α_s in (2+1+1)-Flavor QCD from the Static Energy

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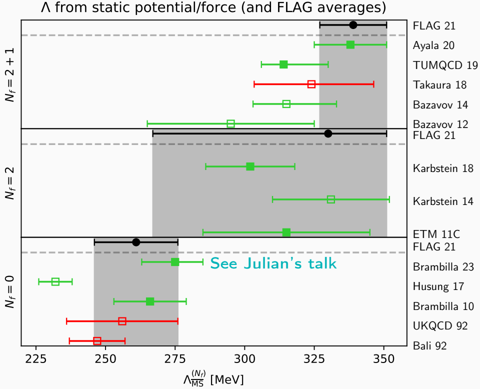
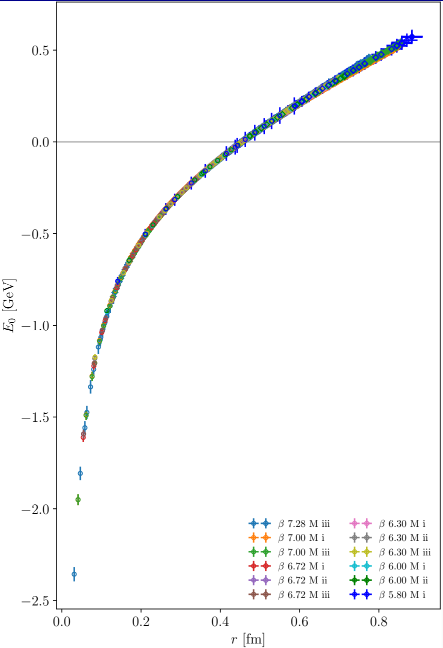
Motivation

- Static energy $E_0(r)$ between a static quark and antiquark
- Defined as a ground state of Wilson loop
- Of major importance for scale setting
- Very well known in perturbation theory
- Physical observable:
  No scheme change required between lattice and continuum
- Detailed previous studies in
  $N_f = 0$ and
  $N_f = 2+1 \Rightarrow \Lambda_{\overline{\mathrm{MS}}}^{N_f=0,3}$
- Very few 2+1+1 extractions



Static energy



- We calculate the static energy in 2+1+1-flavor QCD
- Self-consistently set the scale with

$$r_i^2 F(r_i) = \begin{cases} 1.65, & i = 0^{1} \\ 1.0\ , & i = 1^{2} \\ 0.5\ , & i = 2^{3} \end{cases}, \quad \begin{matrix} r_0 \sim 0.475\,\mathrm{fm} \\ r_1 \sim 0.3106\,\mathrm{fm} \\ r_2 \sim 0.145\,\mathrm{fm} \end{matrix}$$

- Take massive charm quark into account $1/m_c \sim 0.15 fm$
- Aim to extract $\Lambda_{\overline{\mathrm{MS}}}$ from the small distance behavior
- Scale setting and charm effect done in: TUMQCD, PRD107 (2023)

Static energy in perturbation theory

- Determined from the large-time behavior of Wilson loops

$$E(r) = -\lim_{T\to\infty} \frac{\ln\langle \mathrm{Tr}(W_{r\times T})\rangle}{T}, \qquad W_{r\times T} = P\left\{ \exp\left(i \oint_{r\times T} dz_\mu \, gA_\mu \right) \right\}$$

- Perturbatively known to $\mathrm{N^3LL}$ [1]:

$$E_0(r) = \Lambda_\mathrm{s} - \frac{C_\mathrm{F}\alpha_\mathrm{s}}{r}\left(1 + \#\alpha_\mathrm{s} + \#\alpha_\mathrm{s}^2 + \#\alpha_\mathrm{s}^3 + \#\alpha_\mathrm{s}^3 \ln\alpha_\mathrm{s} + \#\alpha_\mathrm{s}^4 \ln^2\alpha_\mathrm{s} + \#\alpha_\mathrm{s}^4 \ln\alpha_\mathrm{s} \ldots\right)$$

- In the dimensional regularization requires a renormalon subtraction
- In the lattice regularization diverges as $1/a$ towards continuum limit
- Both regularization problems can be absorbed into a constant term
- Integrating the static force eliminates the leading renormalon

$$E_0(r) = \int_{r^*}^{r} \mathrm{d}r' F(r') + \mathrm{const}$$

- The ultra-soft scale $\mu_\mathrm{us} \to \ln\alpha_\mathrm{s}(1/r)$ term, can be resummed
- Set scale as $\mu = 1/r$

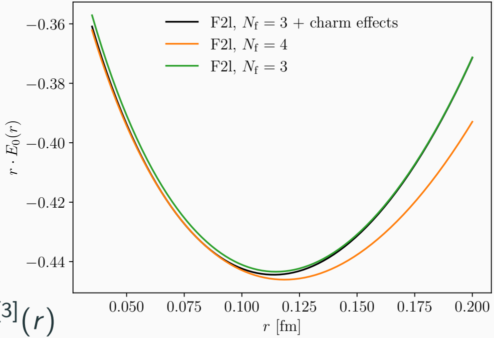
[1] For review of perturbative results, see e.g. X. Tormo Mod. Phys. Lett. A28 (2013)

Charm quark mass effects in perturbation theory

- Effects due to finite mass of a heavy quark give correction $\delta V_{\mathrm{m}}^{(N_f)}(r)$

$$E_{0,m}^{(N_f)}(r) = \int_{r^*}^{r} \mathrm{d}r' F^{(N_f)}(r') + \delta V_{\mathrm{m}}^{(N_f)}(r) + \mathrm{const}$$

$$E_{0,m}^{(N_f)} \to E_0^{(N_f)} \quad \text{for } r \gg 1/m$$
$$E_{0,m}^{(N_f)} \to E_0^{(N_f+1)} \quad \text{for } r \ll 1/m$$



- Finite mass corrections known up to two loops[1]
$\delta V_{\mathrm{m}}^{(N_f)}(r) = \delta V_{\mathrm{m}}^{(N_f),[2]}(r) + \delta V_{\mathrm{m}}^{(N_f),[3]}(r)$

[1] D. Eiras, J. Soto, PRD61 (2000); M. Melles PRD62 (2000); A. H. Hoang hep-ph/0008102 (2000)

Lattice simulations

- Use state of the art 2+1+1 HISQ[1] ensembles from MILC[2]
- Three different light quark masses, physical strange and charm sea
- Six lattice spacings $a \approx 0.032 - 0.15$
- Relation to physical units via f_{p4s} scale
- Fix coulomb gauge
  $\rightarrow$ Static energy as ground state of Wilson line correlation function
  $\rightarrow$ Off-axis separations accessible
- Static quarks with and without one step of HYP-smearing
- Measure static energy for separations $r \approx 0.03 - 0.9$fm.

[1] E. Follana, *et.al.*, PRD75 (200/); [2] A. Bazavov, *et.al.*, PRD98 7 (2018)

Static energy on the lattice

- We compute E_0 from Wilson line correlators in Coulomb gauge $W(\mathrm{r},\tau,a)=\prod_{u=0}^{\tau/a-1}U_4(\mathrm{r},ua,a)$

$$C(\mathrm{r},\tau,a)=\left\langle \frac{1}{N_\sigma^3}\sum_x \sum_{y=R(r)} \frac{1}{N_c N_r}\mathrm{Tr}\, W^\dagger(x+y,\tau,a)W(x,\tau,a)\right\rangle$$

$$C(\mathrm{r},\tau,a)=\mathrm{e}^{-\tau E_0(\mathrm{r},a)}\left(C_0(\mathrm{r},a)+\sum_{n=1}^{N_{\mathrm{st}}-1}C_n(\mathrm{r},a)\prod_{m=1}^{n}\mathrm{e}^{-\tau\Delta_m(\mathrm{r},a)}\right)+\cdots$$

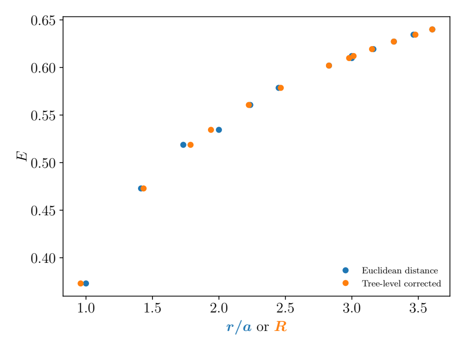
- Vary the fit range with N_{st} and $|\mathrm{r}|$

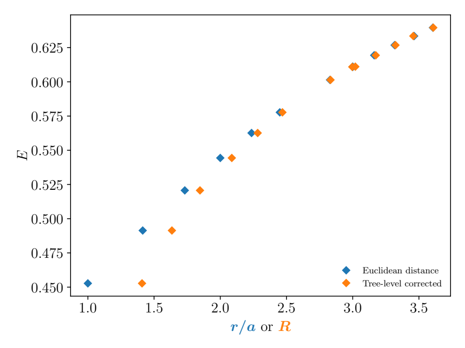
$$\begin{array}{lll}
|\mathrm{r}|+0.2\ \mathrm{fm}\le\tau_{\mathrm{min},1}\le 0.3\ \mathrm{fm} & \text{for } N_{\mathrm{st}}=1, & \Rightarrow \text{ prior values}\\
\frac{2}{3}|\mathrm{r}|+0.1\ \mathrm{fm}\le\tau_{\mathrm{min},2}\le\tau_{\mathrm{min},1}-2a & \text{for } N_{\mathrm{st}}=2, & \Rightarrow \text{ our pick}\\
\frac{1}{3}|\mathrm{r}|\qquad\le\tau_{\mathrm{min},3}\le\tau_{\mathrm{min},2}-2a & \text{for } N_{\mathrm{st}}=3, & \Rightarrow \text{ cross-check}
\end{array}$$

- We use Bayesian fits with loose linear priors
- Priors for E_1-E_0 come from pure gauge [1]

[1] K. Juge, *et.al.*, PRL90 (2003) [2] A. Hasenfratz, *et.al.*, PRD64 (2001);

Discretization effects: Tree-level

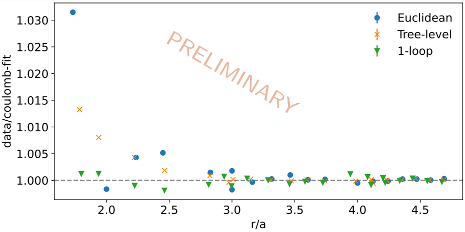


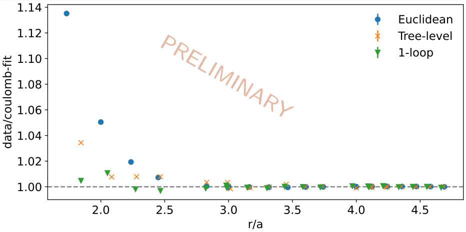


- $E_0(r,a)$ is available only at discrete distances and is direction dependent
- Tree-level improved distance defined with lattice gluon propagator $D_{\mu\nu}(k)$

$$E(r,a) = -C_F g_0^2 \int \frac{\mathrm{d}^3 k}{(2\pi)^3} e^{ik\cdot r} D_{44}(k) = -\frac{C_F g_0^2}{4\pi} \frac{1}{r_I}$$

Discretization effects: 1-loop





- Ongoing effort to calculate the 1-loop improvement[1]
- Lattice perturbation theory can be too complicated to do by hand
- Use HPsrc and HiPPy programs to numerically calculate the diagrams
- Promising results, but needs fine tuning before prime time in analysis

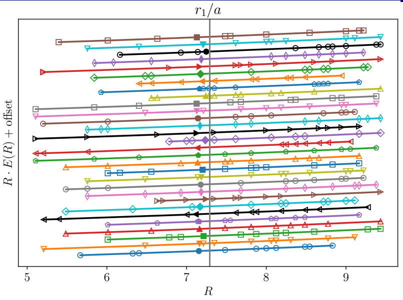
[1] G. v. Hippel, V.L, S. Steinbeißer, in preparation TUM-EFT 171/22

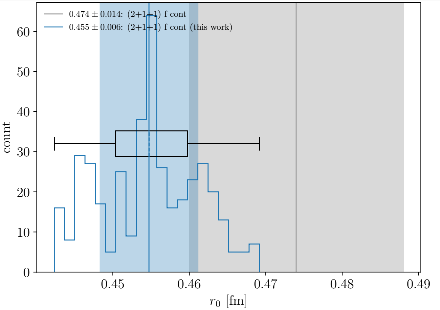
Lattice scales extraction procedure

- Static energy allows for determination of lattice scales r_i and the string tension σ
- $r_2 \sim 1/m_c$, scales r_i expected to be affected by charm differently
- Locally fit the data using Cornell ansatz

$$E(R, a) = -\frac{A}{R} + B + \sigma R$$

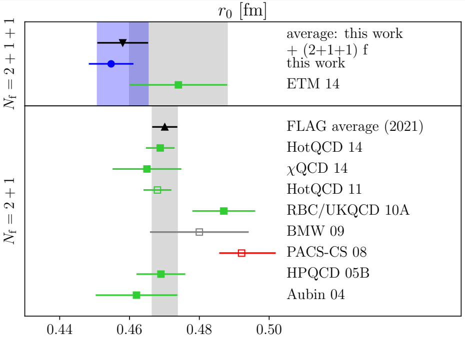
- Asymmetric random picking for systematics
- Smooth the data with Allton ansatz
- Leading discretization effects $\alpha_s^2 a^2$ and a^4 → continuum limit

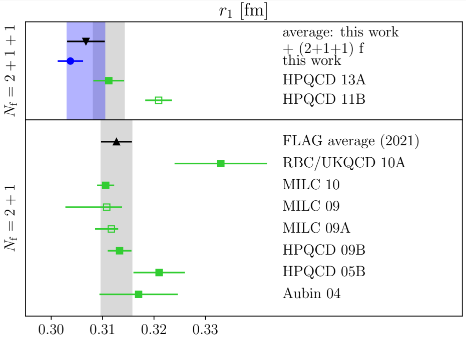




[1] R. Sommer, NPB411 (1994); [2] C. Bernard, *et.al.*, PRD62 (2000); [3] A. Bazavov, *et.al.*, PRD97 (2018)

Extracted scales compared to literature





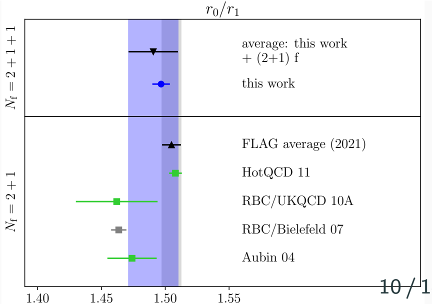
$$r_0 = 0.4547(64)\ \text{fm},$$

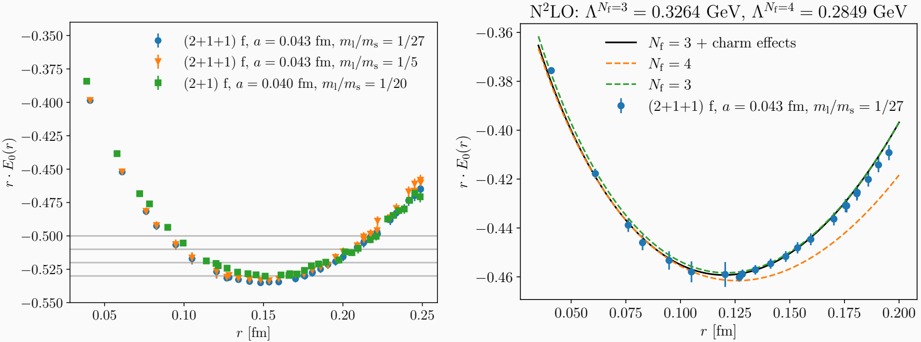
$$r_1 = 0.3037(25)\ \text{fm}, \quad r_0/r_1 = 1.4968(69),$$

$$r_2 = 0.1313(41)\ \text{fm}, \quad r_1/r_2 = 2.313(69).$$

$$\sqrt{\sigma r_0^2} = 1.077 \pm 0.016 \quad (A = A_{r_0}),$$

$$\sqrt{\sigma r_0^2} = 1.110 \pm 0.016 \quad (A = \pi/12).$$

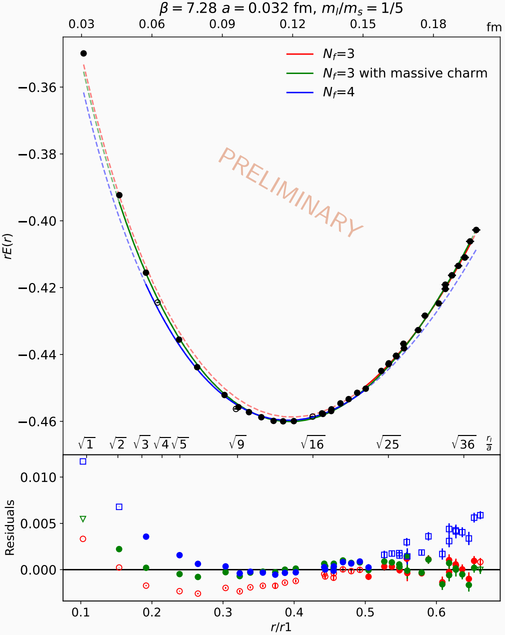




- Clearly visible difference between the behaviors of 2+1[1] and 2+1+1[2]
- Curve with charm effects follows the data better than curves without

[1] A. Bazavov, *et.al.*, PRD100 (2019) [2] TUMQCD, PRD107 (2023)

Fitting lambda



- Fit $N_f = 4$ static potential to $r < 1/m_c$, $N_f = 3$ to $r > 1/m_c$
- Fit $N_f = 3$ plus massive charm to whole range
- Choose fit range with these limits using AIC
- Limits for this talk
  - Limit to 2-loops for consistency
  - Tree-level improvement
  - Limit to off-axis points
  - Focus on finest ensemble

Some (preliminary) results

- Reminder on scales:

  2+1+1 $r_1 = 0.3037(25)\text{fm}$

  2+1 $r_1 = 0.3106(17)\text{fm}$

- We get very preliminary results:

N_f	$r_1\Lambda^{(N_f)}$	$\Lambda^{(N_f)}$ [MeV]	"(de)couple"
$3+1$	$0.501(19)_{\rm lat}(43)_{\rm soft-scale}$	$0.325(13)_{\rm lat}(28)_{\rm soft-scale}$	$0.283(31)$ $[\Lambda^{(4)}]$
4	$0.457(22)_{\rm lat}(32)_{\rm soft-scale}$	$0.297(14)_{\rm lat}(21)_{\rm soft-scale}$	$0.339(25)$ $[\Lambda^{(3)}]$

- Comparing to literature results

N_f	TUMQCD19 r_1	TUMQCD19 MeV	FLAG21
3	0.494^{+24}_{-13}	314^{+15}_{-8}	339(12)
4			297(10)

Conclusions

- We have computed the static energy $E_0(r)$ with 2+1+1 flavors
- Determined scales r_0, r_1, and r_2, their ratios, and string tension σ
- All these scales have been measured simultaneously
- We can see charm decoupling well in the data
  - Perturbative charm effects give better description of the data
  - Scales closer to inverse charm mass differ more from their 2+1 flavor counterparts
- Initial, promising results on the $\Lambda_{\overline{\mathrm{MS}}}$
- One loop improvement coming soon

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Thank you for your attention!