Heavy Quarkonium Decays in the Refined Gribov-Zwanziger Theory

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Color-octet decays of heavy quarkonium

- Quarkonium decay rates are often sensitive to the $Q\bar{Q}g$ content of the quarkonium state. **Color-octet** contributions appear as expectation values of operators $\mathcal{O}_8(^{2S+1}L_J)$ in the NRQCD factorization formula.

  Bodwin, Braaten, Lepage, PRD51, 1125 (1995)

  χ_{cJ} decays : color-singlet and color-octet appear at leading order in v

$$\Gamma(\chi_{QJ} \to \mathrm{LH}) = \Gamma(Q\bar{Q}(^3P_J^{[1]}))\langle \chi_{QJ}|\mathcal{O}_1(^3P_J)|\chi_{QJ}\rangle + \Gamma(Q\bar{Q}(^3S_1^{[8]}))\langle \chi_{QJ}|\mathcal{O}_8(^3S_1)|\chi_{QJ}\rangle,$$

  η_Q decays : color-octet gives first correction to two-photon branching fraction

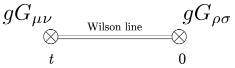
$$[\mathrm{Br}(\eta_Q \to \gamma\gamma)]^{-1} = \frac{\Gamma(Q\bar{Q}(^1S_0^{[1]}) \to gg)}{\Gamma(Q\bar{Q}(^1S_0^{[1]}) \to \gamma\gamma)} + \frac{\Gamma(Q\bar{Q}(^3S_1^{[8]}) \to gg)}{\Gamma(Q\bar{Q}(^1S_0^{[1]}) \to \gamma\gamma)} \frac{\langle \eta_Q|\mathcal{O}_8(^3S_1)|\eta_Q\rangle}{\langle \eta_Q|\mathcal{O}_1(^1S_0)|\eta_Q\rangle}$$

- Color-octet matrix elements are sensitive to the dynamics of the gluon in the $Q\bar{Q}g$ state, and can be used to probe the nonperturbative nature of QCD.

Color-octet decays of heavy quarkonium

- Color-octet matrix elements can be related to moments of QCD field-strength correlators, as first discovered in potential NRQCD.

Brambilla, Eiras, Pineda, Soto, Vairo, PRD67, 034018 (2003)



$$\mathcal{E}_n = \frac{1}{2N_c}\int_0^\infty dt\, t^n \langle g\boldsymbol{E}^{a,i}(t)\Phi_{ab}(t,0)g\boldsymbol{E}^{b,i}(0)\rangle,$$

$$\mathcal{B}_n = \frac{1}{2N_c}\int_0^\infty dt\, t^n \langle g\boldsymbol{B}^{a,i}(t)\Phi_{ab}(t,0)g\boldsymbol{B}^{b,i}(0)\rangle.$$

$c_F = 1 + O(\alpha_s)$

Then $\frac{m_c^2\langle\chi_{cJ}|\mathcal{O}_8(^3S_1)|\chi_{cJ}\rangle}{\langle\chi_{cJ}|\mathcal{O}_1(^3P_J)|\chi_{cJ}\rangle} = \frac{1}{3(d-1)N_c}\mathcal{E}_3$, $\frac{\langle\eta_c|\mathcal{O}_8(^3S_1)|\eta_c\rangle}{\langle\eta_c|\mathcal{O}_1(^1S_0)|\eta_c\rangle} = -\frac{c_F^2}{2m_c^2N_c}\mathcal{B}_1$.

- Usually, $\mathcal{E}_n$ and $\mathcal{B}_n$ are difficult to determine because they are
  - sensitive to the *low-energy dynamics of gluons*: perturbative QCD or OPE give scaleless moments which vanish in dimensional regularization.
  - *UV divergent*: must be renormalized perturbatively in DR.
    $\frac{\partial}{\partial \log\Lambda^2}\mathcal{E}_3 = \frac{6\alpha_s C_F}{\pi} + O(\alpha_s^2)$, $\mathcal{B}_1|_{\text{cutoff}} \sim \frac{2\alpha_s C_F}{\pi}\int^\Lambda dk\, k + O(\alpha_s^2)$.
- The **Refined Gribov-Zwanziger theory** provides a way to compute gluon correlators that is perturbatively renormalizable while preserving the nonperturbative dynamics of nonabelian gauge theories.

Dudal, Sorella, Vandersickel, Verschelde, PRD77, 071501 (2008)
Dudal, Gracey, Sorella, Vandersickel, Verschelde, PRD78, 065047 (2008)

Refined Gribov-Zwanziger Theory

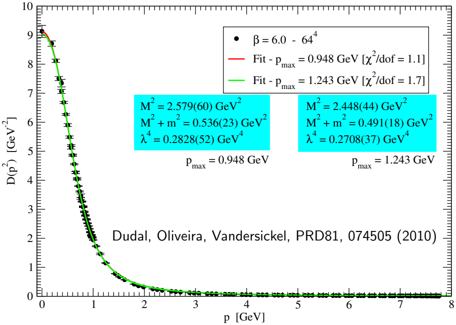
- Gribov (1977) : *Faddeev-Popov gauge-fixing is incomplete* because a gauge orbit can intersect with the gauge condition *more than once!* The solution is to restrict the functional integral in a region which intersects each orbit only once. This *modifies the gluon propagator*:
Gribov, NPB139, 1 (1978)
$$\frac{1}{k^2} \to \frac{k^2}{k^4+\lambda^4} = \frac{1}{2}\left(\frac{1}{k^2+i\lambda^2}+\frac{1}{k^2-i\lambda^2}\right),$$
The gluon is confined, as it does not have a Källén-Lehmann representation due to the imaginary "mass" square $\pm i\lambda^2$.
See Phys.Rept.520, 175 (2012) for a review
- Zwanziger showed that this can be incorporated into a local and renormalizable action now called the *Gribov-Zwanziger action*.
Zwanziger, NPB323, 513 (1989)
- More nonperturbative effects can be included in the GZ theory, such as *dimension-2 condensates*. This further *modifies the gluon propagator*:
$$\frac{1}{k^2} \to \frac{k^2+M^2}{k^4+(M^2+m^2)k^2+\lambda^4}$$
: complex poles in k^2 with a small real part
Dudal, Gracey, Sorella, Vandersickel, Verschelde, PRD78, 065047 (2008)
This is done in the mean-field approach and also results in a local and renormalizable action called the **Refined Gribov-Zwanziger** action.
Dudal, Sorella, Vandersickel, PRD84, 065039 (2011)

Features of the Refined Gribov-Zwanziger Theory

- Gluon propagator is modified by *dimensionful parameters*: recovers usual perturbative QCD at high energies, perturbative renormalization is possible while calculations are infrared finite.
- Tree-level calculation reproduces lattice measurement of gluon propagator. Parameters can be determined from fit to lattice data.

Dudal, Gracey, Sorella, Vandersickel, Verschelde, PRD78, 065047 (2008)

Renormalized Gluon Propagator - $\mu = 3$ GeV

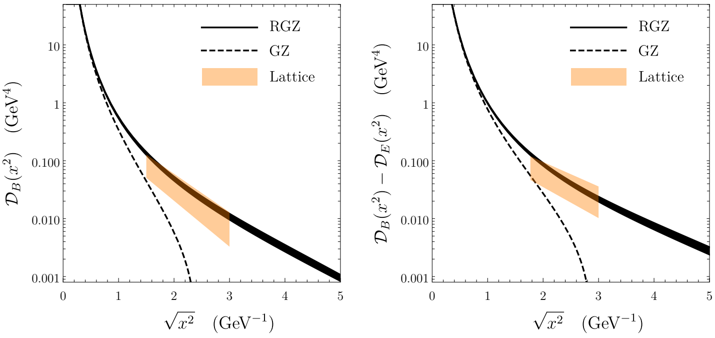


- Perturbative calculations are then expected to be valid as weak field expansions.
- Caveats: only in Euclidean space - timelike physics only. No Fermions.

Field-Strength Correlators

- Compute correlators $\mathcal{D}_B(\tau^2) = \frac{1}{(d-1)(d-2)} \langle g\boldsymbol{B}^{a,i}(\tau)\Phi_{ab}(\tau,0) g\boldsymbol{B}^{b,i}(0)\rangle$ and $\mathcal{D}_E(\tau^2) = \frac{1}{2(d-1)} \langle g\boldsymbol{E}^{a,i}(\tau)\Phi_{ab}(\tau,0) g\boldsymbol{E}^{b,i}(0)\rangle$ at tree level. From lattice QCD results we expect *exponentially decaying behaviors* $\sim e^{-\tau/\lambda_{\rm corr}}$ with fixed correlation length $\lambda_{\rm corr}$ at long distances.

Bali, Brambilla, Vairo, PLB421, 265, (1998)



- Only the **RGZ theory is compatible with lattice results**.

Dimensionless moment $\mathcal{E}_3$

Compute $\mathcal{E}_3$ at leading order in the RGZ theory. Use dimensional regularization in $d = 4 - 2\epsilon$ dimensions

$$\mathcal{E}_3 = \frac{3g^2 C_F}{2\pi^2}\left[\frac{1}{\epsilon} + \log(2\Lambda^2/M^2) - \frac{2}{3}\right] + \frac{3g^2 C_F}{2\pi^2}\mathrm{Re}\left[\frac{m^2 - M^2 - Q^2}{Q^2}\log\left(\frac{m^2 + M^2 - Q^2}{M^2}\right)\right] + O(\alpha_s^2, \epsilon)$$

Λ : $\overline{\mathrm{MS}}$ scale

$Q^2 \equiv i\sqrt{4\lambda^4 - (M^2 + m^2)^2}$

A UV pole appears as expected from NRQCD factorization.
After renormalization:

$$\mathcal{E}_3^{(\Lambda)} = \frac{3g^2 C_F}{2\pi^2}\left[\log(2\Lambda^2/M^2) - \frac{2}{3}\right] + \frac{3g^2 C_F}{2\pi^2}\mathrm{Re}\left[\frac{m^2 - M^2 - Q^2}{Q^2}\log\left(\frac{m^2 + M^2 - Q^2}{M^2}\right)\right] + O(\alpha_s^2)$$

in the $\overline{\mathrm{MS}}$ scheme.

Dimension-2 moments $\mathcal{E}_1$ and $\mathcal{B}_1$

A gauge-invariant definition of the dimension-2 condensate has been known:

Zwanziger, NPB345, 461, (1990)

$$\langle (gA)^2 \rangle_{\min} = \langle (gA)^2 \rangle|_{\partial\cdot A=0} = -\frac{1}{2}\langle (gG^a_{\mu\nu})(D^2)^{-1}(gG^a_{\mu\nu})\rangle + O((gG)^3).$$

This leads to $\langle (gA)^2 \rangle_{\min} = \frac{2N_c}{d-2}(\mathcal{E}_1 + \mathcal{B}_1) + O(\alpha_s^2)$.

$\mathcal{E}_1 = 0$ at tree level because it first appears from gluon vertices. Hence

$$\mathcal{B}_1 = \frac{d-2}{2N_c}\langle (gA)^2 \rangle_{\min}.$$

Explicit calculation reproduces this result. A finite result $\langle (gA)^2 \rangle_{\min} = -\frac{9}{13}\frac{N_c^2-1}{N_c}m^2$ has been obtained in the effective potential method after subtraction of a UV pole. From this we obtain

Verschelde, Knecht, Van Acoleyen, Vanderkelen, PLB516, 307, (2001)

$$\mathcal{B}_1 = -\frac{18C_F}{13N_c}m^2 + \frac{3g^2C_Fm^2}{8\pi^2} + O(\alpha_s^2).$$

This result is valid in dimensional regularization due to appearance of a quadratic UV divergence.

Numerical Results for Moments

Use m, M, and λ determined from lattice gluon propagator.
Dudal, Oliveira, Vandersickel, PRD81, 074505 (2010)
Estimate higher order corrections from lattice measurements of field-strength correlators at long distances.
Bali, Brambilla, Vairo, PLB421, 265, (1998)
All results are from dimensional regularization and logarithmic divergences are renormalized in the $\overline{\mathrm{MS}}$ scheme.

- $\mathcal{E}_3^{(\Lambda=1\ \mathrm{GeV})} = 1.46^{+0.32}_{-0.39}$: appears in $\chi_{QJ} \to \mathrm{LH}$.
- $\mathcal{B}_1 = 1.03^{+0.15}_{-0.12}\ \mathrm{GeV}^2$: appears in $\eta_Q \to \mathrm{LH}$.

We can also compute

- $\mathcal{E}_1 = 0.01^{+0.13}_{-0.09}\ \mathrm{GeV}^2$: appears in $\chi_{QJ} \to \gamma\gamma$.

from comparison with RGZ and lattice QCD results at long distances, and

- $i\mathcal{E}_2 = -0.28^{+0.18}_{-0.14}\ \mathrm{GeV}$: appears in corrections to J/ψ, η_c, Υ, η_b wavefunctions at the origin

from straightforward computation in the RGZ theory.

Numerical Results for Moments

Comparison with lattice and phenomenology based determinations:

moment	**RGZ Theory** **This work**	lattice Based on Bali, Brambilla, Vairo PLB421, 265, (1998)	phenomenology Brambilla, Chung, Müller, Vairo JHEP04(2020)095
$\mathcal{E}_3^{(\Lambda=1\ \text{GeV})}$	$1.46^{+0.32}_{-0.39}$	$1.61^{+1.26}_{-1.29}$	$2.05^{+0.94}_{-0.65}$
$\mathcal{B}_1$ (GeV^2)	$1.03^{+0.15}_{-0.12}$	$0.35^{+0.13}_{-0.09}$	-
$\mathcal{E}_1$ (GeV^2)	$0.01^{+0.13}_{-0.09}$	$-0.08^{+0.19}_{-0.21}$	-0.20 ± 0.91
$i\mathcal{E}_2$ (GeV)	$-0.28^{+0.18}_{-0.14}$	$-0.41^{+0.39}_{-0.42}$	$0.77^{+1.30}_{-1.21}$

Overall RGZ results have smaller uncertainties.
Results are consistent except for $\mathcal{B}_1$. Large $\mathcal{B}_1$ from RGZ comes from $\langle (gA)^2 \rangle_{\text{min}}$, which is necessary for low-energy behavior of gluon propagator.
Lattice results from long-distance behaviors only, no scheme/scale dependence can be determined.

χ_{cJ} decays

$\Gamma(\chi_{cJ} \to \mathrm{LH})$ in units of MeV :

Decay	RGZ Theory (This work)	PDG
$\chi_{c0} \to \mathrm{LH}$	$13.5^{+0.1+6.6}_{-0.1-3.0} \pm 4.1$	10.6 ± 0.6
$\chi_{c1} \to \mathrm{LH}$	$0.41^{+0.10+0.10}_{-0.12-0.07} \pm 0.12$	0.55 ± 0.04
$\chi_{c2} \to \mathrm{LH}$	$2.15^{+0.10+0.00}_{-0.12-0.19} \pm 0.65$	1.60 ± 0.09

uncertainties from $\mathcal{E}_3$, scale, v^2 corrections

Uncertainties from the color-octet matrix element are small.

χ_{bJ} decays

$\Gamma(\chi_{bJ}(1P) \to \text{LH})$ in units of MeV : no measurements yet, but can be compared with theory calculations of $\Gamma(\chi_{bJ} \to \gamma\Upsilon)$ and measurements of $\text{Br}(\chi_{bJ} \to \gamma\Upsilon)$.

Decay	RGZ Theory	Segovia, Steinbeißer, Vairo Using $\Gamma(\chi_{bJ} \to \gamma\Upsilon)$ from weakly coupled pNRQCD PRD99, 074011 (2019)
$\chi_{b0}(1P) \to \text{LH}$	$0.762^{+0.007+0.320}_{-0.009-0.066} \pm 0.076$	1.4 ± 0.2
$\chi_{b1}(1P) \to \text{LH}$	$0.052^{+0.007+0.035}_{-0.009-0.006} \pm 0.005$	0.069 ± 0.009
$\chi_{b2}(1P) \to \text{LH}$	$0.139^{+0.007+0.004}_{-0.009-0.084} \pm 0.014$	0.195 ± 0.021

uncertainties from $\mathcal{E}_3$, scale, v^2 corrections

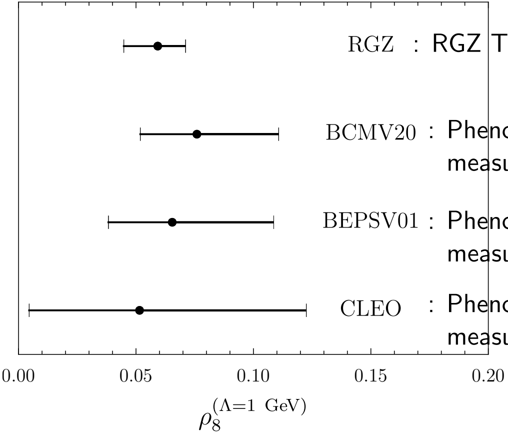
Determination based on $\Gamma(\chi_{bJ} \to \gamma\Upsilon)$ yields much larger decay widths, especially for $J = 0$ and 2. The differences are reduced if we compute $\Gamma(\chi_{bJ} \to \gamma\Upsilon)$ from the Cornell potential model.

See Zhang, Bai, Feng, Sang, Zhou, PRD108, 114030 (2023)

Color-octet matrix element in P-wave quarkonium decays

Comparison of color-octet matrix element determinations.

Define dimensionless ratio $\rho_8^{(\Lambda)} \equiv \frac{m_Q^2 \langle \chi_{QJ} | \mathcal{O}_8(^3S_1) | \chi_{QJ} \rangle^{(\Lambda)}}{\langle \chi_{QJ} | \mathcal{O}_1(^3P_J) | \chi_{QJ} \rangle}$.



RGZ : RGZ Theory (this work)

BCMV20 : Phenomenological determination from measurements of χ_{cJ} decay rates

Brambilla, Chung, Müller, Vairo, JHEP04(2020)095

BEPSV01 : Phenomenological determination from measurements of χ_{cJ} decay rates

Brambilla, Eiras, Pineda, Soto, Vairo, PRL88, 012003 (2001)

CLEO : Phenomenological determination from measurements of $\chi_{bJ}(1P) \to D_0 + X$

Evolved down from determination at $\Lambda = m_b$.

CLEO, PRD78, 092007 (2008)

based on theory calculation in Bodwin, Braaten, Lee, PRD76, 054001 (2007)

η_c decays

- Loop corrections to $c\bar{c}({}^1S_0^{[1]}) \to gg$ converge slowly due to a renormalon arising from the mixing $c\bar{c}({}^3S_1^{[8]}) \to c\bar{c}({}^1S_0^{[1]})g$.
  Bodwin and Chen, PRD64, 114008 (2001)
- The loop corrections can be resummed in the large-n_f limit: this trades the nonconverging perturbation series with the color-octet matrix element. As a result, the unknown color-octet matrix element was a major source of uncertainty in resummation calculations of η_c decays. RGZ result for $\frac{\langle \eta_c | \mathcal{O}_8({}^3S_1) | \eta_c \rangle}{\langle \eta_c | \mathcal{O}_1({}^1S_0) | \eta_c \rangle}$ compared to previous estimates :

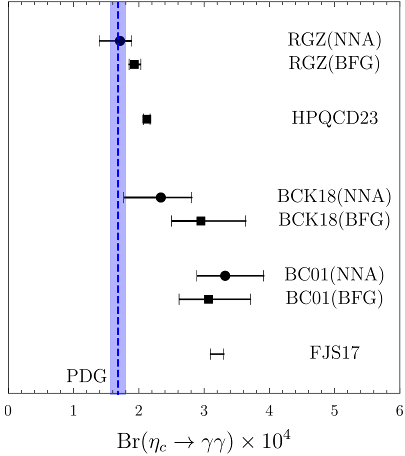
RGZ Theory This work	Petrelli et al NPB514, 245 (1998)	Bodwin and Chen PRD64, 114008 (2001)
$-\frac{c_F^2 \mathcal{B}_1}{2m_c^2 N_c} = -0.10 \pm 0.01$	$\pm \frac{v^3}{2N_c} \approx \pm 0.028$	$\pm \frac{v^3 C_F}{\pi N_c} \approx \pm 0.023$

- Central value from RGZ theory exceeds previously estimated ranges, but the uncertainty is much smaller.

η_c decays

Compute two-photon branching fraction from

$$[\mathrm{Br}(\eta_c \to \gamma\gamma)]^{-1} = \frac{\Gamma(c\bar{c}(^1S_0^{[1]}) \to gg)}{\Gamma(c\bar{c}(^1S_0^{[1]}) \to \gamma\gamma)} + \frac{\Gamma(c\bar{c}(^3S_1^{[8]}) \to gg)}{\Gamma(c\bar{c}(^1S_0^{[1]}) \to \gamma\gamma)} \frac{\langle \eta_c | \mathcal{O}_8(^3S_1) | \eta_c \rangle}{\langle \eta_c | \mathcal{O}_1(^1S_0) | \eta_c \rangle}$$



: RGZ Theory (this work)

: Lattice QCD with relativistic charm
PRD108, 014513 (2023)

: Resummation with two-loop correction
Brambilla, Chung, Komijani, PRD98, 114020 (2018)

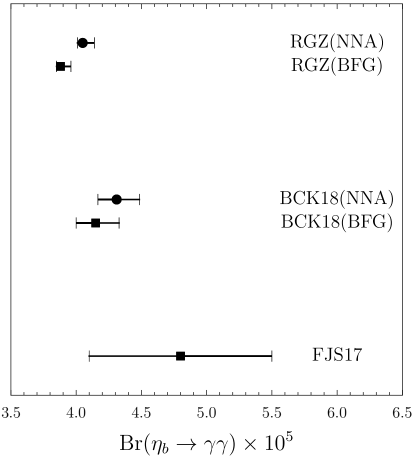
: Resummation with one-loop correction
Bodwin and Chen, PRD64, 114008 (2001)

: Fixed-order two-loop calculation
Feng, Jia, Sang, PRL119, 252001 (2017)

η_b decays

Compute two-photon branching fraction from

$$[\mathrm{Br}(\eta_b \to \gamma\gamma)]^{-1} = \frac{\Gamma(b\bar{b}(^1S_0^{[1]}) \to gg)}{\Gamma(b\bar{b}(^1S_0^{[1]}) \to \gamma\gamma)} + \frac{\Gamma(b\bar{b}(^3S_1^{[8]}) \to gg)}{\Gamma(b\bar{b}(^1S_0^{[1]}) \to \gamma\gamma)} \frac{\langle \eta_b | \mathcal{O}_8(^3S_1) | \eta_b \rangle}{\langle \eta_b | \mathcal{O}_1(^1S_0) | \eta_b \rangle}$$



: RGZ Theory (this work)

: Resummation with two-loop correction

Brambilla, Chung, Komijani, PRD98, 114020 (2018)

: Fixed-order two-loop calculation

Feng, Jia, Sang, PRL119, 252001 (2017)

Conclusions

- Heavy quarkonium decay rates are sensitive to *nonperturbative low-energy dynamics* of gluons through *color-octet* contributions. They can be computed through moments of gluon field-strength correlators which must be determined nonperturbatively.
- **Refined Gribov-Zwanziger** theory provides a way to compute gluon field-strength correlators including *nonperturbative effects* while allowing for perturbative renormalization in dimensional regularization.
- RGZ theory can give color-octet matrix elements for P-wave decays that are *much more precise* than phenomenological determinations.
- The dimension-2 condensate in the RGZ theory leads to the prediction that the effect of *chromomagnetic transition in η_c decays* are much larger than previous estimates. This also helps improve agreement with experiment.