Quarkonia meet Dark Matter

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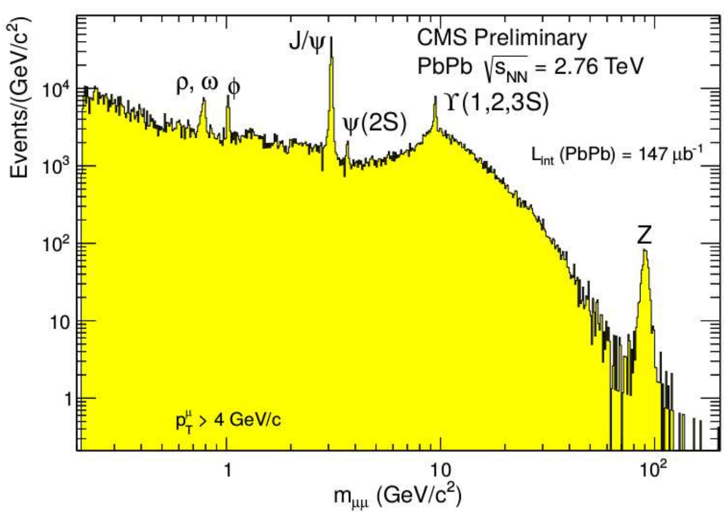
Quarkonium in a quark-gluon plasma

Quarkonium as a quark-gluon plasma probe

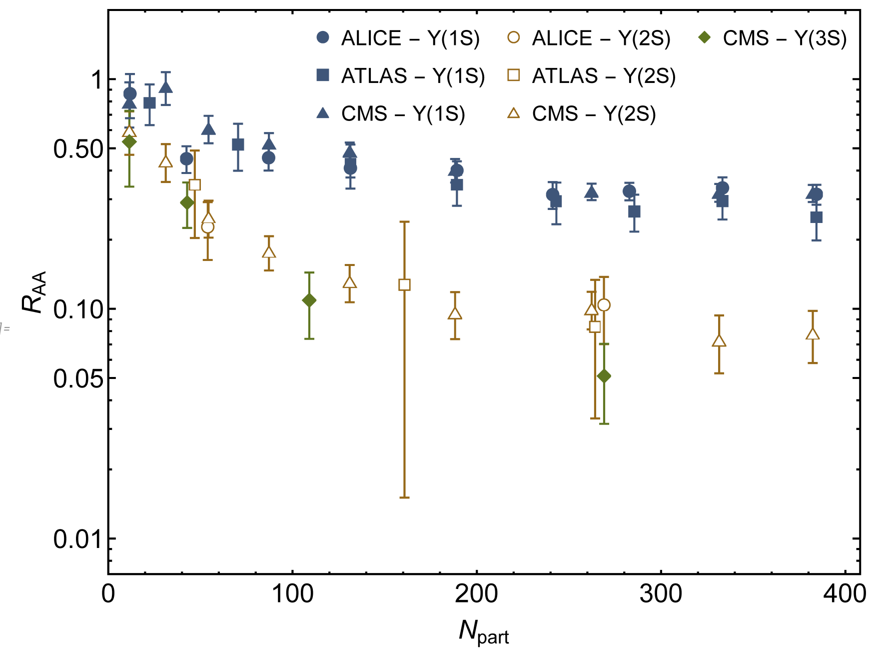
In 1986, Matsui and Satz suggested quarkonium as an ideal quark-gluon plasma probe.
○ `Matsui Satz PLB 178 (1986) 416`

- Heavy quarks are formed early in heavy-ion collisions: $1/M \sim 0.1$ fm < 0.6 fm.
- Heavy quarkonium formation is sensitive to the medium.
- The dilepton signal (happening after the quark-gluon plasma has faded away) makes the quarkonium a clean experimental probe.



Υ suppression @ LHC

Out[ ]=



R_{AA} is the nuclear modification factor = yield of quarkonium in PbPb / yield in pp.

- CMS PLB 790 (2019) 270
  ALICE PLB 822 (2021) 136579
  ATLAS PRC 107 (2023) 054912

Energy scales

Quarkonium in a medium is characterized by several energy scales:

- the scales of a non-relativistic bound state
  (v is the relative heavy-quark velocity; $v \sim \alpha_{\rm s}$ for a Coulombic bound state):
  M (mass),
  Mv (momentum transfer, inverse distance),
  Mv^2 (kinetic energy, binding energy, potential V), ...

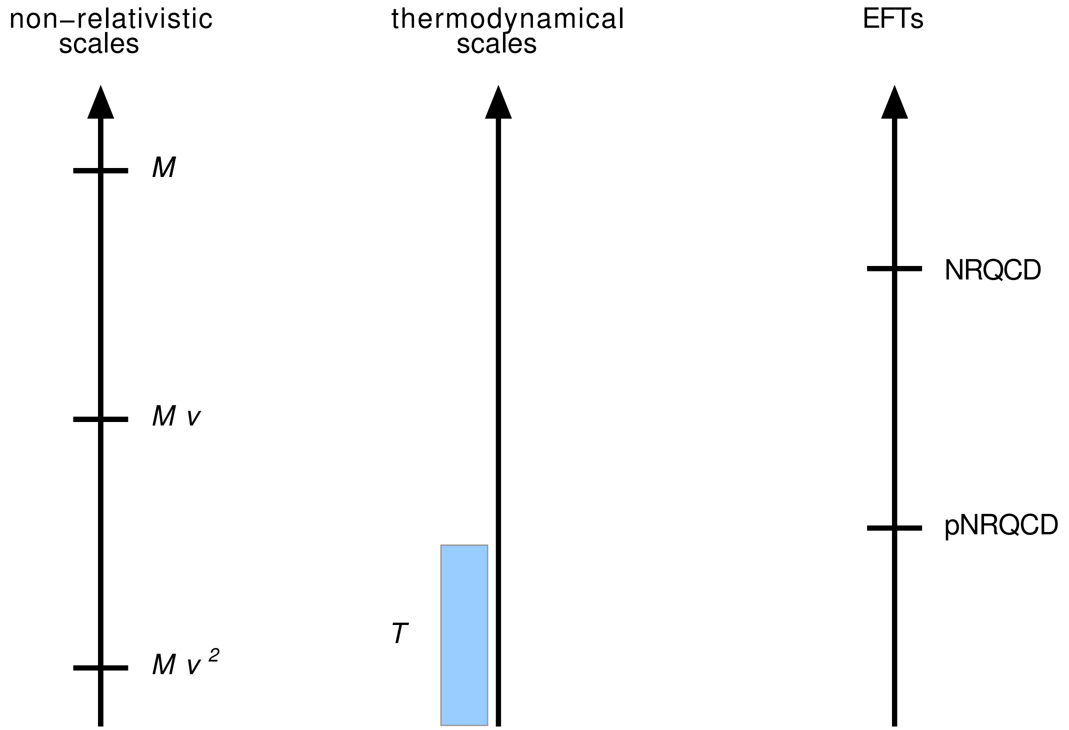
- the thermodynamical scales:
  T (temperature), ...

  T stands for a generic inverse correlation length characterizing the medium. For definiteness we assume that the system is locally in thermal equilibrium so that a slowly varying time-dependent temperature can be defined.

The non-relativistic scales are hierarchically ordered: $M \gg Mv \gg Mv^2$

Non-relativistic EFTs of QCD

The existence of a hierarchy of energy scales calls for a description of the system in terms of a hierarchy of EFTs. We assume $T \gtrsim Mv^2 \sim 400$ MeV for Υ.



- Brambilla Pineda Soto Vairo RMP 77 (2005) 1423
  Brambilla Ghiglieri Petreczky Vairo PRD 78 (2008) 014017

pNRQCD

Fields:

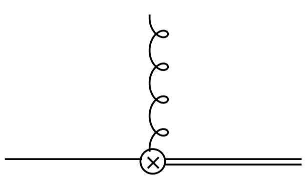
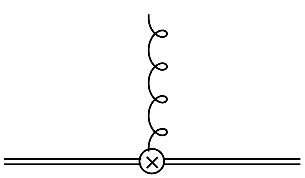
- $S^\dagger$ creates a quark-antiquark pair in a color singlet configuration.
- $O^\dagger$ creates a (unbound) quark-antiquark pair in a color octet configuration.
- gluons and light quarks.

Propagators:

- singlet and octet governed (in a Coulombic system) by

  the Hamiltonians $h_s = \dfrac{\mathbf{p}^2}{M} - \dfrac{4}{3}\dfrac{\alpha_s}{r} + \dots$ and $h_o = \dfrac{\mathbf{p}^2}{M} + \dfrac{\alpha_s}{6r} + \dots$, respectively.

Electric-dipole interactions:

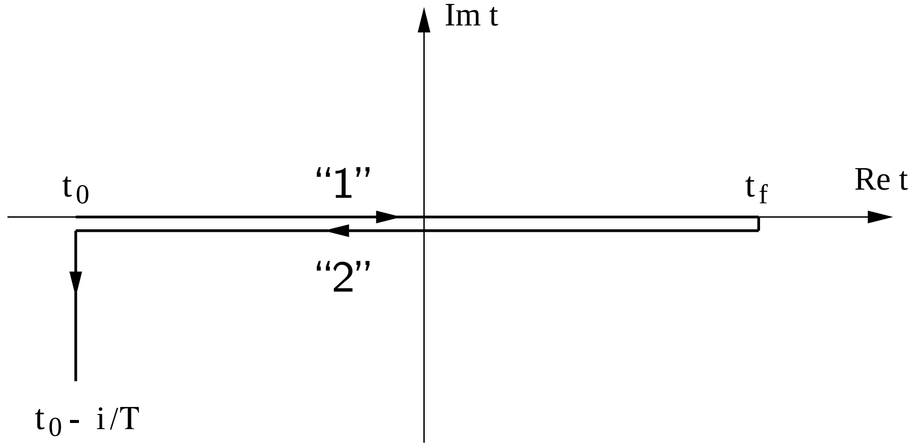
-  $= O^\dagger \mathbf{r} \cdot g\mathbf{E}\, S$  $= O^\dagger \{\mathbf{r} \cdot g\mathbf{E},\, O\}$

Equation of motion:

$$i\partial_t S = h_s S + \mathbf{r} \cdot g\mathbf{E}^a \frac{O^a}{\sqrt{6}} + \dots$$

Real-time formalism

Temperature is introduced via the partition function.
Sometimes it is useful to work in the real-time formalism.



In real time, the degrees of freedom double ("1" and "2"), however, the advantages are

- the framework becomes very close to the one for $T = 0$ EFTs;
- in the heavy-particle sector, the second degrees of freedom, labeled "2", decouple from the physical degrees of freedom, labeled "1".

This usually leads to a simpler treatment with respect to alternative calculations in imaginary time formalism.

Dissociation mechanisms

- Annihilation
  happens after the disappearance of the quark-gluon plasma and is responbile for the dilepton signal.
- Gluodissociation
  is the dissociation of quarkonium by absorption of a gluon from the medium; this is the dominant mechanism for $Mv^2 \gg m_D$.
- Dissociation by inelastic parton scattering
  is the dissociation of quarkonium by scattering with light degrees of freedom from the medium; this is the dominant mechanism for $Mv^2 \ll m_D$.
- ...

Quarkonium annihilation

Annihilation in pNRQCD is described by the imaginary local potentials:

$$\mathcal{L}_{\text{pNRQCD}} \supset$$

$$\frac{i}{M^2} \int d^3r \, \mathrm{Tr}\Big\{ S^\dagger \delta^3(\boldsymbol{r}) \, N \left[2\mathrm{Im}(f_{[\mathbf{1}]}(^1S_0)) - \boldsymbol{S}^2 \left(\mathrm{Im}(f_{[\mathbf{1}]}(^1S_0)) - \mathrm{Im}(f_{[\mathbf{1}]}(^3S_1))\right)\right] S$$

$$+ \, O^\dagger \delta^3(\boldsymbol{r}) \, T_F \left[2\mathrm{Im}(f_{[\mathbf{adj}]}(^1S_0)) - \boldsymbol{S}^2 \left(\mathrm{Im}(f_{[\mathbf{adj}]}(^1S_0)) - \mathrm{Im}(f_{[\mathbf{adj}]}(^3S_1))\right)\right] O \Big\}$$

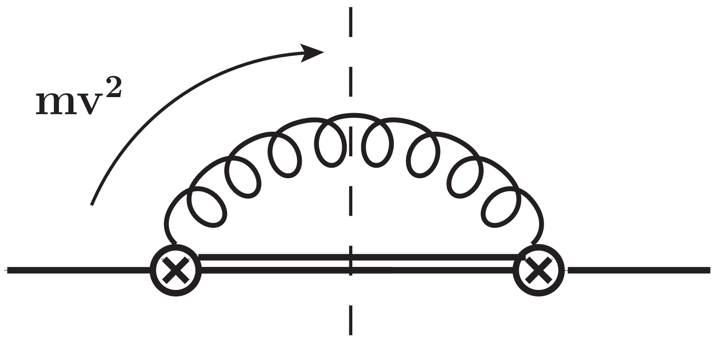
which leads to the (leading order) annihilation widths:

$$\Gamma^{n\mathrm{S}}_{\text{ann}} = \frac{4\mathrm{Im}(f_{[\mathbf{1}]}(^1S_0))}{M^2} |\psi_{nS}(\mathbf{0})|^2$$

Suppressed by at least $\alpha_s v^2$ with respect to bound-state dissociation.

Gluodissociation

From the optical theorem, the gluodissociation width follows from cutting the gluon propagator in the following pNRQCD self-energy diagram (Σ_{11} of 11 type)



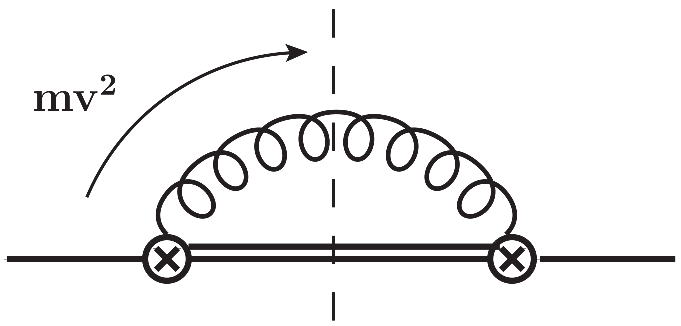
For a quarkonium at rest with respect to the medium, the width has the form

$$\Gamma^{nl} = \int_{q_{\min}} \frac{d^3q}{(2\pi)^3} \, n_{\rm B}(q) \, \sigma^{nl}_{\rm gluo}(q) \, .$$

- $\sigma^{nl}_{\rm gluo}$ is the in-vacuum cross section $(Q\overline{Q})_{nl} + g \to Q + \overline{Q}$.
- Gluodissociation is also known as singlet-to-octet break up.

○ Brambilla Ghiglieri Vairo Petreczky PRD 78 (2008) 014017

$1S$ gluodissociation at LO



The LO gluodissociation cross section for $1S$ Coulombic states is

$$\sigma^{1S}_{\mathrm{gluo\,LO}}(q) = \frac{\alpha_{\mathrm{s}} C_F}{3} 2^{10}\pi^2\rho(\rho+2)^2 \frac{E_1^4}{Mq^5}\left(t(q)^2+\rho^2\right)\frac{\exp\left(\frac{4\rho}{t(q)}\arctan\left(t(q)\right)\right)}{e^{\frac{2\pi\rho}{t(q)}}-1}$$

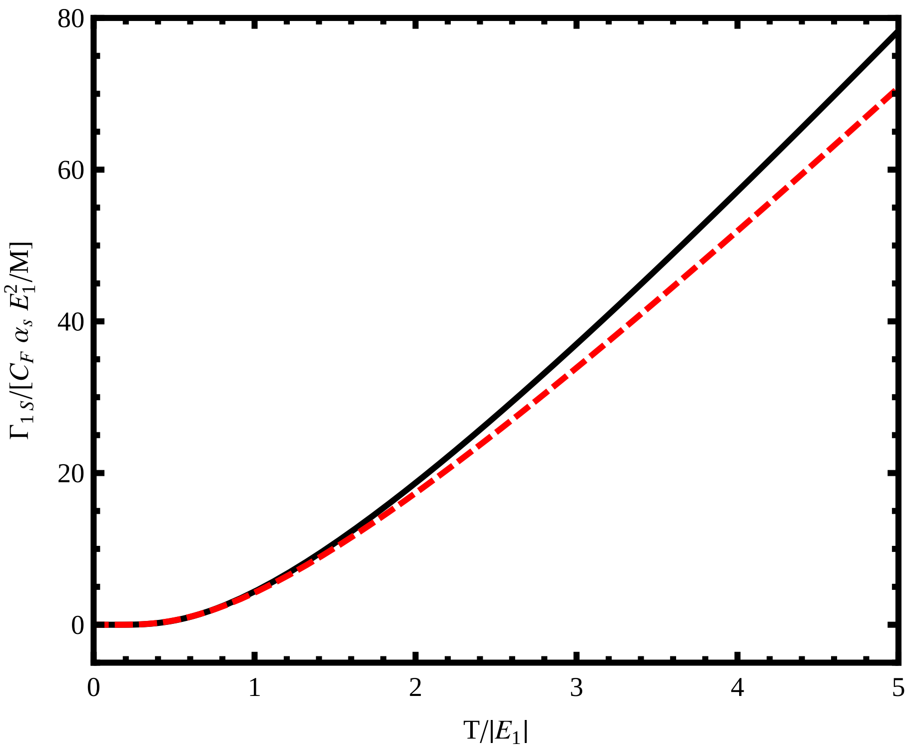
where $\rho \equiv 1/(N_c^2-1)$, $t(q) \equiv \sqrt{q/|E_1|-1}$ and $E_1 = -MC_F^2\alpha_{\mathrm{s}}^2/4$.

- `Brambilla Escobedo Ghiglieri Vairo JHEP 1112 (2011) 116`
  `Brezinski Wolschin PLB 707 (2012) 534`

The Bhanot–Peskin approximation corresponds to the large N_c limit, i.e. to neglecting final state interactions (the rescattering of a $Q\overline{Q}$ pair in a color octet configuration).

- `Peskin NPB 156 (1979) 365, Bhanot Peskin NPB 156 (1979) 391`

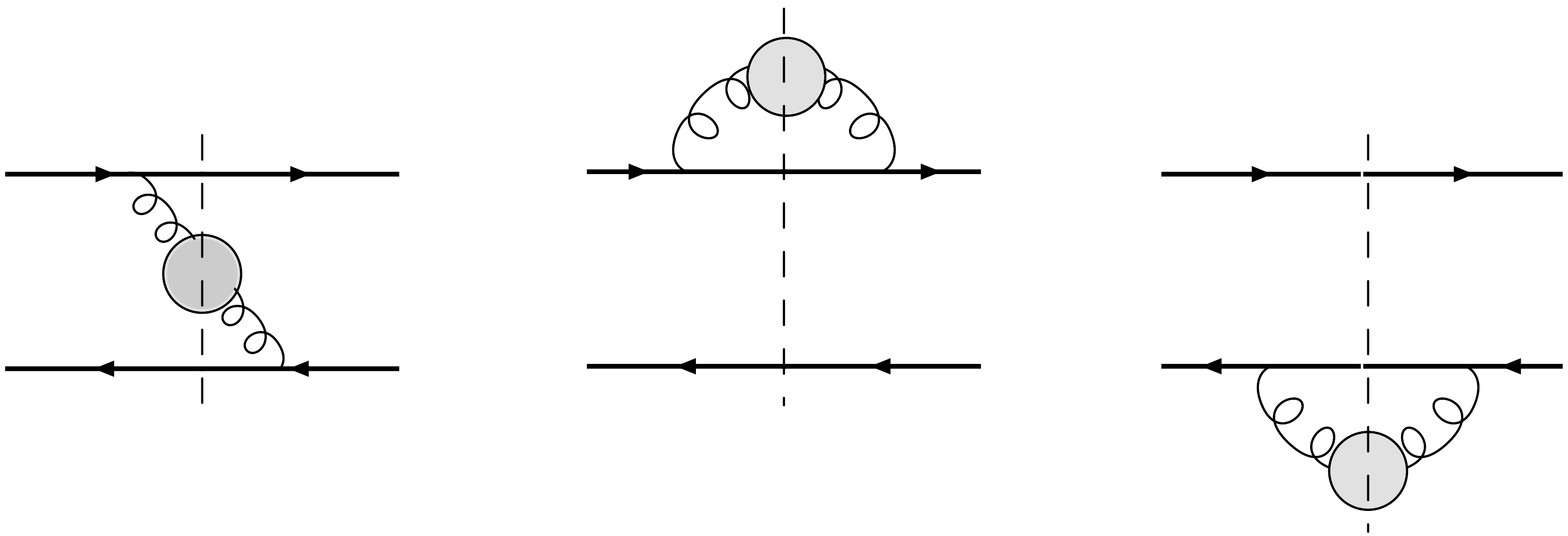
Gluodissociation width vs Bhanot–Peskin width



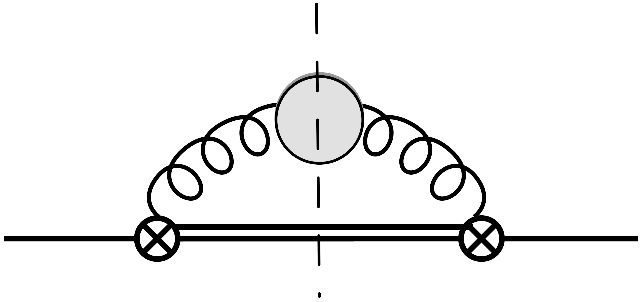
○ Brambilla Escobedo Ghiglieri Vairo JHEP 1112 (2011) 116

Quarkonium dissociation by parton scattering

The thermal width follows from cutting the gluon self-energy in the following NRQCD diagrams with external particles of type 1 (momentum of the gluon $\gtrsim Mv$)



and/or pNRQCD 11 diagram (momentum of the gluon $\ll Mv$)



- Dissociation by inelastic parton scattering is also known as Landau damping.

Quarkonium dissociation by parton scattering

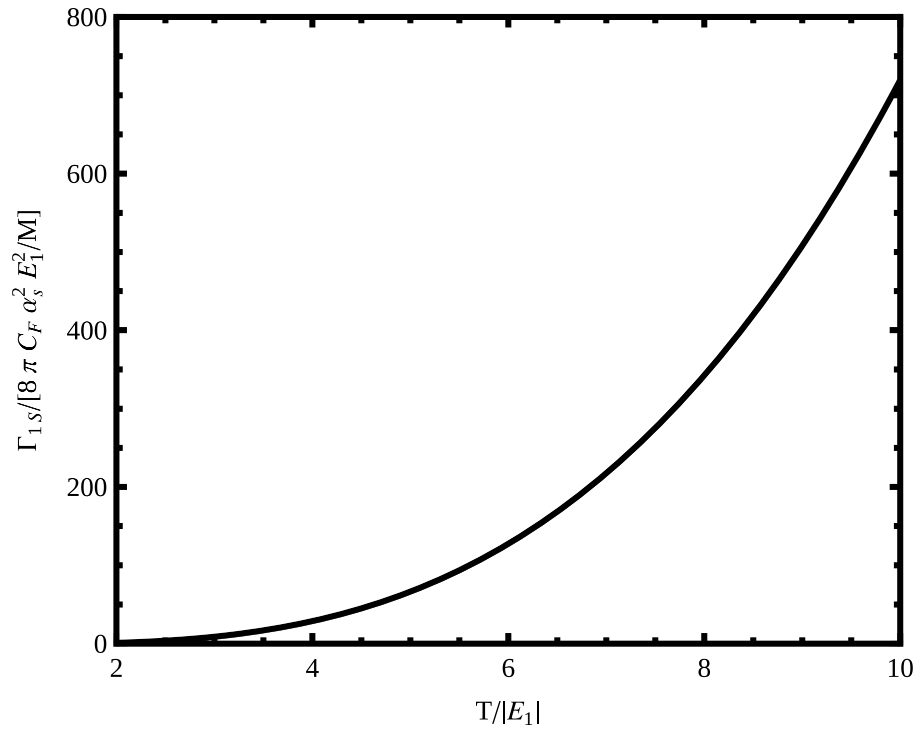
For a quarkonium at rest with respect to the medium, the thermal width has the form

$$\Gamma^{nl} = \sum_p \int_{q_{\min}} \frac{d^3q}{(2\pi)^3}\, f_p(q)\, [1 \pm f_p(q)]\, \sigma_p^{nl}(q)$$

where the sum runs over the different incoming light partons and $f_g = n_{\rm B}$ or $f_q = n_{\rm F}$.

- σ_p^{nl} is the in-medium cross section $(Q\overline{Q})_{nl} + p \to Q + \overline{Q} + p$.
- The convolution formula correctly accounts for Pauli blocking in the fermionic case (minus sign).
- The formula differs from the gluodissociation formula.

Dissociation width



$m_D a_0 = 0.5$

$|E_1|/m_D = 0.5$

$n_f = 3$

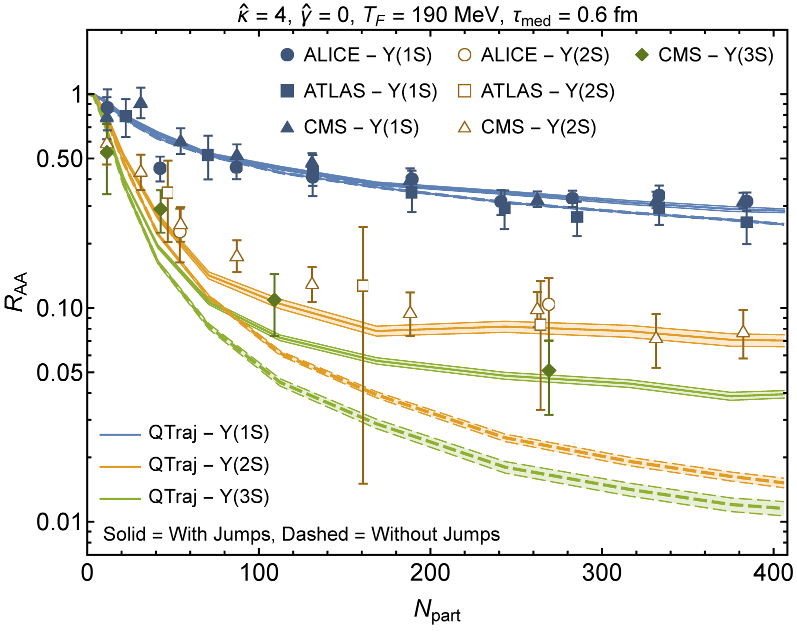
- Brambilla Escobedo Ghiglieri Vairo JHEP 05 (2013) 130
  Vairo EPJ Web Conf 71 (2014) 00135

Evolution equation and quarkonium suppression

The evolution equation is (under some conditions) a Lindblad equation

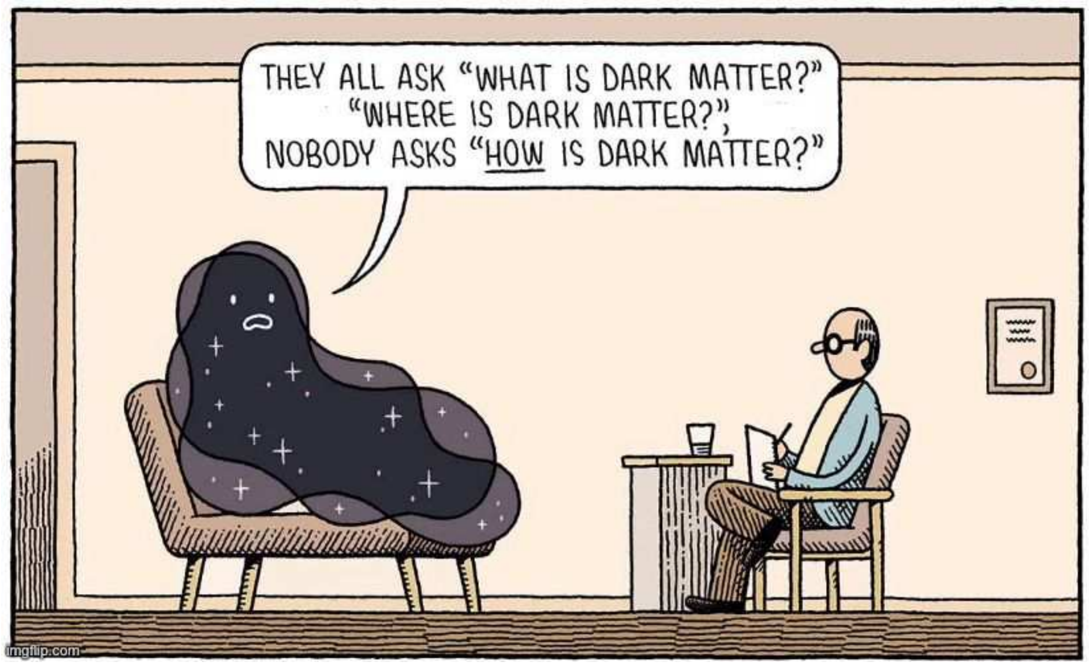
$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_i \left(C_i \rho C_i^\dagger - \frac{1}{2}\{C_i^\dagger C_i, \rho\}\right)$$

In the Brownian limit ($T \gg Mv^2$), we obtain the nuclear modification factor R_{AA}:



○ Brambilla et al JHEP 08 (2022) 303

Dark matter in the early universe



WIMP and freeze-out mechanism

Weakly interacting massive particles (WIMPs) are candidates for dark matter.
The dark matter energy density can be computed via the thermal freeze-out mechanism, where at a temperature of the universe, T, much larger than the dark matter mass M, weak interactions between dark matter and Standard Model particles are sufficient to keep dark matter in thermal equilibrium. As the universe expands and cools down, dark matter pair annihilation becomes ineffective at freeze out $T \approx M/25 \ll M$. Dark matter pairs are non-relativistic from there on. Thermal freeze-out may occur entirely within an extended dark sector and lead to the observed dark matter energy density:

$$\Omega_{\mathrm{DM}} h^2 = 0.1200 \pm 0.0012$$

- `Planck coll AA 641 (2020) A1`

The evolution equation of the dark matter number density n depends on the thermal average $\langle \sigma_{\mathrm{eff}} \rangle$ compraising annihilation, formation, dissociation and transition effects:

$$(\partial_t + 3H)n = -\frac{1}{2}\langle \sigma_{\mathrm{eff}} v_{\mathrm{rel}} \rangle (n^2 - n_{\mathrm{eq}}^2)$$

Near-threshold effects modify the annihilation cross section significatly.

An abelian model

A simple model is dark QED:

$$\mathcal{L} = \bar{X}(i\not{D} - M)X - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \mathcal{L}_{\text{portal}}$$

where X is a dark fermion coupled to a dark photon, i.e. a U(1) gauge field.
$\mathcal{L}_{\text{portal}}$ includes the SM to dark matter coupling, which is small, and may be neglected when computing cross sections.

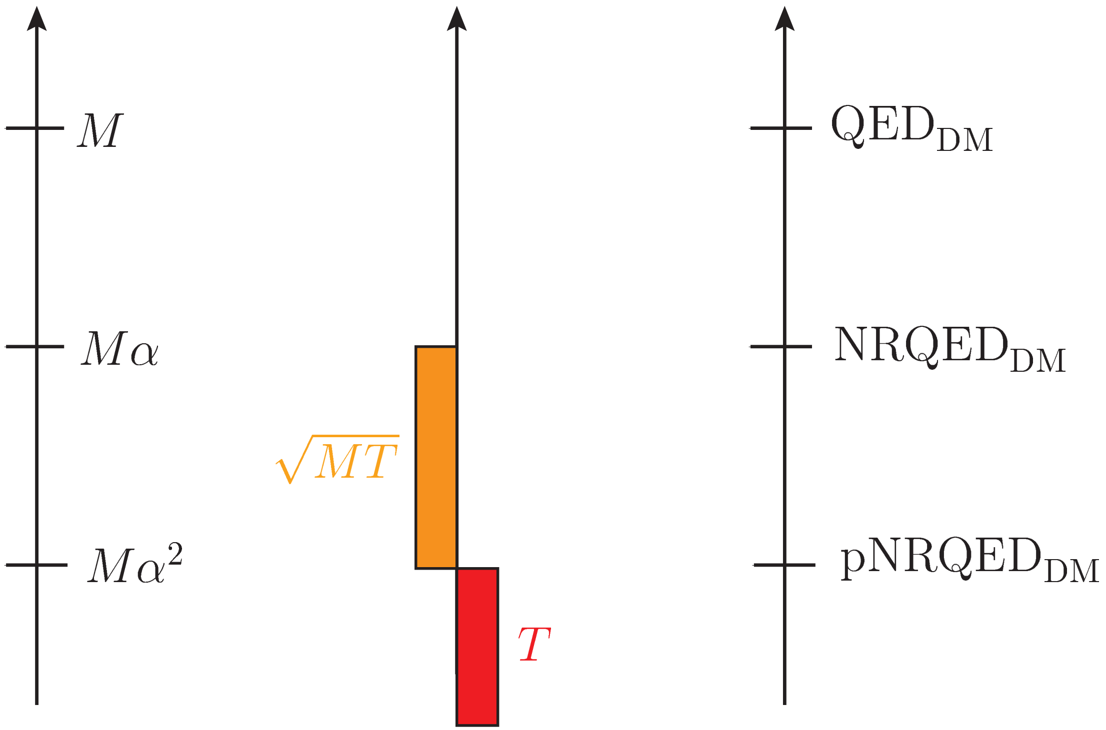
We assume the hierarchy of energy scales

$$M \gg M\alpha \gtrsim \sqrt{MT} \gg M\alpha^2 \gtrsim T$$

where $M\alpha$ and $M\alpha^2$ are the threshold/bound-state scales.
The scale $\sqrt{MT}$ is the average X momentum in the thermal average.

Energy scales and non-relativistic EFTs

The hierarchy of energy scales calls for a hierarchy of EFTs.



Bound-state annihilation

We consider a bound state with quantum numbers n annihilating into dark photons:

$$(X\bar{X})_n \to \gamma\gamma$$

The (leading order) annihilation width follows from the imaginary part of the pNRQED$_{\mathrm{DM}}$ potential (just the abelian part of the pNRQCD potential):

$$\Gamma^{n\mathrm{S}}_{\mathrm{ann}} = \frac{4\mathrm{Im}(f(^1S_0))}{M^2}|\Psi_{nS}(\mathbf{0})|^2$$

Ψ_n is the S-wave bound state wave function.

Annihilation cross section

We consider a scattering state of momentum $\boldsymbol{p} = \dfrac{M\boldsymbol{v}_{\mathrm{rel}}}{2}$ annihilating into dark photons:

$$(X\bar{X})_p \to \gamma\gamma$$

From the imaginary part of the pNRQED$_{\mathrm{DM}}$ potential, the annihilation cross section is

$$(\sigma_{\mathrm{ann}} v_{\mathrm{rel}})(\boldsymbol{p}) = \frac{\mathrm{Im}(f(^1S_0))}{M^2} \, |\Psi_{\boldsymbol{p}0}(\boldsymbol{0})|^2 = (\sigma^{\mathrm{NR}}_{\mathrm{ann}} v_{\mathrm{rel}}) \, S_{\mathrm{ann}}(\zeta)$$

where $(\sigma^{\mathrm{NR}}_{\mathrm{ann}} v_{\mathrm{rel}}) = \pi\alpha^2/M^2$ and $\Psi_{\boldsymbol{p}0}$ is the S-wave component of the scattering state wave function. This is also called Sommefeld factor

$$S_{\mathrm{ann}}(\zeta) \equiv |\Psi_{\boldsymbol{p}0}(\boldsymbol{0})|^2 = \frac{2\pi\zeta}{1 - e^{-2\pi\zeta}} \,, \qquad \zeta \equiv \frac{\alpha}{v_{\mathrm{rel}}} = \frac{1}{2} \frac{M\alpha}{p}$$

The Sommerfeld factor encodes the soft contribution to the scattering, which significantly modifies the cross section for $v_{\mathrm{rel}} \lesssim \alpha$.

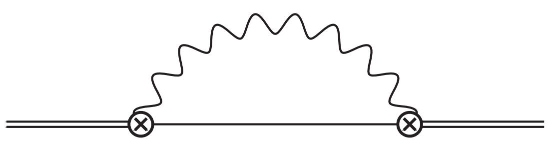
- Jengo JHEP 05 (2009) 024, Cassel JPG 37 (2010) 105009

Bound state formation

We consider the process

$$(X\bar{X})_p \to \gamma + (X\bar{X})_n$$

From the imaginary part of the 11 self energy diagram



we get

$$(\sigma_{\mathsf{bsf}}\, v_{\mathsf{rel}})(\boldsymbol{p}) \equiv \sum_n (\sigma^n_{\mathsf{bsf}}\, v_{\mathsf{rel}})(\boldsymbol{p}) = \frac{g^2}{3\pi} \sum_n \left[1 + n_{\mathsf{B}}(\Delta E^p_n)\right] |\langle n|\boldsymbol{r}|\boldsymbol{p}\,\rangle|^2 (\Delta E^p_n)^3$$

where $\Delta E^p_n \equiv E_p - E_n$ and n_{B} is the Bose–Einstein distribution of the outgoing photon.

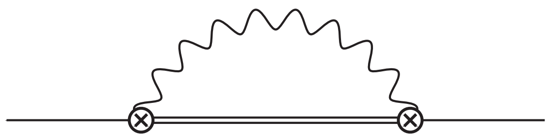
- `von Harling Petraki JCAP 1412 (2014) 033`

Bound state dissociation

We consider the process

$$\gamma + (X\bar{X})_n \to (X\bar{X})_p$$

From the imaginary part of the 11 self energy diagram



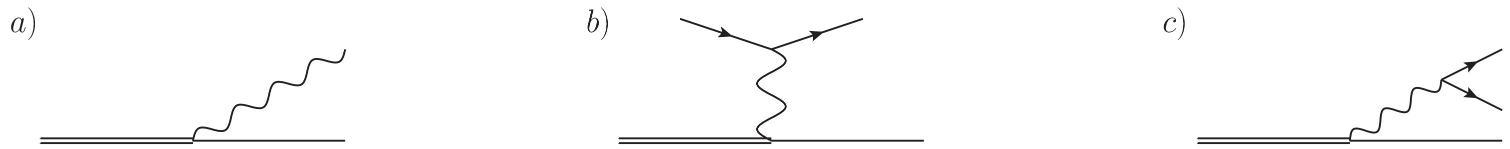
we get

$$\begin{aligned}
\Gamma^n_{\mathsf{bsd}} &= \frac{g^2}{3\pi}\int \frac{d^3p}{(2\pi)^3}\, n_{\mathsf{B}}(\Delta E^p_n) \left|\sum_\ell \langle n|\boldsymbol{r}|\boldsymbol{p}\ell\rangle\right|^2 (\Delta E^p_n)^3 \\
&= 2\int_{|\boldsymbol{k}|\geq|E^b_n|} \frac{d^3k}{(2\pi)^3}\, n_{\mathsf{B}}(|\boldsymbol{k}|)\, \sigma^n_{\mathsf{ion}}(|\boldsymbol{k}|)
\end{aligned}$$

◦ von Harling Petraki JCAP 1412 (2014) 033

Dissociation by light scattering

Dissociation by light scattering is excluded from the adopted version of QED$_{\mathrm{DM}}$ because there are no light degrees of freedom coupled to the dark photon:



In QED$_{\mathrm{DM}}$ with no light fermions only *a)* is possible.
In other models *b)* and *c)* may compete or be more important than *a)* if $m_D \gtrsim Mv^2$.

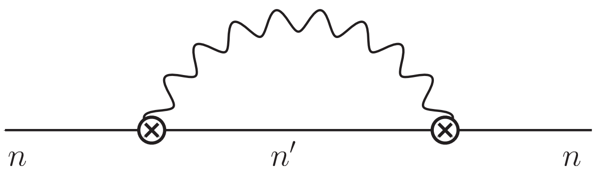
○ Binder Blobel Harz Mukaida JHEP 09 (2020) 086

Transitions

Bound-state transitions are given by the processes

$$(X\bar{X})_n \leftrightarrow (X\bar{X})_{n'} + \gamma$$

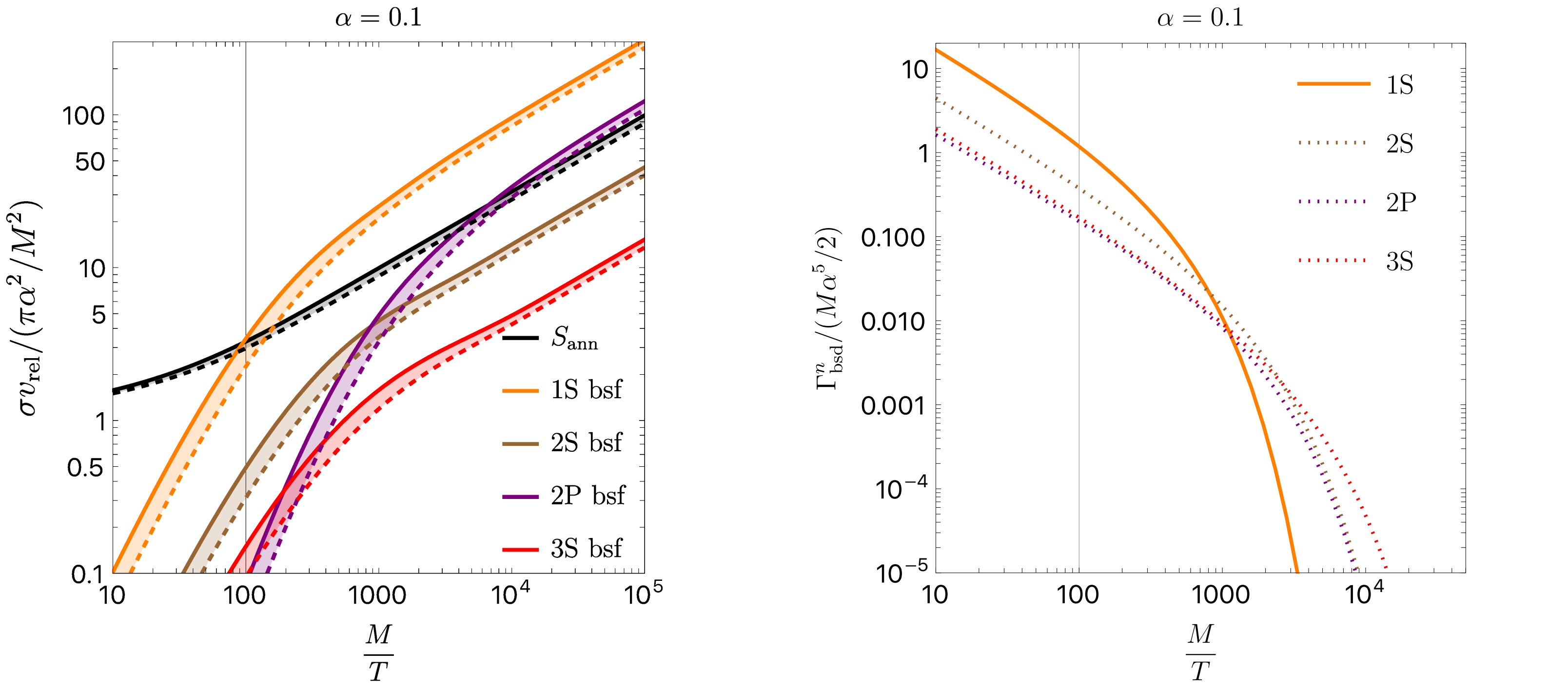
From the imaginary part of the 11 self energy diagram



we get

$$\begin{aligned}
\Gamma_{\text{ex.}}^{\text{1S}\to\text{2P}} &= \frac{2^7}{3^7}\frac{M\alpha^5}{e^{\frac{3M\alpha^2}{16T}}-1} & \Gamma_{\text{de-ex.}}^{\text{2P}\to\text{1S}} &= \frac{2^7}{3^8}\frac{M\alpha^5}{1-e^{-\frac{3M\alpha^2}{16T}}} \\
\Gamma_{\text{ex.}}^{\text{1S}\to\text{3P}} &= \frac{1}{2^6}\frac{M\alpha^5}{e^{\frac{2M\alpha^2}{9T}}-1} & \Gamma_{\text{de-ex.}}^{\text{3P}\to\text{1S}} &= \frac{1}{3}\frac{1}{2^6}\frac{M\alpha^5}{1-e^{-\frac{2M\alpha^2}{9T}}} \\
\dots &\quad \dots & \dots &\quad \dots
\end{aligned}$$

Annihilation, bound-state formation and dissociation plots



- Biondini Brambilla Qerimi Vairo JHEP 07 (2023) 006

Boltzmann equation

The time evolution equation of the dark matter number density can be described by the Boltzmann equation

$$(\partial_t + 3H)n = -\frac{1}{2}\langle\sigma_{\text{eff}}\, v_{\text{rel}}\rangle(n^2 - n_{\text{eq}}^2)$$

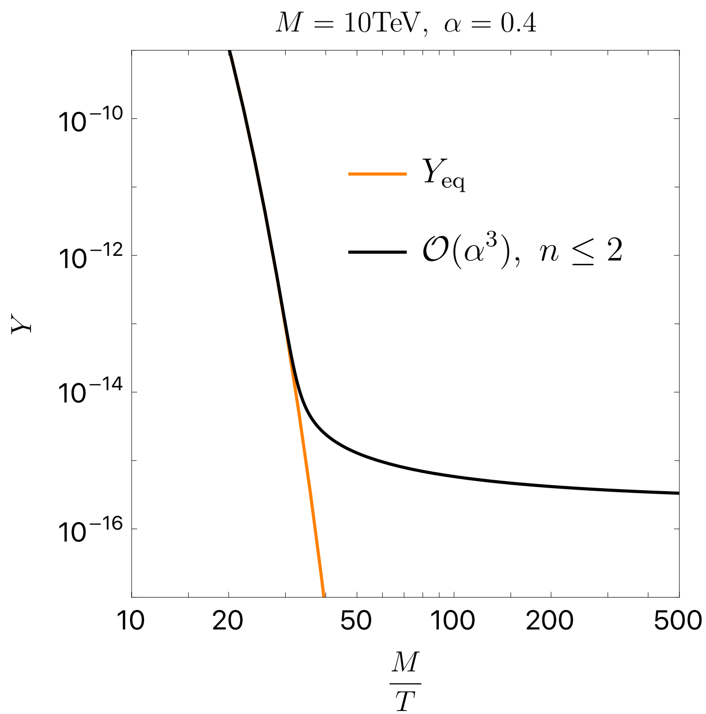
with the thermally averaged effective cross section defined as

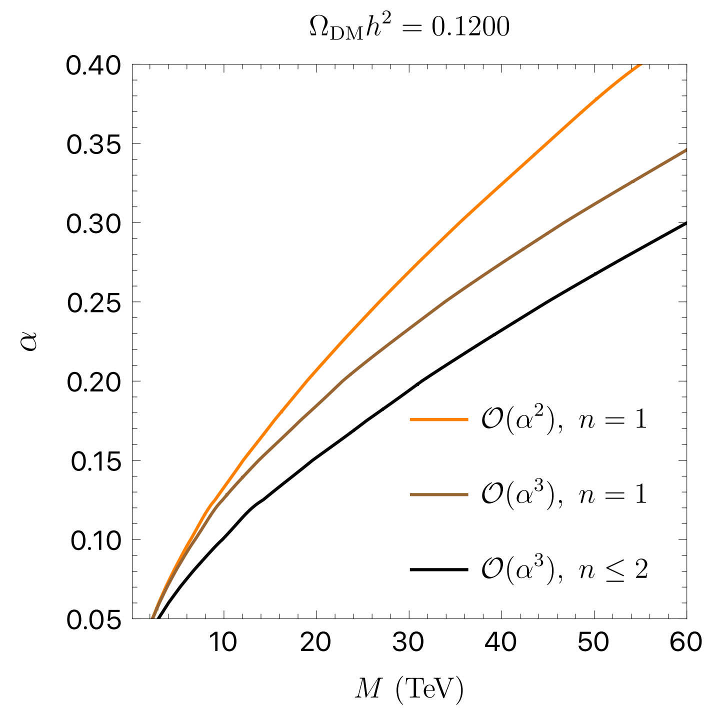
$$\langle\sigma_{\text{eff}}\, v_{\text{rel}}\rangle = \langle\sigma_{\text{ann}} v_{\text{rel}}\rangle + \sum_n \langle\sigma_{\text{bsf}}^n\, v_{\text{rel}}\rangle\, \frac{\Gamma_{\text{ann}}^n}{\Gamma_{\text{ann}}^n + \Gamma_{\text{bsd}}^n}$$

At small temperatures, the bound-state to bound-state transitions become as large as or larger than the dissociation widths and further modify the effective cross section.

- `Garny Heisig PRD 105 (2022) 055004`

Dark matter yield and parameter space





The yield is $Y = n/s$ with s the entropy density.

- Biondini Brambilla Qerimi Vairo JHEP 07 (2023) 006

Conclusion I: from quarkonia to dark matter

Many results from quarkonium physics can be taken over/adapted to similar settings relevant for dark matter scenarios involving WIMPs and the freeze-out mechanism. In the simple framework of $\mathrm{QED}_{\mathrm{DM}}$, annihilation, bound-state formation and dissociation widths and cross sections correspond to the abelian limit of results derived for the quarkonium case.

More importantly the framework of non-relativistic EFTs provides a way to

- establish under which dynamical regime results are valid,
- systematically improve them in the regime of validity.

Conclusion II: when are results valid?

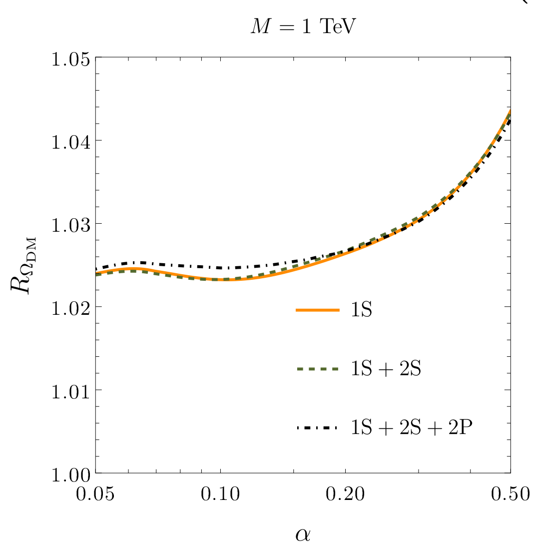
Results are valid as long as the hierarchy $M \gg M\alpha \gtrsim \sqrt{MT} \gg M\alpha^2 \gtrsim T$ is fulfilled, which guarantees that the potential is Coulombic and the multipole expansion holds.

- In QED$_{\text{DM}}$ with $\mathcal{L}_{\text{portal}} = 0$ the condition $M\alpha^2 \gtrsim T$ may be relaxed to $M \gg T$, as thermal effects to the potential are $1/M$ suppressed due to the absence of polarization corrections to the temporal photon propagator.
  The static potential is therefore a Coulomb potential also if just $M \gg T$.
- In dark sectors with asymptotic freedom (QCD-like), in order to apply weak-coupling expansions the coupling has to remain small from M to T. If the temperature range is large this forces $\alpha(2M)$ to be rather small.
  E.g. for $M/T = 10^5$, $\alpha(2M) \lesssim 0.04$ for SU(3).
- In dark sectors with asymptotic freedom (QCD-like), if $T \gtrsim M\alpha^2$ the static potential gets thermal contributions carried by polarization corrections to the temporal gluon (present due to the self-interaction of the gauge bosons).
  E.g. this is relevant for transitions between states with large principal quantum number n. In a Coulombic system, the binding energy is proportional to $M\alpha^2/n^2$ rather than $M\alpha^2$. For some n, $T > M\alpha^2/n^2$ and the Coulombic description of the state breaks down, as the potential gets thermal corrections that may be more important than the Coulomb part.

Conclusion III: how to improve results

Cross sections and decay widths are systematically improvable.

- They are improvable in the hard part.
  E.g. Including subleading corrections of order α^3 to the paradarkonium width and annihilation cross section requires including the orthodarkonium width and annihilation cross section in the effective cross section.
- They are improvable in the soft part.
  E.g. Corrections in $1/M$ to the potential can be included order by order. They modify the wave functions starting from relative order α^2, v_{rel}^2.
- Among higher order corrections are corrections due to the relative motion of the medium with respect to the center of mass of the $(X\bar{X})$ pair.



- Biondini Brambilla Qerimi Vairo arXiv:2402.12787

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Europe/Berlin timezone