





Mapping Techniques

Hall Probes

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Contents of this lecture

Introduction

- What is a Hall sensor, how does it work and why do we use it for field mapping?

Challenges

- What are physical effects limiting the performance of Hall sensor measurements?

Calibration techniques

- How can we characterize these effects?

Mapper systems

- What are the additional challenges for mapper systems?

Post-processing

- How can we overcome the limitations in the post-processing?

Outlook

Literature

Topic	Author	Reference
Hall Effect devices	R. S. Popovic	[1]
Hall probes: physics and application to magnetometry	S. Sanfilippo	[2]
Design of three axis Hall sensors	S. C. Wouter	[3]
Magnetic field reconstruction in 3D	M. Liebsch	[4]

Other references are listed in the bibliography.

- [1] R.S. Popović. "Hall-effect devices". In: *Sensors and Actuators* 17.1 (1989), pp. 39 –53. ISSN: 0250-6874. DOI: [https://doi.org/10.1016/0250-6874\(89\)80063-0](https://doi.org/10.1016/0250-6874(89)80063-0).
- [2] S. Sanfilippo. "Hall probes: physics and application to magnetometry". In: *CERN Accelerator School: Course on Magnets*. Mar. 2011, pp. 423–462. arXiv: 1103.1271 [physics.acc-ph].
- [3] Silke Christina Wouters. "New Type of Three-Axis Hall Sensor Designed for High-Accuracy Magnetic Field Measurements". PhD Thesis. ETH Zürich, 2016. DOI: <https://doi.org/10.3929/ethz-a-010749366>.
- [4] Melvin Liebsch. "Inference of Boundary Data from Magnetic Measurements of Accelerator Magnets". en. PhD thesis. Darmstadt: Technische Universität, 2022, ix, 150 Seiten. DOI: <https://doi.org/10.26083/tuprints-00021144>.

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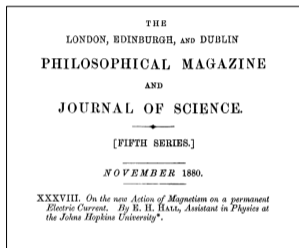
- What are the additional challenges for mapper systems?

Post-processing

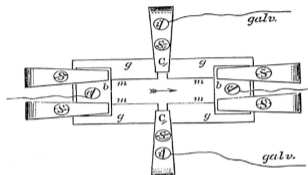
- How can we overcome the limitations in the post-processing?

Outlook

Edwin Hall's discovery



The first Hall plate (from [5])



m gold-leaf
b brass contacts
g glass plate



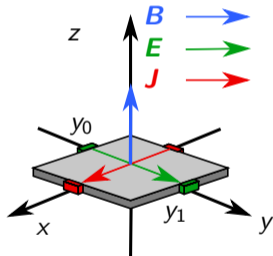
Edwin Herbert Hall (1855-1938) (from [6])

- Hall's apparatus has made the **interaction** between the **magnetic field** and the **electric current** measurable
- This was ***eighteen years before the electron was discovered!***

Galvanomagnetic effects

: Physical effects arising in matter carrying current in the presence of a magnetic field.

The Hall effect



Charge carriers experience the Lorentz force:

$$\mathbf{F} \propto \mathbf{v}_d \times \mathbf{B}$$

The electric field in the material is:

$$\mathbf{E} = \rho \mathbf{J} + R_H (\mathbf{B} \times \mathbf{J}) + \mathcal{O}(|\mathbf{B}|^2)$$

The measured quantity is the **Hall voltage**:

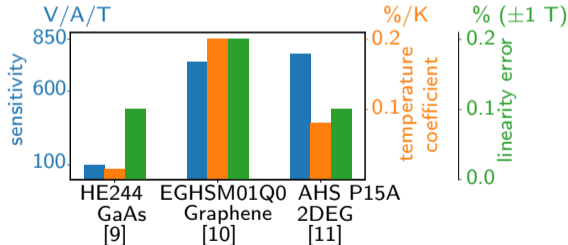
$$U_{\text{Hall}} = \int_{y_0}^{y_1} \mathbf{E} \cdot \mathbf{e}_y dy \approx s_{\text{Hall}} I_{\text{Hall}} (\mathbf{n}_{\text{Hall}} \cdot \mathbf{B})$$

\mathbf{B} :	Magnetic flux density
\mathbf{E} :	Electric field
\mathbf{J} :	Current density
\mathbf{v}_d :	Drift velocity
\mathbf{n}_{Hall} :	Normal vector
I_{Hall} :	Hall current
s_{Hall} :	Sensitivity
ρ :	Resistivity
R_H :	Hall coefficient

Hall sensor technology

- **Semiconductor technology** (emerging in the 50s) made Hall **sensors** feasible
- Today's Hall sensors are about **1000 times more sensitive** than Edwin Hall's gold leaf
- **Low doped semiconductors** (InSb, Si, GaAs) are established
- Recently **graphene** sensors have been brought on the market [7]
- A **promising new approach** is to exploit the Hall effect in very thin (**2D electron gas**) layers \Rightarrow 2DEG-sensors [8]

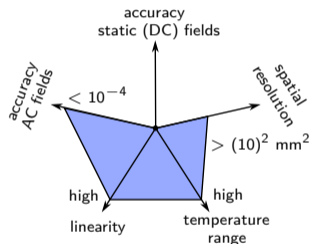
	sensitivity	temperature stability	field range
carrier concentration	↓	↑	↑
mobility	↑		



A comparison

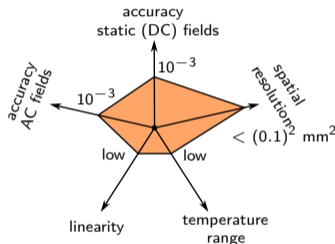
Induction coils

$$U_{\text{ind}} = - \frac{d\Phi}{dt}$$



Hall Sensors

$$U_{\text{Hall}} \approx s n_{\text{Hall}} \cdot B$$



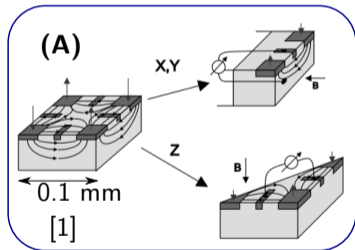
- Induction coils are **not sensitive** to DC fields
- A **movement** is required to induce a voltage
- Sensitivity of induction coil is **proportional to the coil surface**
- This impairs the **spatial resolution**

Advantages of Hall sensors:

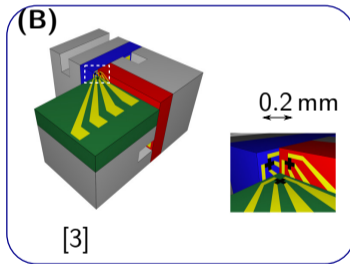
- ⇒ **Small spatial resolution** ⇒ field mapping in **high resolution**
- ⇒ **No integration** ⇒ **no drift!**

Three component measurements

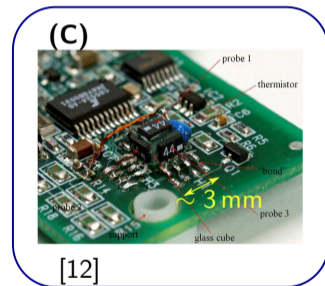
- **Three components measurements** are possible in two ways
- Three component Hall sensor on **single chip (A)**
- **Hall cube**: Three 1D sensors on an orthogonal cube **(B) & (C)**



Three axes on single chip



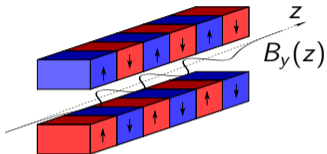
Advanced orthogonal cube



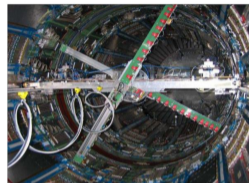
Standard orthogonal cube

Field mapping in accelerator magnet technology

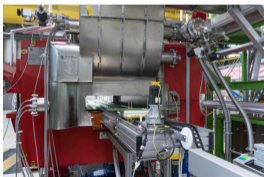
Insertion devices



Detector magnets



Spectrometer magnets



Strongly curved magnets



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Outlook

Challenges for Hall effect devices

One component measurements

- Temperature effects
- Offset voltage
- Sensor noise
- Non-linearity
- The planar Hall effect

⇒ **Will be discussed in detail**

Three component measurements

- Orthogonality
- Sensor positions

Temperature effects

The **temperature coefficient** for s_{Hall} is:

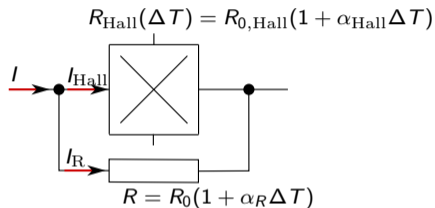
$$TC_I := \frac{1}{s_{\text{Hall}}} \frac{\partial s_{\text{Hall}}}{\partial T}$$

Type	TC_I
Si:	$0.8 \times 10^{-3} \text{ K}^{-1}$
GaAs:	$0.3 \times 10^{-3} \text{ K}^{-1}$
HE244 (@ 25 °C):	$0.15 \times 10^{-3} \text{ K}^{-1}$

Mitigation measures (to obtain 10^{-4})

- **Stabilize Hall temperature** $|\Delta T| < 1 \text{ K}$
- **Calibrate** TC_I , **measure** T and **correct** s_{Hall}
- **Biasing current method** (5.6.2 [1])

Biasing current method



$$s_{\text{Hall}} = s_{0,\text{Hall}}(1 + \alpha_s \Delta T)$$
$$\frac{R_{\text{Hall}}}{R + R_{\text{Hall}}} = \frac{\alpha_s}{\alpha_{\text{Hall}} - \alpha_R}$$

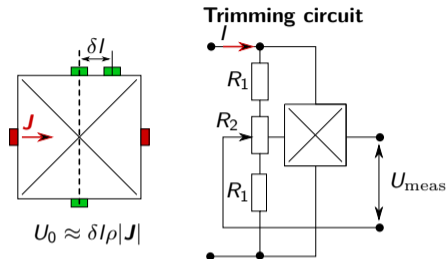
Problem: Magneto-resistance effect for high fields.

Offset voltage

- Fabrication tolerances result in a **zero field offset voltage** U_0

$$U_{\text{Hall}} \approx s_{\text{Hall}} I (\mathbf{n}_{\text{Hall}} \cdot \mathbf{B}) + U_0$$

- Correction with a **trimming circuit** possible
- Problem: Trimming is not stable!** Thermal drifts, mechanical shocks and aging!
- Precise measurements require an offset correction **before each measurement**
- Another mitigation measure is the **spinning current technique**



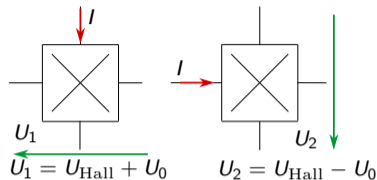
Zero Gauss Chamber



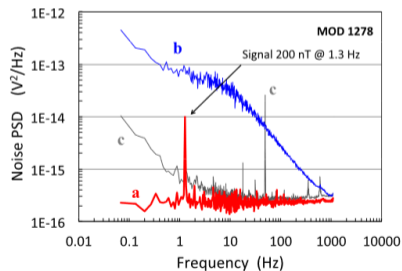
The spinning current technique

- For symmetric Hall sensors, **current and sensing contacts** can be **interchanged**
- Averaging two measurements with interchanged current/sensing contacts **effectively cancels out the offset voltage**
- The $1/f$ sensor noise is attributed to **conductance fluctuations** [13]
- If the switching frequency is large enough (typ. ~ 10 kHz), the spinning current technique also yields a $1/f$ **noise cancellation** [14]

- (a) power spectral density achieved by spinning current technique
(b) low frequency noise of the device
(c) low frequency device of pre-amplifier and connection circuit



$1/f$ noise cancellation [14]



Non-linearity

- **Geometrical properties** and **material effects** give rise to a field dependent sensitivity

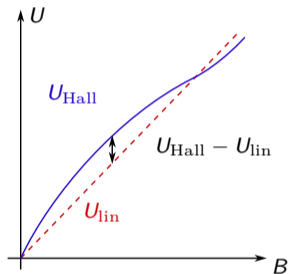
$$s_{\text{Hall}} = f(B)$$

- This yields a **non-linear** Hall voltage U_{Hall}
- The **non-linearity** NL is defined as

$$NL = \frac{U_{\text{Hall}}(B) - U_{\text{lin}}(B)}{U_{\text{lin}}(B)}$$

- $U_{\text{lin}}(B)$ is the **best fit** linear response
- **Typical values** are $0.05\% < NL < 0.5\%$

To obtain a 10^{-4} accuracy, the non-linearity needs to be **calibrated!**



The planar Hall effect (PHE)

- The **non-vanishing thickness** of the device gives rise to the so-called **planar Hall effect PHE**
- The voltage related to the PHE is (see Figure left)

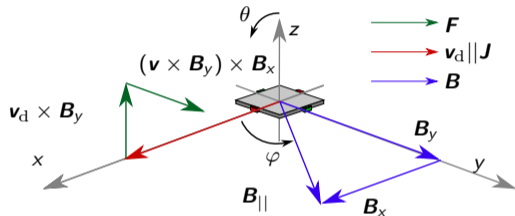
$$U_{\text{PHE}} \propto (\mathbf{J} \cdot \mathbf{B})(\mathbf{e}_y \cdot \mathbf{B})$$

- It has a **double angular dependence**

$$U_{\text{PHE}} = s_{\text{PHE}} I |\mathbf{B}|^2 \sin(\theta) \sin(2\varphi)$$

- Mitigation: Average the output of **two 90 degree rotated Hall plates**

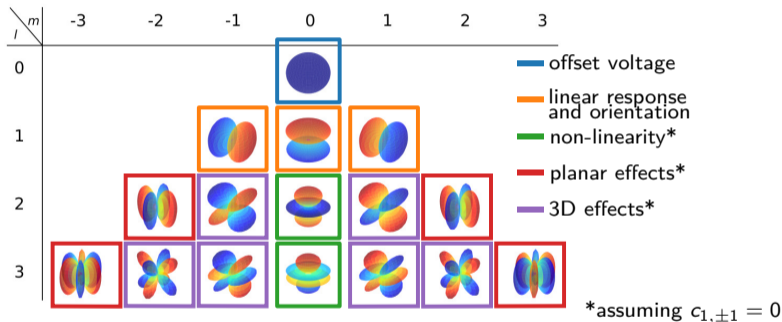
An illustrative description of the PHE



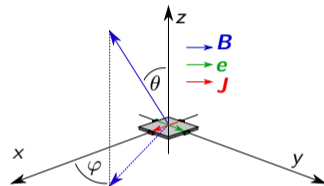
$\mathbf{B}_{||}$: Planar field component
 θ : Polar angle
 φ : Azimuth angle
 s_{PHE} : Planar Hall coefficient

Hall effects in three dimensions

Physical effects can be **decoupled** by expanding the Hall voltage in **spherical harmonic functions** [12]



$$U_{\text{Hall}}(|\mathbf{B}|, T, \theta, \varphi) = \sum_{l=0}^L \sum_{m=-l}^l s_{l,m}(T) \underbrace{|\mathbf{B}|^l Y_l^m(\theta, \varphi)}_{\text{solid harmonics}}.$$



This model includes:

- Offset
- Non-linearity
- Planar-Hall effect
- 3D Hall effects
- Temperature

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Outlook

Calibration techniques

- A calibration is the comparison of measured values delivered by a device under test with those of a **reference of known accuracy**.
- The **difference** to the calibration reference can be used to **correct the parameters** s_l, m of the sensor model.
- The calibration references are:

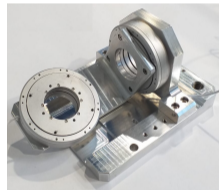
(A) zero Gauss chamber



(B) nuclear magnetic resonance (NMR) sensor



(C) piezo-electric rotary stages



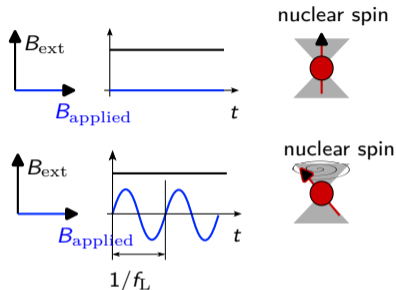
parameters	offset voltage	non-linearity	orthogonality	planar and 3D effects
reference	(A)	(B)	(C)	(B) + (C)

References - Nuclear magnetic resonance

- The spin of a nucleus tends to align to a strong magnetic field \mathbf{B}_{ext}
- Energy levels are **quantized!** The nucleus can absorb electromagnetic waves of the frequency

$$f_L = \frac{\gamma |B|}{2\pi}$$

- γ is the **gyromagnetic ratio**

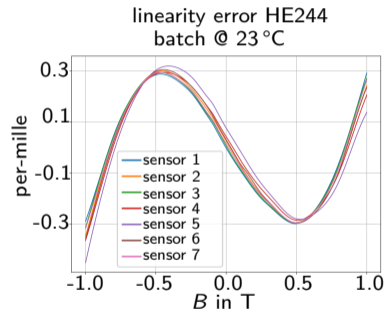


Limitations

- Uniform fields (typical required homogeneity ~ 2 per-mille/cm @ 1 T.)
- Moderate to high fields, typ. $|B| > 14$ mT
- Typ. temperature $T \approx 25^\circ\text{C}$

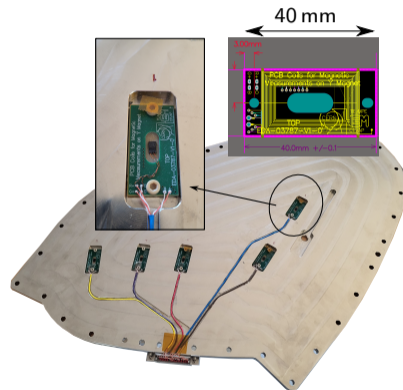
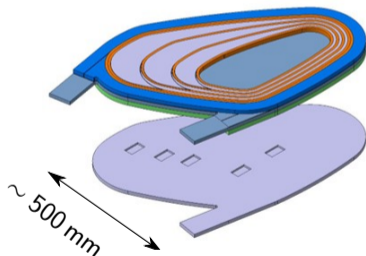
Calibration at room temperature

- Calibration dipole provides field reference
- Magnet in-homogeneity needs to be calibrated (Hall sensor and NMR **not** at same spot!)
- Magnet $B(I)$ curve is non-linear (iron-saturation)
- NMR provides the B -field reference



In-situ calibration at very low temperatures

- The NMR is **not suited** for measurements at **cryogenic temperatures**
- A **hybrid** of **Hall sensor** and **induction coil** was developed by C. Petrone for this purpose (see [15])



In-situ calibration at very low temperatures

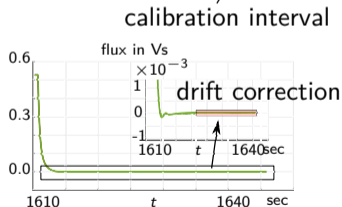
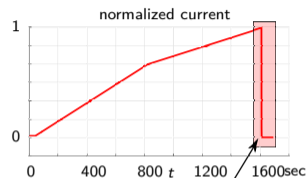
- The induction coil provides the reference

$$\Phi(t) = \int_{t=t_0}^T U_{\text{ind}}(t) dt + \Phi(t_0)$$

- $\Phi(t_0) = -\Phi(T)$, if: $B(T) = 0$
- The **field reference** is (A : coil surface)

$$B(t) = \frac{\Phi(t)}{A}$$

- **Limitations:**
 - ⇒ Short time intervals $T < 30$ s (integrator drift)!
 - ⇒ No information about remanent fields!



Orthogonality errors – 3D effects

- The measured voltages are expanded into the solid harmonics functions

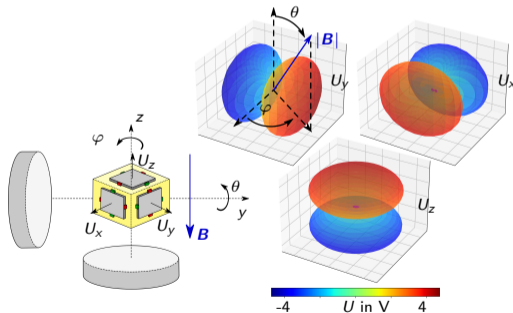
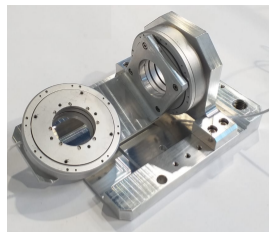
$$U(|\mathbf{B}|, T, \theta, \varphi) = \sum_{l=0}^L \sum_{m=-l}^l s_{l,m}(T) \underbrace{|\mathbf{B}|^l Y_l^m(\theta, \varphi)}_{\text{solid harmonics}}.$$

- **Orientation** ($l = 1$):

$$\varphi = -\arg(s_{1,1}), \quad \theta = \arg\left(s_{1,0} + j\sqrt{2}|s_{1,1}|\right).$$

- **Sensitivity, non-linearity, PHE, 3D effects:** Encoded in the coefficients $s_{l,m}$!

- Reference $|\mathbf{B}|$ is provided by NMR



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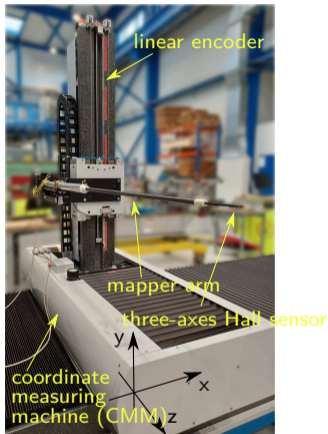
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Outlook

Mapper systems: An example



The Hall probe mapper system at CERN

Stages of a coordinate measuring machine (CMM)

Linear encoders with a resolution of $0.1 \mu\text{m}$

Measurement precision of the linear encoder of $5 \mu\text{m}$

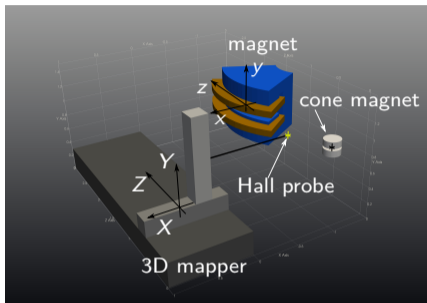
Measurable volume of $3 \times 1 \times 1 \text{ m}^3$

Specified and validated positioning accuracy of the stage of 0.1 mm

Measurements on the fly possible due to the distance trigger generation of up to 100 Hz

Nominal speed $v = 20 \text{ mm s}^{-1}$

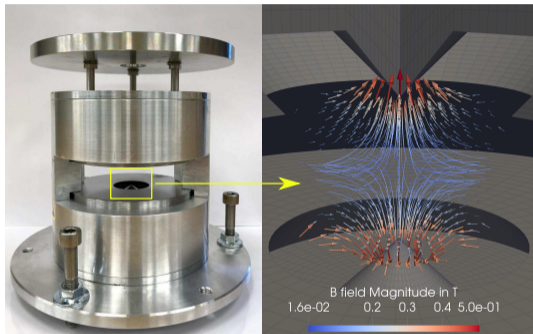
Fiducialization



- (1) A magnet coordinate system is constructed from optical measurements of the magnet geometry.
- (2) The mapper orientation vectors \mathbf{X} , \mathbf{Y} are \mathbf{Z} are determined by measuring the stage movement along the three axes.
- (3) The origin of the mapper coordinates $(x_0, y_0, z_0)^T$ can now be determined, if the mapper coordinates are known for any point in the magnet coordinates.

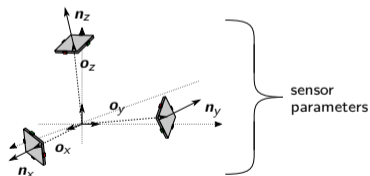
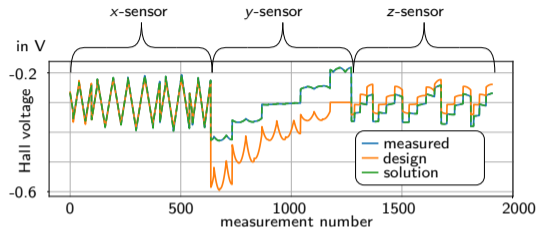
$$\underbrace{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}_{\text{magnet coordinates}} = \underbrace{\begin{pmatrix} \mathbf{X}^T \\ \mathbf{Y}^T \\ \mathbf{Z}^T \end{pmatrix}}_{\text{mapper orientation}} \cdot \underbrace{\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}}_{\text{mapper coordinates}} + \underbrace{\begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}}_{\text{origin of the mapper coordinates}}$$

The cone quadrupole magnet



⇒ Distinct zero field position.

⇒ Allows for the calibration of sensor distances.



Structural vibrations

- Mapper arm can be modeled as **cantilever Beam** (w : displacement)

$$\underbrace{(L w)(z, t)}_{\text{stiffness}} + \underbrace{(C \dot{w})(z, t)}_{\text{damping}} + \underbrace{(M \ddot{w}(z, t))}_{\text{mass}} = \underbrace{p(z, t)}_{\text{load}}$$

- **Leveraging effect:** Deformation $w \propto L^3$
- **Resonance frequency:** First natural frequency $f_1 \propto 1/L^2$
- **Typical values:** $f_1 < 10 \text{ Hz}$ @ $L = 3 \text{ m}$
- Challenging for measurements on the fly: Vibration perturbs measured signal at **low and medium frequencies!**

Finite Element Analysis

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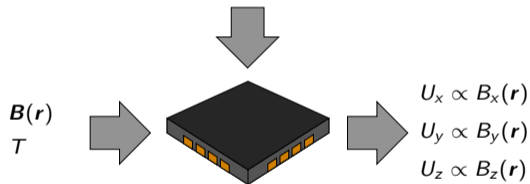
The paradigm shift

Semiconductor optimization

- Non-linearity
- Noise
- Temperature coefficient
- Active volume

Circuit optimization

- Offset voltage
- Noise
- Planar Hall effect

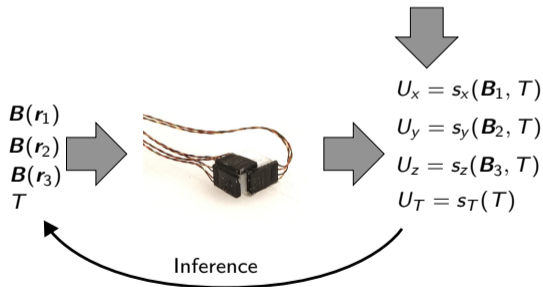


⇒ Research topic of sensor developers

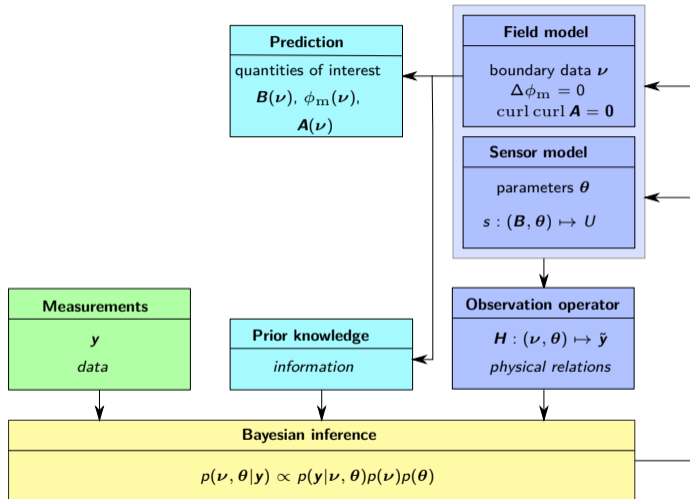
Such sensors are difficult to find and expensive!

Sensor calibration

- Offset voltages
- Orientations
- Positions
- Non-linearity
- Planar and 3D effects
- Temperature drift



The blue-print



The field model - The representation formula

The representation formula

$$\phi_m(\mathbf{r}) = \underbrace{\int_{\partial\Omega} g(\mathbf{r}') u^*(\mathbf{r}, \mathbf{r}') d\mathbf{r}'}_{\text{single layer potential}} - \underbrace{\int_{\partial\Omega} u(\mathbf{r}') \partial_{n'} u^*(\mathbf{r}, \mathbf{r}') d\mathbf{r}'}_{\text{double layer potential}}.$$

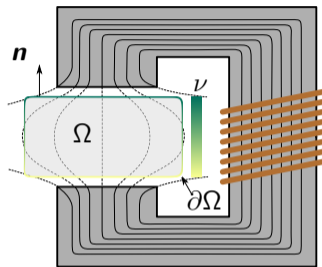
u Dirichlet data “potential at boundary”

g Neumann data “normal flux through boundary”

u^* Greens function $\Delta u^* = \delta$

\Rightarrow **Dirichlet and Neumann data are linearly dependent!**

\Rightarrow Problem **must** be formulated in u **or** g !



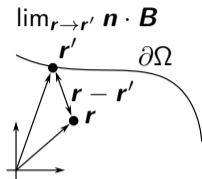
Magnetostatics in Ω

$$\operatorname{div} \mathbf{B} = 0, \quad \operatorname{curl} \mathbf{H} = \mathbf{0}.$$

$$\mathbf{H} = -\operatorname{grad} \phi_m, \quad \mathbf{B} = \operatorname{curl} \mathbf{A}.$$

$$\operatorname{grad} \operatorname{div} \phi_m = 0, \quad \operatorname{curl} \operatorname{curl} \mathbf{A} = \mathbf{0}.$$

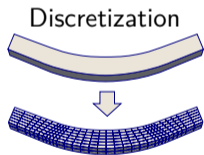
Boundary element methods (BEM)



The **Neumann** to **Dirichlet** map

$$(D \mathbf{u})(\mathbf{r}') = ((1/2 I - K') \mathbf{g})(\mathbf{r}').$$

- K' Adjoint double layer operator
- D Hypersingular operator
- I Identity operator



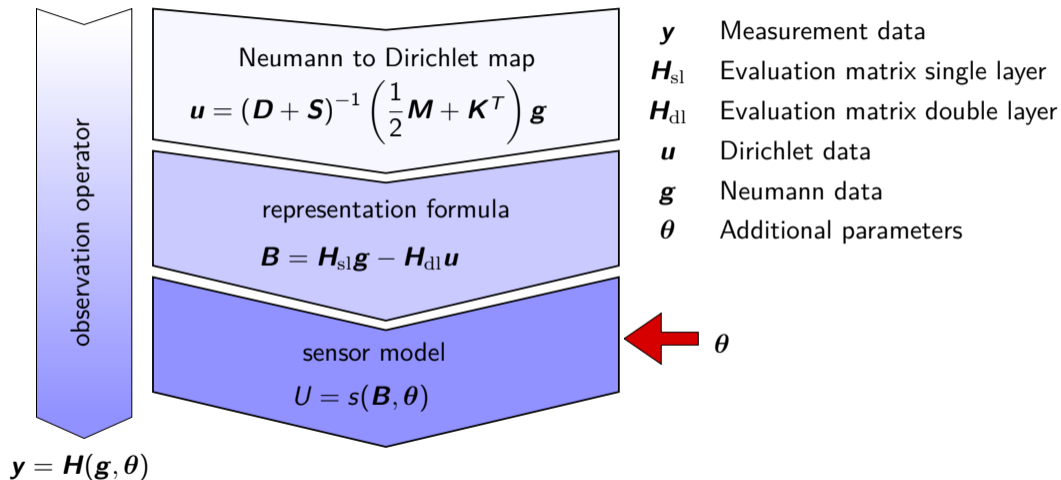
The discrete map

$$(D + S) \mathbf{u} = \left(\frac{1}{2} M + K^T \right) \mathbf{g}.$$

- K discrete double layer operator
- D discrete Hypersingular operator
- M mass matrix
- S Gauge matrix
- \mathbf{u} DoFs Dirichlet data
- \mathbf{g} DoFs Neumann data

⇒ See [16] for details about the integral operators!

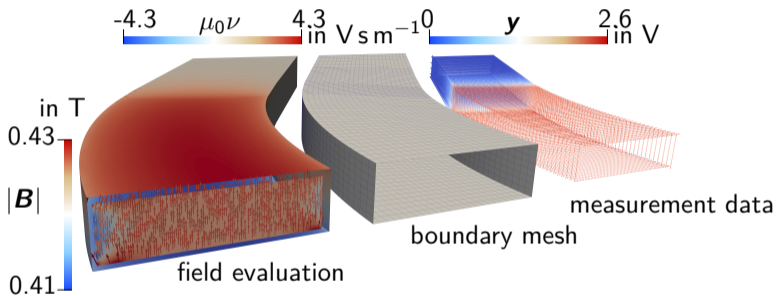
The observation operator



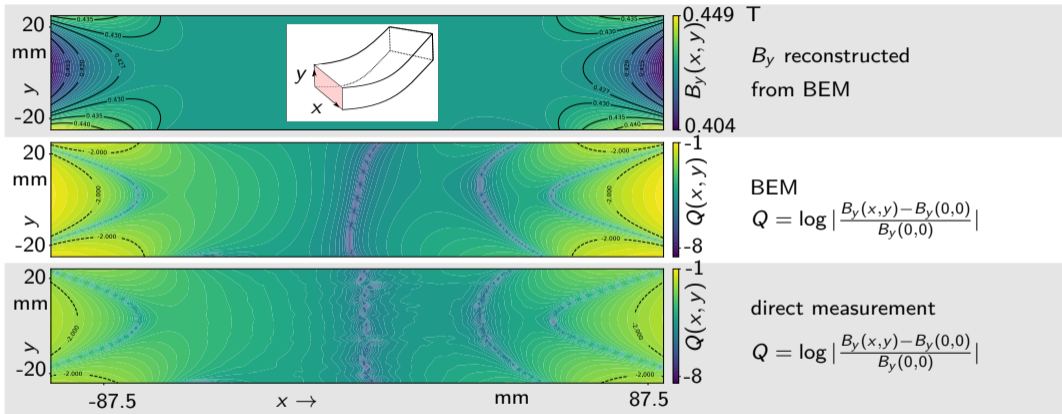
Example: Field map in strongly curved magnet



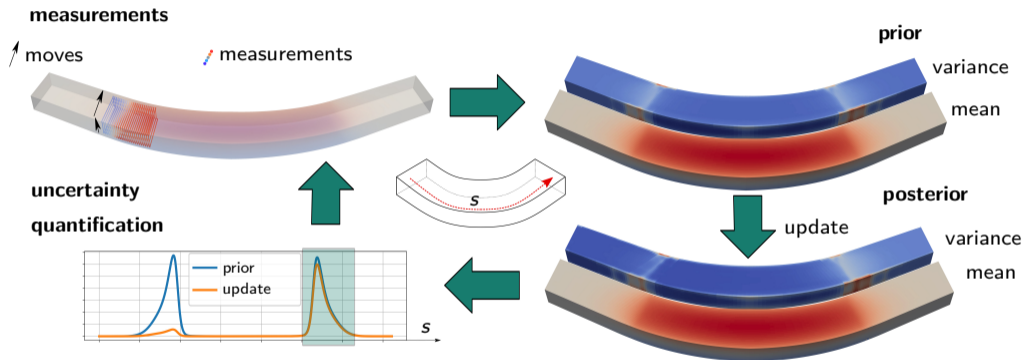
See [17].



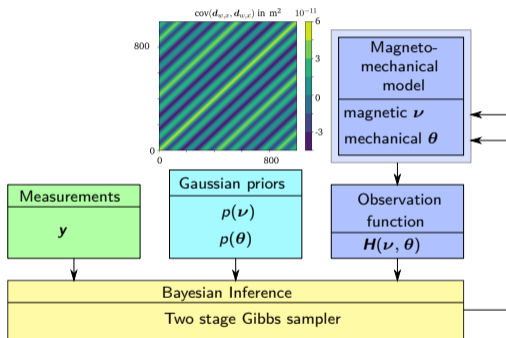
Smoothing property



Active learning



Magneto-mechanical models

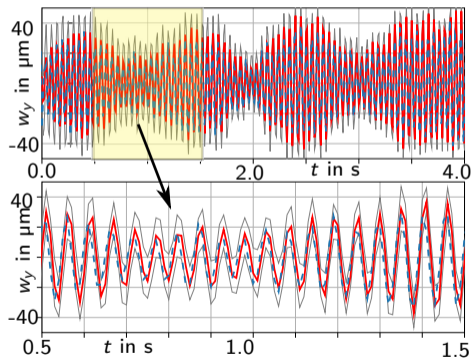


See [18].

Red: Mean vertical position estimated from 1000 samples.

Blue: Optical measurement with Leica laser tracker.

Gray: Maximum and minimum values of the samples.



Contents of this lecture

Introduction

- What is a Hall sensor, how does it work and why do we use it for field mapping?

Challenges

- What are physical effects limiting the performance of Hall sensor measurements?

Calibration techniques

- How can we characterize these effects?

Mapper systems

- What are the additional challenges for mapper systems?

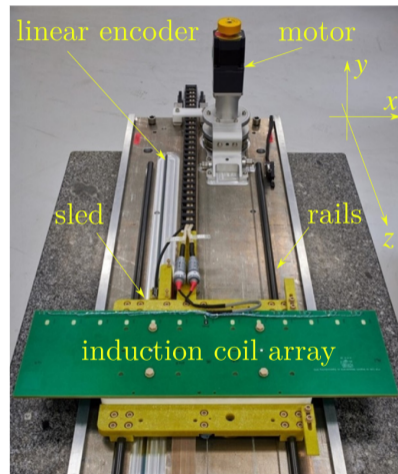
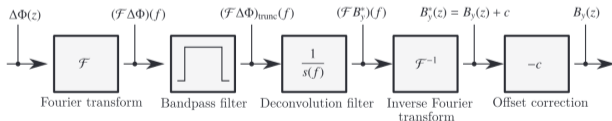
Post-processing

- How can we overcome the limitations in the post-processing?

Outlook

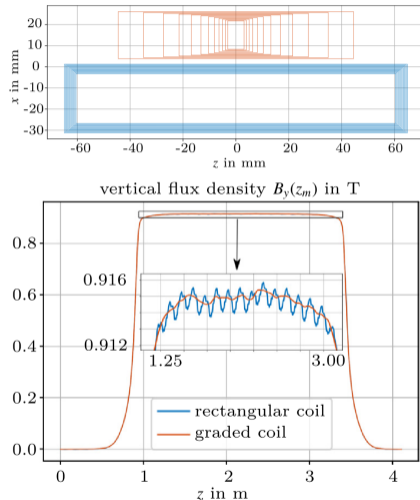
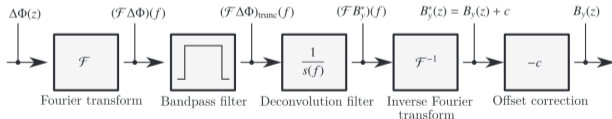
Outlook

- The blue-print is **not limited** to Hall sensor measurements!
- The **translating fluxmeter** (right) measures the vertical field profile
- Standard coil design yields a **low spatial resolution**
- **PCB technology** allows for precise coil layout design
- The graded induction coil (right) allows for a **robust deconvolution** [19]



Outlook

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Summary

We have learned that:

- There are **many physical effects which are affecting** the Hall voltage.
- **Calibration** helps us to characterize these effects.
- We do not measure a field, **we measure a voltage**.
- **Apply physical relations**, wherever You can!
 - ⇒ *We do not need to measure everything!*
 - ⇒ *Even an imperfect sensor provides valuable information!*
 - ⇒ *Imply all available information!*

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