Periodicity five Lattice for a cpEDM Prototype Ring

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Outline

- Introduction
- Prototype Ring: periodicity five design proposal
- Periodicity five lattice with a bending radius of 7.2 m
  - Simulation results for the electrostatic beam operation
  - Simulation results for the frozen spin operation
- Conclusions
Introduction: Prototype ring design

• Motivation

- PTR is required as an intermediate step to address critical questions, gain experience and rule out show stoppers

- Stage 1 all electric ring with an energy of ≈30 MeV → stage 2 frozen spin conditions with an energy of 45 MeV

- The present design for the prototype is a ‘square’ ring with four 8 m long straight sections and a bending radius of 8.86 m → it consists of four unit cells, each of them bending 90°. Each unit cell consists of a focusing structure F-B-D-B-F, where F is a focusing quadrupole, D is a defocusing quadrupole, and B is an electric or magnetic bending unit

- CW and CCW operation are needed
Introduction: Prototype ring design

- Basic parameters of the PTR design:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$E$ only</th>
<th>$E$ &amp; $B$, frozen spin</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bending radius</td>
<td>8.86</td>
<td>8.86</td>
<td>m</td>
</tr>
<tr>
<td>Kinetic energy</td>
<td>30</td>
<td>45</td>
<td>MeV</td>
</tr>
<tr>
<td>$\beta = v/c$</td>
<td>0.247</td>
<td>0.299</td>
<td></td>
</tr>
<tr>
<td>$\gamma$ (kinetic)</td>
<td>1.032</td>
<td>1.048</td>
<td></td>
</tr>
<tr>
<td>Momentum</td>
<td>239</td>
<td>294</td>
<td>MeV/c</td>
</tr>
<tr>
<td>Electric field $E$</td>
<td>6.67</td>
<td>7.00</td>
<td>MV/m</td>
</tr>
<tr>
<td>Magnetic field $B$</td>
<td>0.0285</td>
<td>0.0327</td>
<td>T</td>
</tr>
<tr>
<td>r.m.s. emittances $\epsilon_x = \epsilon_y$</td>
<td>1</td>
<td>1</td>
<td>$\pi$ mm mrad</td>
</tr>
<tr>
<td>Transverse acceptance $a_x = a_y$</td>
<td>$\geq 10$</td>
<td>$\geq 10$</td>
<td>$\pi$ mm mrad</td>
</tr>
</tbody>
</table>

- Basic layout of the PTR design:

[Diagram showing ring design with dimensions 8 m and 29 m]
Prototype Ring: periodicity five design proposal

• Motivation

- The lattice described has a bending radius of 7.2 m and half of the COSY circumference
- The ratio between bending length to straight section lengths is reduced significantly
- Weak vertical focusing \(\rightarrow\) one quadrupole family need to achieve stability
- Stable lattice in the horizontal plane without gradients
- Horizontal optics properties dominated by the focusing from the bending \(\rightarrow\) just little impact from the quadrupoles
- For the all electric operation the horizontal tune is above 2 \(\rightarrow\) works well with higher periodicity
PTR periodicity five design with bending radius of 7.2 m

- Design of the ring

![Diagram of the ring design](image)

**Lattice structure**

**Basic parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$E$ only</th>
<th>$E&amp;B$ frozen spin</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bending radius $\rho$</td>
<td>7.20</td>
<td>7.20</td>
<td>m</td>
</tr>
<tr>
<td>Circumference $C$</td>
<td>91.79</td>
<td>91.79</td>
<td>m</td>
</tr>
<tr>
<td>Kinetic energy $E_k$</td>
<td>25.538</td>
<td>37.034</td>
<td>MeV</td>
</tr>
<tr>
<td>relativistic $\beta = v/c$</td>
<td>0.2287</td>
<td>0.2730</td>
<td></td>
</tr>
<tr>
<td>relativistic $\gamma$</td>
<td>1.02722</td>
<td>1.03947</td>
<td></td>
</tr>
<tr>
<td>Momentum $p$</td>
<td>220.4</td>
<td>266.2</td>
<td>MeV/c</td>
</tr>
<tr>
<td>Electric field $E$</td>
<td>7.00</td>
<td>7.00</td>
<td>MV/m</td>
</tr>
<tr>
<td>Magnetic field $B$</td>
<td>0</td>
<td>0.03779</td>
<td>T</td>
</tr>
</tbody>
</table>
Simulation Results

- Working points of some of the quads settings - tune diagram

- Red lines \(\rightarrow\) frozen spin beam operations
- Green lines \(\rightarrow\) electrostatic beam operations
- The solid and the dashed lines are for finite quadrupole components of quadrupole family QE and QC, respectively
Simulation Results

• Electro-static operation at 25.54 MeV-Twiss parameters and horizontal dispersion from Mathematica and BMAD simulations

➤ Case a

![Graph showing electro-static operation parameters for Case a]

QE = -0.02 m⁻¹  QC = 0.03960 m⁻¹

➤ Case b

![Graph showing electro-static operation parameters for Case b]

QE = 0.02 m⁻¹  QC = -0.04030 m⁻¹
Simulation Results

- Electro-static operation at 25.54 MeV-Twiss parameters and horizontal dispersion from Mathematica and BMAD simulations

Case c

- QE = 0.04 m^{-1}  
  \[ QC = -0.07530 \ m^{-1} \]

Case d

- QE = 0.06 m^{-1}  
  \[ QC = -0.10753 \ m^{-1} \]
Simulation Results

- Electro-static operation at 25.54 MeV- Spin tracking and analytical evaluation of the spin tune
  
  - Spin tune for the purely electric case: \( Q_E = G\gamma - \frac{G+1}{\gamma} = -0.877191 \)
  
  - From simulations: \( Q_S = -\frac{\alpha}{2\pi} = \frac{1}{2\pi} \arccos(S_z) - 1 = -0.877191 \)

  With \( S_z = 0.7168057 \) and \( \alpha = 2\pi - \arccos(S_z) \)

  ![Diagram showing spin tune](image)

  showing good agreement with simulations...
Simulation Results

- Frozen spin operation at 37.03 MeV-Twiss parameters and horizontal dispersion from Mathematica and BMAD simulations

Case a

- QE= -0.02 m\(^{-1}\)
- QC= 0.03960 m\(^{-1}\)

Case b

- QE= 0.02 m\(^{-1}\)
- QC= -0.04030 m\(^{-1}\)
Simulation Results

- Frozen spin operation at 37.03 MeV-Twiss parameters and horizontal dispersion from Mathematica and BMAD simulations

➤ Case c

➤ Case d
Simulation Results

- Frozen spin operation at 37.03 MeV-Spin Tracking results
Conclusions

- Different quads family settings for the periodicity five PTR ring have been studied:
  - Different scenarios for the possible working points have been showed
  - Evaluation of the lattice functions with BMAD benchmarked with Mathematica results

- Possible operations with the all electrstring ring as first step → spin tune evaluation agrees with BMAD simulations

- Possible operations with frozen spin condition → the simulation confirms the frozen spin is maintained

Thank you for the attention!