EPOS4 and Quarkonium

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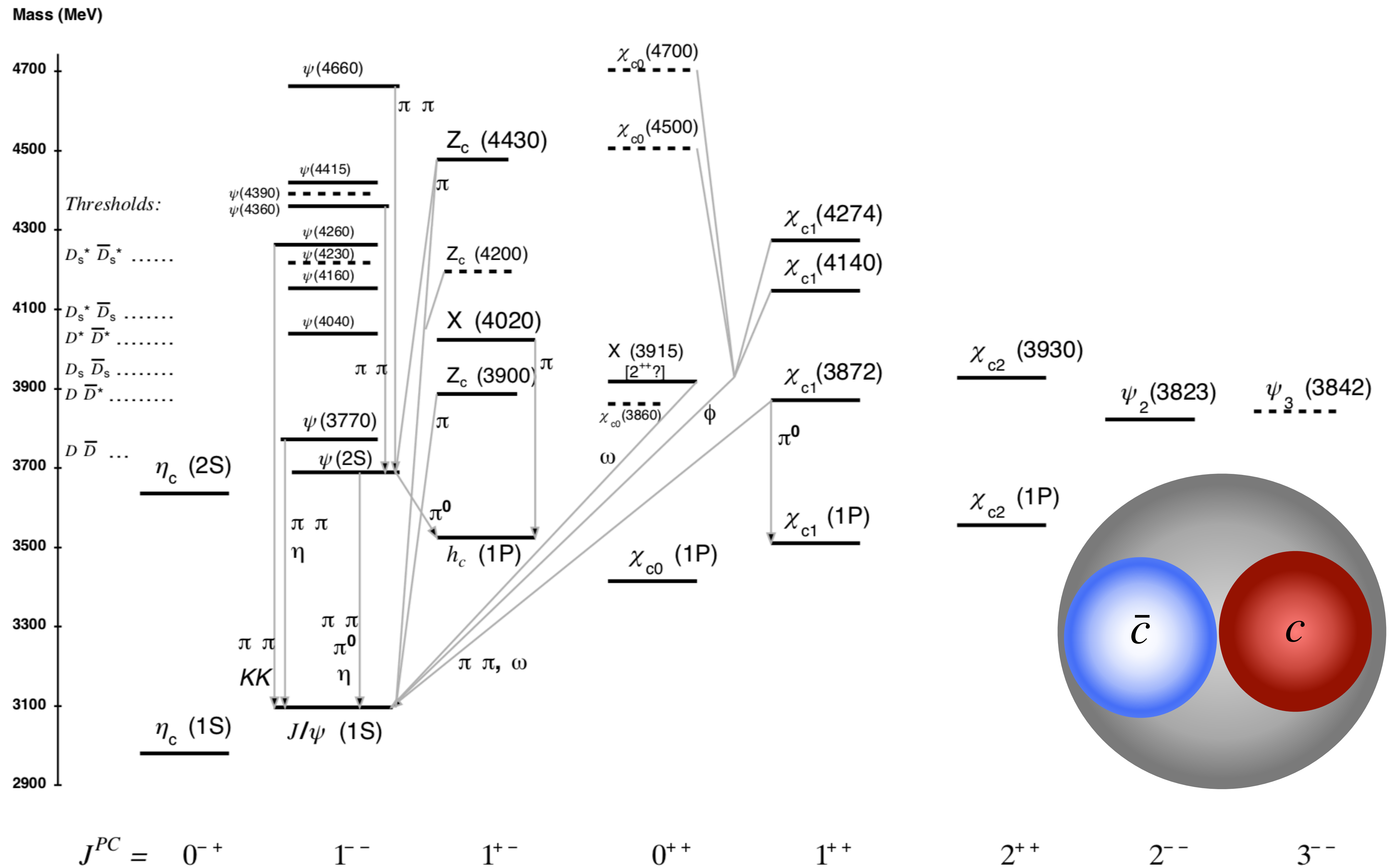


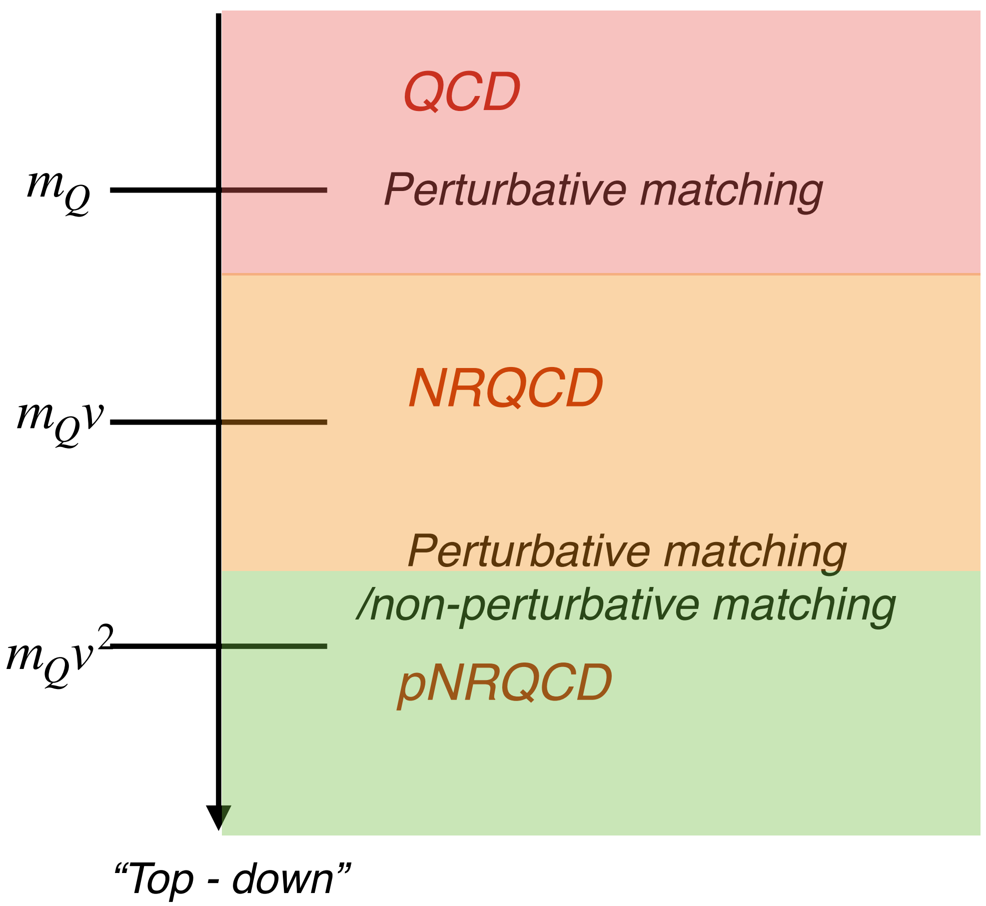
- *Static properties of charmonium, bottomonium, B_c in vacuum*
- *Charmonium, bottomonium, B_c production in pp collisions with EPOS4.*
- *Quarkonium production in AA collisions with EPOS4.*

Quarkonium static properties in a vacuum

From QCD to the potential model

$$m_c \sim 1.5 \text{ GeV}, m_b \sim 4.7 \text{ GeV}$$

Separation of scales: $m_Q \gg m_Q v \gg m_Q v^2$



$$\begin{aligned}\mathcal{L}_{pNRQCD} &= \int d^3r \mathrm{Tr}\Big[S^\dagger(i\partial_0 - H_S)S + O^\dagger(i\partial_0 - H_O)O\Big] \\ &+ V_A(r)\mathrm{Tr}[O^\dagger \mathbf{r}\cdot g\mathbf{E}S + S^\dagger \mathbf{r}\cdot g\mathbf{E}O] \\ &+ \frac{V_B(r)}{2}\mathrm{Tr}[O^\dagger \mathbf{r}\cdot g\mathbf{E}O + O^\dagger O\mathbf{r}\cdot g\mathbf{E}] + \mathcal{L}'_g + \mathcal{L}'_l\end{aligned}$$

Singlet field S; Octet field O.

$$\begin{aligned}H_S &= \{c_1^s(r), \frac{\mathbf{p}^2}{2\mu}\} + c_2^s(r)\frac{\mathbf{P}^2}{2M} + V_S^{(0)} + \frac{V_S^{(1)}}{m_Q} + \frac{V_S^{(2)}}{m_Q^2}, \\ H_O &= \{c_1^o(r), \frac{\mathbf{p}^2}{2\mu}\} + c_2^o(r)\frac{\mathbf{P}^2}{2M} + V_O^{(0)} + \frac{V_O^{(1)}}{m_Q} + \frac{V_O^{(2)}}{m_Q^2}.\end{aligned}$$

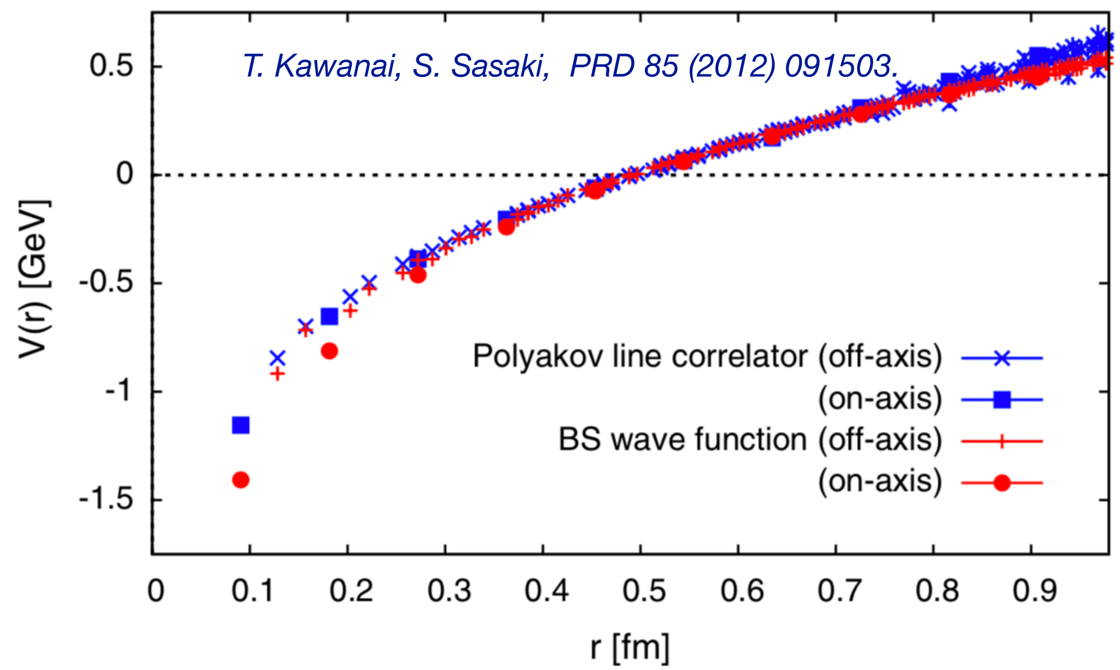
See for e.g.:
W.E. Caswell, G.P. Lepage, Phys. Lett. B 167 (1986) 437.
N. Brambilla, A. Pineda, J. Soto, A. Vairo, Nucl. Phys. B 566 (2000) 275.

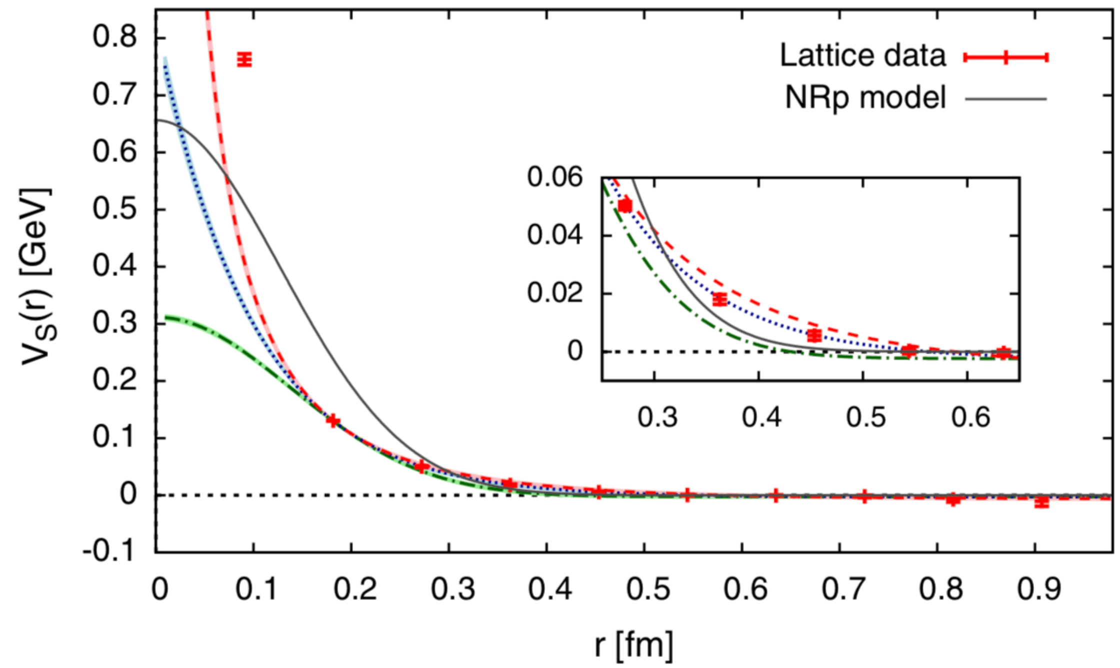
The potential model can be used to study the static properties of quarkonium !

Quarkonium static properties in a vacuum

Two-body Schroedinger equation: $\left[\frac{\hat{p}_1^2}{2m_1} + \frac{\hat{p}_2^2}{2m_2} + V(\mathbf{r}_1, \mathbf{r}_2, \mathbf{s}_1, \mathbf{s}_2)\right]\psi = E\psi$

Cornell potential + Spin-spin interaction





States	$\eta_c(1S)$	$J/\psi(1S)$	$h_c(1P)$	$\chi_c(1P)$	$\eta_c(2S)$	$\psi(2S)$	$h_c(2P)$	$\chi_c(2P)$
$M_{Exp.}$(GeV)	2.981	3.097	3.525	3.556	3.639	3.686	-	3.927
$M_{Th.}$(GeV)	2.967	3.102	3.480	3.500	3.654	3.720	3.990	4.000
$\langle r\rangle$(fm)	0.365	0.427	0.635	0.655	0.772	0.802	0.961	0.980

States	$\eta_b(1S)$	$\Upsilon(1S)$	$h_b(1P)$	$\chi_b(1P)$	$\eta_b(2S)$	$\Upsilon(2S)$	$\chi_b(2P)$	$\Upsilon(3S)$
$M_{Exp.}$(GeV)	9.398	9.460	9.898	9.912	9.999	10.023	10.269	10.355
$M_{Th.}$(GeV)	9.397	9.459	9.845	9.860	9.957	9.977	10.221	10.325
$\langle r\rangle$(fm)	0.200	0.214	0.377	0.387	0.465	0.474	0.603	0.680

JZ, *K. Zhou, S. Chen, P. Zhuang, PPNP.* 114 (2020) 103801.

Can explain the exp. mass very well!

Quarkonium static properties in a vacuum

Two-body Schroedinger equation:

$$\left[\frac{\hat{p}_1^2}{2m_1}+\frac{\hat{p}_2^2}{2m_2}+V(\mathbf{r}_1,\mathbf{r}_2,\mathbf{s}_1,\mathbf{s}_2)\right]\psi = E\psi$$

Cornell potential + Spin-spin interaction

Citation: P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

$\boxed{B_c^+}$

Quantum numbers sh

VALUE (MeV)		
6274.9 ± 0.8	**OUR AVERAG**	
6274.28 ± 1.40 ± 0.32		
6274.0 ± 1.8 ± 0.4		
6276.28 ± 1.44 ± 0.36		
6273.7 ± 1.3 ± 1.6		
6275.6 ± 2.9 ± 2.5		
6300 ± 14 ± 5		
6400 ± 390 ± 130		

• • • We do not use the followi

$I(J^P) = 0(0^-)$

I, J, P need confirmation.

Citation: P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

[Particle Data Group], Phys. Rev. D 98, no.3, 030001 (2018)

$\boxed{B_c(2S)^\pm}$

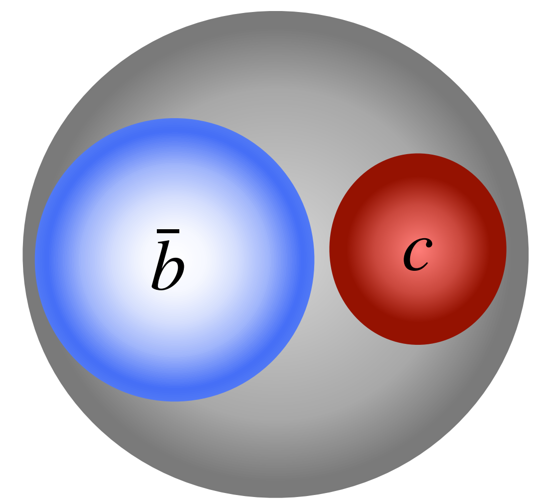
$I(J^P) = 0(0^-)$

OMITTED FROM SUMMARY TABLE

Quantum numbers neither measured nor confirmed.

$B_c(2S)^\pm$ MASS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*		*TECN*	*COMMENT*
6871.6±1.1 OUR AVERAGE					
6872.1±1.3±0.8	24	[1,2] AAIJ	19Y	LHCB	pp at 7, 8, 13 TeV
6871.0±1.4±0.8	51	[3,4] SIRUNYAN	19M	CMS	pp at 13 TeV
• • • We do not use the following data for averages, fits, limits, etc. • • •					
not seen		[5] AAIJ	18AL	LHCB	pp at 8 TeV
6842 ±4 ±5	57	[6,7] AAD	14AQ	ATLS	pp at 7, 8 TeV



B_c	$^1S_0$	$^3S_1$	$^1P_1$	$^3P_{0,1,2}$	2^1S_0	2^3S_1
$M_{Th.}$(GeV)	6.282	6.347	6.726	6.738	6.886	6.915
$M_{Exp.}$(GeV)	6.275	-	-	-	6.872	-

The mass of B_c can be predicted with the same parameters!

Quarkonium static properties in a vacuum

Two-body Dirac equation: *P. A. M. Dirac, Yeshiva University, New York, 1964*

$$\mathcal{S}_1\Psi = \left[\gamma_5\left(\gamma^\mu(p_\mu - A_\mu) + m + S\right)\right]_1 \Psi = 0,$$
$$\mathcal{S}_2\Psi = \left[\gamma_5\left(\gamma^\mu(p_\mu - A_\mu) + m + S\right)\right]_2 \Psi = 0.$$

Meson	J^P	M_E (GeV)	M_T (GeV)	D_R	r_{rms} (fm)
D^0	0^-	1.865	1.940	4.0%	0.41
D^{*0}	1^-	2.007	2.066	3.0%	0.47
D^+	0^-	1.870	1.940	3.8%	0.41
D^{*+}	1^-	2.010	2.066	2.8%	0.47
D_s	0^-	1.968	2.028	3.1%	0.40
D_s^*	1^-	2.112	2.157	2.1%	0.45
η_c	0^-	2.984	2.990	0.2%	0.32
$\eta_c(2S)$	0^-	3.637	3.609	−0.8%	0.63
h_{c1}	1^+	3.525	3.506	−0.5%	0.54
J/ψ	1^-	3.097	3.123	0.8%	0.37
$\psi(2S)$	1^-	3.686	3.701	0.4%	0.68
χ_{c0}	0^+	3.415	3.442	0.8%	0.48
χ_{c1}	1^+	3.511	3.504	−0.2%	0.53
χ_{c2}	2^+	3.556	3.519	−1.0%	0.56
B^-	0^-	5.279	5.326	0.5%	0.43
B^{-*}	1^-	5.325	5.371	0.9%	0.46
B^0	0^-	5.280	5.326	0.9%	0.43
B^{0*}	1^-	5.325	5.371	0.9%	0.46
B_s	0^-	5.367	5.408	0.8%	0.41
B_s^*	1^-	5.415	5.458	0.8%	0.44
η_b	0^-	9.399	9.378	−0.2%	0.18
$\eta_b(2S)$	0^-	9.999	9.964	−0.3%	0.44
h_{b1}	1^+	9.899	9.918	0.2%	0.38
$\Upsilon(1S)$	1^-	9.460	9.507	0.5%	0.22
$\Upsilon(2S)$	1^-	10.023	10.025	0.0%	0.47
χ_{b0}	0^+	9.859	9.878	0.2%	0.35
χ_{b1}	1^+	9.893	9.912	0.2%	0.37
χ_{b2}	2^+	9.912	9.929	0.2%	0.38

Taking Pauli reduction and scale transformation in center-of-mass frame———Schroedinger-like equation:

$$\left[p^2 + \Phi_{12}\right]\psi = b^2\psi$$

$$\begin{aligned}\Phi_{ij} = {} & 2m_{ij}S + S^2 + 2\epsilon_{ij}A - A^2 + \Phi_D + \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\Phi_{SS} \\ & + \mathbf{L}_{ij}\cdot(\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j)\Phi_{SO} + \mathbf{L}_{ij}\cdot(\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j)\Phi_{SOD} + i\mathbf{L}_{ij}\cdot(\boldsymbol{\sigma}_i\times\boldsymbol{\sigma}_j)\Phi_{SOX} \\ & + (\boldsymbol{\sigma}_i\cdot\hat{\mathbf{r}}_{ij})(\boldsymbol{\sigma}_j\cdot\hat{\mathbf{r}}_{ij})\mathbf{L}_{ij}\cdot(\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j)\Phi_{SOT} + (3(\boldsymbol{\sigma}_i\cdot\hat{\mathbf{r}}_{ij})(\boldsymbol{\sigma}_j\cdot\hat{\mathbf{r}}_{ij}) - \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)\Phi_T.\end{aligned}$$

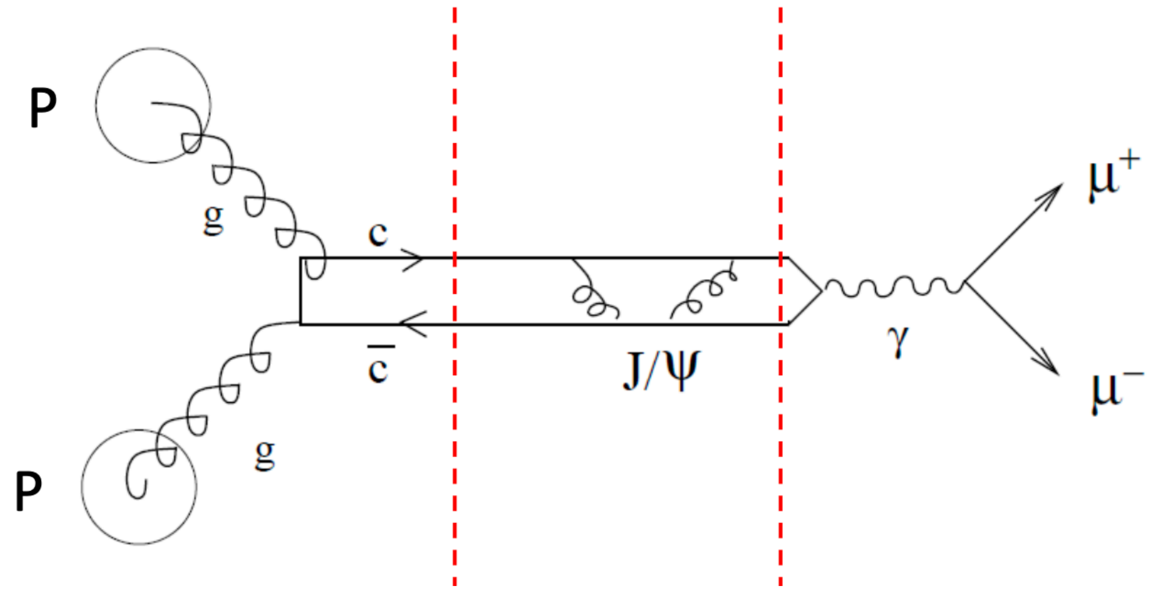
Darwin, spin-spin, spin-orbit and tensor terms are included self-consistently.

hyperfine interactions. see: Michael's talk, Tue. 14:55

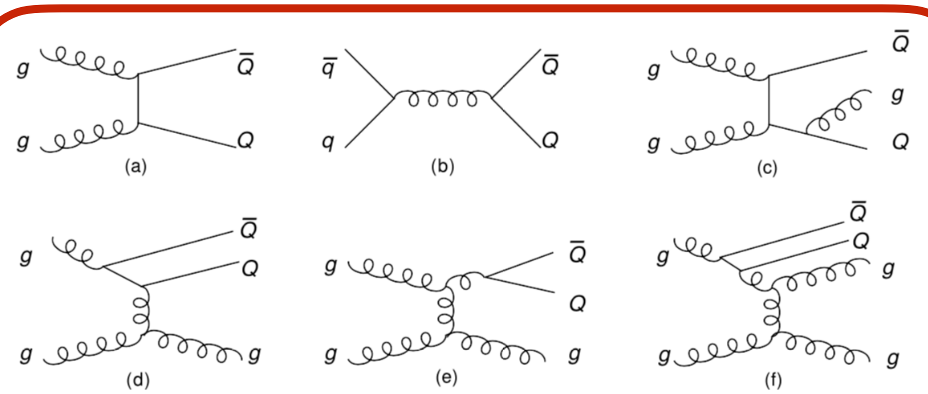
Gives the relativistic correction and hyperfine mass shift!

H. Crater, J. Yoo and C. Wong. PRD 79. 034011(2009)
S. Shi, X. Guo and P. Zhuang. PRD 88. 014021(2013)

Quarkonium production in pp collisions



Perturbative part *Non-perturbative part* *Decays*



LO, NLO,…

- ✦ *Color evaporation model (CEM)*

  R.Vogt, V. Cheung, Y. Ma, H. Fritzsch,…

- ✦ *Color singlet model (CSM)*

  C.H. Chang, E. Berger, D. Jones, R. Baier,…

- ✦ *Color octet model (COM)*

  G.T. Bodwin, E. Braaten, T.C. Yuan, G. Lepage,…

- ✦ *Non-relativistic QCD model (NRQCD)*

  Y. Ma. H.S. Shao, K.Chao, R. Venugopala, M.Butenschoen, B.Kniehl ,C.H. Chang, J. Wang…

- ✦ ***Wigner density matrix formalism***

  *T. Song, **JZ**, P.B. Gossiaux, E. Bratkovskaya, J. Aichelin,…*

Quarkonium production in pp collisions

Wigner density matrix formalism —-> density matrix projection

$$P_\Phi(t) = \mathrm{Tr}[\rho_\Phi \hat{\rho}^{(N)}]$$

density matrix of the quarkonium

density matrix of N heavy quarks and antiquarks system

In phase-space, the differential production probability:

$$\frac{dP_i}{d^3\mathbf{R}d^3\mathbf{P}} = \sum \int \frac{d^3r d^3p}{(2\pi)^6} W_i(\mathbf{r},\mathbf{p}) \prod_{j>2} \int \frac{d^3r_j d^3p_j}{(2\pi)^{3(N-2)}} W^{(N)}(\mathbf{r}_1,\mathbf{p}_1,\mathbf{r}_2,\mathbf{p}_2,\ldots,\mathbf{r}_N,\mathbf{p}_N).$$

Wigner density of the quarkonium

Wigner representation of the ensemble of N heavy quarks produced in a pp collision

JZ, *P.B. Gossiaux, T. Song, E. Bratkovskaya, J. Aichelin. arXiv: 2312.11349.*

D. Villar, ***JZ,*** *J. Aichelin, P.B. Gossiaux. Phys.Rev.C 107 (2023) 5, 054913*

T. Song, J. Aichelin, E. Bratkovskaya. PRC 96 (2017) 1, 014907.

T. Song, J. Aichelin, ***JZ,*** *P.B. Gossiaux, E. Bratkovskaya. PRC 108 (2023) 5, 054908*

Quarkonium Wigner function

The Wigner function of quarkonium can be constructed by their wave function.

$$\left[-\frac{1}{2\mu}\left(\frac{d^2}{dr^2}+\frac{2}{2}\frac{d}{dr}\right)+\frac{l(l+1)}{2\mu r^2}+V(r)\right]R_{nl}(r)=ER_{nl}(r),$$

Approximate the wave function by a 3-D isotropic harmonic oscillator wave function

$$\psi_{nlm}(r,\theta,\phi)=R_{nl}(r)Y_{l,m}(\theta,\phi) \qquad R_{nl}(r)=\left[\frac{2(n!)}{\sigma^3\Gamma(n+l+3/2)}\right]^{\frac{1}{2}}\left(\frac{r}{\sigma}\right)^l e^{-\frac{r^2}{2\sigma^2}}L_n^{l+1/2}\left(\frac{r^2}{\sigma^2}\right),$$

Widths are chosen to match the root-mean-square radius $\langle r^2\rangle$ of the real quarkonium wave function !

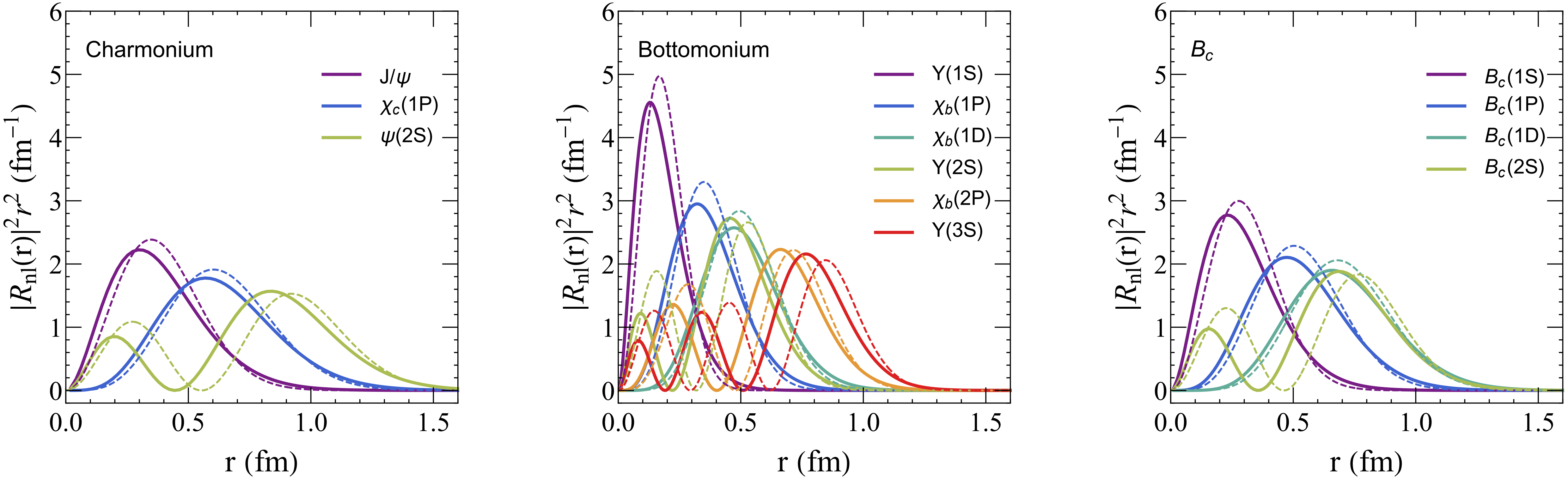
$$\langle r^2\rangle_{1S}=3\sigma^2/2,\quad \langle r^2\rangle_{1P}=5\sigma^2/2,\quad \langle r^2\rangle_{1D}=7\sigma^2/2,$$
$$\langle r^2\rangle_{2S}=7\sigma^2/2,\quad \langle r^2\rangle_{2P}=9\sigma^2/2,\quad \langle r^2\rangle_{3S}=11\sigma^2/2.$$

Real quarkonium wavefunction by solving the Schroeding eq.

	J/ψ	$\chi_c(1P)$	$\psi(2S)$	$\Upsilon(1S)$	$\chi_b(1P)$	$\chi_b(1D)$	$\Upsilon(2S)$	$\chi_b(2P)$	$\Upsilon(3S)$	$B_c(1S)$	$B_c(1P)$	$B_c(1D)$	$B_c(2S)$
$\langle r^2\rangle(\mathrm{fm}^2)$	0.182	0.453	0.714	0.042	0.153	0.284	0.236	0.410	0.520	0.115	0.316	0.542	0.497
$\sigma(\mathrm{fm})$	0.348	0.426	0.452	0.167	0.247	0.285	0.260	0.302	0.307	0.277	0.356	0.393	0.377

Parameter in the sotropic harmonic oscillator wave function

Quarkonium Wigner function



Dashed lines: 3-D isotropic harmonic oscillator

Solid lines: solve the two-body Schroedinger equation

The ground states and low lying excited states can be well reproduced by the 3-D isotropic harmonic oscillator, while the difference increases for higher excited states !

Quarkonium Wigner function

Then, the Wigner function can be constructed via a Wigner transformation in the spherical coordinate.

JZ, *P.B. Gossiaux, T. Song, E. Bratkovskaya, J. Aichelin. arXiv: 2312.11349.*

$$
\begin{aligned}
W_{1\mathrm{S}}(\mathbf{r},\mathbf{p}) &= 8e^{-\xi}, \\
W_{1\mathrm{P}}(\mathbf{r},\mathbf{p}) &= \frac{8}{3}e^{-\xi}\Big(2\xi-3\Big), \\
W_{1\mathrm{D}}(\mathbf{r},\mathbf{p}) &= \frac{8}{15}e^{-\xi}\Big(15+4\xi^2-20\xi+8[p^2r^2-(\mathbf{p}\cdot\mathbf{r})^2]\Big), \\
W_{2\mathrm{S}}(\mathbf{r},\mathbf{p}) &= \frac{8}{3}e^{-\xi}\Big(3+2\xi^2-4\xi-8[p^2r^2-(\mathbf{p}\cdot\mathbf{r})^2)]\Big), \\
W_{2\mathrm{P}}(\mathbf{r},\mathbf{p}) &= \frac{8}{15}e^{-\xi}\Big(-15+4\xi^3-22\xi^2+30\xi-8(2\xi-7)[p^2r^2-(\mathbf{p}\cdot\mathbf{r})^2]\Big), \\
W_{3\mathrm{S}}(\mathbf{r},\mathbf{p}) &= \frac{8}{315}e^{-\xi}\Big(315+42\xi^4-336\xi^3+924\xi^2-840\xi \\
&- [2009+32p^2r^2+336r^4/\sigma^4-1400r^2/\sigma^2-896p^2\sigma^2+224p^4\sigma^4][p^2r^2-(\mathbf{p}\cdot\mathbf{r})^2] \\
&- [686+608p^2r^2+112r^2/\sigma^2-896p^2\sigma^2+224p^4\sigma^4-672(\mathbf{p}\cdot\mathbf{r})^2](\mathbf{p}\cdot\mathbf{r})^2\Big),
\end{aligned}
$$

$$\xi = \frac{r^2}{\sigma^2} + p^2\sigma^2.$$

*Wigner function of excited states depends not only on the |**r**| and |**p**|, but also the angle between them.*

Quarkonium production in pp collisions

Wigner density matrix formalism —-> density matrix projection

$$P_\Phi(t) = \mathrm{Tr}[\rho_\Phi \hat{\rho}^{(N)}]$$

density matrix of the quarkonium

density matrix of N heavy quarks and antiquarks system

In phase-space, the differential production probability:

$$\frac{dP_i}{d^3\mathbf{R}d^3\mathbf{P}} = \sum \int \frac{d^3rd^3p}{(2\pi)^6} W_i(\mathbf{r},\mathbf{p}) \prod_{j>2} \int \frac{d^3r_jd^3p_j}{(2\pi)^{3(N-2)}} W^{(N)}(\mathbf{r}_1,\mathbf{p}_1,\mathbf{r}_2,\mathbf{p}_2,...,\mathbf{r}_N,\mathbf{p}_N).$$

Wigner density of the quarkonium

Wigner representation of the ensemble of N heavy quarks produced in a pp collision

Assume that the unknown quantal N-body Wigner density can be replaced by the average of classical phase space distributions: $W^{(N)} \approx \langle W^{(N)}_{\mathrm{classical}} \rangle$*. Classical momentum space distribution of the heavy quarks can be provided by EPOS4, PYTHIA, …. The relative distance in their center-of-mass frame is given by a Gaussian distribution.*

$$W^{(2)}(\mathbf{r},\mathbf{p}) \sim r^2 \exp\left(-\frac{r^2}{2\sigma^2_{\mathrm{Q\bar{Q}}}}\right) f^{\mathrm{EPOS4}}_{\mathrm{Q\bar{Q}}}(\mathbf{p}),$$

Only one "free" parameter $\sigma_{Q\bar{Q}}$

$$\sigma_{Q\bar{Q}} \sim 1/p_r$$

—> EPOS4

EPOS4

EPOS4: A Monte Carlo tool for simulating high-energy scatterings

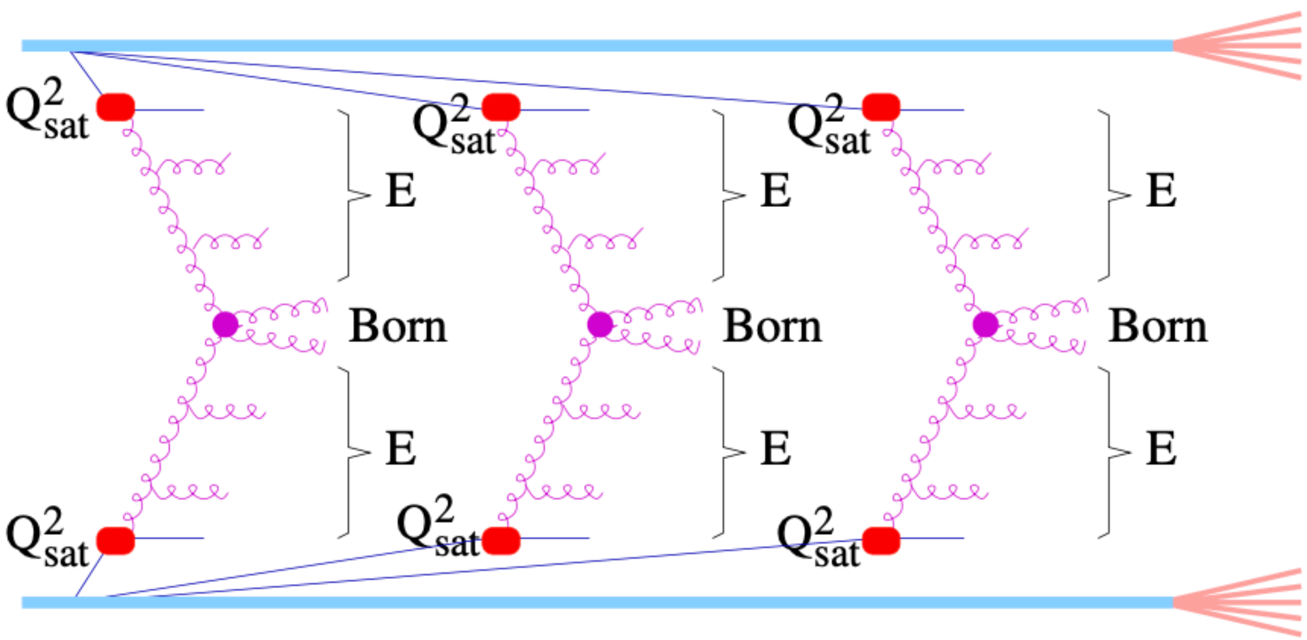
https://klaus.pages.in2p3.fr/epos4/

*An abbreviatation of **E**nergy conserving quantum mechanical multiple scattering approach, based on **P**arton (parton ladders), **O**ff-shell remnants, and **S**aturation of parton ladders.*

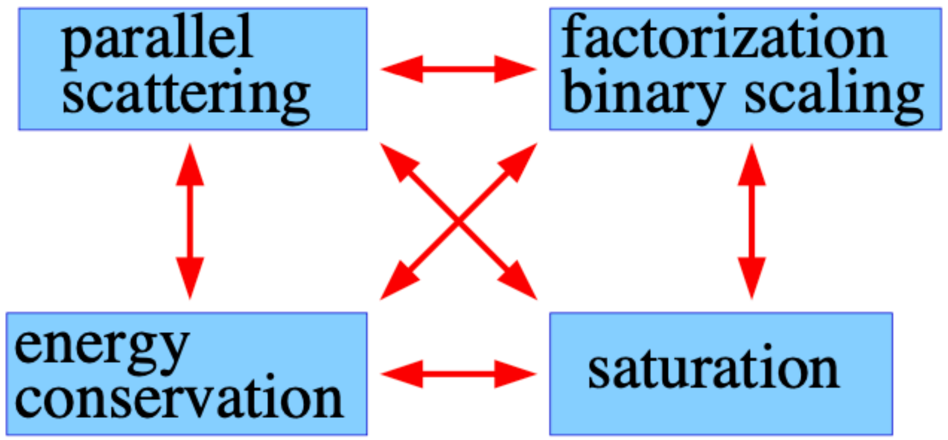
K. Werner. PRC 108 (2023) 6, 064903

K. Werner, B. Guiot, PRC 108 (2023) 3, 034904

K. Werner, PRC 109 (2024) 1, 014910



e.g. three parallel scatterings



S-matrix theory (to deal with parallel scatterings happens in high energy collisions)

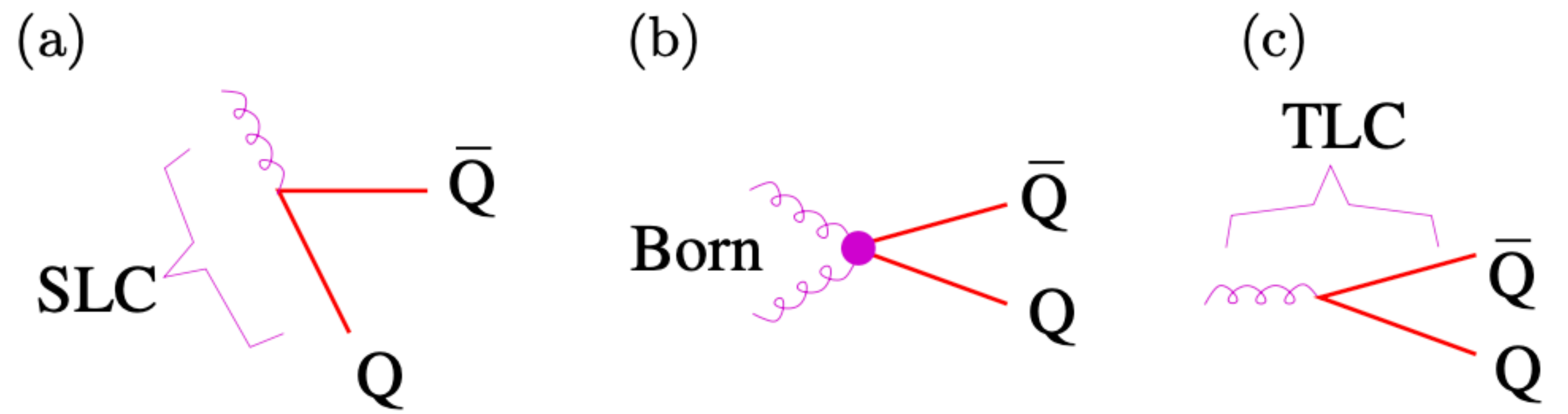
For each one we have a parton evolution according to some evolution function, such as DGLAP .

Consistently accommodate these four crucial concepts is realized in the EPOS4!

EPOS4: heavy quark production

K. Werner, B. Guiot, Phys.Rev.C 108 (2023) 3, 034904

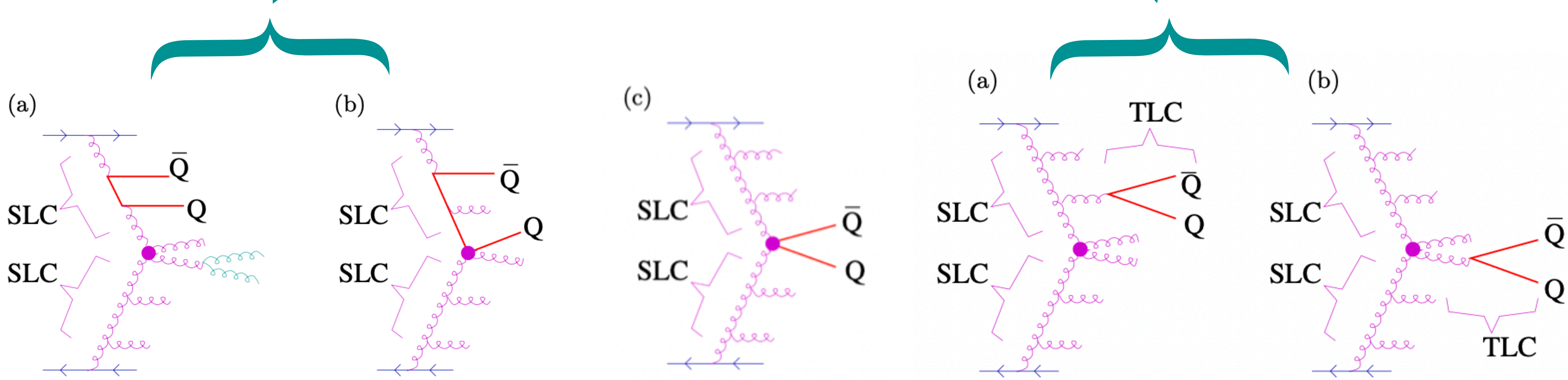
Heavy quarks are produced initialy via:



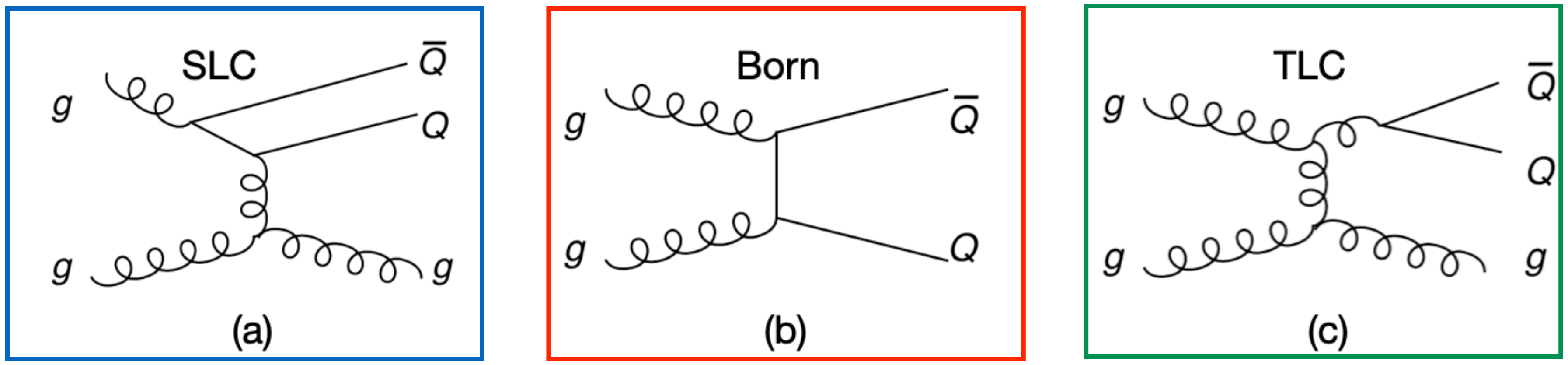
$$g + g \rightarrow Q + \bar{Q}$$
$$q + \bar{q} \rightarrow Q + \bar{Q}$$

Space-like cascade

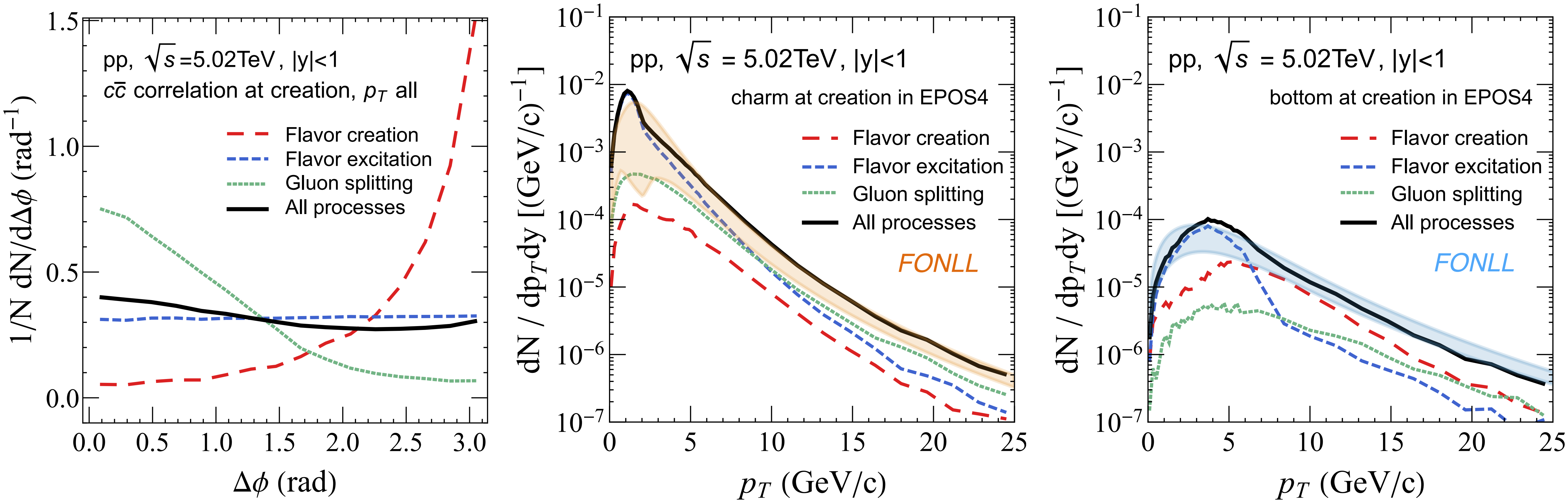
Time-like cascade



EPOS4: heavy quark production



Flavor excitation *Flavor creation* *Gluon splitting*



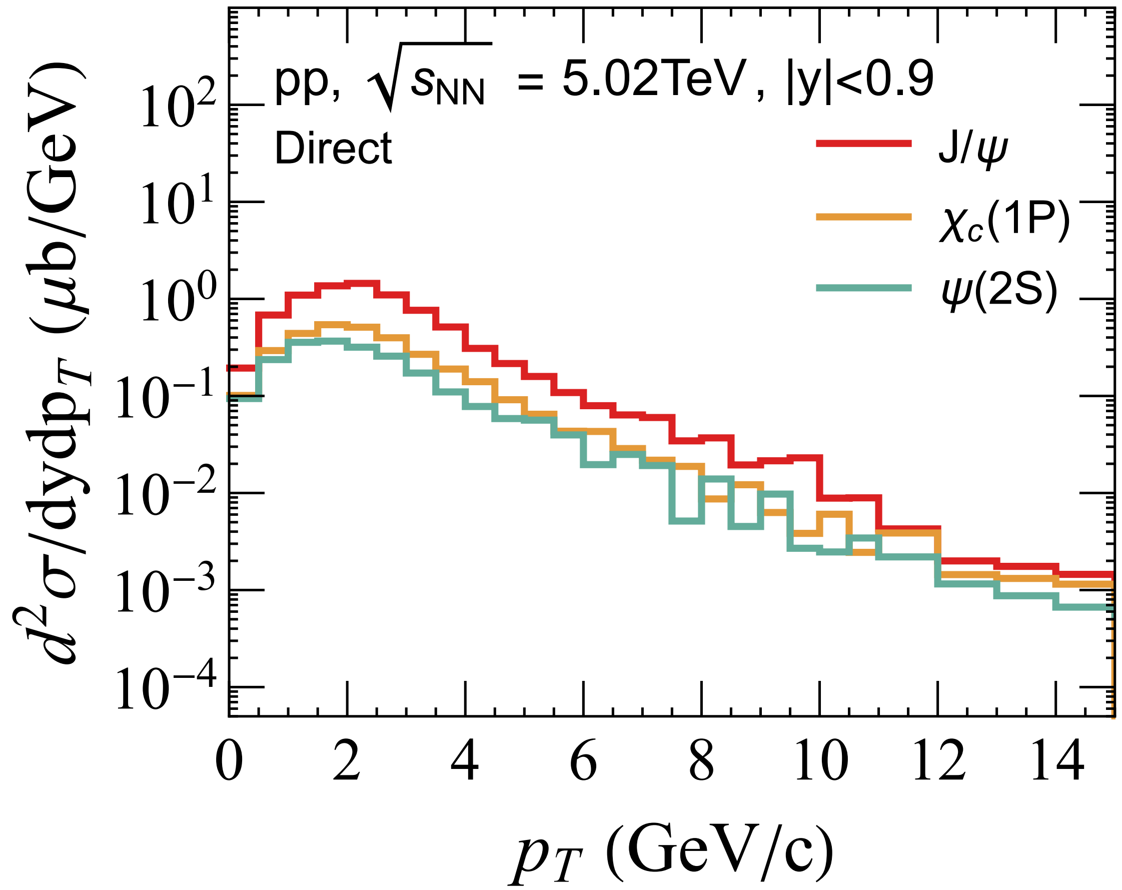
Different correlations from different processes.

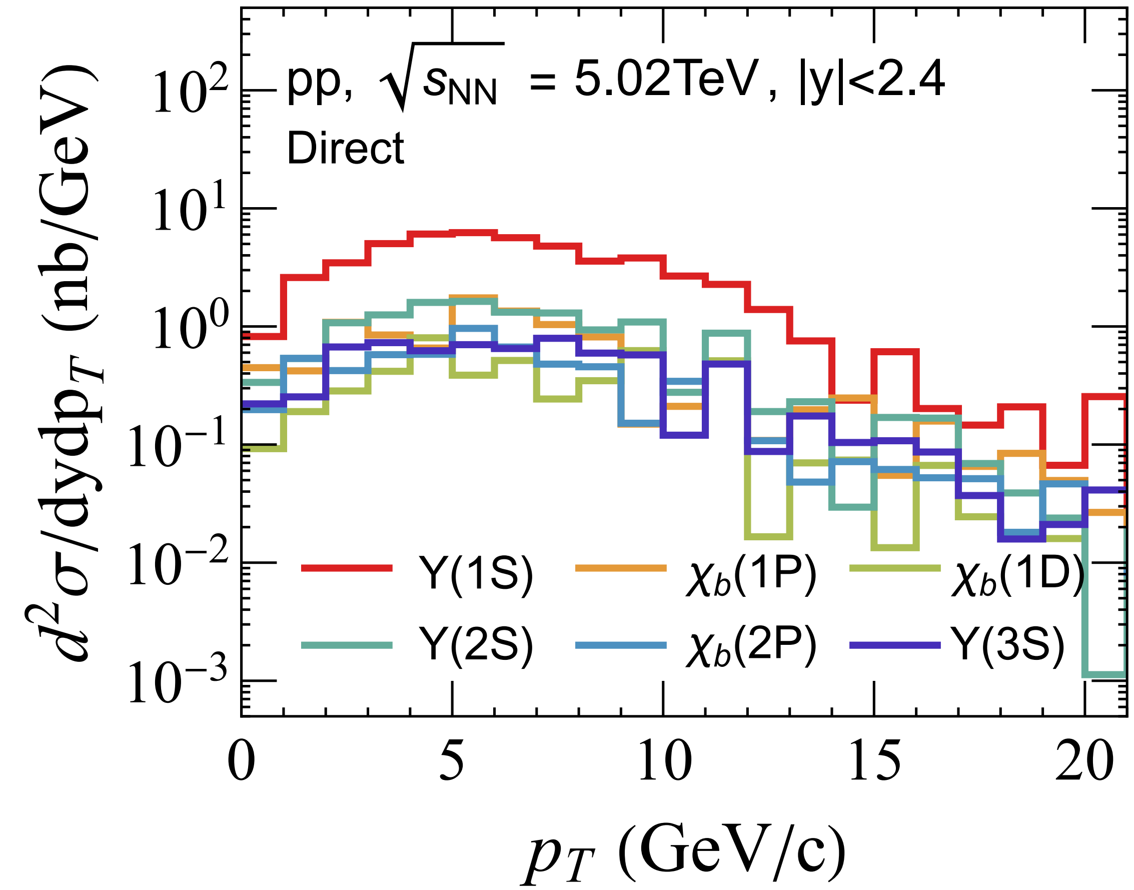
Flavor excitation dominates at low pT while gluon splitting becomes important at high pT.

Quarkonium production

Wigner density matrix formalism

$$\frac{dP_i}{d^3\mathbf{R}d^3\mathbf{P}} = \sum \int \frac{d^3rd^3p}{(2\pi)^6} W_i(\mathbf{r},\mathbf{p}) \prod_{j>2} \int \frac{d^3r_jd^3p_j}{(2\pi)^{3(N-2)}} W^{(N)}(\mathbf{r}_1,\mathbf{p}_1,\mathbf{r}_2,\mathbf{p}_2,...,\mathbf{r}_N,\mathbf{p}_N).$$





$\sigma_{c\bar{c}} = 0.4$ fm

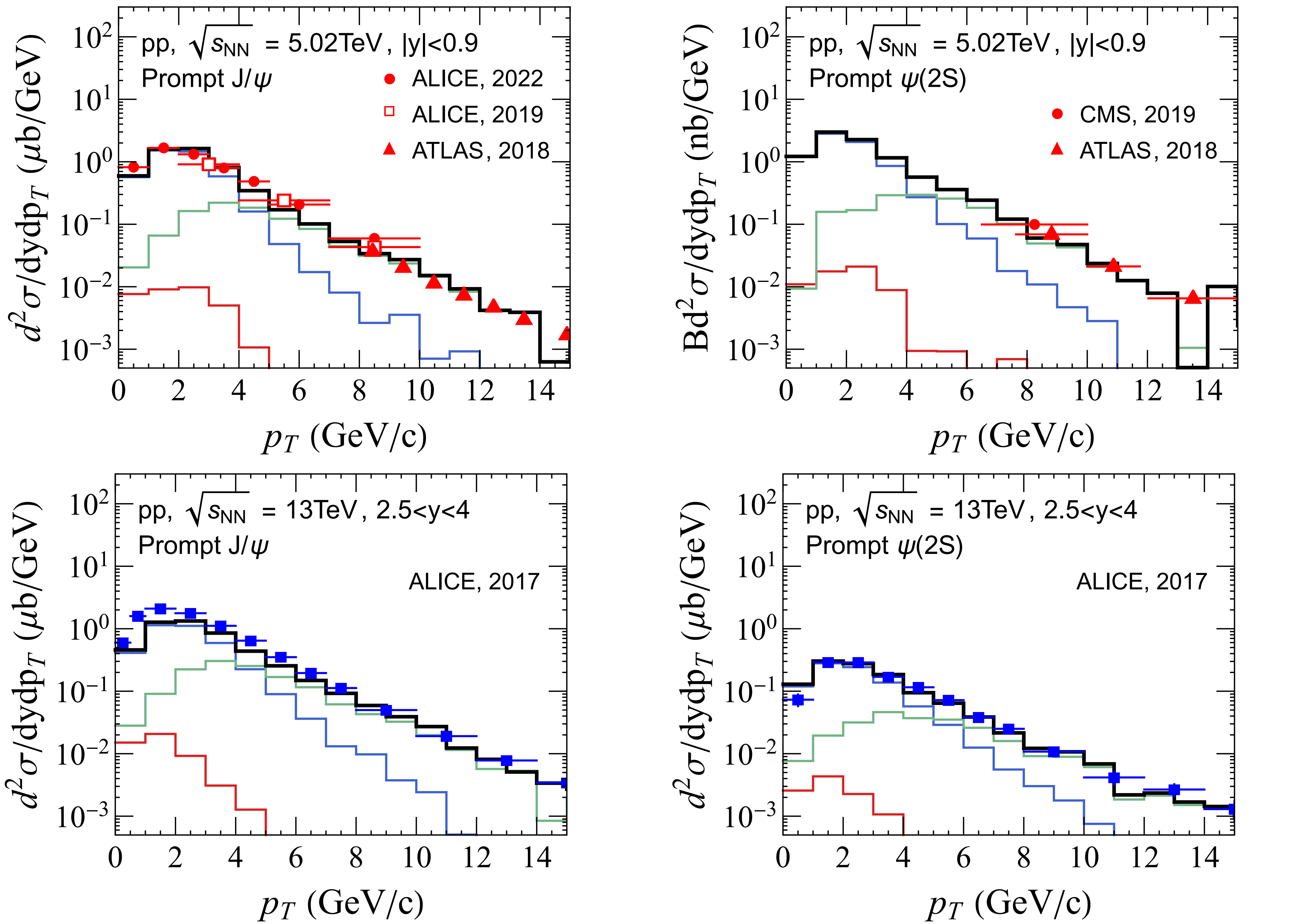
$\sigma_{b\bar{b}} = 0.2$ fm

Charmonium production

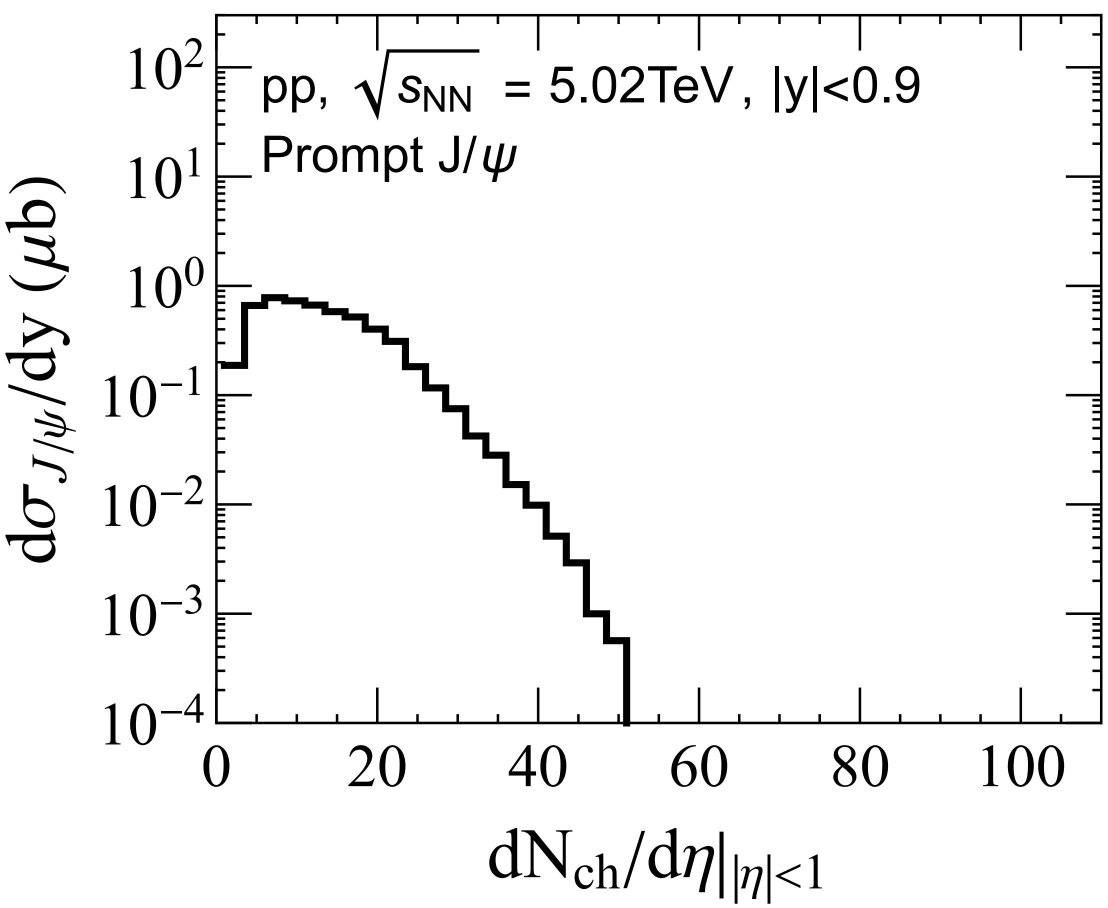
Prompt $J/\psi = J/\psi + \chi_c \times 30\% + \psi(2S) \times 61\%$

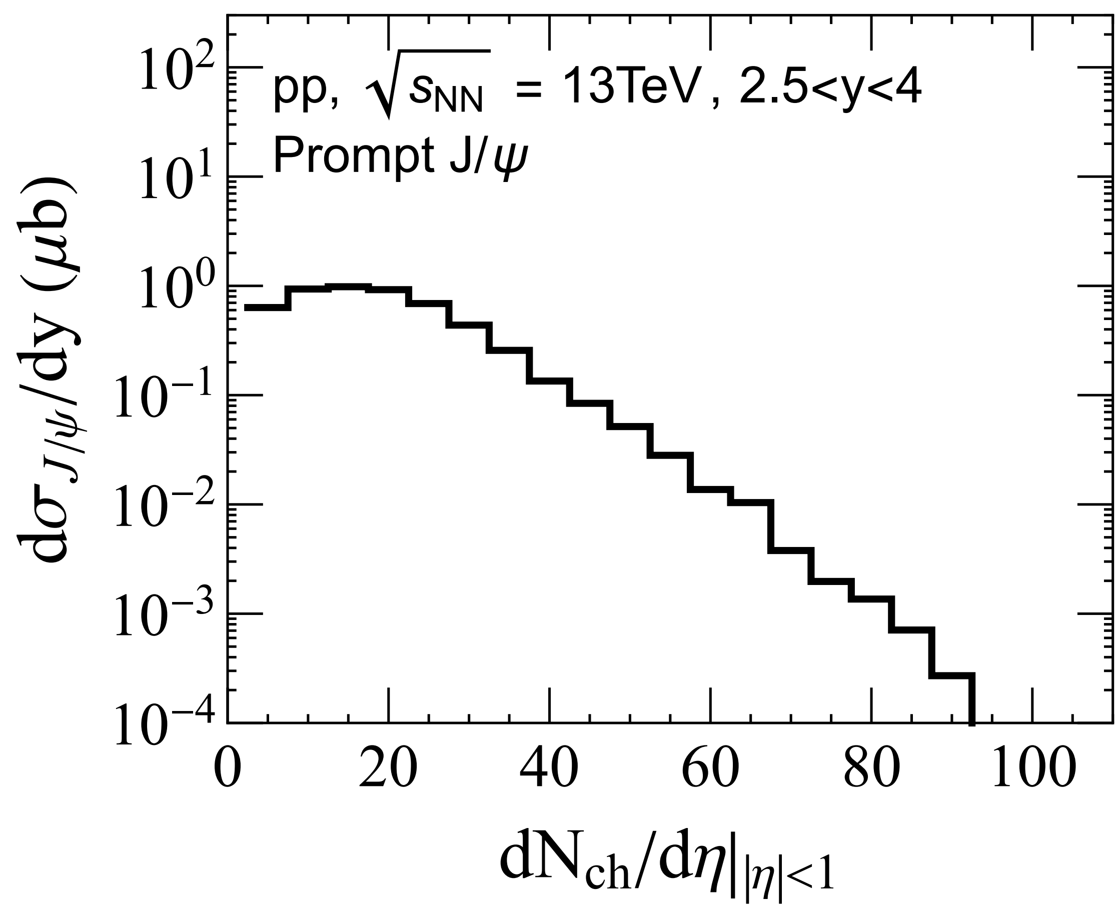
Prompt $\psi(2S) = \psi(2S)$

Black: prompt; Red: flavor creation; Blue: flavor excitation; Green: gluon splitting



Charmonium production



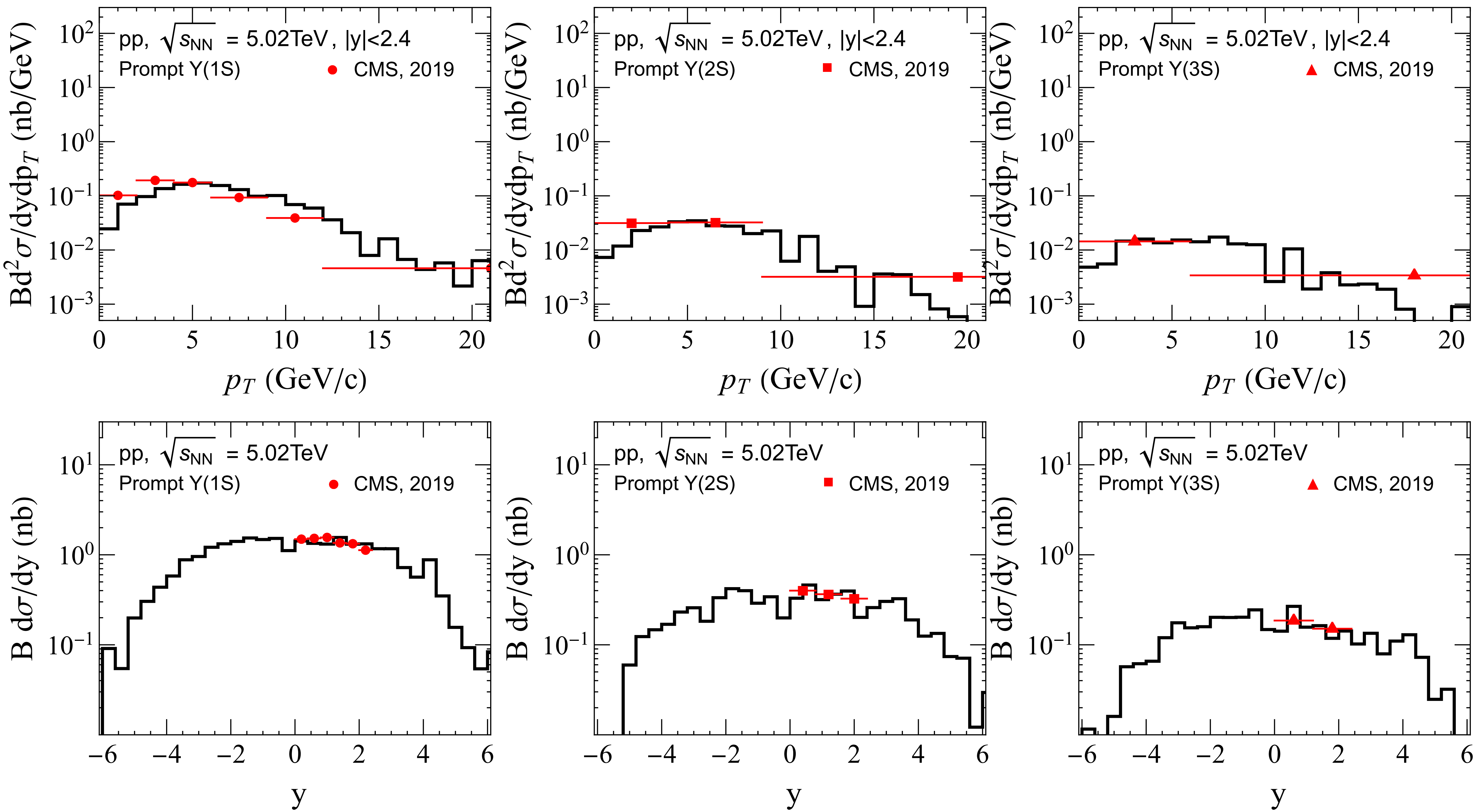


Bottomonium production

Prompt $\Upsilon(1S)$ = $\Upsilon(1S)$ + $\chi_b(1P)$ x 23% + $\chi_b(1D)$ x 20% + $\Upsilon(2S)$ x 7% + $\chi_b(2P)$ x 7% + $\Upsilon(3S)$ x 1%

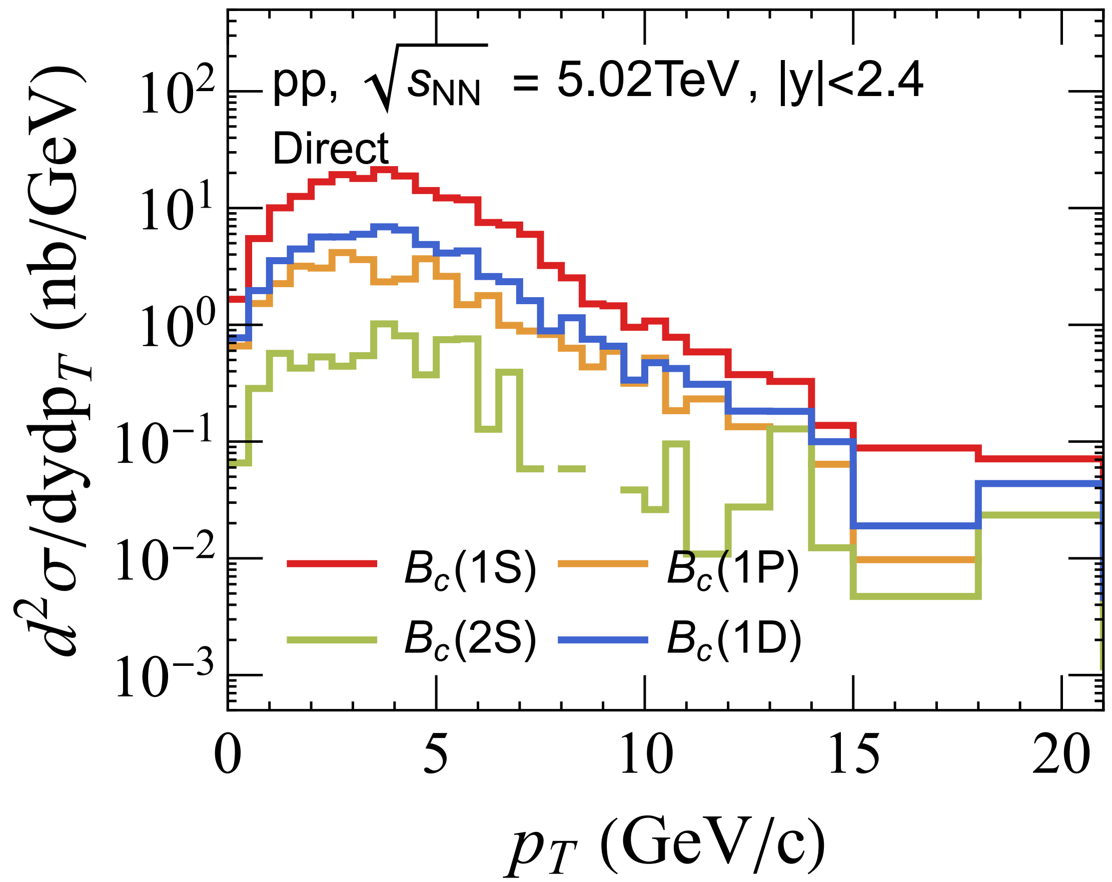
Prompt $\Upsilon(2S)$ = $\Upsilon(2S)$ + $\chi_b(2P)$ x 9.3% + $\Upsilon(3S)$ x 10.6%

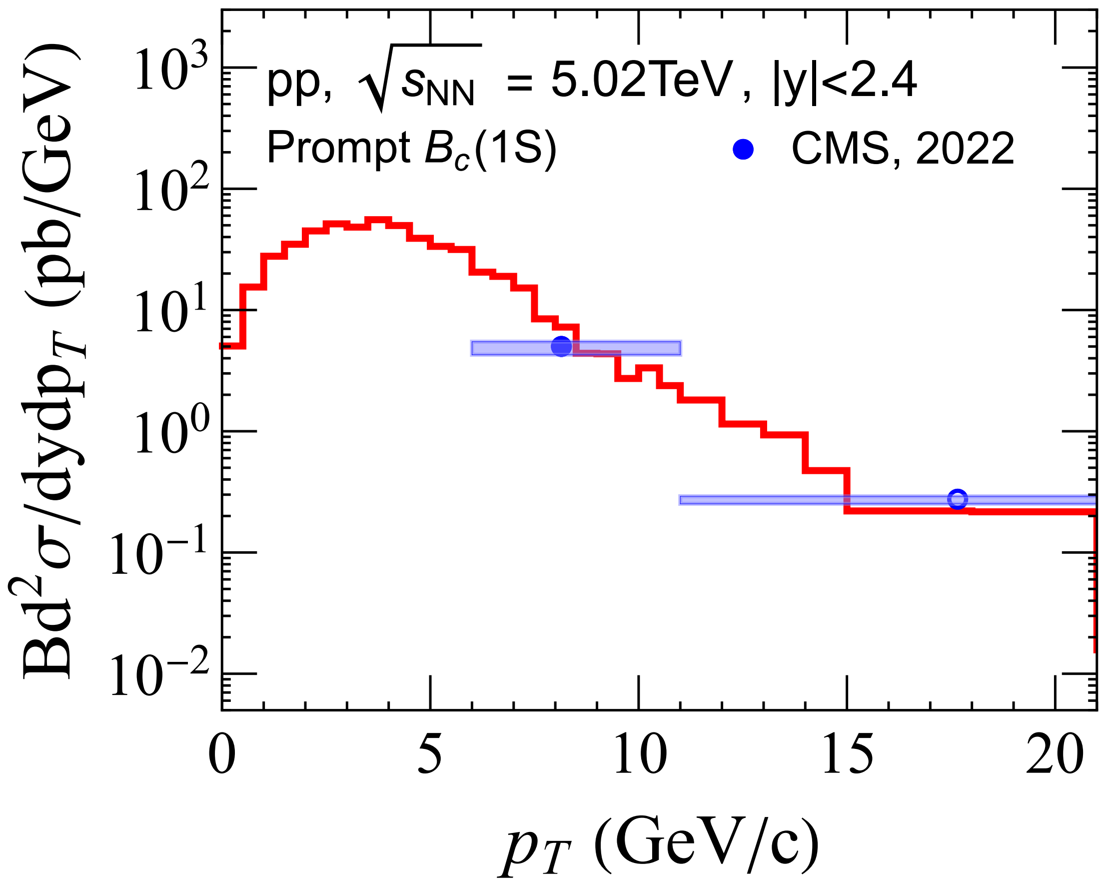
Prompt $\Upsilon(3S)$ = $\Upsilon(3S)$



Bc production

Prompt $B_c(1S) = B_c(1S) + B_c(1P) \times 50\% + B_c(1D) \times 50\% + B_c(2S) \times 50\%$





Summary

✤ *Quarkonium static properties can be described by the two-body Schrödinger/ Dirac equation. These static properties are crucial to their productions.*

✤ *Wigner density matrix formalism is used to investigate the quarkonium production in pp collisions in EPOS4. The Wigner density of the quarkonia is approximated by analytical 3-D isotropic harmonic oscillator Wigner densities with the same root-mean-square radius given by the solution of the Schrödinger equation. This approach reproduces quite well the available experimental transverse momentum and rapidity distributions.*

✤ *Wigner density matrix formalism has also been used/extended to study the quarkonium production in heavy-ion collisions.* *[Phys.Rev.C 107 (2023) 5, 054913]*

A small QGP is created in high energy pp collisions!

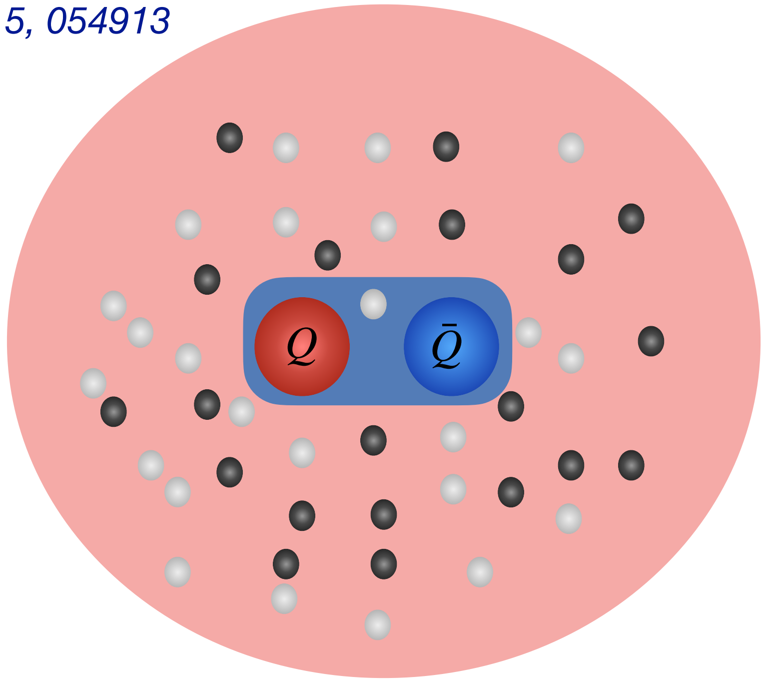
JZ, *J.Aichelin, P.B. Gossiaux, K.Werner, arXiv: 2310.08684*

Thanks for your attention!

Microscopic Model for Quarkonia Production in AA collisions

*D. Villar, **JZ,** J. Aichelin, P.B. Gossiaux. Phys.Rev.C 107 (2023) 5, 054913*

Open quantum system



$$\hat{H}_{tot} = \hat{H}_s \otimes I_e + I_s \otimes \hat{H}_e + \hat{H}_{int}\,,$$

Subsystem **Environment** **Interaction**

$$\frac{d\hat{\rho}_{tot}}{dt} = -\,i[\hat{H}_{tot},\, \hat{\rho}_{tot}]$$ *von Neumann equation*

Probability that at time t the state Φ is produced:

$$P^{\Phi}(t) = \mathrm{Tr}[\rho^{\Phi}\hat{\rho}_{tot}(t)] \qquad \rho^{\Phi} = |\,\Phi > < \Phi\,|$$

Rate of decay and formation of state Φ is:

$$\Gamma = \frac{dP^{\Phi}}{dt} = \underbrace{\mathrm{Tr}\left[\frac{d\rho^{\Phi}}{dt}\rho_{tot}\right]}_{\Gamma_{\mathrm{local}}} \underbrace{- i\mathrm{Tr}\left[\rho^{\Phi}[H_{tot}, \rho_{tot}]\right]}_{\Gamma_{\mathrm{coll}}}$$

$$P^{\Phi}(t) = P^{\Phi}(0) + \int_0^t (\Gamma_{\mathrm{local}} + \Gamma_{\mathrm{coll}})dt$$

Remler equation

Annals Phys. 136 (1981)

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Deuteron and entropy production in relativistic heavy ion collisions

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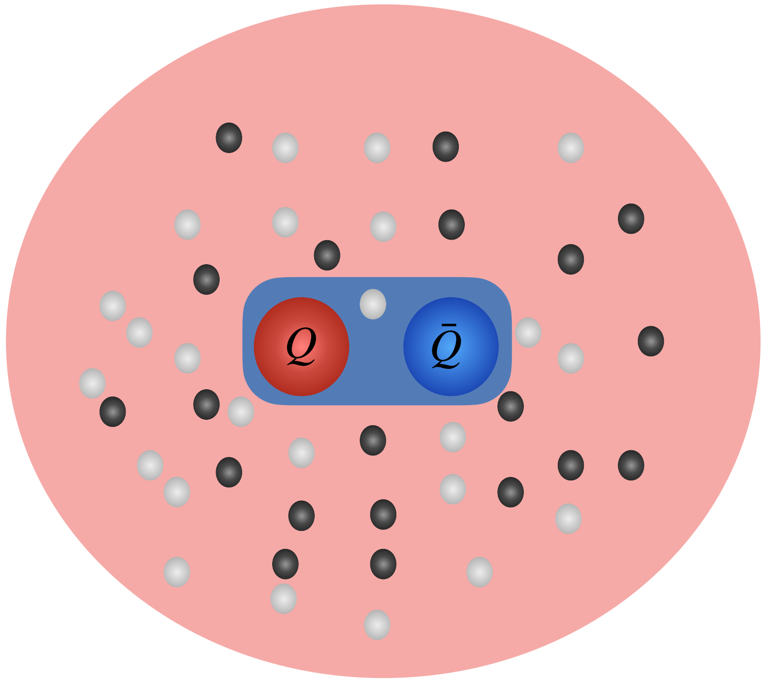
Microscopic Model for Quarkonia Production in AA collisions

Rate Hamiltonian is decomposed into:



$$H_{tot} = H_{1,2} + H_{N-2} + \sum_{i\geq 3} (V_{1i} + V_{2i})$$

$$H_{1,2} = K_1 + K_2 + V_{12}$$

$$H_{N-2} = \sum_{i\geq 3} K_i + \sum_{i>j\geq 3} V_{ij}$$

With: $[H_{N-2}, \rho^{\Phi}] = 0 \quad [H_{1,2}, \rho^{\Phi}] = 0$

$$\Gamma_{\text{coll}} = -i\text{Tr}[\rho^{\Phi}[H_{tot}, \rho_{tot}]] = -i\sum_{i\geq 3} \text{Tr}[\rho^{\Phi}\underline{[V_{1i} + V_{2i}, \rho_{tot}]}]$$

still impossible to deal with exactly at the quantum n-body level

An equivalent N-body Wigner density W_N, or Wigner representation:

$$W_N(\{\mathbf{r}_i\}, \{\mathbf{p}_i\}, t) = \frac{1}{h^{3N}} \int d^3\mathbf{y}_1 \ldots d^3\mathbf{y}_N e^{i\mathbf{p}_1\cdot\mathbf{y}_1} \ldots e^{i\mathbf{p}_N\cdot\mathbf{y}_N} \langle \mathbf{r}_1 + \frac{\mathbf{y}_1}{2}, \ldots, \mathbf{r}_N + \frac{\mathbf{y}_N}{2} | \rho_{tot}(t) | \mathbf{r}_1 - \frac{\mathbf{y}_1}{2}, \ldots, \mathbf{r}_N - \frac{\mathbf{y}_N}{2} \rangle$$

Microscopic Model for Quarkonia Production in AA collisions

Rate Hamiltonian is decomposed into:

$$\frac{\partial}{\partial t}\rho_{tot}(t) = -i\sum_i [K_i, \rho_{tot}(t)] \boxed{-i\sum_{j>i}[V_{ij}, \rho_{tot}(t)]}$$

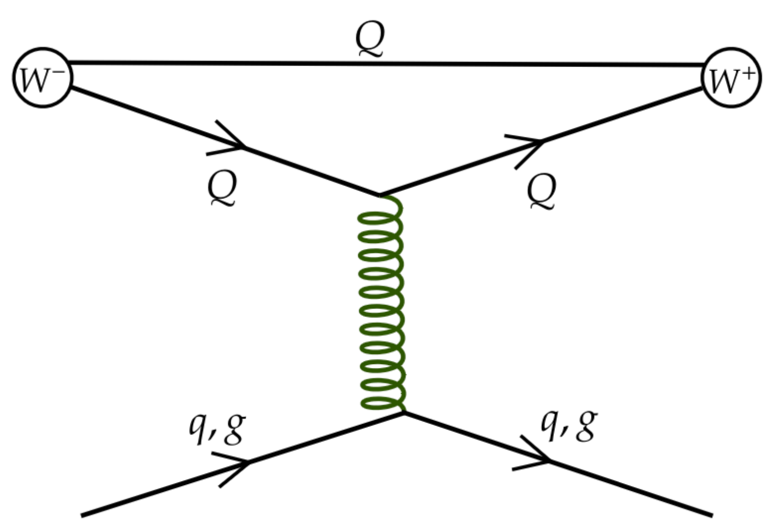
Can be modelized from the trajectories evolution in Wigner space

$$\frac{\partial}{\partial t}W_N(t) = \sum_i \mathbf{v}_i \cdot \partial_{r_i} W_N(\{\mathbf{r}_i\}, \{\mathbf{p}_i\}, t) + \sum_{j>i}\sum_n \delta(t - t_{ij}(n))\Big[W_N(\{\mathbf{r}_j\}, \{\mathbf{p}_j\}, t+\epsilon) - W_N(\{\mathbf{r}_j\}, \{\mathbf{p}_j\}, t-\epsilon)\Big]$$

Free streaming *Scattering at time $t_{ij}(n)$ for nth collisions between i and j*

$$\Gamma_{\mathrm{coll}} = -i\sum_{i\geq 3}\mathrm{Tr}[\rho^{\Phi}[V_{1i} + V_{2i}, \rho_{tot}]]$$

$$\approx \sum_k^2 \sum_{i\geq 3}^N \sum_n \delta(t - t_{ki}(n))\int \prod_{j=1}^N d^3\mathbf{r}_j d^3\mathbf{p}_j W^{\Phi}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{p}_1, \mathbf{p}_2)\Big[W_N(\{\mathbf{r}_j\}, \{\mathbf{p}_j\}, t+\epsilon) - W_N(\{\mathbf{r}_j\}, \{\mathbf{p}_j\}, t-\epsilon)\Big]$$



QGP thermal partons

$$\boxed{W_N \approx W_N^{\mathrm{C(classical)}} = \prod_{i=1}^N \delta(\mathbf{r}_i - \mathbf{r}_i^*(t))\delta(\mathbf{p}_i - \mathbf{p}_i^*(t))}$$

Microscopic Model for Quarkonia Production in AA collisions

Defining the bound state formation probability at a given time t:

$$\omega_{1,2}(n,t) = \int \prod_{j=1}^{N} d^3\mathbf{r}_j d^3\mathbf{p}_j W^{\Phi}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{p}_1,\mathbf{p}_2) W_N^C(\{\mathbf{r}_j\},\{\mathbf{p}_j\},t)$$

$$= W^{\Phi}(\mathbf{r}_1(t),\mathbf{p}_1(t),\mathbf{r}_2(t),\mathbf{p}_2(t))$$

substituting the semi-classical N-body Wigner density

The contribution of this Q and parton i n-th collision to the Φ production rate:

$$\Gamma_{1,2;i}(n,t) = \delta(t - t_{1i}(n))\Big[\omega_{1,2}(n,t+\epsilon) - \omega_{1,2}(n,t-\epsilon)\Big] \qquad \Gamma_{1,2;i}(n,t) = \Gamma_{2,1;i}(n,t)$$

$$\Gamma_{\mathrm{coll}} = \sum_{k}^{2} \sum_{i\geq 3}^{N} \sum_{n} \Gamma_{k,3-k;i}(n,t)$$

This form is suitable for Monte Carlo implementations

Extend to the system with N_Q Q quarks and $N_{\bar{Q}}$ $\bar{Q}$ quarks—>sum over all possible $Q\bar{Q}$ pairs

$$\Gamma_{\mathrm{coll}} = \sum_{k=1}^{N_Q} \sum_{l=N_Q+1}^{N_Q+N_{\bar{Q}}} \sum_{i>N_Q+N_{\bar{Q}}}^{N} \sum_{n} \Big(\Gamma_{k,l;i}(n,t) + \Gamma_{l,k;i}(n,t)\Big)$$

Microscopic Model for Quarkonia Production in AA collisions

For the local rate:

$$\Gamma_{\mathrm{local}} = \mathrm{Tr}\left[\frac{d\rho^{\Phi}}{dt}\rho_{tot}\right] \approx \int \prod_{j=1}^{N} d^3\mathbf{r}_j d^3\mathbf{p}_j \dot{W}^{\Phi}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{p}_1, \mathbf{p}_2, T(t)) W_N^C(\{\mathbf{r}_j\}, \{\mathbf{p}_j\}, t)$$

$$= \dot{\sigma}(T(t))\partial_\sigma W^{\Phi}(\mathbf{r}_1(t), \mathbf{p}_1(t), \mathbf{r}_2(t), \mathbf{p}_2(t), \sigma(T))$$

The local rate is non zero if the temperature changes with time and therefore the temperature dependent Gaussian width becomes time dependent.

Here we take the Gaussian Wigner density:

$$W^{1S}(r,p) = 8e^{-\frac{r^2}{\sigma^2}-\frac{p^2\sigma^2}{\hbar^2}}$$

$$W^{1P}(r,p) = \frac{8}{3}e^{-\frac{r^2}{\sigma^2}-\frac{p^2\sigma^2}{\hbar^2}}\left(2\frac{r^2}{\sigma^2} + 2\frac{p^2\sigma^2}{\hbar^2} - 3\right)$$

$$W^{2S}(r,p) = \frac{8}{3}e^{-\frac{r^2}{\sigma^2}-\frac{p^2\sigma^2}{\hbar^2}}\left(2\frac{r^4}{\sigma^4} + 4\frac{r^2}{\sigma^2}(\frac{p^2\sigma^2}{\hbar^2} - 3) + (3 - 4\frac{p^2\sigma^2}{\hbar^2} + 2\frac{p^4\sigma^4}{\hbar^4})\right)$$

$$\begin{aligned}\sigma^2 &= \frac{2}{3}\langle r^2\rangle, \quad for\ 1S\ wave,\\ \sigma^2 &= \frac{2}{5}\langle r^2\rangle, \quad for\ 1P\ wave,\\ \sigma^2 &= \frac{2}{7}\langle r^2\rangle, \quad for\ 2S\ wave.\end{aligned}$$

With this, the local rate can be obtained(for 1S): $\Gamma^{1S}_{\mathrm{local}} = 16\dot{\sigma}(T(t))\left(\frac{r^2}{\sigma^3} - \frac{\sigma p^2}{\hbar^2}\right)e^{-\frac{r^2}{\sigma^2}-\frac{p^2\sigma^2}{\hbar^2}}$