Tetraquark states in hidden charm and bottom sector

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T. Bhavsar, Manan Shah. Smruti Patel, P C Vinodkumar, Nuclear Physics A 1000 (2020) 121856.
Smruti Patel, P C Vinodkumar, et al., Eur. Phys. J. C (2016) 76:356.

Outline

- Introduction
- Theoretical Methodology
- Mass Spectra of Tetraquark
- Radiative Decay Width
- Hadronic Decay Width
- Leptonic Decay Width
- Summary

Introduction

- **The recent experimental observations particularly in the hidden charm sector have generated renewed interest in the study of tetraquark spectroscopy*. The discovery of many high-precision experimental observations of various tetraquark states have necessitated reconsideration of the parameters involved in the previous studies.**

- **The large number of tetraquark states with their masses and decay widths have been recorded experimentally, but new investigation of the exotic meson, molecular and multiquark systems have created great interest in the four-quark spectroscopy.**

- **Status of some of these states e.g. X(4140), X(4274), Z_b(10610) and Y_b(10890) are still controversial and there exist disparities related to their decay properties.**

*M. Ablikim, (BESIII Collaboration) Physical Review Letters 129, 112003 (2022)

Introduction

ψ(4230) and ψ(4260) are two nearby ψ states initially were referred to as Y(4230) and Y(4260) states.

On the other hand, BESIII [1] suggested that Y(4260) is not a simple peak. This state is a combination of two resonance Y(4220) and Y(4330) [1].

According to Segovia et al. the Y(4260) is not a pure charmonium state [2].

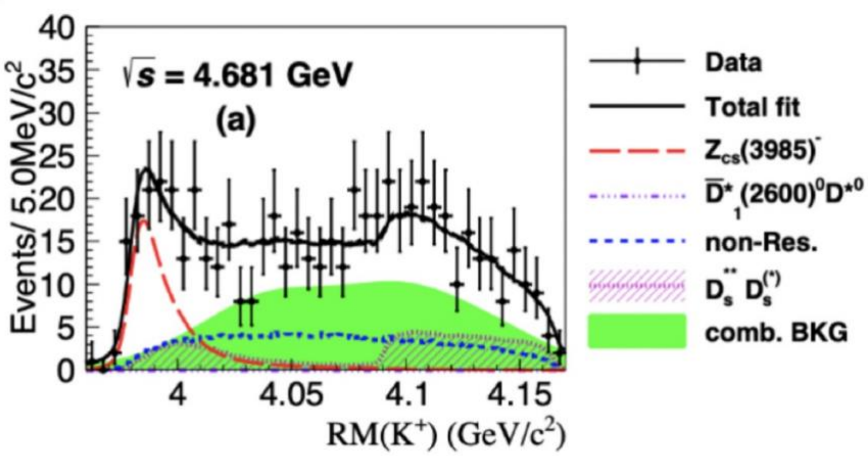
•	$Z_c(3900)$ was $X(3900)$	$1^+(1^{+-})$
	$Z_{cs}(4000)$	$1/2(1^+)$
•	$X(4020)^\pm$	$1^+(?^{?-})$
	$X(4050)^\pm$	$1^-(?^{?+})$
	$X(4055)^\pm$	$1^+(?^{?-})$
	$X(4100)^\pm$	$1^-(?^{??})$
	$Z_c(4200)$ was $X(4200)^\pm$	$1^+(1^{+-})$
	$Z_{cs}(4220)^+$	$1/2(1^+)$
	$R_{c0}(4240)$ was $X(4240)^\pm$	$1^+(0^{--})$
	$X(4250)^\pm$	$1^-(?^{?+})$
•	$Z_c(4430)$ was $X(4430)^\pm$	$1^+(1^{+-})$

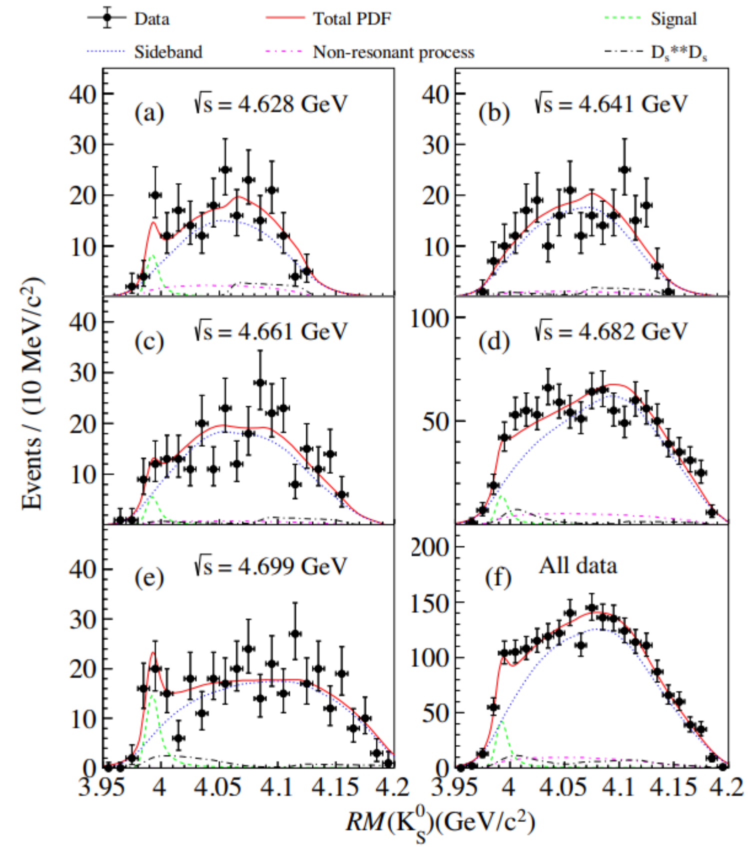
[1] M. Ablikim, et al., BESIII Collaboration, Phys. Rev. Lett. 118 (2017) 092001.
[2] J. Segovia, A.M. Yasser, D.R. Entem, F. Fernandez, Phys. Rev. D 78 (2008) 114033.

Introduction

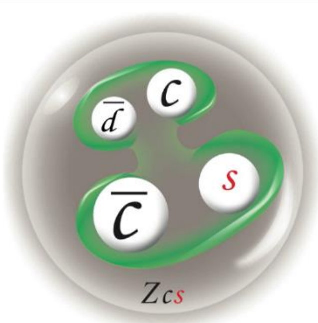
$T^{\theta}_{\psi s1}(3985)^+$ in $D_s^- D^{*0} + D^0 D_s^{*-}$ by BESIII

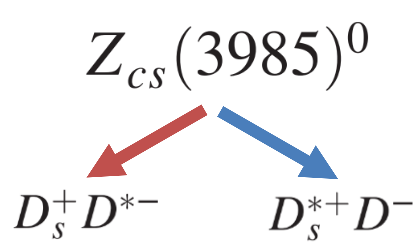
PRL 126 (2021) 102201





BESIII Collaboration [3]





[3] M. Ablikim, (BESIII Collaboration) Physical Review Letters 129, 112003 (2022)

Theoretical Methodology

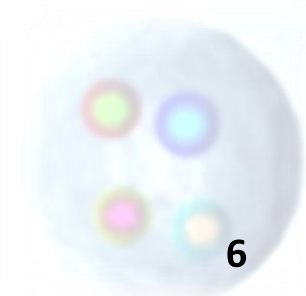
- **To first approximation, the confining part of the interaction is believed to provide the zeroth-order quark dynamics inside the meson through the quark Lagrangian density**

$$\mathcal{L}_q^0(x) = \bar{\psi}_q(x)\left[\frac{i}{2}\gamma^\mu \overleftrightarrow{\partial_\mu} - V(r) - m_q\right]\psi_q(x) \qquad (1)$$

- **For the present study, we assume that the colour quarks and anti quarks are independently confined by an average potential of the form [4, 5]**

$$V(r) = \frac{1}{2}(1+\gamma_0)(\lambda r^{0.1} + V_0) \qquad (2)$$

- **The bound constituent quark and antiquark inside the meson are in definite energy states.**



[4] Manan Shah, Bhavin Patel, P C Vinodkumar., PHYSICAL REVIEW D 90 (2014) 014009.
[5] Manan Shah, Bhavin Patel, and P. C. Vinodkumar, Eur. Phys. J. C 76 (2016) 36.

Dirac formalism

- **The Dirac equation is obtained from $\mathcal{L}_q^0(x)$ as**

$$[\gamma^0 E_q - \vec{\gamma}.\vec{P} - m_q - V(r)]\psi_q(\vec{r}) = 0 \qquad (3)$$

- **The solution of Dirac equation can be written as two component (positive and negative energies in the zeroth order) form as**

$$\psi_{nlj}(r) = \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} \qquad (4)$$

Where the positive and negative energy solutions are written as

$$\psi_A^{(+)}(\vec{r}) = N_{nlj} \begin{pmatrix} \frac{ig(r)}{r} \\ \frac{(\sigma.\hat{r})f(r)}{r} \end{pmatrix} \mathcal{Y}_{ljm}(\hat{r}) \qquad (5)$$

$$\psi_B^{(-)}(\vec{r}) = N_{nlj} \begin{pmatrix} \frac{i(\sigma.\hat{r})f(r)}{r} \\ \frac{g(r)}{r} \end{pmatrix} (-1)^{j+m_j-l} \mathcal{Y}_{ljm}(\hat{r}) \qquad (6)$$

N_{nlj} **is the overall normalization constant.**

Dirac formalism

- The reduced radial part g(r) and f(r) of the Dirac spinor of $\psi_{nlj}(r)$ are the solutions of the equations given by

$$\frac{d^2g(r)}{dr^2} + \left[(E_D + m_q)[E_D - m_q - V(r)] - \frac{\kappa(\kappa+1)}{r^2}\right] g(r) = 0$$

$$\frac{d^2f(r)}{dr^2} + \left[(E_D + m_q)[E_D - m_q - V(r)] - \frac{\kappa(\kappa-1)}{r^2}\right] f(r) = 0$$

- It can be transformed into a convenient dimensionless form given as

$$\frac{d^2g(\rho)}{d\rho^2} + \left[\epsilon - \rho^{0.1} - \frac{\kappa(\kappa+1)}{\rho^2}\right] g(\rho) = 0 \qquad \frac{d^2f(\rho)}{d\rho^2} + \left[\epsilon - \rho^{0.1} - \frac{\kappa(\kappa-1)}{\rho^2}\right] f(\rho) = 0$$

where $\rho = (r/r_0)$ is a dimensionless variable with an arbitrary scale factor chosen conveniently as

$$r_0 = \left[(m_q + E_D)\frac{\lambda}{2}\right]^{-\frac{10}{21}},$$

and ϵ is a corresponding dimensionless energy eigenvalue defined as

$$\epsilon = (E_D - m_q - V_0)(m_q + E_D)^{\frac{1}{21}}\left(\frac{2}{\lambda}\right)^{\frac{20}{21}}$$

Dirac formalism

- **The mass of the specific $^{2S+1}L_J$ states of $Q\bar{q}$ system is expressed as**

$$M_{^{2S+1}L_J} = M_{Q\bar{q}}\left(n_1 l_1 j_1, n_2 l_2 j_2\right) + \langle V^{j_1 j_2}_{Q\bar{q}}\rangle + \langle V^{LS}_{Q\bar{q}}\rangle + \langle V^{T}_{Q\bar{q}}\rangle$$

- **The spin-spin part is defined here as**

$$\langle V^{j_1 j_2}_{Q\bar{q}}(r)\rangle = \frac{\sigma\ \langle j_1 j_2 J M | \hat{j}_1 . \hat{j}_2 | j_1 j_2 J M\rangle}{(E_Q + m_Q)(E_{\bar{q}} + m_{\bar{q}})}$$

- **The tensor and spin-orbit parts of confined one-gluon exchange potential (COGEP) [4-6] are given as**

$$V^{T}_{Q\bar{q}}(r) = -\frac{\alpha_s}{4}\frac{N_Q^2 N_{\bar{q}}^2}{(E_Q + m_Q)(E_{\bar{q}} + m_{\bar{q}})} \otimes\ \lambda_Q . \lambda_{\bar{q}}\left(\left(\frac{D_1''(r)}{3} - \frac{D_1'(r)}{3\,r}\right) S_{Q\bar{q}}\right)$$

$$\begin{aligned} V^{LS}_{Q\bar{q}}(r) \quad &= \quad \frac{\alpha_s}{4}\frac{N_Q^2 N_{\bar{q}}^2}{(E_Q + m_Q)(E_{\bar{q}} + m_{\bar{q}})}\frac{\lambda_Q . \lambda_{\bar{q}}}{2\,r} \\ &\otimes\left[\left[\vec{r} \times (\hat{p_Q} - \hat{p_q}) . (\sigma_Q + \sigma_q)\right]\left(D_0'(r) + 2D_1'(r)\right)\right. \\ &\left. + \left[\vec{r} \times (\hat{p_Q} + \hat{p_q}) . (\sigma_i - \sigma_j)\right]\left(D_0'(r) - D_1'(r)\right)\right] \end{aligned}$$

[4] Manan Shah, Bhavin Patel, P.C. Vinodkumar, Phys. Rev. D 90 (2014) 014009.
[5] Manan Shah, Bhavin Patel, P.C. Vinodkumar, Eur. Phys. J. C 76 (2016) 36.
[6] Manan Shah, Bhavin Patel, P.C. Vinodkumar, Phys. Rev. D 93 (2016) 094028.

Mass spectra of S- wave of D_s and D meson

Table 3.4: S-wave D_s ($c\bar{s}$) spectrum (in MeV).

nL	J^P	State	$M_{Q\bar{q}}$	$\langle V_{Q\bar{q}}^{j_1 j_2}\rangle$	Present	Experiment Meson	Experiment Mass[98]	[142]	[144]	[117]	[149]
1S	1^-	1^3S_1	2113.2	0.73	2113.9	D_s^*	2112.3 ± 0.5	. . .	2111	2117	2107
	0^-	1^1S_0	1970.1	-1.84	1968.3	D_s	1968.49 ± 0.32	. . .	1969	1970	1969
2S	1^-	2^3S_1	2717.3	0.46	2717.8	$D_s^*(2710)$	2710^{+12}_{-7} [150, 151]	2728	2731	2723	2714
	0^-	2^1S_0	2634.6	-1.06	2633.5	$D_s(2632)$	2632.5 ± 1.7 [105]	2656	2688	2684	2640
3S	1^-	3^3S_1	3263.5	0.33	3263.8		. . .	3200	3242	3180	. . .
	0^-	3^1S_0	3203.2	-0.75	3202.4		. . .	3140	3219	3158	. . .
4S	1^-	4^3S_1	3781.4	0.25	3781.6			.	3669	3571	. . .
	0^-	4^1S_0	3732.7	-0.57	3732.1			.	3652	3556	. . .

PHYSICAL REVIEW D 90, 014009 (2014)

Table 3.5: S-wave D ($c\bar{s}$) spectrum (in MeV).

nL	J^P	State	$M_{Q\bar{q}}$	$\langle V_{Q\bar{q}}^{j_1 j_2}\rangle$	Present	Experiment Meson	Experiment Mass[98]	[142][a]	[144][b]	[118][c]	[149][d]	[97][e]	[124][f]	QSR[g]
1S	1^-	1^3S_1	2009.54	0.99	2010.53	D^*	2010.28±0.13		2010	2018	2010	2038	2013	2000±20 [122]
	0^-	1^1S_0	1869.57	-2.58	1867.00	D	1864.86±0.13		1871	1865	1867	1874	1890	1900±30 [122]
2S	1^-	2^3S_1	2605.29	0.57	2605.86	$D^*(2600)$	2608.7±2.4±2.5 [60]	2639	2632	2639	2636	2645	2708	2612±6 [119]
	0^-	2^1S_0	2523.05	-1.33	2521.72	$D(2550)$	2539.4±4.5±6.8 [60]	2567	2581	2598	2555	2583	2642	2539±8 [119]
3S	1^-	3^3S_1	3147.50	0.39	3147.89			3125	3096	3110		3111	3103	
	0^-	3^1S_0	3087.21	-0.90	3086.31			3065	3062	3087		3068	3064	
4S	1^-	4^3S_1	3662.99	0.29	3663.28				3482	3514			3395	
	0^-	4^1S_0	3614.22	-0.66	3613.56				3452	3498			3299	

Eur. Phys. J. C 76 (2016) 36

[4] Manan Shah, Bhavin Patel, P.C. Vinodkumar, Phys. Rev. D 90 (2014) 014009.
[5] Manan Shah, Bhavin Patel, P.C. Vinodkumar, Eur. Phys. J. C 76 (2016) 36.

Mass spectra of S- wave of B and B_s meson

Table 3.6: S-wave B ($b\bar{q}$, $q \in u, d$) spectrum (in MeV).

						Experiment								
nL	J^P	State	$M_{Q\bar{q}}$	$\langle V_{Q\bar{q}}^{j_1 j_2} \rangle$	Present	Meson	Mass[98]	[143]	[144]	[145]	[146]	[147]	[61]	[62]
1S	1^-	1^3S_1	5360.21	-34.98	5325.23	B^*	5325.2 ± 0.4	5330	5326	5330	5324	5325	5325	5321
	0^-	1^1S_0	5191.38	87.97	5279.36	B	5279.58 ± 0.17	5280	5280	5266	5279	5277	5279	5291
2S	1^-	2^3S_1	5847.79	-23.89	5823.90			5870	5906	5946	5920	5848		
	0^-	2^1S_0	5748.46	55.76	5804.22			5830	5890	5930	5886	5822		
3S	1^-	3^3S_1	6272.08	-18.49	6253.58			6240	6387	6396	6347	6136		
	0^-	3^1S_0	6199.61	42.46	6242.07			6210	6379	6387	6320	6117		
4S	1^-	4^3S_1	6664.61	-15.18	6649.43				6786	6779		6351		
	0^-	4^1S_0	6606.55	34.61	6641.17			6520	6781	6773		6335		

PHY. REV. D 93, 094028 (2016)

Table 3.7: S-wave B_s ($b\bar{s}$) spectrum (in MeV).

						Experiment								
nL	J^P	State	$M_{Q\bar{q}}$	$\langle V_{Q\bar{q}}^{j_1 j_2} \rangle$	Present	Meson	Mass[98]	[143]	[144]	[145]	[146]	[147]	[61]	[62]
1S	1^-	1^3S_1	5451.61	-36.19	5415.42	B_s^*	$5415.4^{+2.4}_{-2.1}$	5430	5414	5417	5421	5417	5430	5409
	0^-	1^1S_0	5277.53	88.92	5366.45	B_s	5366.77 ± 0.24	5370	5372	5355	5373	5366	5380	5382
2S	1^-	2^3S_1	5982.04	-25.89	5956.15			5970	5992	6016	6019	5966		
	0^-	2^1S_0	5879.22	60.02	5939.24			5930	5976	5998	5985	5939		
3S	1^-	3^3S_1	6447.69	-20.48	6427.20			6340	6475	6449	6449	6274		
	0^-	3^1S_0	6372.25	46.88	6419.14			6310	6467	6441	6421	6254		
4S	1^-	4^3S_1	6881.33	-17.03	6864.30				6879	6818		6504		
	0^-	4^1S_0	6820.60	38.75	6859.35			6620	6874	6812		6487		

[6] Manan Shah, Bhavin Patel, P.C. Vinodkumar, Phys. Rev. D 93 (2016) 094028.

Tetraquark System

- **Here, the tetraquark state is treated as three two-body interactive systems, $c\ (or\ b) - q$, $\bar{c}\ (or\ \bar{b}) - \bar{q}$ and diquark-antidiquark [7-9].**

- **The diquark - antidiquark picture for tetraquark configuration is one of the most promising approach to understand the structure of many exotic mesonic states as the diquark-antidiquark structure is a strongly correlated system.**

- **The confinement masses of quarks and antiquarks are obtained through a mean-field linear potential of the form [7],**

$$V(r) = \frac{1}{2}(1+\gamma_0)(\lambda r + V_0)$$

[7] Tanvi Bhavsar, Manan Shah, Smruti Patel, P. C. Vinodkumar, Nuclear Physics A 1000 (2020) 121856.
[8] Smruti Patel, Manan Shah, P. C. Vinodkumar, Eur. Phys. J.A (2014) 50:131
[9] Smruti Patel, P. C. Vinodkumar, Eur. Phys. J. C (2016) 76:356

Tetraquark System

- In the diquark-antidiquark structure, the masses of the diquark/diantiquark system are given by

$$m_d = E_Q + E_q + E_d + \langle V_{SD} \rangle_{Qq} - E_{CM}$$

$$m_{\bar{d}} = E_{\bar{Q}} + E_{\bar{q}} + E_{\bar{d}} + \langle V_{SD} \rangle_{\bar{Q}\bar{q}} - E_{CM}$$

- Further, the same procedure is adopted to compute the binding energy of the diquark-antidiquark bound system as

$$m_{d-\bar{d}} = E_d + E_{\bar{d}} + E_{d\bar{d}} + \langle V_{SD} \rangle_{d\bar{d}}$$

[7] Tanvi Bhavsar, Manan Shah, Smruti Patel, P. C. Vinodkumar, Nuclear Physics A 1000 (2020) 121856.

Mass Spectra of hidden Charm Sector

Mass spectra of $cs\bar{c}\bar{s}$, $cq\bar{c}\bar{q}$ and $cs\bar{c}\bar{q}$ states in the diquark − antidiquark picture for $L_d = 0$ and $L_{\bar{d}} = 0$ (in MeV).

state	S_d	L_d	$S_{\bar{d}}$	$L_{\bar{d}}$	J_d	$J_{\bar{d}}$	$J = J_d + J_{\bar{d}}$	J^{PC}	state notation	Masses of		
										$cs\bar{c}\bar{s}$	$cq\bar{c}\bar{q}$	$cs\bar{c}\bar{q}$
1s	0	0	0	0	0	0	0	0^{++}	$^1S_0$	3967	3739	3852
	1	0	0	0	1	0	1	$1^{+\pm}$	$^3S_1$	4097	3877	3981
	1	0	1	0	1	1	0	0^{++}	$^1S_0$	4214	4001	4106
							1	1^{+-}	$^3S_1$	4217	4004	4110
							2	2^{++}	$^5S_2$	4229	4018	4123
2s	0	0	0	0	0	0	0	0^{++}	$^1S_0$	4505	4325	4414
	1	0	0	0	1	0	1	$1^{+\pm}$	$^3S_1$	4601	4425	4510
	1	0	1	0	1	1	0	0^{++}	$^1S_0$	4689	4516	4601
							1	1^{+-}	$^3S_1$	4691	4518	4604
							2	2^{++}	$^5S_2$	4700	4528	4612

[7] Tanvi Bhavsar, Manan Shah, Smruti Patel, P. C. Vinodkumar, Nuclear Physics A 1000 (2020) 121856.

Mass Spectra of hidden Charm Sector

Mass spectra of $cs\bar{c}\bar{s}$ and $cq\bar{c}\bar{q}$ states in the diquark - antidiquark picture for $L_d = 1$ and $L_{\bar{d}} = 0$ (in MeV).

State	S_d	L_d	$S_{\bar{d}}$	$L_{\bar{d}}$	J_d	$J_{\bar{d}}$	$J = J_d + J_{\bar{d}}$	J^{PC}	state notation	Masses of		
										$cs\bar{c}\bar{s}$	$cq\bar{c}\bar{q}$	$cs\bar{c}\bar{q}$
1P	0	1	0	0	1	0	1	1^{--}	$^1P_1$	4416	4217	4315
	1	1	0	0	0	0	0	$0-+$	$^3P_0$	4428	4269	4328
					1		1	$1-+$	$^3P_1$	4439	4278	4339
					2		2	$2-+$	$^3P_2$	4445	4283	4344
	1	1	1	0	0	1	1	1^{--}	$^1P_1$	4466	4343	4404
					1	1	0	$0-+$	$^3P_0$	4385	4296	4339
							1	$1-+$	$^3P_1$	4413	4314	4362
							2	$2-+$	$^3P_2$	4487	4355	4420
					2	1	1	$1--$	$^5P_1$	4414	4308	4359
							2	$2--$	$^5P_2$	4420	4320	4369
							3	$3--$	$^5P_3$	4555	4392	4474

[7] Tanvi Bhavsar, Manan Shah, Smruti Patel, P. C. Vinodkumar, Nuclear Physics A 1000 (2020) 121856.

Mass Spectra of Bottom Sector

Table 1 Mass spectra of four-quark states in the diquark–antidiquark picture (for $L_1 = 0$, $L_2 = 0$) (in GeV)

S_d	L_d	$S_{\bar{d}}$	$L_{\bar{d}}$	J_d	$J_{\bar{d}}$	J	J^{PC}	$^{2s+1}X_J$	M_{cw}	V_{SS}	V_{LS}	V_T	M_J
0	0	0	0	0	0	0	0^{++}	$^1S_0$	10.309	0.0	0.0	0.0	10.309
1	0	0	0	1	0	1	1^{+-}	$^3S_1$	10.316	0.0	0.0	0.0	10.316
1	0	1	0	1	1	0	0^{++}	$^1S_0$	10.323	−0.179	0.0	0.0	10.143
						1	1^{+-}	$^3S_1$	10.323	−0.089			10.233
						2	2^{++}	$^5S_1$	10.323	0.089			10.413

Table 4 First radially excited mass spectra of four-quark states in the diquark–antidiquark picture (for $L_1 = 0$, $L_2 = 0$) (in GeV)

S_d	L_d	$S_{\bar{d}}$	$L_{\bar{d}}$	J_d	$J_{\bar{d}}$	J	J^{PC}	$^{2s+1}X_J$	M_{cw}	V_{SS}	V_{LS}	V_T	M_J
0	0	0	0	0	0	0	0^{++}	$^1S_0$	10.702	0.0	0.0	0.0	10.702
1	0	0	0	1	0	1	1^{+-}	$^3S_1$	10.709	0.0	0.0	0.0	10.709
1	0	1	0	1	1	0	0^{++}	$^1S_0$	10.716	−0.066	0.0	0.0	10.650
						1	1^{+-}	$^3S_1$	10.716	−0.033			10.683
						2	2^{++}	$^5S_1$	10.716	0.033			10.750

[9] Smruti Patel, P. C. Vinodkumar, Eur. Phys. J. C (2016) 76:356

Mass Spectra of Bottom Sector

Table 2 Mass spectra of four-quark states in the diquark–antidiquark picture ($L_1 = 1, L_2 = 0$) (in GeV)

S_d	L_d	$S_{\bar{d}}$	$L_{\bar{d}}$	J_d	$J_{\bar{d}}$	J	J^{PC}	$^{2s+1}X_J$	M_{cw}	V_{SS}	V_{LS}	V_T	M_J
0	1	0	0	1	0	1	1^{--}	$^1P_1$	10.917	0.0	0.0	0.014	10.931
1	1	0	0	0	0	0	0^{-+}	$^3P_0$	10.917	0.000	−0.0059	−0.0286	10.883
				1		1	1^{-+}	$^3P_1$		0.000	−0.0029	−0.011	10.921
				2		2	2^{-+}	$^3P_2$		0.000	0.0029	−0.0256	10.913
1	1	1	0	0	1	1	1^{--}	$^1P_1$	10.925	−0.019	0.0	−0.0233	10.882
				1	1	0	0^{-+}	$^3P_0$		−0.0095	−0.0059	−0.0467	10.862
						1	1^{-+}	$^3P_1$			−0.0029	−0.011	10.900
						2	2^{-+}	$^3P_2$			0.0029	−0.026	10.892
				2	1	1	1^{--}	$^5P_1$		0.0095	−0.0088	−0.072	10.853
						2	2^{--}	$^5P_2$			−0.0029	0.0256	10.957
						3	3^{--}	$^5P_3$			0.006	−0.037	10.903

[9] Smruti Patel, P. C. Vinodkumar, Eur. Phys. J. C (2016) 76:356

Decay Properties of Tetra quark States

- The predictions of the decay widths play a crucial role in the identification of the structure and quark compositions of the exotic states.

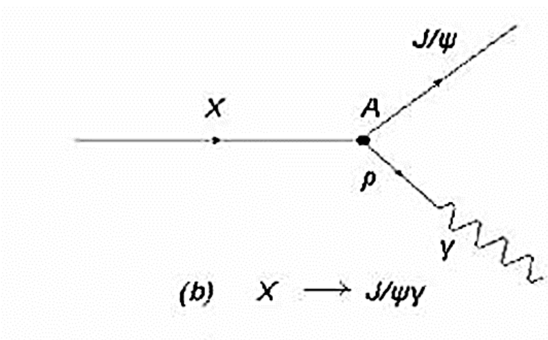
- The prominent exotic state with $J^{PC} = 1^{--}$ was first observed by the Belle collaboration [10,11] and to date, it remains to be confirmed by independent experiments.

[10] K.F.Chen et al. (BelleCollaboration), Phys. Rev. Lett. 100, 112001 (2008). arXiv:0710.2577 [hep-ex]
[11] I. Adachi et al. (Belle Collaboration), arXiv: 1209.6450 [hep-ex]

Radiative Decay width of $J^{PC}=1^{--}$ state

- **We study the radiative decays of these states using the idea of vector meson dominance (VMD) which describes the interactions between photons and hadronic matter [12].**

- **The transition matrix element for the radiative decay of $Y_b \rightarrow \chi_b + \gamma$ or $X \rightarrow J/\psi + \gamma$ is given with the use of VMD by**



$$\langle \chi_b \gamma | Y_b \rangle = \langle \gamma | \rho \rangle \frac{1}{m_\rho^2} \langle \chi_b \, \rho | Y_b \rangle$$

$$\langle J/\psi \gamma | X \rangle = \langle \gamma | \rho \rangle \frac{1}{m_\rho^2} \langle J/\psi \rho | X \rangle = \frac{f_\rho}{m_\rho^2} A$$

[12] J. J. Sakurai, Currents and Mesons (University of Chicago Press, Chicago,1969)

Radiative Decay width of $J^{PC}=1^{--}$ state

- And the decay width is given by

$$\Gamma(Y_b \to \chi_b + \gamma) = 2|A^2|(\frac{f_\rho}{m_\rho{}^2})^2 \frac{1}{8\pi M_{Y_b}{}^2} \frac{(\lambda)^{\frac{1}{2}}}{2M_{Y_b}}$$

$$\Gamma(X \to J/\psi\gamma) = 2|A^2|\left(\frac{f_\rho}{m_\rho^2}\right)^2 \frac{1}{8\pi M_X^2} \frac{\sqrt{\lambda(M_X, M_\psi, 0)}}{2M_X}$$

- Here, λ is the centre of mass momentum and $f_\rho = 0.152\ GeV^2$ [13,14].

e.g. $a \to b\ c$

$$\lambda = (M_a)^4 + (M_b)^4 + (M_c)^4 - 2(M_a M_b)^2 - 2(M_a M_c)^2 - 2(M_b M_c)^2$$

[13] A.Deandrea, G. Nardulli, A. D. Polosa, Riv ,Nuovo Cim. 23N, 11, 1(2000).
[14] A. Deandrea, G. Nardulli, A. D. Polosa, Phys. Rev. D 68, 034002 (2003).

Hadronic Decay width of $J^{PC}=1^{--}$ state

- **We have studied the hadronic decay of the 1^{--} P-wave Y_b and X (or ψ) states. We discuss the two-body hadronic decays, i.e. $Y_b(q) \rightarrow B * q(k)^{-} B * q(l)$.**

- **These are Zweig-allowed processes and involve essentially the quark rearrangements. For calculating dominant two-body hadronic decay widths of the 1^{--} tetraquark state, the vertices are given as [15,16]**

$$
\begin{aligned}
Y_b &\longrightarrow B\bar{B} = F(k^{\mu} - l^{\nu}), \\
Y_b &\longrightarrow B\bar{B}^{*} = \frac{F}{M}\epsilon^{\mu\nu\rho\sigma}k_{\rho}l_{\sigma}, \\
Y_b &\longrightarrow B^{*}\bar{B}^{*} = F(g^{\mu\rho}(q+l)^{\nu} - g^{\mu\nu}(k+q)^{\rho} \\
&\qquad +g^{\rho\nu}(q+k)^{\mu}),
\end{aligned}
$$

[15] M. E. Peskin, D.V. Schroeder, An Introduction to Quantum Field Theory. Addison G Wesley (1995).
[16] Ruilin Zhu, Phys. Rev. D 94 (2016) 054009.

Hadronic Decay width of $J^{PC}=1^{--}$ state

- **The corresponding decay widths [15] are given by**

$$\Gamma(Y_b \longrightarrow B\bar{B}) = \frac{F^2|\vec{k}|^3}{2M^2\pi},$$

$$\Gamma(Y_b \longrightarrow B\bar{B}^*) = \frac{F^2|\vec{k}|^3}{4M^2\pi},$$

$$\Gamma(Y_b \longrightarrow B^*\bar{B}^*) = \frac{F^2|\vec{k}|^3(48|\vec{k}|^4 - 104M^2|\vec{k}|^2 + 27M^4)}{2\pi(M^3 - 4|\vec{k}|^2M)^2}$$

Here $|\vec{k}| = |\vec{K}|$ is the center of mass momentum given by

$$|\vec{k}| = \frac{\sqrt{M^2-(M_k+M_l)^2}\sqrt{M^2-(M_k+M_l)^2}}{2M}$$

Here, M is the mass of the decaying particle and M_k, M_l are the masses of the decay products.

[15] M. E. Peskin, D.V. Schroeder, An Introduction to Quantum Field Theory. Addison G Wesley (1995).

Hadronic Decay width of $J^{PC}=1^{--}$ state

- **The hadronic decay widths for ψ states [16, 17] are computed using the relations given by**

$$\Gamma\left(T_{c\bar{c}}\left[1^{--}\right] \longrightarrow D_q \bar{D}_q\right) = \frac{F^2_{T_{c\bar{c}}[1^{--}]D_q\bar{D}_q} |\vec{K}|^3}{6M^2\pi}$$

$$\Gamma\left(T_{c\bar{c}}\left[1^{--}\right] \longrightarrow D_q\, \bar{D}_q^*\right) = \frac{F^2_{T_{c\bar{c}}[1^{--}]D_q\bar{D}_q^*} |\vec{K}|^3}{12M^2\pi}$$

$$\Gamma\left(T_{c\bar{c}}\left[1^{--}\right] \longrightarrow D_q^*\, \bar{D}_q^*\right) = \frac{F^2_{T_{c\bar{c}}[1^{--}]\longrightarrow D_q^* \bar{D}_q^*} |\vec{K}|^3 \left(M^4 - \frac{104}{9} M^2 |\vec{K}|^2 + \frac{48}{9} |\vec{K}|^4\right)}{2\,\pi\, M^2 \left(M^2 - 4|\vec{K}|^2\right)^2}$$

[16] Ruilin Zhu, Phys. Rev. D 94 (2016) 054009.
[17] R.S. Azevedo, M. Nielsen, Braz. J. Phys. 34 (2004) 1.

Leptonic Decay width of $J^{PC}=1^{--}$ state

- **In the conventional $b\bar{b}$ (or $c\bar{c}$) systems, the decay widths are determined by the wave functions at the origin for the ground state, while for the P-waves the derivations of these wave functions at the origin are used.**

- **The partial electronic decay widths of the tetraquark states made up of diquarks and antidiquarks are given by the well-known VanRoyen–Weisskopf formula for P-waves [18],**

$$\Gamma\left(T_{(c\bar{c}\ or\ b\bar{b})}\ [1^{--}] \longrightarrow e^+e^-\right) = \frac{24\alpha^2\langle e_Q\rangle^2}{M_{Y_b^4}}\sigma^2|R'_{11}(0)|^2$$

- **Here, α is the fine structure coupling constant, σ <1 and $\langle e_Q\rangle$ is the effective charge [19] of Qq diquark system given by**

$$\langle e_Q\rangle = \left|\frac{m_Q e_q - m_q e_Q}{m_Q + m_q}\right|$$

[18] A. Ali, arXiv:1108.2197v1 [hep-ph]
[19] J. L. Domenech-Garret, M.A. Sanchis-Lozano, Comput. Phys. Commun. 180, 768 (2009).

Radiative and Leptonic Decay width of $J^{PC}=1^{--}$ state

Radiative and leptonic decay widths of 1^{--} states.

Experimental state	Experimental Mass (MeV)	Predicted Mass (MeV)	Quark composition	Decay modes	Predicted Decay width	Experimental Decay width
$\psi(4230)$	~ 4230	4217	$cq\bar{c}\bar{q}$	$\Gamma_{(\psi(4230 \to J/\psi\gamma))}$ $\Gamma_{e^+e^-}$	1.932 MeV 0.391 keV	$\cdots$ $\cdots$
$\psi(4260)$	4230 ± 8	4263	mixed $cq\bar{c}\bar{q}$	$\Gamma_{(\psi(4260 \to J/\psi\gamma))}$ $\Gamma_{e^+e^-}$	1.959 MeV 0.374 keV	$\cdots$ $\cdots$
$\psi(4360)$	4359 ± 13	4359	$cs\bar{c}\bar{q}$	$\Gamma_{(\psi(4360 \to J/\psi\gamma))}$ $\Gamma_{e^+e^-}$	2.009 MeV 0.369 keV	$\cdots$ $\cdots$
$\psi(4390)$	~ 4390	4389	mixed $cs\bar{c}\bar{q}$	$\Gamma_{(\psi(4390 \to J/\psi\gamma))}$ $\Gamma_{e^+e^-}$	2.023 MeV 0.386 keV	$\cdots$ $\cdots$
$\psi(4415)$	4421 ± 4	4416	$cs\bar{c}\bar{s}$	$\Gamma_{(\psi(4415 \to J/\psi\gamma))}$ $\Gamma_{e^+e^-}$	2.035 MeV 0.498 keV	$\cdots$ 0.58 ± 0.07

[7] Tanvi Bhavsar, Manan Shah, Smruti Patel, P. C. Vinodkumar, Nuclear Physics A 1000 (2020) 121856.

Hadronic Decay width of $J^{PC}=1^{--}$ state

Hadronic decay widths of 1^{--} states.

	$\psi(4230)$	$\psi(4260)$	$\psi(4360)$	$\psi(4390)$	$\psi(4415)$
Masses (Our) (MeV)	4217	4263	4359	4404	4416
Width (MeV)					
$\Gamma(\psi \rightarrow D^0 \bar{D}^0)$	3.014	7.225	18.875	25.306	27.104
$\Gamma(\psi \rightarrow D^0 \bar{D}^0*)$	2.420	0.608	1.801	3.914	4.557
$\Gamma(\psi \rightarrow D^{0*} \bar{D}^0*)$	59.297	42.028	11.736	2.428	0.859
$\Gamma(\psi \rightarrow D_s \bar{D}_s)$	32.729	24.366	10.181	5.214	4.098
$\Gamma(\psi \rightarrow D_s \bar{D}_s*)$	31.133	25.648	15.680	11.707	10.726

[7] Tanvi Bhavsar, Manan Shah, Smruti Patel, P. C. Vinodkumar, Nuclear Physics A 1000 (2020) 121856.

Hadronic Decay width of $J^{PC}=1^{--}$ state

Hadronic decay widths of bottom tetraquark (in keV)

State	Decay mode	F	$\lvert \vec{k} \rvert$	Γ
	$\Upsilon_{bq} \rightarrow B\bar{B}$	1.35	1.31	5.500
$\Upsilon_b(10882)$	$\Upsilon_{bq} \rightarrow B\bar{B}^*$	3.12	1.22	11.87
	$\Upsilon_{bq} \rightarrow B^*\bar{B}^*$	0.92	1.11	40.86
	$\Upsilon_{bq} \rightarrow B\bar{B}$	1.35	1.25	4.800
$\Upsilon_b(10853)$	$\Upsilon_{bq} \rightarrow B\bar{B}^*$	3.12	1.15	10.00
	$\Upsilon_{bq} \rightarrow B^*\bar{B}^*$	0.92	1.04	30.63
	$\Upsilon_{bq} \rightarrow B\bar{B}$	1.35	1.41	6.800
$\Upsilon_b(10931)$	$\Upsilon_{bq} \rightarrow B\bar{B}^*$	3.12	1.32	14.91
	$\Upsilon_{bq} \rightarrow B^*\bar{B}^*$	0.92	1.23	57.23

[9] Smruti Patel, P. C. Vinodkumar, Eur. Phys. J. C (2016) 76:356

Radiative and Leptonic Decay width of $J^{PC}=1^{--}$ state

Radiative decay widths (in keV))

State	$\Gamma \to \chi_b + \gamma$	$\Gamma \to \eta_b + \gamma$
$\Upsilon_b(10882)$	0.173	0.247
$\Upsilon_b(10853)$	0.169	0.243
$\Upsilon_b(10931)$	0.179	0.252

Di-leptonic decay widths (in keV)

State	$\Gamma_{ee[bl]}$	$\Gamma_{ee[\mathrm{bh}]}$
$\Upsilon_b(10882)$	0.0251	0.123
$\Upsilon_b(10853)$	0.0254	0.125
$\Upsilon_b(10931)$	0.0246	0.121

[9] Smruti Patel, P. C. Vinodkumar, Eur. Phys. J. C (2016) 76:356

Summary

- $Z_c(3985) \rightarrow J^{PC} = 1^{++}\ or\ 1^{+-}\ cs\bar{c}\bar{q}$ tetraquark (Ds* D) or (Ds D*)

- According to our analysis $Z_c(4600)$ is the radial excitation of $cs\bar{c}\bar{q}$ tetraquark state having mass 4604 MeV. According to our analysis $Z_c(4430)$ is the radial excited state of $cq\bar{c}\bar{q}$ tetraquark state having mass 4425 MeV, which in accordance with the results suggested by [20, 21].

- Hadronic decays are not seen for ψ(4260) state experimentally but its full width is measured around 55 ±19 MeV. Our computed hadronic decay widths for $D_s\,\bar{D}_s$ and $D_s\,\bar{D}_s^*$ are higher than it's full width. So, according to the present study these are not possible decays for ψ(4260) state. Experimentally, we don't have much information about the state ψ(4360) but the full width and leptonic decay width of this state are measured [22]. The hadronic decays of this state are not even seen experimentally but the full width of 96 ± 7 suggests us to look for other decays channels.

- According to the present analysis, ψ(4415) fit to be a pure hidden charm hidden strange $cs\bar{c}\bar{s}$ tetraquark state having mass 4416 MeV. Our predicted radiative and leptonic decay widths for this state are 2.035 MeV and 0.498 keV ($\Gamma_{ee}^{exp}(\psi(4415)) = 0.58 \pm 0.07$ keV) respectively.

[20] L. Maiani, F. Piccinini, A.D. Polosa, V. Riquer, Phys. Rev. D 89 (2014) 114010.
[21] M. Nielsen, F.S. Navarra, Mod. Phys. Lett. A 29 (2014) 1430005.
[22] M. Tanabashi, et al., Particle Data Group, Phys. Rev. D 98 (2018) 030001.

Summary

- We predicted the $Z_b(10650)$ state as the first radial excitation of either the $X_b(10143)$ $(0++)$ state or the $X_b(10233)$ (1^{+-}) state.

- Our prediction regarding $Z_b(10650)$ state is just a straightforward extension to the beauty sector and we observe that the $Z_b(10650)$ is also a radially excited state of a still unmeasured X_b state.

- As the $Y_b(10890)$ state with quantum number 1^{--} is of keen interest, in this study we have predicted three P-wave 1^{--} states in the mass region around 10.850–10.931 GeV.

  We have observed that the P-wave Y_b state has mass 10.853 GeV as the Y_b state. The calculated partial electronic decay widths for the P-wave Y_b is about 0.03 – 0.12 keV, which is in agreement with the experiment decay width 0.043 ± 0.004 keV [23].

[23] B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 102, 012001 (2009).

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