Polarized TMD Fragmentation Functions for J/ψ Production

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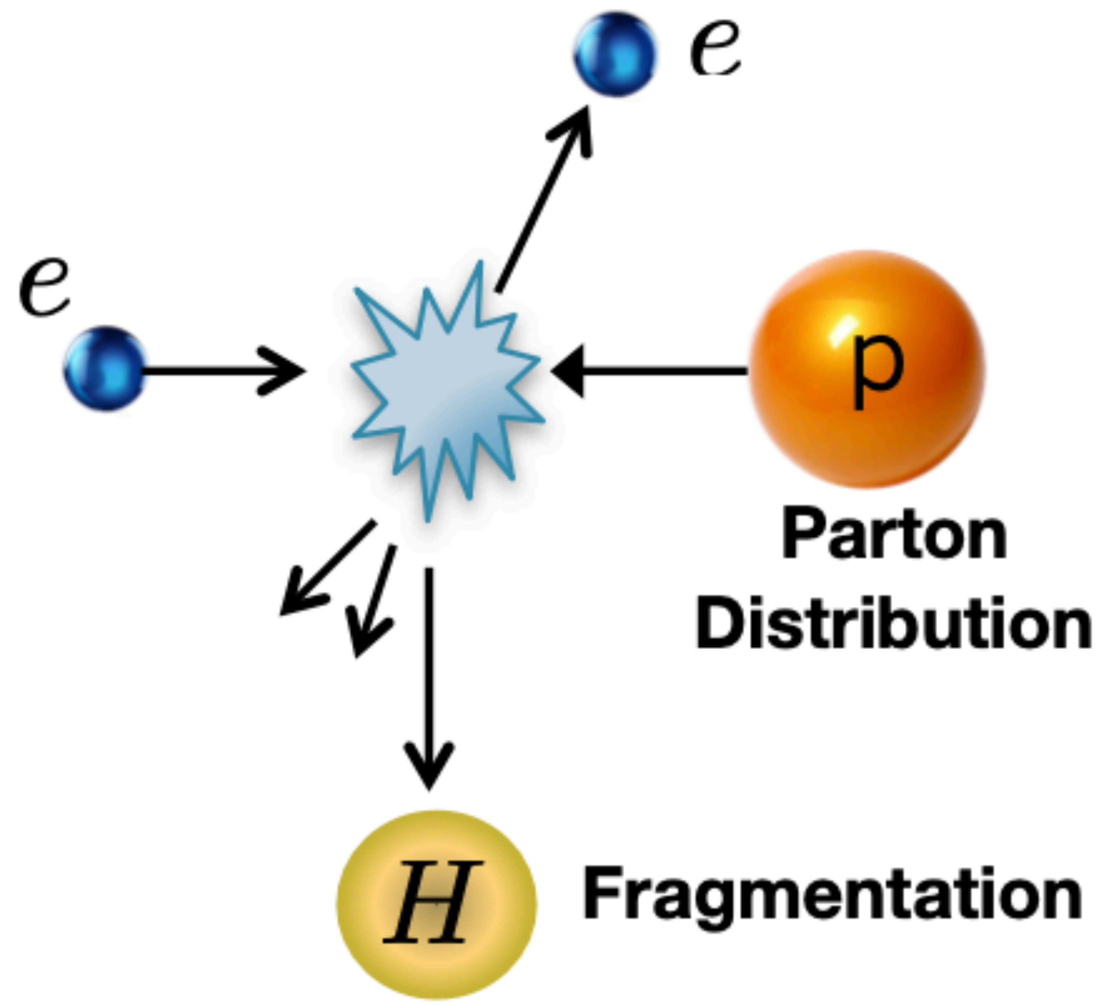
Quarkonium Working Group

IISER Mohali, India, 2/28/2024

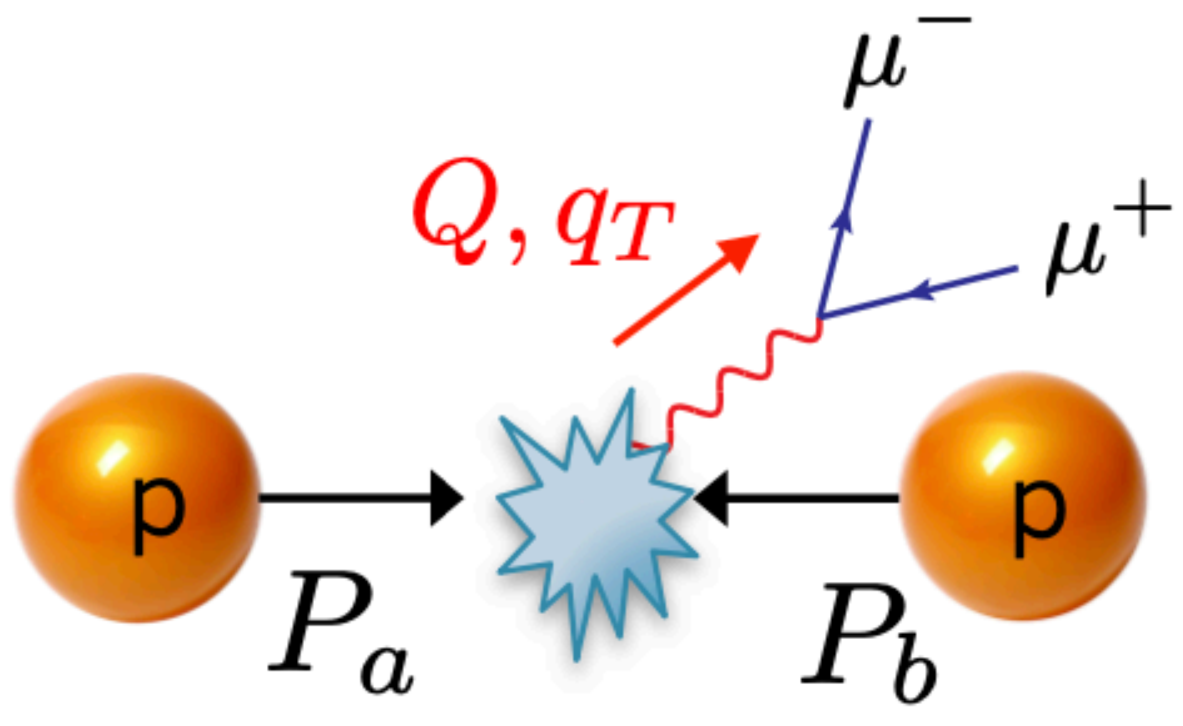
Transverse Momentum Dependent (TMD) Factorization

relevant when identified hadron is produced with moderate p_T

Semi-Inclusive DIS

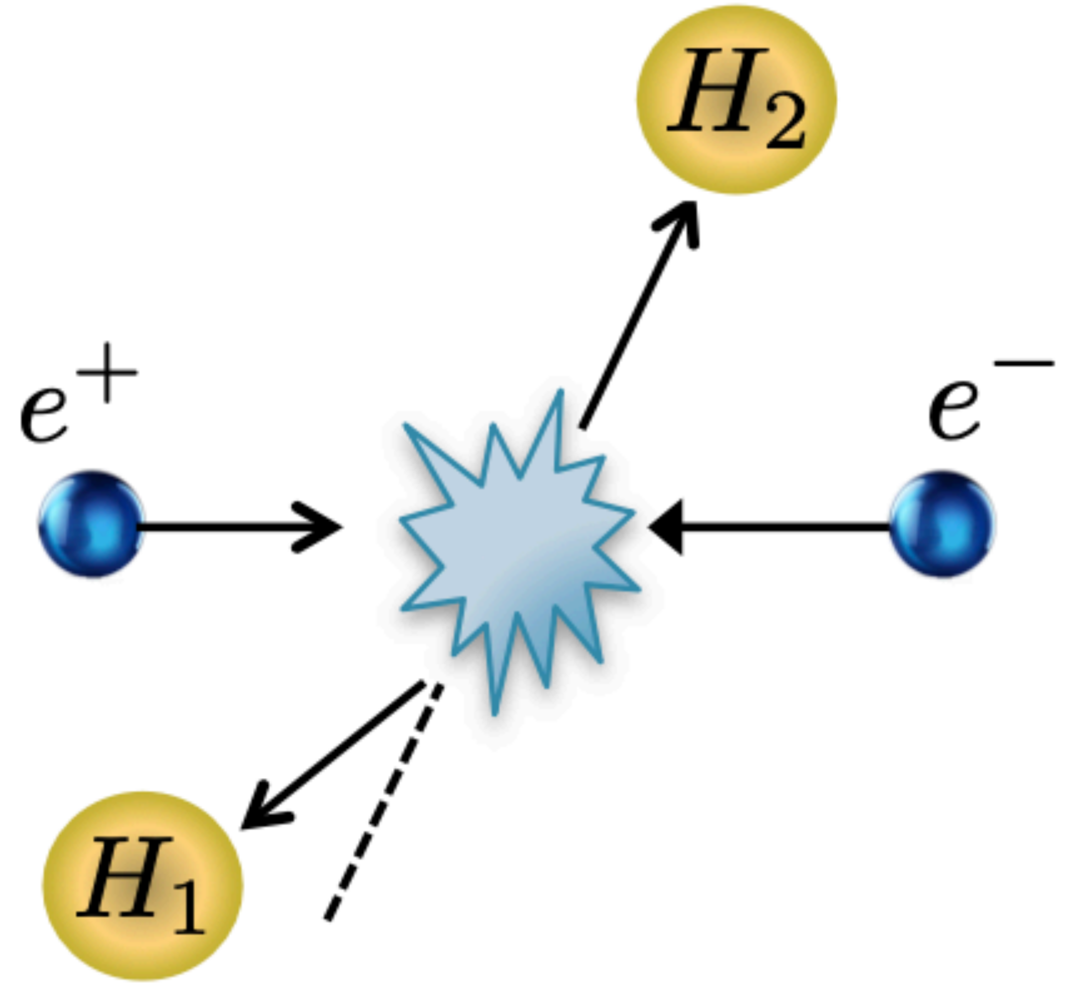


Drell-Yan



$q_T \ll Q$

Dihadron in e+e-



Review: R. Boussarie, et. al., “TMD Handbook”, arXiv:2304.0332

Semi-Inclusive Deep Inelastic Scattering (SIDIS)

$$e^-(l) + p(P) \to e^-(l') + h(P_h) + X$$

collinear factorization

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\mathrm{d}y\mathrm{d}z_h} = \sum_{i,j} \int_x^1 d\xi \int_{z_h}^1 d\zeta \, f_{i/p}(\xi)\, D_{h/j}(\zeta)\, \frac{\mathrm{d}\hat{\sigma}_{ij}(\xi,\zeta)}{\mathrm{d}x\mathrm{d}y\mathrm{d}z_h} \left[1 + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{Q^2}\right)\right]$$

TMD factorization

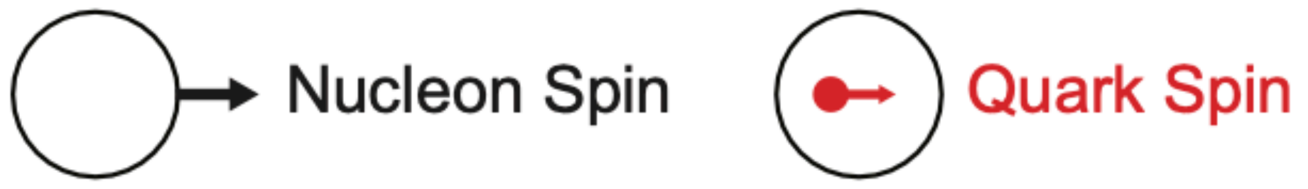
$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\mathrm{d}y\mathrm{d}z_h\mathrm{d}^2\mathbf{P}_{hT}} = \sum_i \hat{\sigma}_{ii}^{\mathrm{TMD}}(Q,x,y) \int \mathrm{d}^2\mathbf{p}_T\, \mathrm{d}^2\mathbf{k}_T\, \delta^{(2)}\left(\mathbf{P}_{hT} - z_h\mathbf{k}_T - \mathbf{p}_T\right) f_{i/p}(x,\mathbf{k}_T)\, D_{h/i}(z_h,\mathbf{p}_T)$$

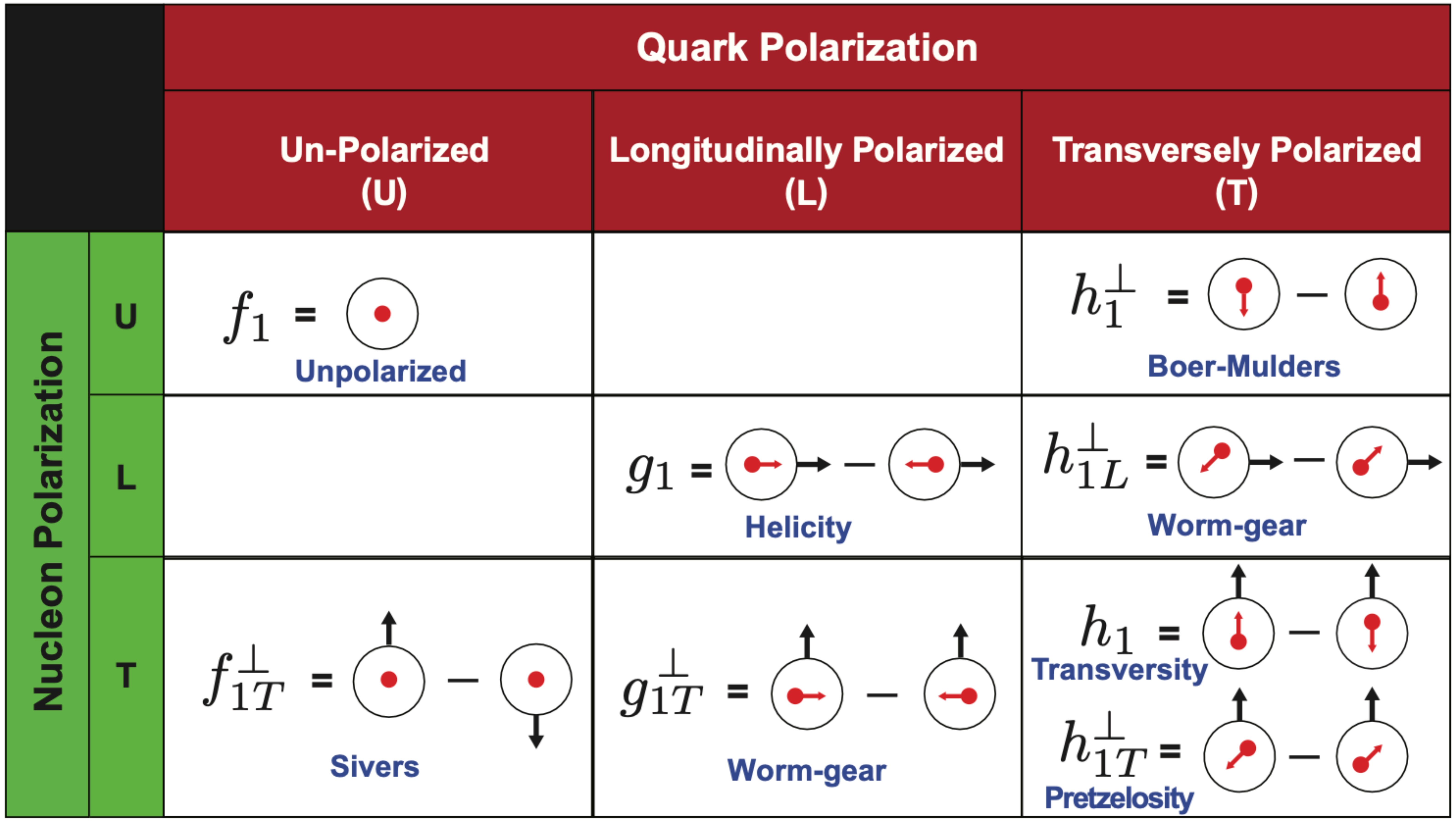
$$\times \left[1 + \mathcal{O}\left(\frac{P_{hT}^2}{Q^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{Q^2}\right)\right]$$

TMDPDF

TMDFF

Leading Quark TMDPDFs





richer spin structure than collinear TMDs: $f_1\,,\, g_1\,,\, h_1$

Non-Relativistic QCD (NRQCD) Factorization Formalism

Bodwin, Braaten, Lepage, PRD 51 (1995) 1125

$$\sigma(gg \to J/\psi + X) = \sum_n \sigma(gg \to c\bar{c}(n) + X)\langle \mathcal{O}^{J/\psi}(n)\rangle$$

$$n - {}^{2S+1}L_J^{(1,8)}$$

double expansion in α_s, v

NRQCD long-distance matrix element (LDME)

$$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[1]})\rangle \sim v^3$$

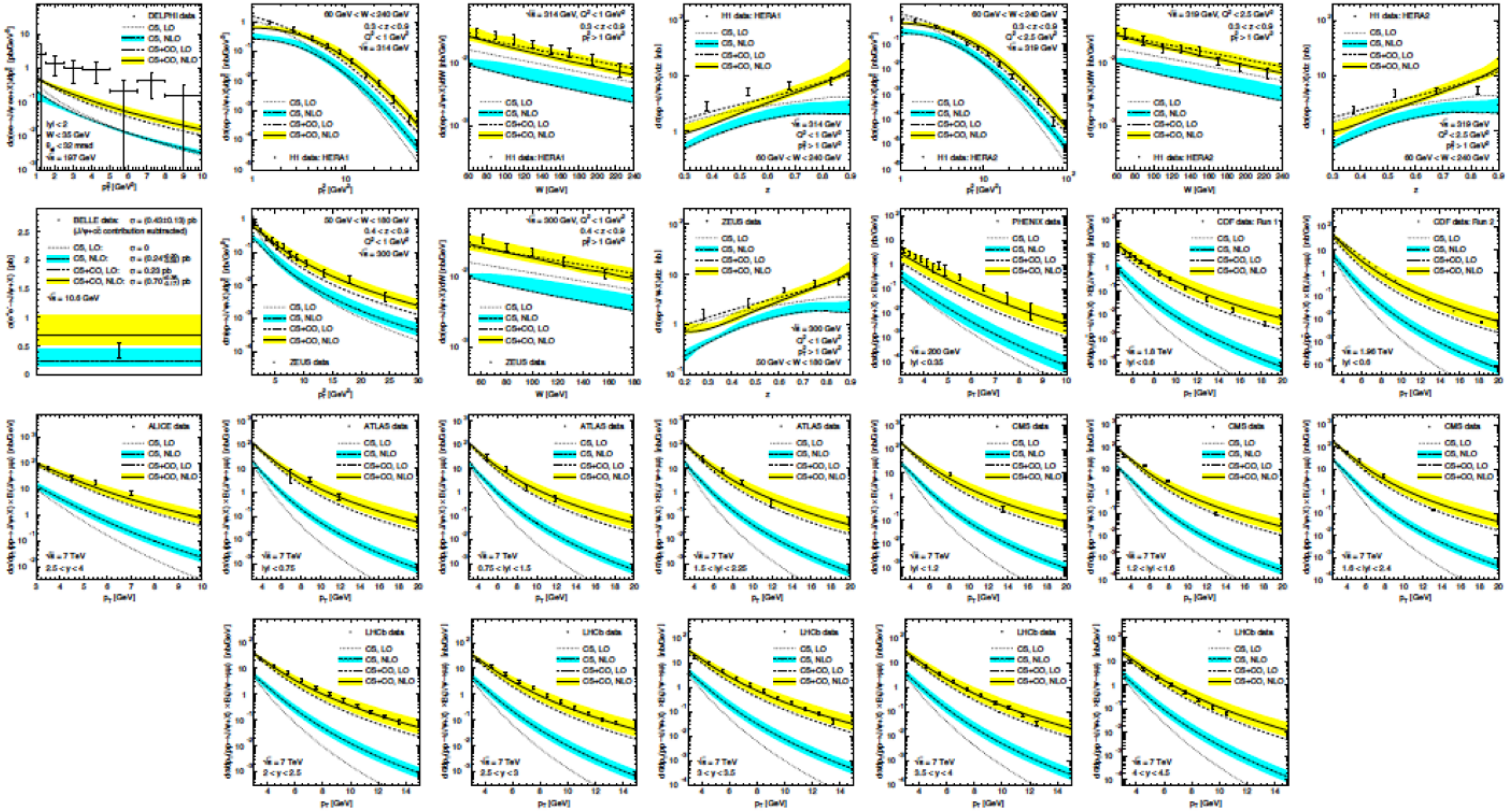
CSM - lowest order in v

$$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]})\rangle, \langle \mathcal{O}^{J/\psi}({}^1S_0^{[8]})\rangle, \langle \mathcal{O}^{J/\psi}({}^3P_J^{[8]})\rangle \sim v^7$$

color-octet mechanisms

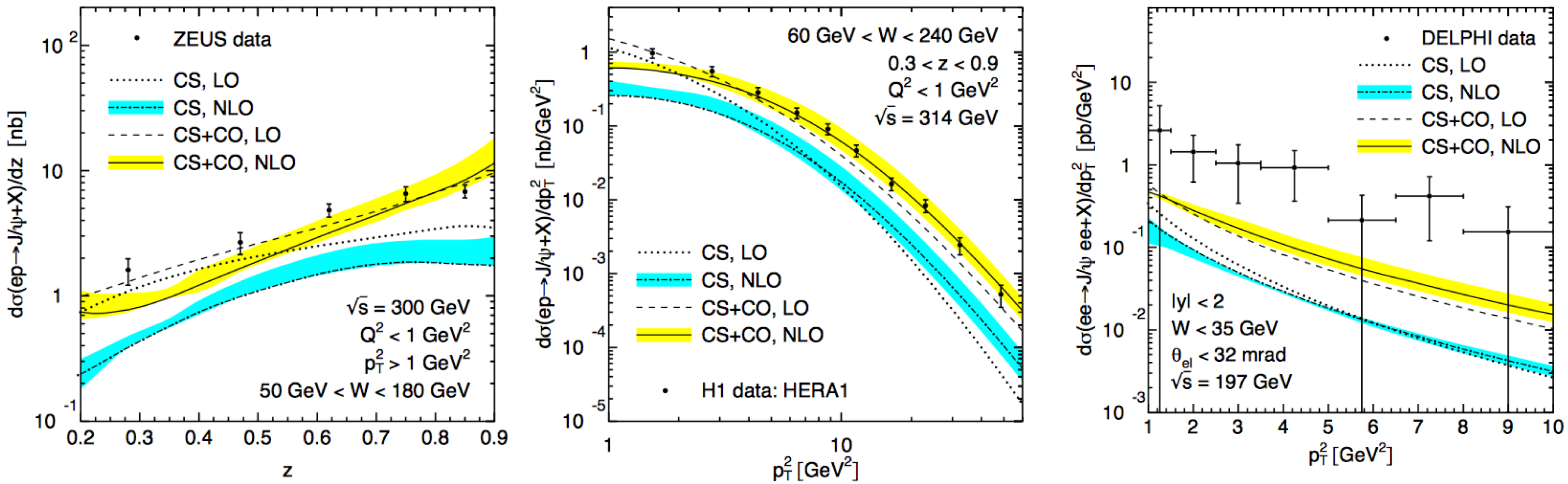
Global Fits with NLO CSM + COM

Butenschoen and Kniehl, PRD 84 (2011) 051501



$e^+e^-, \gamma\gamma, \gamma p, p\bar{p}, pp \to J/\psi + X$ fit to 194 data points, 26 data sets

NLO: CSM + COM Required to Fit Data



$$ep \to J/\psi + X$$

$$\gamma^*\gamma^* \to J/\psi + X$$

Status of NRQCD approach to J/ψ Production

NLO: COM + CSM required for most processes

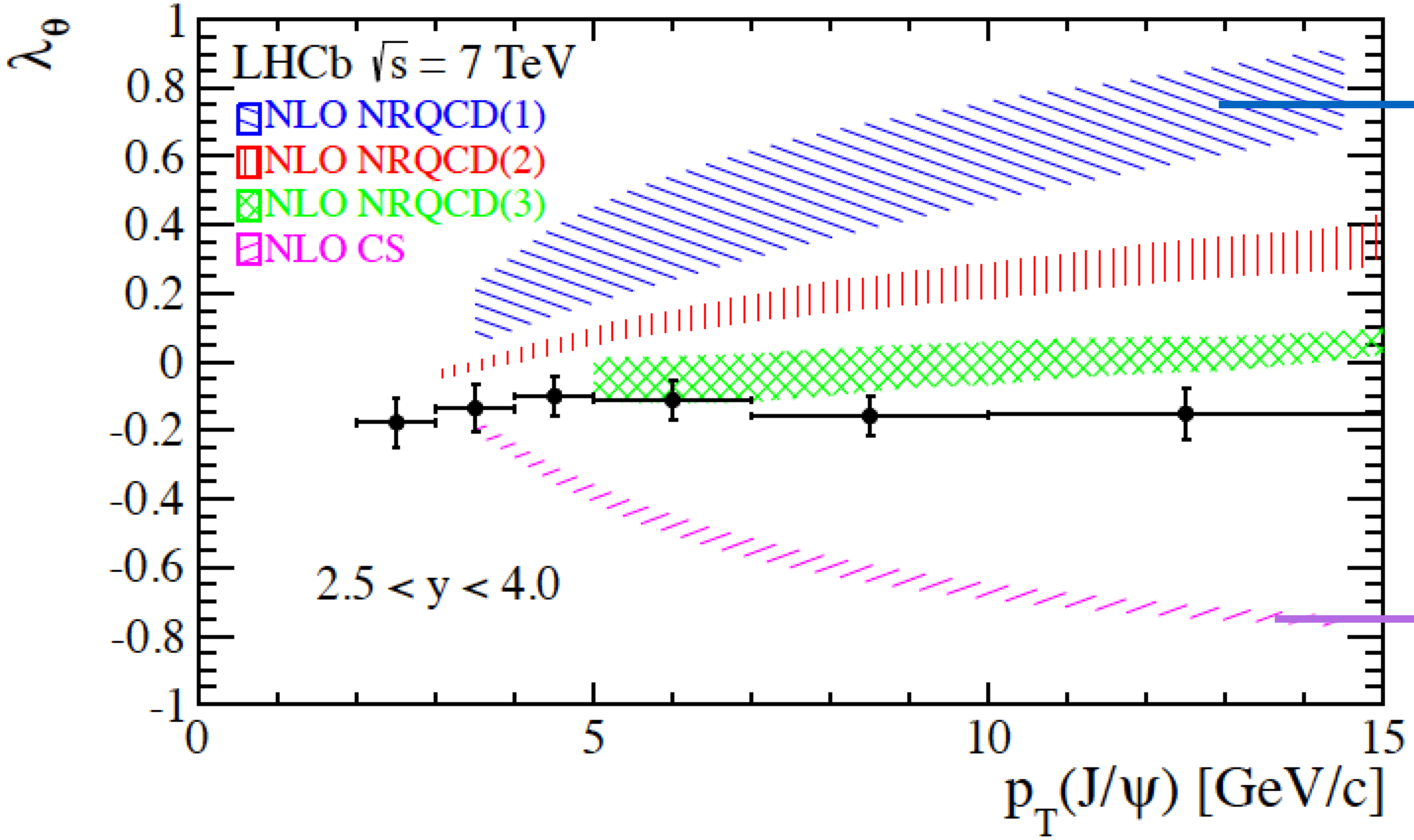
extracted LDME satisfy NRQCD v-scaling

$$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[1]}) \rangle = 1.32 \text{ GeV}^3$$

$\langle \mathcal{O}^{J/\psi}({}^1S_0^{[8]}) \rangle$	$(4.97 \pm 0.44) \times 10^{-2}$ GeV^3
$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]}) \rangle$	$(2.24 \pm 0.59) \times 10^{-3}$ GeV^3
$\langle \mathcal{O}^{J/\psi}({}^3P_0^{[8]}) \rangle$	$(-1.61 \pm 0.20) \times 10^{-2}$ GeV^5

$$\chi^2_{\text{d.o.f.}} = 857/194 = 4.42$$

Polarization of J/ψ at LHCb



LDME from Global fits

CSM

Values of LDMEs

	$\langle\mathcal{O}^{J/\psi}(^3S_1^{[1]})\rangle$ $\times$ GeV^3	$\langle\mathcal{O}^{J/\psi}(^3S_1^{[8]})\rangle$ $\times 10^{-2}$ GeV^3	$\langle\mathcal{O}^{J/\psi}(^1S_0^{[8]})\rangle$ $\times 10^{-2}$ GeV^3	$\langle\mathcal{O}^{J/\psi}(^3P_0^{[8]})\rangle/m_c^2$ $\times 10^{-2}$ GeV^3
B & K	1.32 ± 0.20	0.224 ± 0.59	4.97 ± 0.44	-0.72 ± 0.88
Chao et al.	1.16 ± 0.20	0.30 ± 0.12	8.9 ± 0.98	0.56 ± 0.21
Bodwin et al.	1.32 ± 0.20	1.1 ± 1.0	9.9 ± 2.2	0.49 ± 0.44

[7] M. Butenschoen, B. Kniehl ’11
[8] M. Butenschoen, B. Kniehl ‘13
[9] K. Chao, Y. Ma, H. Shao, K. Wang, Y. Zhang, ’12
[10] G. Bodwin, H. Chung, U. Kim, J. Lee, ‘14

$\langle\mathcal{O}^{J/\psi}(^3S_1^{[8]})\rangle$ poorly constrained by different fits

TMDFFs for Quarkonium

M. Echevarria, et. al., JHEP 10 (2020) 164, arXiv:2007.05547

p_T dependence of TMDFFs for quarkonia calculable in NRQCD

evaluated TMDFF for unpolarized light quarks, quarkonia

light quark fragmentation dominates for large Q $\frac{d\sigma(\gamma^* g)}{d\sigma(\gamma^* q)} \sim \left(\frac{M}{Qv^2}\right)^2$

light quark fragmentation $\propto \langle \mathcal{O}^{J/\psi}(^3S_1^{[8]}) \rangle$ at LO

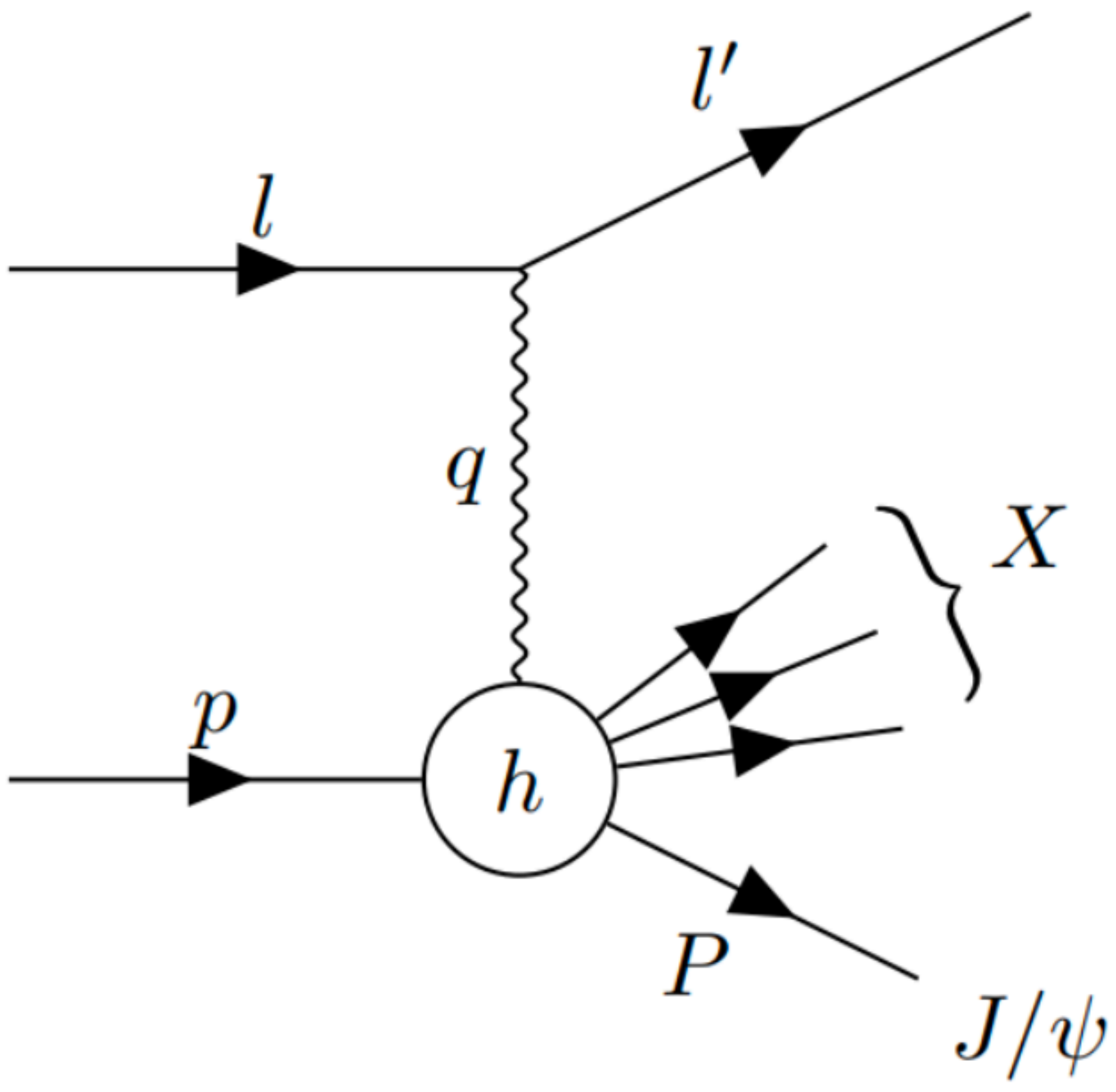
J/ψ leptoproduction at $Q > M/v^2 \sim 10$ GeV has never been measured

Kinematic variables

$$x_B = \frac{Q^2}{2p{\cdot}q} \qquad q^2 = -Q^2$$

$$y = \frac{p{\cdot}q}{p{\cdot}l} \qquad z = \frac{p{\cdot}P_{J/\psi}}{p{\cdot}q}$$

$$l + p \to l' + P + X$$

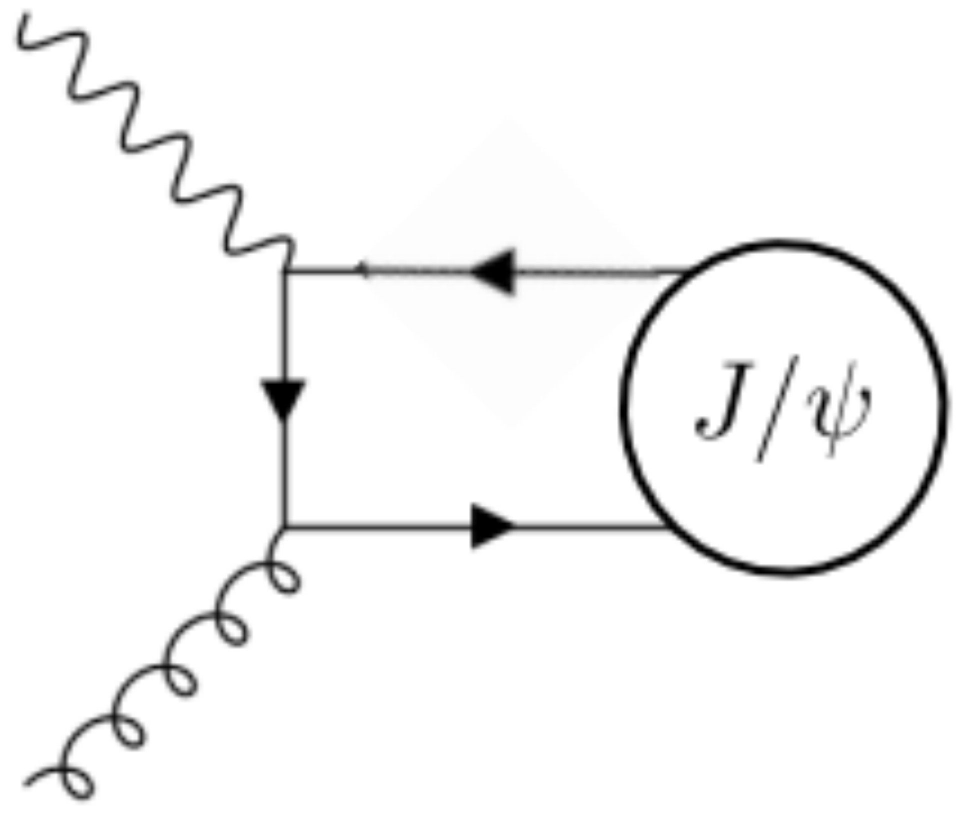


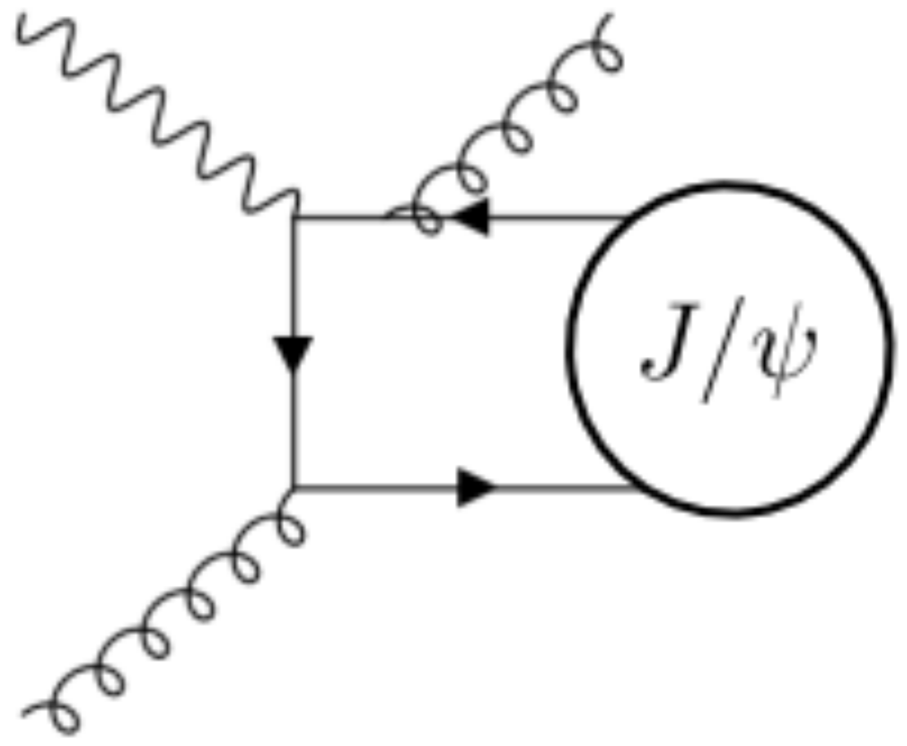
p_T defined relative to proton-photon axis

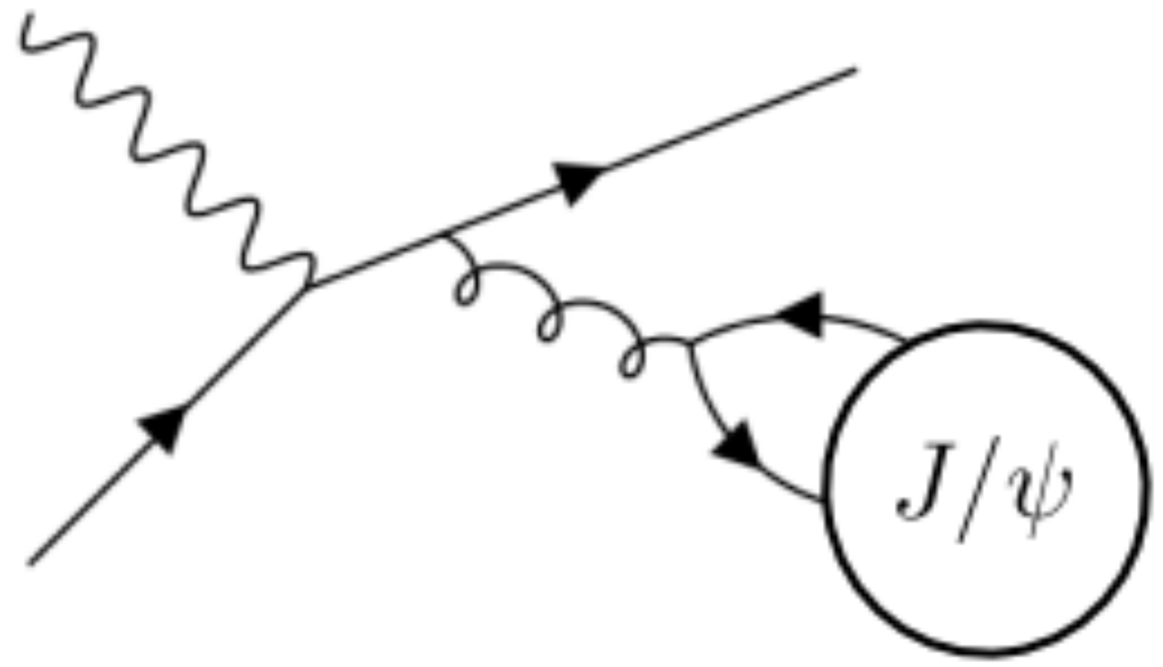
Production Mechanisms

photon-gluon fusion

light quark fragmentation







$$\left\langle \mathcal{O}^{J/\psi}({}^{[1]}S_0^{[8]}) \right\rangle \sim \alpha_s v^7$$

$$\left\langle \mathcal{O}^{J/\psi}({}^{[3]}P_J^{[8]}) \right\rangle \sim \alpha_s v^7$$

$$\left\langle \mathcal{O}^{J/\psi}({}^{[3]}S_1^{[1]}) \right\rangle \sim \alpha_s^2 v^3$$

$$\left\langle \mathcal{O}^{J/\psi}({}^{[3]}S_1^{[8]}) \right\rangle \sim \alpha_s^2 v^7$$

$$\propto \delta(1-z)\delta^{(2)}(\mathbf{P}_\perp)$$

$$\delta^{(2)}(\mathbf{P}_T) \to \frac{1}{\pi \langle P_T^2 \rangle} e^{-\mathbf{P}_T^2/\langle P_T^2 \rangle}$$

$$\delta(1-z) \to \frac{1}{\sqrt{\pi \langle \bar{z} \rangle}} e^{-(1-z)^2/\langle \bar{z} \rangle}.$$

shape function effects

S. Fleming and T. Mehen, Phys. Rev. D 57 (1998) 1846, hep-ph/9707365

Light quark TMDFFs

$$\Delta_{q\to J/\Psi} = \frac{1}{2N_C z}\mathrm{Tr}\left[\int \frac{\mathrm{db}^-}{2\pi}\mathrm{e}^{\mathrm{ib^-P^+/z}}\sum_{\mathrm{X}}\ \Gamma^{\alpha\alpha'}\langle 0|\mathrm{W}_{\mathrm{n}}^{\dagger}(\mathrm{b})\psi_{\mathrm{i}}^{\alpha 0}|\mathrm{J}/\psi,\mathrm{X}\rangle\langle \mathrm{J}/\psi,\mathrm{X}|\overline{\psi}_{\mathrm{i}}^{0\alpha'}\mathrm{W}_{\mathrm{n}}(0)|0\rangle\right]$$

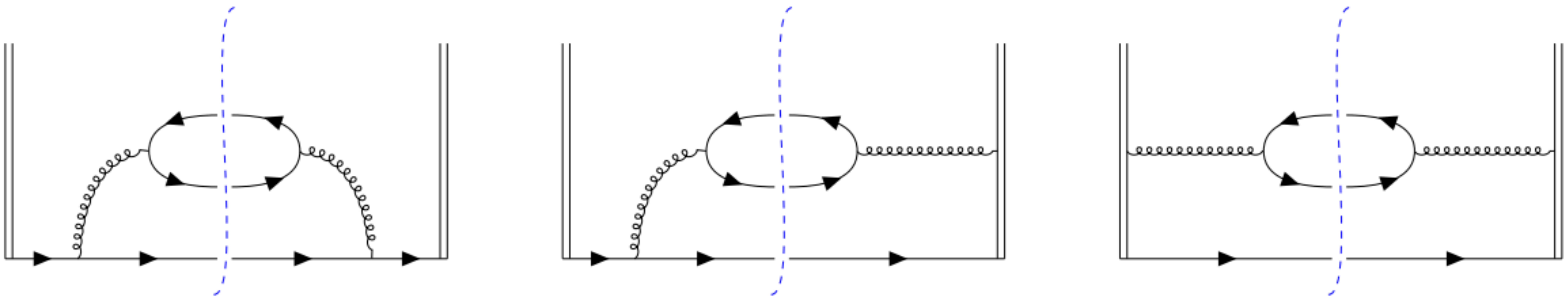
Light quark polarization: $\Gamma \in \frac{\gamma^+}{2}, \frac{\gamma^+\gamma_5}{2}, \frac{1}{2}\sigma^{\mu+}\gamma_5$

Hadron Polarization (Spin-1): $\epsilon^i\epsilon^{*j} = \frac{1}{3}\delta^{ij} - \frac{i}{2}\epsilon^{ijk}S_k - T^{ij}$

$$\vec{S} = (S_T^x, S_T^y, S_L) \qquad T_{ij} = \frac{1}{2}\begin{pmatrix} -\frac{2}{3}S_{LL} + S_{TT}^{xx} & S_{TT}^{xy} & S_{LT}^{x} \\ S_{TT}^{yx} & -\frac{2}{3}S_{LL} - S_{TT}^{xx} & S_{LT}^{y} \\ S_{LT}^{x} & S_{LT}^{y} & \frac{4}{3}S_{LL} \end{pmatrix}$$

A. Bachetta, P.J Mulders, Phys. Rev. D 62, 114004 (2000), hep-ph/0007120

Light quark TMDFFs in NRQCD



unpolarized: M. Echevarria, et. al., JHEP 10 (2020) 164, arXiv:2007.05547

polarized light quark/quarkonium: M. Copeland , et. al., accepted in PRD, arXiv:2308..08605

Assume: $\Delta_{i\to J/\psi}(z,\boldsymbol{k}_\perp) \to \sum_{L,s,c} d_{i\to c\bar{c}}(z,\boldsymbol{k}_\perp)\langle\mathcal{O}^{J/\psi}(^{2s+1}L_J^{[c]})\rangle$

Classification of quark to spin-1 TMD FFs

		Quark polarization		
		Unpolarized	Longitudinal	Transverse
Hadron polarization	U	D_1		$H_1^{\perp}$
	L		G_1	$H_{1L}^{\perp}$
	T	$D_{1T}^{\perp}$	$G_{1T}^{\perp}$	H_1, $H_{1T}^{\perp}$
	LL	D_{1LL}		$H_{1LL}^{\perp}$
	LT	D_{1LT}	G_{1LT}	$H_{1LT}^{\perp}$, H'_{1LT}
	TT	D_{1TT}	G_{1TT}	$H_{1TT}^{\perp}$, H'_{1TT}

A. Bachetta, P.J Mulders, Phys. Rev. D 62, 114004 (2000), hep-ph/0007120

$$G_{1L}(z,\mathbf{k}_T;\mu) = \frac{\alpha_s^2(\mu)}{3\pi N_c M^3}\frac{\mathbf{k}_T^2 z^2(2-z)}{[z^2\mathbf{k}_T^2 + M^2(1-z)]^2}\left\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]})\right\rangle$$

$$G_{1T}^{\perp}(z,\mathbf{k}_T;\mu) = \frac{2\alpha_s^2(\mu)}{3\pi N_c M}\frac{z(z-1)}{[z^2\mathbf{k}_T^2 + M^2(1-z)]^2}\left\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]})\right\rangle$$

$$D_1(z,\mathbf{k}_T;\mu) = \frac{2\alpha_s^2(\mu)}{9\pi N_c M^3 z}\frac{\mathbf{k}_T^2 z^2(z^2-2z+2) + 2M^2(z-1)^2}{[z^2\mathbf{k}_T^2 + M^2(1-z)]^2}\left\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]})\right\rangle$$

$$D_{1LL}(z,\mathbf{k}_T;\mu) = \frac{2\alpha_s^2(\mu)}{9\pi N_c M^3 z}\frac{\mathbf{k}_T^2 z^2(z^2-2z+2) - 4M^2(z-1)^2}{[z^2\mathbf{k}_T^2 + M^2(1-z)]^2}\left\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]})\right\rangle$$

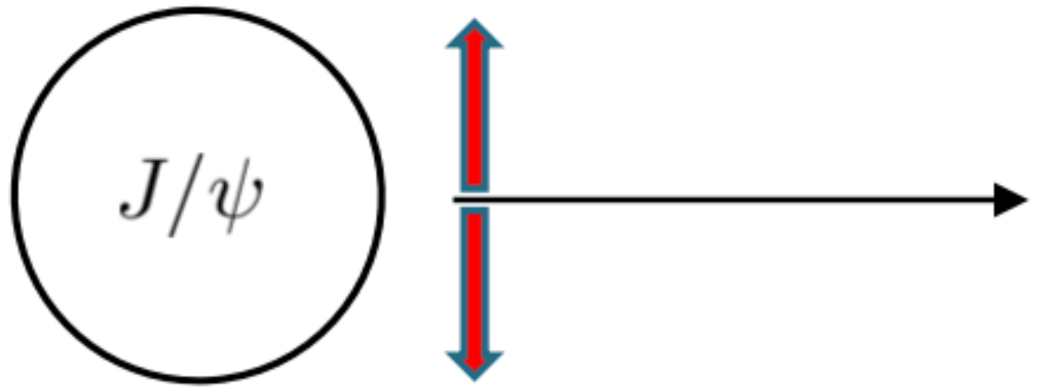
$$D_{1LT}(z,\mathbf{k}_T;\mu) = \frac{2\alpha_s^2(\mu)}{3\pi N_c M}\frac{(2-z)(1-z)}{[z^2\mathbf{k}_T^2 + M^2(1-z)]^2}\left\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]})\right\rangle$$

$$D_{1TT}(z,\mathbf{k}_T;\mu) = \frac{2\alpha_s^2(\mu)}{3\pi N_c M}\frac{z(z-1)}{[z^2\mathbf{k}_T^2 + M^2(1-z)]^2}\left\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]})\right\rangle$$

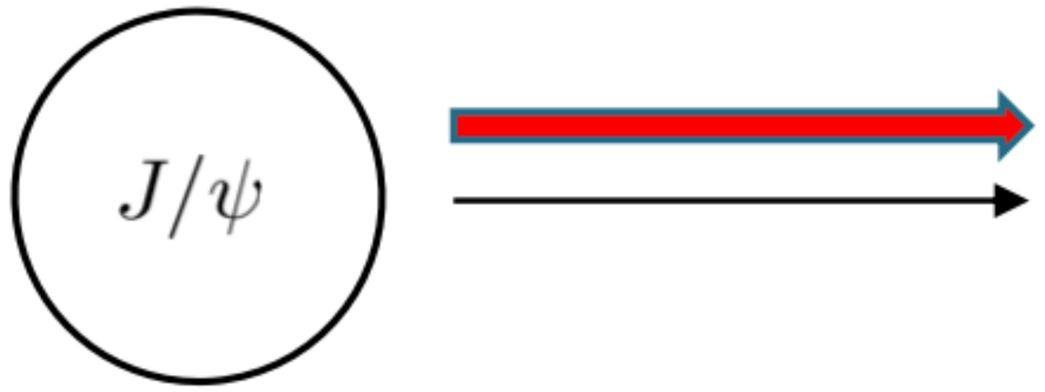
Polarized and unpolarized SIDIS cross sections

$$\frac{d\sigma_{UU}(l + H \to l' + J/\psi + X)}{dx\ dz\ dy\ d^2\mathbf{P}_\perp} = \frac{4\pi\alpha^2 s}{Q^4}\left(1 - y + \frac{y^2}{2}\right)\left\{\mathbf{I}[f_1 D_1] + S_{LL}\mathbf{I}[f_1 D_{LL}]\right\}$$

$$\frac{d\sigma_{LL}(l + H \to l' + J/\psi + X)}{dx\ dz\ dy\ d^2\mathbf{P}_\perp} = \frac{4\pi\alpha^2 s}{Q^4} 2\lambda_c S_{qL}\, y\left(1 - \frac{y}{2}\right) x\left\{\mathbf{I}[g_{1L} D_1] + S_{LL}\mathbf{I}[g_{1L} D_{1LL}]\right\}$$



$S_{LL} = 1/2$



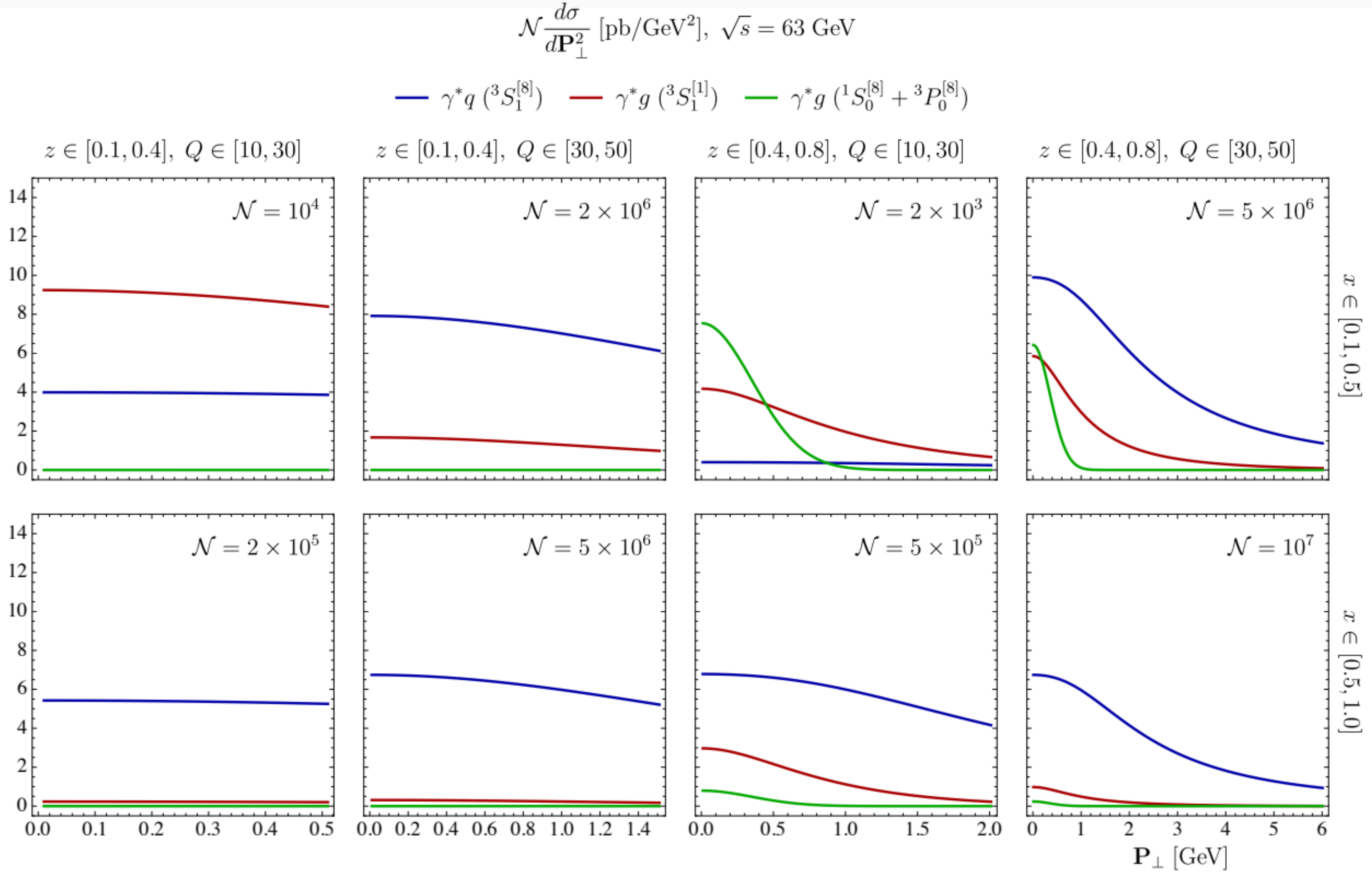
$S_{LL} = -1$

$I[f_1 D_1]$ denotes p_T convolution of f_1 and D_1

Following Echevarria, et. al., we split phase space into 8 bins

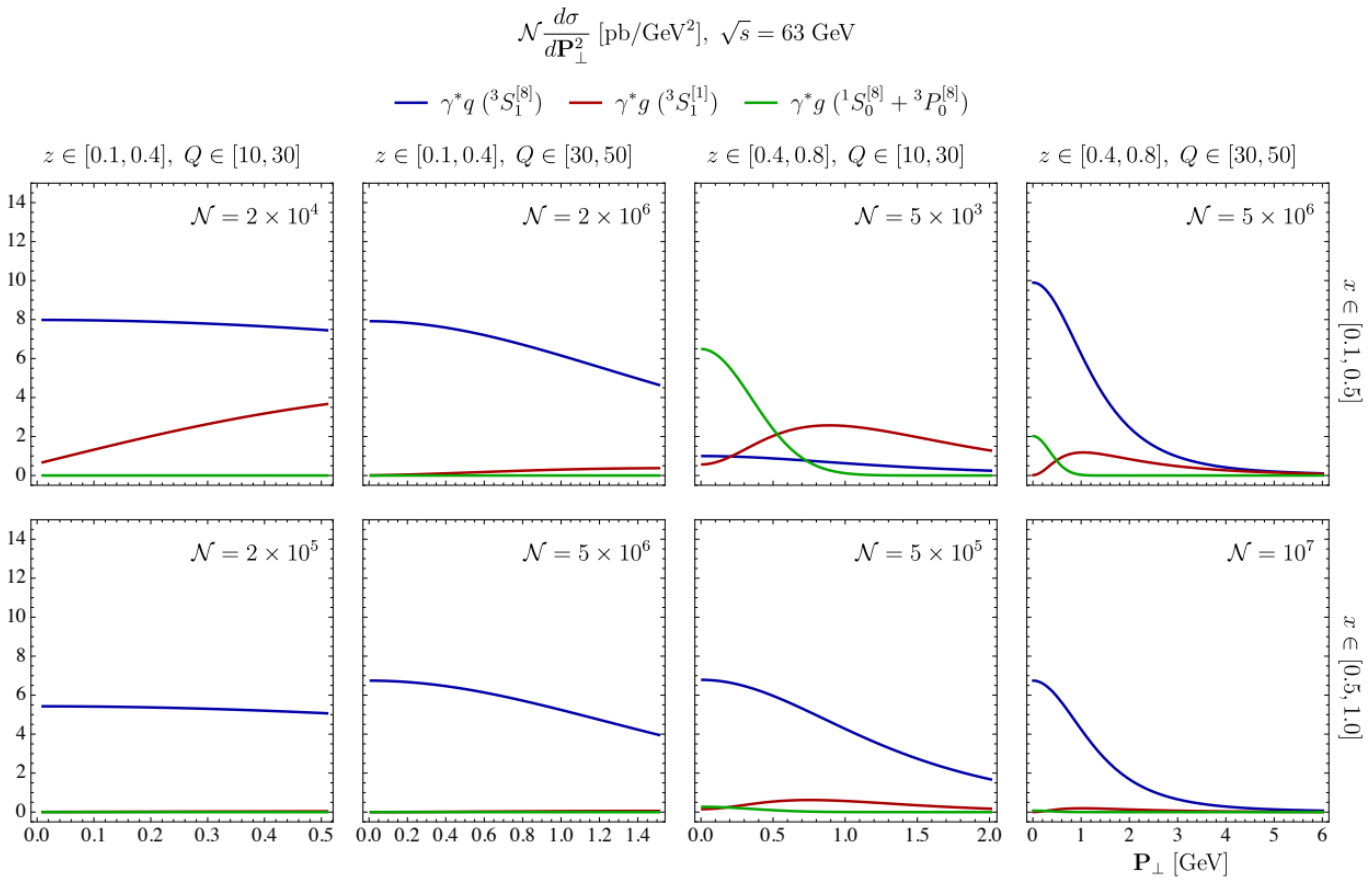
$x \in [0.1, 0.5], [0.5, 1]$ $z \in [0.1, 0.4], [0.4, 0.8]$ $Q \in [10, 30], [30, 50]$

Unpolarized J/ψ



M. Copeland , et. al., arXiv:2308..08605

Longitudinally Polarized J/ψ



M. Copeland , et. al., arXiv:2308..08605

Summary

calculated polarized TMDFFs for J/ψ in NRQCD

included shape function effects for color-octet photon-gluon fusion

new potential tests of NRQCD, help extracting gluon TMDPDFs

To Do:

NLO calculations: for unpolarized gluons TMDFFs, see

M. Echevarria, et. al., JHEP 12 (2023) 181, arXiv:2308.120332

NLO calculations for polarized quarks, gluons M. Copeland , et. al., in preparation

Feasibility of measuring at such large Q?

Other processes, e.g., quarkonium production in $e^+ e^-$