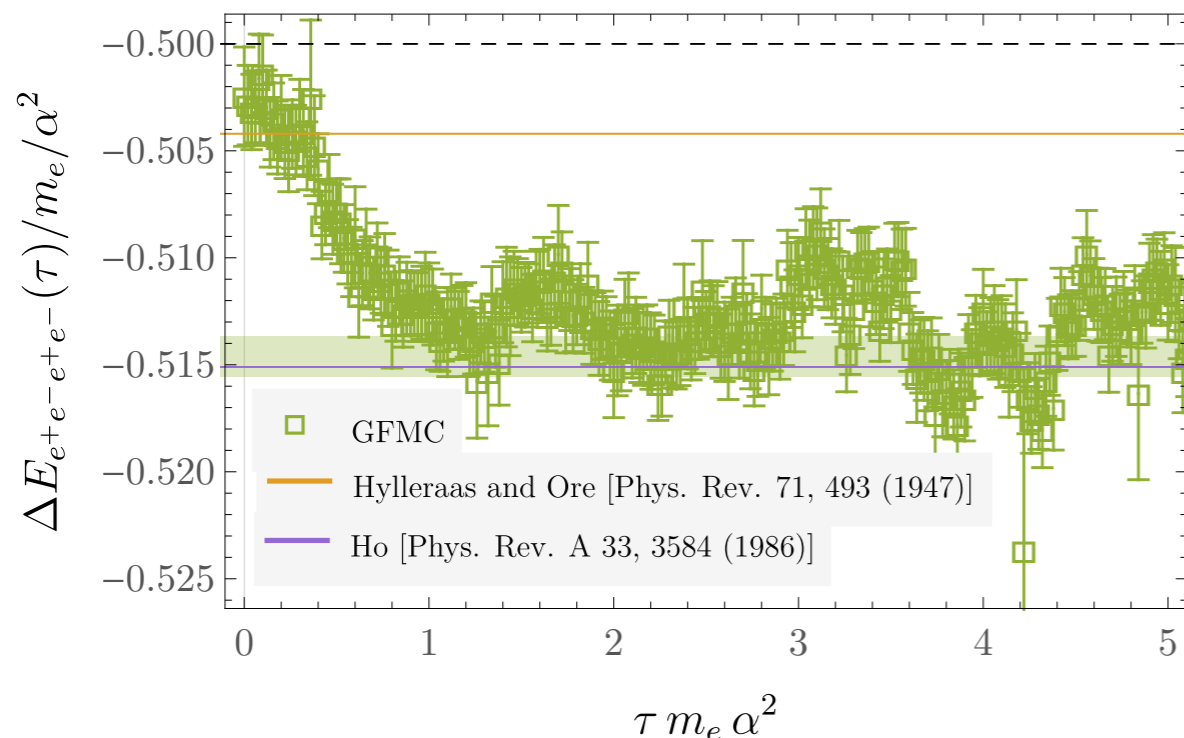
$$\langle e^+(r_1)e^-(r_2)e^+(r_3)e^-(r_4) | V | e^+(r_1)e^-(r_2)e^+(r_3)e^-(r_4) \rangle$$

$$= \alpha \left(\frac{1}{r_{13}} + \frac{1}{r_{24}} - \frac{1}{r_{12}} - \frac{1}{r_{34}} - \frac{1}{r_{14}} - \frac{1}{r_{23}} \right)$$

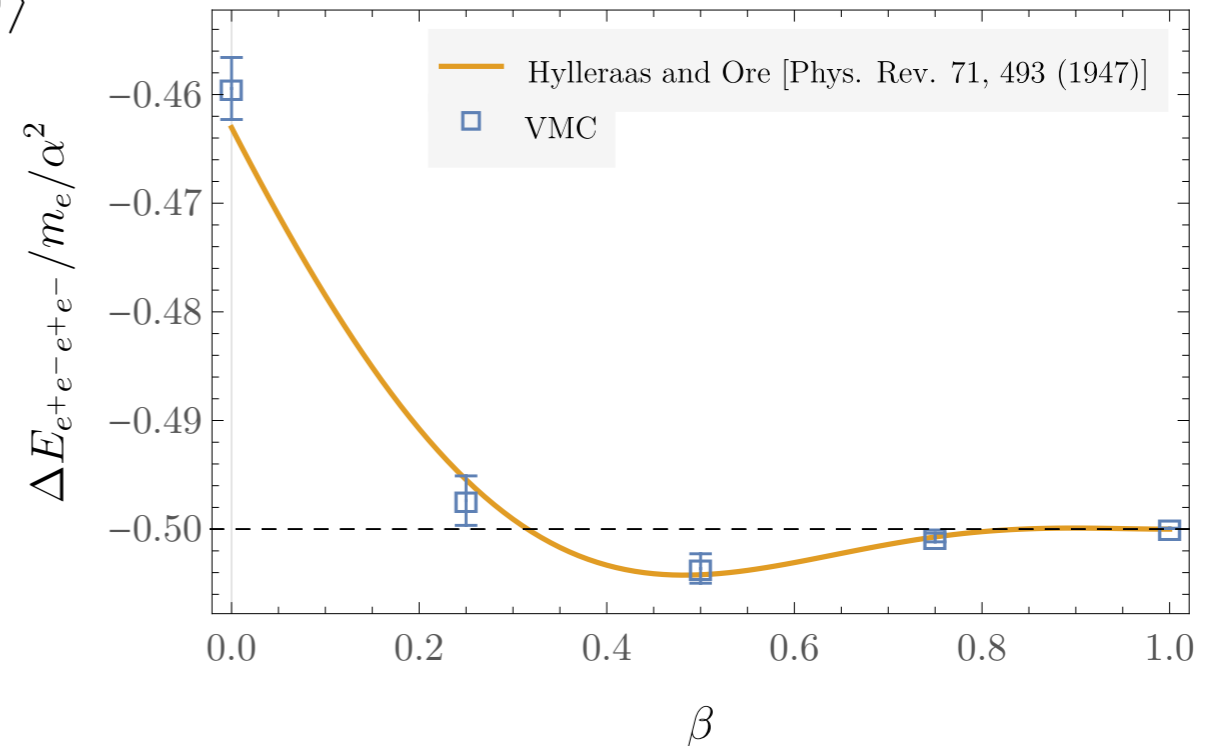
Hylleraas variational wavefunction demonstrates positronium molecule binds known from 1947

$$\psi \propto e^{-(r_{12}+r_{23}+r_{14}+r_{34})/2}$$

$$\times \cosh [\beta(r_{12} - r_{23} - r_{14} + r_{23})/2]$$



BA and Wagman, *in preparation*



Applying GFMC to Hylleraas wavefunctions reproduces state-of-the-art variational results for positronium molecule binding:

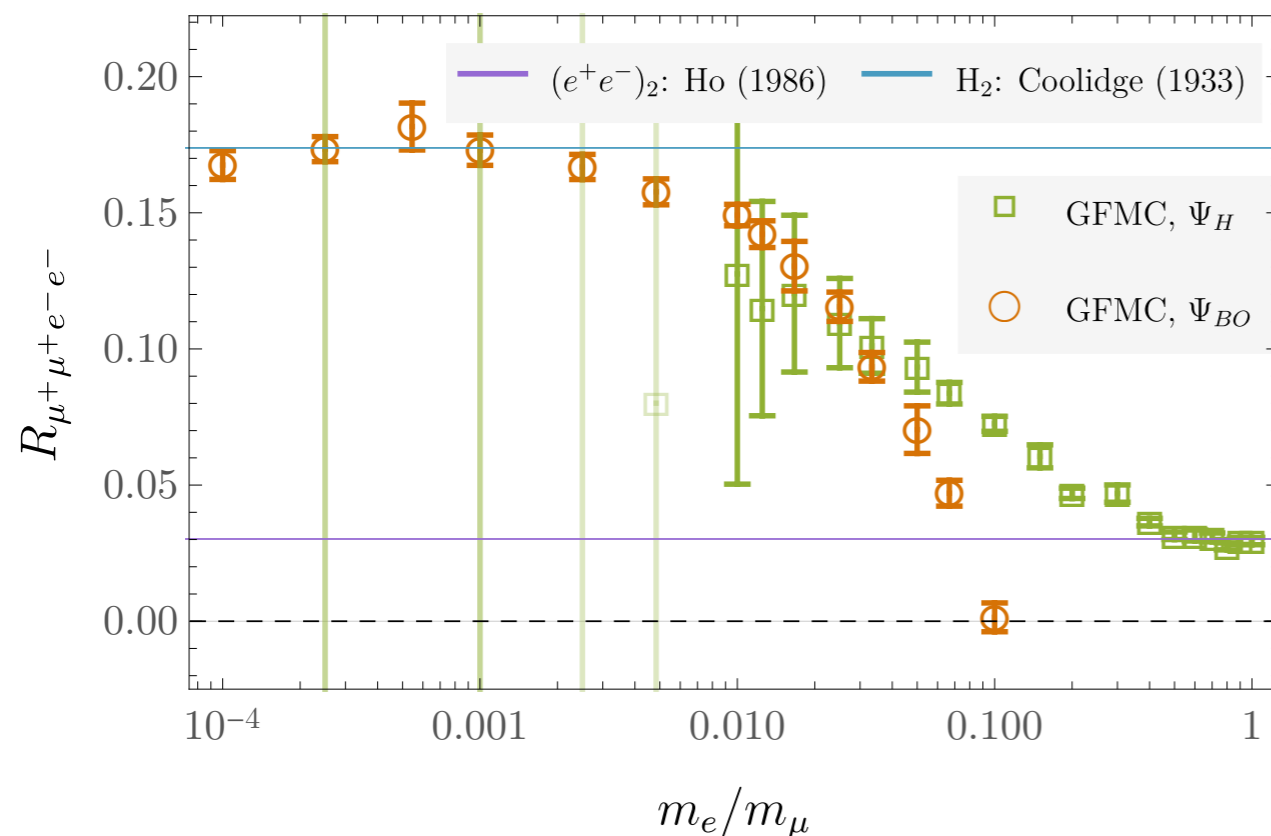
$$\Delta E_{\mu^+\mu^+e^-e^-} - 2\Delta E_{\mu^+e^-} = -0.40(3) \text{ eV}$$

“I like your paper on molecules”

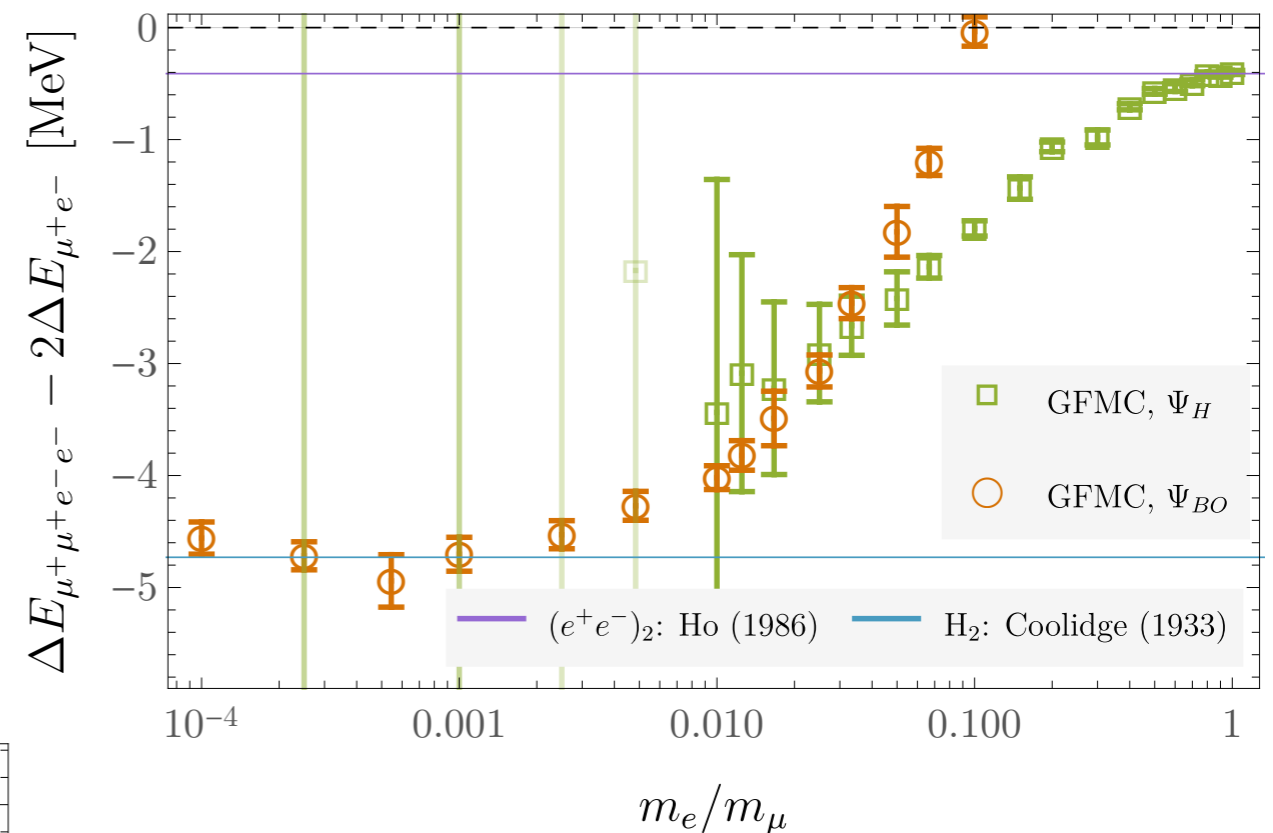
Same methods can be used to study the case of large unequal mass differences in order to understand transition to BO approximation in H_2

BO wavefunction is Hylleraas-like with fixed separation between heavy partners by delta-function

Coolidge (1933)



BA and Wagman, *in preparation*



BO trial state trial state are fixed distance $R = 1.4/(\alpha m_e)$ and applying GFMC to BO allows R to vary

$$\Delta E_{ppe^-e^-} - 2\Delta E_{pe^-} \sim -4.7 \text{ eV}$$

Meson interactions in pNRQCD

For multi-hadron states, interplay between different color tensors leads to complex dependence of potential on color states:

$$\begin{aligned}
 |\mathbf{1} \otimes \mathbf{1}\rangle &\propto |Q_i \bar{Q}_j Q_k \bar{Q}_l\rangle \delta_{ij} \delta_{kl} & |\mathbf{3} \otimes \bar{\mathbf{3}}\rangle &\propto |Q_i \bar{Q}_j Q_k \bar{Q}_l\rangle \epsilon_{ikm} \epsilon_{jlm} \\
 & & |\mathbf{6} \otimes \bar{\mathbf{6}}\rangle &\propto |Q_i \bar{Q}_j Q_k \bar{Q}_l\rangle (\delta_{ij} \delta_{kl} + \delta_{il} \delta_{kj})
 \end{aligned}$$

Matrix elements of e.g. LO potential:

$$\langle \mathbf{6} \otimes \bar{\mathbf{6}} | V | \mathbf{6} \otimes \bar{\mathbf{6}} \rangle = \frac{\alpha_s}{3} \left(\frac{1}{r_{13}} + \frac{1}{r_{24}} \right) - \frac{5\alpha_s}{6} \left(\frac{1}{r_{12}} + \frac{1}{r_{34}} + \frac{1}{r_{14}} + \frac{1}{r_{23}} \right)$$

$$\langle \mathbf{3} \otimes \bar{\mathbf{3}} | V | \mathbf{3} \otimes \bar{\mathbf{3}} \rangle = -\frac{2\alpha_s}{3} \left(\frac{1}{r_{13}} + \frac{1}{r_{24}} \right) - \frac{\alpha_s}{3} \left(\frac{1}{r_{12}} + \frac{1}{r_{34}} + \frac{1}{r_{14}} + \frac{1}{r_{23}} \right)$$

$$\langle \mathbf{1} \otimes \mathbf{1} | V | \mathbf{1} \otimes \mathbf{1} \rangle = -\frac{4\alpha_s}{3} \left(\frac{1}{r_{12}} + \frac{1}{r_{34}} \right)$$



Product of two color-singlet mesons has 0 meson-meson potential at LO and NLO...

Equal-mass tetraquarks

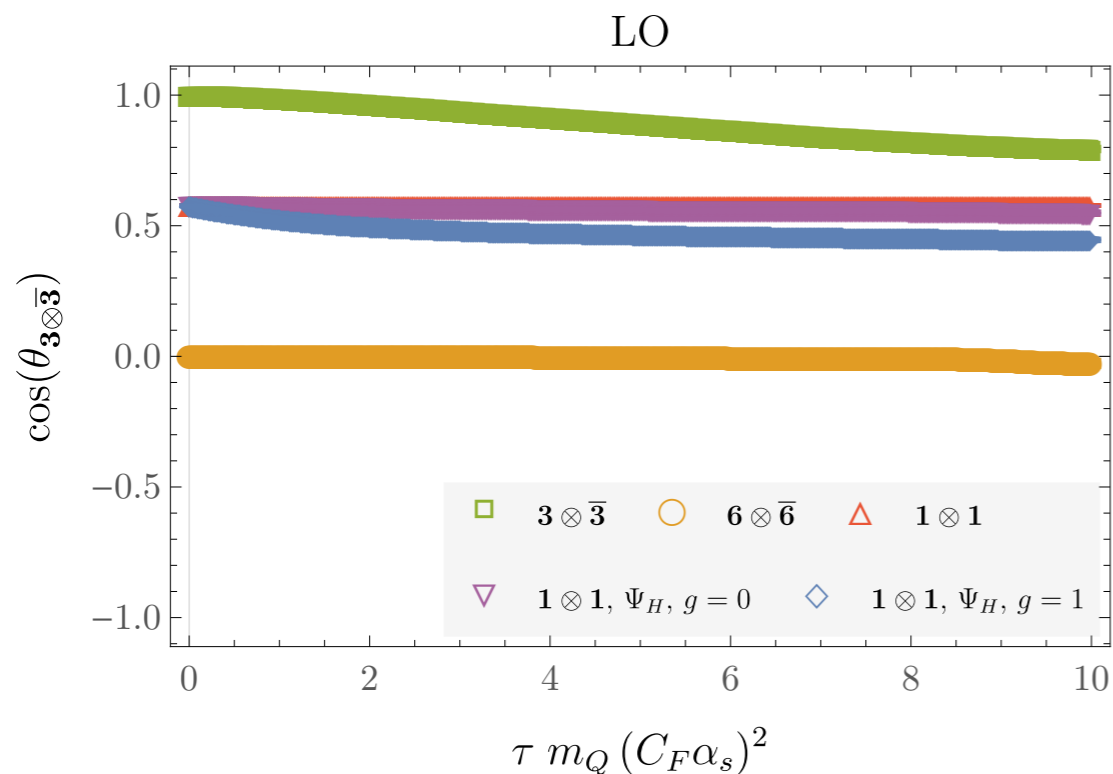
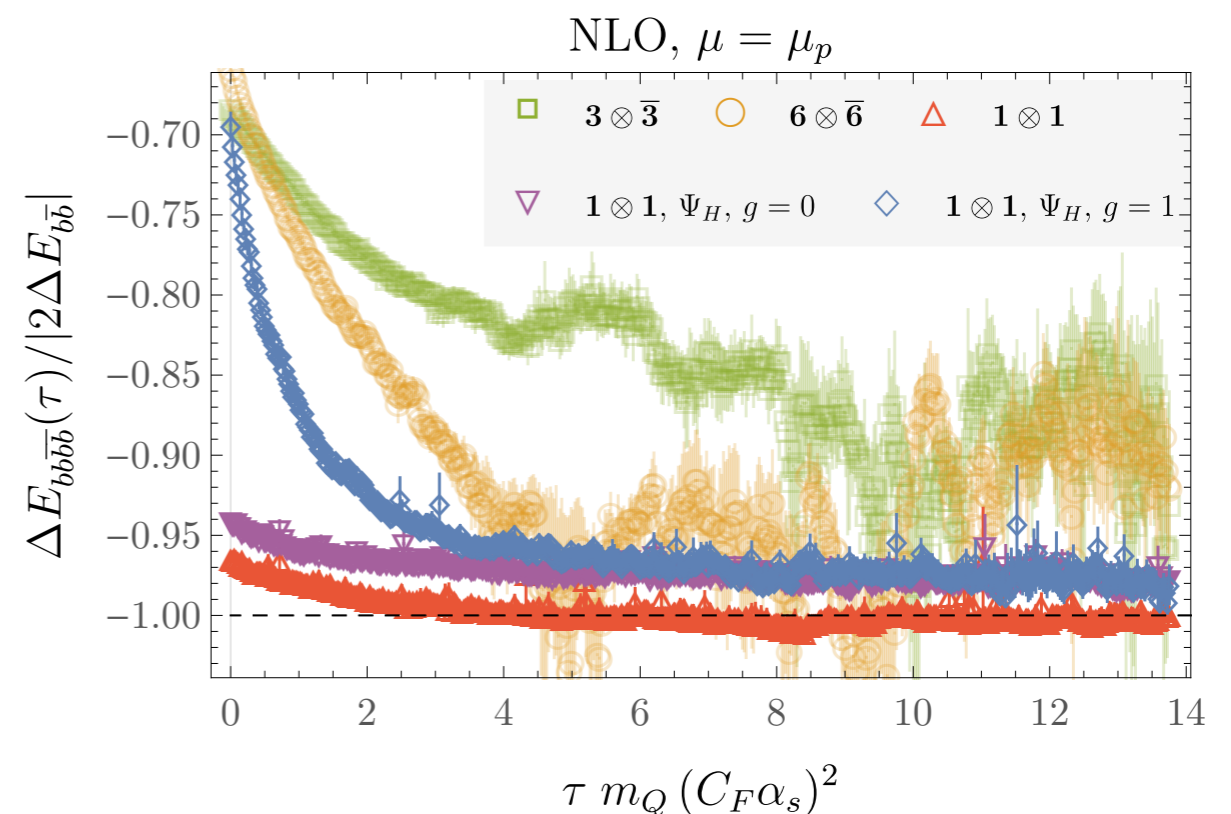
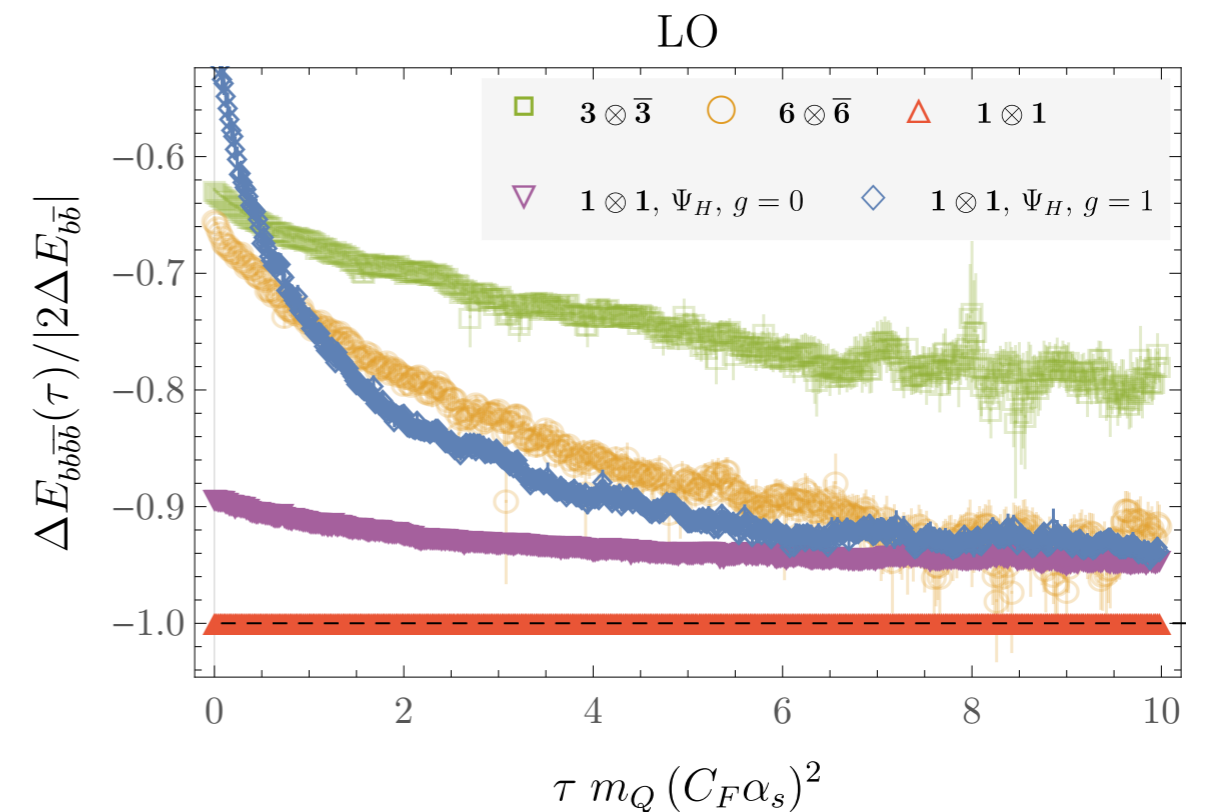
BA and Wagman, *in preparation*

Applying GFMC to pNRQCD for equal-mass heavy quarks leads to energies above the 2-meson threshold

Consistent with Czarnecki, Long, and Voloshin, *Phys. Lett. B* 778 (2018); Eichten, Hughes *PRD* 97 (2018)

We investigate optimal spatial wavefunction for each color state

Hylleras wavefunctions with 1×1 color trial states lead to the strongest bounds



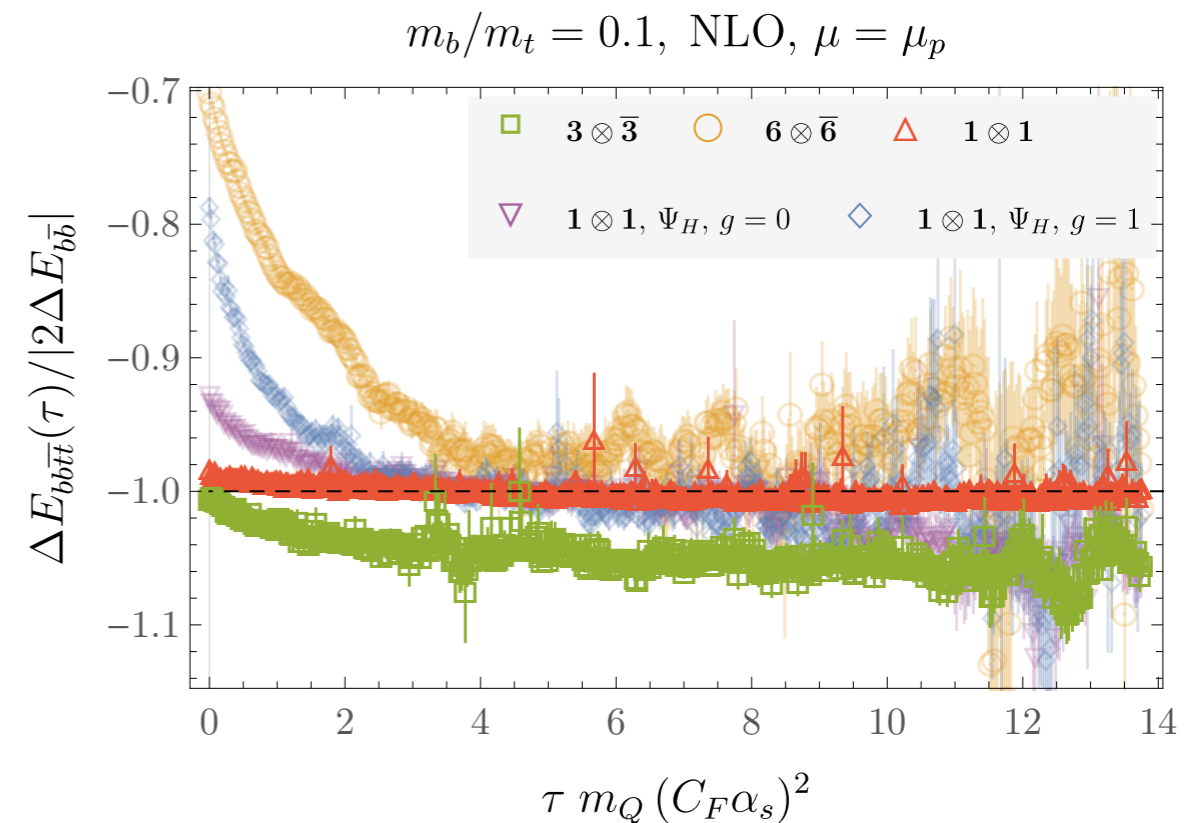
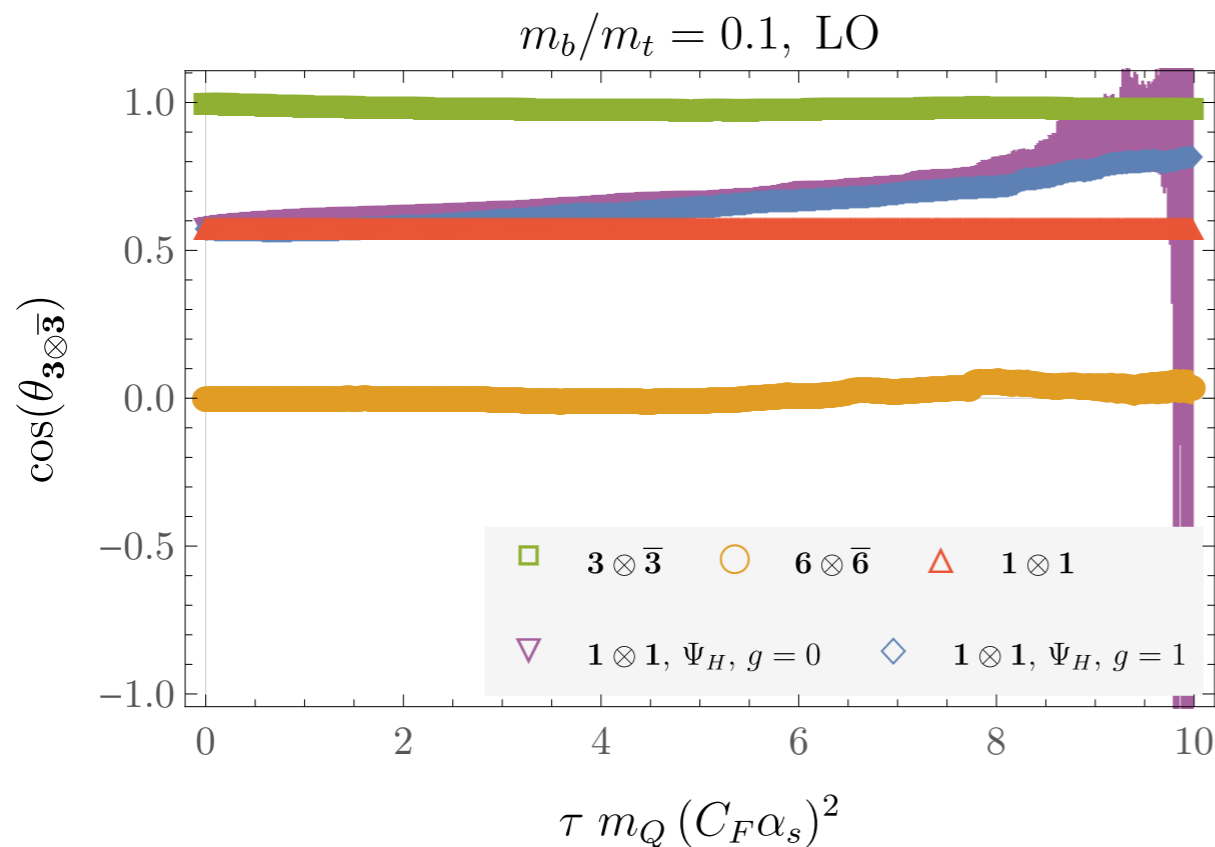
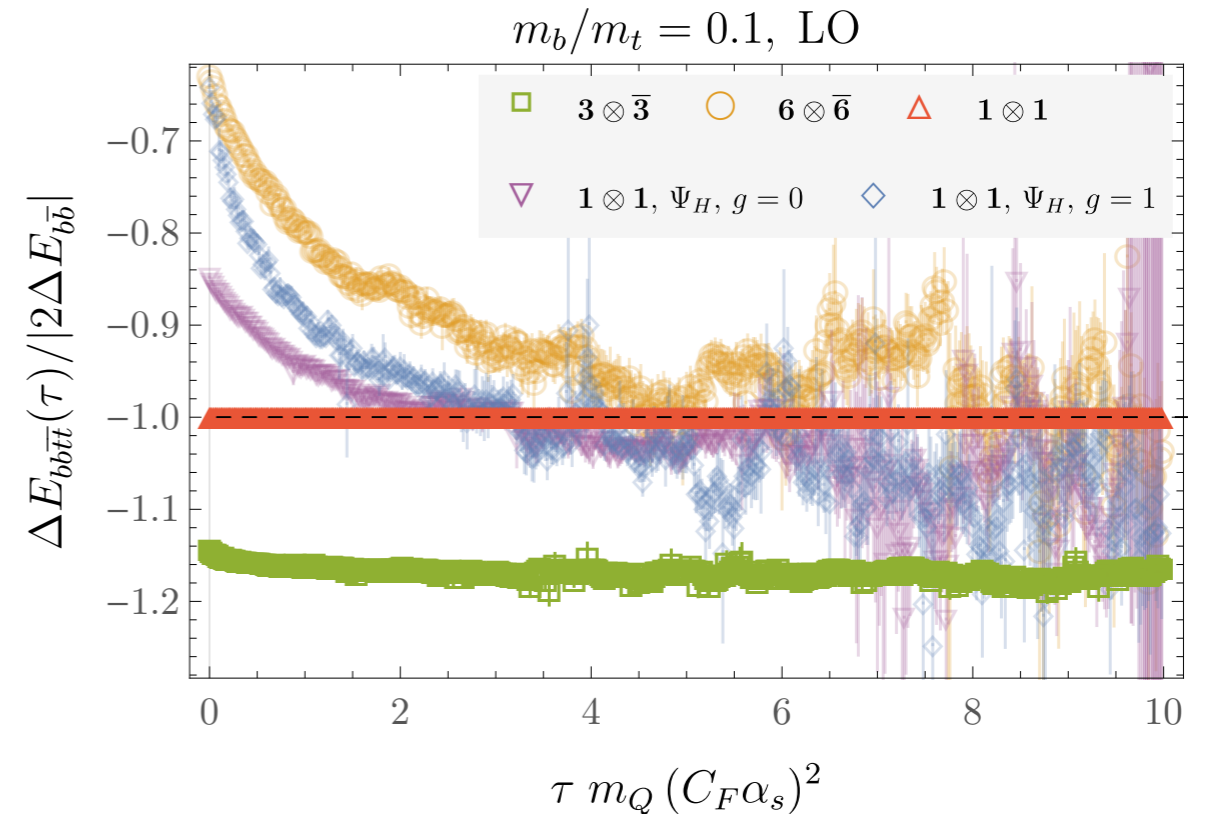
Unequal-mass tetraquarks

BA and Wagman, *in preparation*

Sufficiently unequal heavy tetraquarks display clear signs of binding

Consistent with Czarnecki, Long, and Voloshin, *Phys. Lett. B* 778 (2018)

NNLO requires 3- and 4-body potentials not yet derived

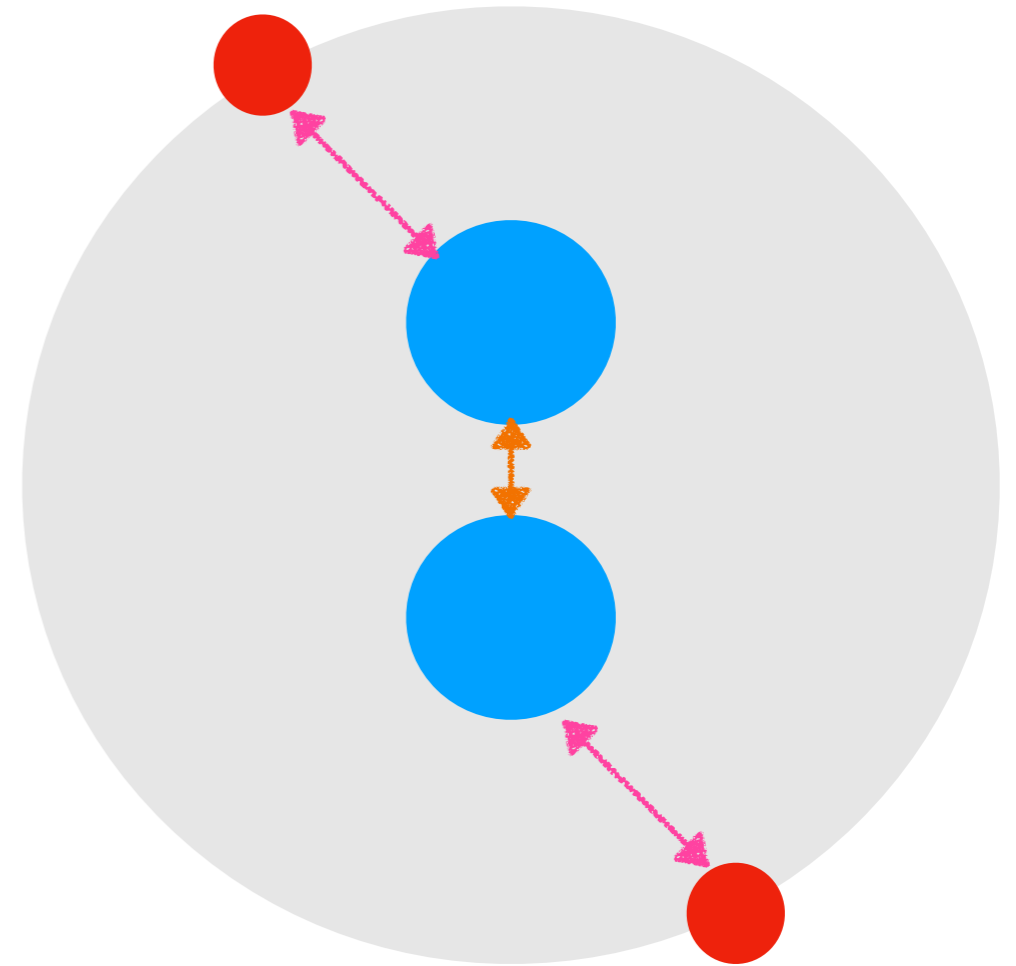


Unequal-mass tetraquark structure

Quarkonium wavefunction known exactly at LO

$$\Psi_T^{Q_i \bar{Q}_j}(\mathbf{R}; a) = \frac{\delta_{ij}}{\sqrt{N_c}} \frac{1}{\sqrt{\pi a^3}} e^{-|\mathbf{r}_{12}|/a_B}$$

Tetraquark wavefunction **not known**, we investigated large set of spatial wavefunctions for each color state



Optimal state: color antisymmetric Hyleraas-like wavefunction

$$\Psi_T^{Q_i \bar{Q}_j Q_k \bar{Q}_l}(\mathbf{R}; a, b) \propto \epsilon_{ijk} \epsilon_{klm} e^{-|\mathbf{r}_{13}|/(2a_{i\bar{i}})} e^{-|\mathbf{r}_{24}|/(2a_{b\bar{b}})} e^{-(|\mathbf{r}_{12}|+|\mathbf{r}_{14}|+|\mathbf{r}_{23}|+|\mathbf{r}_{34}|)/(4a_{i\bar{b}})}$$

Heavy **diquark core** with lighter diquarks twice mass ratio apart

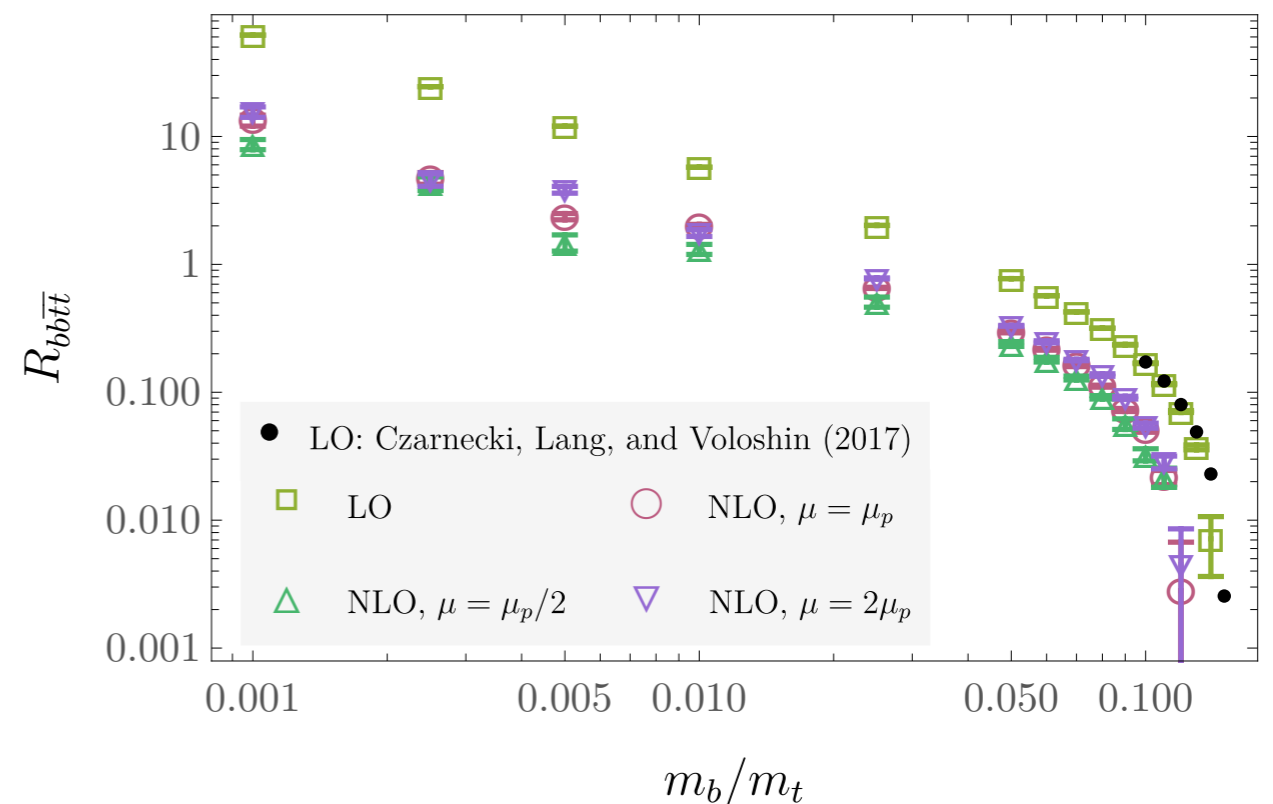
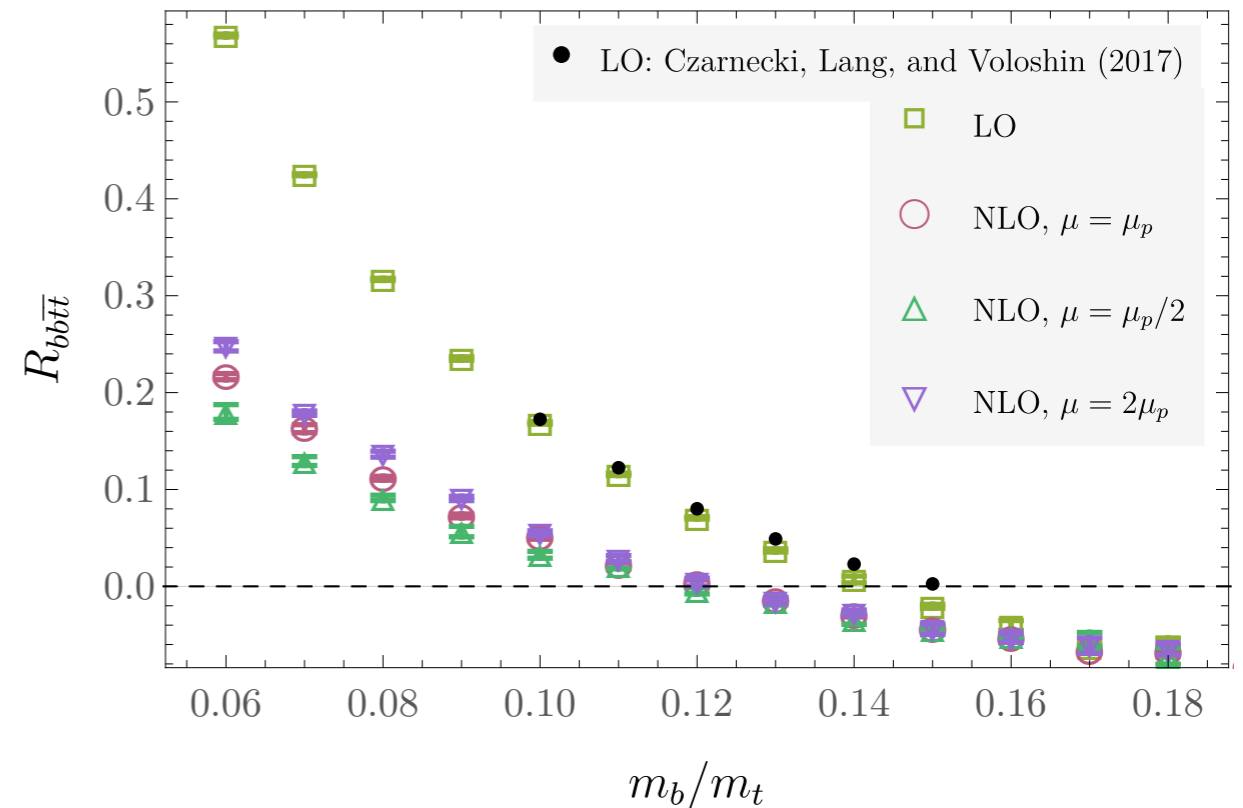
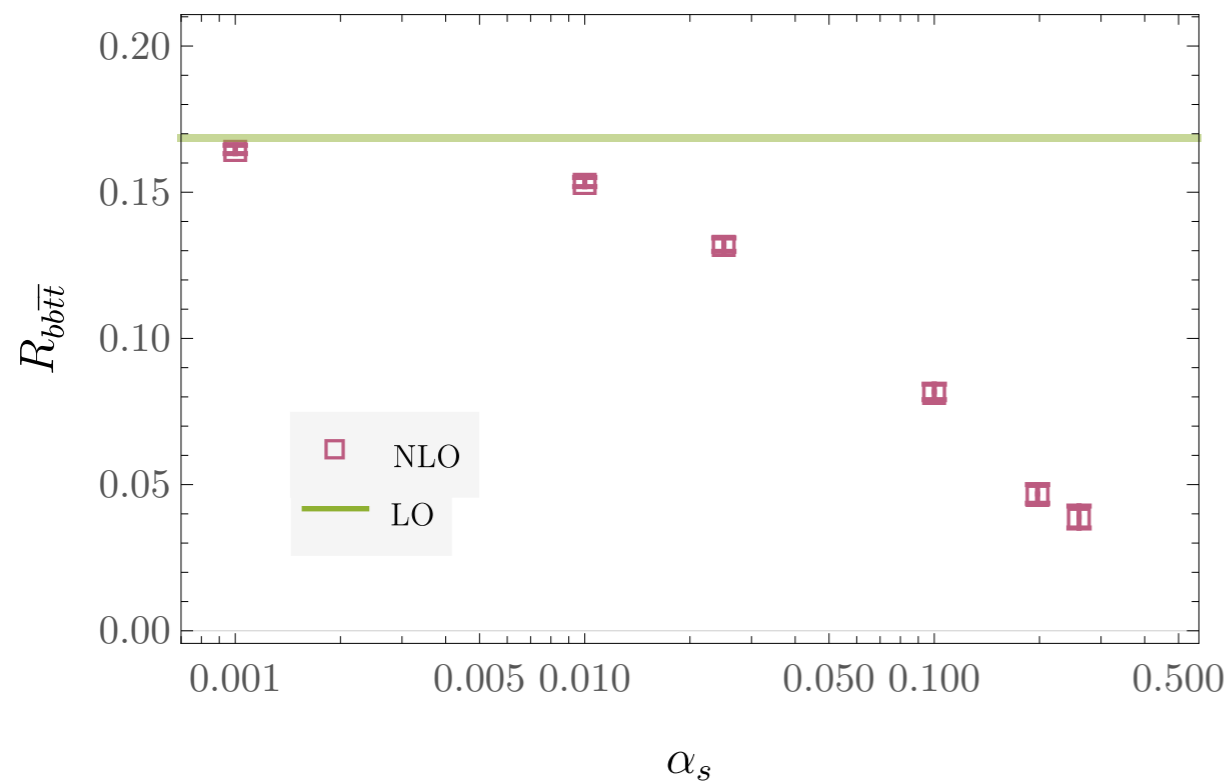
Unequal-mass tetraquarks

BA and Wagman, *in preparation*

Sufficiently unequal heavy tetraquarks display clear signs of binding

Consistent with Czarnecki, Long, and Voloshin, *Phys. Lett. B* 778 (2018)

All-order convergence at infinite mass limit independent of unequal mass ratio



Summary

Developed and validated methods for using pNRQCD+QMC to study multi-hadron systems

In contrast to QED NR equal-mass tetraquarks show no signs of binding to NLO

Unequal mass exhibit clear signs of binding for $m_q/m_Q \sim 0.15$

Outlook

Study heavy dibaryons with these methods and compare with lattice prediction [Mathur, Padmananth, Chakraborty PRL 130 \(2023\)](#)

Investigate pheno relevant exotics, X(6900) as possible P-wave resonance [BA, Brambilla, Mohapatra, Vairo, Wagman](#)

Improve convergence by resummation, RS and SD pieces for mass-splittings [BA, Kronfeld, Wagman and Vaiva, *in preparation*](#)

BACK-UP Slides

Nuclear interactions in pNRQCD

Two-baryon systems comprised of heavy quarks can be studied similarly

$$|Q_i(r_1)Q_j(r_2)Q_k(r_3)Q_l(r_4)Q_m(r_5)Q_n(r_6)\rangle$$

Product of two color-singlet baryons has 0 inter-baryon potential at LO and NLO

$$\langle \mathbf{1} \otimes \mathbf{1} | V | \mathbf{1} \otimes \mathbf{1} \rangle = -\frac{2\alpha_s}{3} \left(\frac{1}{r_{12}} + \frac{1}{r_{23}} + \frac{1}{r_{23}} + \frac{1}{r_{45}} + \frac{1}{r_{46}} + \frac{1}{r_{56}} \right)$$

Other color structures lead to more interesting potentials:

Complete basis: $(Q_i Q_j)(Q_k Q_l)(Q_m Q_n) T_{ijklmn}^{AAA}$

$$\langle \bar{\mathbf{3}} \otimes \bar{\mathbf{3}} \otimes \bar{\mathbf{3}} | V | \bar{\mathbf{3}} \otimes \bar{\mathbf{3}} \otimes \bar{\mathbf{3}} \rangle = -\frac{2\alpha_s}{3} \left(\frac{1}{r_{12}} + \frac{1}{r_{34}} + \frac{1}{r_{56}} \right) - \frac{\alpha_s}{6} \left(\frac{1}{r_{13}} + \frac{1}{r_{14}} + \frac{1}{r_{15}} + \frac{1}{r_{16}} + \dots \right)$$

$$1 \subset \bar{\mathbf{3}} \otimes \bar{\mathbf{3}} \otimes \bar{\mathbf{3}},$$

$$1 \subset \mathbf{6} \otimes \mathbf{6} \otimes \mathbf{6},$$

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$$1 \subset \mathbf{6} \otimes \bar{\mathbf{3}} \otimes \bar{\mathbf{3}}$$

$(Q_i Q_j)(Q_k Q_l)(Q_m Q_n) T_{ijklmn}^{SSS}$

$$\langle \bar{\mathbf{6}} \otimes \bar{\mathbf{6}} \otimes \bar{\mathbf{6}} | V | \bar{\mathbf{6}} \otimes \bar{\mathbf{6}} \otimes \bar{\mathbf{6}} \rangle = \frac{\alpha_s}{3} \left(\frac{1}{r_{12}} + \frac{1}{r_{34}} + \frac{1}{r_{56}} \right) - \frac{5\alpha_s}{12} \left(\frac{1}{r_{13}} + \frac{1}{r_{14}} + \frac{1}{r_{15}} + \frac{1}{r_{16}} + \dots \right)$$

Could these color states describe bound dibaryons?

Rao and Shrock, Phys. Lett. B 116 (1982)

Stay tuned!

Harvey, Nucl. Phys. A 352 (1981)

BA and Wagman, *in preparation*

A simple trial wavefunction

Complete basis for Ψ_T and optimise for $\alpha \Rightarrow$ require **functional form** of basis

Recall **Hydrogen**

$$H = -\frac{1}{2}\nabla_r^2 - \frac{b_0}{|\mathbf{r}|} \Rightarrow E_0 = -\frac{m}{2}b_0^2, \quad \psi_{100}(r) = \sqrt{\frac{b_0^3}{\pi}}e^{-b_0 r}$$

Positronium and quarkonium at LO

$$H = -\frac{1}{2}(\nabla_{r_1}^2 + \nabla_{r_2}^2) - \frac{b_0}{|\mathbf{r}_{12}|} \Rightarrow E_0 = -\frac{m}{4}b_0^2, \quad \Psi_{100}(\mathbf{r}_{1,2}) = \sqrt{\frac{b_0^3}{8\pi}}e^{-b_0|\mathbf{r}_{12}|/2}$$

Ansatz: N -particle **baryons** with only LO (pairwise) potential:

$$H = -\frac{1}{2}\sum_i^N \nabla_{r_i}^2 + V_{\bar{N}}^{\psi\psi}(\mathbf{r}_{1\dots N}) \Rightarrow \Psi_{\min}(\mathbf{r}_1, \dots, \mathbf{r}_N) = \prod_{i<j}^N \psi_{100}(\mathbf{r}_{ij}, a_0 = 2/b_{N_c})$$

Upon analysis this simple product state gives $\sim E_{\min}$ for all N_c

Green's function Monte-Carlo

We now have Ψ_T and E_{\min} but what if \exists lower-lying states?

Limited in VMC by choice and size of basis \Rightarrow need new tool

GFMC: Given Ψ_T from VMC diffuse in time and reach **true ground-state energy**

$$|\Psi_T\rangle = \lim_{\tau \rightarrow \infty} e^{-\hat{H}\tau} |\psi_T\rangle$$

In practice: Take small time $\delta\tau = \tau/L$ for $L \gg 1$ and path integrate

$$\Psi(\tau, \mathbf{R}_N) = \int \prod_{i=0}^{N-1} d\mathbf{R}_i \langle \mathbf{R}_N | e^{-H\delta\tau} | \mathbf{R}_{N-1} \rangle \cdots \langle \mathbf{R}_1 | e^{-H\delta\tau} | \mathbf{R}_0 \rangle \langle \mathbf{R}_0 | \Psi_T \rangle$$

with **Trotter-Suzuki expansion**

$$\langle \mathbf{R}_{i+1} | e^{-H\delta\tau} | \mathbf{R}_i \rangle = e^{-V(\mathbf{R}_{i+1})\delta\tau/2} \langle \mathbf{R}_{i+1} | e^{-T\delta\tau} | \mathbf{R}_i \rangle e^{-V(\mathbf{R}_i)\delta\tau/2} + \mathcal{O}(\delta\tau^3)$$

N.B. $\Psi(\tau, \mathbf{R}_N)$ is not the true wavefunction but a set of amplitudes

Dark Hadrons

Composite dark matter attractive candidate since implicit stability due to global flavour symmetry

$$\mathcal{L}_{\text{dQCD}} = -\frac{1}{2}\text{Tr}G_{\mu\nu}^2 + \bar{q}iDq + m_d\bar{q}q$$

New $SU(N_d)$ gauge sector **confines** at

$$\Lambda_{\text{dQCD}} \sim \exp\left(-\frac{2\pi}{\beta_0\alpha_d}\right)$$

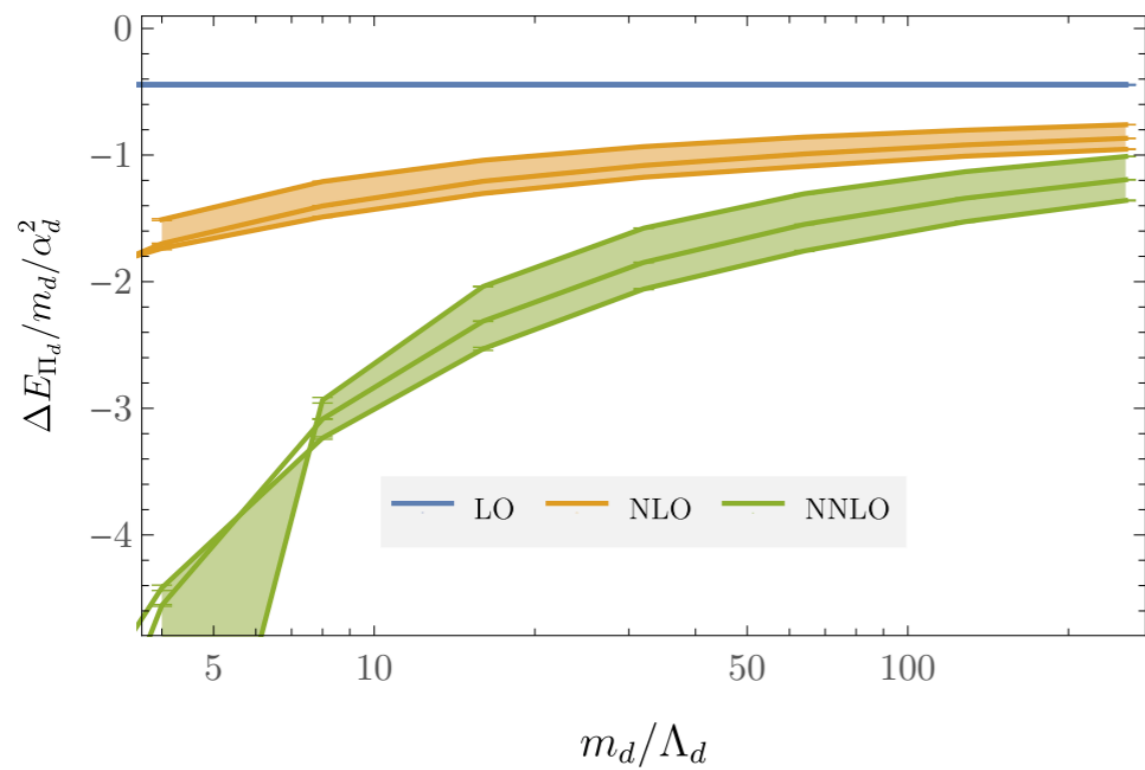
DM **stability** is maintained beyond Gyrs \sim proton stability in SM

SM-DM interactions **suppressed** in the EFT above confinement

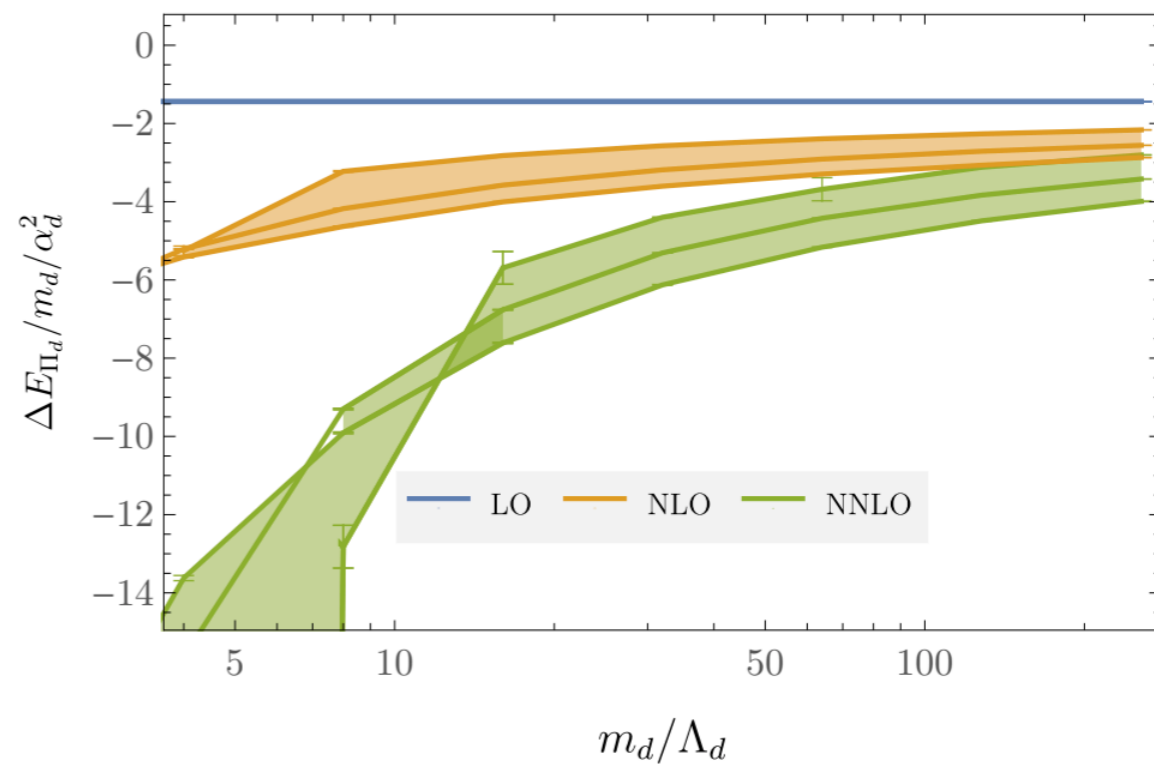


Dark Mesons

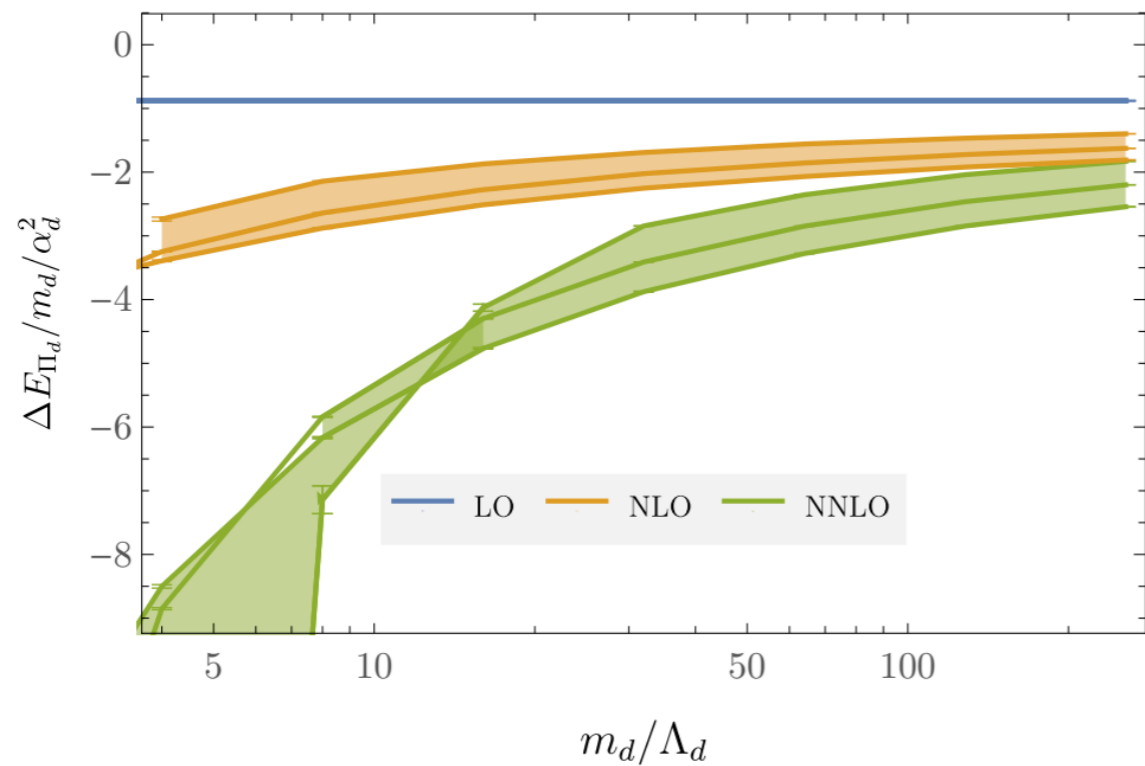
$N_c = 3$



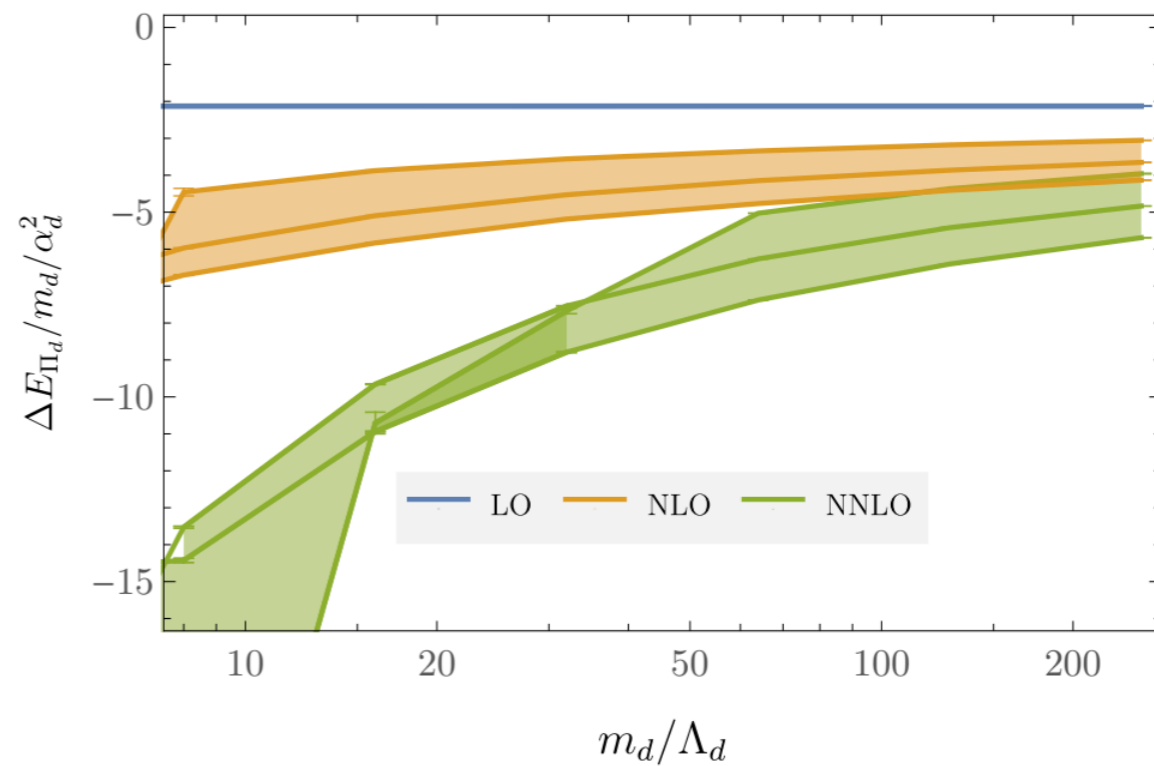
$N_c = 5$



$N_c = 4$

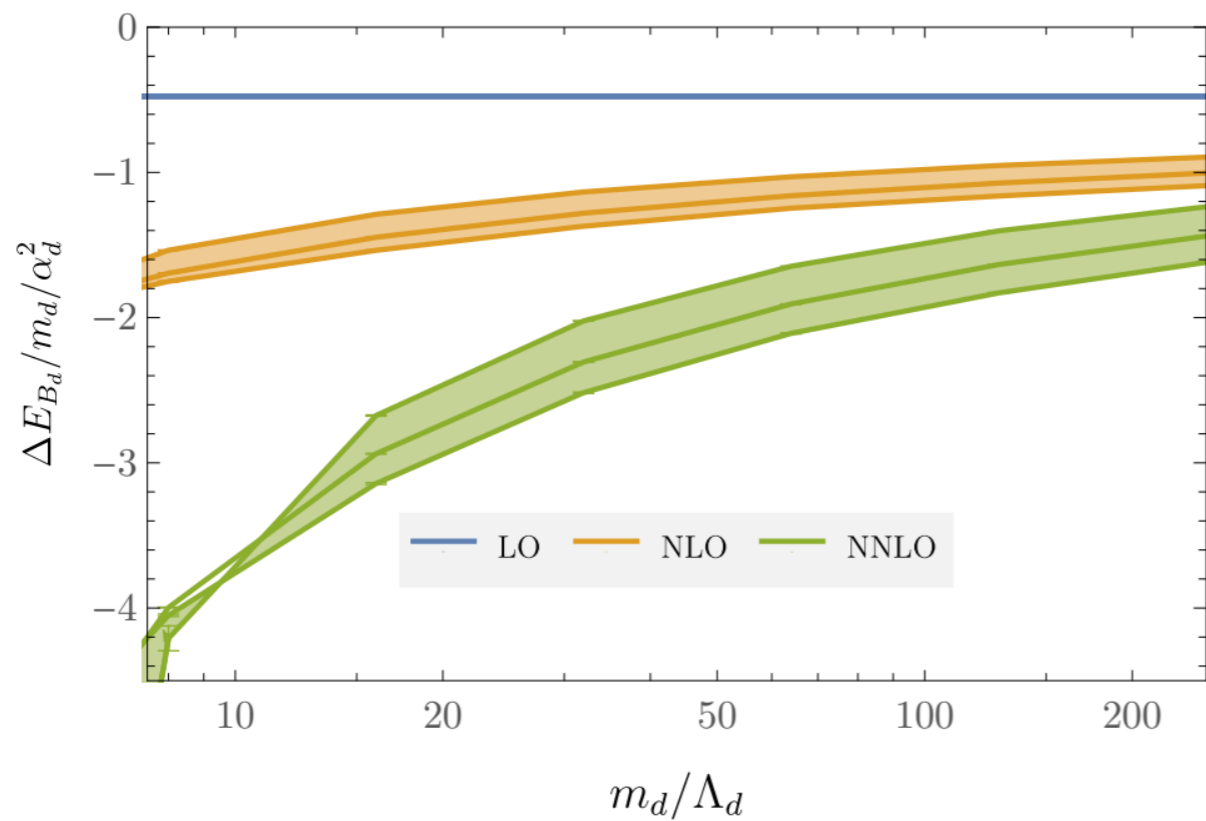


$N_c = 6$

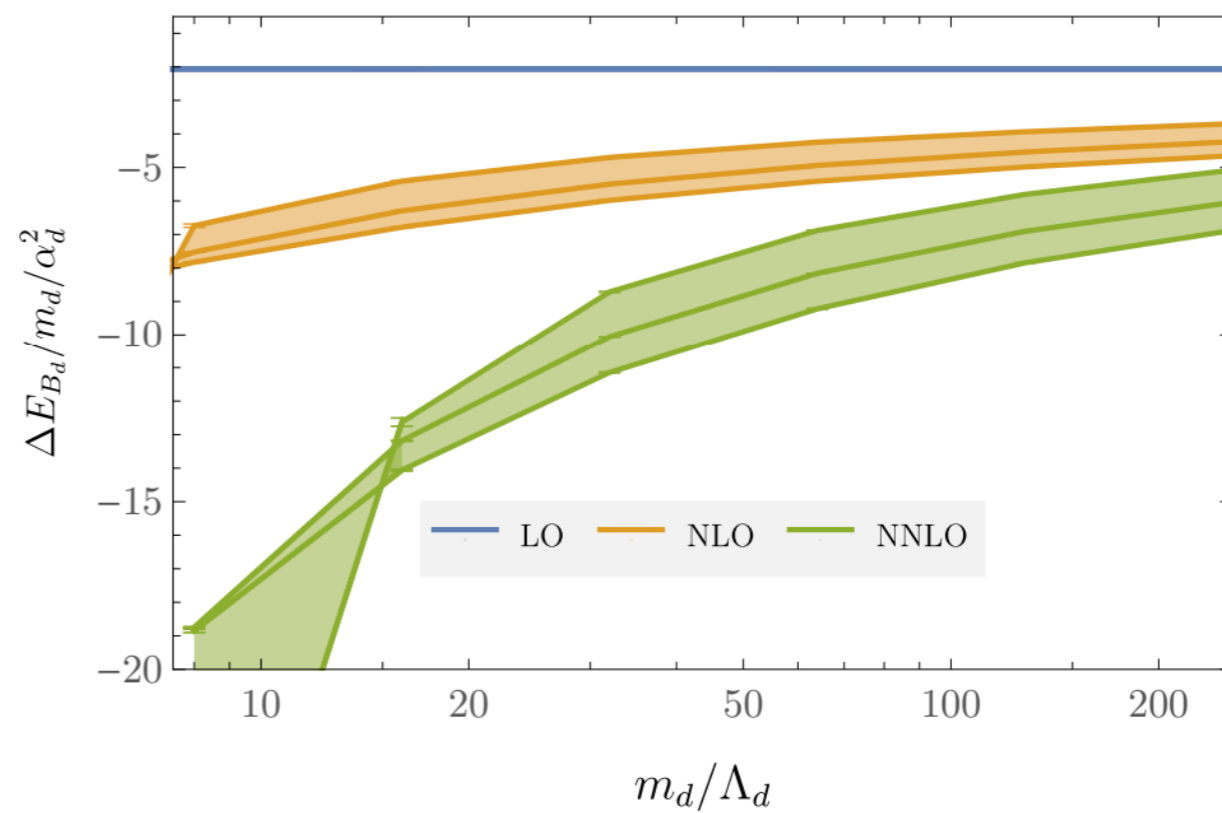


Dark baryons

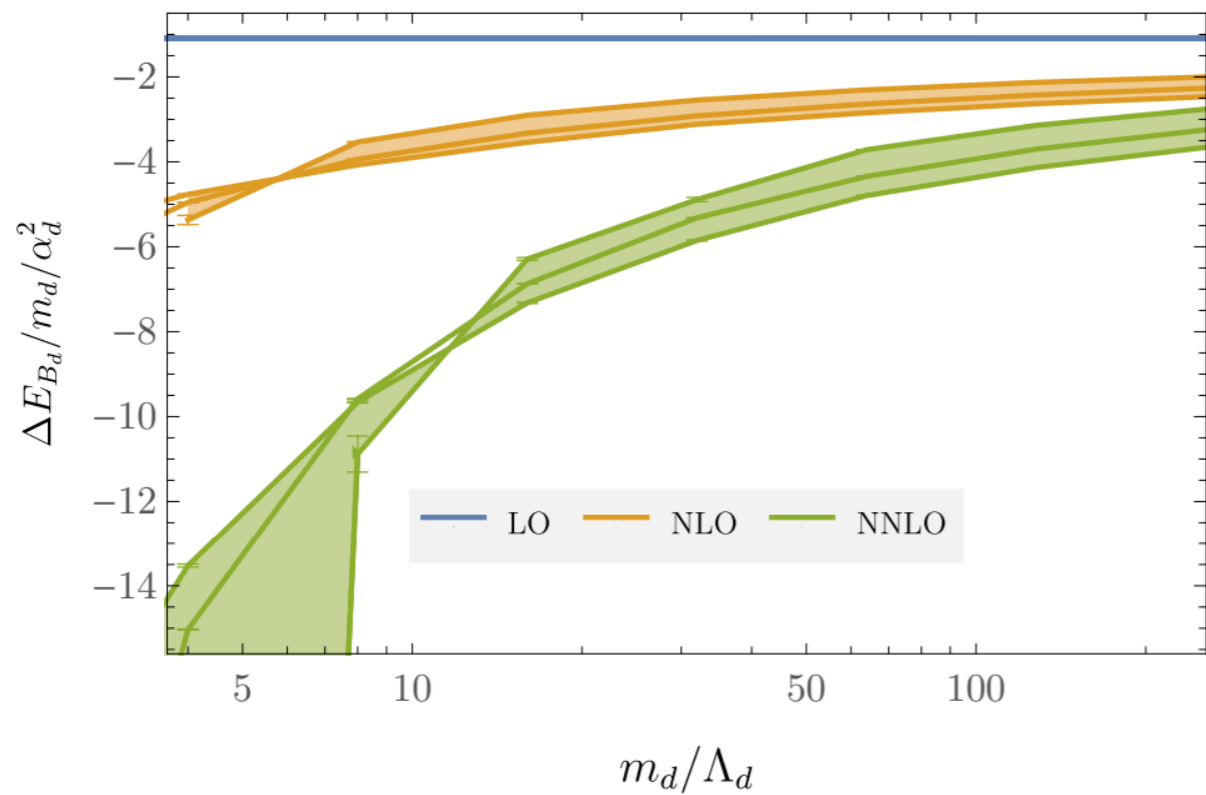
$N_c = 3$



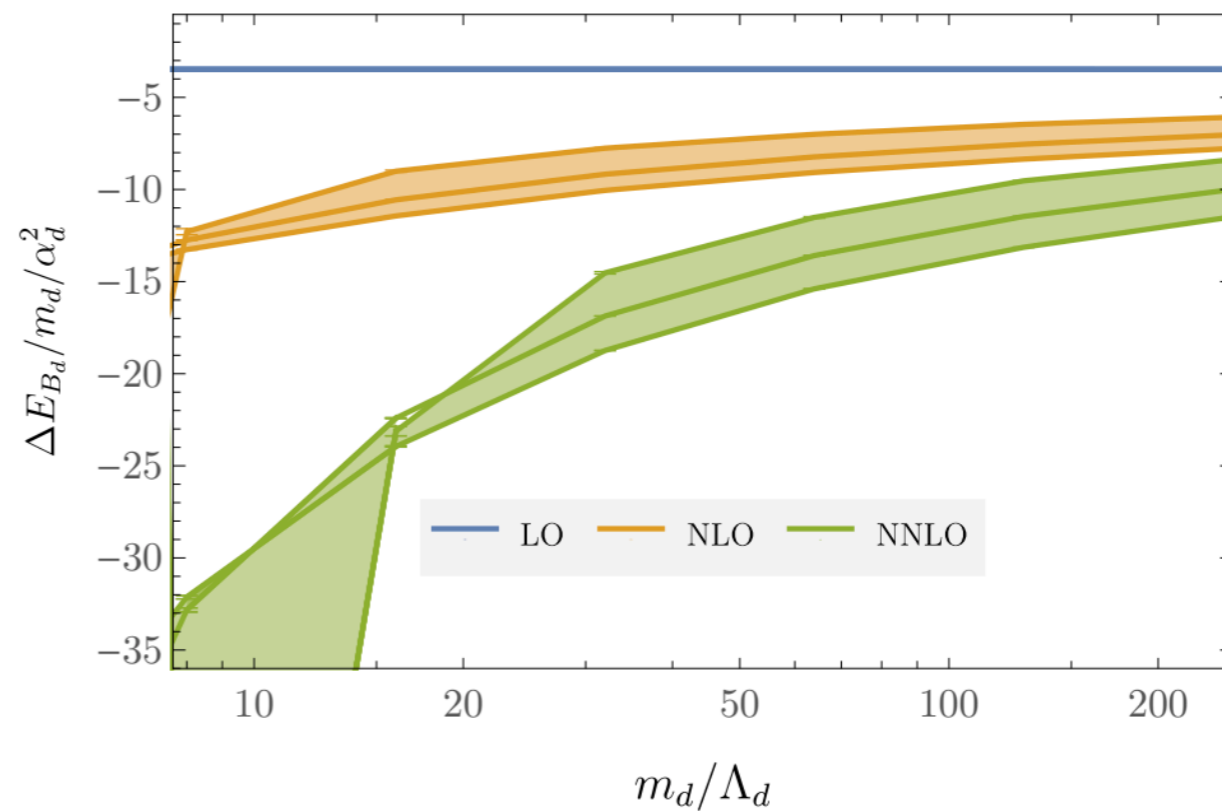
$N_c = 5$



$N_c = 4$



$N_c = 6$



Extra: pNRQCD Hamiltonian

Converting from Lagrangian to Hamiltonian formalism

Quarkonium and baryon **state** can be written

$$|Q\bar{Q}(\mathbf{r}_1, \mathbf{r}_2)\rangle = \frac{1}{\sqrt{N_c}} |\psi_m(\mathbf{r}_1), \chi_n^\dagger(\mathbf{r}_2)\rangle \delta_{mn} \quad |B(\mathbf{r}_1, \dots, \mathbf{r}_{N_c})\rangle \equiv \frac{\epsilon_{i_1 \dots i_{N_c}}}{\sqrt{N_c!}} \psi_{i_1}^\dagger(\mathbf{r}_1) \cdots \psi_{i_{N_c}}^\dagger(\mathbf{r}_{N_c}) |0\rangle$$

A non-relativistic potential **operator** then acts as

$$\begin{aligned} & \hat{V}^{\psi\chi} |\psi_m(\mathbf{r}_1), \chi_n^\dagger(\mathbf{r}_2)\rangle \\ &= \int d^3\mathbf{s}_1 d^3\mathbf{s}_2 \left[\frac{1}{N_c} \delta_{ij} \delta_{kl} V_1^{\psi\chi}(\mathbf{s}_{12}) + \frac{1}{T_F} T_{ij}^a T_{kl}^a V_{\text{Ad}}^{\psi\chi}(\mathbf{s}_{12}) \right] \\ & \quad \times \psi_i^\dagger(t, \mathbf{s}_1) \chi_j(t, \mathbf{s}_2) \chi_k^\dagger(t, \mathbf{s}_2) \psi_l(t, \mathbf{s}_1) |\psi_m(\mathbf{r}_1), \chi_n^\dagger(\mathbf{r}_2)\rangle \\ &= \left[\frac{1}{N_c} \delta_{ij} \delta_{kl} V_1^{\psi\chi}(\mathbf{r}_{12}) + \frac{1}{T_F} T_{ij}^a T_{kl}^a V_{\text{Ad}}^{\psi\chi}(\mathbf{r}_{12}) \right] |\psi_i(\mathbf{r}_1), \chi_j^\dagger(\mathbf{r}_2)\rangle \delta_{km} \delta_{ln} \end{aligned}$$

More generally 3-quark potential acting on **arbitrary** state

$$\begin{aligned} & \hat{V}^{3\psi} |\psi_1(\mathbf{r}_1), \dots, \psi_{N_q}(\mathbf{r}_{N_q})\rangle \\ &= \sum_{\rho \in \{A, S, MA, MS\}} \int d^3\mathbf{s}_1 d^3\mathbf{s}_2 d^3\mathbf{s}_3 V_\rho^{3\psi}(\mathbf{s}_{12}, \mathbf{s}_{13}, \mathbf{s}_{23}) \left(\mathcal{F}_{ijk}^\rho \right)^* \mathcal{F}_{lmn}^\rho \psi_i^\dagger(t, \mathbf{s}_1) \psi_j^\dagger(t, \mathbf{s}_2) \psi_k^\dagger(t, \mathbf{s}_3) \\ & \quad \times \psi_l(t, \mathbf{s}_3) \psi_m(t, \mathbf{s}_2) \psi_n(t, \mathbf{s}_1) |\psi_{n_1}(\mathbf{r}_1), \dots, \psi_{n_{N_q}}(\mathbf{r}_{N_q})\rangle \\ &= \sum_{I \neq J \neq K} \sum_{\rho \in \{A, S, MA, MS\}} V_\rho^{3\psi}(\mathbf{r}_{IJ}, \mathbf{r}_{IK}, \mathbf{r}_{JK}) \left(\mathcal{F}_{m_I m_J m_K}^\rho \right)^* \mathcal{F}_{n_I n_J n_K}^\rho \\ & \quad \times |\psi_{n_1}(\mathbf{r}_1), \dots, \psi_{m_I}(\mathbf{r}_I), \dots, \psi_{m_J}(\mathbf{r}_J), \dots, \psi_{m_K}(\mathbf{r}_K), \dots, \psi_{n_{N_q}}(\mathbf{r}_{N_q})\rangle. \end{aligned}$$

Extra: Variational methods

Trial wave function PDF:
$$P(\mathbf{r}) = \frac{|\Psi_T(\mathbf{r}, \boldsymbol{\alpha})|^2}{\int d\mathbf{r} |\Psi_T(\mathbf{r}, \boldsymbol{\alpha})|^2}$$

Two-step algorithm:

1) Pick initial $(\mathbf{r}, \boldsymbol{\alpha}) \rightarrow$ compute $\langle H \rangle, |\Psi_T(\mathbf{r}, \boldsymbol{\alpha})|^2$

2) $\mathbf{r}_p = \mathbf{r} + \mathbf{x}n_{\text{step}} : |\mathbf{x}| \in [0,1] \rightarrow$ **accept/reject** new \mathbf{r}_p with ratio $P(\mathbf{r}_p)/P(\mathbf{r})$

So far we considered **two classes** of N -particle Hamiltonians:

Meson in $SU(N_c)$

$$\hat{H} = -\frac{1}{2} \sum_i^N \nabla_{\mathbf{r}_i} + \hat{V}^{\psi\chi}$$

N -heavy **baryon** ($N = N_c$)

$$\hat{H} = -\frac{1}{2} \sum_i^N \nabla_{\mathbf{r}_i} + \hat{V}^{\psi\psi} + \hat{V}^{3\psi}$$

Tetraquark state and potential

Action of quark-antiquark potential on four quark-antiquark state

$$\begin{aligned} & \hat{V}^{\psi\chi} |\psi_{n_1}(\mathbf{r}_1), \chi_{n_2}^\dagger(\mathbf{r}_2)\psi_{n_3}(\mathbf{r}_3), \chi_{n_4}^\dagger(\mathbf{r}_4)\rangle \\ &= \int d^3\mathbf{s}_1 d^3\mathbf{s}_2 \left[\frac{1}{N_c} \delta_{ij} \delta_{kl} V_1^{\psi\chi}(\mathbf{s}_{12}) + \frac{1}{T_F} T_{ij}^a T_{kl}^a V_{\text{Ad}}^{\psi\chi}(\mathbf{s}_{12}) \right] \\ & \quad \times \psi_i^\dagger(t, \mathbf{s}_1) \chi_j(t, \mathbf{s}_2) \chi_k^\dagger(t, \mathbf{s}_2) \psi_l(t, \mathbf{s}_1) |\psi_{n_1}(\mathbf{r}_1), \chi_{n_2}^\dagger(\mathbf{r}_2)\psi_{n_3}(\mathbf{r}_3), \chi_{n_4}^\dagger(\mathbf{r}_4)\rangle \\ &= \left[\frac{1}{N_c} \delta_{ij} \delta_{n_1 n_2} V_1^{\psi\chi}(\mathbf{r}_{12}) + \frac{1}{T_F} T_{ij}^a T_{n_1 n_2}^a V_{\text{Ad}}^{\psi\chi}(\mathbf{r}_{12}) \right] |\psi_i(\mathbf{r}_1), \chi_j^\dagger(\mathbf{r}_2)\psi_{n_3}(\mathbf{r}_3), \chi_{n_4}^\dagger(\mathbf{r}_4)\rangle \\ & \quad + \left[\frac{1}{N_c} \delta_{ij} \delta_{n_1 n_4} V_1^{\psi\chi}(\mathbf{r}_{14}) + \frac{1}{T_F} T_{ij}^a T_{n_1 n_4}^a V_{\text{Ad}}^{\psi\chi}(\mathbf{r}_{14}) \right] |\psi_i(\mathbf{r}_1), \chi_{n_2}^\dagger(\mathbf{r}_2)\psi_{n_3}(\mathbf{r}_3), \chi_j^\dagger(\mathbf{r}_4)\rangle \\ & \quad + \left[\frac{1}{N_c} \delta_{ij} \delta_{n_1 n_2} V_1^{\psi\chi}(\mathbf{r}_{32}) + \frac{1}{T_F} T_{ij}^a T_{n_1 n_2}^a V_{\text{Ad}}^{\psi\chi}(\mathbf{r}_{32}) \right] |\psi_{n_1}(\mathbf{r}_1), \chi_j^\dagger(\mathbf{r}_2)\psi_i(\mathbf{r}_3), \chi_{n_4}^\dagger(\mathbf{r}_4)\rangle \\ & \quad + \left[\frac{1}{N_c} \delta_{ij} \delta_{n_3 n_4} V_1^{\psi\chi}(\mathbf{r}_{34}) + \frac{1}{T_F} T_{ij}^a T_{n_3 n_4}^a V_{\text{Ad}}^{\psi\chi}(\mathbf{r}_{34}) \right] |\psi_{n_1}(\mathbf{r}_1), \chi_{n_2}^\dagger(\mathbf{r}_2)\psi_i(\mathbf{r}_3), \chi_j^\dagger(\mathbf{r}_4)\rangle \end{aligned}$$