



QWG

# Decays of $1^{-+}$ Charmoniumlike Hybrid

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Collaborate with Ying Chen, Ming Gong, Xiangyu Jiang, Zhaofeng Liu, Wei Sun.

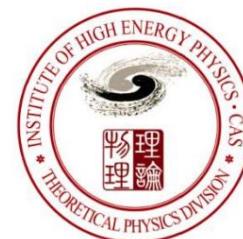
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Based on: arXiv:hep-lat/2306.12884



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# Outline

- Motivation
- Lattice methodology
- Lattice results
- Experimental prediction
- Summary

# Introduction

## ➤ 1. Exotic hadrons:

- tetraquark / pentaquark states and hadronic molecule.
- Glueball
- hybrids ( $q\bar{q}g$ ).

## ➤ 2. Experimental candidates for hybrids

### ➤ Isovector $1^{-+}$ states:

- $\pi_1(1400) / \pi_1(1600)$ ,  $\Gamma_{tot} \sim 240\text{MeV}$
- $\pi_1(2105)$

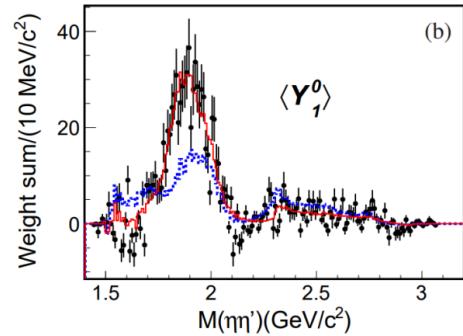
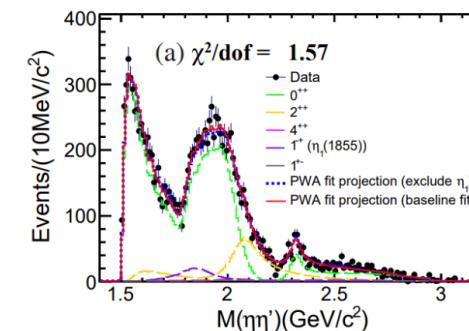
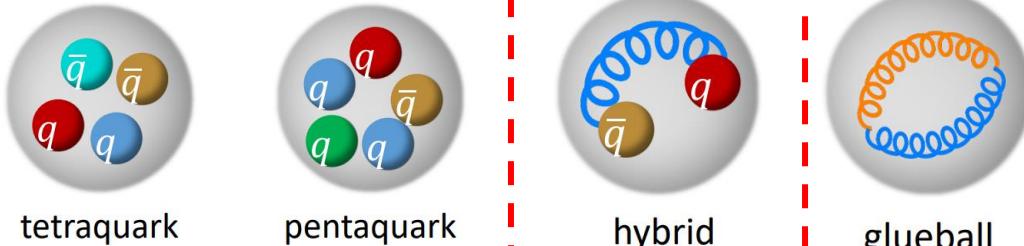
### ➤ Isoscalar $1^{-+}$ states: $\eta_1(1855)$ .

### ➤ charmoniumlike $1^{-+}$ counterpart $\eta_{c1}(c\bar{c}g)$ of $\eta_1(1855)$ . Exists?

## ➤ 3. Insights of the masses of hybrids predicted by Lattice QCD

➤  $m_{\pi_1} \sim 1.8 - 2.0\text{ GeV}$ ,       $m_{\eta_1} \sim 2.0 - 2.3\text{ GeV}$ ,

➤  $m_{\eta_{c1}} \sim 4.2 - 4.4\text{ GeV}$ .



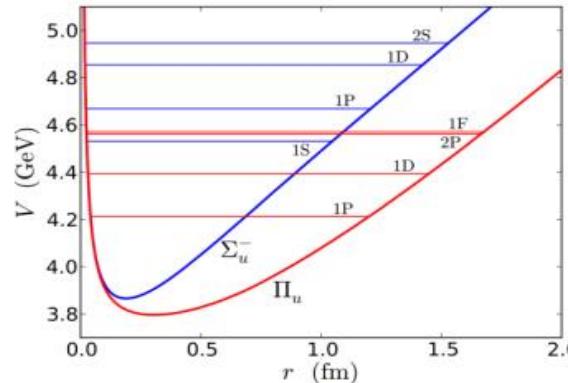
$\eta_1(1855)$  from  $J/\psi \rightarrow \gamma\eta_1 \rightarrow \gamma\eta\eta'$

[BESIII : PRD.107.079901 \(2023\)](#)

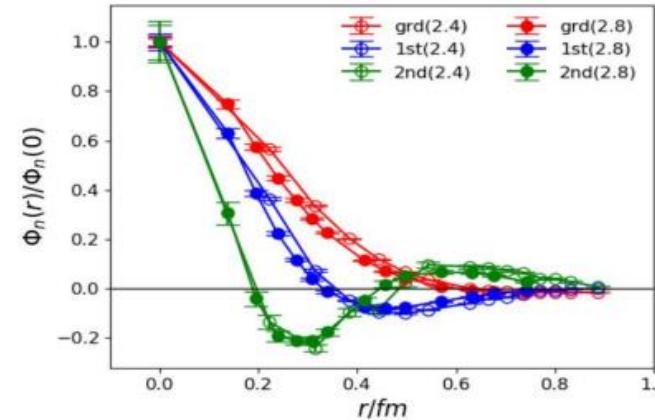
[BESIII : PRL.130.159901 \(2023\)](#)

# Spectrum of charmoniumlike $1^{-+}$

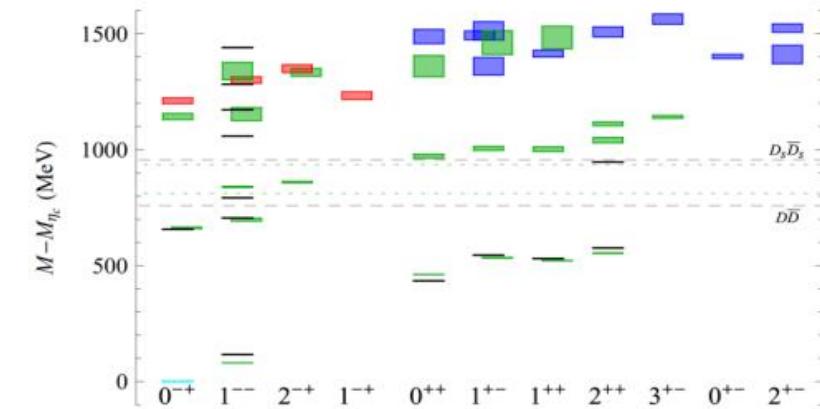
- Super-multiplet  $((0, 1, 2)^-, 1^{--})$  observed around 4.2 GeV
  - Consistent with the phenomenological expectation.
  - Non-consistency appears in the spectrum of excited states
    - Flux-tube:  $\Delta m(2P - 1P) \sim 0.38$  GeV
    - QLQCD:  $\Delta m(2P - 1P) \sim 1.2 - 1.3$  GeV
  - The internal structure of these hybrids reflected by the BS wave functions seems different from the flux-tube picture.



E. Braaten et al.,  
Phys. Rev. D 90 (2014) 014044



Y. Ma et al., Chin. Phys. C 45 (2021) 093111



$$m_{\eta_{c1}} \approx 4.23 \text{ GeV } (N_f = 2 + 1)$$

L. Liu [HSC Collab.], JHEP 07 (2012) 126

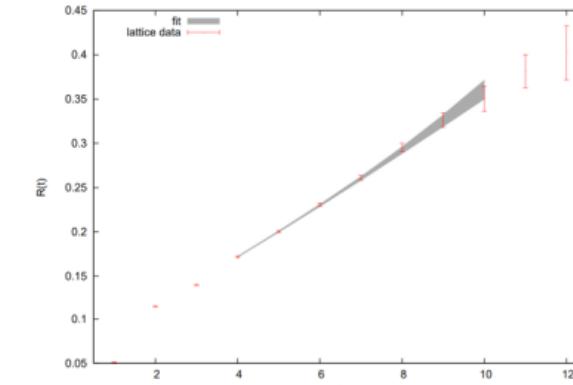
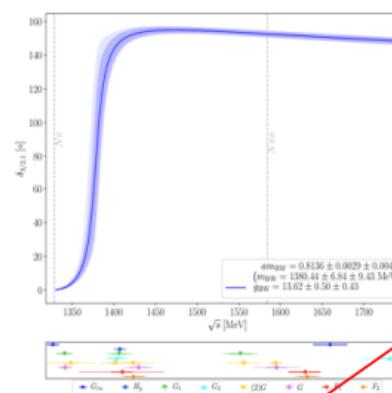
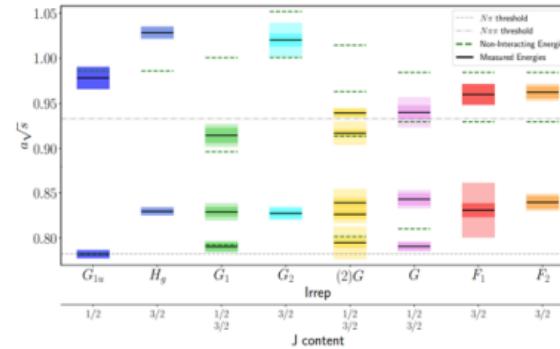
#node	$m(1^{--})$ (GeV)	$m(0^{-+})$ (GeV)	$m(1^{-+})$ (GeV)	$m(2^{-+})$ (GeV)
0	3.109(5)	3.010(4)	-	-
0	3.703(82)	3.672(76)	-	-
0	4.591(69)	4.551(63)	4.309(2)	4.419(3)
1	5.460(31)	5.393(28)	5.693(12)	5.779(12)
2	8.226(99)	8.286(109)	7.661(31)	7.708(29)

# Lattice methodology: M&M method (c. McNeile & C. Michael, PLB 556 (2003) 177)

$N\pi$  scattering and the  $\Delta$  resonance

G. Silvi et al., PRD103 (2021) 094508 (arXiv:2101.00689) and references therein

$N_s^3 \times N_t$	$24^3 \times 48$
$\beta$	3.31
$am_{u,d}$	-0.09530
$am_s$	-0.040
$a$ [fm]	0.1163(4)
$L$ [fm]	2.791(9)
$m_\pi$ [MeV]	255.4(1.6)
$m_\pi L$	3.61(2)
$N_{config}$	600
$N_{meas}$	9600



C. Alexandrou et al.,  
Phys. Rev. D 88 (2013) 031501

$$K\text{-matrix rescaled: } K = \rho^{1/2} \hat{K} \rho^{1/2}$$

$$K \text{ relates to the phase shift: } K^{Jl} = \tan \delta_{Jl}$$

$$\text{Breit Wigner: } \hat{K}^{(3/2,1)} = \frac{\sqrt{s}\Gamma(s)}{(m_{BW}^2 - s)\rho}$$

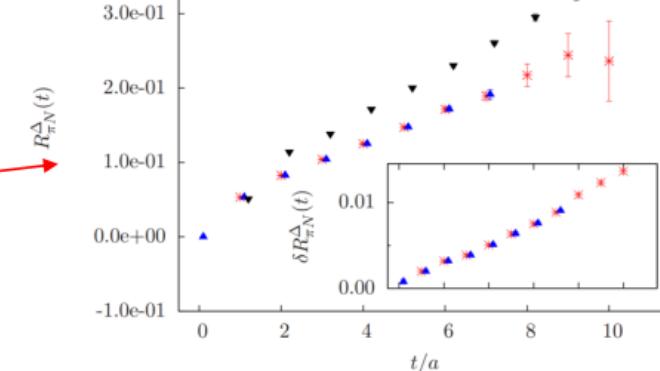
$$\Gamma(s) = \frac{g_{BW}^2 k^3}{6\pi} \frac{1}{s}$$

$$m_\Delta = (1378.3 \pm 6.6 \pm 9.0) \text{ MeV}$$

$$\Gamma_\Delta = (-16.4 \pm 1.0 \pm 1.4) \text{ MeV}$$

$$\Gamma_{\text{EFT}}^{\text{LO}} = \frac{g_{\Delta-\pi N}^2}{48\pi} \frac{E_N + m_N}{E_N + E_\pi} \frac{k^3}{m_N^2}$$

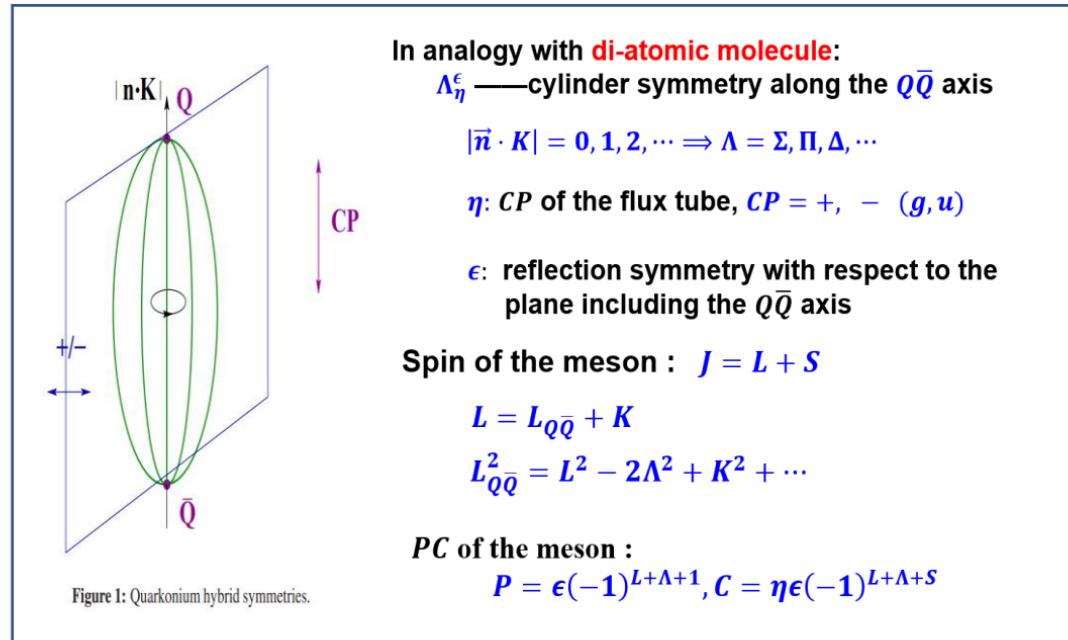
Collaboration	$m_\pi$ [MeV]	Methodology	$m_\Delta$ [MeV]	$g_{\Delta-\pi N}$
Verduci 2014 [38]	266(3)	Distillation, Lüscher	1396(19) <sub>BW</sub>	19.90(83)
Alexandrou et al. 2013 [37]	360	Michael, McNeile	1535(25)	27.0(0.6)(1.5)
Alexandrou et al. 2016 [39]	180	Michael, McNeile	1350(50)	23.7(0.7)(1.1)
Andersen et al. 2018 [41]	280	Stoch. distillation, Lüscher	1344(20) <sub>BW</sub>	37.1(9.2)
Our result	255.4(1.6)	Smeared sources, Lüscher	1380(7)(9) <sub>BW</sub> , 1378(7)(9) <sub>pole</sub>	23.8(2.7)(0.9)
Physical value [5]	139.5704(2)	phenomenology, K-matrix	1232(1) <sub>BW</sub> , 1210(1) <sub>pole</sub>	29.4(3) [79], 28.6(3) [80]



C. Alexandrou et al.,  
Phys. Rev. D 93 (2016) 114515

# Flux tube model

- A vibrational string along the  $Q\bar{Q}$  axis.
- The picture is originated from the lattice QCD formulation.



$$H = -\frac{1}{2\mu} \frac{\partial^2}{\partial r^2} + \frac{L(L+1) - \Lambda^2}{2\mu r^2} + E^1(r)$$

$$E^1(r) = -\frac{4\alpha_s}{3r} + c + br + \frac{\pi}{r} \left(1 - e^{-fb^{1/2}r}\right)$$

TABLE I. Some low-lying meson hybrids.

Flavor	$J^{PC}$ or $J^P$	Mass (GeV) for $f=1$	$\frac{dm}{df}$ (GeV)	$\Delta m^a$ (GeV)	$m^b$ (GeV)
$I=1$	$2^{\pm\mp}, 1^{\pm\mp}, 0^{\pm\mp}, 1^{\pm\pm}$	1.67	0.08	0.19	$\sim 1.9$
$I=\frac{1}{2}$	$2^{\pm}, 1^{\pm}, 0^{\pm}, 1^{\pm}$	1.80	0.10	0.17	$\sim 2.0$
$I=0$	$\left[ \frac{u\bar{u} + d\bar{d}}{\sqrt{2}} \right]$	$2^{\pm\mp}, 1^{\pm\mp}, 0^{\pm\mp}, 1^{\pm\pm}$	1.67	0.08	0.19
$I=0$ ( $s\bar{s}$ )	$2^{\pm\mp}, 1^{\pm\mp}, 0^{\pm\mp}, 1^{\pm\pm}$	1.91	0.12	0.14	$\sim 2.1$
$c\bar{c}$	$2^{\pm\mp}, 1^{\pm\mp}, 0^{\pm\mp}, 1^{\pm\pm}$	4.19	0.18	0.06	$\sim 4.3$
$b\bar{b}$	$2^{\pm\mp}, 1^{\pm\mp}, 0^{\pm\mp}, 1^{\pm\pm}$	10.79	0.28	0.02	$\sim 10.8$

<sup>a</sup>Contribution to the mass from nonadiabatic effects, taken from Ref. 14.

<sup>b</sup>A “best guess” based on the previous columns.

# Hybrid in the Flux tube model

**Flux-tube model has selection rule:**

(P. Page et al., Phys.Rev.D 59 (1999) 034016 )

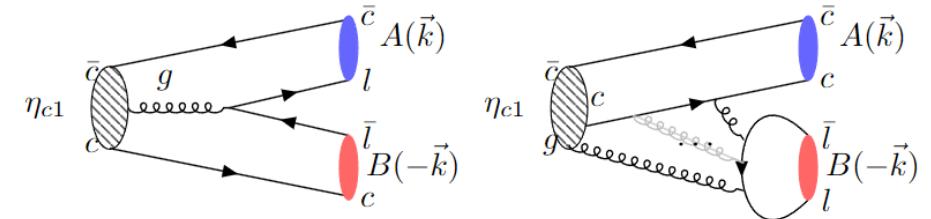
$$\langle AB|H_I|H\rangle \propto \int d^3\vec{r} (\phi_H(\vec{r}) \cdots) \int_0^1 d\xi \cos(\xi\pi) \cdot \phi_A(\xi\vec{r}) \cdot \phi_B((1-\xi)\vec{r})$$

- 1) Modes of two S-wave mesons are suppressed, SP-modes are favored.
- 2) Modes of two identical mesons are prohibited.

Open-charm and closed-charm decay modes

1 ) SP modes :  $D_1\bar{D}$ ,  $\chi_{c1}\eta(\eta')$

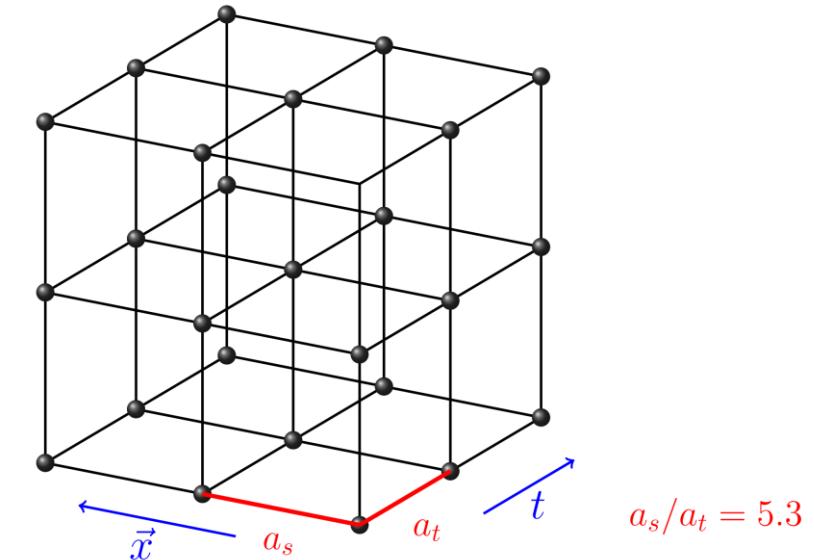
2 ) SS modes :  $D^*\bar{D}$ ,  $D^*\bar{D}^*$ ,  $\eta_c\eta(\eta')$ ,  $J/\psi\omega(\phi)$



# Lattice setup

- Tadpole improved Symanzik's gauge action  
(C. Morningstar, PRD60(1999)034509)
- Anisotropic Lattice
- $N_f = 2$  clover gauge ensembles with degenerate u, d quarks

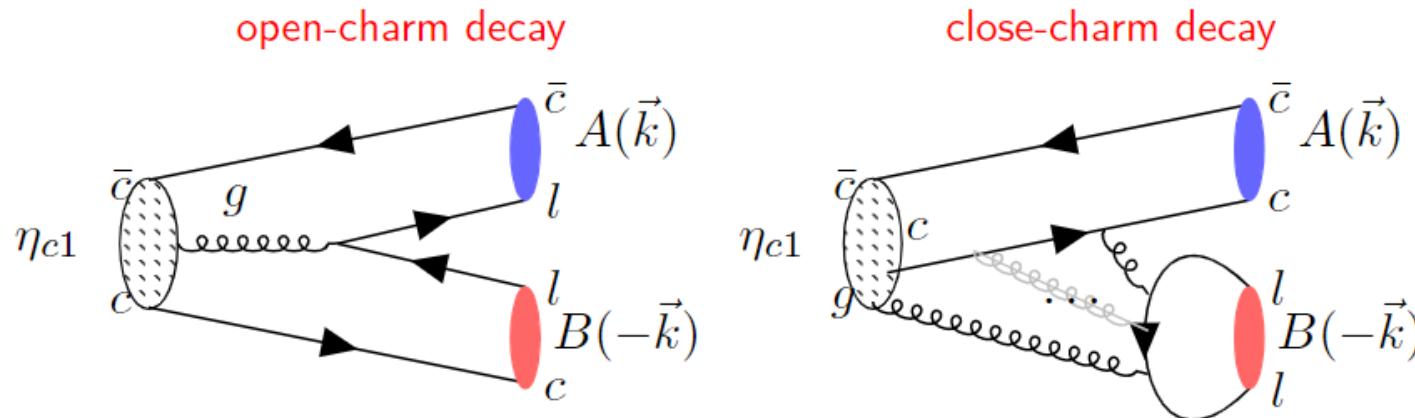
IE	$N_s^3 \times N_t$	$\beta$	$a_t^{-1}(\text{GeV})$	$\xi$	$m_\pi(\text{MeV})$	$N_V$	$N_{\text{cfg}}$
L16	$16^3 \times 128$	2.0	6.894(51)	$\sim 5.3$	$\sim 350$	70	708
L24	$24^3 \times 192$	2.0	6.894(51)	$\sim 5.3$	$\sim 350$	160	171



See: Jiang et al, Phys.Rev.D 107 (2023) 094510

- Distillation method:  
(M.Peardon et al.(HSC.)(2009)PRD80,054506)
- disconnected diagrams are involved

# $\eta_{c1}$ decay modes



open-charm decay:  $D\bar{D}_1$ (S-wave),  $D\bar{D}^*$ (P-wave),  $D^*\bar{D}^*$ (P-wave).  
close-charm decay:  $\chi_{c1}\eta$ (S-wave),  $\eta_c\eta'$ (P-wave),  $J/\psi\omega$ (P-wave).

- The flavor wave functions of the open-charm modes

$$|\mathbf{D}\bar{\mathbf{D}}'\rangle_{(c=+)}^{(I=0)} = \frac{1}{2}(|\mathbf{D}^+\bar{\mathbf{D}}'^-\rangle + |\mathbf{D}^0\bar{\mathbf{D}}'^0\rangle) \pm \frac{1}{2}(|\mathbf{D}^-\bar{\mathbf{D}}'^+\rangle + |\bar{\mathbf{D}}^0\mathbf{D}'^0\rangle)$$

$\xleftarrow{\begin{array}{l} D' = D^*, D_1, \\ "+" \text{ for } D\bar{D}^* \\ "-" \text{ for } D\bar{D}_1 \end{array}}$

$$|\mathbf{D}^*\bar{\mathbf{D}}^*\rangle_{(c=+)}^{(I=0)} = \frac{1}{\sqrt{2}}(|\mathbf{D}^{*+}\bar{\mathbf{D}}^{*-}\rangle + |\mathbf{D}^{0*}\bar{\mathbf{D}}^{0*}\rangle)$$

$\xleftarrow{L + S = \text{even}}$

# Decay amplitudes in M&M method

Effective Lagrangian:

$$\mathcal{L}_I^{cc} \sim -g_{\chi\eta} m_{\eta_{c1}} H_\mu A^\mu \eta - ig_{\eta_c\eta} H_\mu \eta_c \overleftrightarrow{\partial}^\mu \eta + iH_\mu \left( g\psi_\nu \partial^\nu \omega^\mu + g'\omega_\nu \partial^\nu \psi^\mu + g_0 \psi_\nu \overleftrightarrow{\partial}^\mu \omega^\nu \right),$$

$$\begin{aligned} \mathcal{L}_I^{\text{oc}} &\sim g_{D_1 D} m_{\eta_{c1}} H_\mu \frac{1}{2} \left( D_1^{\mu,\dagger} D + D^\dagger D_1^\mu \right) \\ &+ g_{D^* \bar{D}^*} H^\mu \frac{i}{\sqrt{2}} \left( D^{\nu,\dagger} \partial_\nu D_\mu + \partial_\nu D_\mu^\dagger D^\nu \right) \\ &+ \frac{g_{D^* \bar{D}}}{m_{\eta_{c1}}} \epsilon^{\mu\nu\rho\sigma} (\partial_\mu H_\nu) \frac{1}{2} \left[ (\partial_\rho D_\sigma^\dagger) D - D^\dagger (\partial_\rho D_\sigma) \right]. \end{aligned}$$

$|\eta_{c1}\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |AB\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$   
 $\hat{H} = \begin{pmatrix} m_{\eta_{c1}} & x \\ x & E_{AB} \end{pmatrix}$

$\hat{T}(a) = e^{-a\hat{H}} = e^{-a\bar{E}} \begin{pmatrix} e^{-a\Delta/2} & ax \\ ax & e^{a\Delta/2} \end{pmatrix}$   
 $\bar{E} = \frac{m_{\eta_{c1}} + E_{AB}}{2}, \quad \Delta = m_{\eta_{c1}} - E_{AB}$

$x_{AP}^{\lambda'\lambda} = g_{AP} m_{\eta_{c1}} \vec{\epsilon}_\lambda(\vec{0}) \cdot \vec{\epsilon}_{\lambda'}^*(\vec{k}),$   
 $x_{PP}^{\lambda} = 2g_{PP} \vec{\epsilon}_\lambda(\vec{0}) \cdot \vec{k},$   
 $x_{VP}^{\lambda'\lambda} = g_{VP} \vec{\epsilon}_\lambda(\vec{0}) \cdot (\vec{\epsilon}_{\lambda'}^*(\vec{k}) \times \vec{k}),$   
 $x_{D^*\bar{D}^*}^{\lambda'\lambda''\lambda} = 2g_{D^*\bar{D}^*} \vec{\epsilon}_\lambda(\vec{0}) \cdot \left( \vec{k} \times [\vec{\epsilon}_{\lambda'}^*(\vec{k}) \times \vec{\epsilon}_{\lambda''}^*(-\vec{k})] \right),$

$\langle \Omega | \mathcal{O}_{AB} | \eta_{c1} \rangle \approx 0 \quad \langle \Omega | \mathcal{O}_{\eta_{c1}} | AB \rangle \approx 0$

$t \qquad t' \qquad 0$

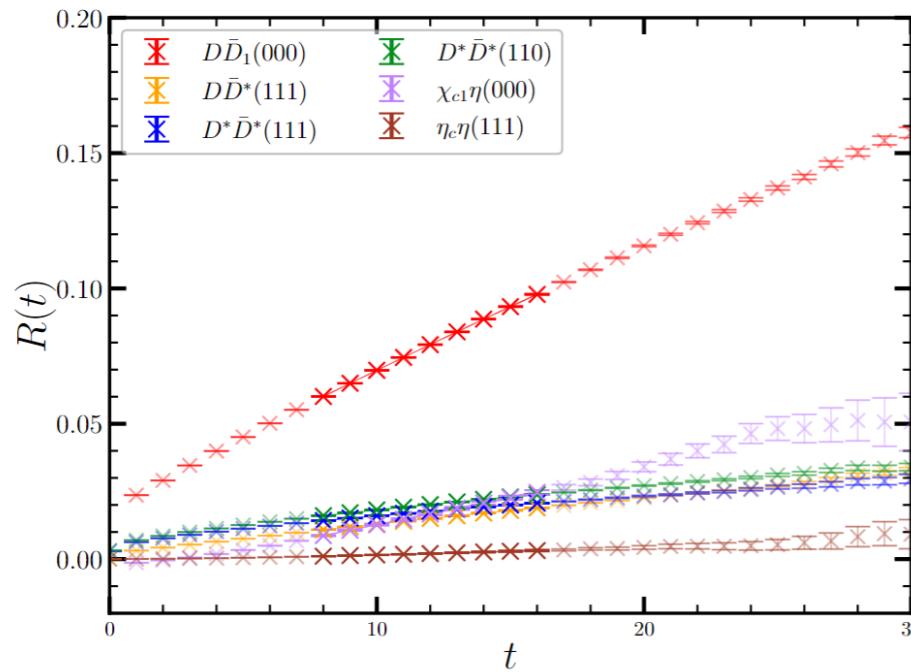
$\langle AB | \hat{H} | \eta_{c1} \rangle = ax$

# Lattice results

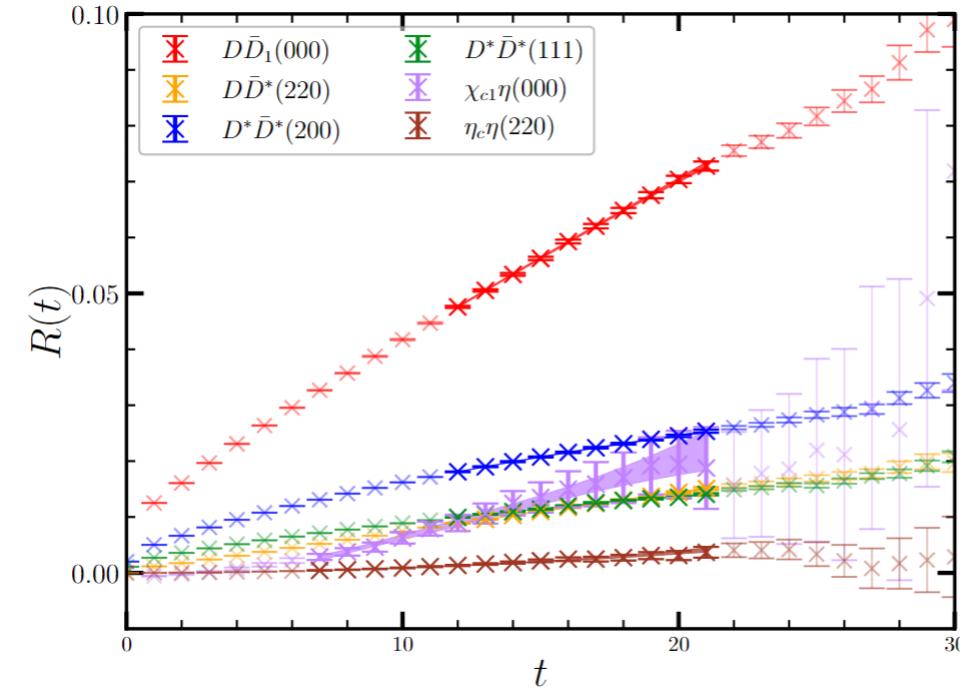
$$\begin{aligned}\mathcal{C}_{\eta_{c1}AB}(t) &= \langle \Omega | \mathcal{O}_{AB}(t) \mathcal{O}_{\eta_{c1}}^+(0) | \Omega \rangle \\ &= \langle \Omega | \mathcal{O}_{AB}(0) e^{-t a \bar{H}} \mathcal{O}_{\eta_{c1}}^+(0) | \Omega \rangle \\ &\rightarrow -axt e^{-t a \bar{E}} \langle \Omega | \mathcal{O}_{AB} | AB \rangle \langle \eta_{c1} | \mathcal{O}_{\eta_{c1}}^+ | \Omega \rangle\end{aligned}$$



transition amplitude
 $\frac{\mathcal{C}_{\eta_{c1}AB}(t)}{\sqrt{\mathcal{C}_{\eta_{c1}}(t)\mathcal{C}_A(t)\mathcal{C}_B(t)}} \rightarrow -(ax)t \left(1 + \frac{1}{24}(a\Delta t)^2\right)$



L16



L24

# Numerical results

Mode (AB)	$\hat{k}$ (IE)	$r_1$ ( $\times 10^{-3}$ )	$g_{AB}$	$g_{AB}$ (ave.)	$\Gamma_{AB}$ (MeV)
$D_1\bar{D}$	(0, 0, 0)(L16)	4.95(5)	4.27(5)	4.6(6)	258(133)
	(0, 0, 0)(L24)	3.10(26)	4.92(41)		
$D^*\bar{D}$	(1, 1, 1)(L16)	1.11(3)	8.35(21)	8.3(7)	88(18)
	(2, 2, 0)(L24)	0.78(7)	8.34(74)		
$D^*\bar{D}^*$	(1, 1, 1)(L16)	1.00(3)	3.44(12)	4.6(1.8)	150(118)
	(1, 1, 0)(L16)	1.15(4)	3.79(12)		
	(2, 0, 0)(L24)	1.05(9)	5.06(42)		
	(1, 1, 1)(L24)	0.67(7)	6.31(58)		
$\chi_{c1}\eta_{(2)}$	(0, 0, 0)(L16)	2.04(26)	1.31(2)	1.35(45)	–
	(0, 0, 0)(L24)	1.18(38)	1.39(45)		
$\eta_c\eta_{(2)}$	(1, 1, 1)(L16)	0.20(6)	0.62(18)	0.55(22)	–
	(2, 2, 0)(L24)	0.10(3)	0.47(12)		

$$\begin{aligned} \overline{|\mathcal{M}(\eta_{c1} \rightarrow AP)|^2} &= \frac{1}{3} g_{AP}^2 m_{\eta_{c1}} \left( 3 + \frac{k_{\text{ex}}^2}{m_A^2} \right), \\ \overline{|\mathcal{M}(\eta_{c1} \rightarrow PP)|^2} &= \frac{4}{3} g_{PP}^2 k_{\text{ex}}^2, \\ \overline{|\mathcal{M}(\eta_{c1} \rightarrow VP)|^2} &= \frac{2}{3} g_{VP}^2 k_{\text{ex}}^2, \\ \overline{|\mathcal{M}(\eta_{c1} \rightarrow D^* \bar{D}^*)|^2} &= \frac{4}{3} g^2 k_{\text{ex}}^2 \frac{m_{\eta_{c1}}^2}{m_{D^*}^2}. \end{aligned}$$

$$\Gamma_{AB} = \frac{1}{8\pi} \frac{\mathbf{k}_{\text{ex}}}{m_{\eta_{c1}}^2} |\overline{|\mathcal{M}(\eta_{c1} \rightarrow AB)|^2}|$$

- $D_1\bar{D}$  dominates.
- $D^*\bar{D}$  and  $D^*\bar{D}^*$  are important.

This observation is in striking contrast to the expectation of the flux-tube model.

# Numerical results

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Due to  $m_{\eta_{c1}} = 4.329(36)\text{GeV}$  from our lattice

$$\overline{|\mathcal{M}(\eta_{c1} \rightarrow AP)|^2} = \frac{1}{3} g_{AP}^2 m_{\eta_{c1}} \left( 3 + \frac{k_{\text{ex}}^2}{m_A^2} \right),$$

$$\overline{|\mathcal{M}(\eta_{c1} \rightarrow PP)|^2} = \frac{4}{3} g_{PP}^2 k_{\text{ex}}^2,$$

$$\overline{|\mathcal{M}(\eta_{c1} \rightarrow VP)|^2} = \frac{2}{3} g_{VP}^2 k_{\text{ex}}^2,$$

$$\overline{|\mathcal{M}(\eta_{c1} \rightarrow D^*\bar{D}^*)|^2} = \frac{4}{3} g^2 k_{\text{ex}}^2 \frac{m_{\eta_{c1}}^2}{m_{D^*}^2}.$$

$$\Gamma_{AB} = \frac{1}{8\pi} \frac{\mathbf{k}_{\text{ex}}}{m_{\eta_{c1}}^2} \overline{|\mathcal{M}(\eta_{c1} \rightarrow AB)|^2}$$

- $D_1\bar{D}$  dominates.
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## Results v.s. Flux-tube model

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	(2, 2, 0)(L24)	0.10(3)	0.47(12)		

### (Isgur& Paton 1985)

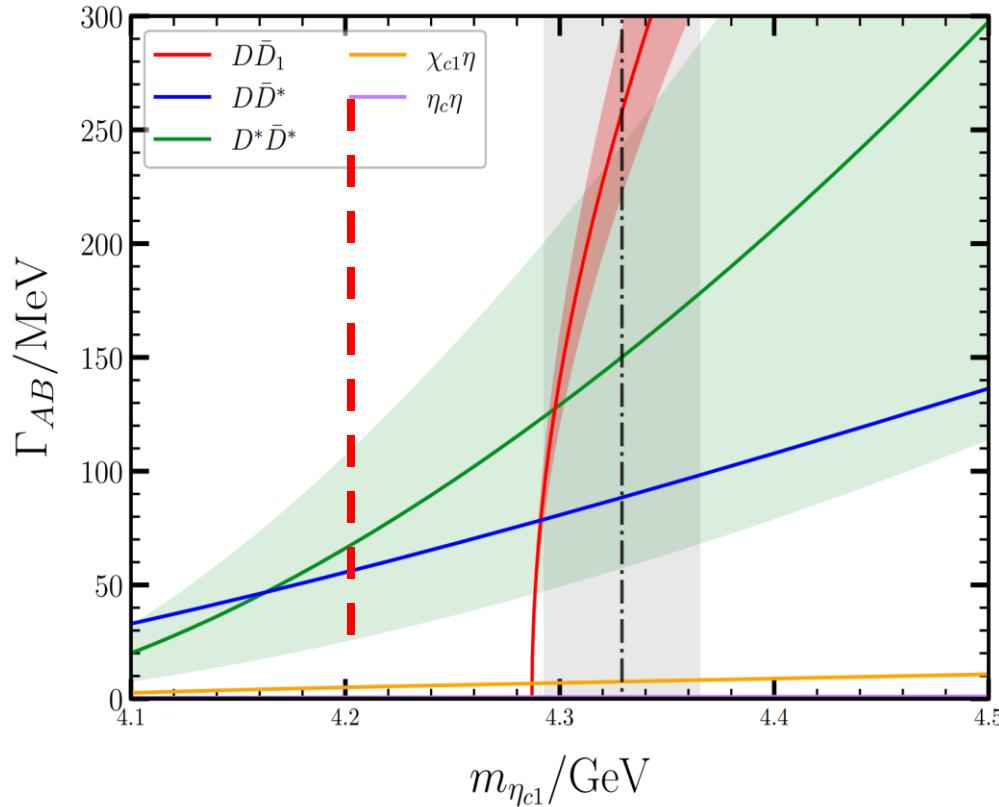
Flux tube model: light Isoscalar hybrid width  $\sim 150$  MeV

$c\bar{c}$  Isoscalar hybrid width  $\sim 30$  MeV

$1^{-+}$	$D^*D$	P	.5	.1
	$D^{**}(2^+)D$	D	—	.5
	$D^{**}(1_L^+)D$	S	—	1.2
		D	—	2.5
	$D^{**}(1_H^+)D$	S	—	25
		D	—	0
	$\Gamma$		.5	29

This observation is in striking contrast to the expectation of the flux-tube model.

## Conclusion



The  $m_{\eta_{c1}}$ -dependence of partial decay widths

$$|\mathbf{D}^* \bar{\mathbf{D}}^* \rangle_{(C=+)}^{(I=0)} = \frac{1}{\sqrt{2}} (|\mathbf{D}^{*+} \bar{\mathbf{D}}^{*-} \rangle + |\mathbf{D}^{0*} \bar{\mathbf{D}}^{0*} \rangle)_{(L=1)}^{(S=1)}$$

$L + S = \text{even}$

- We suggest LHCb, BelleII and BESIII to search for  $\eta_{c1}$  in  $D^* \bar{D}$  and  $D^* \bar{D}^*$  systems !

- For  $m_{\eta_{c1}} = 4329(36)$  MeV, we have

$$\Gamma_{D_1 \bar{D}} = 258(133) \text{ MeV}$$

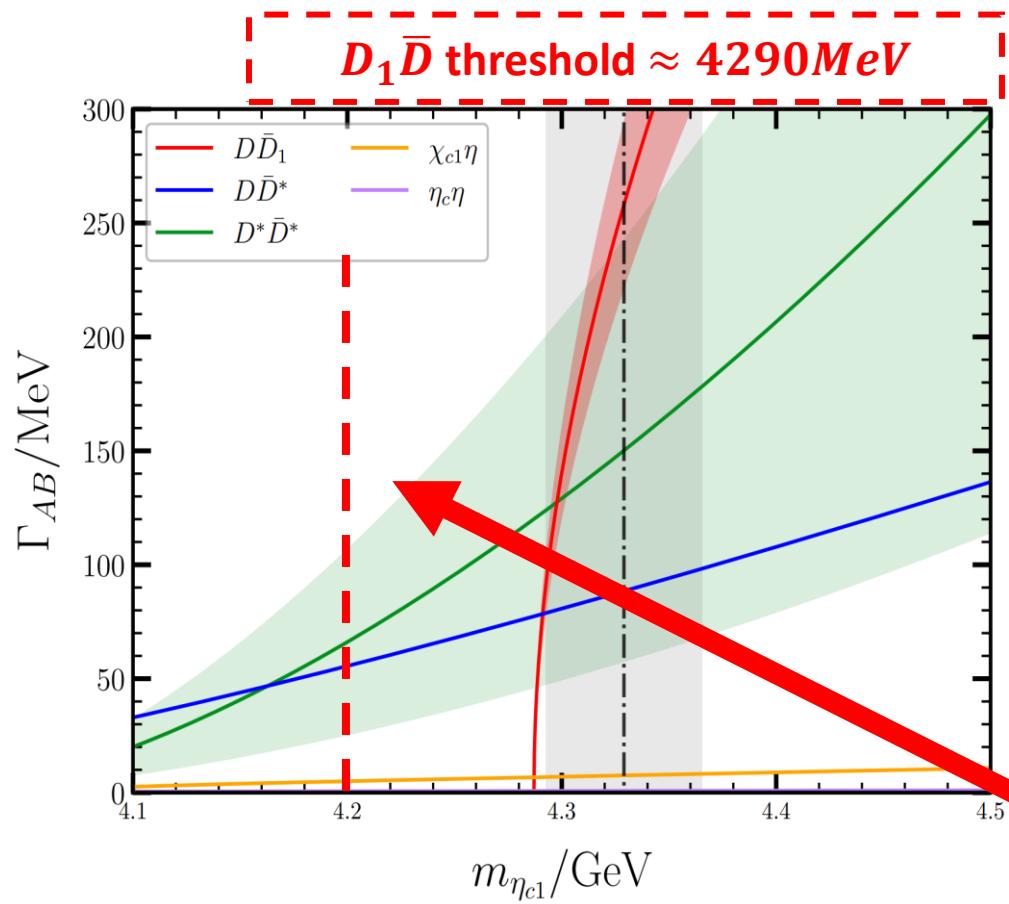
$$\Gamma_{D^* \bar{D}^*} = 150(118) \text{ MeV}$$

$$\Gamma_{D \bar{D}^*} = 88(18) \text{ MeV}$$

$$\Gamma_{\chi c1\eta} = \sin^2 \theta \cdot 44(29) \text{ MeV}$$

$$\Gamma_{\eta c\eta'} = \cos^2 \theta \cdot 0.93(77) \text{ MeV}$$

- Given the mass above,  $\eta_{c1}$  seems too wide to be identified easily in experiments.



$$|D^* \bar{D}^*\rangle_{(C=+)}^{(I=0)} = \frac{1}{\sqrt{2}} (|D^{*+} D^{*-}\rangle + |D^{0*} \bar{D}^{0*}\rangle)_{(L=1)}^{(S=1)}$$

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- For  $m_{\eta_{c1}} = 4329(36)$  MeV, we have

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$$\Gamma_{\eta_c \eta'} = \cos^2 \theta \cdot 0.93(77) \text{ MeV}$$

- Given the mass above,  $\eta_{c1}$  seems **too wide to be identified easily** in experiments.
- However,  $\Gamma_{\eta_{c1}}$  is very sensitive to  $m_{\eta_{c1}}$ .**
- If  $m_{\eta_{c1}} \sim 4.2 \text{ GeV}$ , then  $\Gamma_{\eta_{c1}} \sim 100 \text{ MeV}$ . The dominant decay channels are  $D^* \bar{D}$  and  $D^* \bar{D}^*$ .**
- Especially for  $D^* \bar{D}^*$ , the measurement of the polarization of  $D^*$  and  $\bar{D}^*$  will help distinguish a  $1^{-+}$  states from  $1^{--}$  states.**

- We suggest LHCb, BelleII and BESIII to search for  $\eta_{c1}$  in  $D^* \bar{D}$  and  $D^* \bar{D}^*$  systems !

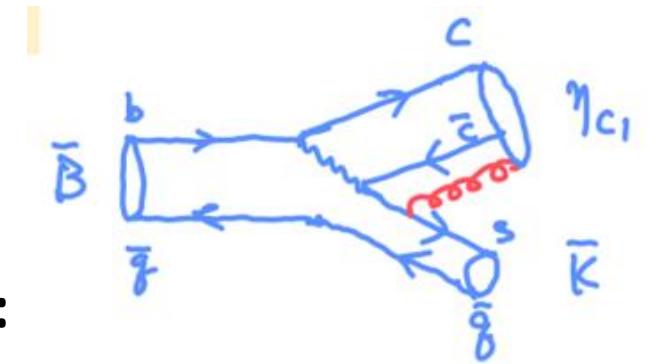
# Possible production in experiments

1)  $\eta_{c1}$  production on  $e^+e^-$  collider experiments: (**BESIII**)

$$e^+e^- \rightarrow \psi(nS) \rightarrow \gamma\eta_{c1}$$

2)  $\eta_{c1}$  production from  **$B$  meson weak decay**

$$B \rightarrow \eta_{c1}\bar{K} \quad (\text{LHCb and Belle}):$$



## Summary

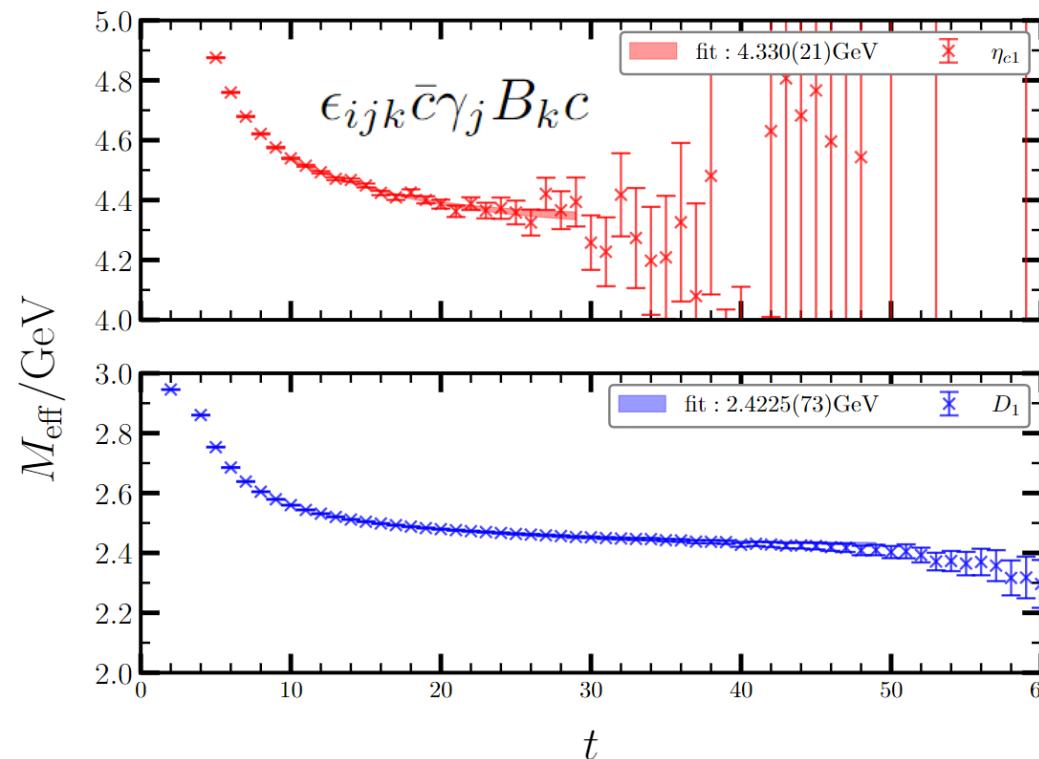
- ✓ We give the first Lattice QCD prediction of the partial decay widths of the charmoniumlike  $\eta_{c1}$ .
- ✓ Disfavor the results of the Flux-tube model (Disfavor the S-wave suppression).
- ✓ We provide the theoretical information for the experimental search for charmoniumlike hybrid  $\eta_{c1}$ .

Thank You

# Backup slides

# Meson spectrum on our lattice

$X$	$\omega$	$\eta_{(2)}$	$\eta_c$	$J/\psi$	$\chi_{c1}$	$D$	$D^*$	$D_1$	$\eta_{c1}$
$m_X(\text{L16})(\text{MeV})$	857(10)	715(10)	2975.0(3)	3099.1(2)	3559.8(1.7)	1882(1)	2023(1)	2423(7)	4330(21)
$m_X(\text{L24})(\text{MeV})$	845(7)	705(21)	2976.3(4)	3099.4(4)	3560.4(6.4)	1881(1)	2019(1)	2430(10)	4328(68)
$m_X(\text{PDG})(\text{MeV})$ [42]	782	547/958	2983	3097	3511	$\sim 1867$	$\sim 2008$	2420	-



Partial-wave operators

$$\begin{aligned} \mathcal{O}_{AB;JLSP}^M(\hat{k}) = & \sum_{M_L, M_S, M_A, M_B} \langle L, M_L; S, M_S | JM \rangle \langle S_A M_A; S_B M_B | S, M_S \rangle \\ & \times \sum_{R \in O_h} Y_{LM_L}^*(R \circ \vec{k}) \mathcal{O}_A^{M_A}(R \circ \vec{k}) \mathcal{O}_B^{M_B}(-R \circ \vec{k}), \end{aligned}$$

Mode (AB)	$E_{AB}$ (MeV)	$a_t \Delta$	$r_0$	$r_1 (\times 10^{-3})$	$r_3 (\times 10^{-7})$
L16 $(m_{\eta_{c1}} = 4330(21) \text{ MeV})$					
$D\bar{D}_1(000)$	4304(7)	$\sim 0.004$	0.02077(38)	4.95(5)	-5.1(1.3)
$D\bar{D}^*(111)$	4301(4)	$\sim 0.004$	0.00210(19)	1.11(3)	-2.2(8)
$D^*\bar{D}^*(111)$	4405(10)	$\sim -0.011$	0.00702(25)	1.00(3)	-4.3(9)
$D^*\bar{D}^*(110)$	4288(4)	$\sim 0.006$	0.00702(25)	1.15(4)	-4.1(9)
$\chi_{c1}\eta_{(2)}(000)$	4262(8)	$\sim 0.010$	-0.0076(18)	2.04(26)	-0.9(6.8)
$\eta_c\eta_{(2)}(111)$	4347(39)	$\sim -0.003$	-0.00057(39)	0.20(6)	1.6(1.6)
L24 $(m_{\eta_{c1}} = 4328(68) \text{ MeV})$					
$D\bar{D}_1(000)$	4310(10)	$\sim 0.003$	0.0110(26)	3.10(26)	-3.3(3.5)
$D\bar{D}^*(220)$	4361(5)	$\sim -0.005$	-0.00027(69)	0.78(7)	-1.07(97)
$D^*\bar{D}^*(200)$	4269(2)	$\sim 0.009$	0.00598(87)	1.05(9)	-3.1(1.2)
$D^*\bar{D}^*(111)$	4216(3)	$\sim 0.016$	0.00234(62)	0.67(7)	-2.50(83)
$\chi_{c1}\eta_{(2)}(000)$	4260(7)	$\sim 0.010$	-0.0057(27)	1.18(38)	-3.0(9.0)
$\eta_c\eta_{(2)}(220)$	4304(25)	$\sim 0.004$	-0.00037(17)	0.10(3)	2.49(77)

# Fitting stability

