THE ROYAL SOCIETY

UNIVERSITY OF OXFORD

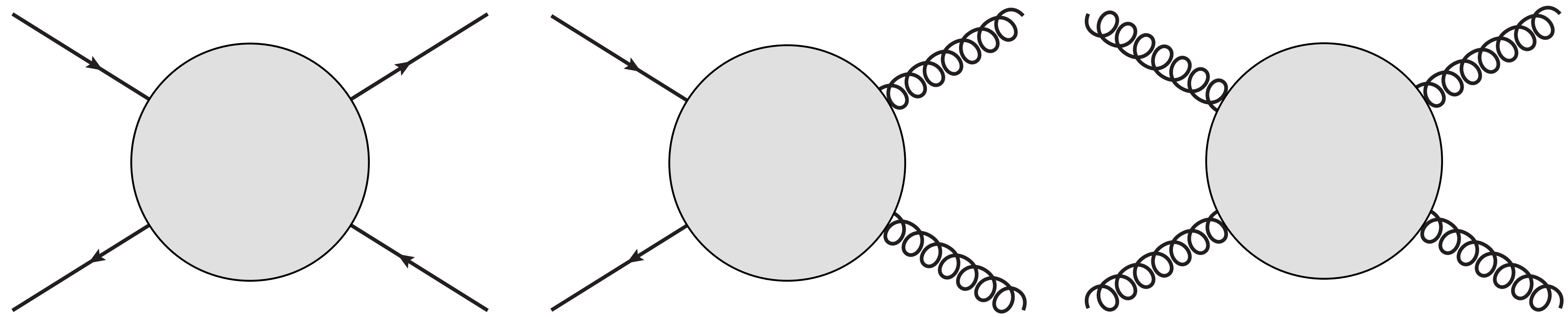
Three-Loop Scattering Amplitudes in full-color QCD

results in the IR and Regge limits

Collaborators: Piotr Bargiela
Amlan Chakraborty
Fabrizio Caola
Andreas von Manteuffel
Lorenzo Tancredi

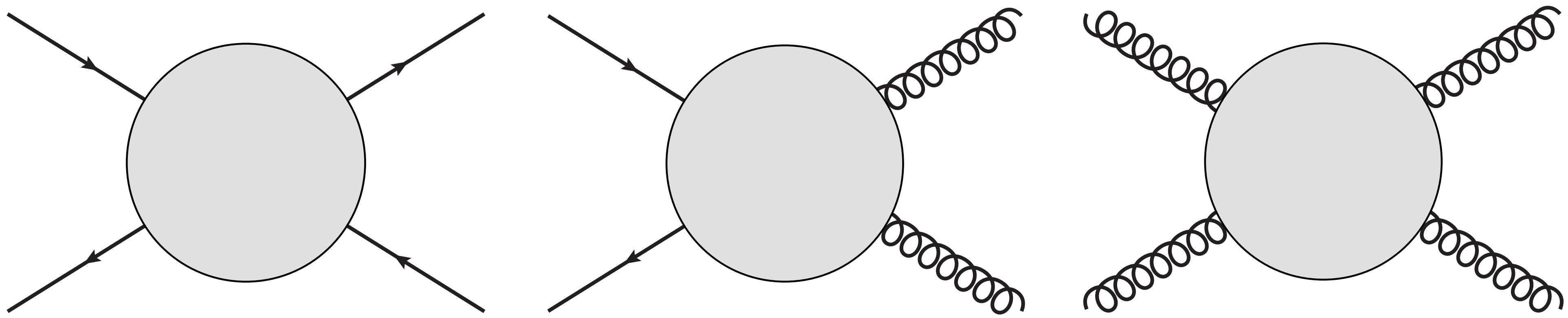
Giulio Gambuti - 26/06/2023 - Loopfest, SLAC

Exact Amplitudes

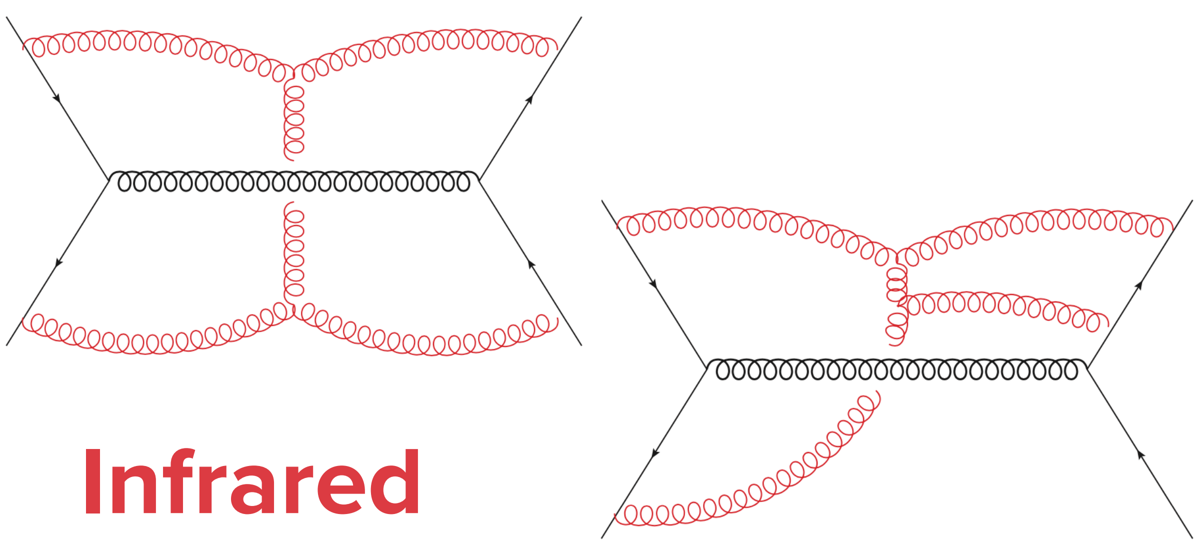


towards N^3LO QCD

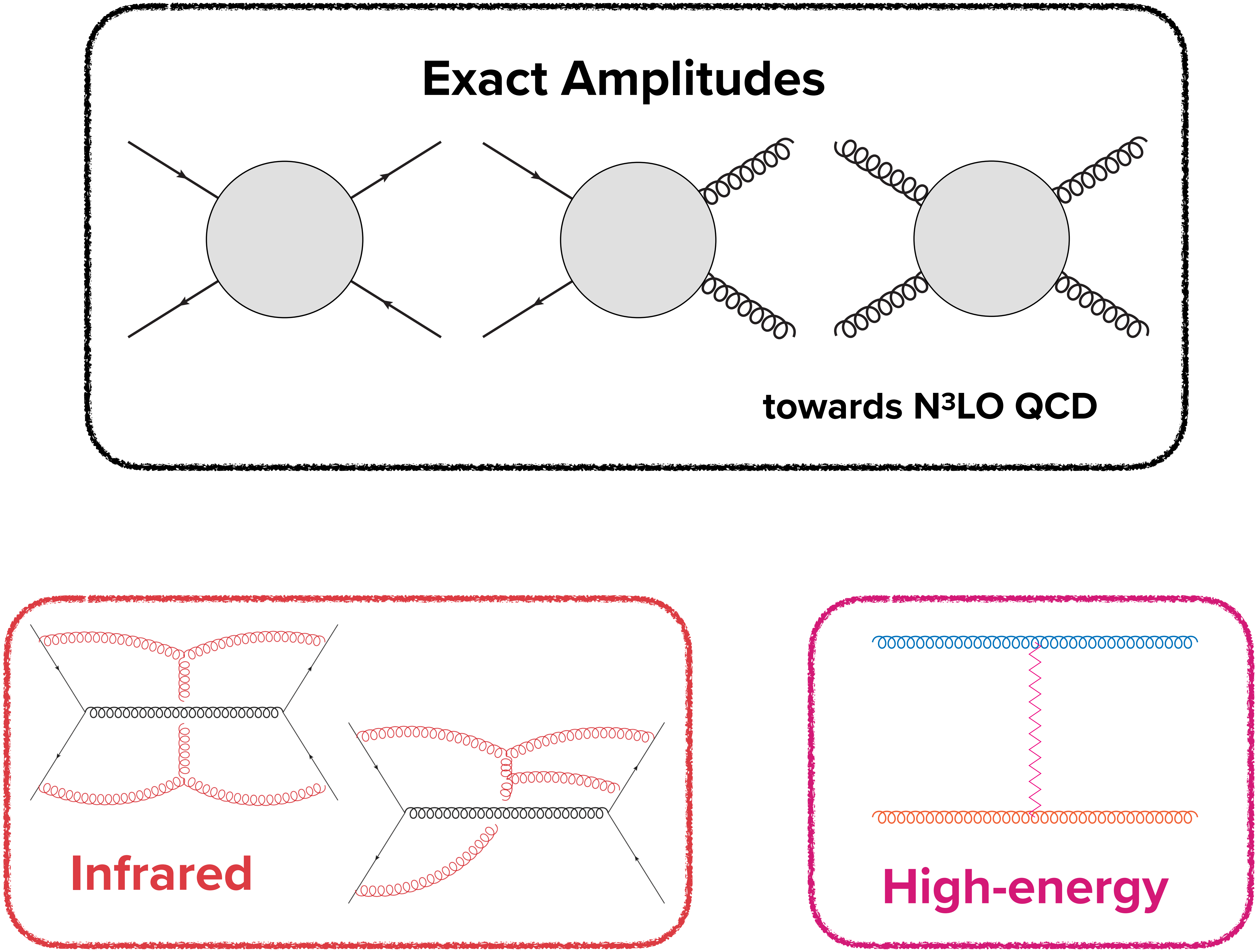
Exact Amplitudes



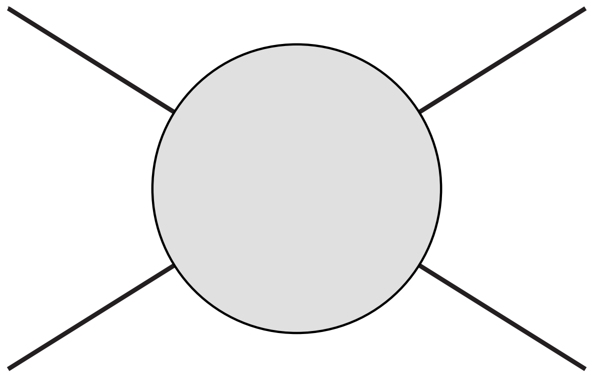
towards N^3LO QCD



Infrared


Exact Amplitudes
towards N³LO QCD
Infrared
High-energy

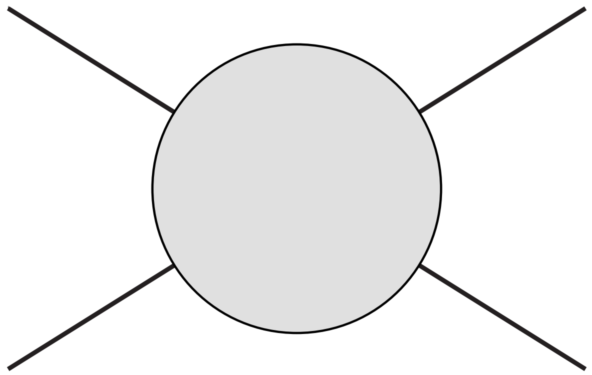
State of the art



2-loop

Anastasiou, Glover, Oleari, Tejeda-Yeomans : 0101304, 0011094
Glover, Oleari, Tejeda-Yeomans : 0102201
Bern, De Freitas, Dixon : 0109078, 0201161, 0304168
Bern, De Freitas, Dixon, Wong : 0202271

State of the art



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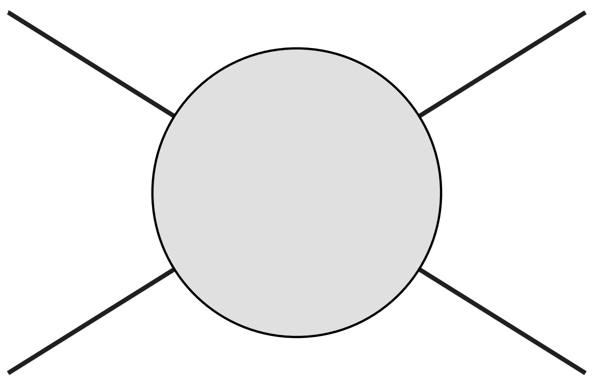
3-loop now complete!

State of the art



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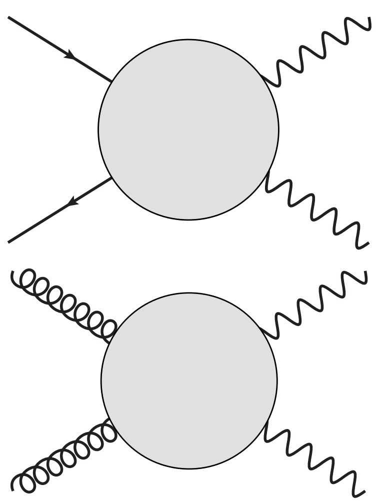
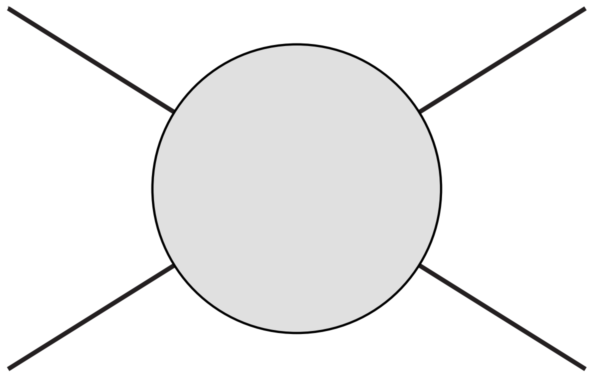


2 partons

Caola, von Manteuffel, Tancredi:
2011.13946(PRL)

Bargiela, Caola, von Manteuffel, Tancredi:
2111.13595(JHEP)

State of the art



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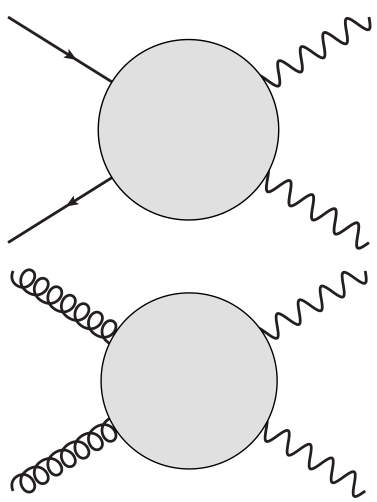

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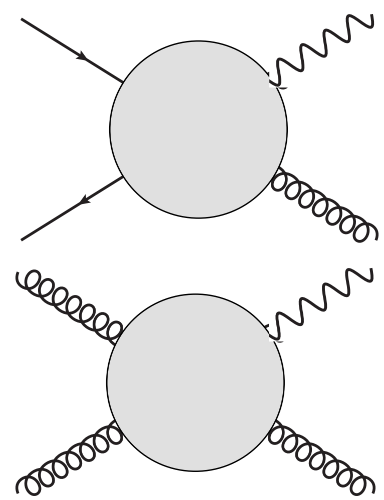
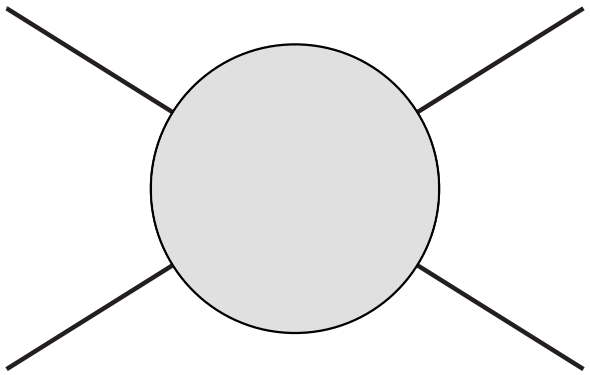


3 partons

Bargiela, Chakraborty, GG:
2212.14069(PRD)

State of the art



2-loop

Anastasiou, Glover, Oleari, Tejeda-Yeomans : 0101304, 0011094
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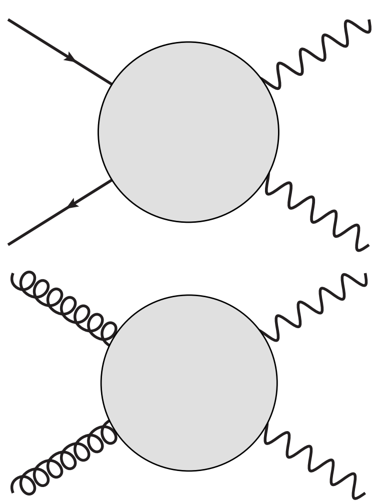
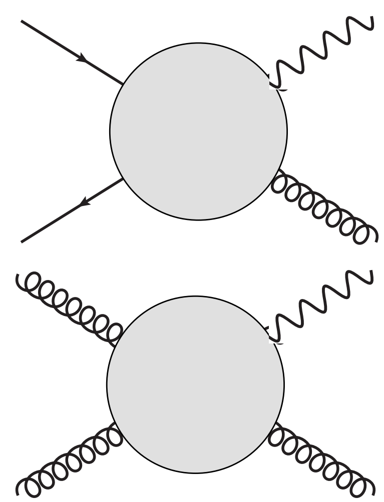
3-loop now complete!



2 partons

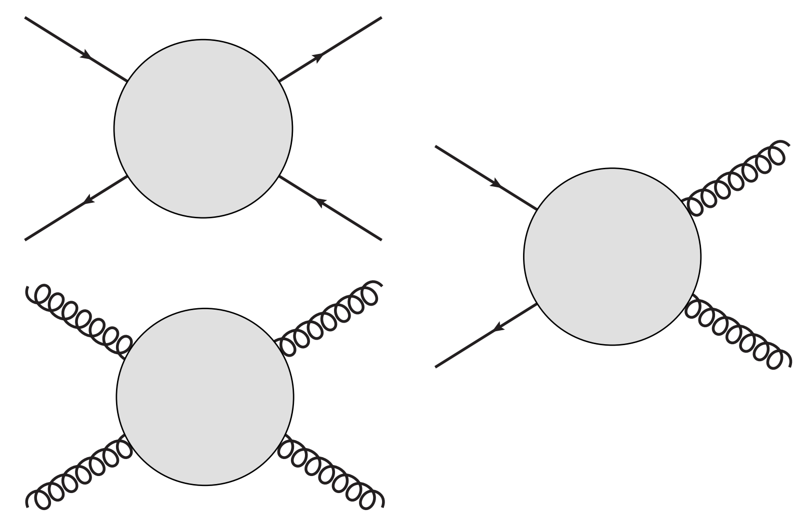
Caola, von Manteuffel, Tancredi: 2011.13946(PRL)

Bargiela, Caola, von Manteuffel, Tancredi: 2111.13595(JHEP)



3 partons

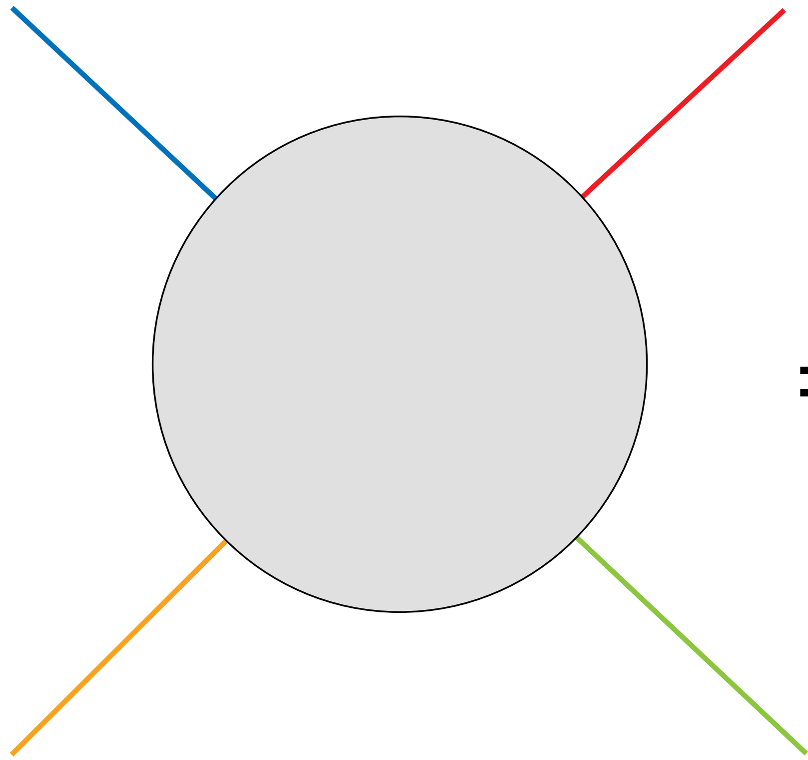
Bargiela, Chakraborty, GG: 2212.14069(PRD)



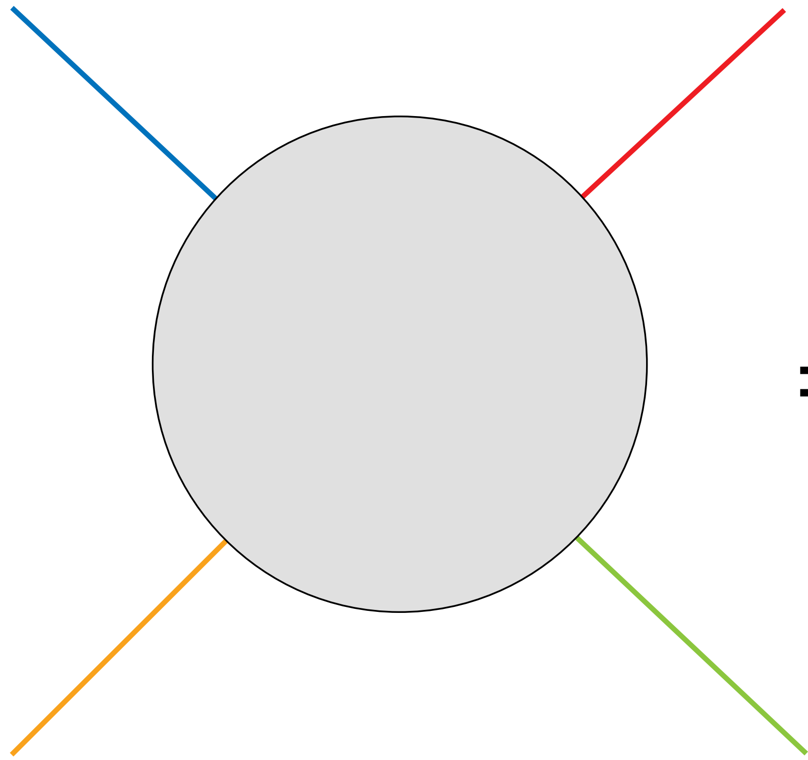
4 partons

Chakraborty, Caola, GG, Tancredi, von Manteuffel: 2108.00055(JHEP), 2207.03503(JHEP), 2112.11097(PRL)

The amplitudes

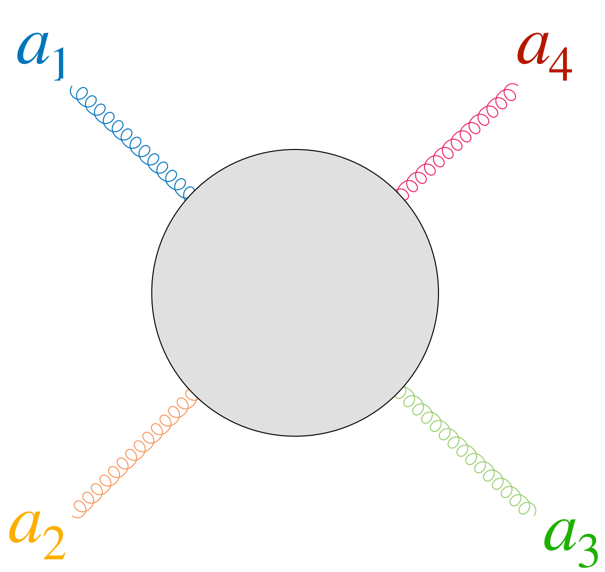


$$= \sum \text{Feynman Diagrams}$$

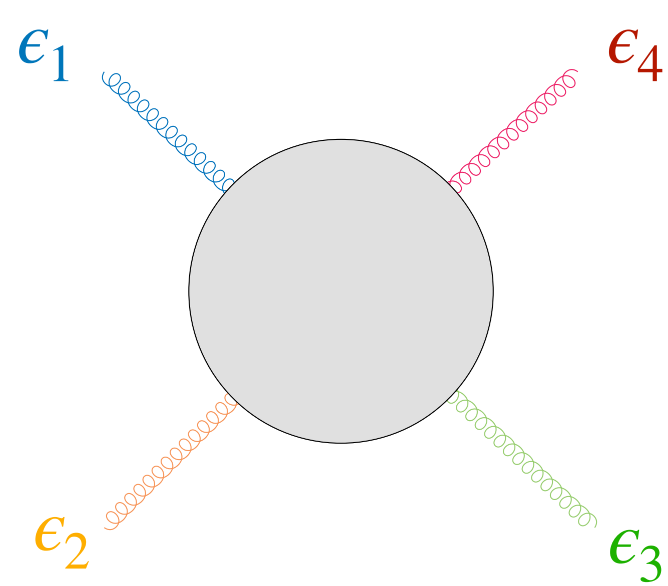


$= \sum$ Feynman Diagrams

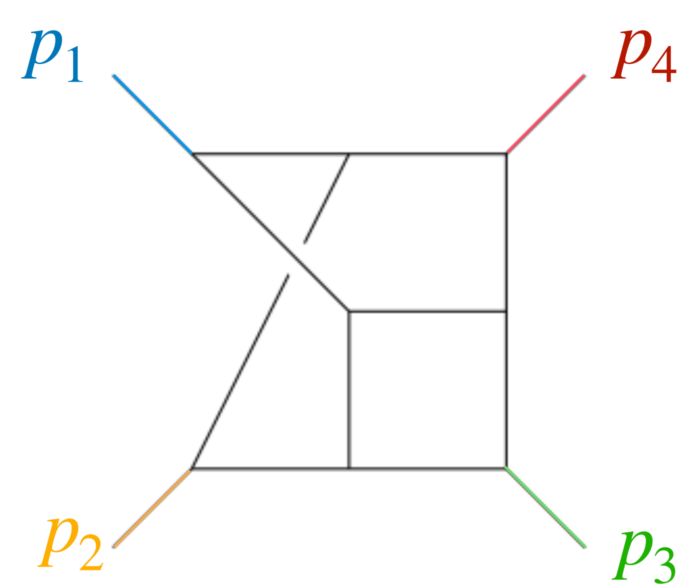
Colour



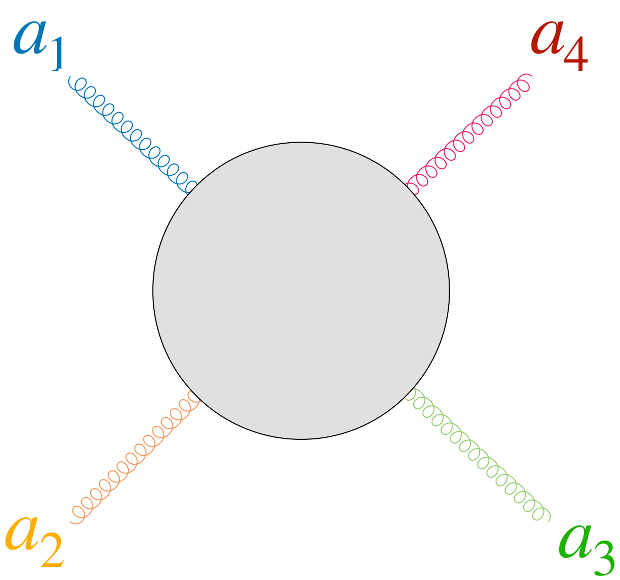
Spin



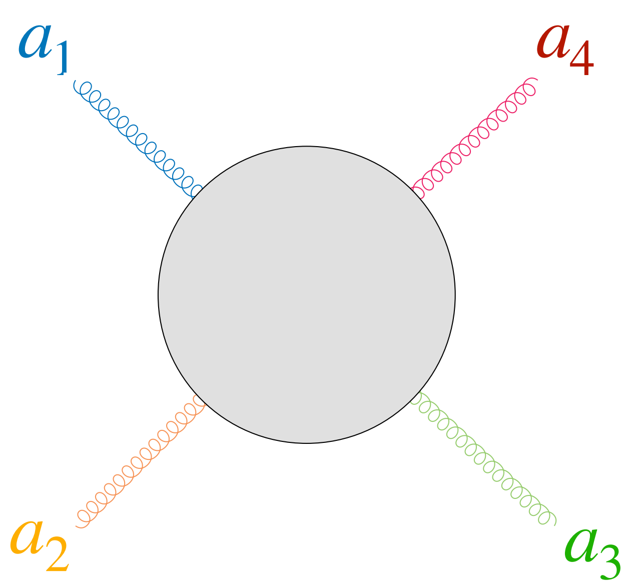
Kinematics



Colour



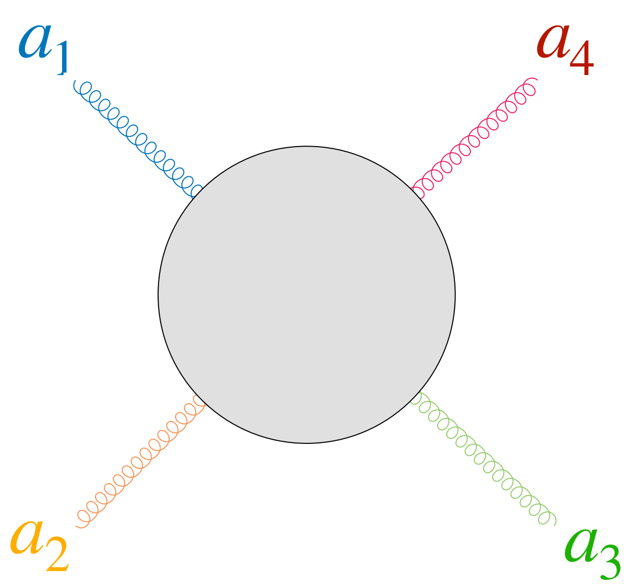
Colour



Colour Decomposition

$$A^{a_1 a_2 a_3 a_4} = \sum_{c=1}^{6} A_c \, \mathscr{C}_c^{a_1 a_2 a_3 a_4}$$

Colour



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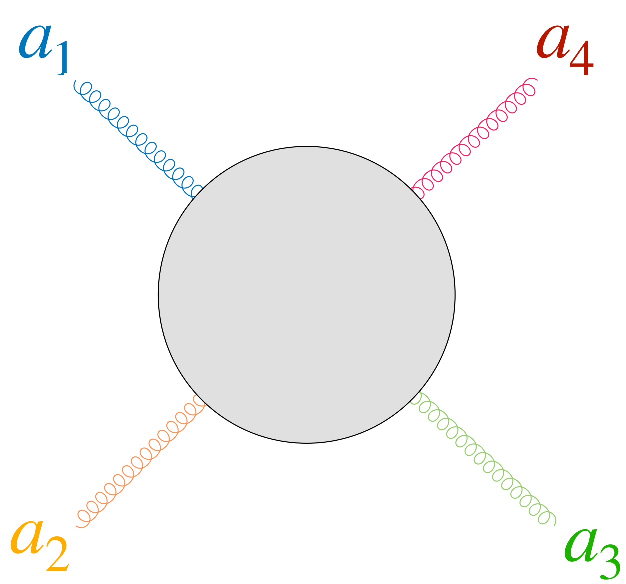
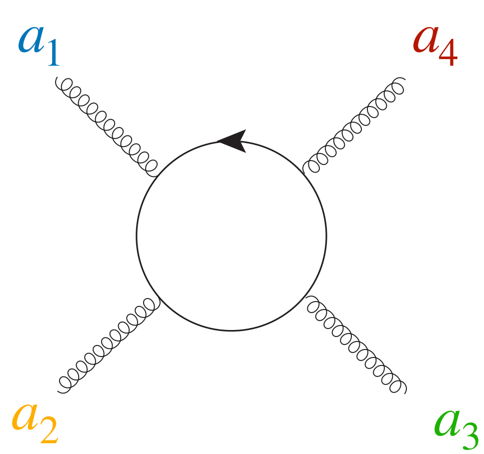
Partial Amplitudes

Colour



$= A_1$

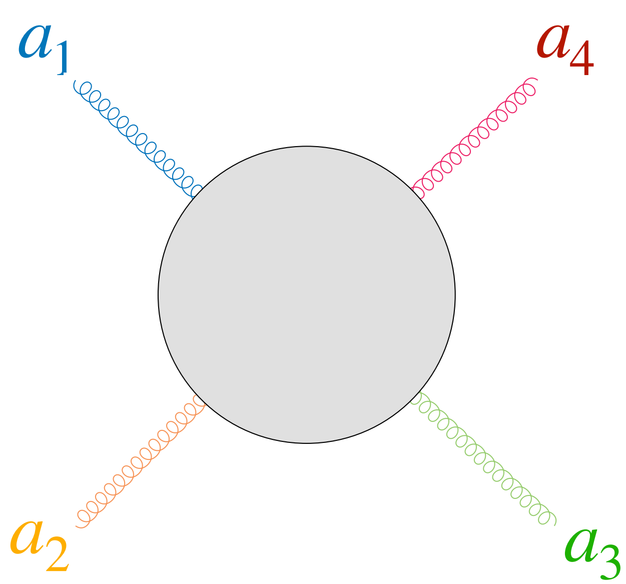


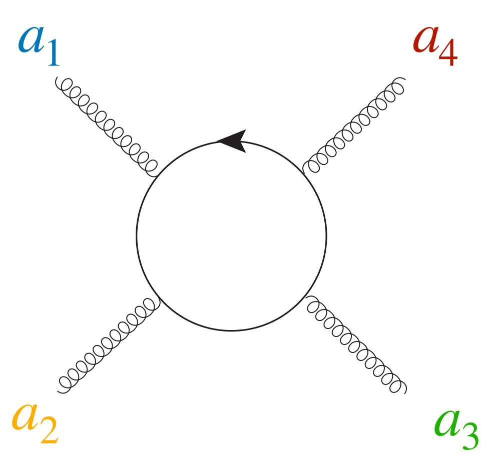
Colour Decomposition

$$A^{a_1 a_2 a_3 a_4} = \sum_{c=1}^{6} A_c \, \mathcal{C}_c^{a_1 a_2 a_3 a_4}$$

Partial Amplitudes

Colour



$= \; A_1$ 

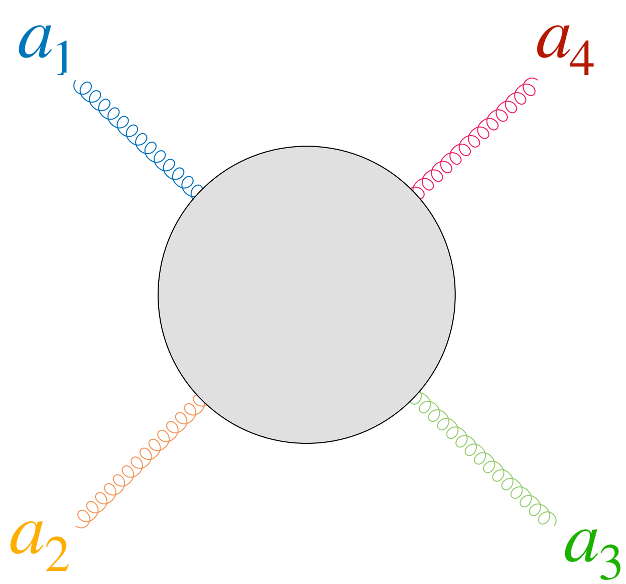
$Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_2}\mathbf{T}^{a_3}\mathbf{T}^{a_4})$

Colour Decomposition

$$A^{a_1a_2a_3a_4} = \sum_{c=1}^{6} A_c \, \mathscr{C}_c^{a_1a_2a_3a_4}$$

Partial Amplitudes

Colour



Colour Decomposition

$$A^{a_1 a_2 a_3 a_4} = \sum_{c=1}^{6} A_c \, \mathcal{C}_c^{a_1 a_2 a_3 a_4}$$

Partial Amplitudes

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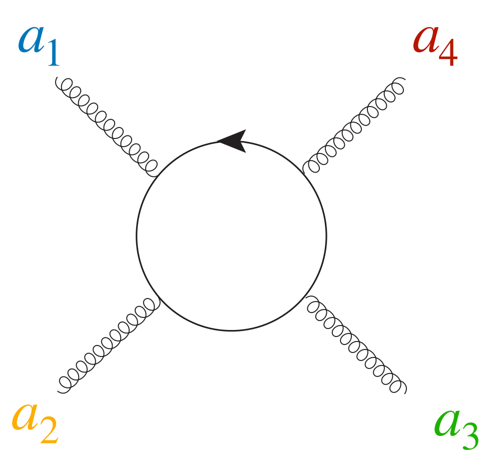
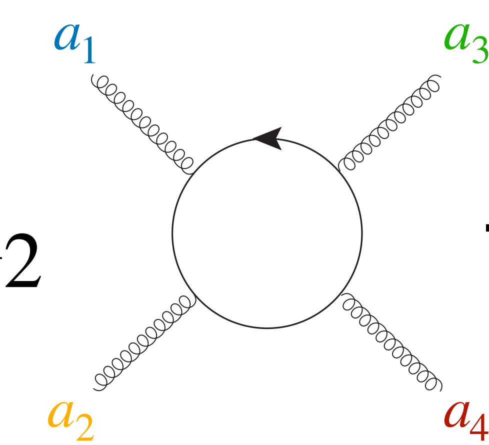
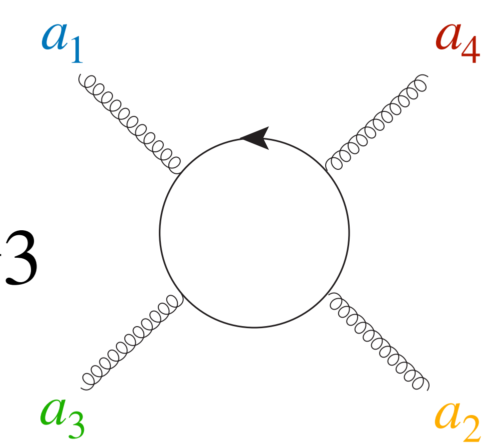


$Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_2}\mathbf{T}^{a_3}\mathbf{T}^{a_4})$

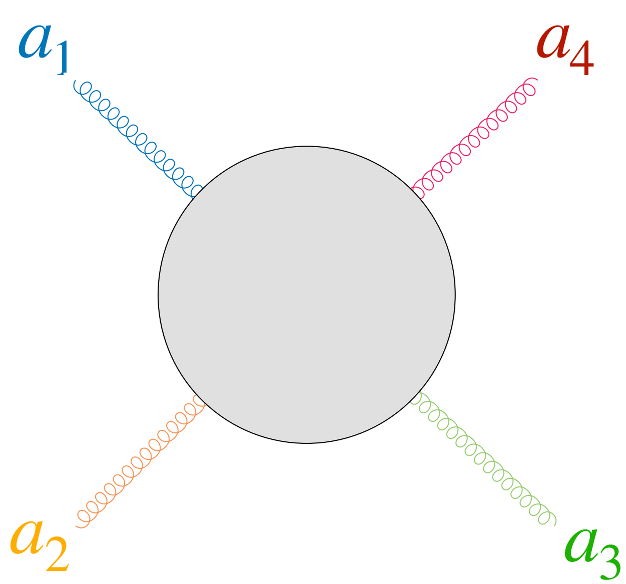
$+ A_2$



$+ A_3$



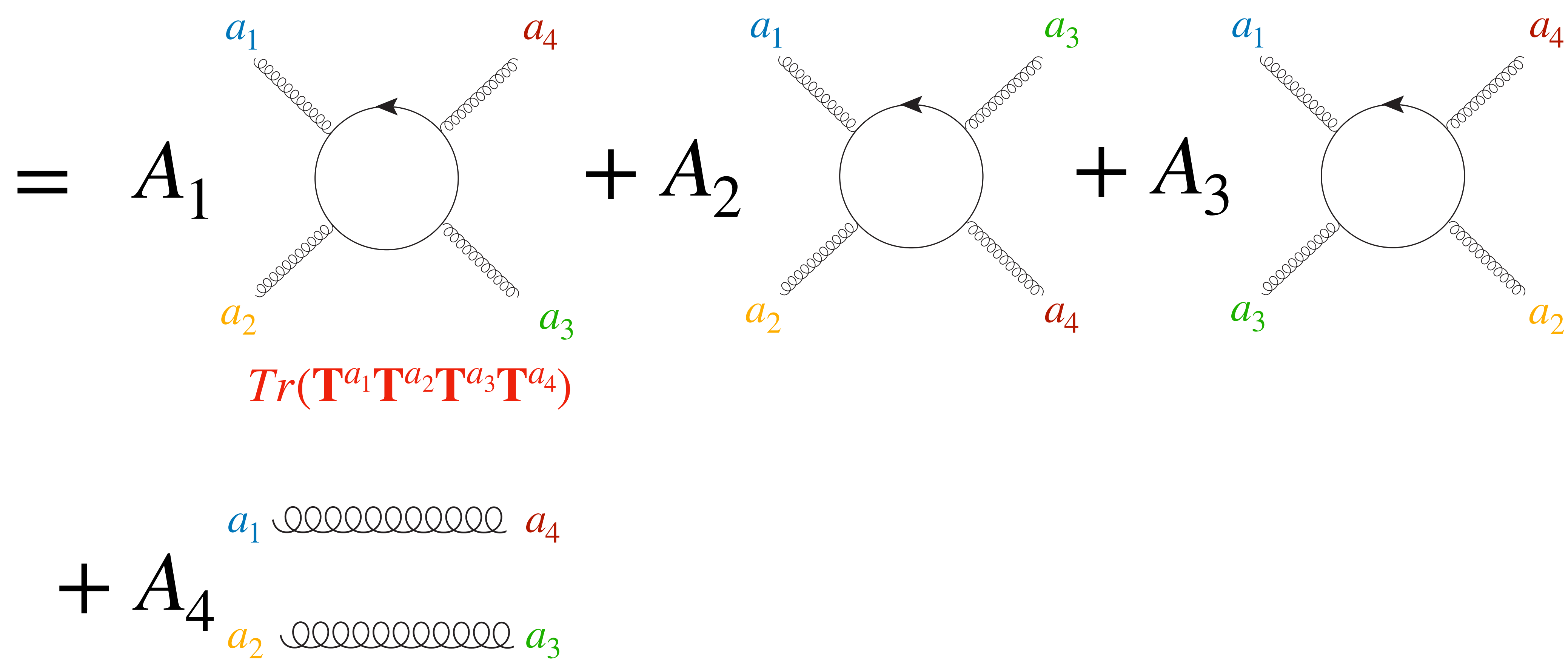
Colour



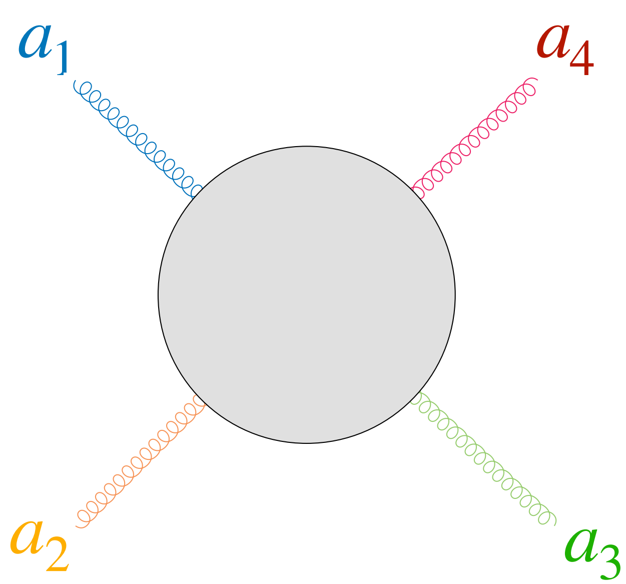
Colour Decomposition

$$A^{a_1 a_2 a_3 a_4} = \sum_{c=1}^{6} A_c \, \mathscr{C}_c^{a_1 a_2 a_3 a_4}$$

Partial Amplitudes



Colour



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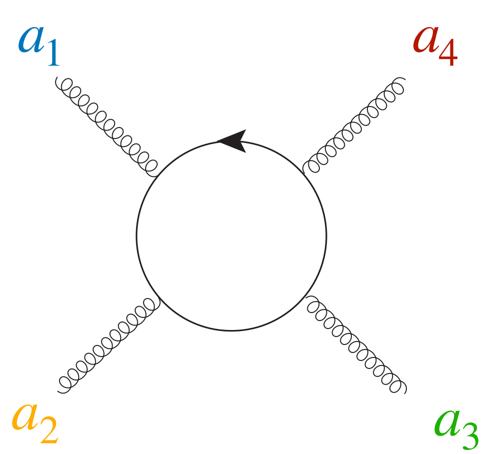
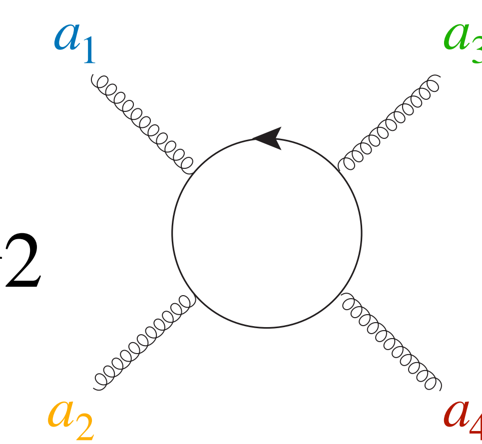
Partial Amplitudes

$= A_1$

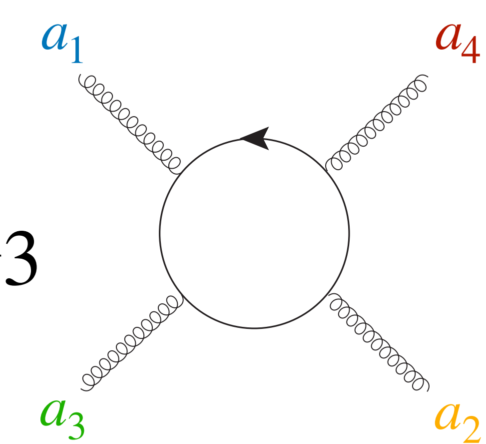


$Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_2}\mathbf{T}^{a_3}\mathbf{T}^{a_4})$

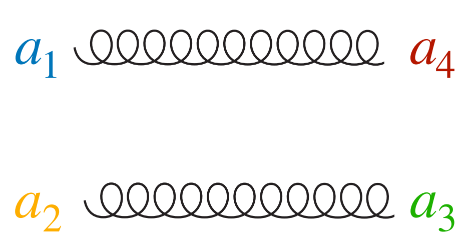
$+ A_2$



$+ A_3$

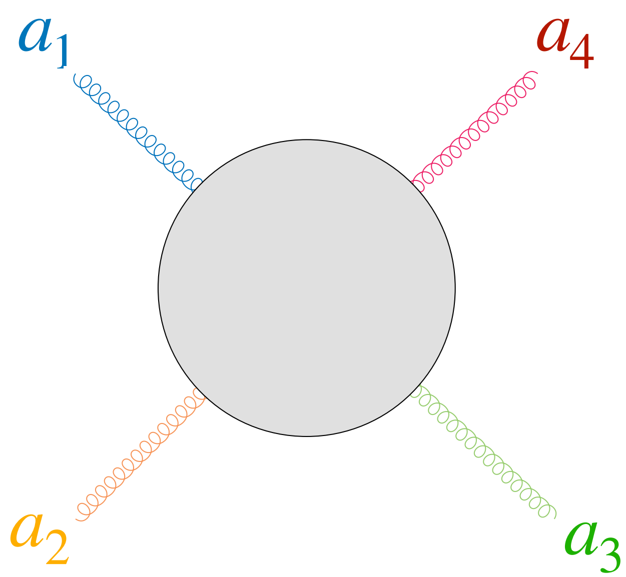


$+ A_4$



$\delta^{a_1 a_4} \delta^{a_2 a_3}$

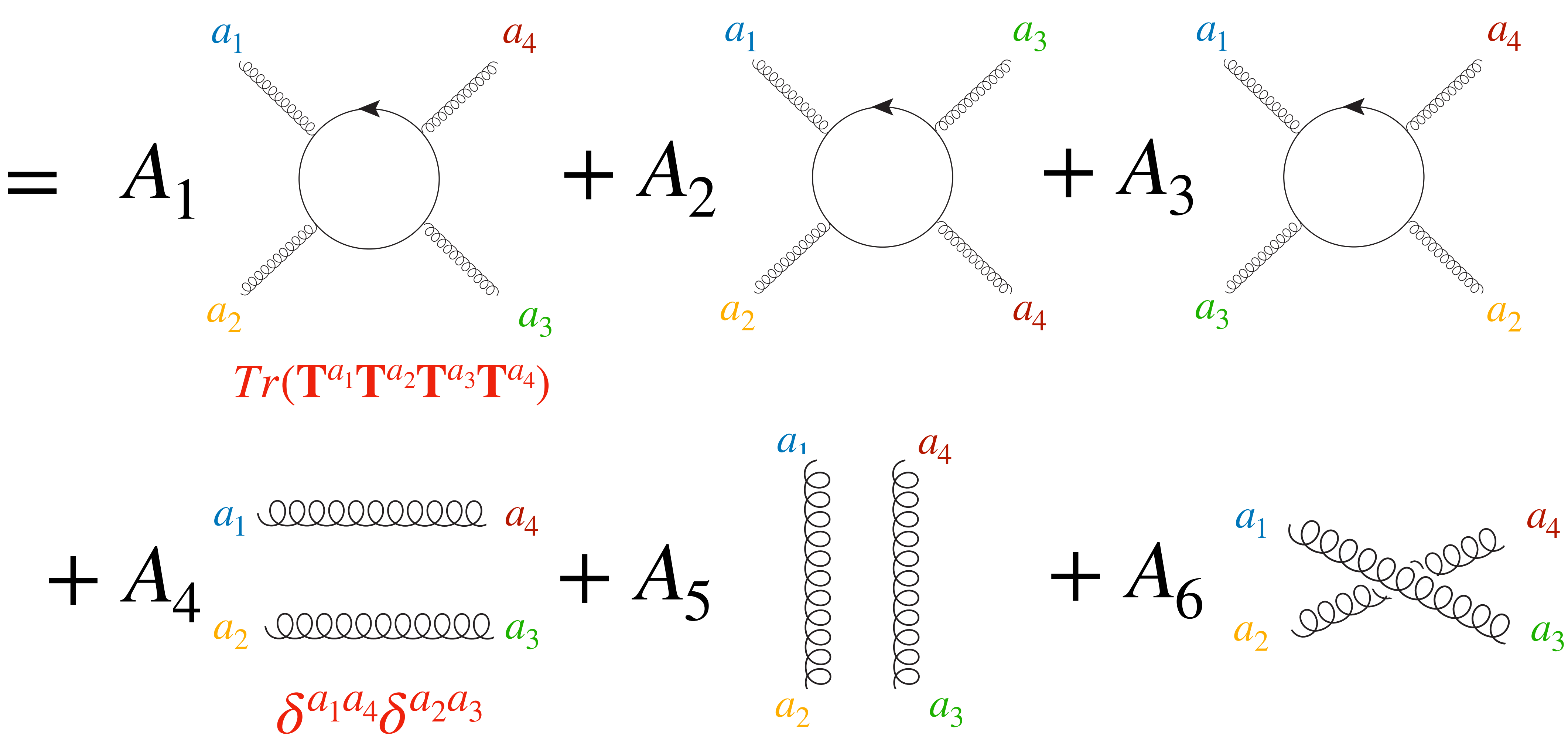
Colour



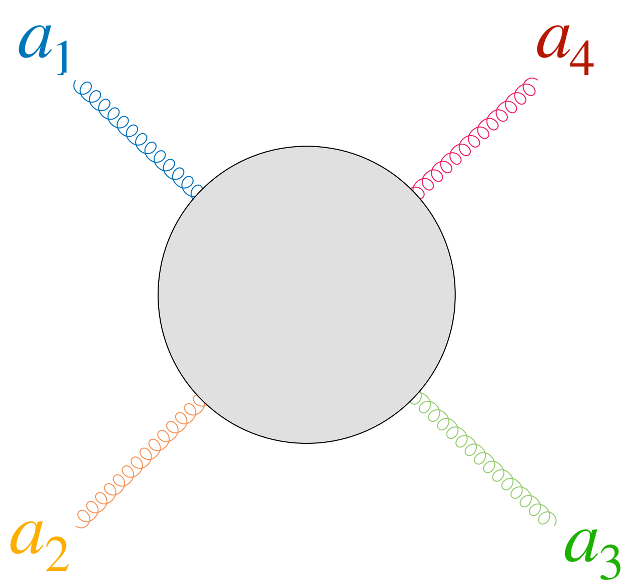
Colour Decomposition

$$A^{a_1a_2a_3a_4} = \sum_{c=1}^{6} A_c \, \mathscr{C}_c^{a_1a_2a_3a_4}$$

Partial Amplitudes



Colour



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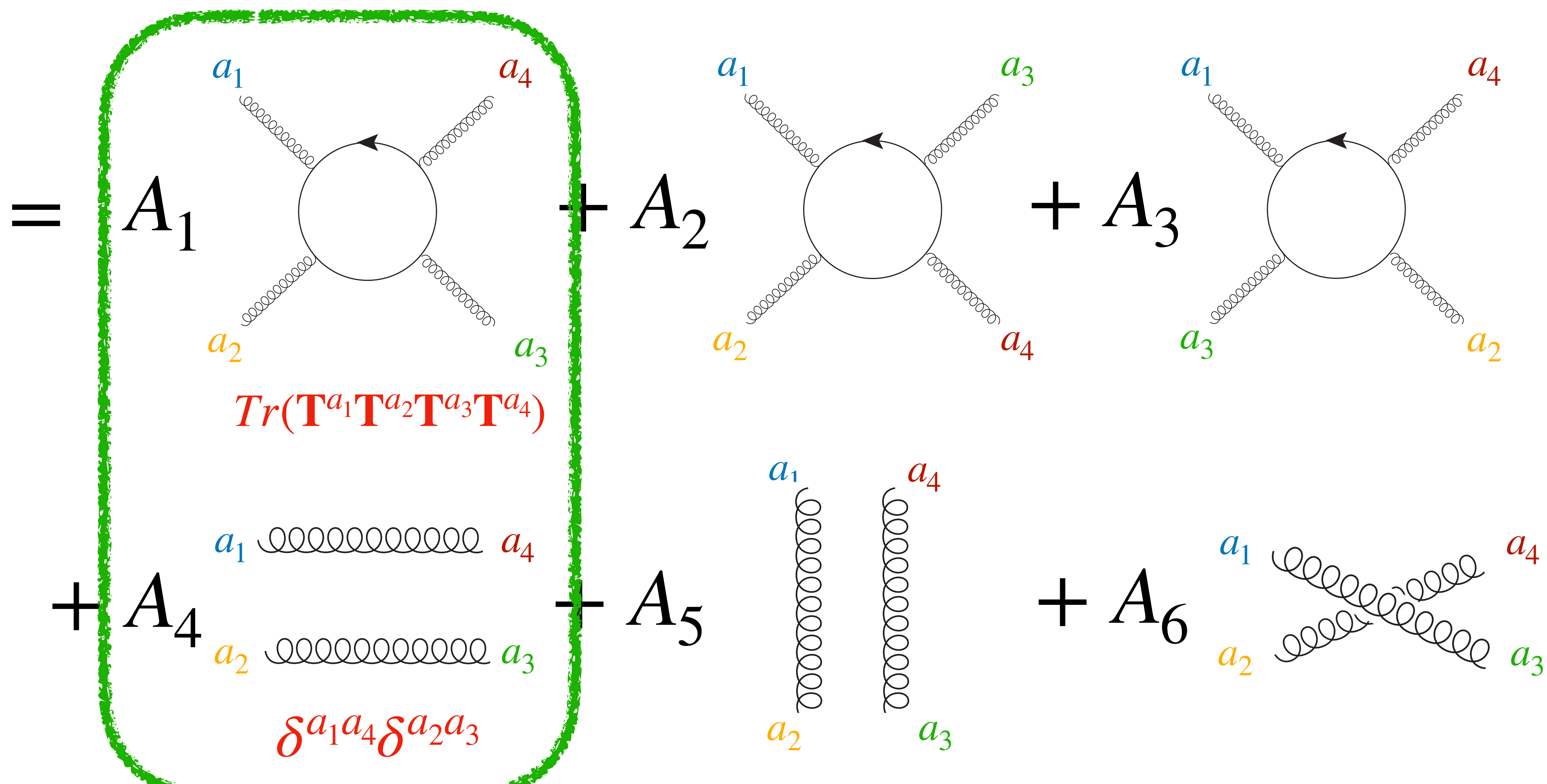
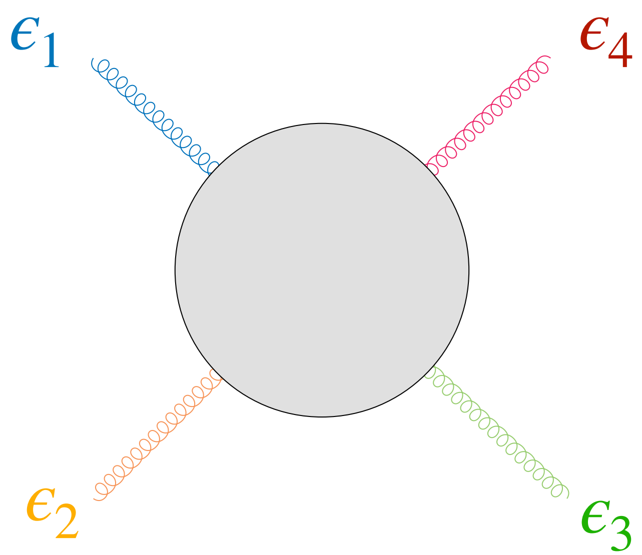
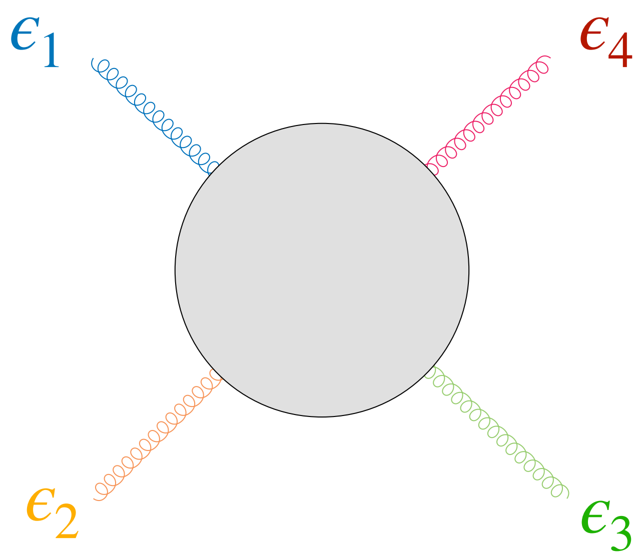


Spin



$$
\begin{aligned}
A_c &= A_c^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4} \\
&= \sum_j F_c^j \; T_j^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}
\end{aligned}
$$

Spin



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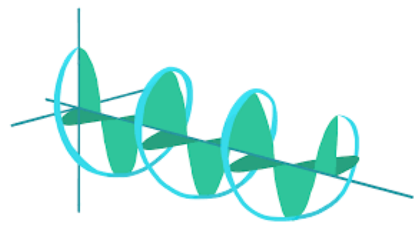
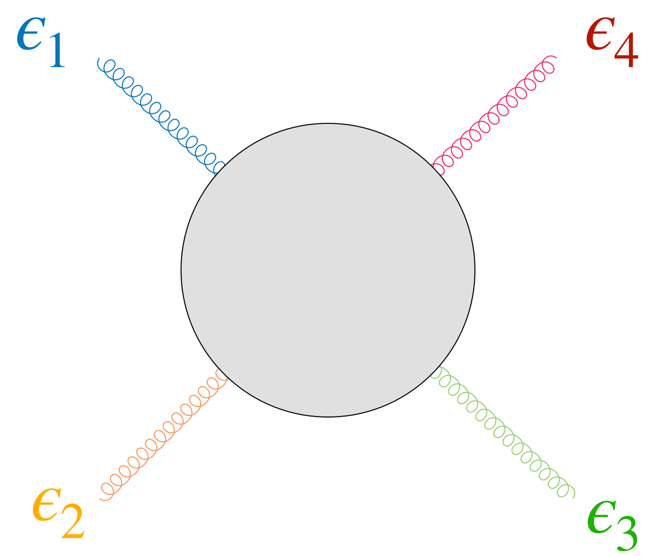
$$= \sum_j F_c^j \; T_j^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$

Transversality $\epsilon_i \cdot p_i = 0$



Spin



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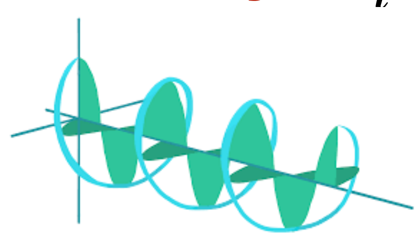
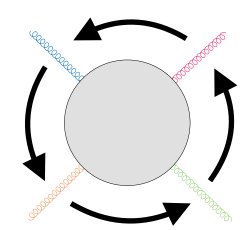
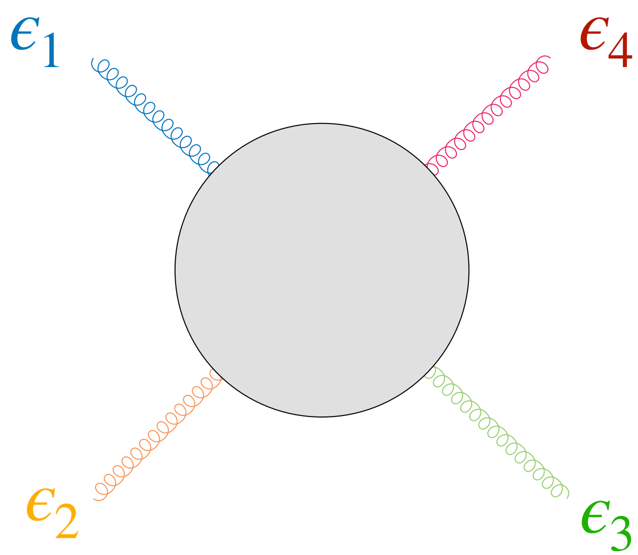


Reference choice $\epsilon_i \cdot p_{i+1} = 0$



Spin



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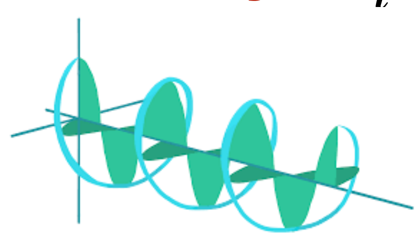
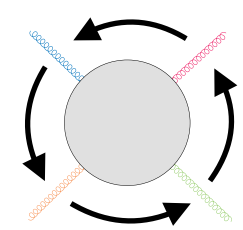
$$= \sum_j F_c^j \; T_j^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$

Transversality $\epsilon_i \cdot p_i = 0$



Reference choice $\epsilon_i \cdot p_{i+1} = 0$



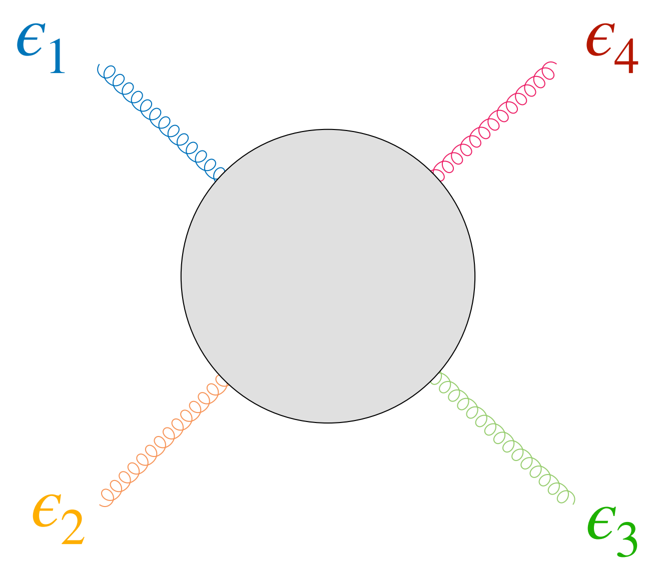
$$p_3^{\mu_1}p_1^{\mu_2}g^{\mu_3\mu_4} \quad p_3^{\mu_1}p_2^{\mu_4}g^{\mu_2\mu_3} \quad p_1^{\mu_2}p_1^{\mu_3}g^{\mu_1\mu_4}$$

$$p_3^{\mu_1}p_1^{\mu_2}p_1^{\mu_3}p_2^{\mu_4} \quad p_3^{\mu_1}p_1^{\mu_3}g^{\mu_2\mu_4} \quad p_1^{\mu_2}p_2^{\mu_4}g^{\mu_1\mu_3} \quad p_1^{\mu_3}p_3^{\mu_4}g^{\mu_1\mu_2}$$

$$g^{\mu_1\mu_2}g^{\mu_3\mu_4} \quad g^{\mu_1\mu_3}g^{\mu_2\mu_4} \quad g^{\mu_1\mu_4}g^{\mu_2\mu_3}$$

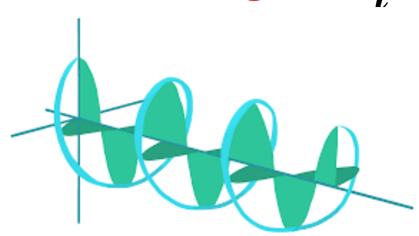
138 → **10**

Spin

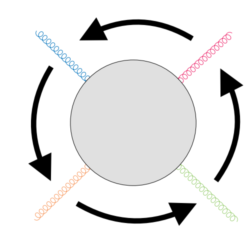


$$A_c = A_c^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$
$$= \sum_j F_c^j \; T_j^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$

Transversality $\epsilon_i \cdot p_i = 0$



Reference choice $\epsilon_i \cdot p_{i+1} = 0$



$$p_3^{\mu_1}p_1^{\mu_2}g^{\mu_3\mu_4} \quad p_3^{\mu_1}p_2^{\mu_4}g^{\mu_2\mu_3} \quad p_1^{\mu_2}p_1^{\mu_3}g^{\mu_1\mu_4}$$

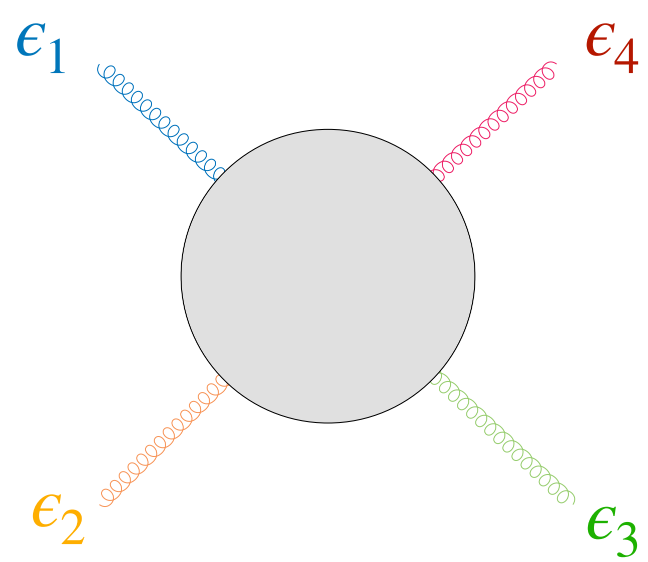
$$p_3^{\mu_1}p_1^{\mu_2}p_1^{\mu_3}p_2^{\mu_4} \quad p_3^{\mu_1}p_1^{\mu_3}g^{\mu_2\mu_4} \quad p_1^{\mu_2}p_2^{\mu_4}g^{\mu_1\mu_3} \quad p_1^{\mu_3}p_3^{\mu_4}g^{\mu_1\mu_2}$$

$$g^{\mu_1\mu_2}g^{\mu_3\mu_4} + g^{\mu_1\mu_3}g^{\mu_2\mu_4} + g^{\mu_1\mu_4}g^{\mu_2\mu_3}$$

138 → 10

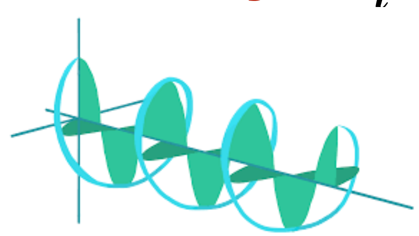
Tancredi, Peraro: 1906.03298, 2012.00820

Spin

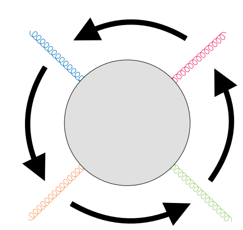


$$A_c = A_c^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$
$$= \sum_j F_c^j \; T_j^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$

Transversality $\epsilon_i \cdot p_i = 0$



Reference choice $\epsilon_i \cdot p_{i+1} = 0$



$p_3^{\mu_1}p_1^{\mu_2}g^{\mu_3\mu_4}$ $\quad p_3^{\mu_1}p_2^{\mu_4}g^{\mu_2\mu_3}$ $\quad p_1^{\mu_2}p_1^{\mu_3}g^{\mu_1\mu_4}$

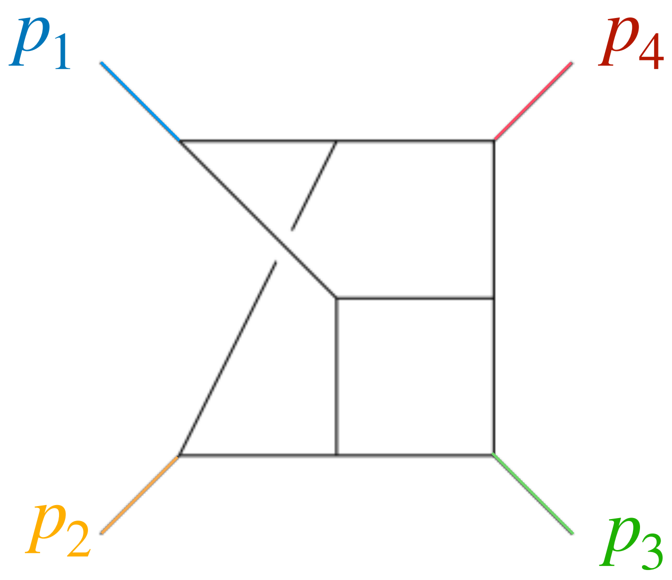
$p_3^{\mu_1}p_1^{\mu_2}p_1^{\mu_3}p_2^{\mu_4}$ $\quad p_3^{\mu_1}p_1^{\mu_3}g^{\mu_2\mu_4}$ $\quad p_1^{\mu_2}p_2^{\mu_4}g^{\mu_1\mu_3}$ $\quad p_1^{\mu_3}p_3^{\mu_4}g^{\mu_1\mu_2}$

$$g^{\mu_1\mu_2}g^{\mu_3\mu_4} + g^{\mu_1\mu_3}g^{\mu_2\mu_4} + g^{\mu_1\mu_4}g^{\mu_2\mu_3}$$

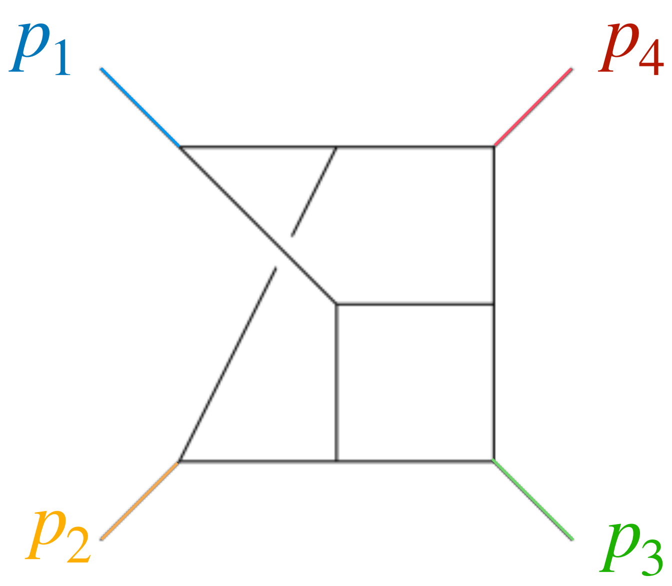
138 → 10 → 8

Tancredi, Peraro: 1906.03298, 2012.00820

Kinematics

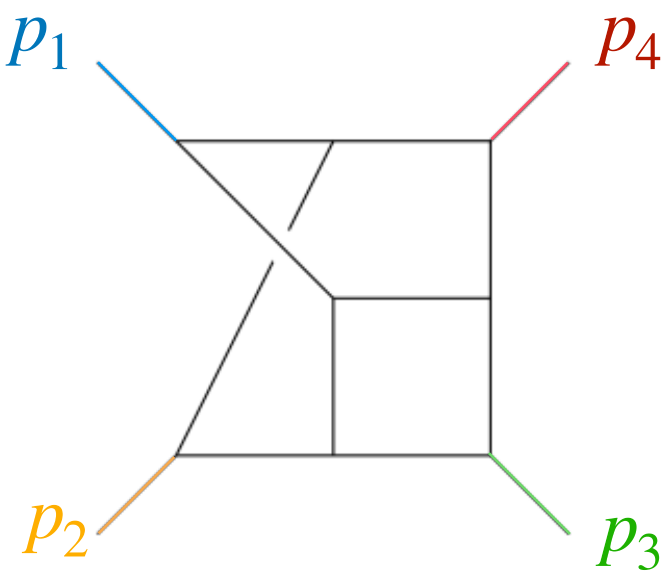


Kinematics



$$A_{\vec{h}}^{a_1 a_2 a_3 a_4} = \sum_{c,i} F_c^i \, T_i^{\vec{h}} \, \mathcal{C}_c^{a_1 a_2 a_3 a_4}$$

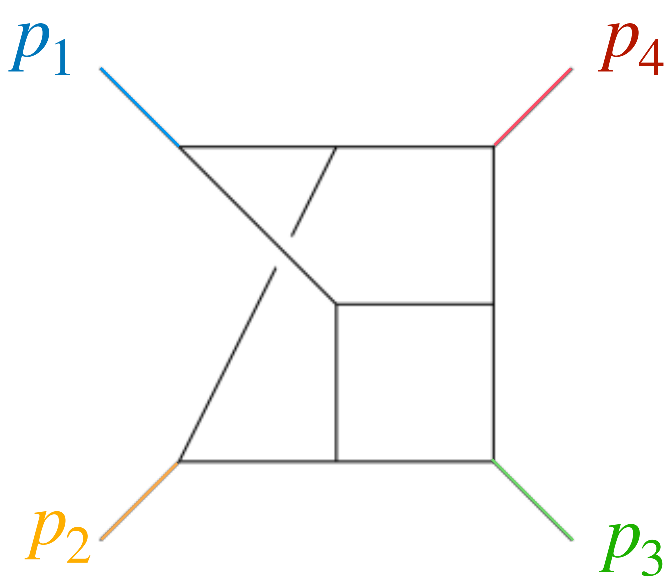
Kinematics



$$A^{a_1 a_2 a_3 a_4}_{\vec{h}} = \sum_{c,i} F^i_c \; T^{\vec{h}}_i \; \mathscr{C}^{a_1 a_2 a_3 a_4}_c$$

True to all orders

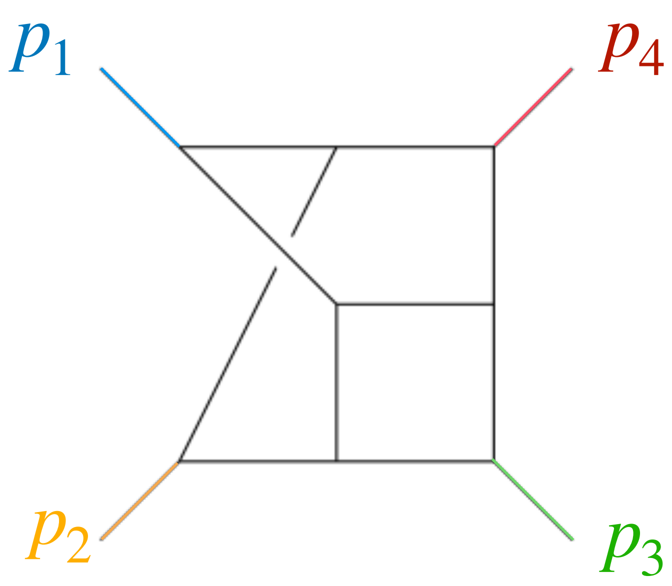
Kinematics



$$A^{a_1 a_2 a_3 a_4}_{\vec{h}} = \sum_{c,i} F^i_c \, T^{\vec{h}}_i \, \mathscr{C}^{a_1 a_2 a_3 a_4}_c$$

True to all orders

Kinematics

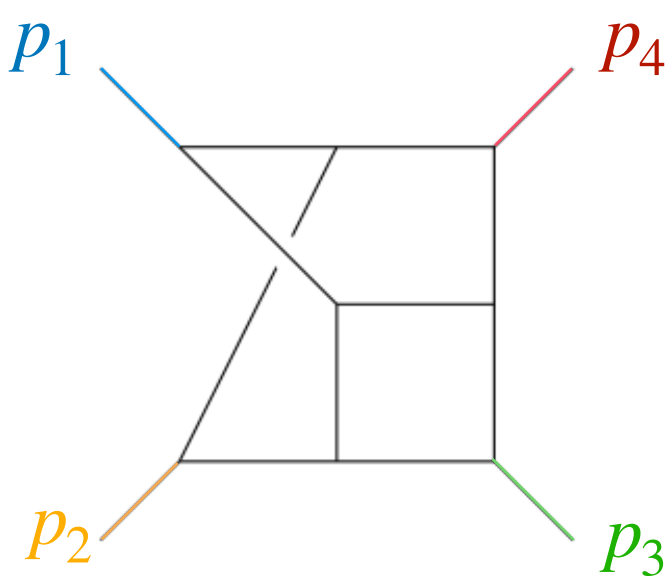


$$A_{\vec{h}}^{a_1a_2a_3a_4} = \sum_{c,i} F_c^i \, T_i^{\vec{h}} \, \mathscr{C}_c^{a_1a_2a_3a_4}$$

True to all orders

$$F_c^i \sim \int \frac{\mathrm{d}^d\{k_j\}}{(2\pi)^{dl}} \frac{\mathcal{N}(\{p\},\{k\})}{D_1^{n_1} D_2^{n_2} \ldots D_N^{n_N}}$$

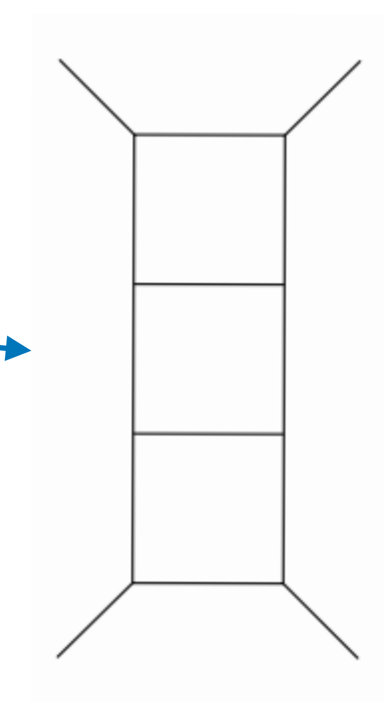
Kinematics



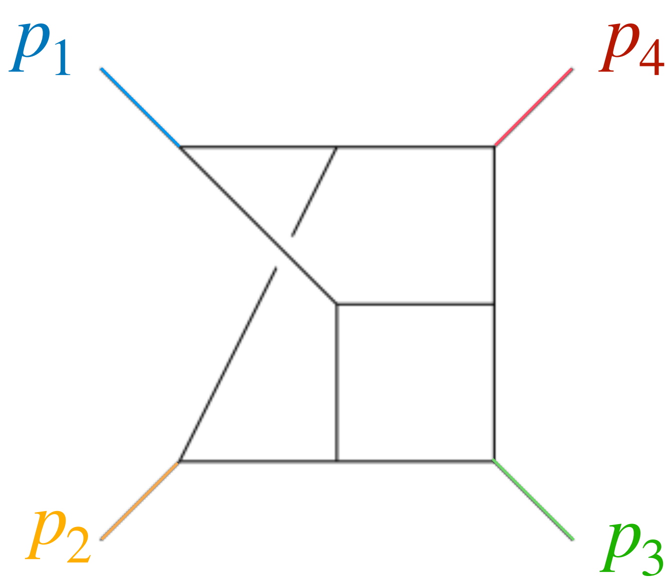
$$A_{\vec{h}}^{a_1 a_2 a_3 a_4} = \sum_{c,i} \boxed{F_c^i} \, T_i^{\vec{h}} \, \mathscr{C}_c^{a_1 a_2 a_3 a_4}$$

True to all orders

$$F_c^i \sim \int \frac{\mathrm{d}^d \{k_j\}}{(2\pi)^{dl}} \frac{\mathcal{N}(\{p\}, \{k\})}{D_1^{n_1} D_2^{n_2} \ldots D_N^{n_N}}$$



Kinematics

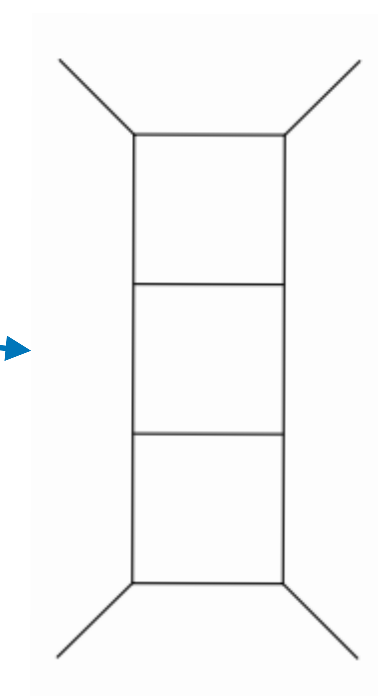


$$A^{a_1a_2a_3a_4}_{\vec{h}} = \sum_{c,i} F^i_c \, T^{\vec{h}}_i \, \mathcal{C}^{a_1a_2a_3a_4}_c$$

True to all orders

$$F^i_c \sim \int \frac{\mathrm{d}^d\{k_j\}}{(2\pi)^{dl}} \frac{\mathcal{N}(\{p\},\{k\})}{D_1^{n_1} D_2^{n_2} \dots D_N^{n_N}}$$

$$(k_1 \cdot p_1)(k_2 \cdot k_3) \; + \; (k_1 \cdot p_2)(k_3 \cdot p_3) \; + \; \dots$$



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Constructing d-log integrands and computing master integrals for three-loop four-particle scattering

Johannes Henn,[a] **Bernhard Mistlberger,**[b] **Vladimir A. Smirnov**[c,e] **and Pascal Wasser**[d]

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[b] *Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, U.S.A.*
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[d] *PRISMA+ Cluster of Excellence, Johannes Gutenberg University, D-55099 Mainz, Germany*
[e] *Moscow Center for Fundamental and Applied Mathematics, Moscow, Russia*

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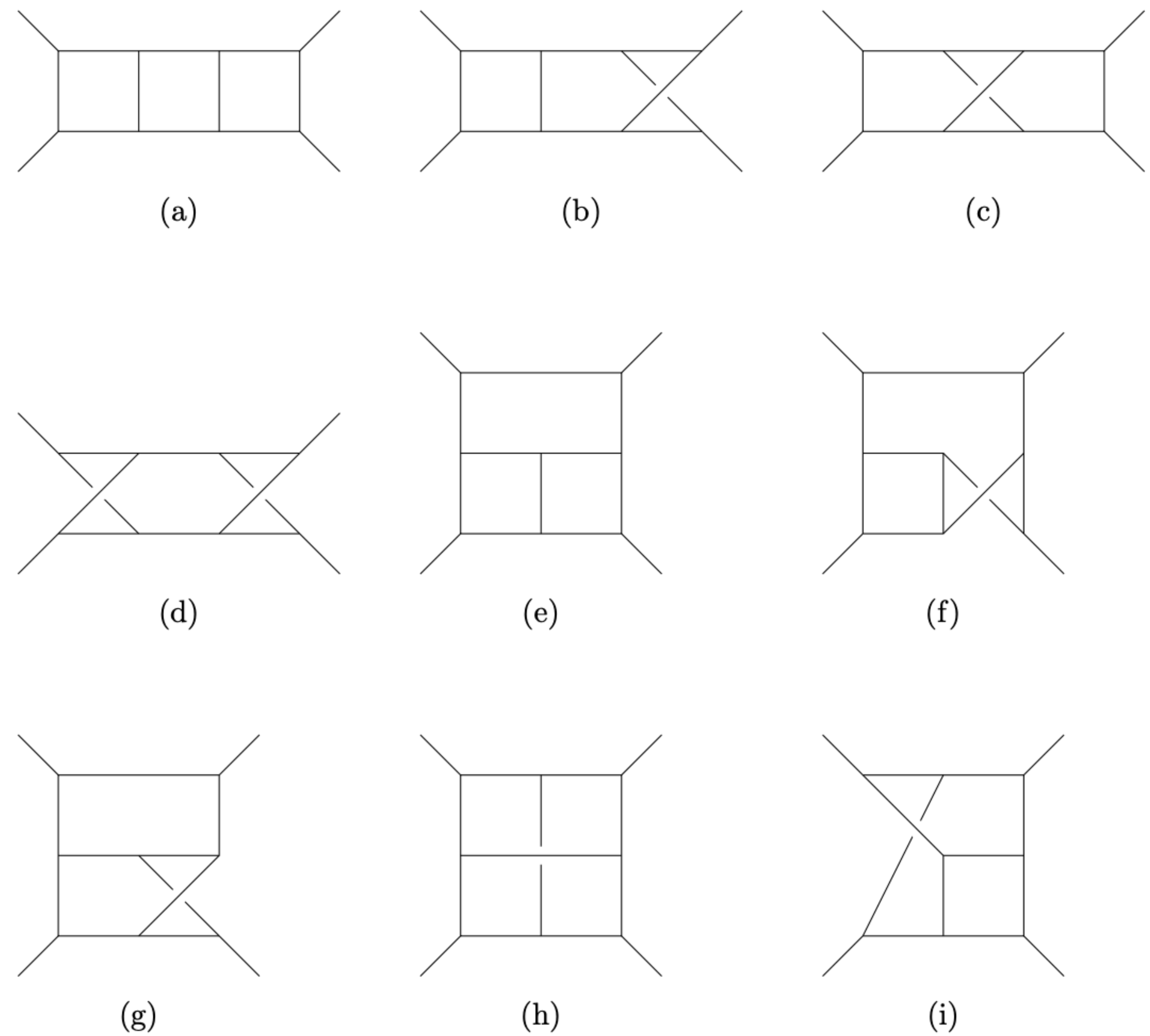
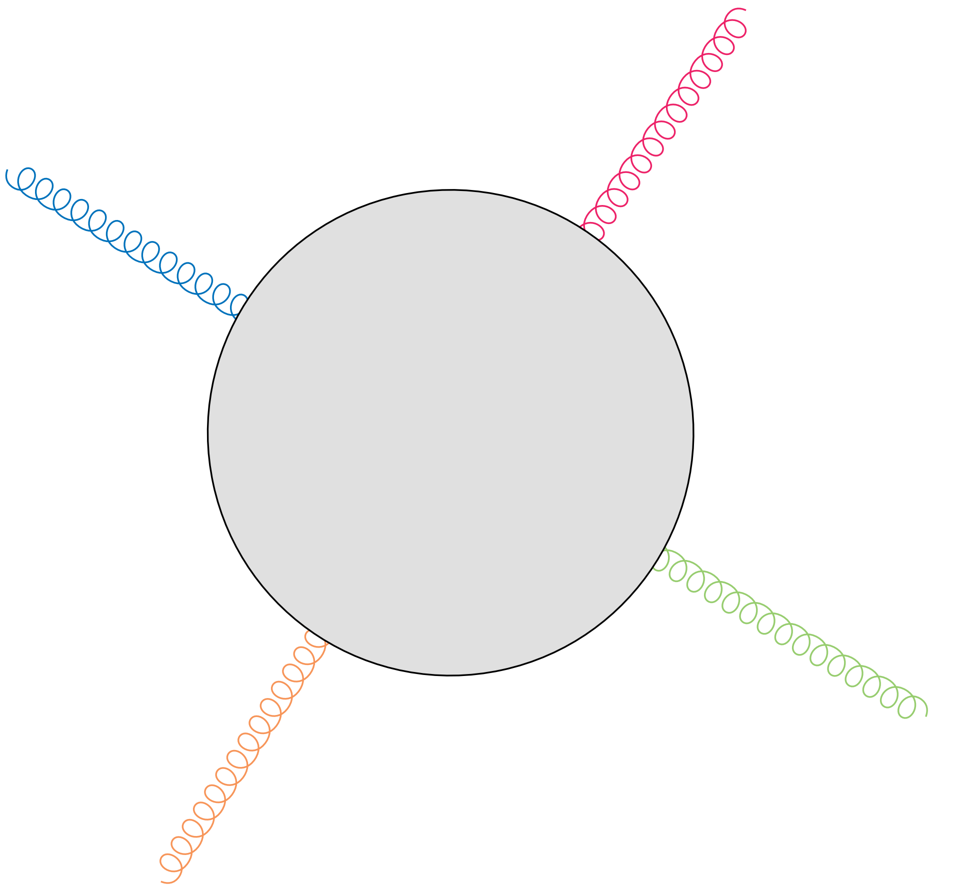
Abstract: We compute all master integrals for massless three-loop four-particle scattering amplitudes required for processes like di-jet or di-photon production at the LHC. We present our result in terms of a Laurent expansion of the integrals in the dimensional regulator up to 8^{th} power, with coefficients expressed in terms of harmonic polylogarithms. As a basis of master integrals we choose integrals with integrands that only have logarithmic poles — called $d\log$ forms. This choice greatly facilitates the subsequent computation via the method of differential equations. We detail how this basis is obtained via an improved algorithm originally developed by one of the authors. We provide a public implementation of this algorithm. We explain how the algorithm is naturally applied in the context of unitarity. In addition, we classify our $d\log$ forms according to their soft and collinear properties.

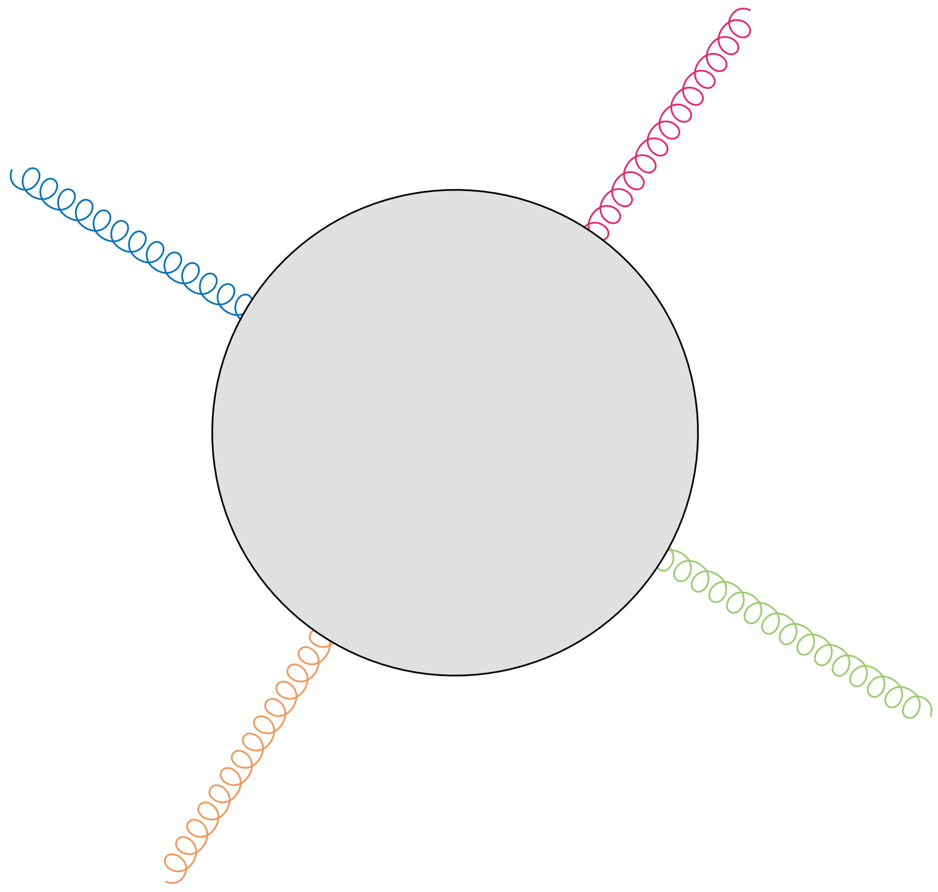


(a) (b) (c) (d) (e) (f) (g) (h) (i)



Feynman Rules

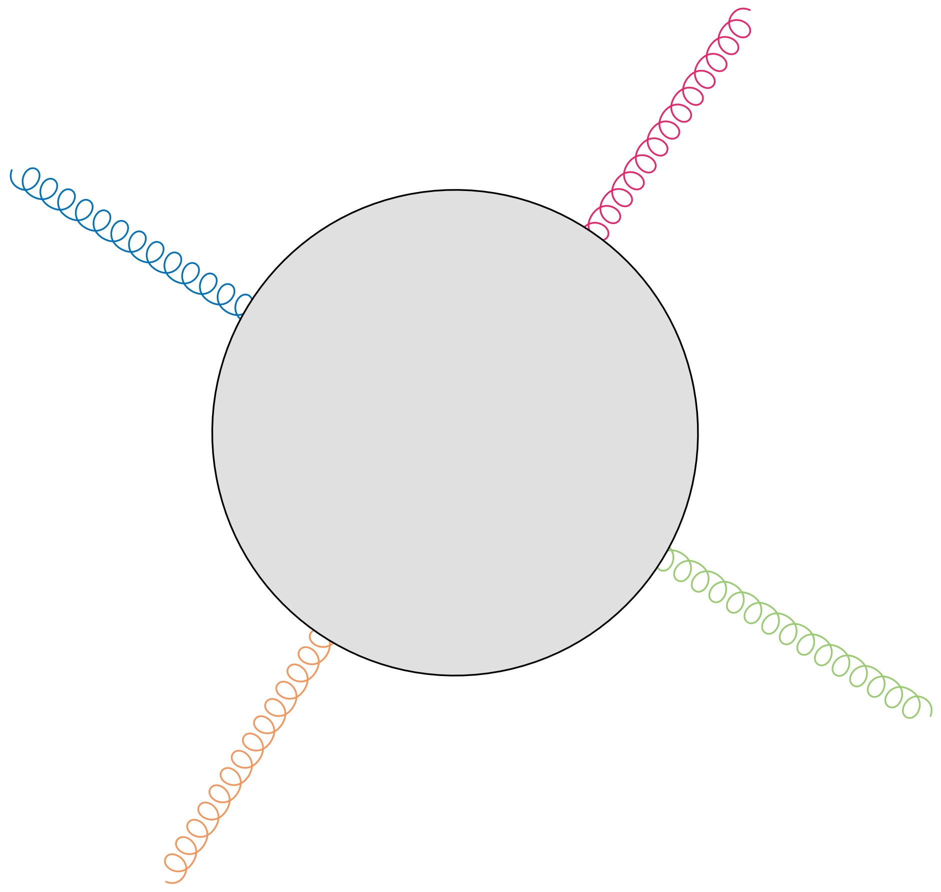
10^7 integrals



Feynman Rules

10^7 integrals

200 GB !!



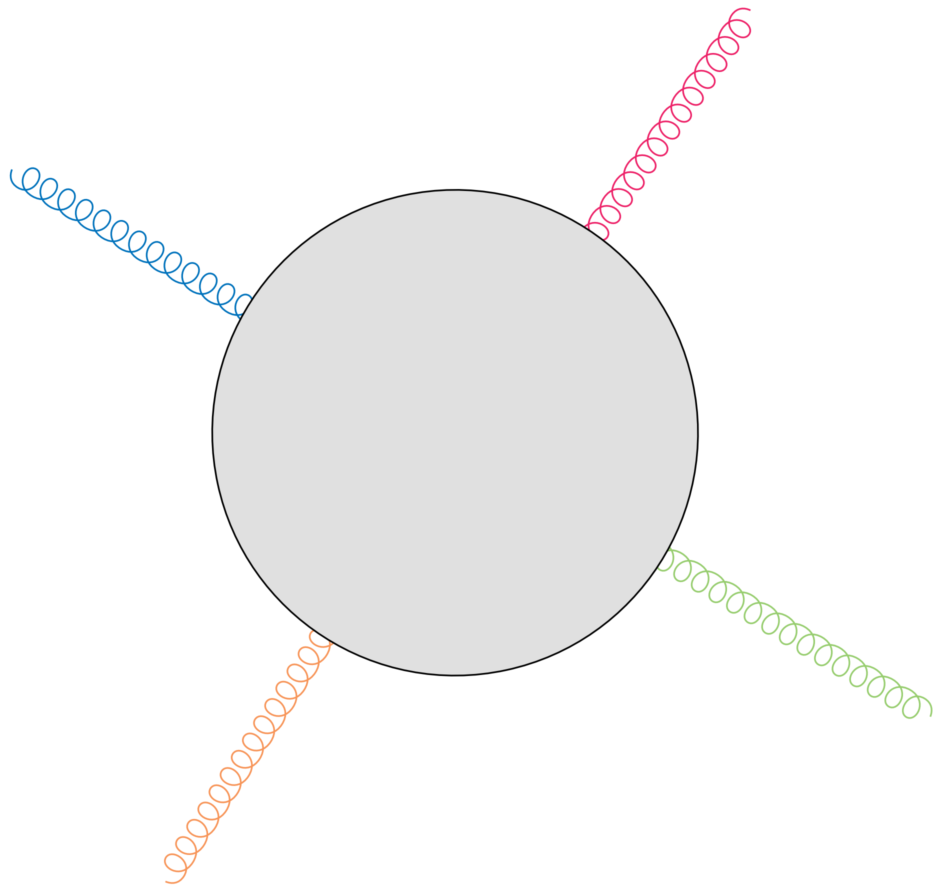
Feynman Rules

10^7 integrals

200 GB !!

Momentum Rerouting

10^6 integrals



Feynman Rules

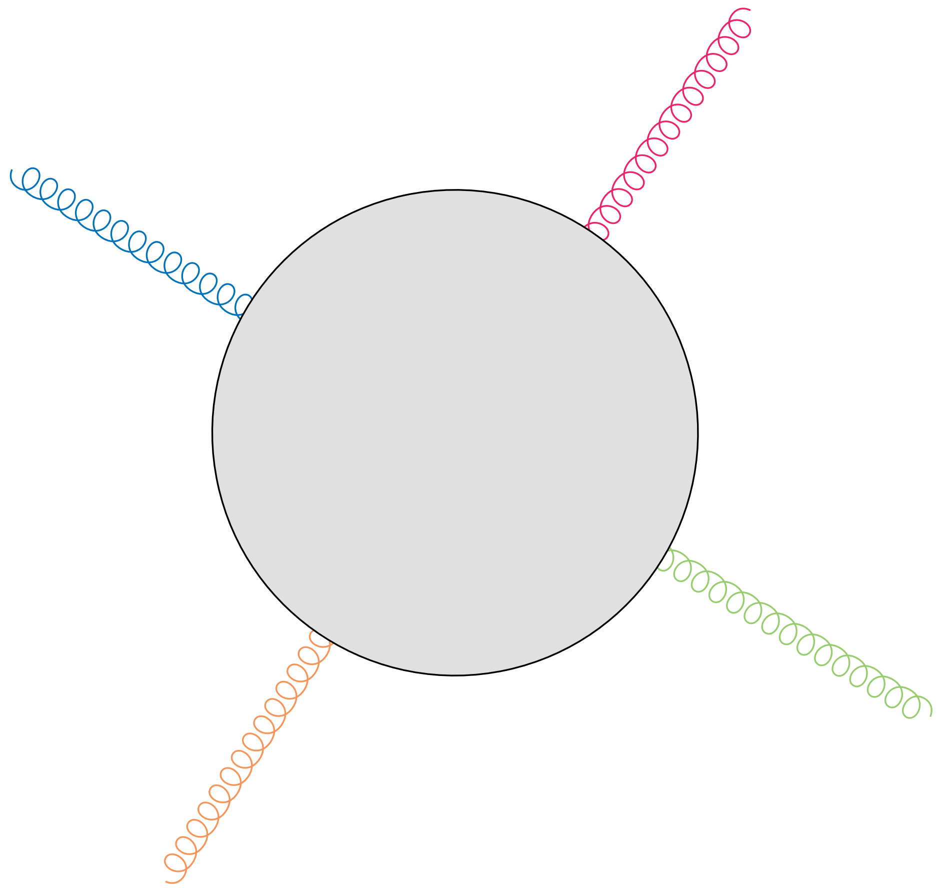
10^7 integrals

200 GB !!

Momentum Rerouting

10^6 integrals

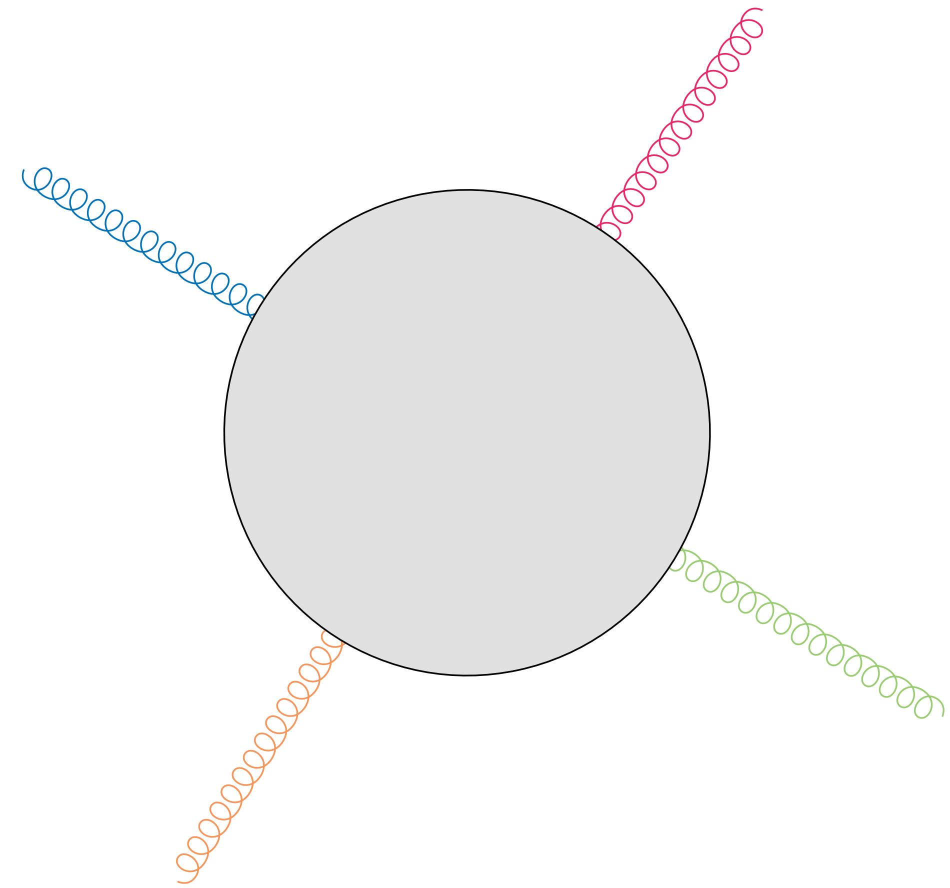
30 GB



Feynman Rules

10^7 integrals

200 GB !!



Momentum Rerouting

10^6 integrals

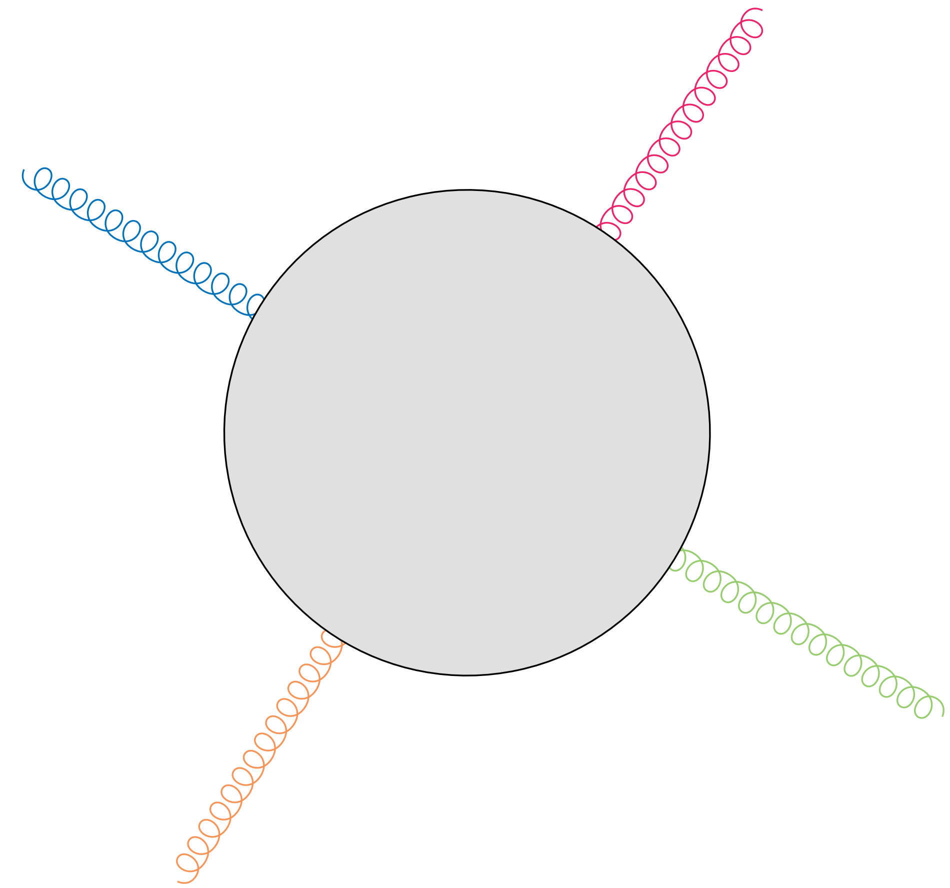
30 GB

FinRed

IBPs

$$F_c^i = \sum_{masters}^{\sim 500} R_i \, M_i$$

1 GB



Feynman Rules

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$$F^i_c = \sum_{masters}^{\sim 500} R_i \, M_i$$

1 GB

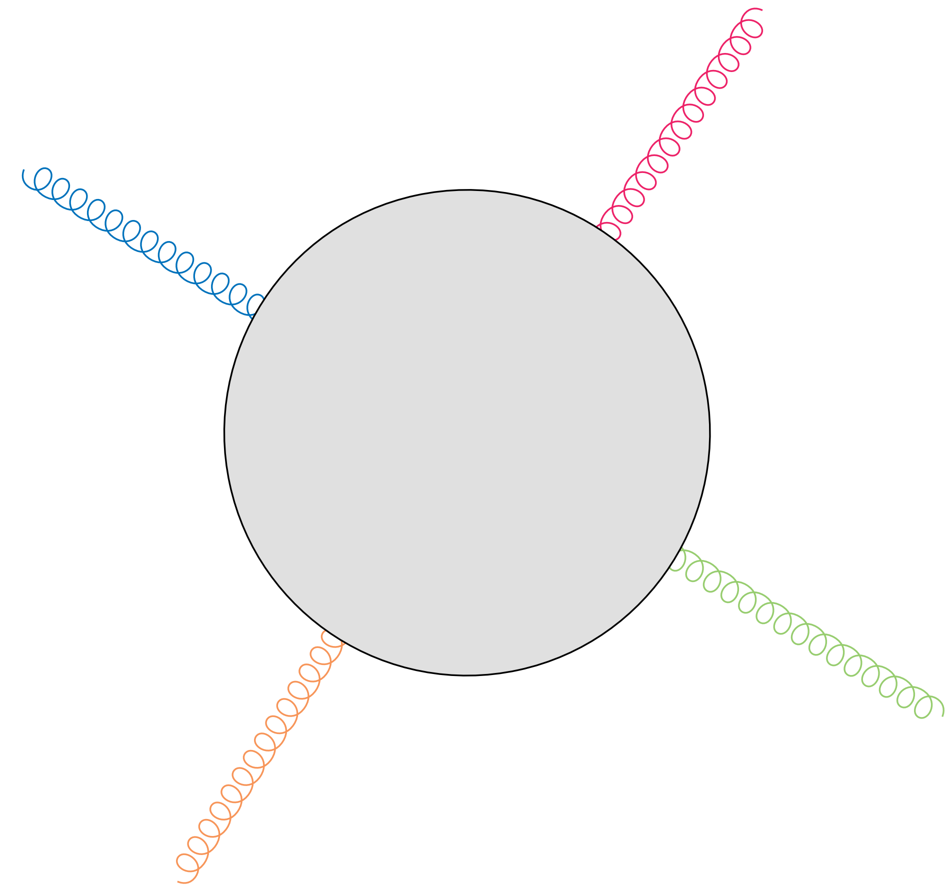
ε expansion

Feynman Rules

10^7 integrals

200 GB !!

Momentum Rerouting



10^6 integrals

30 GB

FinRed

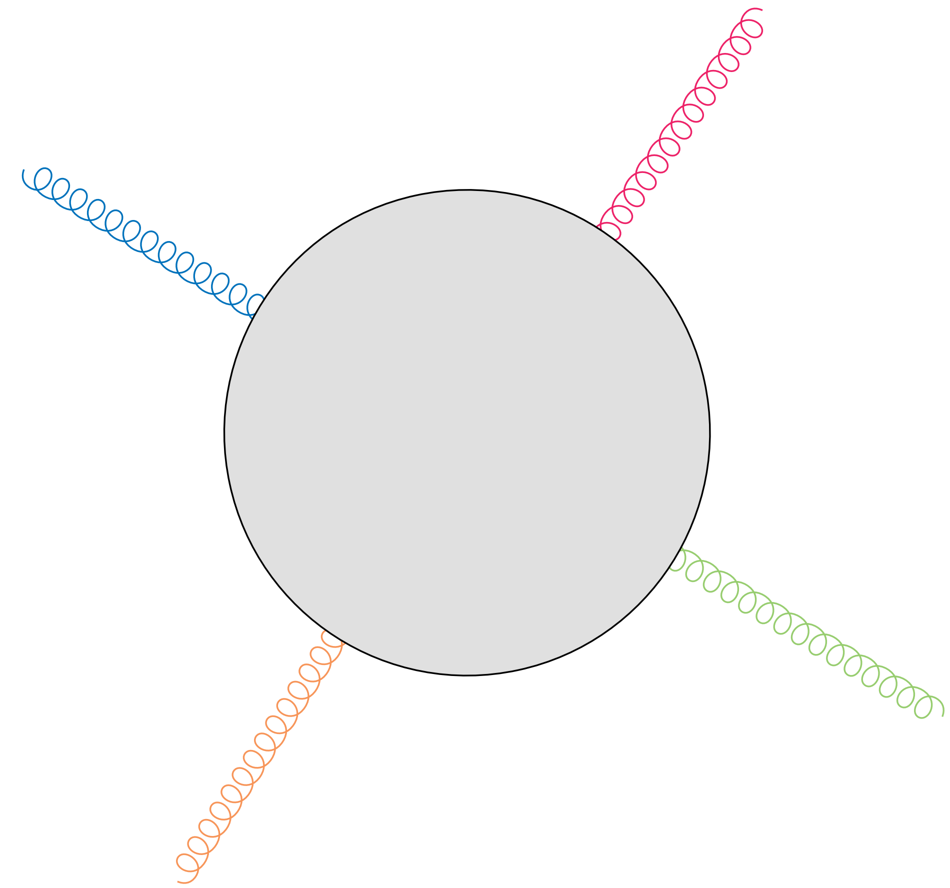
IBPs

$$F_c^i = \sum_{masters}^{\sim 500} R_i \, M_i$$

1 GB

ε expansion

$$F_c^i = \sum \epsilon^{-n} \, r_i \, \mathbb{T}_i$$



Feynman Rules

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ε expansion

$$F_c^i = \sum \epsilon^{-n} \, r_i \, \mathbb{T}_i$$

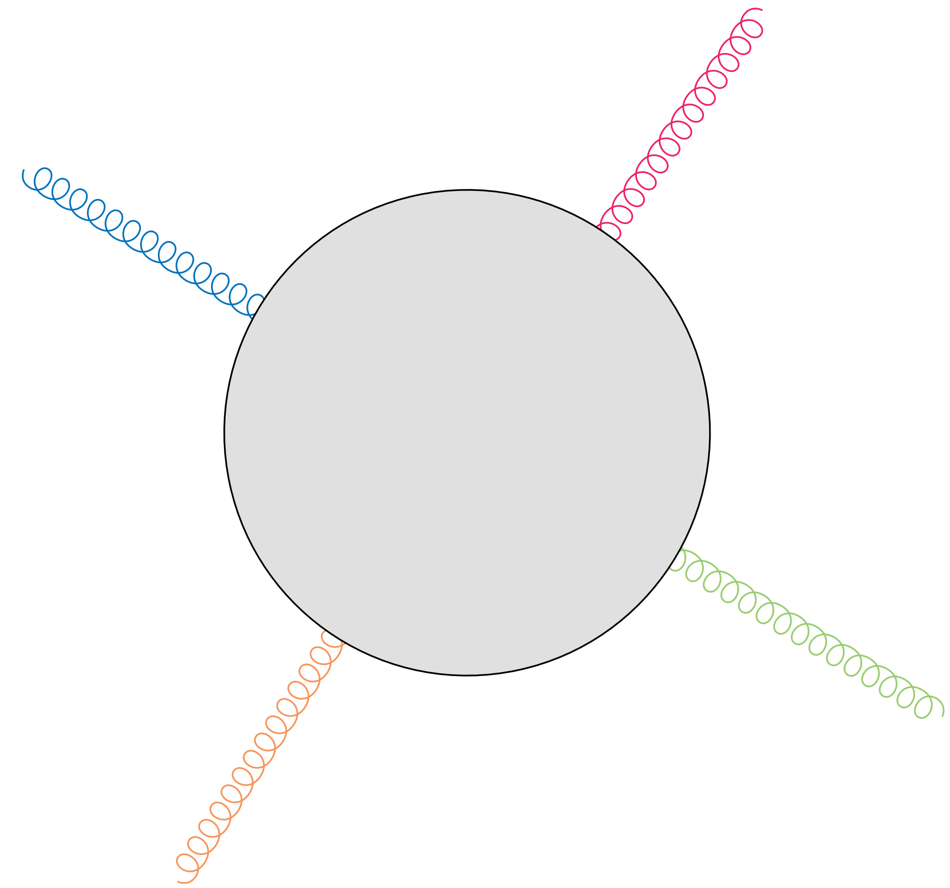
Harmonic Polylogarithms

Feynman Rules

10^7 integrals

200 GB !!

Momentum Rerouting



10^6 integrals

30 GB

FinRed

IBPs

$$F_c^i = \sum_{masters}^{\sim 500} R_i \, M_i$$

1 GB

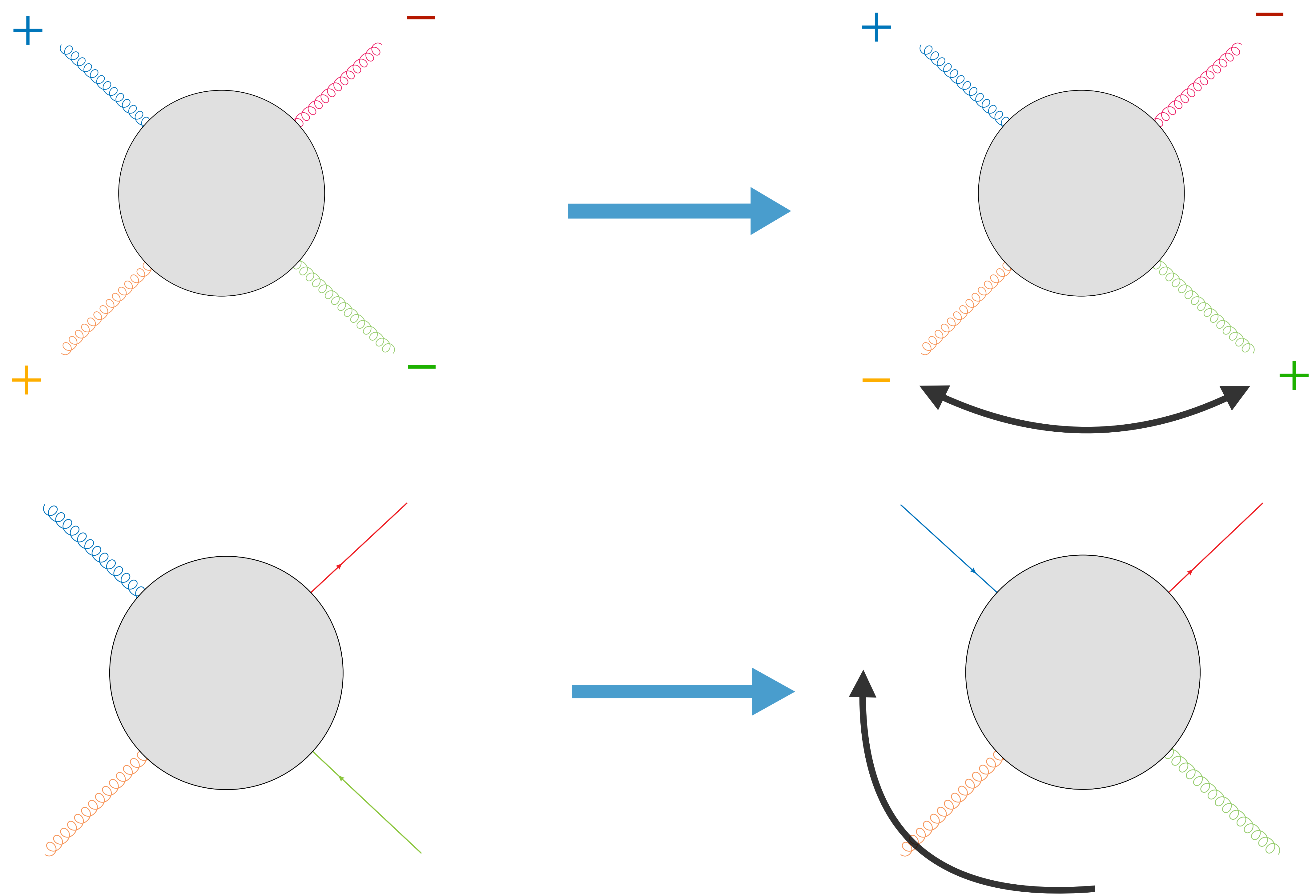
ε expansion

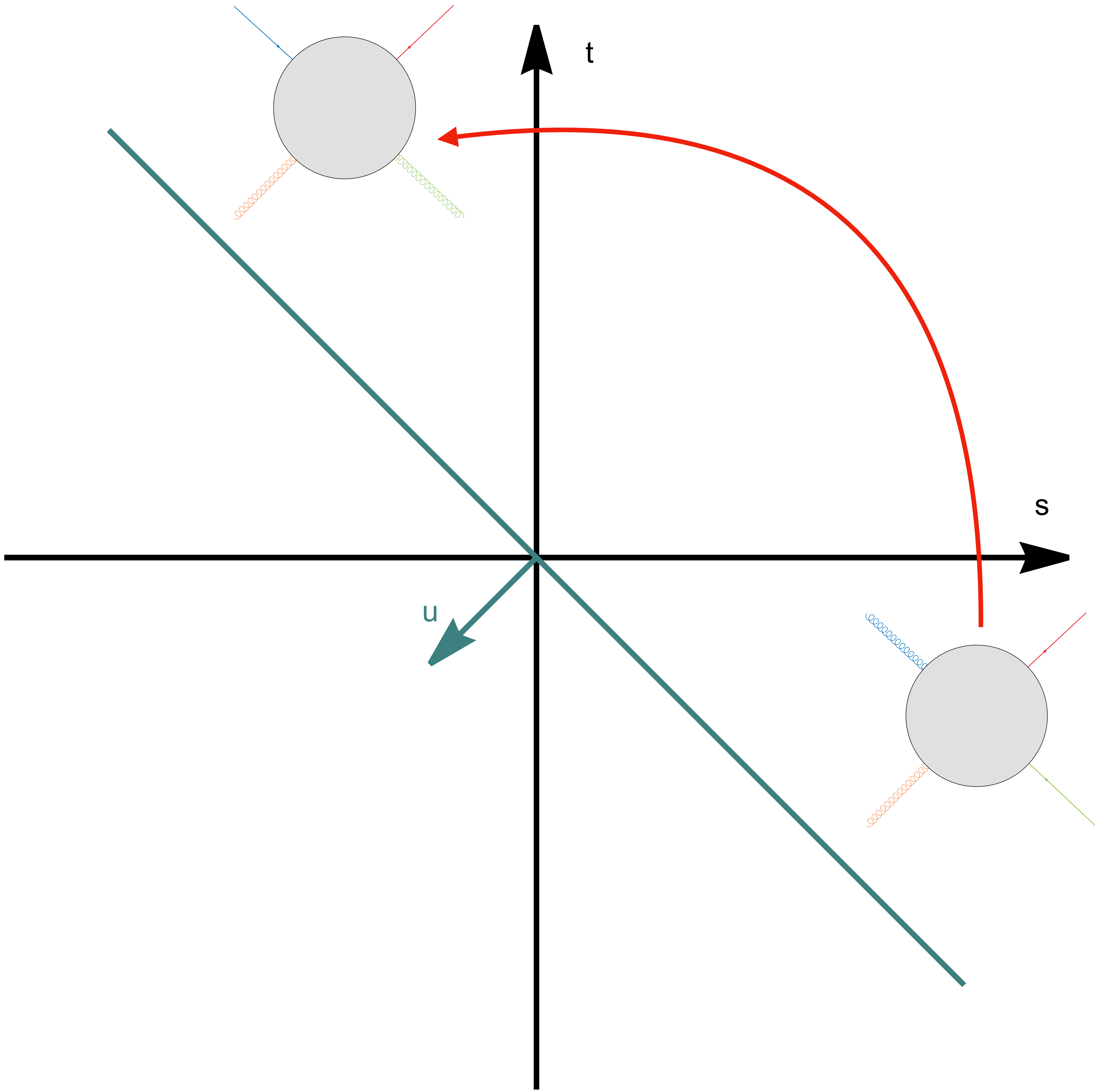
$$F_c^i = \sum \epsilon^{-n} \, r_i \, \mathbb{T}_i$$

Harmonic Polylogarithms

2 MB

Crossing the Amplitudes

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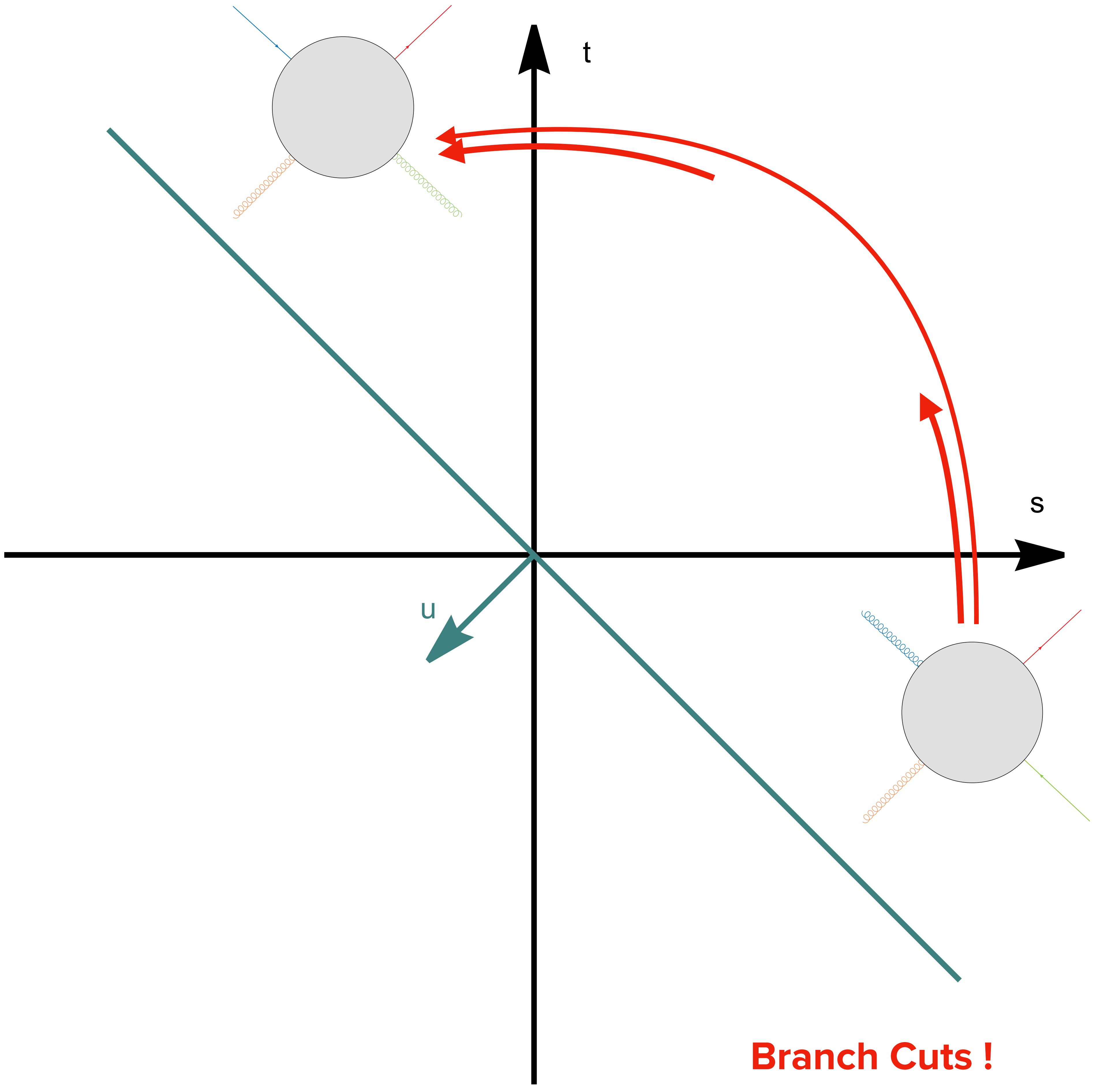
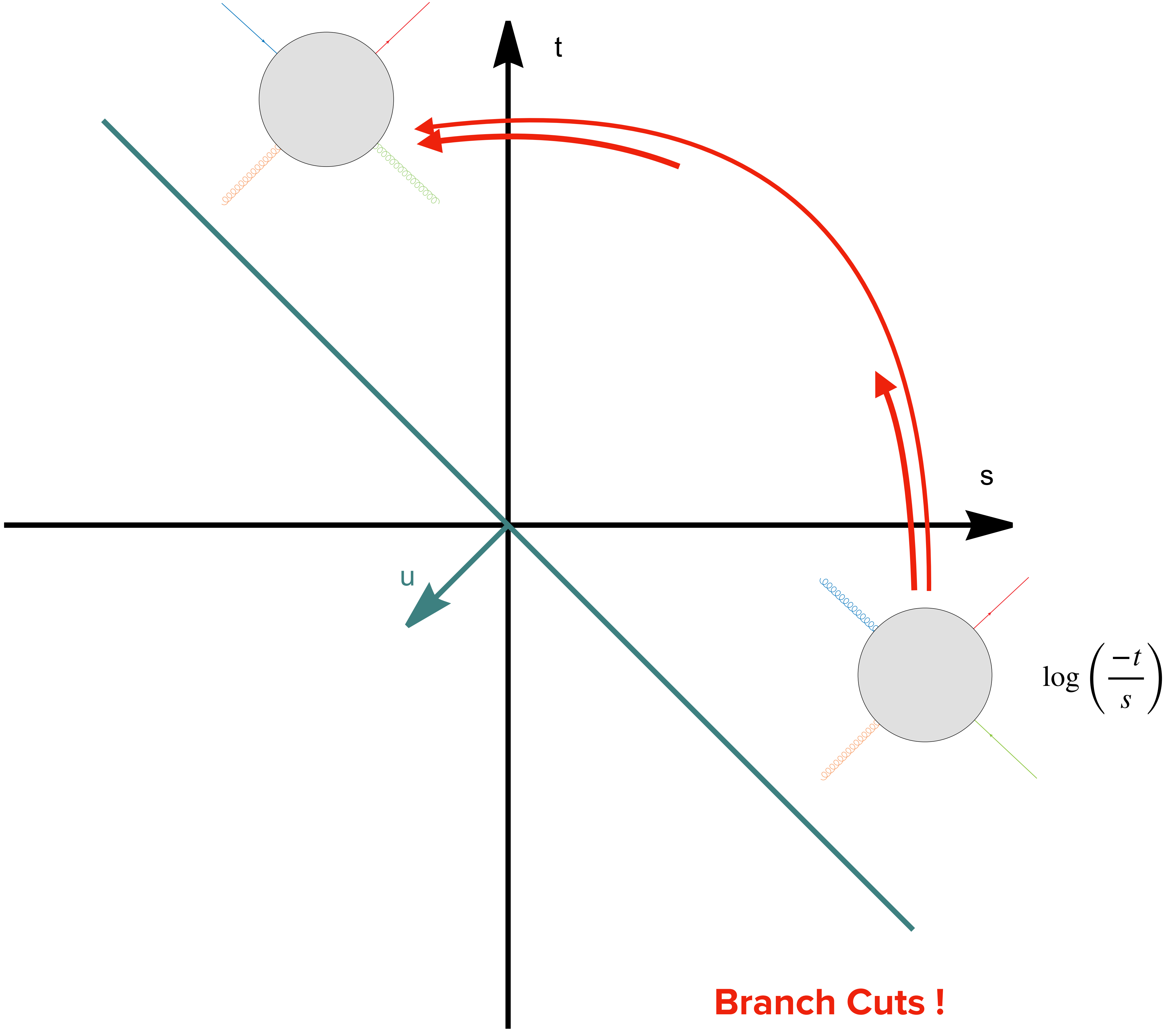
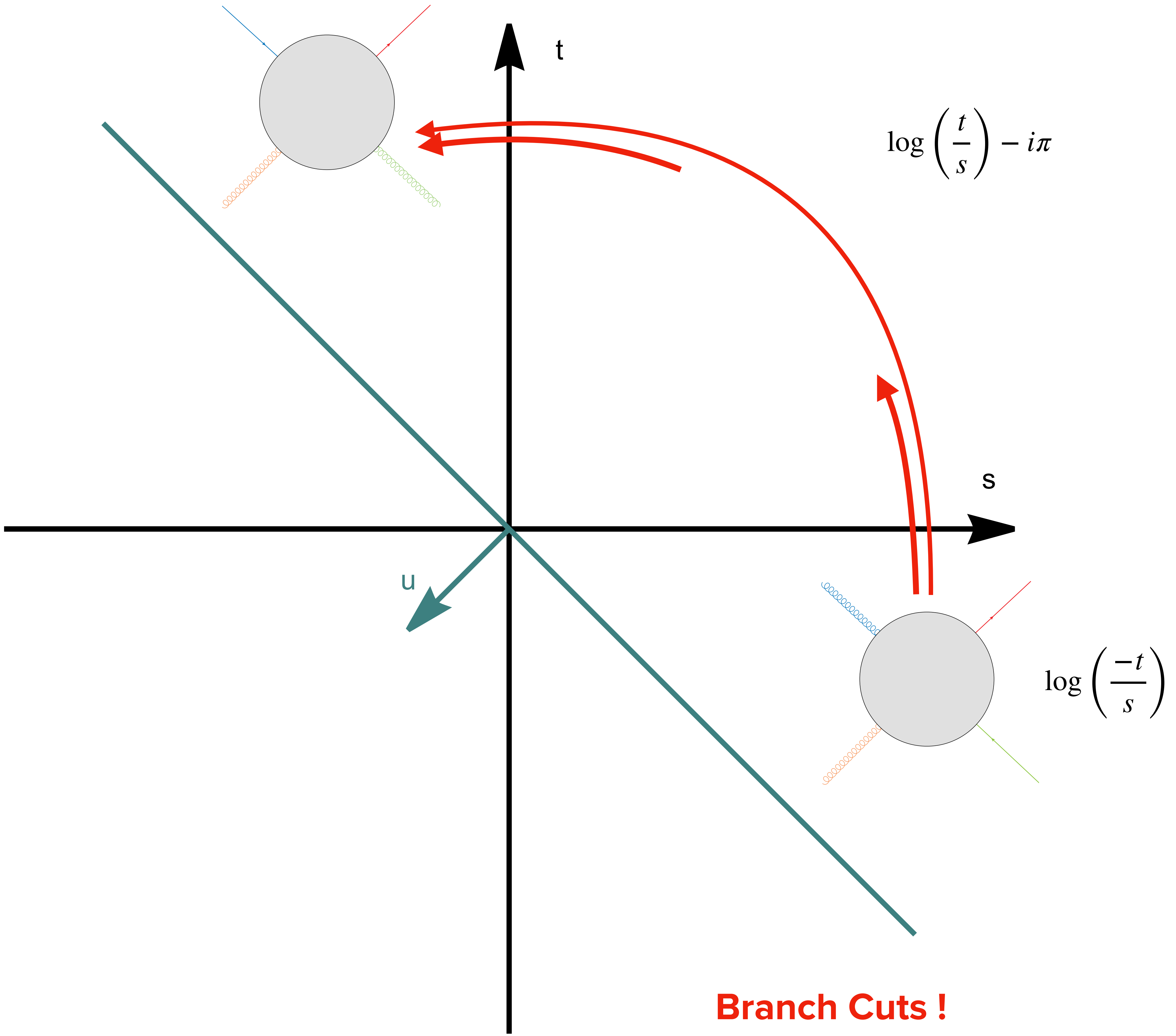
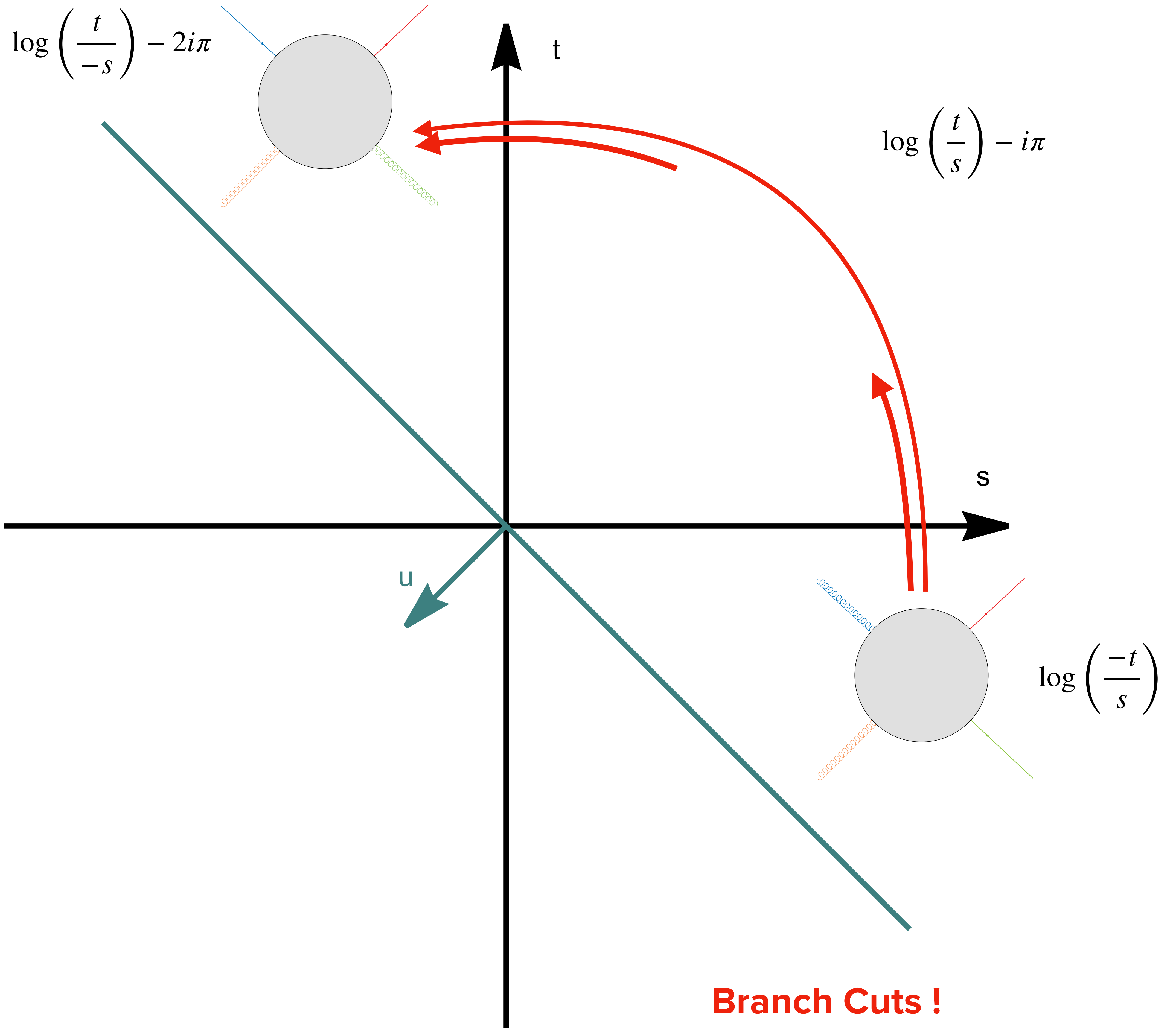


Branch Cuts !

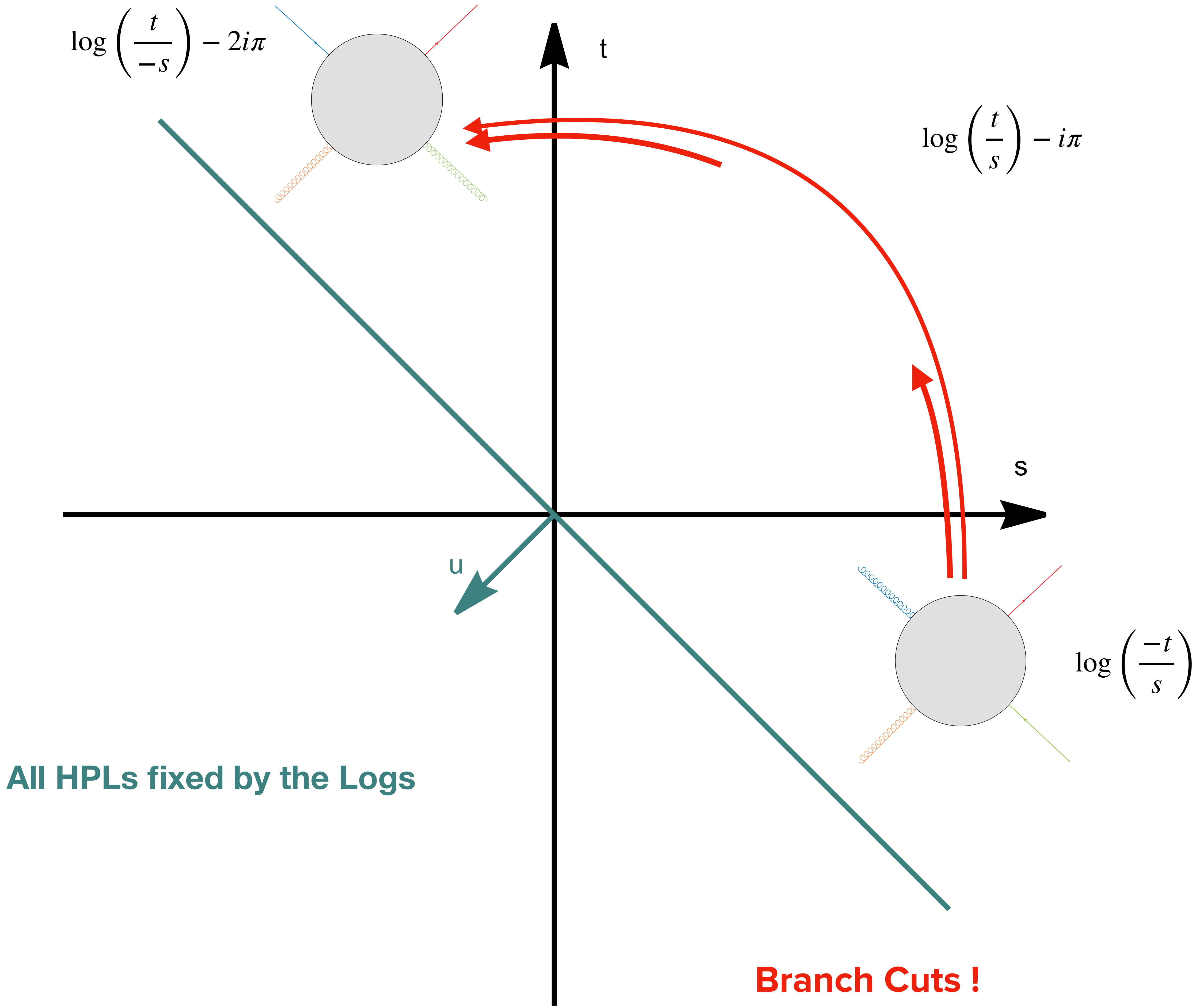

t
s
u
$\log\left(\frac{-t}{s}\right)$
Branch Cuts !



t
$\log\left(\frac{t}{s}\right) - i\pi$
s
u
$\log\left(\frac{-t}{s}\right)$
Branch Cuts !



$\log\left(\frac{t}{-s}\right) - 2i\pi$
t
$\log\left(\frac{t}{s}\right) - i\pi$
s
u
$\log\left(\frac{-t}{s}\right)$
Branch Cuts !



$\log\left(\frac{t}{-s}\right) - 2i\pi$
t
$\log\left(\frac{t}{s}\right) - i\pi$
s
u
$\log\left(\frac{-t}{s}\right)$
All HPLs fixed by the Logs
Branch Cuts !

Infrared Structure

Infrared Factorisation

$$\boldsymbol{\mathcal{H}}_{\mathrm{ren}}(\epsilon, \{p\}) = \boldsymbol{\mathcal{Z}}(\epsilon, \{p\}, \mu)\, \boldsymbol{\mathcal{H}}_{\mathrm{fin}}(\mu, \{p\})$$

Gardi, Magnea: **0901.1091**, **0908.3273**; Becher, Neubert: **0903.1126**

Infrared Factorisation

$$\boldsymbol{\mathcal{H}}_{\text{ren}}(\epsilon, \{p\}) = \boldsymbol{\mathcal{Z}}(\epsilon, \{p\}, \mu)\, \boldsymbol{\mathcal{H}}_{\text{fin}}(\mu, \{p\})$$

IR-divergent

Gardi, Magnea: **0901.1091**, **0908.3273**; Becher, Neubert: **0903.1126**

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$$\boldsymbol{\mathcal{H}}_{\,\mathrm{ren}}(\epsilon, \{p\}) = \boldsymbol{\mathcal{Z}}(\epsilon, \{p\}, \mu)\, \boldsymbol{\mathcal{H}}_{\,\mathrm{fin}}(\mu, \{p\})$$

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IR-finite

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$$\boldsymbol{\mathcal{H}}_{\,\mathrm{ren}}(\epsilon, \{p\}) = \boldsymbol{\mathcal{Z}}(\epsilon, \{p\}, \mu)\, \boldsymbol{\mathcal{H}}_{\,\mathrm{fin}}(\mu, \{p\})$$

IR-divergent

IR "renormalisation"

IR-finite

Gardi, Magnea: **0901.1091**, **0908.3273**; Becher, Neubert: **0903.1126**

Infrared Factorisation

$$\boldsymbol{\mathcal{H}}_{\text{ren}}(\epsilon, \{p\}) = \boldsymbol{\mathcal{Z}}(\epsilon, \{p\}, \mu)\, \boldsymbol{\mathcal{H}}_{\text{fin}}(\mu, \{p\})$$

IR-divergent — IR "renormalisation" — IR-finite

$$\boldsymbol{\mathcal{Z}}(\epsilon, \{p\}, \mu) = \mathbb{P} \exp\left[\int_{\mu}^{\infty} \frac{\mathrm{d}\mu'}{\mu'} \boldsymbol{\Gamma}(\{p\}, \mu')\right]$$

Gardi, Magnea: **0901.1091**, **0908.3273**; Becher, Neubert: **0903.1126**

Infrared Factorisation

$$\mathcal{H}_{\text{ren}}(\epsilon, \{p\}) = \mathcal{Z}(\epsilon, \{p\}, \mu)\, \mathcal{H}_{\text{fin}}(\mu, \{p\})$$

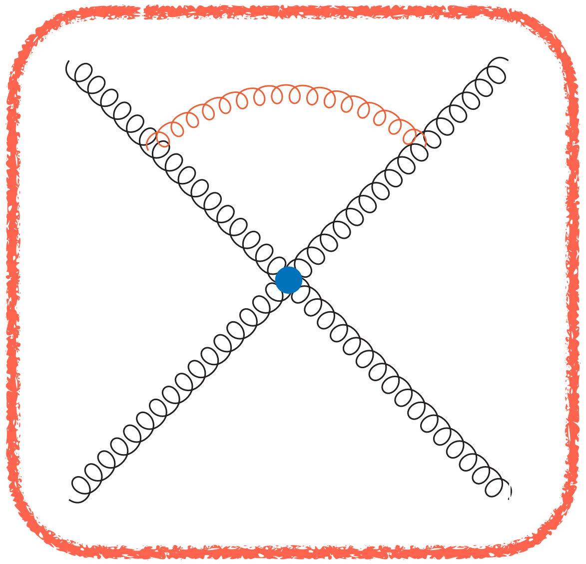
IR-divergent — IR "renormalisation" — IR-finite

$$\mathcal{Z}(\epsilon, \{p\}, \mu) = \mathbb{P}\exp\left[\int_{\mu}^{\infty} \frac{\mathrm{d}\mu'}{\mu'} \mathbf{\Gamma}(\{p\}, \mu')\right]$$

$$\mathbf{\Gamma}(\{p\}, \mu) = \mathbf{\Gamma}_{\text{dipole}}(\{p\}, \mu) + \mathbf{\Delta}_4(\{p\})$$

Gardi, Magnea: **0901.1091**, **0908.3273**; Becher, Neubert: **0903.1126**

Infrared Factorisation



$$\mathcal{H}_{\text{ren}}(\epsilon, \{p\}) = \mathcal{Z}(\epsilon, \{p\}, \mu)\, \mathcal{H}_{\text{fin}}(\mu, \{p\})$$

IR-divergent — IR "renormalisation" — IR-finite

$$\mathcal{Z}(\epsilon, \{p\}, \mu) = \mathbb{P}\exp\left[\int_{\mu}^{\infty} \frac{\mathrm{d}\mu'}{\mu'} \boldsymbol{\Gamma}(\{p\}, \mu')\right]$$

$$\boldsymbol{\Gamma}(\{p\}, \mu) = \boldsymbol{\Gamma}_{\text{dipole}}(\{p\}, \mu) + \boldsymbol{\Delta}_4(\{p\})$$

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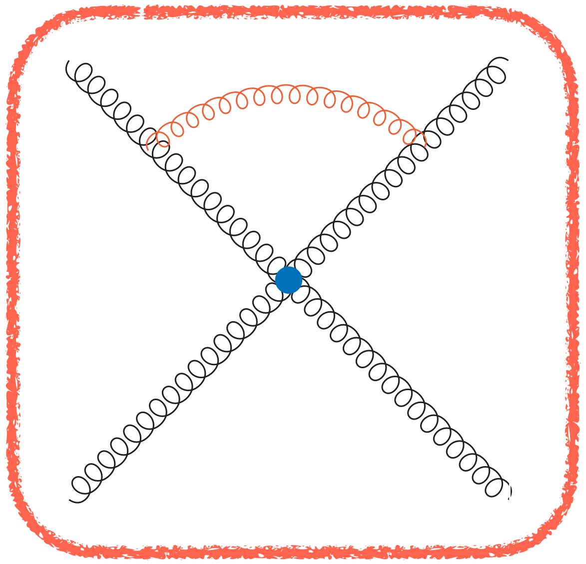
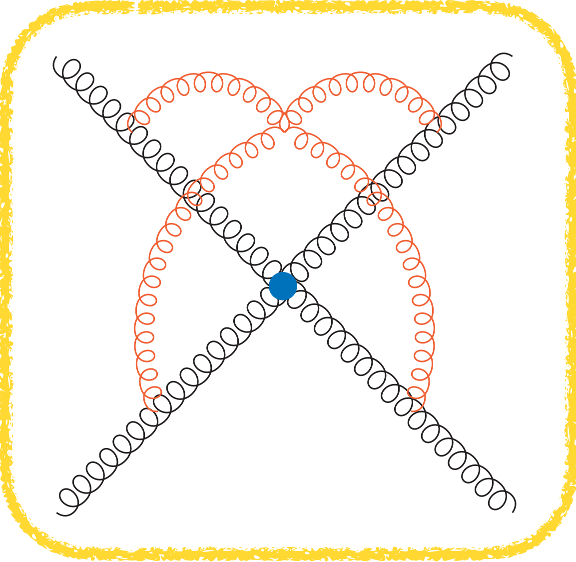
Infrared Factorisation





$$\mathcal{H}_{\rm ren}(\epsilon, \{p\}) = \mathcal{Z}(\epsilon, \{p\}, \mu)\, \mathcal{H}_{\rm fin}(\mu, \{p\})$$

IR-divergent IR "renormalisation" IR-finite

$$\mathcal{Z}(\epsilon, \{p\}, \mu) = \mathbb{P}\exp\left[\int_{\mu}^{\infty} \frac{\mathrm{d}\mu'}{\mu'} \mathbf{\Gamma}(\{p\}, \mu')\right]$$

$$\mathbf{\Gamma}(\{p\}, \mu) = \mathbf{\Gamma}_{\rm dipole}(\{p\}, \mu) + \mathbf{\Delta}_4(\{p\})$$

Gardi, Magnea: **0901.1091**, **0908.3273**; Becher, Neubert: **0903.1126**

$$\mathbf{\Gamma}_{\mathrm{dipole}}(\{p\},\mu) = \sum_{1\leq i<j\leq 4} \mathbf{T}_i^a\ \mathbf{T}_j^a\ \gamma^{\mathrm{cusp}}(\alpha_{\mathrm{s}})\ \log\left(\frac{\mu^2}{-s_{ij}-i\delta}\right) + \sum_{i=1}^{4} \gamma^i(\alpha_{\mathrm{s}})$$

$$\mathbf{\Gamma}_{\text{dipole}}(\{p\},\mu) = \sum_{1\leq i<j\leq 4} \mathbf{T}_i^a\, \mathbf{T}_j^a\, \gamma^{\text{cusp}}(\alpha_{\text{s}})\, \log\left(\frac{\mu^2}{-s_{ij} - i\delta}\right) + \sum_{i=1}^{4} \gamma^i(\alpha_{\text{s}})$$

$$\mathbf{\Delta}_4^{(3)} = 128\, f_{abe}\, f_{cde}\, \left[\mathbf{T}_1^a\, \mathbf{T}_2^c\, \mathbf{T}_3^b\, \mathbf{T}_4^d\, D_1(x) - \mathbf{T}_4^a\, \mathbf{T}_1^b\, \mathbf{T}_2^c\, \mathbf{T}_3^d\, D_2(x)\right]$$

$$- 16\, C\, f_{abe}\, f_{cde} \sum_{i=1}^{4} \sum_{\substack{1\leq j<k\leq 4\\ j,k\neq i}} \left\{\mathbf{T}_i^a, \mathbf{T}_i^d\right\}\, \mathbf{T}_j^b\, \mathbf{T}_k^c\, ,$$

Almelid, Duhr, Gardi: **1507.00047**

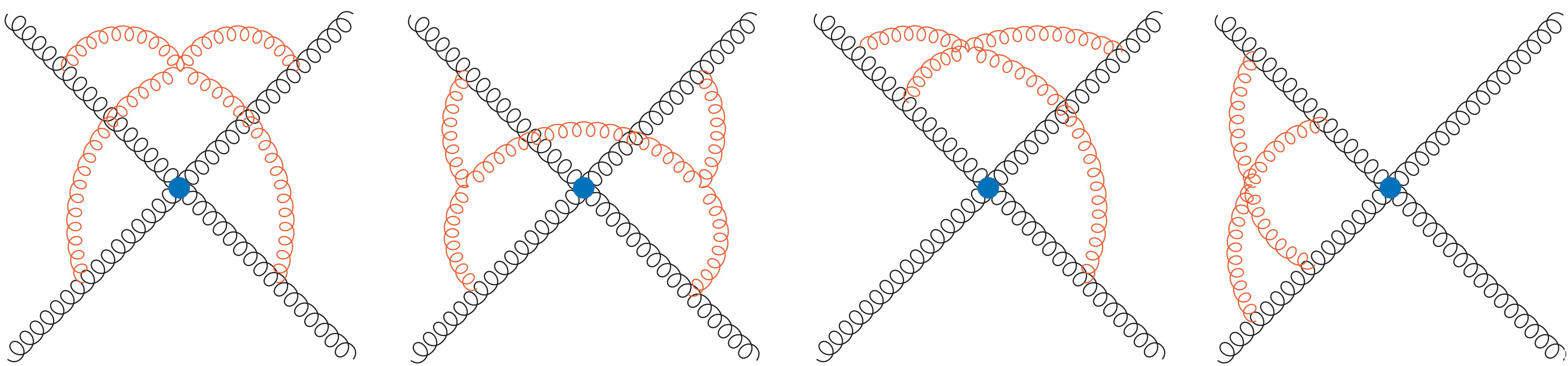
Henn, Mistlberger: **1608.00850**

$$\mathbf{\Gamma}_{\text{dipole}}(\{p\},\mu) = \sum_{1\le i<j\le 4} \mathbf{T}_i^a\, \mathbf{T}_j^a\, \gamma^{\text{cusp}}(\alpha_s)\, \log\left(\frac{\mu^2}{-s_{ij} - i\delta}\right) + \sum_{i=1}^{4} \gamma^i(\alpha_s)$$

$$\mathbf{\Delta}_4^{(3)} = 128\, f_{abe}\, f_{cde}\, \left[\mathbf{T}_1^a\, \mathbf{T}_2^c\, \mathbf{T}_3^b\, \mathbf{T}_4^d\, D_1(x) - \mathbf{T}_4^a\, \mathbf{T}_1^b\, \mathbf{T}_2^c\, \mathbf{T}_3^d\, D_2(x)\right]$$
$$- 16\, C\, f_{abe}\, f_{cde} \sum_{i=1}^{4} \sum_{\substack{1\le j<k\le 4 \\ j,k\neq i}} \left\{\mathbf{T}_i^a, \mathbf{T}_i^d\right\}\, \mathbf{T}_j^b\, \mathbf{T}_k^c\,,$$

Almelid, Duhr, Gardi: **1507.00047**

Henn, Mistlberger: **1608.00850**



$$\mathbf{\Gamma}_{\rm dipole}(\{p\},\mu) = \sum_{1\leq i<j\leq 4} \mathbf{T}_i^a\,\mathbf{T}_j^a\,\gamma^{\rm cusp}(\alpha_s)\,\log\left(\frac{\mu^2}{-s_{ij}-i\delta}\right) + \sum_{i=1}^{4}\gamma^i(\alpha_s)$$

$$\mathbf{\Delta}_4^{(3)} = 128\,f_{abe}\,f_{cde}\,\left[\mathbf{T}_1^a\,\mathbf{T}_2^c\,\mathbf{T}_3^b\,\mathbf{T}_4^d\,D_1(x) - \mathbf{T}_4^a\,\mathbf{T}_1^b\,\mathbf{T}_2^c\,\mathbf{T}_3^d\,D_2(x)\right]$$
$$-16\,C\,f_{abe}\,f_{cde}\sum_{i=1}^{4}\sum_{\substack{1\leq j<k\leq 4\\ j,k\neq i}}\left\{\mathbf{T}_i^a,\mathbf{T}_i^d\right\}\mathbf{T}_j^b\,\mathbf{T}_k^c\,,$$

Almelid, Duhr, Gardi: **1507.00047**

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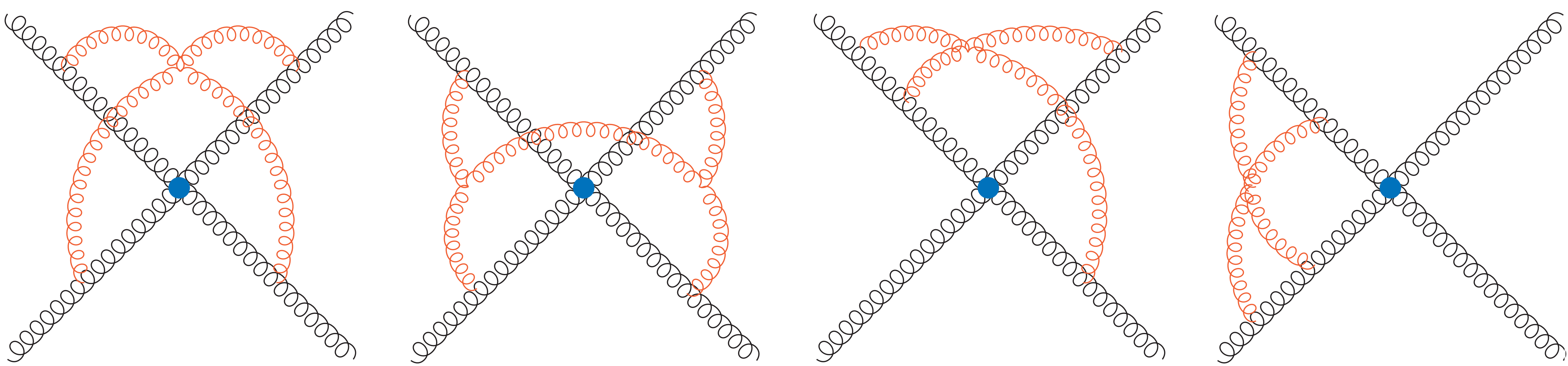
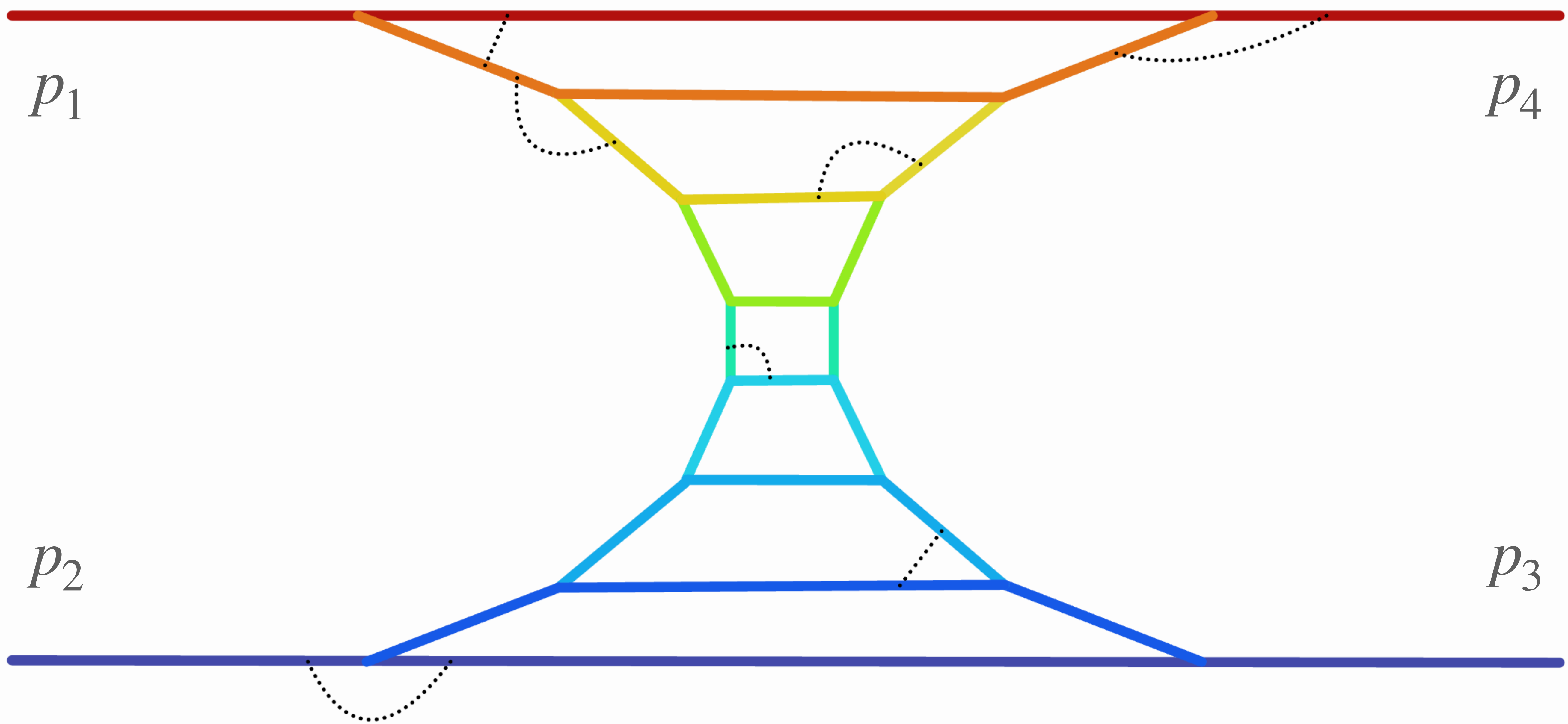
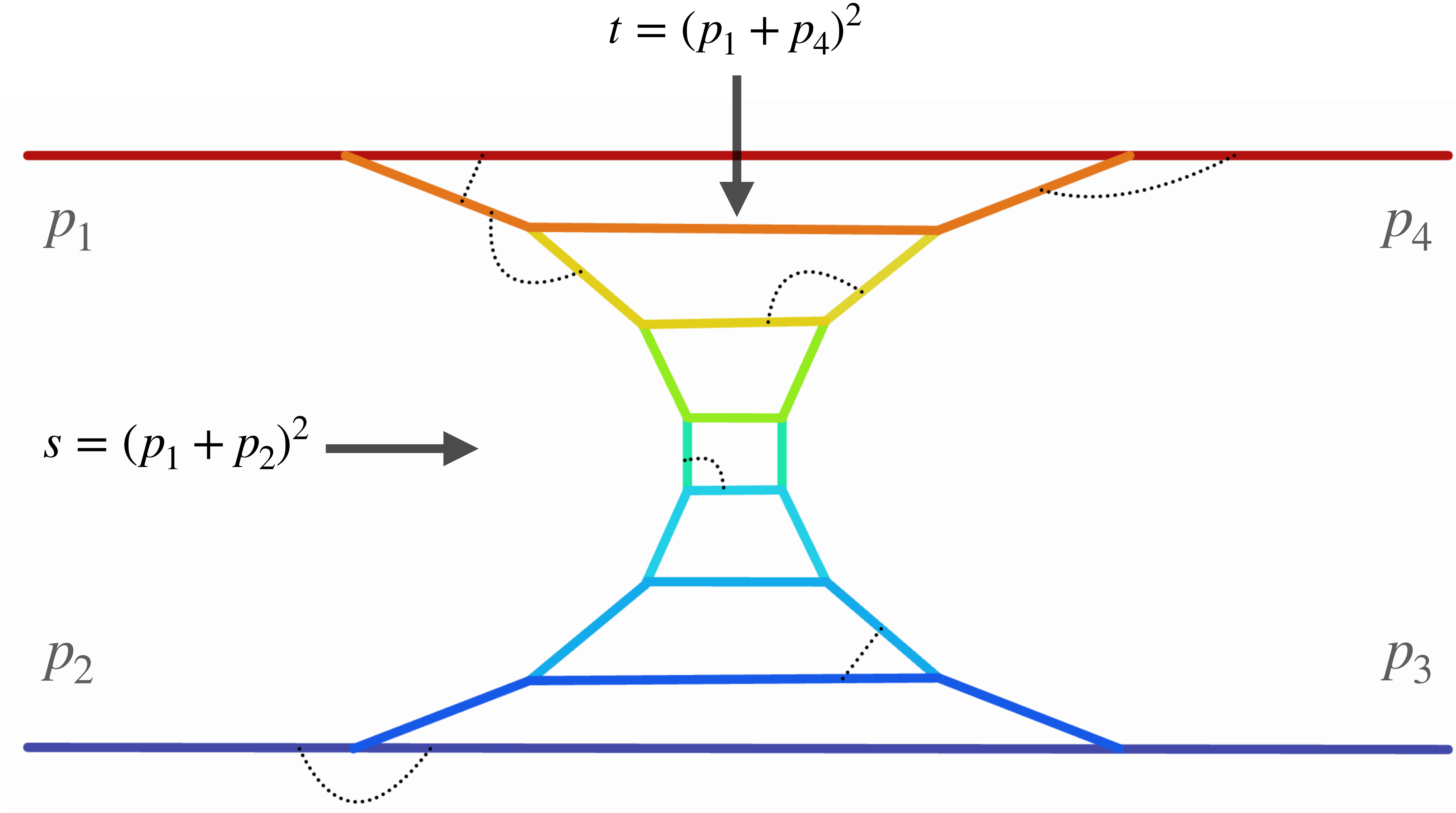


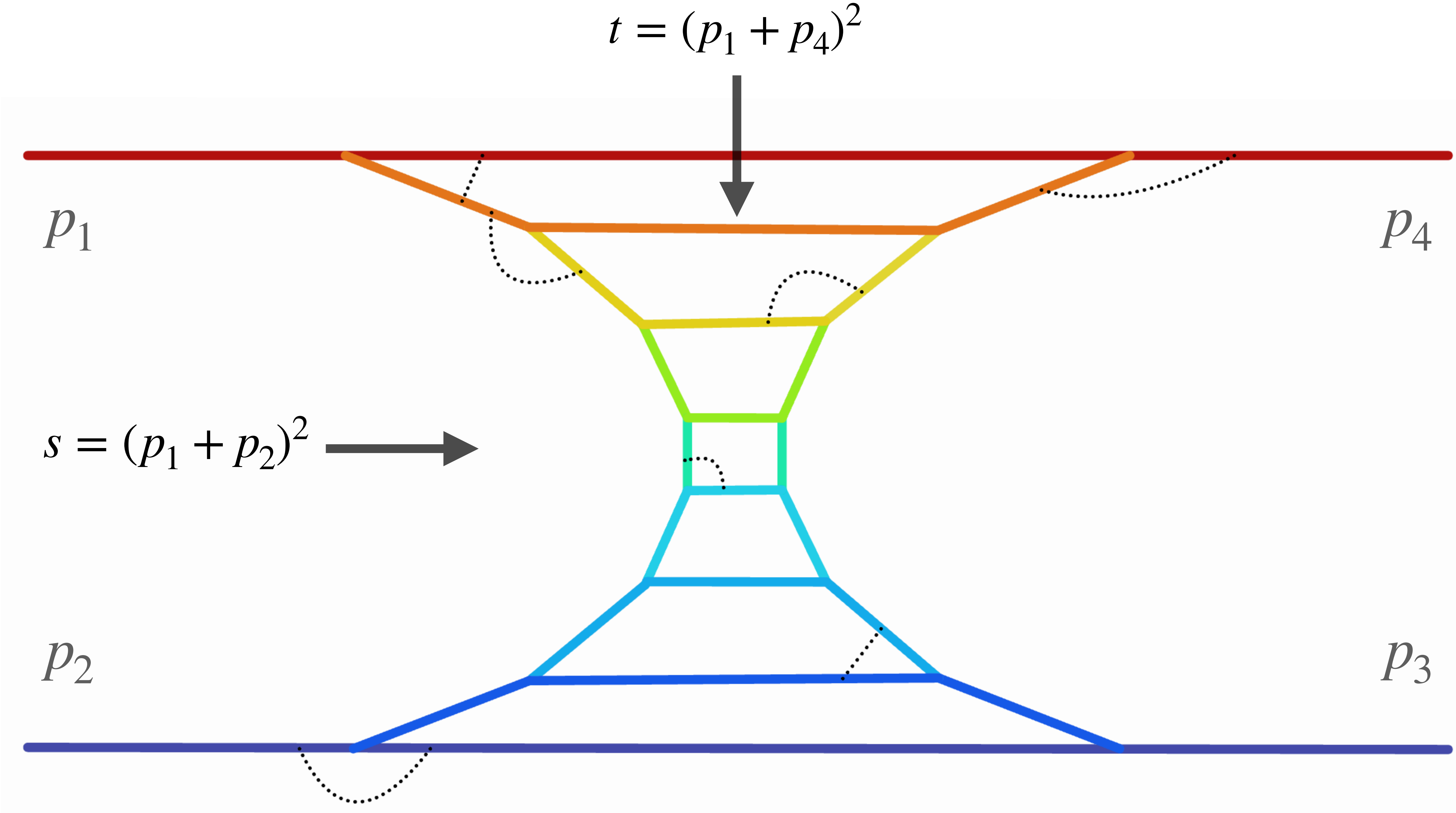
Now confirmed in QCD:

Chakraborty, Caola, GG, Tancredi, von Manteuffel:
2207.03503, 2112.11097

High Energy Limit

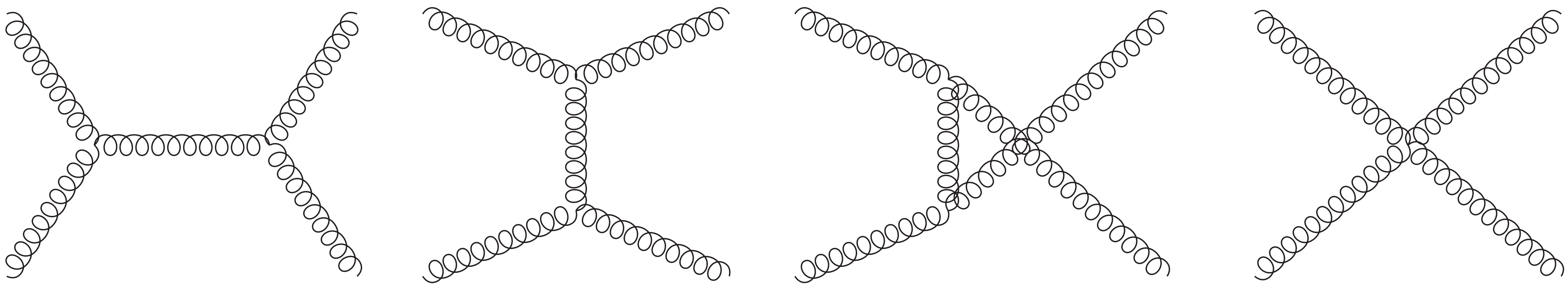

p_1
p_4
p_2
p_3


$t = (p_1 + p_4)^2$
p_1
p_4
$s = (p_1 + p_2)^2$
p_2
p_3

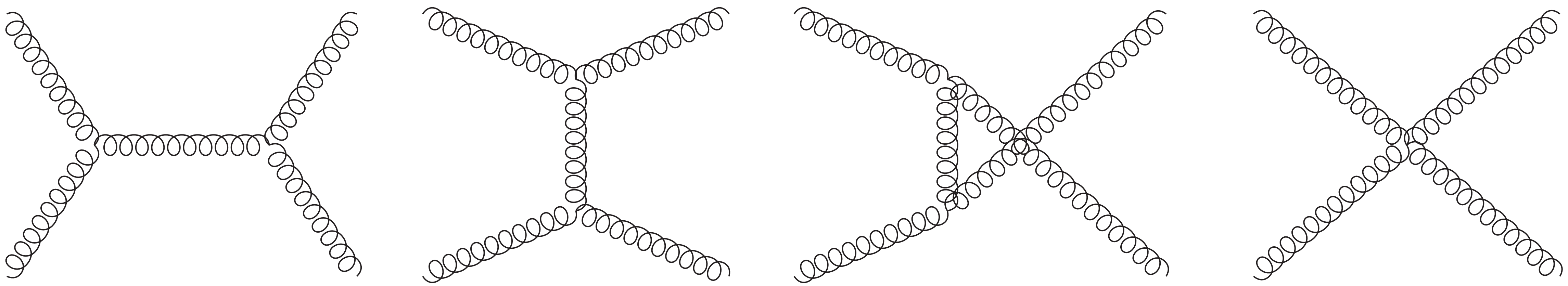


Regge Limit: $s \gg |t|$

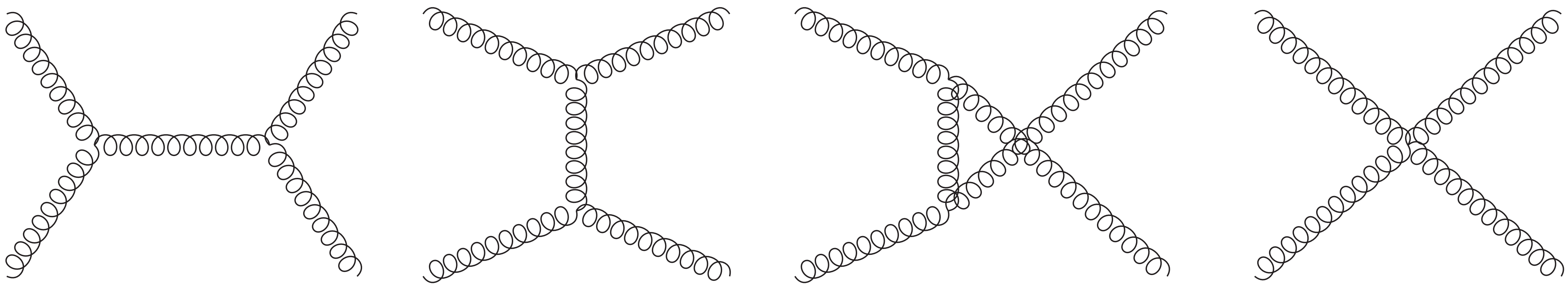
$$s \gg |t|$$



$$s \gg |t|$$

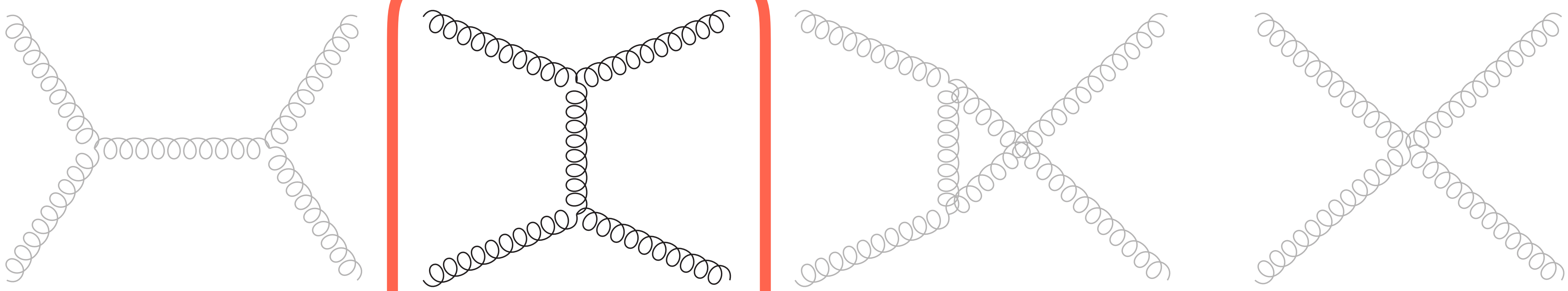


$$s \gg |t|$$



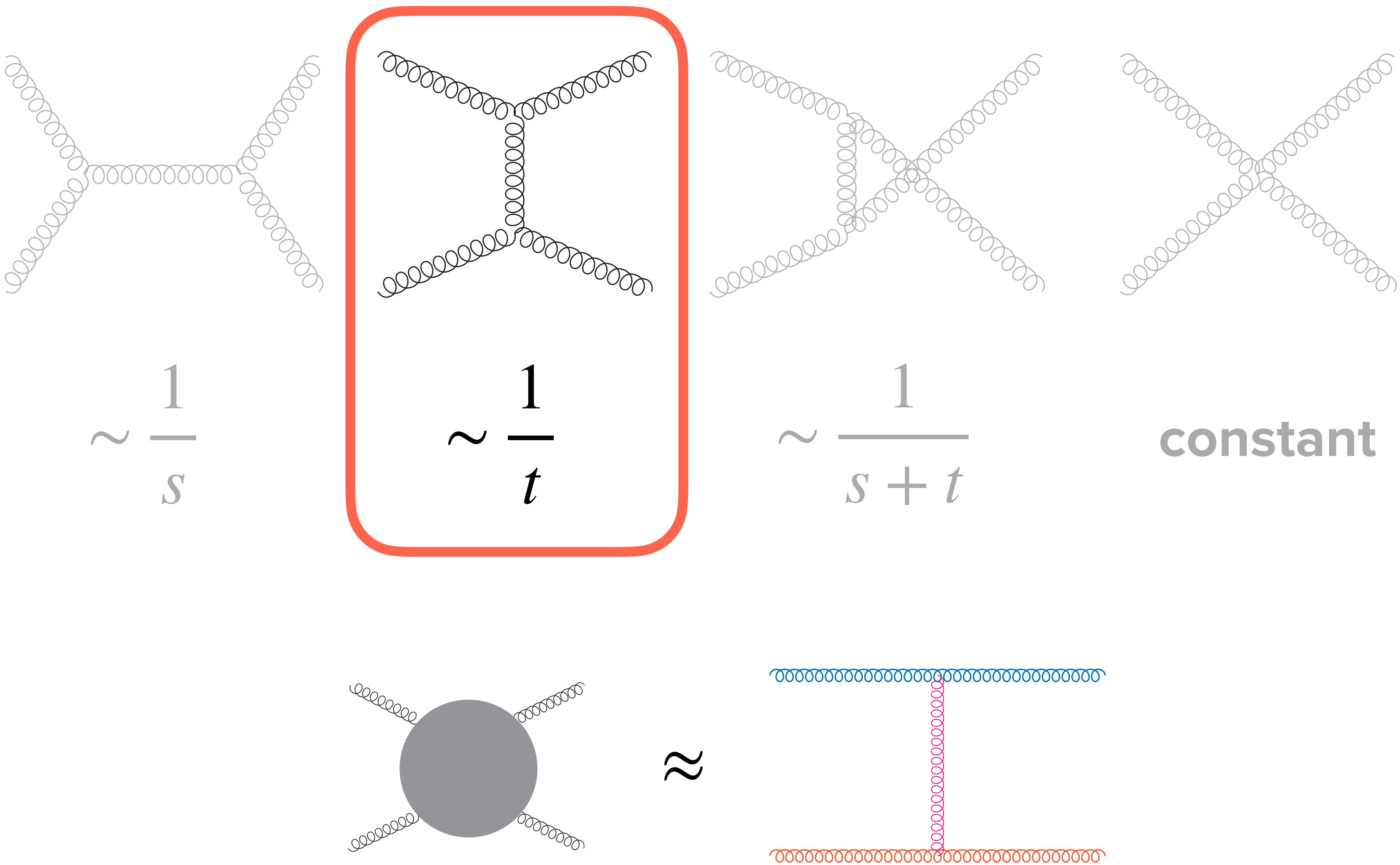
$\sim \frac{1}{s}$ $\sim \frac{1}{t}$ $\sim \frac{1}{s+t}$ **constant**

$$s \gg |t|$$



$\sim \frac{1}{s}$ $\sim \frac{1}{t}$ $\sim \frac{1}{s+t}$ **constant**

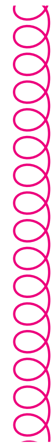
$$s \gg |t|$$



Reggeized gluon

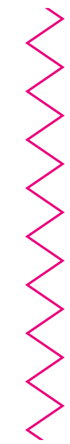
$$\log^{\#}\left(\frac{-t}{s}\right) = L^{\#}$$

Reggeized gluon



$$\frac{1}{t}$$

$$\log^{\#}\left(\frac{-t}{s}\right) = L^{\#}$$



$$e^{L\,\tau_g(\alpha)} \quad \frac{1}{t}$$

Reggeized gluon

$$\frac{1}{t}$$

$$\log^{\#}\left(\frac{-t}{s}\right) = L^{\#}$$

$$e^{L\,\tau_g(\alpha)} \quad \frac{1}{t}$$

$$\tau_g(\alpha) = \alpha\tau^{(1)} + \alpha^2\tau^{(2)} + \alpha^3\tau^{(3)}$$

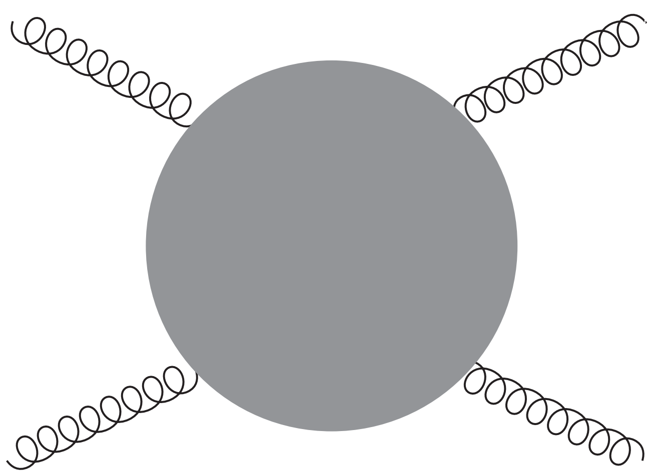
Reggeized gluon

$$\frac{1}{t}$$

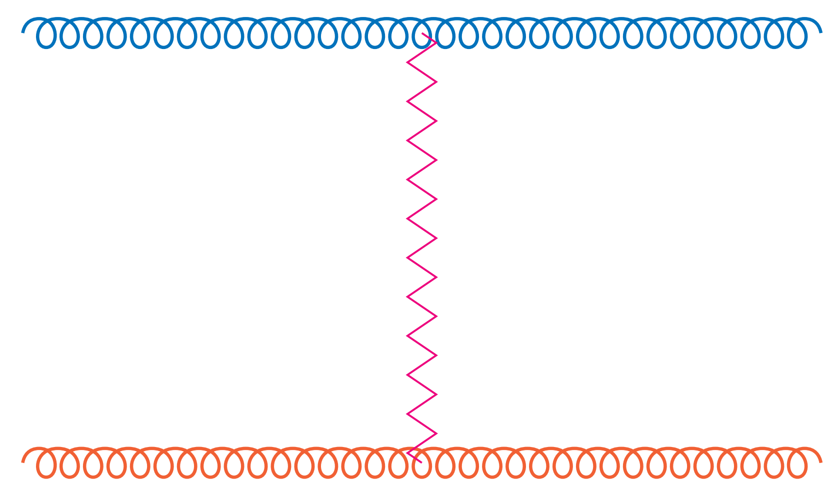
$$\log^{\#}\left(\frac{-t}{s}\right) = L^{\#}$$

$$e^{L\,\tau_g(\alpha)} \quad \frac{1}{t}$$

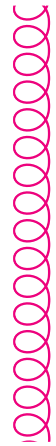
Regge Trajectory $\tau_g(\alpha) = \alpha\tau^{(1)} + \alpha^2\tau^{(2)} + \alpha^3\tau^{(3)}$



$\overset{\text{LL}}{=}$



Reggeized gluon



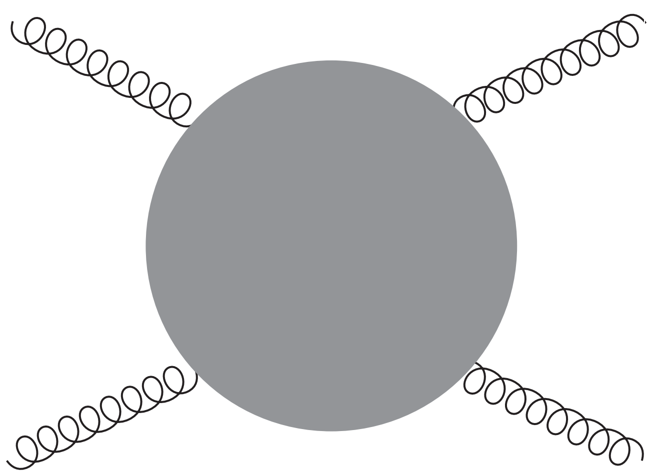
$$\frac{1}{t}$$

$$\log^{\#}\left(\frac{-t}{s}\right) = L^{\#}$$

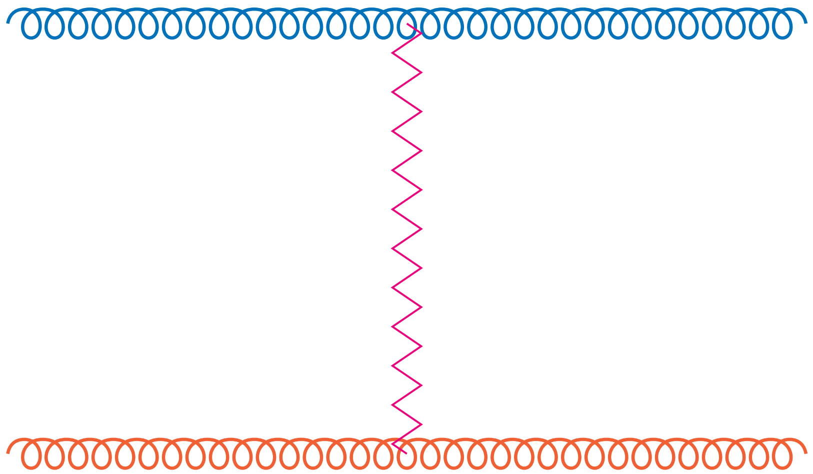
$$e^{L\,\tau_g(\alpha)}\ \frac{1}{t}$$

Regge Trajectory

$$\tau_g(\alpha) = \alpha\tau^{(1)} + \alpha^2\tau^{(2)} + \alpha^3\tau^{(3)}$$

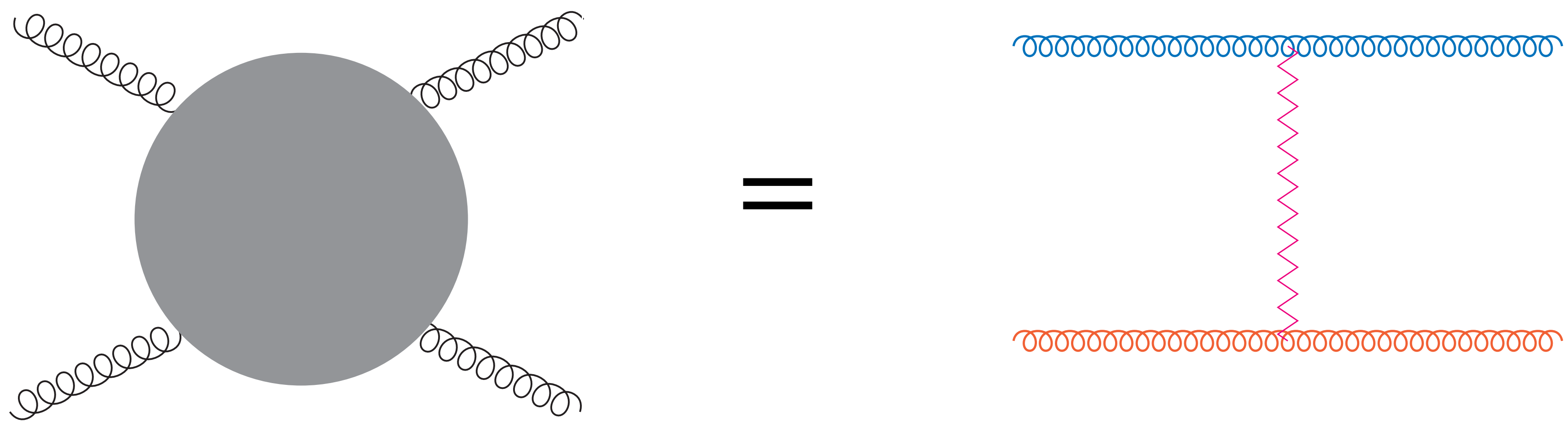


$\overset{\text{LL}}{=}$

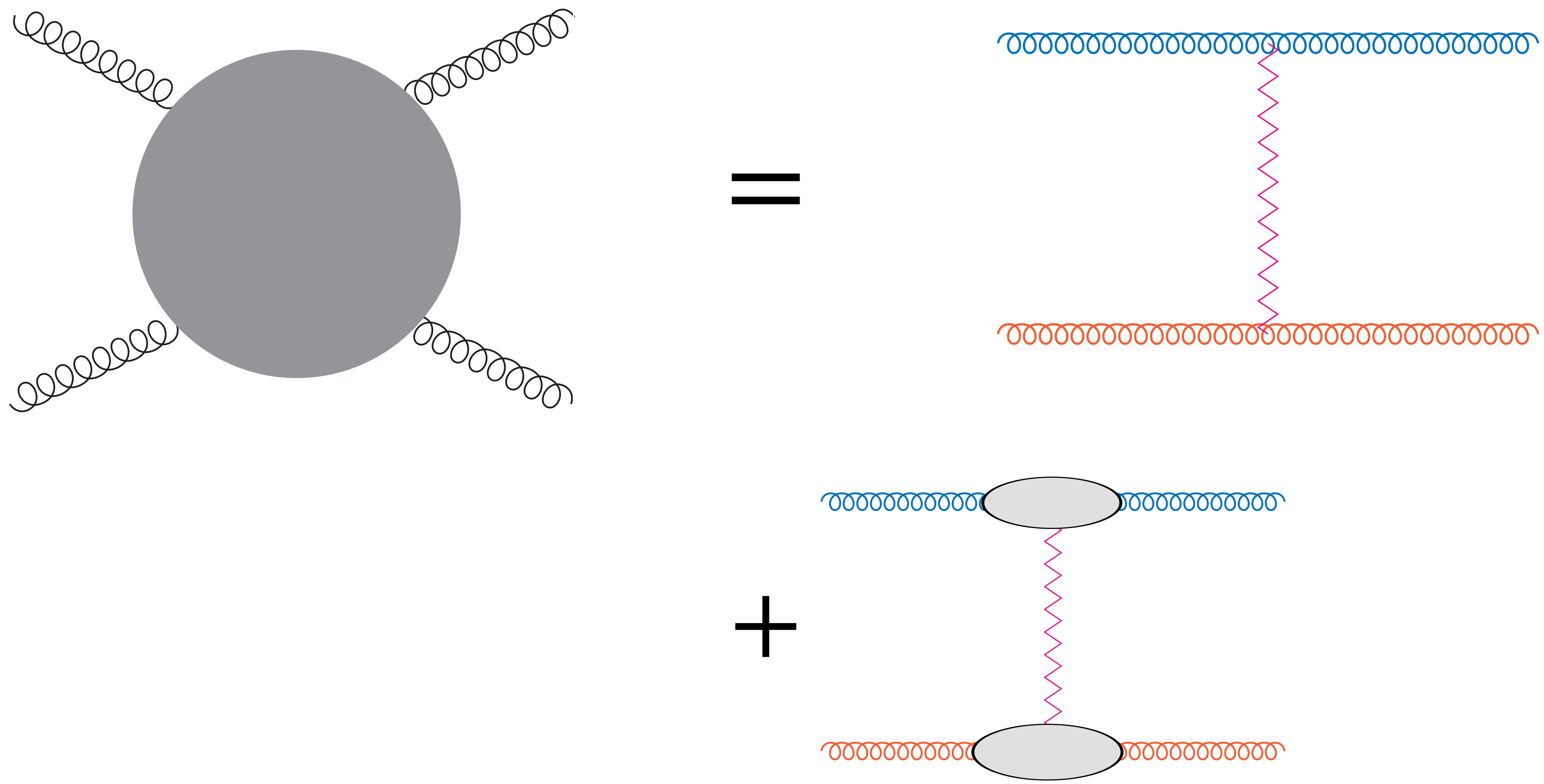


$$e^{L\,\tau_g(\alpha)} = \ \dots\ +\ L\alpha^3\tau^{(3)}\ +\ \dots$$

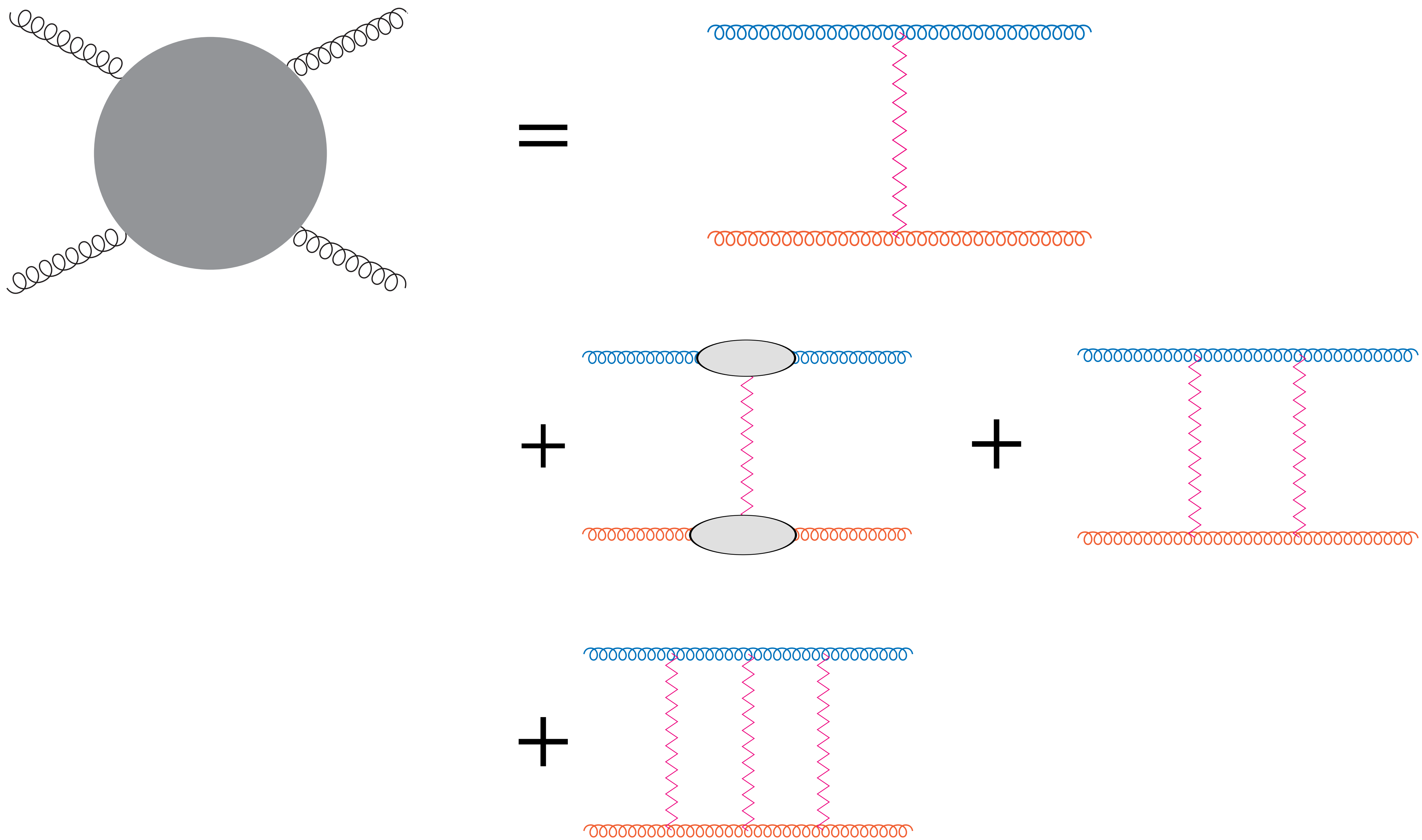
The NNLL Story



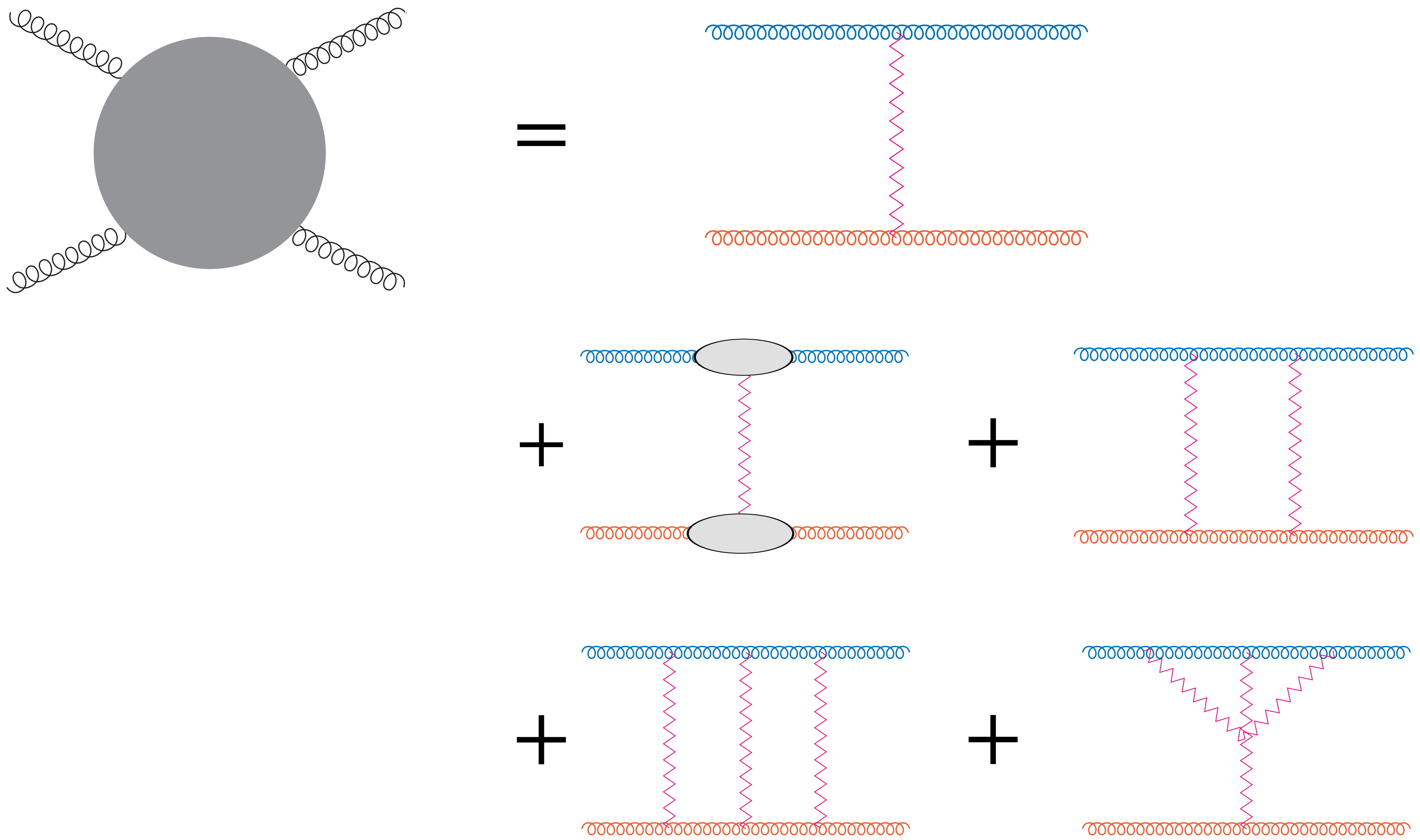
The NNLL Story



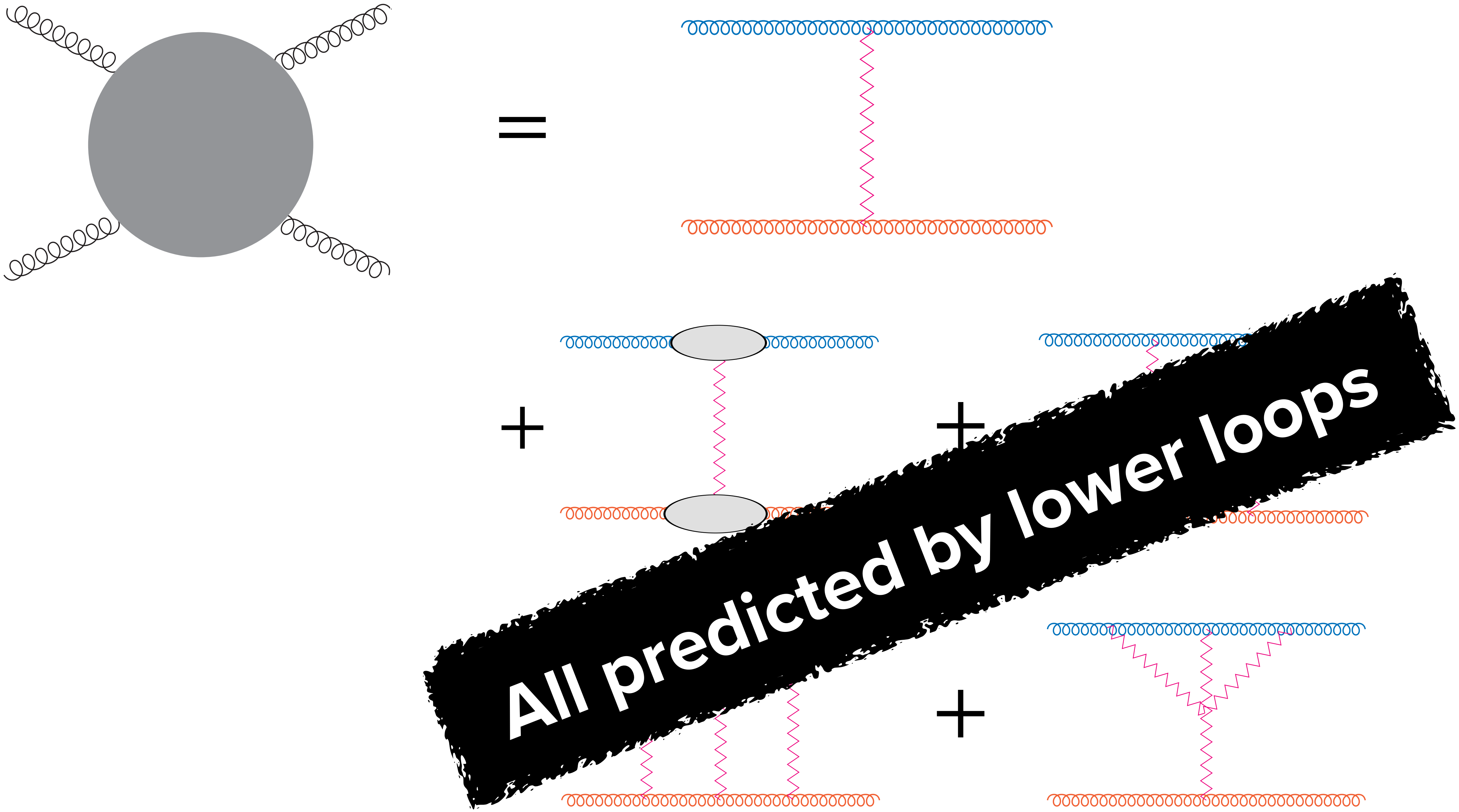
The NNLL Story



The NNLL Story



The NNLL Story

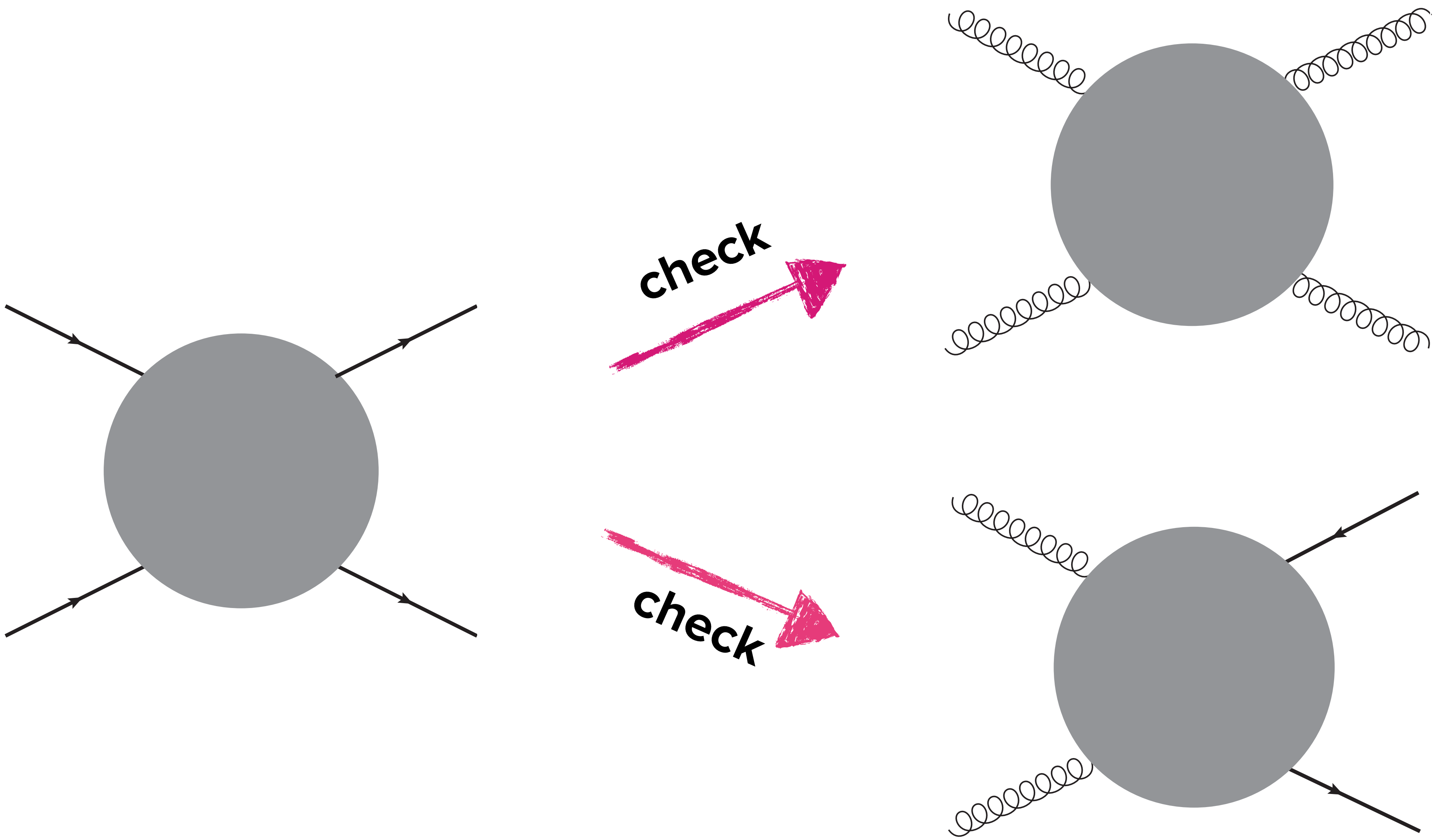


Falcioni, Gardi, Maher, Milloy, Vernazza: 2112.11098

Regge Limit as a check

full 2-loop

a single 3-loop amplitude



contribution to N^3LO

contribution to N^3LO

See Federico's Talk for a second step..

contribution to N^3LO

See Federico's Talk for a second step..

confirmation of QCD quadrupole radiation

contribution to N^3LO

See Federico's Talk for a second step..

confirmation of QCD quadrupole radiation

3-loop gluon Regge Trajectory

contribution to N^3LO

See Federico's Talk for a second step..

confirmation of QCD quadrupole radiation

3-loop gluon Regge Trajectory

playground for new ideas

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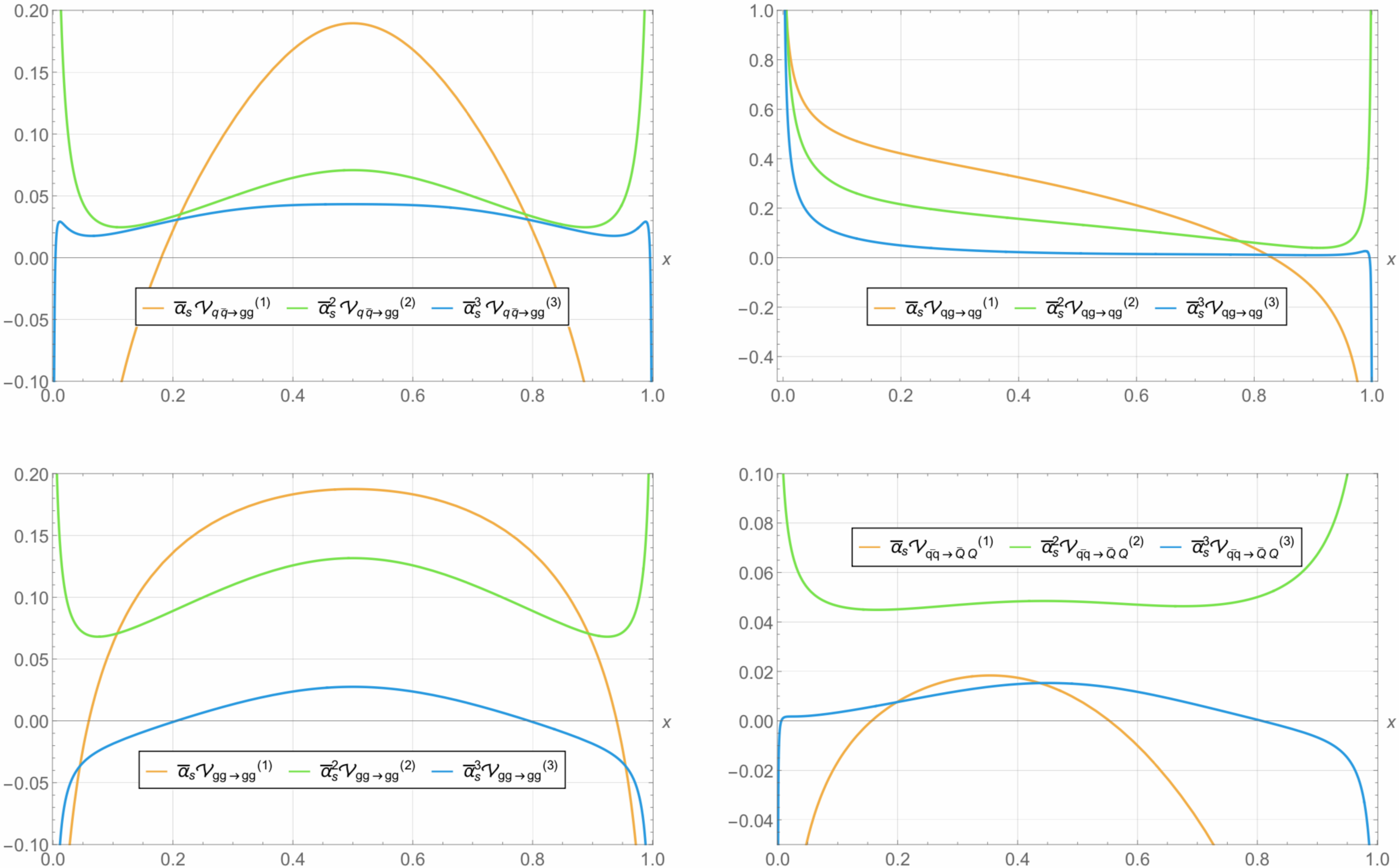
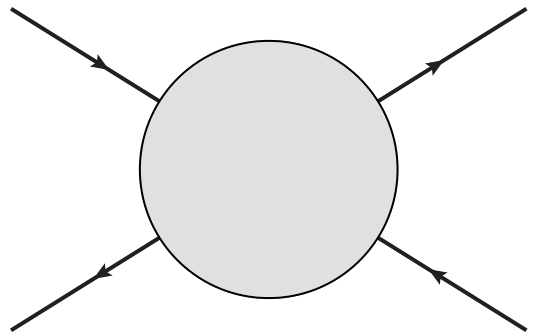
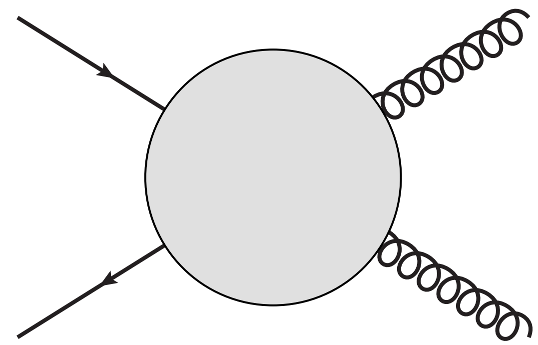
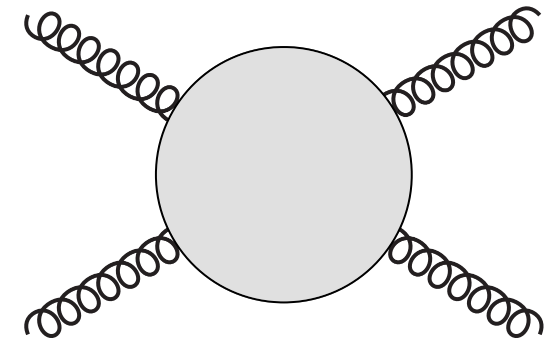
UNIVERSITY OF OXFORD

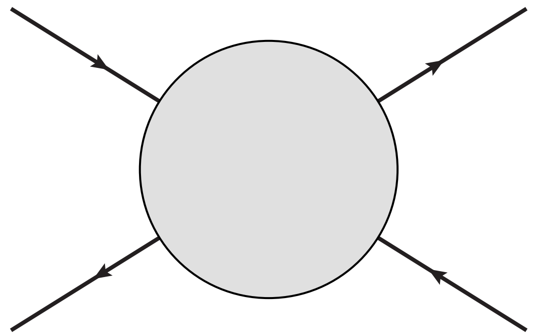
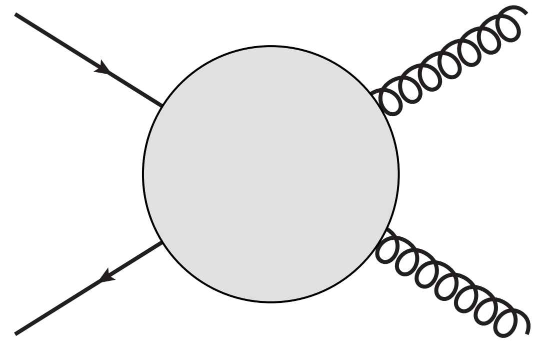
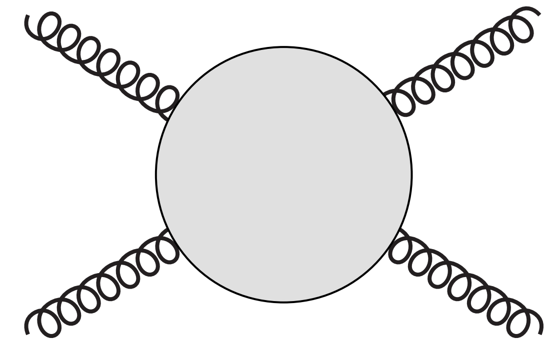
Thank you !!

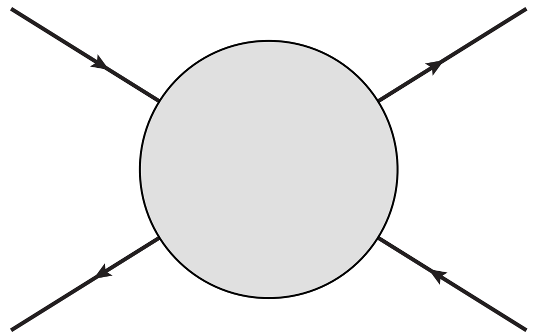
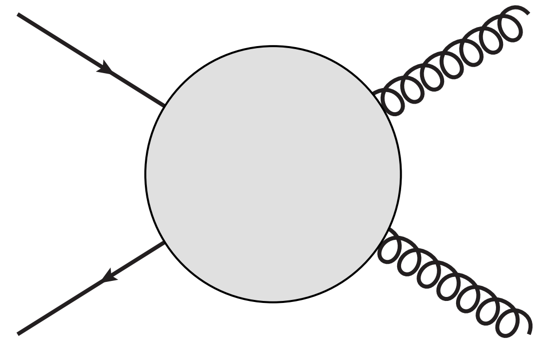
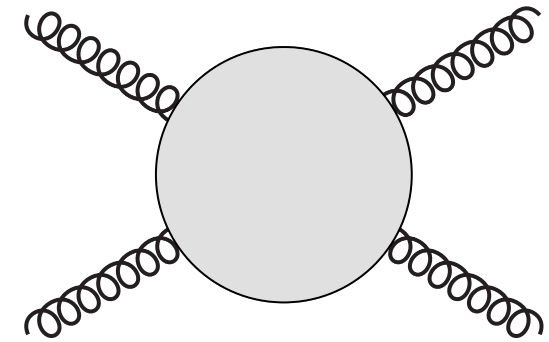
Backup Slides

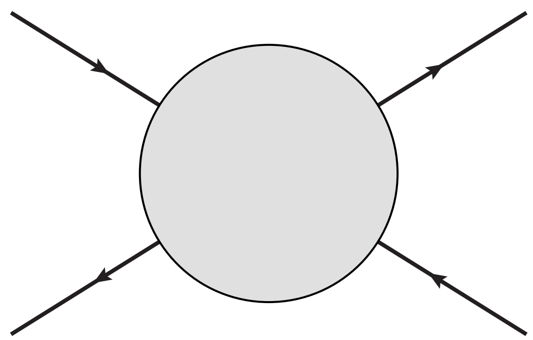
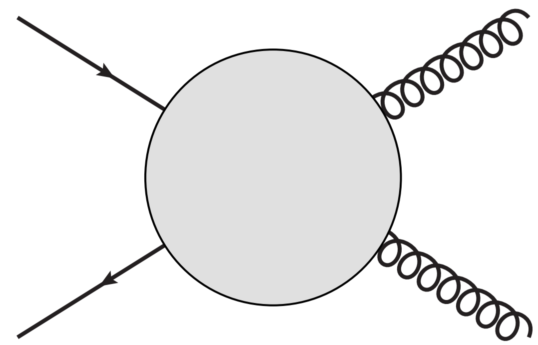
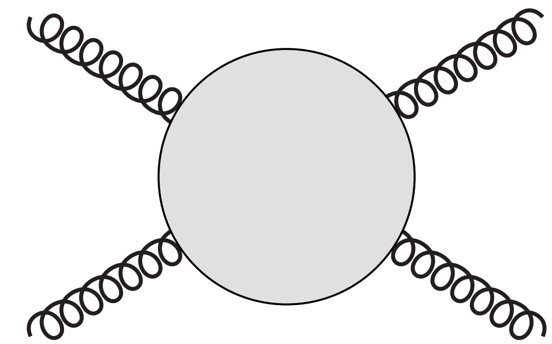


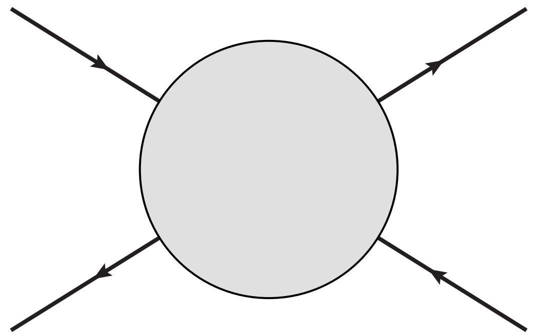
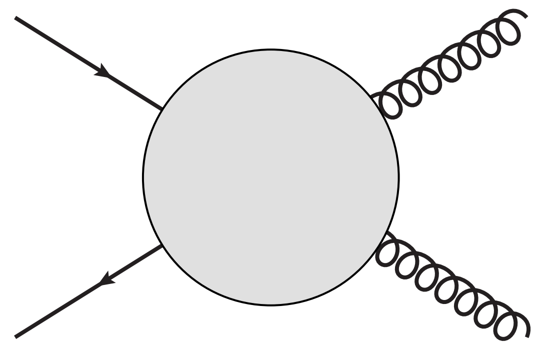
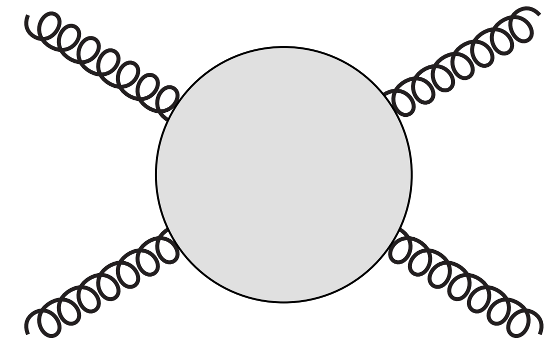
$\overline{\alpha}_s \mathcal{V}_{q\bar{q}\to gg}{}^{(1)}$ $\overline{\alpha}_s^2 \mathcal{V}_{q\bar{q}\to gg}{}^{(2)}$ $\overline{\alpha}_s^3 \mathcal{V}_{q\bar{q}\to gg}{}^{(3)}$
$\overline{\alpha}_s \mathcal{V}_{qg\to qg}{}^{(1)}$ $\overline{\alpha}_s^2 \mathcal{V}_{qg\to qg}{}^{(2)}$ $\overline{\alpha}_s^3 \mathcal{V}_{qg\to qg}{}^{(3)}$
$\overline{\alpha}_s \mathcal{V}_{gg\to gg}{}^{(1)}$ $\overline{\alpha}_s^2 \mathcal{V}_{gg\to gg}{}^{(2)}$ $\overline{\alpha}_s^3 \mathcal{V}_{gg\to gg}{}^{(3)}$
$\overline{\alpha}_s \mathcal{V}_{\bar{q}q\to \bar{Q}Q}{}^{(1)}$ $\overline{\alpha}_s^2 \mathcal{V}_{\bar{q}q\to \bar{Q}Q}{}^{(2)}$ $\overline{\alpha}_s^3 \mathcal{V}_{\bar{q}q\to \bar{Q}Q}{}^{(3)}$
x

# of QCD Feynman diagrams			
tree level			
1-loop			
2-loop			
3-loop			

# of QCD Feynman diagrams			
tree level	1	3	4
1-loop			
2-loop			
3-loop			

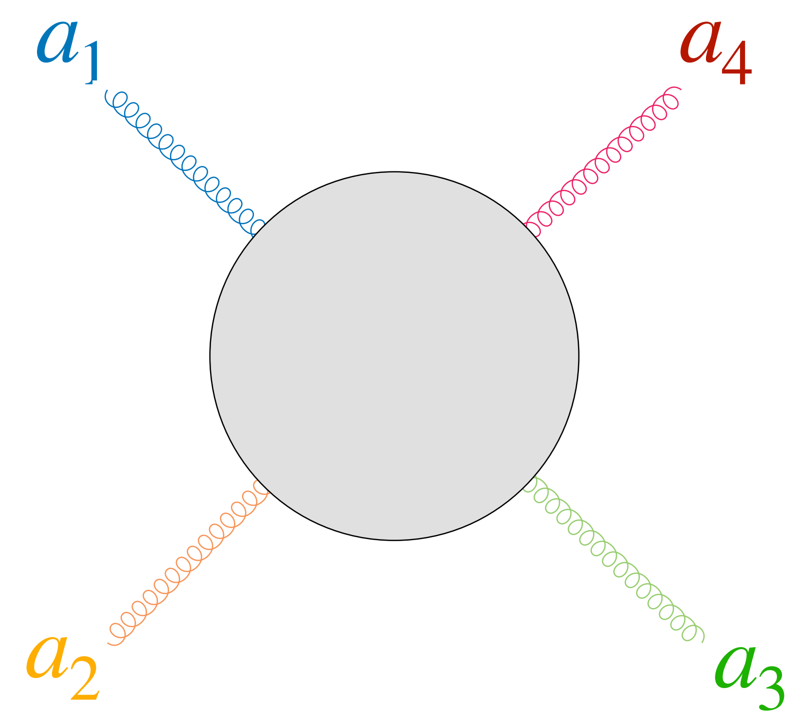
# of QCD Feynman diagrams			
tree level	1	3	4
1-loop	9	3	81
2-loop			
3-loop			

# of QCD Feynman diagrams			
tree level	1	3	4
1-loop	9	3	81
2-loop	158	595	1771
3-loop			

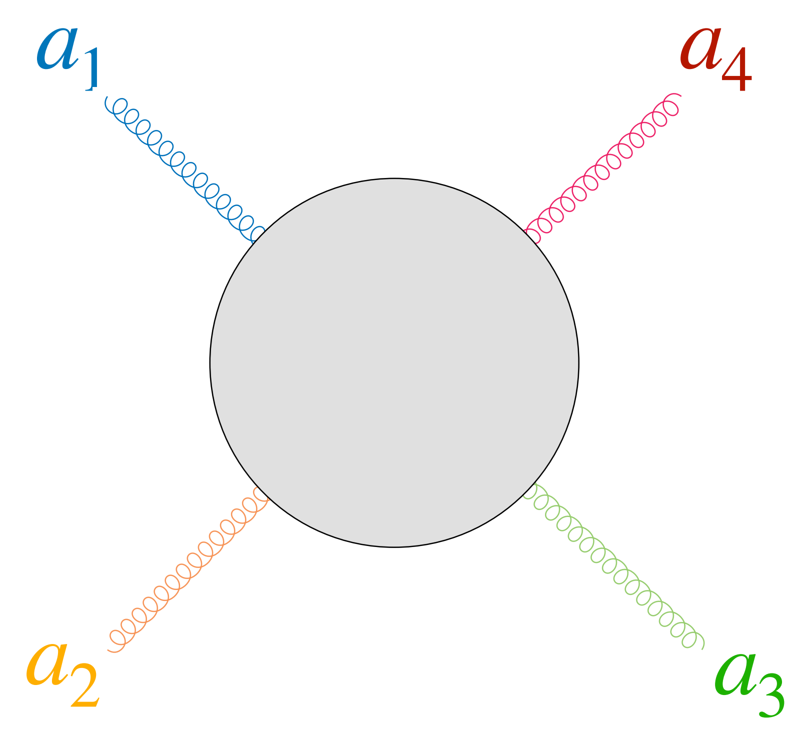
# of QCD Feynman diagrams			
tree level	1	3	4
1-loop	9	3	81
2-loop	158	595	1771
3-loop	358	14971	48723 !?!

Colour Decomposition

$$A = \sum_{i=1}^{6} A_i \mathscr{C}_i$$



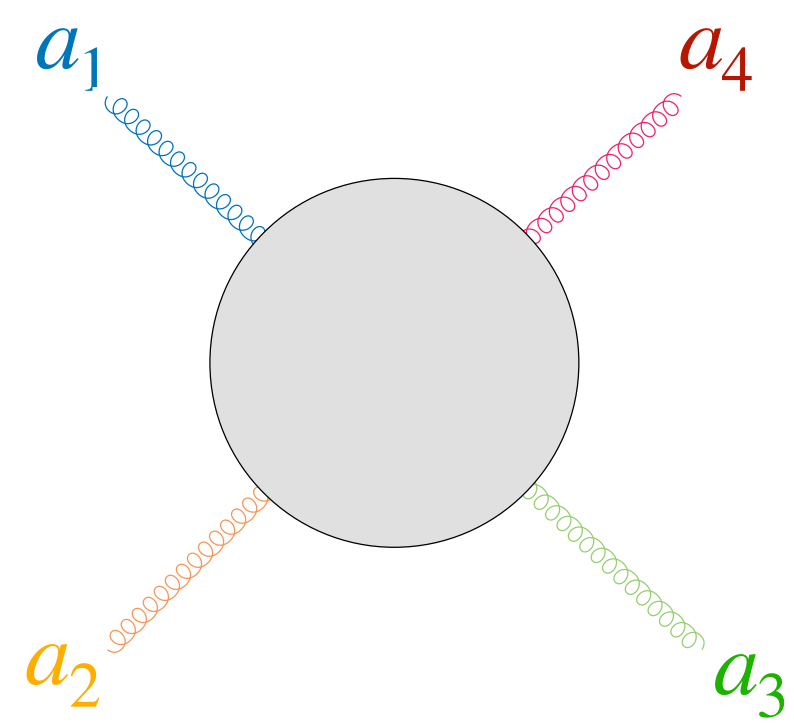
Colour Decomposition



$$A = \sum_{i=1}^{6} A_i \mathscr{C}_i$$

$$\mathscr{C}_1 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_2}\mathbf{T}^{a_3}\mathbf{T}^{a_4})$$

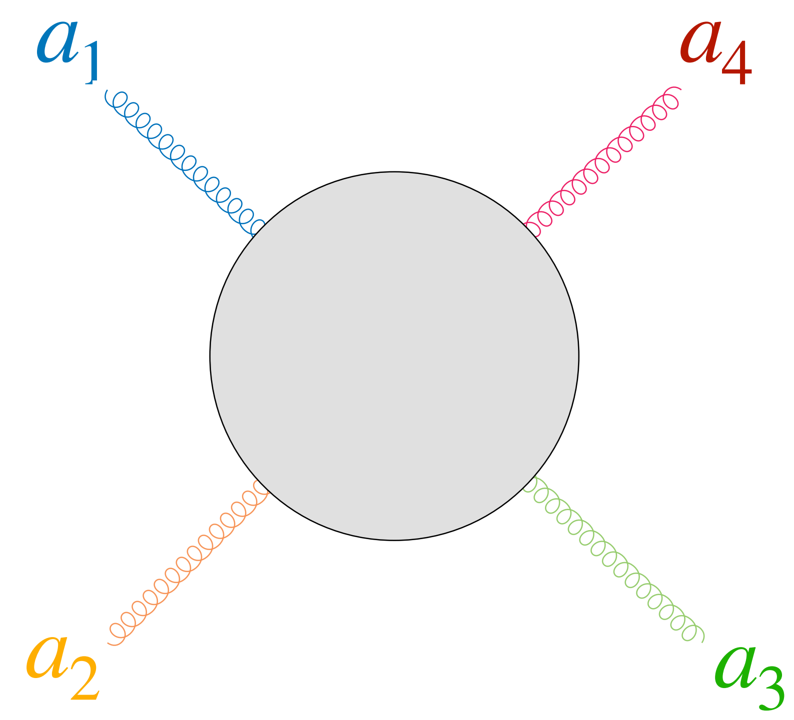
Colour Decomposition



$$A = \sum_{i=1}^{6} A_i \mathscr{C}_i$$

$$\mathscr{C}_1 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_2}\mathbf{T}^{a_3}\mathbf{T}^{a_4}) \; + Tr(\mathbf{T}^{a_4}\mathbf{T}^{a_3}\mathbf{T}^{a_2}\mathbf{T}^{a_1})$$

Colour Decomposition



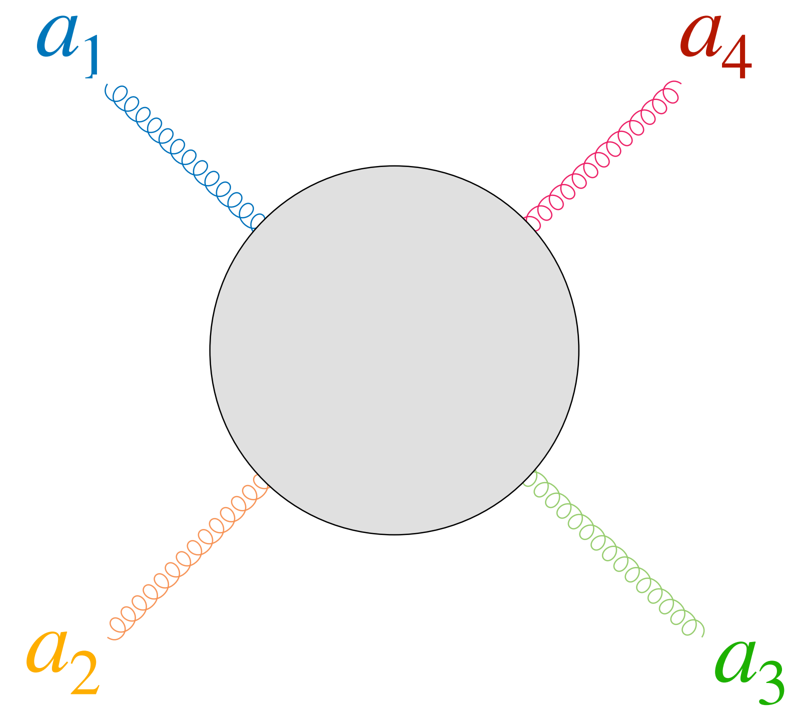
$$A = \sum_{i=1}^{6} A_i \mathscr{C}_i$$

$$\mathscr{C}_1 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_2}\mathbf{T}^{a_3}\mathbf{T}^{a_4}) + Tr(\mathbf{T}^{a_4}\mathbf{T}^{a_3}\mathbf{T}^{a_2}\mathbf{T}^{a_1})$$

$$\mathscr{C}_2 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_2}\mathbf{T}^{a_4}\mathbf{T}^{a_3}) + Tr(\mathbf{T}^{a_3}\mathbf{T}^{a_4}\mathbf{T}^{a_2}\mathbf{T}^{a_1})$$

$$\mathscr{C}_3 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_4}\mathbf{T}^{a_2}\mathbf{T}^{a_3}) + Tr(\mathbf{T}^{a_3}\mathbf{T}^{a_2}\mathbf{T}^{a_4}\mathbf{T}^{a_1})$$

Colour Decomposition



$$A = \sum_{i=1}^{6} A_i \mathscr{C}_i$$

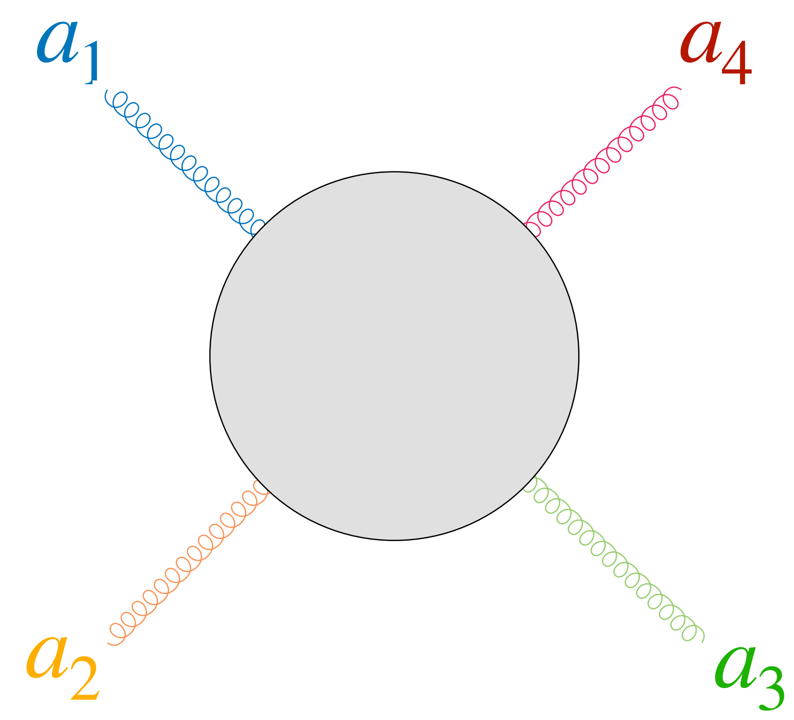
$$\mathscr{C}_1 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_2}\mathbf{T}^{a_3}\mathbf{T}^{a_4}) + Tr(\mathbf{T}^{a_4}\mathbf{T}^{a_3}\mathbf{T}^{a_2}\mathbf{T}^{a_1})$$

$$\mathscr{C}_2 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_2}\mathbf{T}^{a_4}\mathbf{T}^{a_3}) + Tr(\mathbf{T}^{a_3}\mathbf{T}^{a_4}\mathbf{T}^{a_2}\mathbf{T}^{a_1})$$

$$\mathscr{C}_3 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_4}\mathbf{T}^{a_2}\mathbf{T}^{a_3}) + Tr(\mathbf{T}^{a_3}\mathbf{T}^{a_2}\mathbf{T}^{a_4}\mathbf{T}^{a_1})$$

$$\mathscr{C}_4 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_2})\, Tr(\mathbf{T}^{a_3}\mathbf{T}^{a_4})$$

Colour Decomposition



$$A = \sum_{i=1}^{6} A_i \mathscr{C}_i$$

$$\mathscr{C}_1 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_2}\mathbf{T}^{a_3}\mathbf{T}^{a_4}) + Tr(\mathbf{T}^{a_4}\mathbf{T}^{a_3}\mathbf{T}^{a_2}\mathbf{T}^{a_1})$$

$$\mathscr{C}_2 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_2}\mathbf{T}^{a_4}\mathbf{T}^{a_3}) + Tr(\mathbf{T}^{a_3}\mathbf{T}^{a_4}\mathbf{T}^{a_2}\mathbf{T}^{a_1})$$

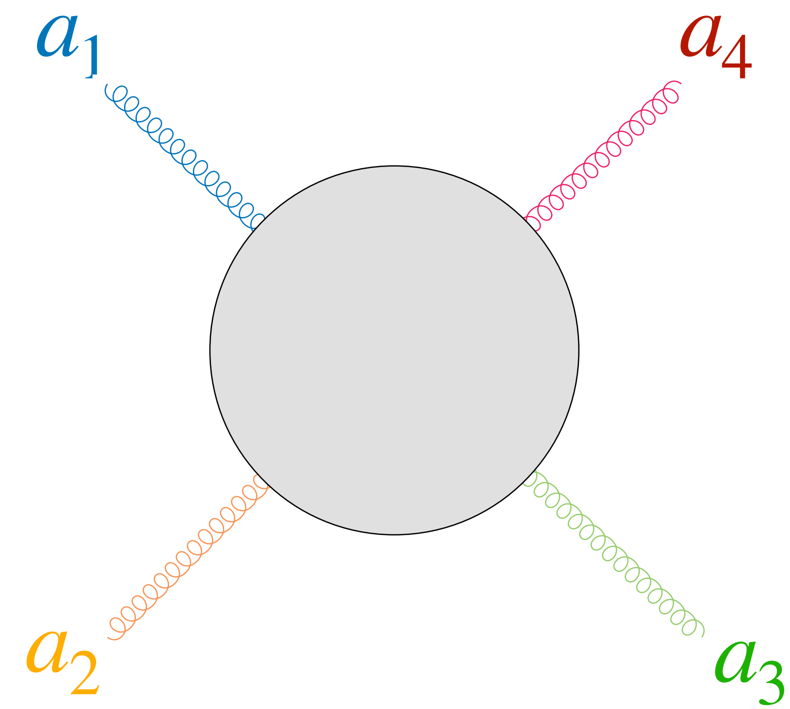
$$\mathscr{C}_3 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_4}\mathbf{T}^{a_2}\mathbf{T}^{a_3}) + Tr(\mathbf{T}^{a_3}\mathbf{T}^{a_2}\mathbf{T}^{a_4}\mathbf{T}^{a_1})$$

$$\mathscr{C}_4 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_2})\, Tr(\mathbf{T}^{a_3}\mathbf{T}^{a_4})$$

$$\mathscr{C}_5 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_3})\, Tr(\mathbf{T}^{a_2}\mathbf{T}^{a_4})$$

$$\mathscr{C}_6 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_4})\, Tr(\mathbf{T}^{a_2}\mathbf{T}^{a_3})$$

Colour Decomposition



$$A = \sum_{i=1}^{6} A_i \mathscr{C}_i$$

$$\mathscr{C}_1 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_2}\mathbf{T}^{a_3}\mathbf{T}^{a_4}) + Tr(\mathbf{T}^{a_4}\mathbf{T}^{a_3}\mathbf{T}^{a_2}\mathbf{T}^{a_1})$$

$$\mathscr{C}_2 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_2}\mathbf{T}^{a_4}\mathbf{T}^{a_3}) + Tr(\mathbf{T}^{a_3}\mathbf{T}^{a_4}\mathbf{T}^{a_2}\mathbf{T}^{a_1})$$

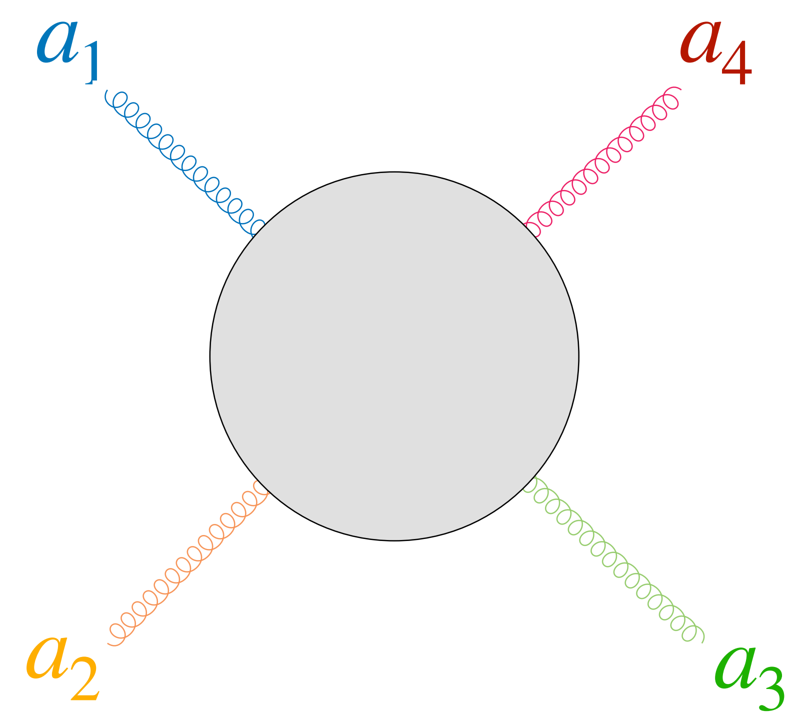
$$\mathscr{C}_3 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_4}\mathbf{T}^{a_2}\mathbf{T}^{a_3}) + Tr(\mathbf{T}^{a_3}\mathbf{T}^{a_2}\mathbf{T}^{a_4}\mathbf{T}^{a_1})$$

$$\mathscr{C}_4 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_2})\, Tr(\mathbf{T}^{a_3}\mathbf{T}^{a_4})$$

$$\mathscr{C}_5 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_3})\, Tr(\mathbf{T}^{a_2}\mathbf{T}^{a_4})$$

$$\mathscr{C}_6 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_4})\, Tr(\mathbf{T}^{a_2}\mathbf{T}^{a_3})$$

Colour Decomposition



$$A = \sum_{i=1}^{6} A_i \mathscr{C}_i$$

$$\mathscr{C}_1 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_2}\mathbf{T}^{a_3}\mathbf{T}^{a_4}) + Tr(\mathbf{T}^{a_4}\mathbf{T}^{a_3}\mathbf{T}^{a_2}\mathbf{T}^{a_1})$$

$$\mathscr{C}_2 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_2}\mathbf{T}^{a_4}\mathbf{T}^{a_3}) + Tr(\mathbf{T}^{a_3}\mathbf{T}^{a_4}\mathbf{T}^{a_2}\mathbf{T}^{a_1})$$

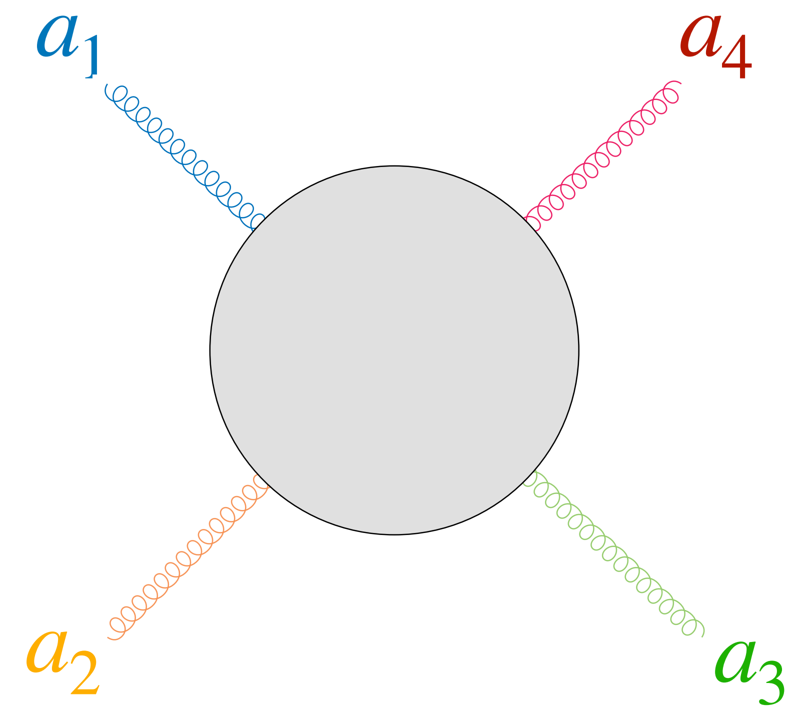
$$\mathscr{C}_3 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_4}\mathbf{T}^{a_2}\mathbf{T}^{a_3}) + Tr(\mathbf{T}^{a_3}\mathbf{T}^{a_2}\mathbf{T}^{a_4}\mathbf{T}^{a_1})$$

$$\mathscr{C}_4 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_2})\, Tr(\mathbf{T}^{a_3}\mathbf{T}^{a_4})$$

$$\mathscr{C}_5 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_3})\, Tr(\mathbf{T}^{a_2}\mathbf{T}^{a_4})$$

$$\mathscr{C}_6 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_4})\, Tr(\mathbf{T}^{a_2}\mathbf{T}^{a_3})$$

Colour Decomposition



$$A = \sum_{i=1}^{6} A_i \mathscr{C}_i$$

$$\mathscr{C}_1 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_2}\mathbf{T}^{a_3}\mathbf{T}^{a_4}) + Tr(\mathbf{T}^{a_4}\mathbf{T}^{a_3}\mathbf{T}^{a_2}\mathbf{T}^{a_1})$$

$$\mathscr{C}_2 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_2}\mathbf{T}^{a_4}\mathbf{T}^{a_3}) + Tr(\mathbf{T}^{a_3}\mathbf{T}^{a_4}\mathbf{T}^{a_2}\mathbf{T}^{a_1})$$

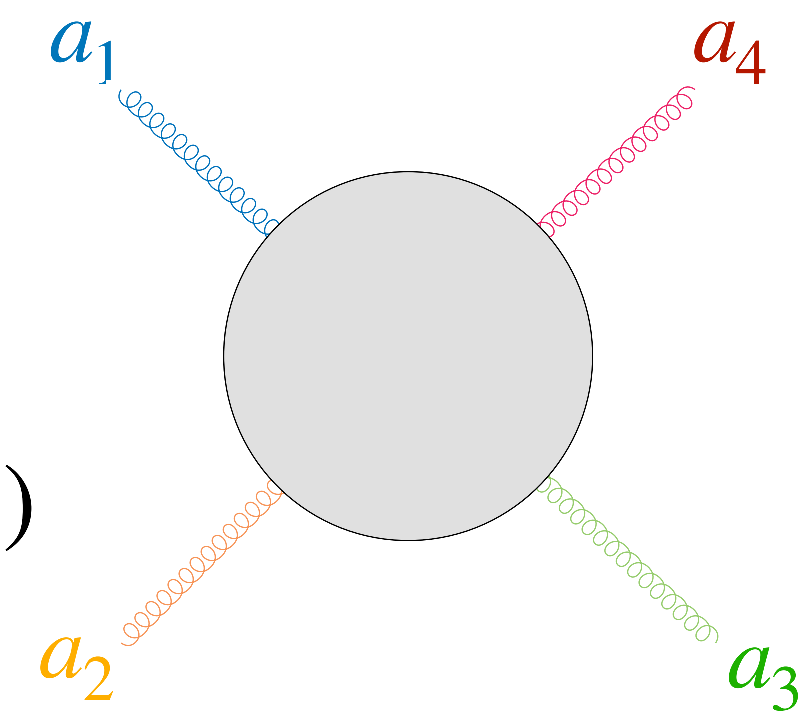
$$\mathscr{C}_3 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_4}\mathbf{T}^{a_2}\mathbf{T}^{a_3}) + Tr(\mathbf{T}^{a_3}\mathbf{T}^{a_2}\mathbf{T}^{a_4}\mathbf{T}^{a_1})$$

$$\mathscr{C}_4 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_2})\, Tr(\mathbf{T}^{a_3}\mathbf{T}^{a_4})$$

$$\mathscr{C}_5 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_3})\, Tr(\mathbf{T}^{a_2}\mathbf{T}^{a_4})$$

$$\mathscr{C}_6 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_4})\, Tr(\mathbf{T}^{a_2}\mathbf{T}^{a_3})$$

Colour Decomposition



$$A = \sum_{i=1}^{6} A_i \mathscr{C}_i = A_1 \mathscr{C}_1 + A_4 \mathscr{C}_4 + (perms)$$

$$\mathscr{C}_1 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_2}\mathbf{T}^{a_3}\mathbf{T}^{a_4}) + Tr(\mathbf{T}^{a_4}\mathbf{T}^{a_3}\mathbf{T}^{a_2}\mathbf{T}^{a_1})$$

$$\mathscr{C}_2 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_2}\mathbf{T}^{a_4}\mathbf{T}^{a_3}) + Tr(\mathbf{T}^{a_3}\mathbf{T}^{a_4}\mathbf{T}^{a_2}\mathbf{T}^{a_1})$$

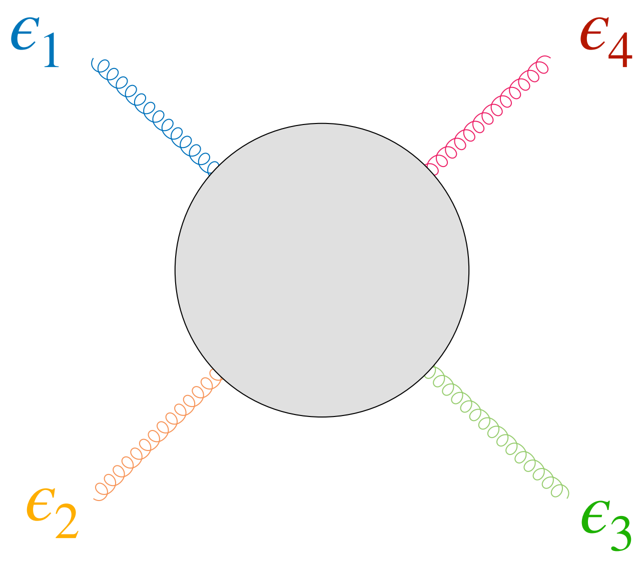
$$\mathscr{C}_3 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_4}\mathbf{T}^{a_2}\mathbf{T}^{a_3}) + Tr(\mathbf{T}^{a_3}\mathbf{T}^{a_2}\mathbf{T}^{a_4}\mathbf{T}^{a_1})$$

$$\mathscr{C}_4 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_2})\, Tr(\mathbf{T}^{a_3}\mathbf{T}^{a_4})$$

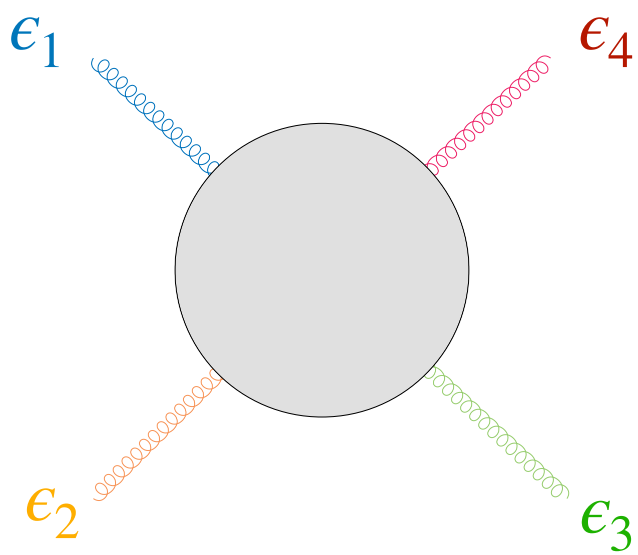
$$\mathscr{C}_5 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_3})\, Tr(\mathbf{T}^{a_2}\mathbf{T}^{a_4})$$

$$\mathscr{C}_6 = Tr(\mathbf{T}^{a_1}\mathbf{T}^{a_4})\, Tr(\mathbf{T}^{a_2}\mathbf{T}^{a_3})$$

Spin

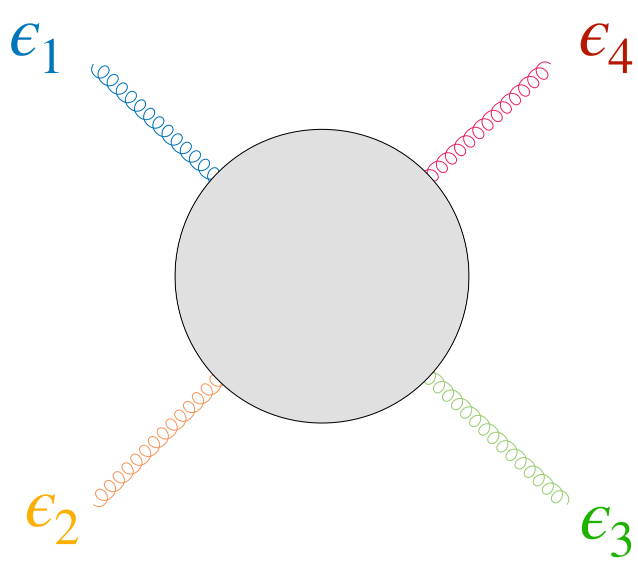


Spin



$$A_c = A_c^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$

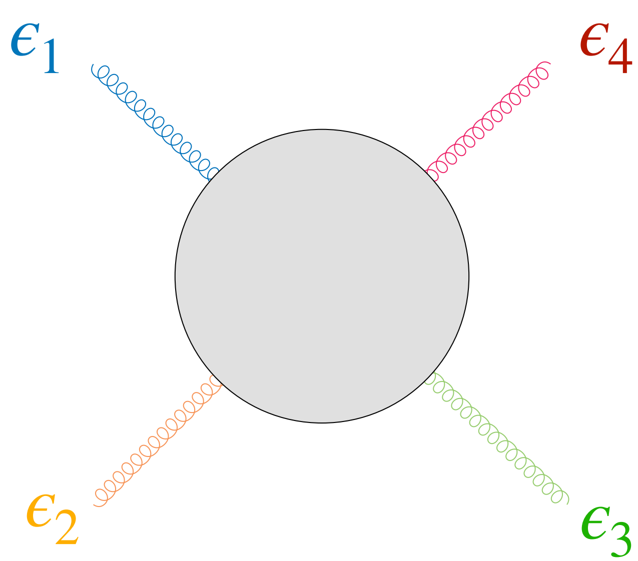
Spin



$$A_c = A_c^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$

$$= \sum_j F_c^j \; T_j^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$

Spin

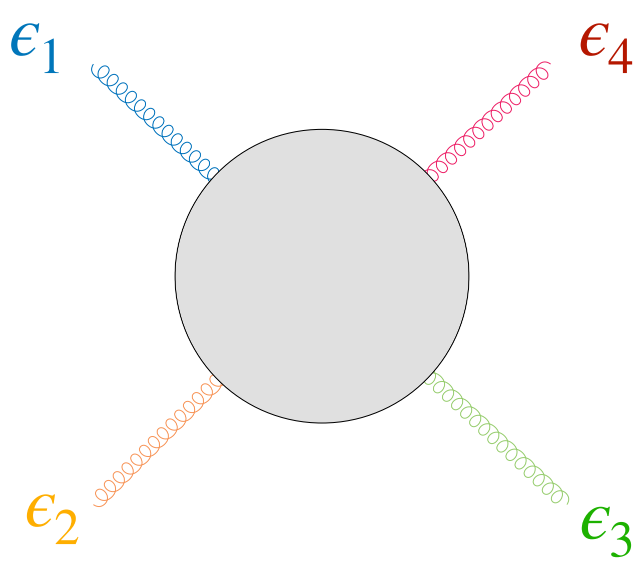


$$A_c = A_c^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$

$$= \sum_j F_c^j \; T_j^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$

$$p_1^{\mu_1}p_2^{\mu_2}p_3^{\mu_3}p_3^{\mu_4}$$

Spin

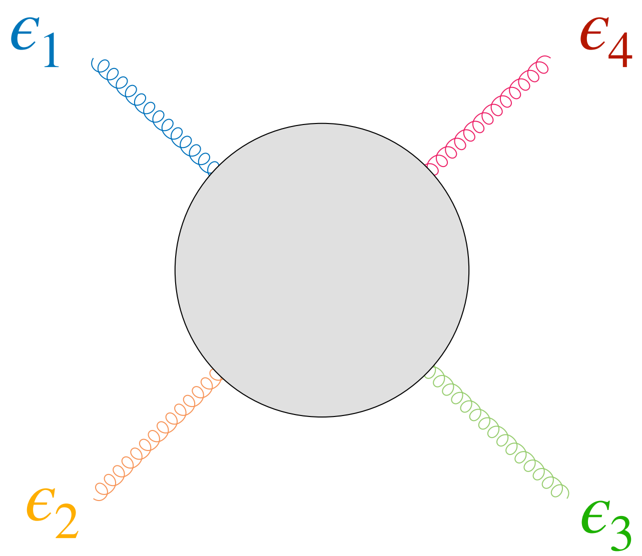


$$
\begin{aligned}
A_c &= A_c^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4} \\
&= \sum_j F_c^j \; T_j^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}
\end{aligned}
$$

$$p_1^{\mu_1}p_2^{\mu_2}p_3^{\mu_3}p_3^{\mu_4}$$
$$p_2^{\mu_1}p_2^{\mu_2}p_3^{\mu_3}p_2^{\mu_4}$$
$$p_3^{\mu_1}p_1^{\mu_2}p_1^{\mu_3}p_2^{\mu_4}$$
$$p_2^{\mu_1}p_1^{\mu_2}p_3^{\mu_3}p_2^{\mu_4}$$
$$p_3^{\mu_1}p_2^{\mu_2}p_1^{\mu_3}p_2^{\mu_4}$$

$\vdots$ **81**

Spin



$$A_c = A_c^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$

$$= \sum_j F_c^j \; T_j^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$

$$p_1^{\mu_1}p_2^{\mu_2}p_3^{\mu_3}p_3^{\mu_4}$$

$$p_2^{\mu_1}p_2^{\mu_2}p_3^{\mu_3}p_2^{\mu_4}$$

$$p_3^{\mu_1}p_1^{\mu_2}p_1^{\mu_3}p_2^{\mu_4}$$

$$p_2^{\mu_1}p_1^{\mu_2}p_3^{\mu_3}p_2^{\mu_4}$$

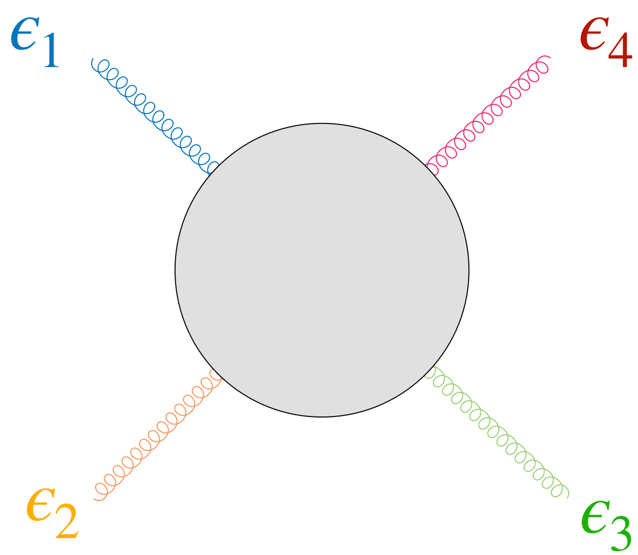
$$p_3^{\mu_1}p_2^{\mu_2}p_1^{\mu_3}p_2^{\mu_4}$$

$$\vdots$$

81

$$p_3^{\mu_1}p_3^{\mu_2}g^{\mu_3\mu_4}$$

Spin



$$A_c = A_c^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$
$$= \sum_j F_c^j \; T_j^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$

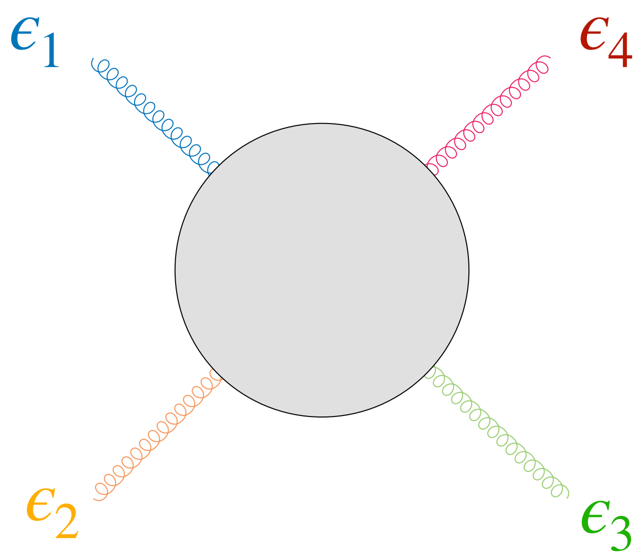
$$p_1^{\mu_1}p_2^{\mu_2}p_3^{\mu_3}p_3^{\mu_4}$$
$$p_2^{\mu_1}p_2^{\mu_2}p_3^{\mu_3}p_2^{\mu_4}$$
$$p_3^{\mu_1}p_1^{\mu_2}p_1^{\mu_3}p_2^{\mu_4}$$
$$p_2^{\mu_1}p_1^{\mu_2}p_3^{\mu_3}p_2^{\mu_4}$$
$$p_3^{\mu_1}p_2^{\mu_2}p_1^{\mu_3}p_2^{\mu_4}$$
$$\vdots$$

81

$$p_3^{\mu_1}p_3^{\mu_2}g^{\mu_3\mu_4}$$
$$g^{\mu_1\mu_2}p_1^{\mu_3}p_1^{\mu_4}$$
$$g^{\mu_1\mu_3}p_1^{\mu_2}p_2^{\mu_4}$$
$$\vdots$$

54

Spin



$$A_c = A_c^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$
$$= \sum_j F_c^j \; T_j^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$

$p_1^{\mu_1}p_2^{\mu_2}p_3^{\mu_3}p_3^{\mu_4}$

$p_2^{\mu_1}p_2^{\mu_2}p_3^{\mu_3}p_2^{\mu_4}$

$p_3^{\mu_1}p_1^{\mu_2}p_1^{\mu_3}p_2^{\mu_4}$

$p_2^{\mu_1}p_1^{\mu_2}p_3^{\mu_3}p_2^{\mu_4}$

$p_3^{\mu_1}p_2^{\mu_2}p_1^{\mu_3}p_2^{\mu_4}$

⋮ **81**

$p_3^{\mu_1}p_3^{\mu_2}g^{\mu_3\mu_4}$

$g^{\mu_1\mu_2}p_1^{\mu_3}p_1^{\mu_4}$

$g^{\mu_1\mu_3}p_1^{\mu_2}p_2^{\mu_4}$

⋮ **54**

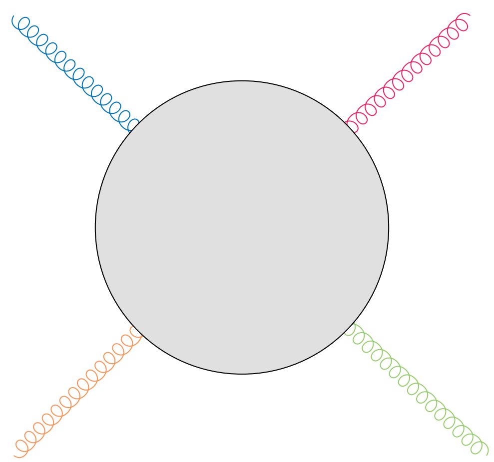
$g^{\mu_1\mu_2}g^{\mu_3\mu_4}$

$g^{\mu_1\mu_3}g^{\mu_2\mu_4}$

$g^{\mu_1\mu_4}g^{\mu_2\mu_3}$

3

Spin



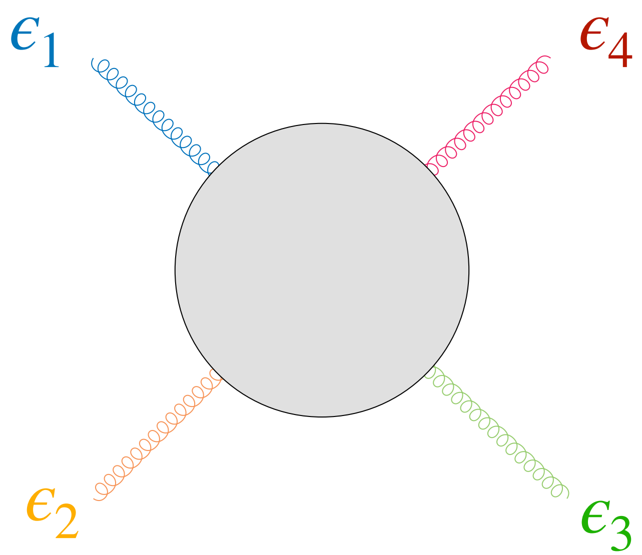
$$A_c = A_c^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$

$$= \sum_j F_c^j \; T_j^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$

$p_1^{\mu_1}p_2^{\mu_2}p_3^{\mu_3}p_3^{\mu_4}$	$p_3^{\mu_1}p_3^{\mu_2}g^{\mu_3\mu_4}$	$g^{\mu_1\mu_2}g^{\mu_3\mu_4}$
$p_2^{\mu_1}p_2^{\mu_2}p_3^{\mu_3}p_2^{\mu_4}$	$g^{\mu_1\mu_2}p_1^{\mu_3}p_1^{\mu_4}$	$g^{\mu_1\mu_3}g^{\mu_2\mu_4}$
$p_3^{\mu_1}p_1^{\mu_2}p_1^{\mu_3}p_2^{\mu_4}$	$g^{\mu_1\mu_3}p_1^{\mu_2}p_2^{\mu_4}$	$g^{\mu_1\mu_4}g^{\mu_2\mu_3}$
$p_2^{\mu_1}p_1^{\mu_2}p_3^{\mu_3}p_2^{\mu_4}$	⋮ **54**	**3**
$p_3^{\mu_1}p_2^{\mu_2}p_1^{\mu_3}p_2^{\mu_4}$		
⋮ **81**		

138

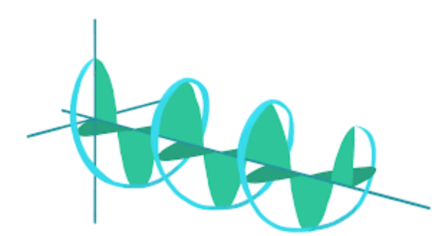
Spin



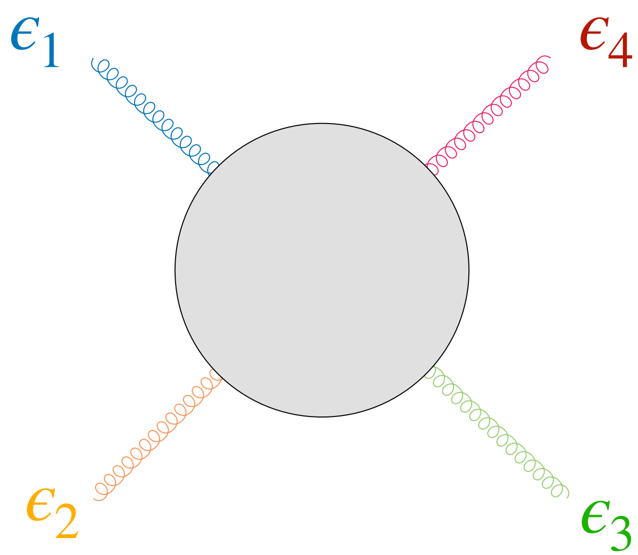
$$A_c = A_c^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$
$$= \sum_j F_c^j \; T_j^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$

$p_1^{\mu_1}p_2^{\mu_2}p_3^{\mu_3}p_3^{\mu_4}$	$p_3^{\mu_1}p_3^{\mu_2}g^{\mu_3\mu_4}$	$g^{\mu_1\mu_2}g^{\mu_3\mu_4}$
$p_2^{\mu_1}p_2^{\mu_2}p_3^{\mu_3}p_2^{\mu_4}$	$g^{\mu_1\mu_2}p_1^{\mu_3}p_1^{\mu_4}$	$g^{\mu_1\mu_3}g^{\mu_2\mu_4}$
$p_3^{\mu_1}p_1^{\mu_2}p_1^{\mu_3}p_2^{\mu_4}$	$g^{\mu_1\mu_3}p_1^{\mu_2}p_2^{\mu_4}$	$g^{\mu_1\mu_4}g^{\mu_2\mu_3}$
$p_2^{\mu_1}p_1^{\mu_2}p_3^{\mu_3}p_2^{\mu_4}$	$\vdots$ **54**	**3**
$p_3^{\mu_1}p_2^{\mu_2}p_1^{\mu_3}p_2^{\mu_4}$		
$\vdots$ **81**		

Transversality $\epsilon_i \cdot p_i = 0$



Spin



$$A_c = A_c^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$

$$= \sum_j F_c^j \; T_j^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$

~~$p_1^{\mu_1}p_2^{\mu_2}p_3^{\mu_3}p_3^{\mu_4}$~~

$p_2^{\mu_1}p_2^{\mu_2}p_3^{\mu_3}p_2^{\mu_4}$

$p_3^{\mu_1}p_1^{\mu_2}p_1^{\mu_3}p_2^{\mu_4}$

$p_2^{\mu_1}p_1^{\mu_2}p_3^{\mu_3}p_2^{\mu_4}$

$p_3^{\mu_1}p_2^{\mu_2}p_1^{\mu_3}p_2^{\mu_4}$

⋮ **81**

$p_3^{\mu_1}p_3^{\mu_2}g^{\mu_3\mu_4}$

$g^{\mu_1\mu_2}p_1^{\mu_3}p_1^{\mu_4}$

$g^{\mu_1\mu_3}p_1^{\mu_2}p_2^{\mu_4}$

⋮ **54**

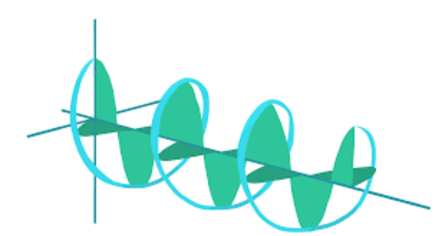
$g^{\mu_1\mu_2}g^{\mu_3\mu_4}$

$g^{\mu_1\mu_3}g^{\mu_2\mu_4}$

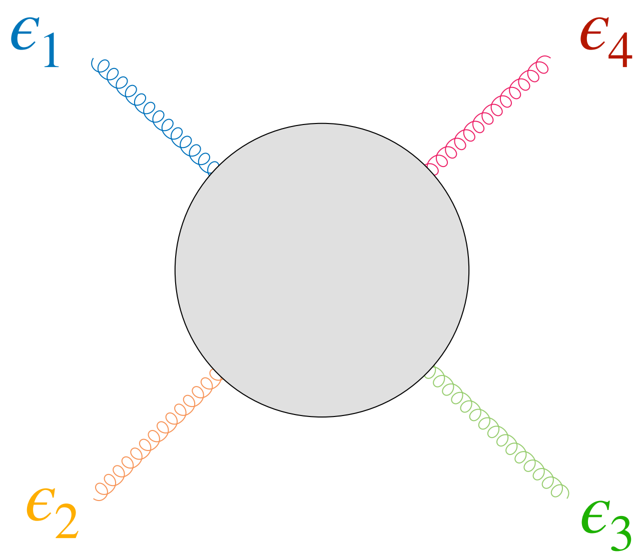
$g^{\mu_1\mu_4}g^{\mu_2\mu_3}$

3

Transversality $\epsilon_i \cdot p_i = 0$



Spin



$$A_c = A_c^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$

$$= \sum_j F_c^j \; T_j^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$

~~$p_1^{\mu_1}p_2^{\mu_2}p_3^{\mu_3}p_3^{\mu_4}$~~

~~$p_2^{\mu_1}p_2^{\mu_2}p_3^{\mu_3}p_2^{\mu_4}$~~

$p_3^{\mu_1}p_1^{\mu_2}p_1^{\mu_3}p_2^{\mu_4}$

$p_2^{\mu_1}p_1^{\mu_2}p_3^{\mu_3}p_2^{\mu_4}$

~~$p_3^{\mu_1}p_2^{\mu_2}p_1^{\mu_3}p_2^{\mu_4}$~~

$\vdots$ **81**

$p_3^{\mu_1}p_3^{\mu_2}g^{\mu_3\mu_4}$

$g^{\mu_1\mu_2}p_1^{\mu_3}p_1^{\mu_4}$

$g^{\mu_1\mu_3}p_1^{\mu_2}p_2^{\mu_4}$

$\vdots$ **54**

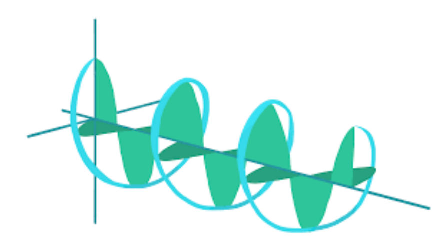
$g^{\mu_1\mu_2}g^{\mu_3\mu_4}$

$g^{\mu_1\mu_3}g^{\mu_2\mu_4}$

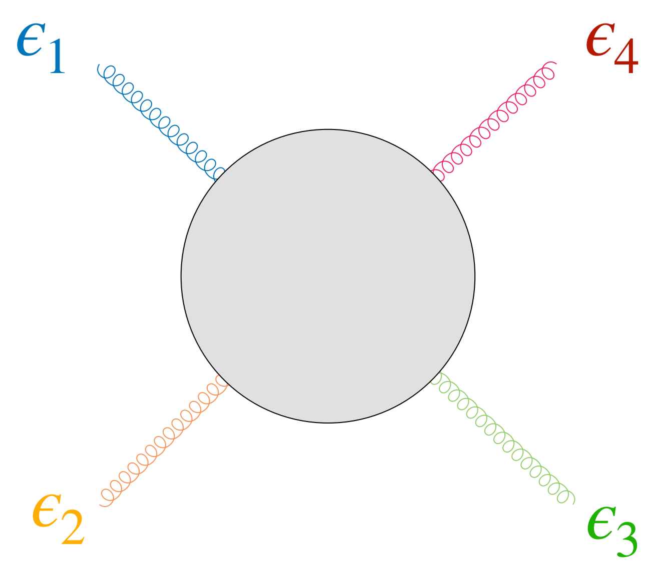
$g^{\mu_1\mu_4}g^{\mu_2\mu_3}$

3

Transversality $\epsilon_i \cdot p_i = 0$



Spin

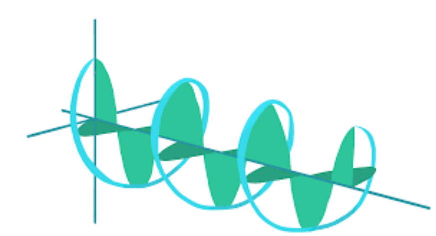


$$A_c = A_c^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$

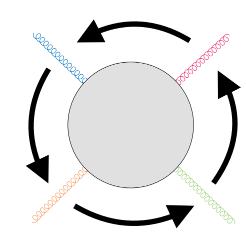
$$= \sum_j F_c^j \; T_j^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$

~~$p_1^{\mu_1}p_2^{\mu_2}p_3^{\mu_3}p_3^{\mu_4}$~~	$p_3^{\mu_1}p_3^{\mu_2}g^{\mu_3\mu_4}$	$g^{\mu_1\mu_2}g^{\mu_3\mu_4}$
~~$p_2^{\mu_1}p_2^{\mu_2}p_3^{\mu_3}p_2^{\mu_4}$~~	$g^{\mu_1\mu_2}p_1^{\mu_3}p_1^{\mu_4}$	$g^{\mu_1\mu_3}g^{\mu_2\mu_4}$
$p_3^{\mu_1}p_1^{\mu_2}p_1^{\mu_3}p_2^{\mu_4}$	$g^{\mu_1\mu_3}p_1^{\mu_2}p_2^{\mu_4}$	$g^{\mu_1\mu_4}g^{\mu_2\mu_3}$
$p_2^{\mu_1}p_1^{\mu_2}p_3^{\mu_3}p_2^{\mu_4}$	⋮ **54**	**3**
~~$p_3^{\mu_1}p_2^{\mu_2}p_1^{\mu_3}p_2^{\mu_4}$~~		
⋮ **81**		

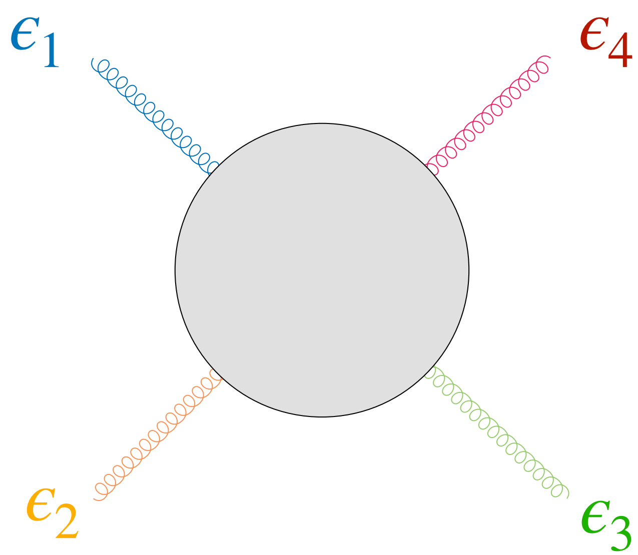
Transversality $\epsilon_i \cdot p_i = 0$



Reference choice $\epsilon_i \cdot p_{i+1} = 0$



Spin



$$A_c = A_c^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$

$$= \sum_j F_c^j \; T_j^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$

~~$p_1^{\mu_1}p_2^{\mu_2}p_3^{\mu_3}p_3^{\mu_4}$~~

~~$p_2^{\mu_1}p_2^{\mu_2}p_3^{\mu_3}p_2^{\mu_4}$~~

$p_3^{\mu_1}p_1^{\mu_2}p_1^{\mu_3}p_2^{\mu_4}$

$p_2^{\mu_1}p_1^{\mu_2}p_3^{\mu_3}p_2^{\mu_4}$

~~$p_3^{\mu_1}p_2^{\mu_2}p_1^{\mu_3}p_2^{\mu_4}$~~

⋮ **81**

~~$p_3^{\mu_1}p_3^{\mu_2}g^{\mu_3\mu_4}$~~

$g^{\mu_1\mu_2}p_1^{\mu_3}p_1^{\mu_4}$

$g^{\mu_1\mu_3}p_1^{\mu_2}p_2^{\mu_4}$

⋮ **54**

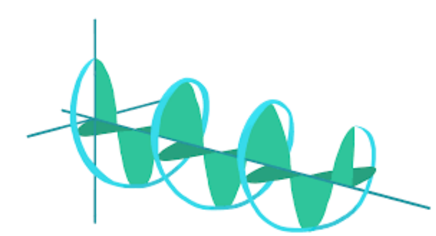
$g^{\mu_1\mu_2}g^{\mu_3\mu_4}$

$g^{\mu_1\mu_3}g^{\mu_2\mu_4}$

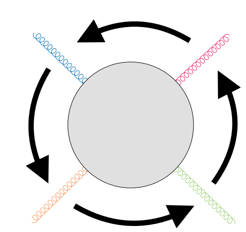
$g^{\mu_1\mu_4}g^{\mu_2\mu_3}$

3

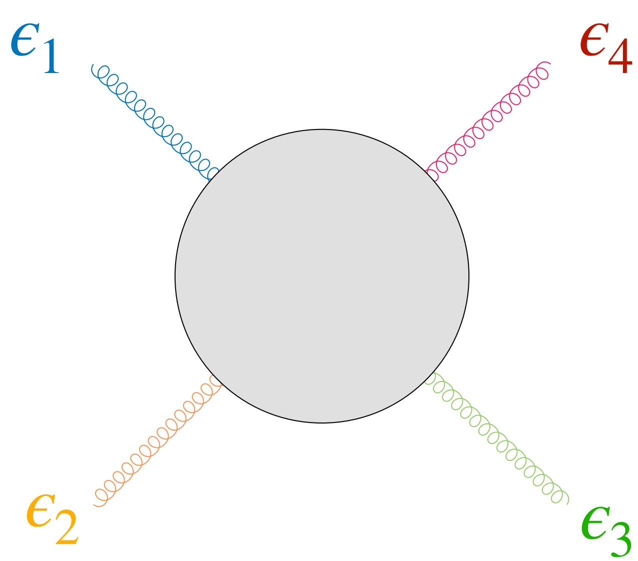
Transversality $\epsilon_i \cdot p_i = 0$



Reference choice $\epsilon_i \cdot p_{i+1} = 0$



Spin

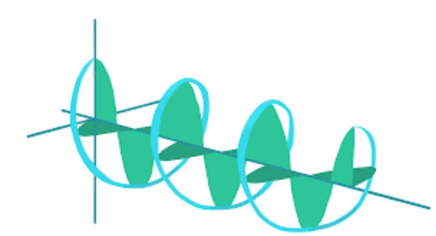


$$A_c = A_c^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$

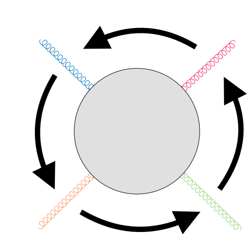
$$= \sum_j F_c^j \; T_j^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$

~~$p_1^{\mu_1}p_2^{\mu_2}p_3^{\mu_3}p_3^{\mu_4}$~~	~~$p_3^{\mu_1}p_3^{\mu_2}g^{\mu_3\mu_4}$~~	$g^{\mu_1\mu_2}g^{\mu_3\mu_4}$
~~$p_2^{\mu_1}p_2^{\mu_2}p_3^{\mu_3}p_2^{\mu_4}$~~	~~$g^{\mu_1\mu_2}p_1^{\mu_3}p_1^{\mu_4}$~~	$g^{\mu_1\mu_3}g^{\mu_2\mu_4}$
$p_3^{\mu_1}p_1^{\mu_2}p_1^{\mu_3}p_2^{\mu_4}$	$g^{\mu_1\mu_3}p_1^{\mu_2}p_2^{\mu_4}$	$g^{\mu_1\mu_4}g^{\mu_2\mu_3}$
~~$p_2^{\mu_1}p_1^{\mu_2}p_3^{\mu_3}p_2^{\mu_4}$~~	⋮ **54**	**3**
~~$p_3^{\mu_1}p_2^{\mu_2}p_1^{\mu_3}p_2^{\mu_4}$~~		
⋮ **81**		

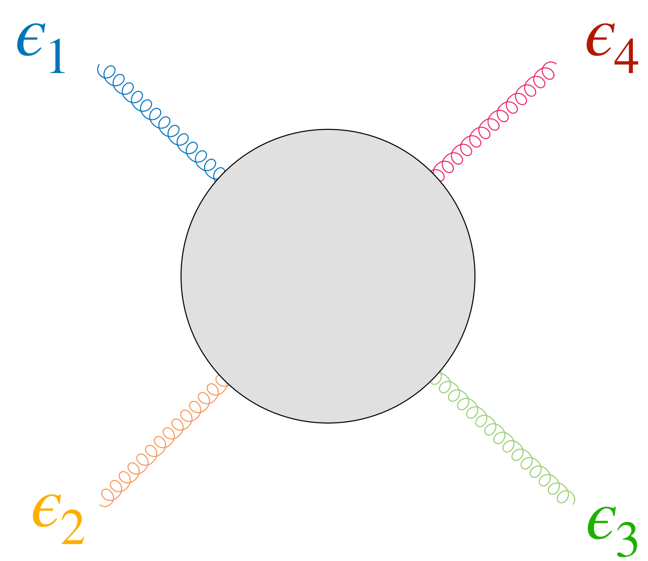
Transversality $\epsilon_i \cdot p_i = 0$



Reference choice $\epsilon_i \cdot p_{i+1} = 0$



Spin

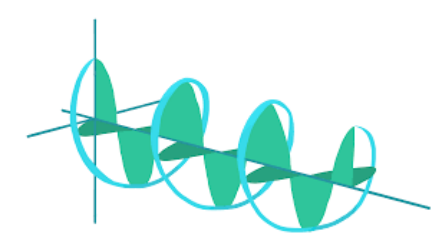


$$A_c = A_c^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$

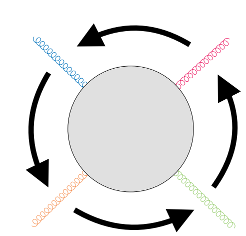
$$= \sum_j F_c^j \; T_j^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$

$p_1^{\mu_1}p_2^{\mu_2}p_3^{\mu_3}p_3^{\mu_4}$	$p_3^{\mu_1}p_3^{\mu_2}g^{\mu_3\mu_4}$	$g^{\mu_1\mu_2}g^{\mu_3\mu_4}$
$p_2^{\mu_1}p_2^{\mu_2}p_3^{\mu_3}p_2^{\mu_4}$	$g^{\mu_1\mu_2}p_1^{\mu_3}p_1^{\mu_4}$	$g^{\mu_1\mu_3}g^{\mu_2\mu_4}$
$p_3^{\mu_1}p_1^{\mu_2}p_1^{\mu_3}p_2^{\mu_4}$	$g^{\mu_1\mu_3}p_1^{\mu_2}p_2^{\mu_4}$	$g^{\mu_1\mu_4}g^{\mu_2\mu_3}$
$p_2^{\mu_1}p_1^{\mu_2}p_3^{\mu_3}p_2^{\mu_4}$	⋮ **54**	**3**
$p_3^{\mu_1}p_2^{\mu_2}p_1^{\mu_3}p_2^{\mu_4}$		
⋮ **81**		

Transversality $\epsilon_i \cdot p_i = 0$

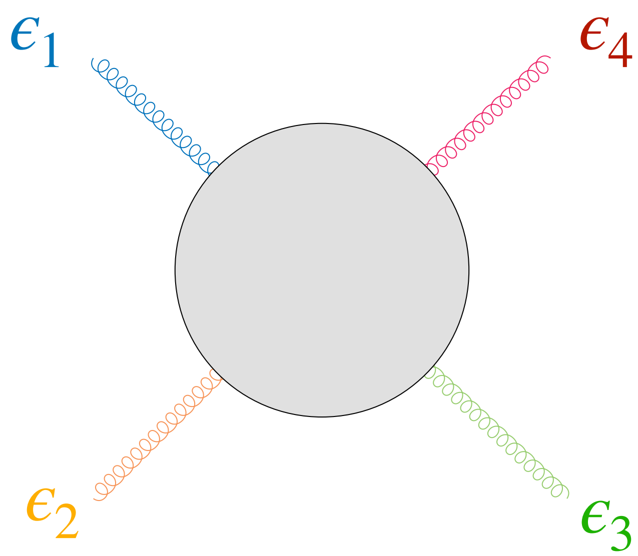


Reference choice $\epsilon_i \cdot p_{i+1} = 0$



138 → 10

Spin



$$A_c = A_c^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$

$$= \sum_j F_c^j \; T_j^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$

$$p_3^{\mu_1}p_1^{\mu_2}p_1^{\mu_3}p_2^{\mu_4}$$

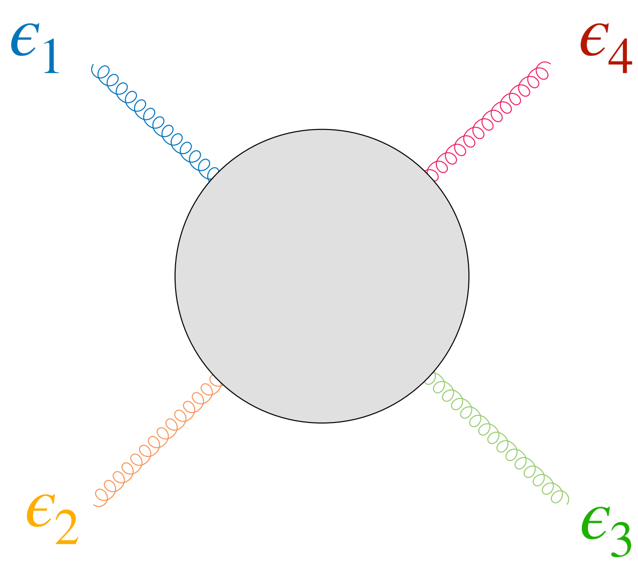
$$p_3^{\mu_1}p_1^{\mu_2}g^{\mu_3\mu_4} \qquad p_3^{\mu_1}p_2^{\mu_4}g^{\mu_2\mu_3} \qquad p_1^{\mu_2}p_1^{\mu_3}g^{\mu_1\mu_4}$$

$$p_3^{\mu_1}p_1^{\mu_3}g^{\mu_2\mu_4} \qquad p_1^{\mu_2}p_2^{\mu_4}g^{\mu_1\mu_3} \qquad p_1^{\mu_3}p_3^{\mu_4}g^{\mu_1\mu_2}$$

$$g^{\mu_1\mu_2}g^{\mu_3\mu_4} \qquad g^{\mu_1\mu_3}g^{\mu_2\mu_4} \qquad g^{\mu_1\mu_4}g^{\mu_2\mu_3}$$

138 → 10

Spin



$$A_c = A_c^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$
$$= \sum_j F_c^j \; T_j^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$

$$p_3^{\mu_1}p_1^{\mu_2}p_1^{\mu_3}p_2^{\mu_4}$$

$$p_3^{\mu_1}p_1^{\mu_2}g^{\mu_3\mu_4} \qquad p_3^{\mu_1}p_2^{\mu_4}g^{\mu_2\mu_3} \qquad p_1^{\mu_2}p_1^{\mu_3}g^{\mu_1\mu_4}$$

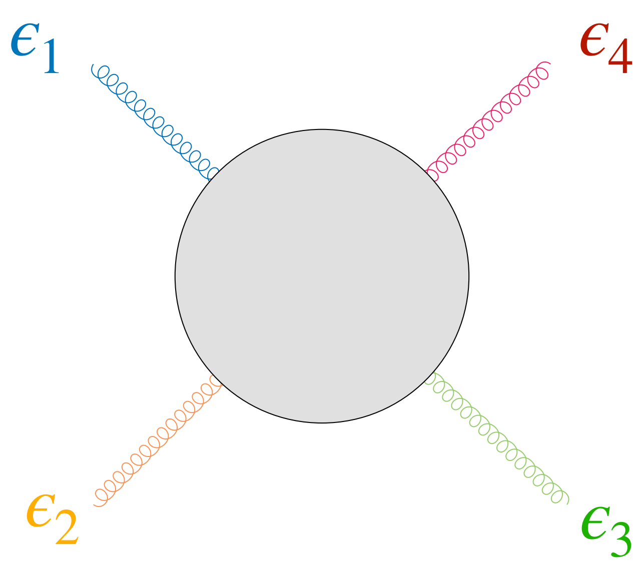
$$p_3^{\mu_1}p_1^{\mu_3}g^{\mu_2\mu_4} \qquad p_1^{\mu_2}p_2^{\mu_4}g^{\mu_1\mu_3} \qquad p_1^{\mu_3}p_3^{\mu_4}g^{\mu_1\mu_2}$$

$$g^{\mu_1\mu_2}g^{\mu_3\mu_4} \qquad g^{\mu_1\mu_3}g^{\mu_2\mu_4} \qquad g^{\mu_1\mu_4}g^{\mu_2\mu_3}$$

138 → 10

Tancredi, Peraro: 1906.03298, 2012.00820

Spin



$$A_c = A_c^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$
$$= \sum_j F_c^j \, T_j^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$

$$p_3^{\mu_1}p_1^{\mu_2}p_1^{\mu_3}p_2^{\mu_4}$$

$$p_3^{\mu_1}p_1^{\mu_2}g^{\mu_3\mu_4} \qquad p_3^{\mu_1}p_2^{\mu_4}g^{\mu_2\mu_3} \qquad p_1^{\mu_2}p_1^{\mu_3}g^{\mu_1\mu_4}$$

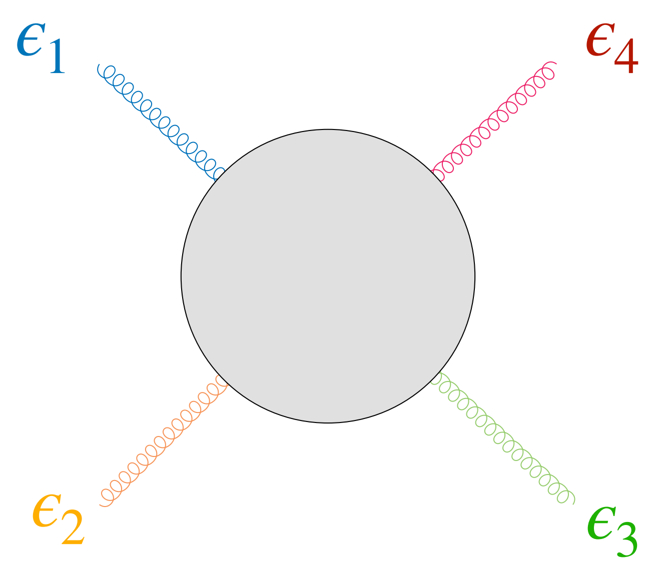
$$p_3^{\mu_1}p_1^{\mu_3}g^{\mu_2\mu_4} \qquad p_1^{\mu_2}p_2^{\mu_4}g^{\mu_1\mu_3} \qquad p_1^{\mu_3}p_3^{\mu_4}g^{\mu_1\mu_2}$$

$$g^{\mu_1\mu_2}g^{\mu_3\mu_4} + g^{\mu_1\mu_3}g^{\mu_2\mu_4} + g^{\mu_1\mu_4}g^{\mu_2\mu_3}$$

138 → 10

Tancredi, Peraro: 1906.03298, 2012.00820

Spin



$$A_c = A_c^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$

$$= \sum_j F_c^j \; T_j^{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}$$

$$p_3^{\mu_1}p_1^{\mu_2}p_1^{\mu_3}p_2^{\mu_4}$$

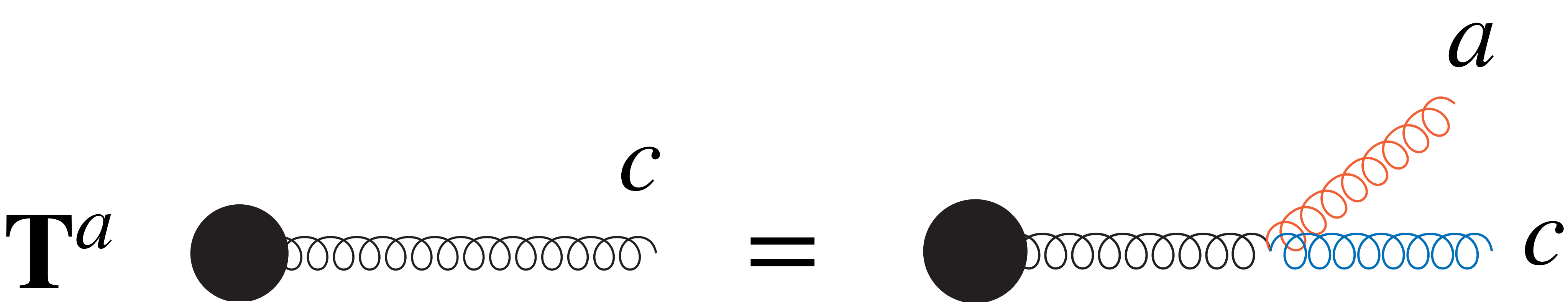
$$p_3^{\mu_1}p_1^{\mu_2}g^{\mu_3\mu_4} \qquad p_3^{\mu_1}p_2^{\mu_4}g^{\mu_2\mu_3} \qquad p_1^{\mu_2}p_1^{\mu_3}g^{\mu_1\mu_4}$$

$$p_3^{\mu_1}p_1^{\mu_3}g^{\mu_2\mu_4} \qquad p_1^{\mu_2}p_2^{\mu_4}g^{\mu_1\mu_3} \qquad p_1^{\mu_3}p_3^{\mu_4}g^{\mu_1\mu_2}$$

$$g^{\mu_1\mu_2}g^{\mu_3\mu_4} + g^{\mu_1\mu_3}g^{\mu_2\mu_4} + g^{\mu_1\mu_4}g^{\mu_2\mu_3}$$

138 → 10 → 8

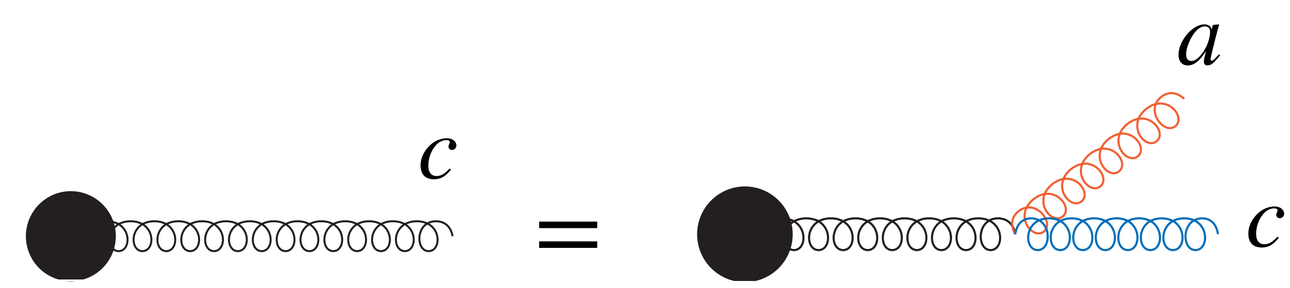
Tancredi, Peraro: 1906.03298, 2012.00820



$$\mathbf{T}^a X^c = -i f^a{}_{cc'} X^{c'}$$

Dipole

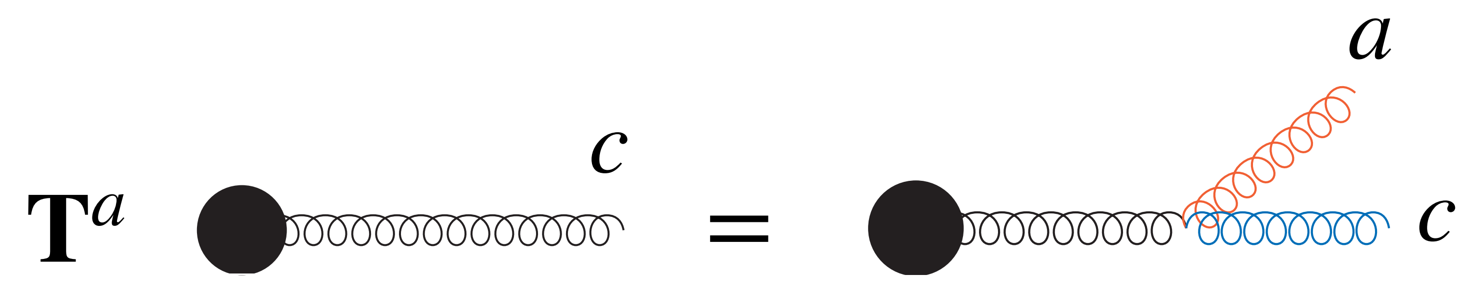
$\mathbf{T}^a$



$$\mathbf{T}^a X^c = -\, i f^a{}_{cc'} X^{c'}$$

$$\boldsymbol{\Gamma}_{\text{dipole}}(\{p\},\mu) = \sum_{1\le i<j\le 4} \mathbf{T}_i^a\, \mathbf{T}_j^a\, \gamma^{\text{cusp}}(\alpha_{\text{s}})\, \log\left(\frac{\mu^2}{-s_{ij} - i\delta}\right) + \sum_{i=1}^{4} \gamma^i(\alpha_{\text{s}})$$

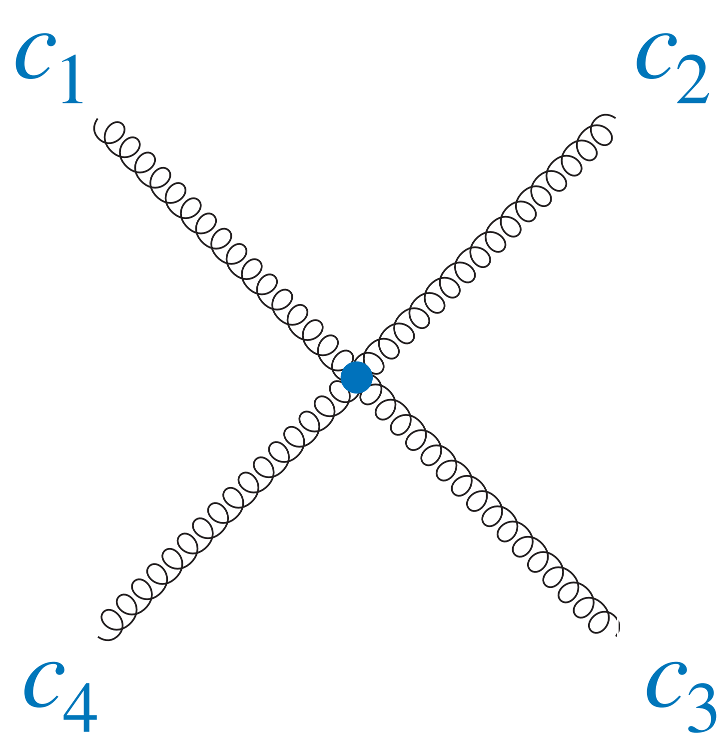
Dipole



$$\mathbf{T}^a X^c = -\, i f^a{}_{cc'} X^{c'}$$

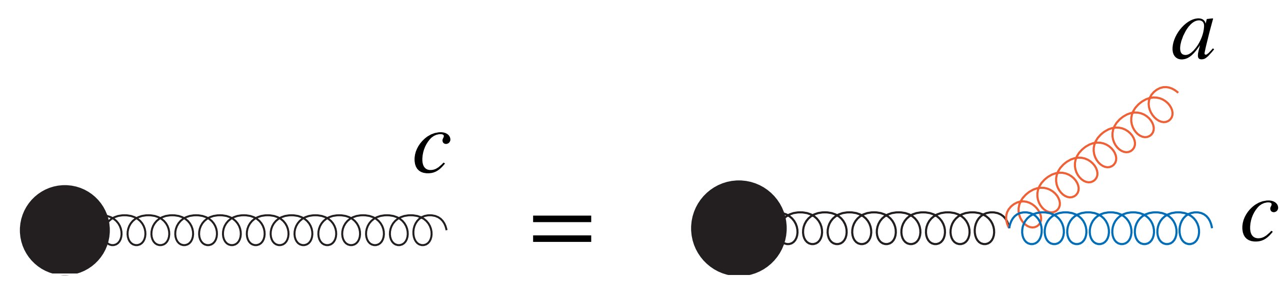
$$\boldsymbol{\Gamma}_{\text{dipole}}(\{p\},\mu) = \sum_{1\le i<j\le 4} \mathbf{T}_i^a\, \mathbf{T}_j^a\, \gamma^{\text{cusp}}(\alpha_s)\, \log\left(\frac{\mu^2}{-s_{ij} - i\delta}\right) + \sum_{i=1}^{4} \gamma^i(\alpha_s)$$

$$\mathbf{T}_1^a\, \mathbf{T}_2^a$$



Dipole

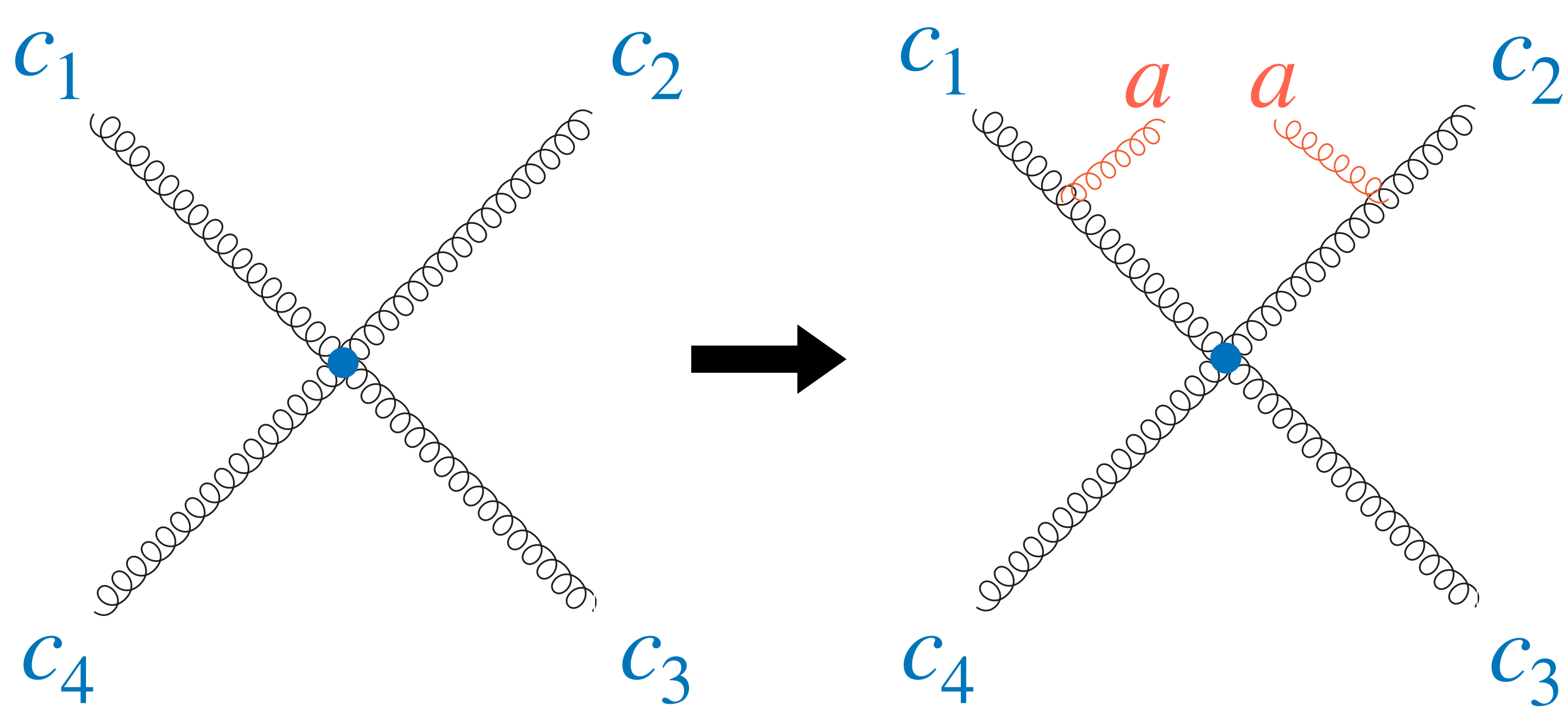
$\mathbf{T}^a$



$$\mathbf{T}^a\, X^c = -\, i f^a{}_{cc'}\, X^{c'}$$

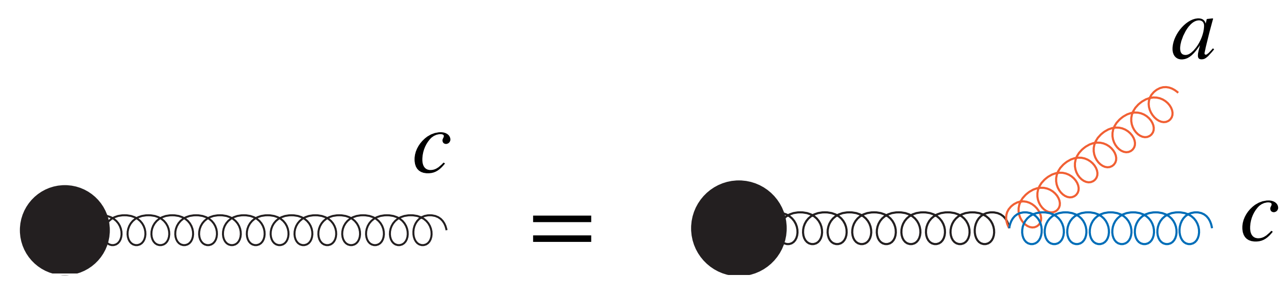
$$\boldsymbol{\Gamma}_{\text{dipole}}(\{p\},\mu) = \sum_{1\le i<j\le 4} \mathbf{T}_i^a\, \mathbf{T}_j^a\, \gamma^{\text{cusp}}(\alpha_{\text{s}})\, \log\left(\frac{\mu^2}{-s_{ij}-i\delta}\right) + \sum_{i=1}^{4} \gamma^i(\alpha_{\text{s}})$$

$\mathbf{T}_1^a\,\mathbf{T}_2^a$



Dipole

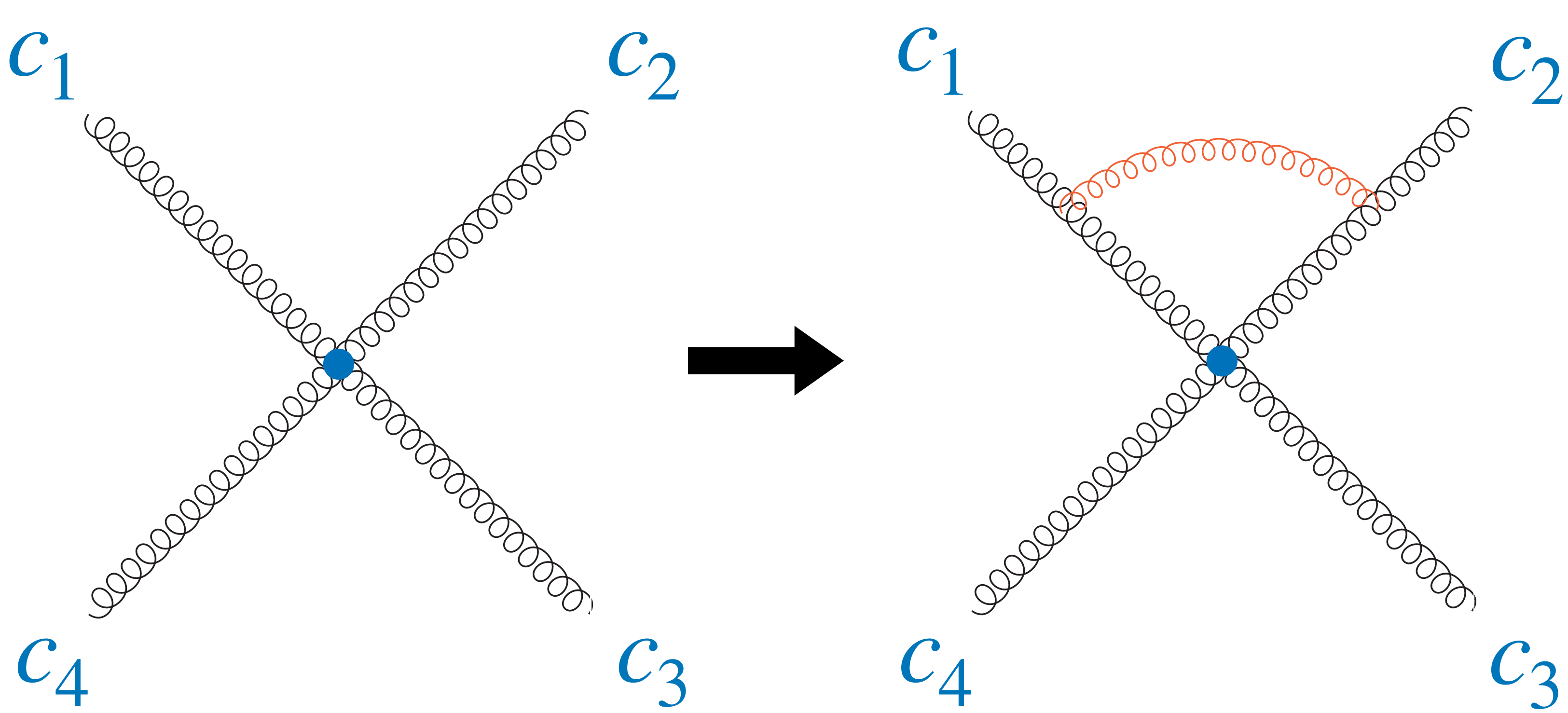
$\mathbf{T}^a$



$$\mathbf{T}^a\, X^c = -\, i f^a{}_{cc'}\, X^{c'}$$

$$\Gamma_{\text{dipole}}(\{p\},\mu) = \sum_{1\le i<j\le 4} \mathbf{T}_i^a\, \mathbf{T}_j^a\, \gamma^{\text{cusp}}(\alpha_s)\, \log\left(\frac{\mu^2}{-s_{ij} - i\delta}\right) + \sum_{i=1}^{4} \gamma^i(\alpha_s)$$

$\mathbf{T}_1^a\, \mathbf{T}_2^a$



$$F_c^i = \sum \epsilon^{-n} \, r_i \, \mathbb{T}_i$$

Divergent Logarithms

$$\sim \log^{\#}\left(\frac{-t}{s}\right) = L^{\#}$$

$$F_c^i = \sum \epsilon^{-n}\, r_i\, \mathbb{T}_i$$

Divergent Logarithms

$$\sim \log^{\#}\left(\frac{-t}{s}\right) = L^{\#}$$

Spoiled expansion

$$\mathbf{A}^{(0)} + \alpha \mathbf{A}^{(1)} + \alpha^2 \mathbf{A}^{(2)} + \alpha^3 \mathbf{A}^{(3)} + \ldots$$

$$F_c^i = \sum \epsilon^{-n} \, r_i \, \mathbb{T}_i$$

Divergent Logarithms

$$\sim \log^{\#}\left(\frac{-t}{s}\right) = L^{\#}$$

Spoiled expansion

$$\mathbf{A}^{(0)} + \alpha \mathbf{A}^{(1)} + \alpha^2 \mathbf{A}^{(2)} + \alpha^3 \mathbf{A}^{(3)} + \ldots$$

$$\begin{aligned}
A^{(0)} &= \# \\
A^{(1)} &= \# L + \# \\
A^{(2)} &= \# L^2 + \# L + \# \\
A^{(3)} &= \# L^3 + \# L^2 + \# L + \# \\
& \quad \vdots \qquad \vdots \qquad \vdots
\end{aligned}$$

$$F_c^i = \sum \epsilon^{-n}\, r_i\, \mathbb{T}_i$$

Divergent Logarithms

$$\sim \log^{\#}\left(\frac{-t}{s}\right) = L^{\#}$$

Spoiled expansion

$$\mathbf{A}^{(0)} + \alpha \mathbf{A}^{(1)} + \alpha^2 \mathbf{A}^{(2)} + \alpha^3 \mathbf{A}^{(3)} + \ldots$$

LL

$$A^{(0)} = \#$$
$$A^{(1)} = \# L + \#$$
$$A^{(2)} = \# L^2 + \# L + \#$$
$$A^{(3)} = \# L^3 + \# L^2 + \# L + \#$$
$$\vdots \qquad \vdots \qquad \vdots$$

$$F_c^i = \sum \epsilon^{-n} \, r_i \, \mathbb{T}_i$$

Divergent Logarithms

$$\sim \log^{\#}\left(\frac{-t}{s}\right) = L^{\#}$$

Spoiled expansion

$$\mathbf{A}^{(0)} + \alpha \mathbf{A}^{(1)} + \alpha^2 \mathbf{A}^{(2)} + \alpha^3 \mathbf{A}^{(3)} + \ldots$$

LL

NLL

$$\begin{aligned}
A^{(0)} &= \# \\
A^{(1)} &= \# L + \# \\
A^{(2)} &= \# L^2 + \# L + \# \\
A^{(3)} &= \# L^3 + \# L^2 + \# L + \# \\
&\;\;\vdots \qquad\quad \vdots \qquad\quad \vdots
\end{aligned}$$

$$F_c^i = \sum \epsilon^{-n} \, r_i \, \mathbb{T}_i$$

Divergent Logarithms

$$\sim \log^{\#}\left(\frac{-t}{s}\right) = L^{\#}$$

Spoiled expansion

$$\mathbf{A}^{(0)} + \alpha \mathbf{A}^{(1)} + \alpha^2 \mathbf{A}^{(2)} + \alpha^3 \mathbf{A}^{(3)} + \ldots$$

LL | NLL | NNLL

$$\begin{aligned}
A^{(0)} &= \# \\
A^{(1)} &= \# L + \# \\
A^{(2)} &= \# L^2 + \# L + \# \\
A^{(3)} &= \# L^3 + \# L^2 + \# L + \# \\
&\;\;\vdots \qquad\quad \vdots \qquad\quad \vdots
\end{aligned}$$

$$A^{\pm,(0)} = \#$$

$$A^{\pm,(1)} = \# L \ + \#$$

$$A^{\pm,(2)} = \# L^2 + \# L \ + \#$$

$$A^{\pm,(3)} = \# L^3 + \# L^2 + \# L \ + \#$$

LL

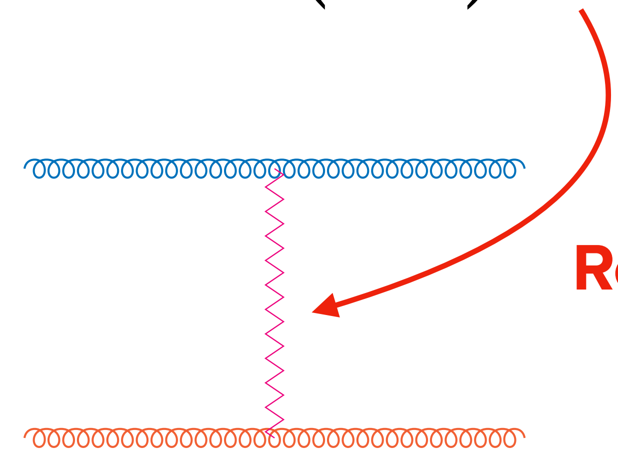
$$A^{\pm,(0)} = \#$$

$$A^{\pm,(1)} = \# L + \#$$

$$A^{\pm,(2)} = \# L^2 + \# L + \#$$

$$A^{\pm,(3)} = \# L^3 + \# L^2 + \# L + \#$$

$$A^{\pm}_{LL} = \left(\frac{s}{-t}\right)^{C_A \tau_g} A^{\pm,tree}$$



Reggeized Gluon

LL ... NLL

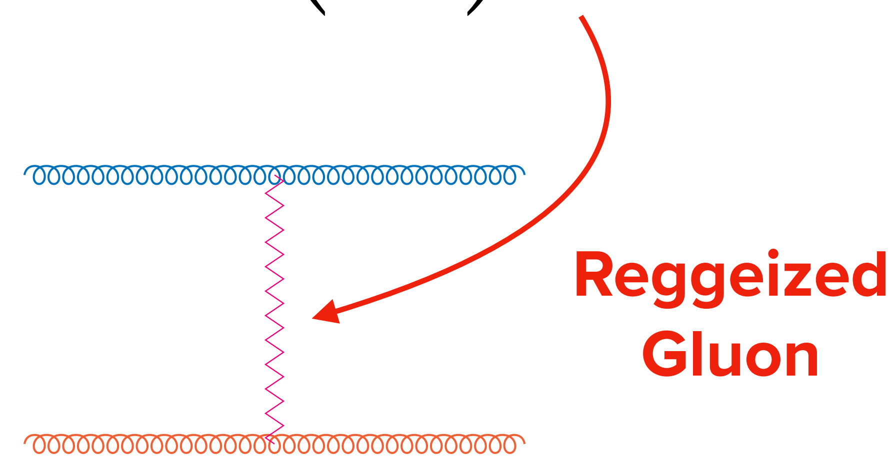
$$A^{\pm,(0)} = \#$$

$$A^{\pm,(1)} = \# L + \#$$

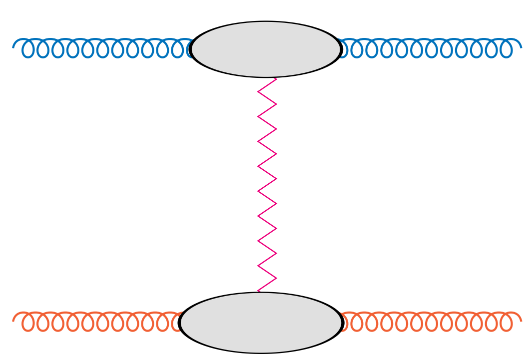
$$A^{\pm,(2)} = \# L^2 + \# L + \#$$

$$A^{\pm,(3)} = \# L^3 + \# L^2 + \# L + \#$$

$$A^{\pm}_{LL} = \left(\frac{s}{-t}\right)^{C_A \tau_g} A^{\pm,tree}$$

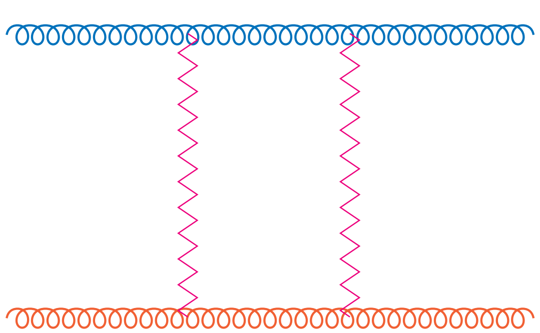


Regge Trajectory @2loops $\tau_g(\alpha_s)$

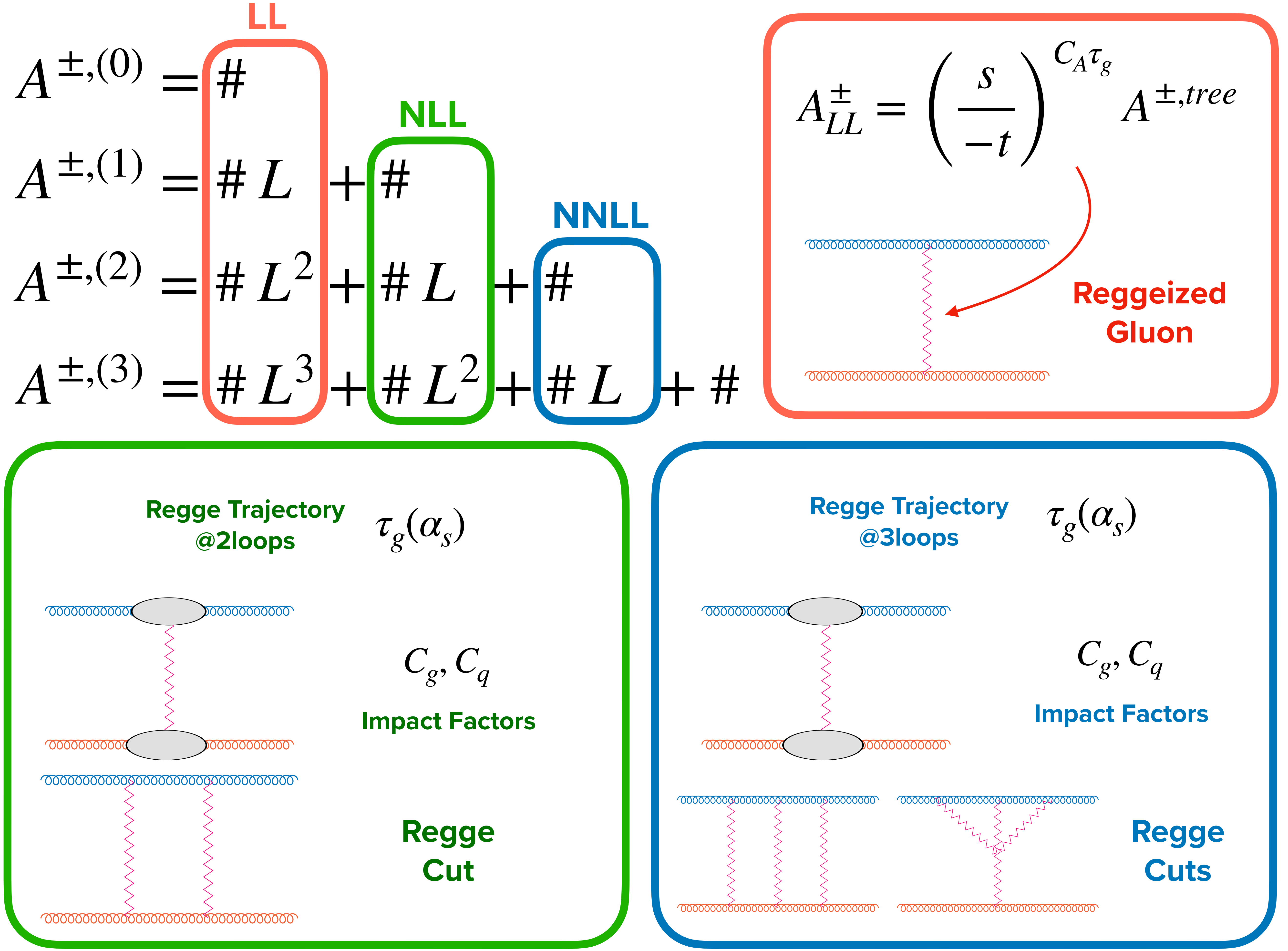


C_g, C_q

Impact Factors



Regge Cut


LL
NLL
NNLL
$A^{\pm,(0)} = \#$
$A^{\pm,(1)} = \# L + \#$
$A^{\pm,(2)} = \# L^2 + \# L + \#$
$A^{\pm,(3)} = \# L^3 + \# L^2 + \# L + \#$
$A^{\pm}_{LL} = \left(\frac{s}{-t}\right)^{C_A \tau_g} A^{\pm,tree}$
Reggeized Gluon
Regge Trajectory @2loops
$\tau_g(\alpha_s)$
C_g, C_q
Impact Factors
Regge Cut
Regge Trajectory @3loops
$\tau_g(\alpha_s)$
C_g, C_q
Impact Factors
Regge Cuts

LL — **NLL** — **NNLL**

$$A^{\pm,(0)} = \#$$
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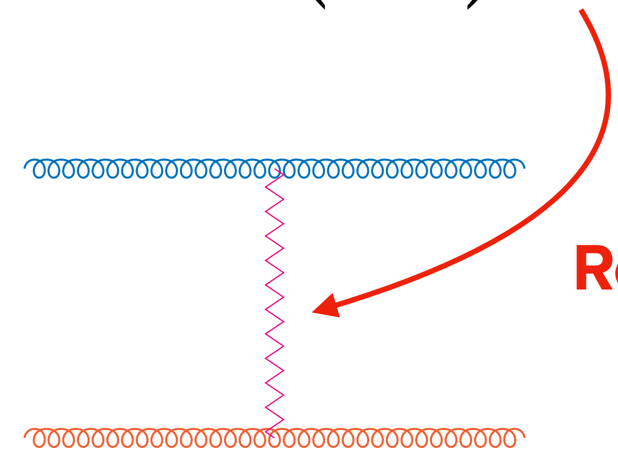
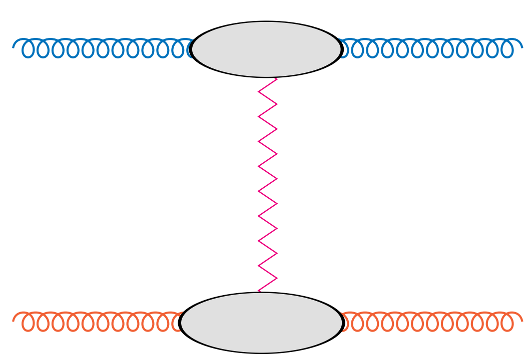
$$A^{\pm}_{LL} = \left(\frac{s}{-t}\right)^{C_A \tau_g} A^{\pm,tree}$$

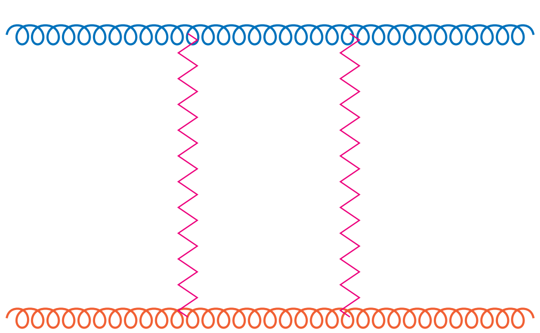


Regge Trajectory @2loops $\tau_g(\alpha_s)$



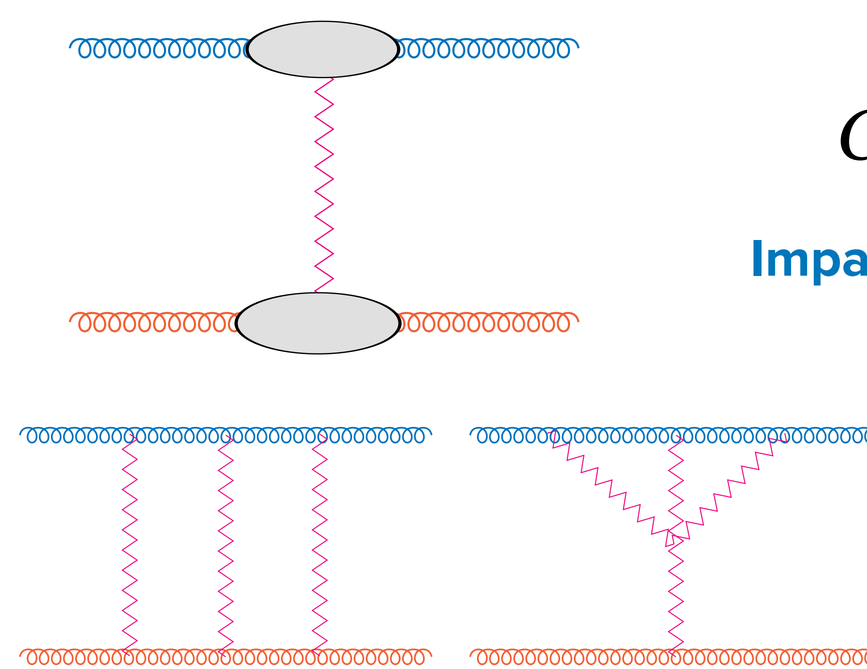
C_g, C_q

Impact Factors



Regge Cut

Regge Trajectory @3loops $\tau_g(\alpha_s)$



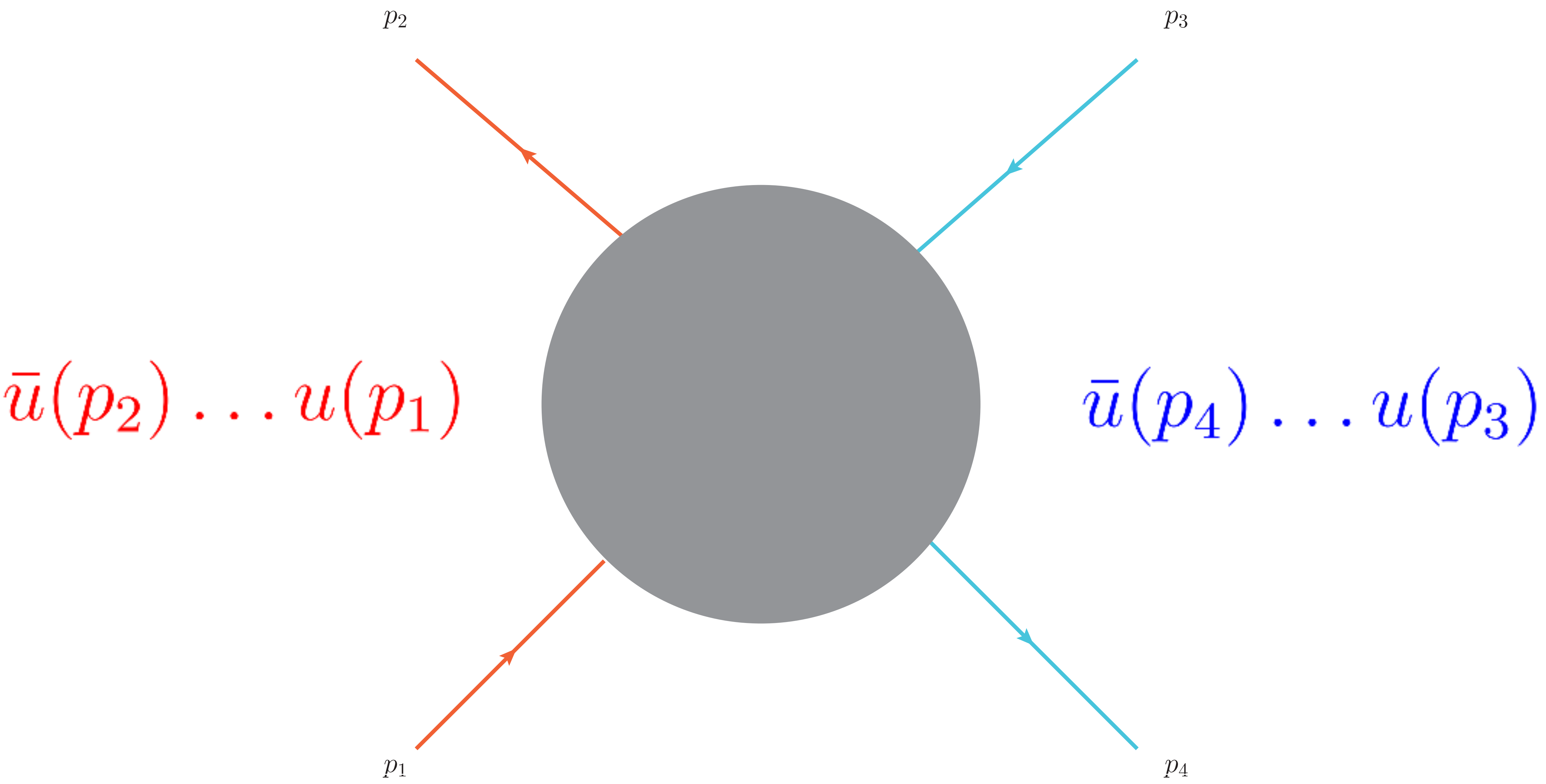
C_g, C_q

Impact Factors

Regge Cuts

Falcioni, Gardi, Maher, Milloy, Vernazza: 2112.11098

qQ → qQ



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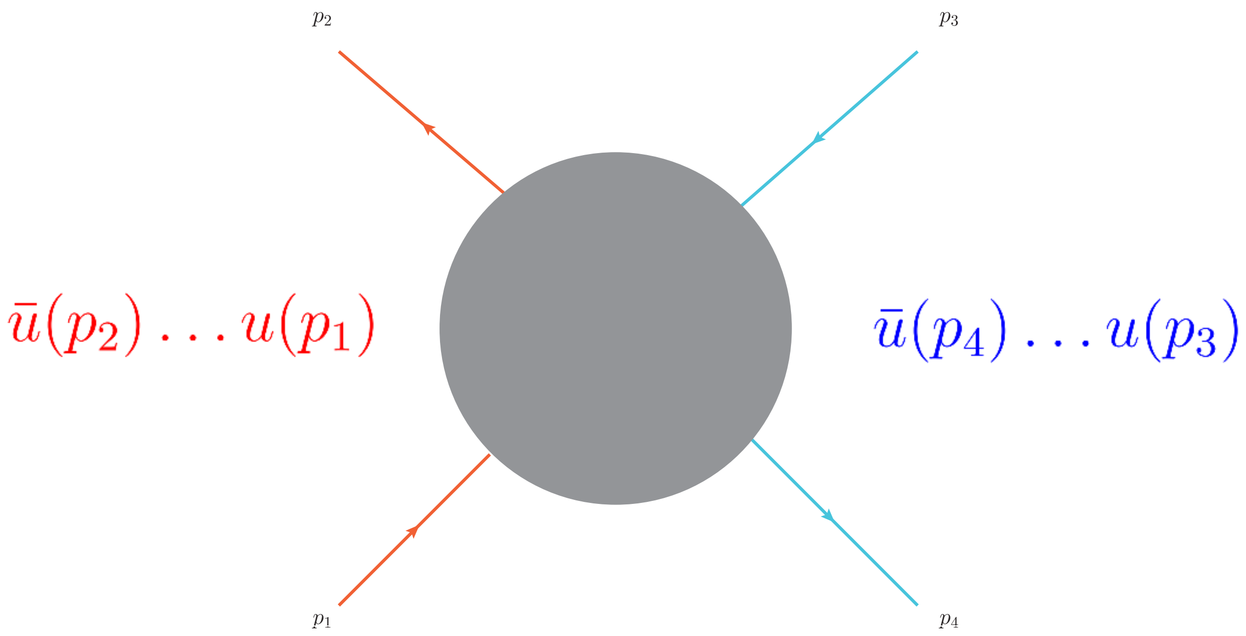
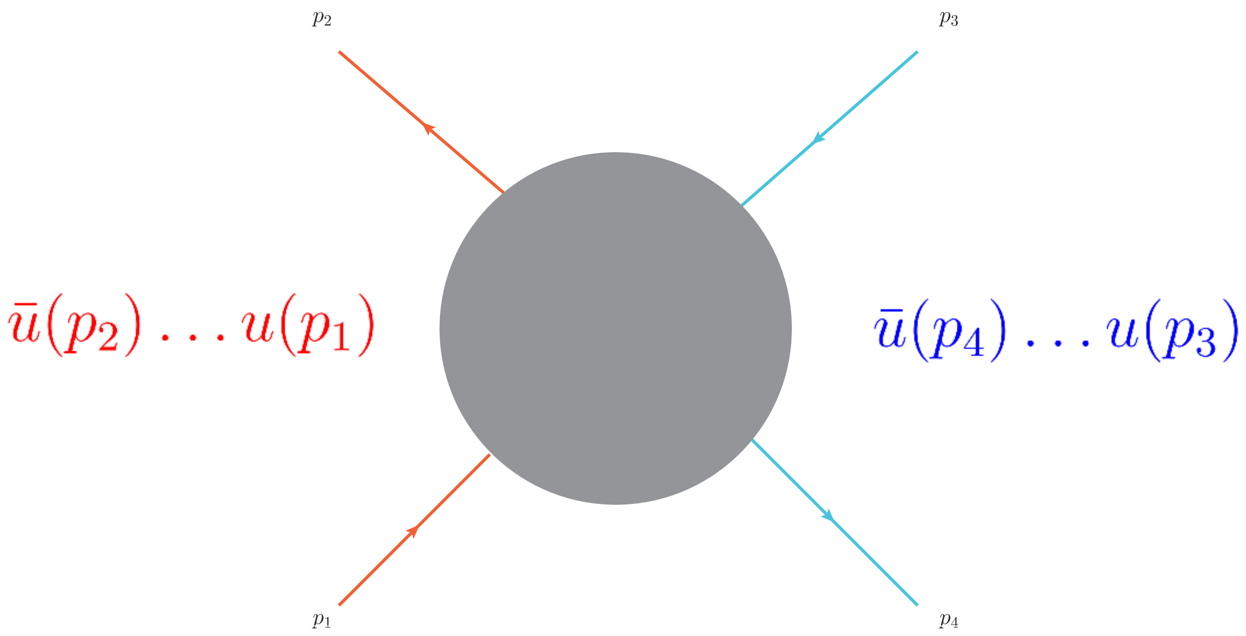


qQ → qQ

$$A^X = \sum_{i=1}^{N} F_i T_i^X$$



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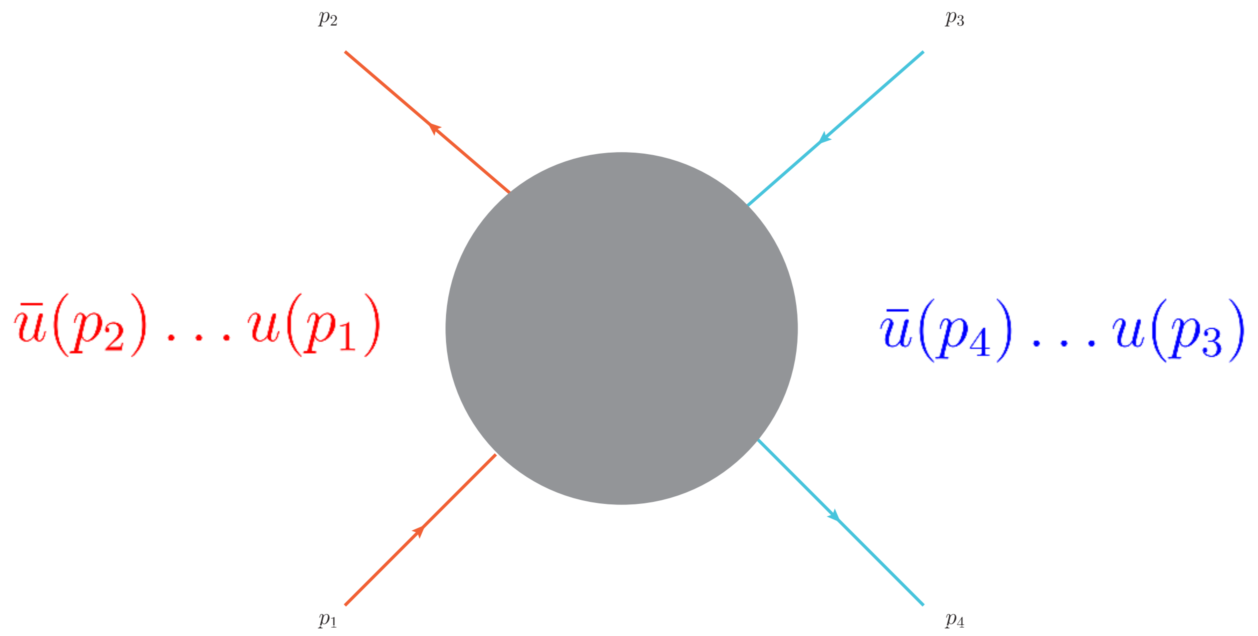

$$T_1 = \bar{u}(p_2)\gamma_{\mu_1}u(p_1) \;\times\; \bar{u}(p_4)\gamma^{\mu_1}u(p_3)$$

$$T_2 = \bar{u}(p_2)\not{p}_3 u(p_1) \;\times\; \bar{u}(p_4)\not{p}_1 u(p_3)$$

qQ → qQ

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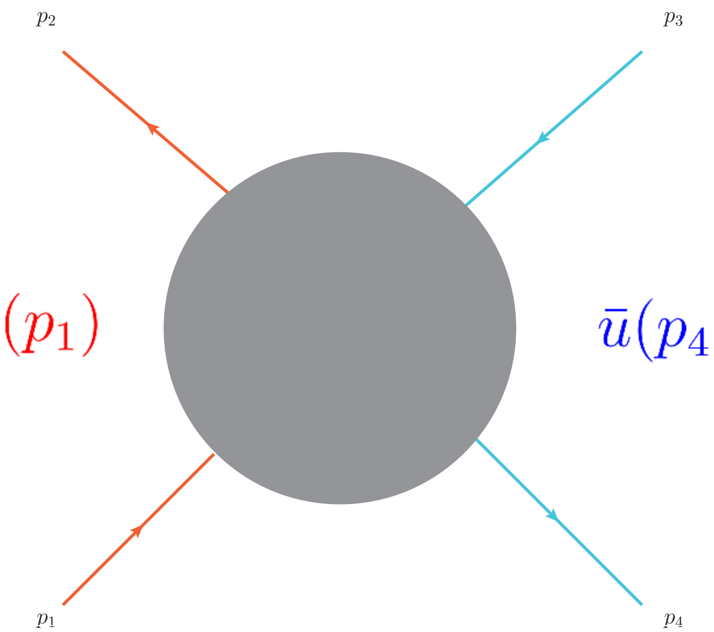
$\bar{u}(p_2)\ldots u(p_1)$



$\bar{u}(p_4)\ldots u(p_3)$

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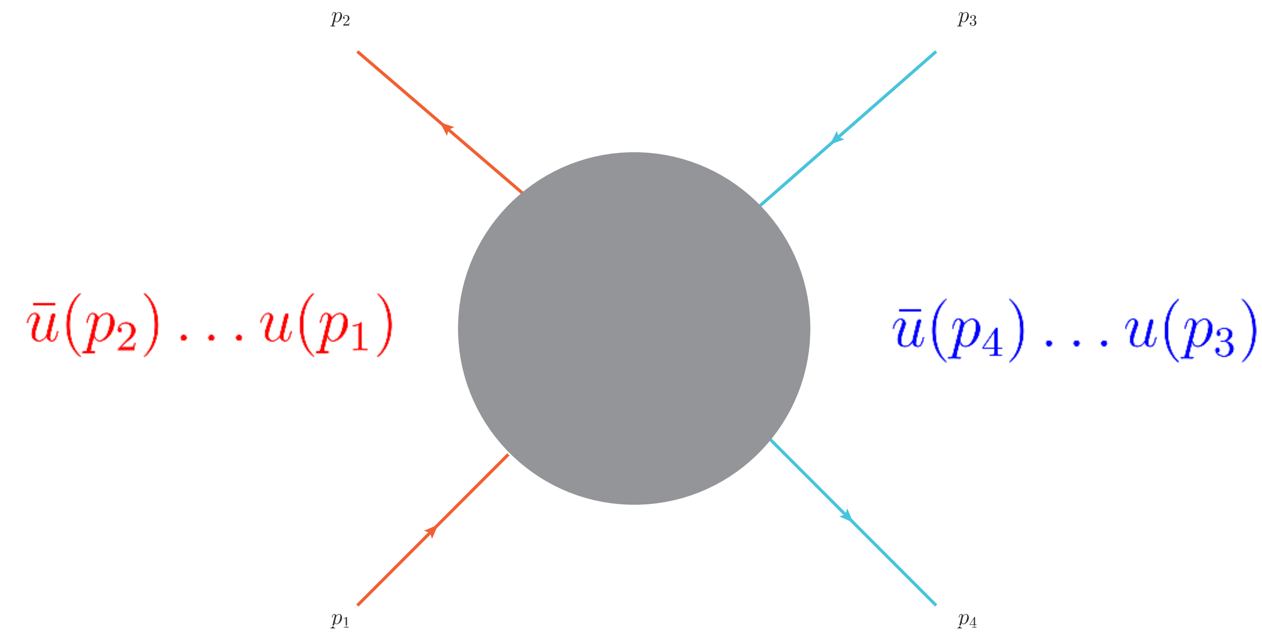

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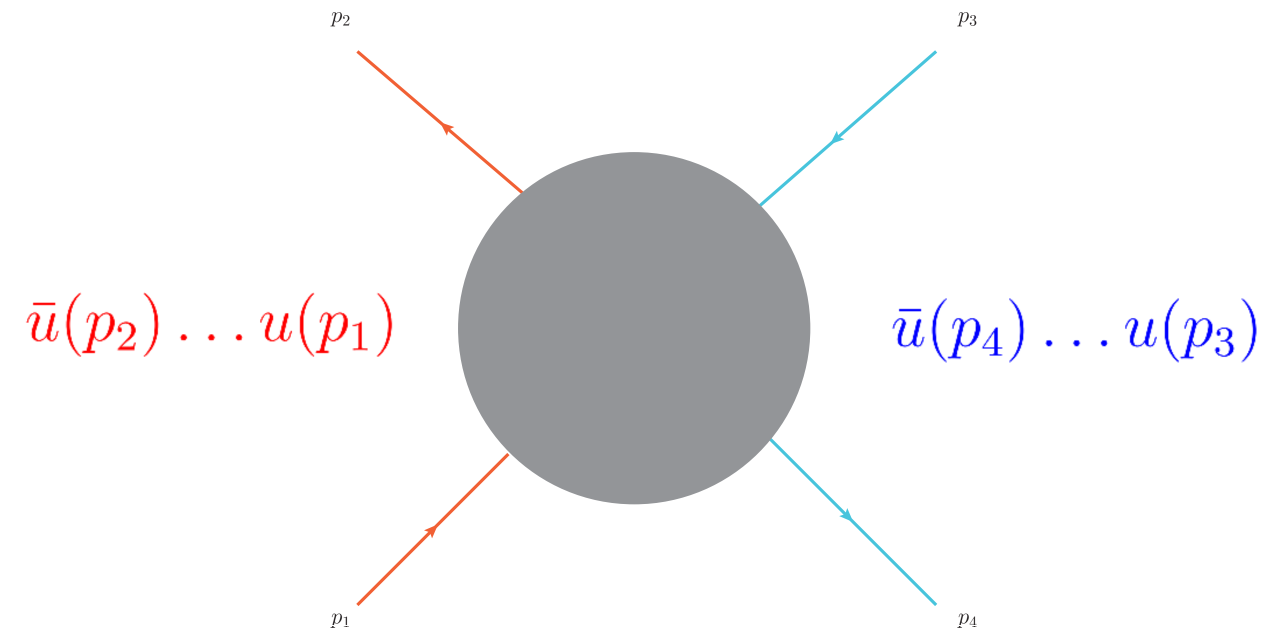
$$T_4 = \bar{u}(p_2)\gamma_{\mu_1}\not{p}_3\gamma_{\mu_3}u(p_1) \;\times\; \bar{u}(p_4)\gamma^{\mu_1}\not{p}_1\gamma^{\mu_3}u(p_3)$$

$$T_5 = \bar{u}(p_2)\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\gamma_{\mu_5}u(p_1) \;\times\; \bar{u}(p_4)\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}\gamma^{\mu_5}u(p_3)$$

$$T_6 = \bar{u}(p_2)\gamma_{\mu_1}\gamma_{\mu_2}\not{p}_3\gamma_{\mu_4}\gamma_{\mu_5}u(p_1) \;\times\; \bar{u}(p_4)\gamma^{\mu_1}\gamma^{\mu_2}\not{p}_1\gamma^{\mu_4}\gamma^{\mu_5}u(p_3)$$

qQ → qQ

$$A^X = \sum_{i=1}^{N} F_i T_i^X$$



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$$T_7 = \bar{u}(p_2)\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\gamma_{\mu_5}\gamma_{\mu_6}\gamma_{\mu_7}u(p_1) \;\times\; \bar{u}(p_4)\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}\gamma^{\mu_5}\gamma^{\mu_6}\gamma^{\mu_7}u(p_3)$$

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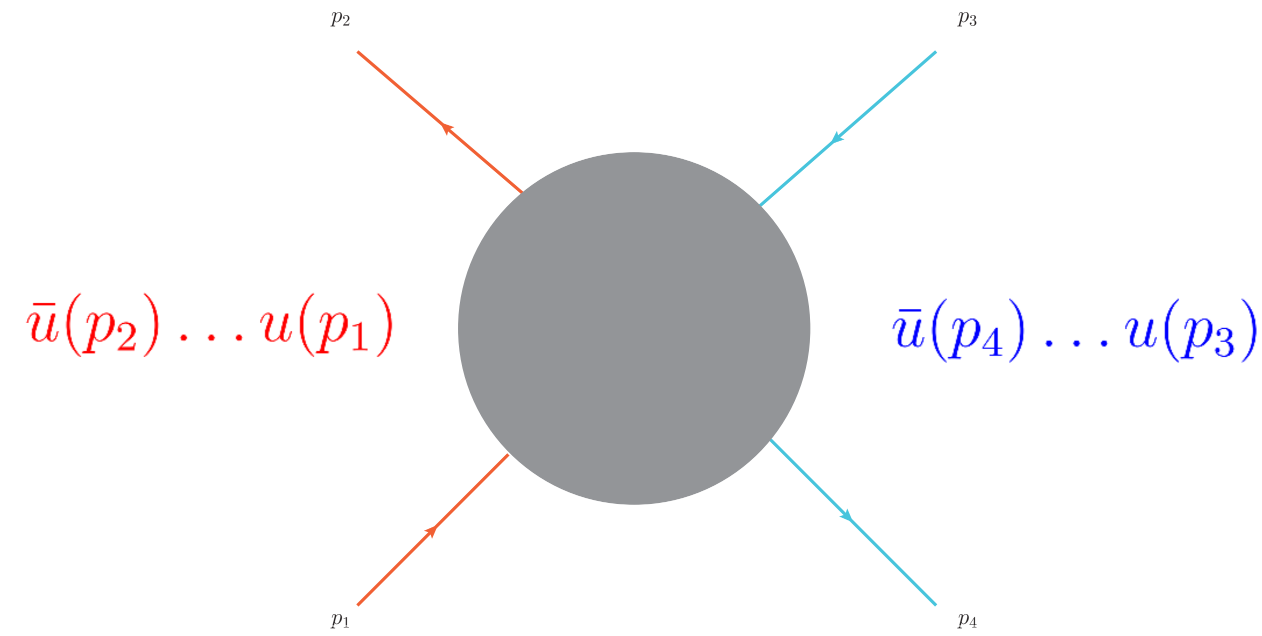
qQ → qQ

$$A^X = \sum_{i=1}^{N} F_i T_i^X$$



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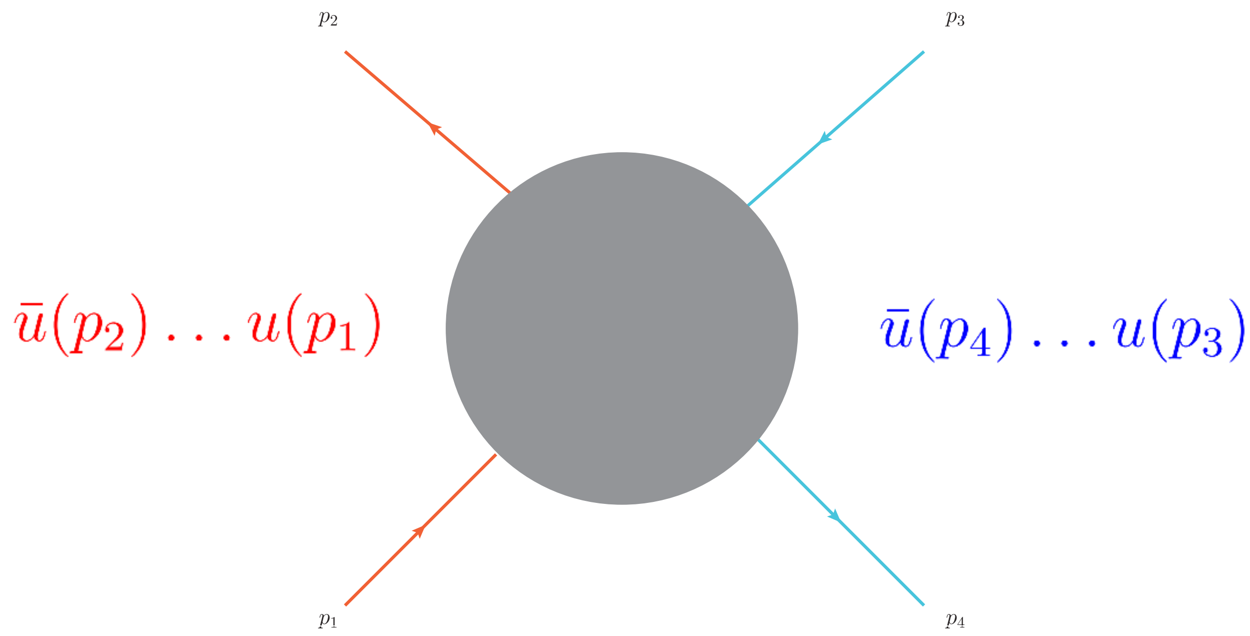
$$\vdots$$

qQ → qQ

$$A^X = \sum_{i=1}^{N} F_i T_i^X$$



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in d=4

$$T_3 = \bar{u}(p_2)\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}u(p_1) \times \bar{u}(p_4)\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}u(p_3)$$

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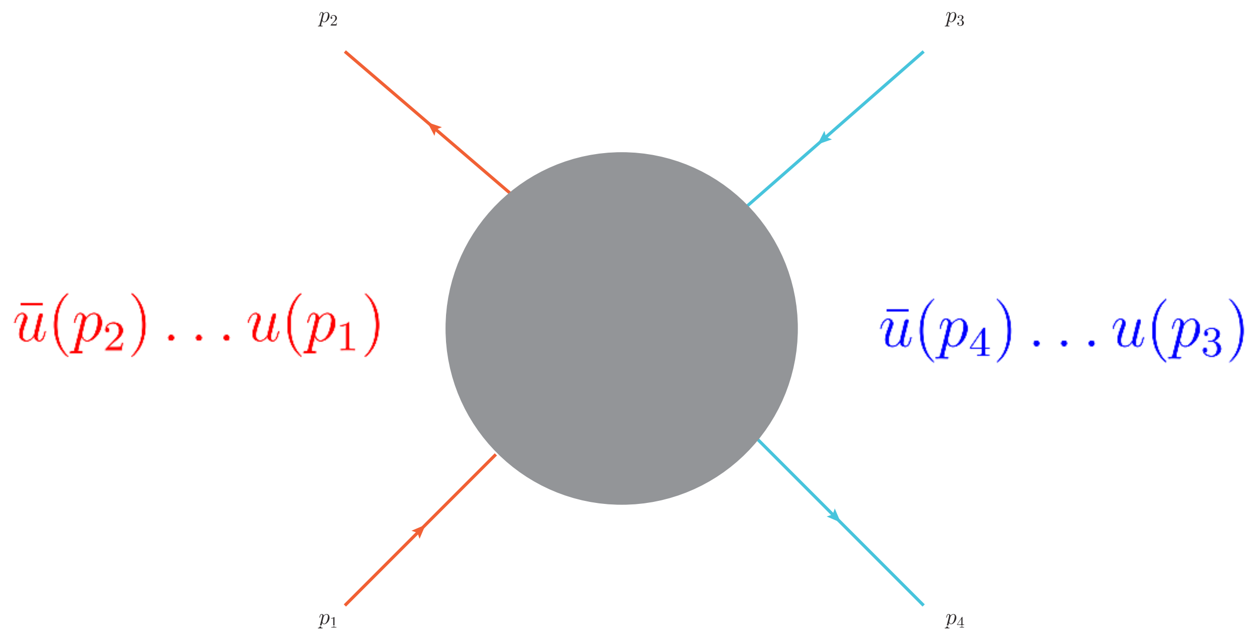
⋮

L. Tancredi, T. Peraro: arXiv:1906.03298, arXiv:2012.00820

qQ → qQ

$$A^X = \sum_{i=1}^{N} F_i T_i^X$$



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$$T_3 = T_3^4 + (d-4) T_3^{-2\epsilon}$$

$$T_4 = T_4^4 + (d-4) T_4^{-2\epsilon}$$

$$T_5 = T_5^4 + (d-4) T_5^{-2\epsilon}$$

$$T_6 = T_6^4 + (d-4) T_6^{-2\epsilon}$$

$$T_7 = T_7^4 + (d-4) T_7^{-2\epsilon}$$

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L. Tancredi, T. Peraro: arXiv:1906.03298, arXiv:2012.00820

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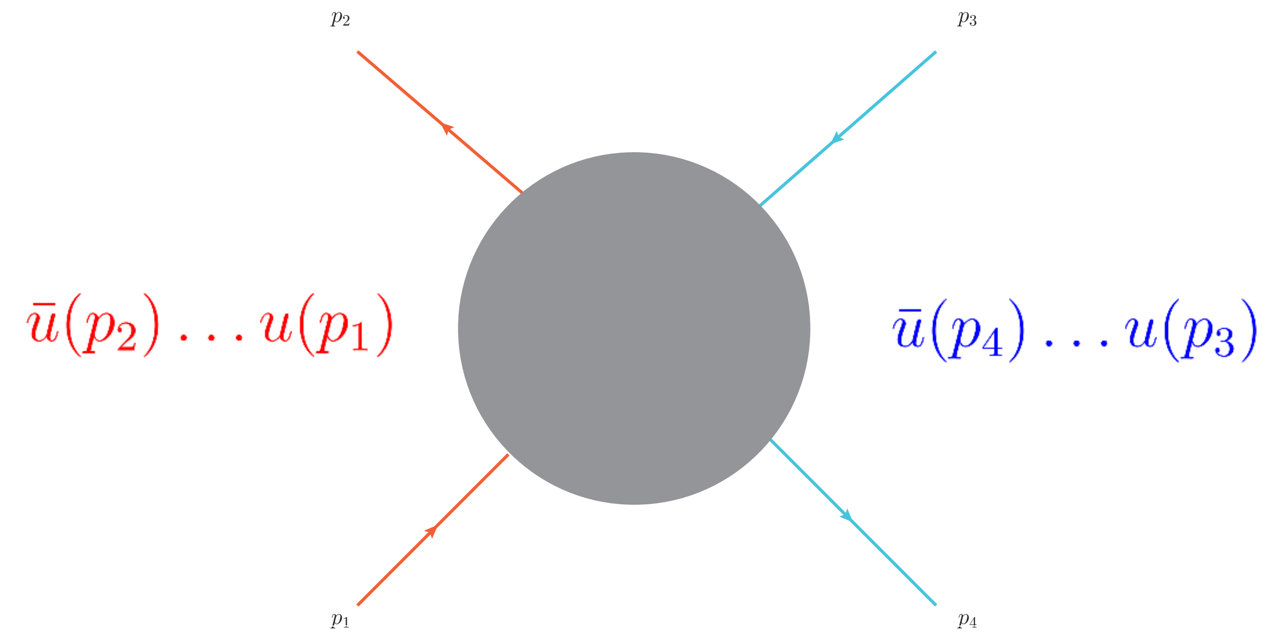

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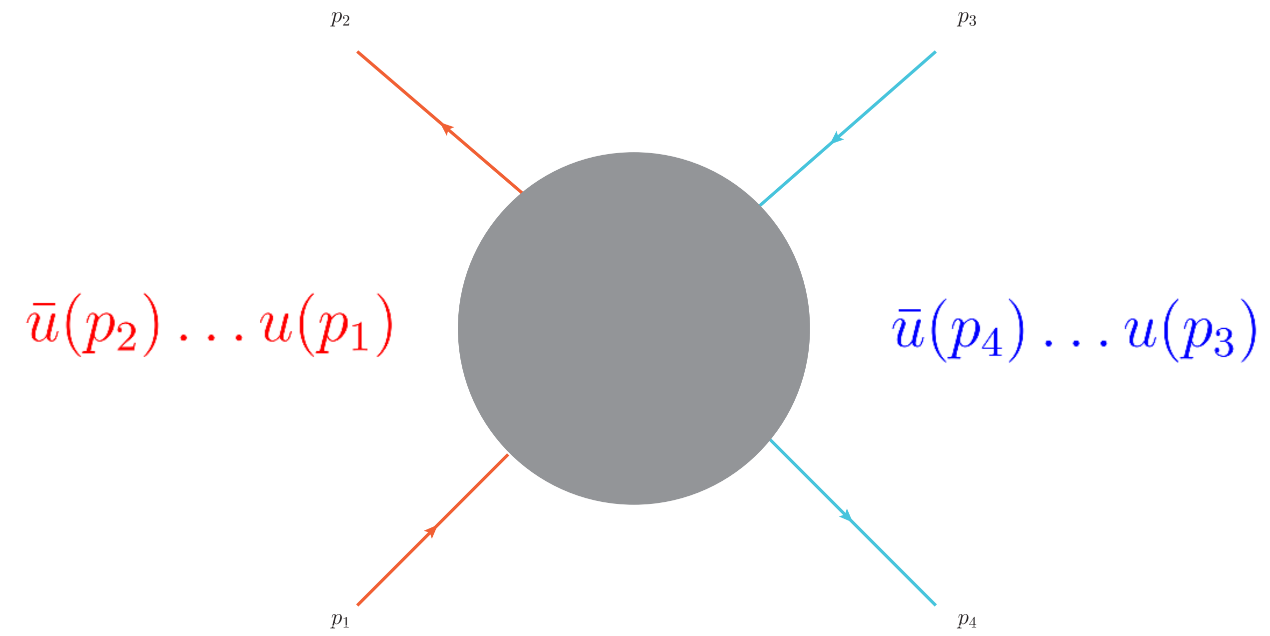

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L. Tancredi, T. Peraro: arXiv:1906.03298, arXiv:2012.00820

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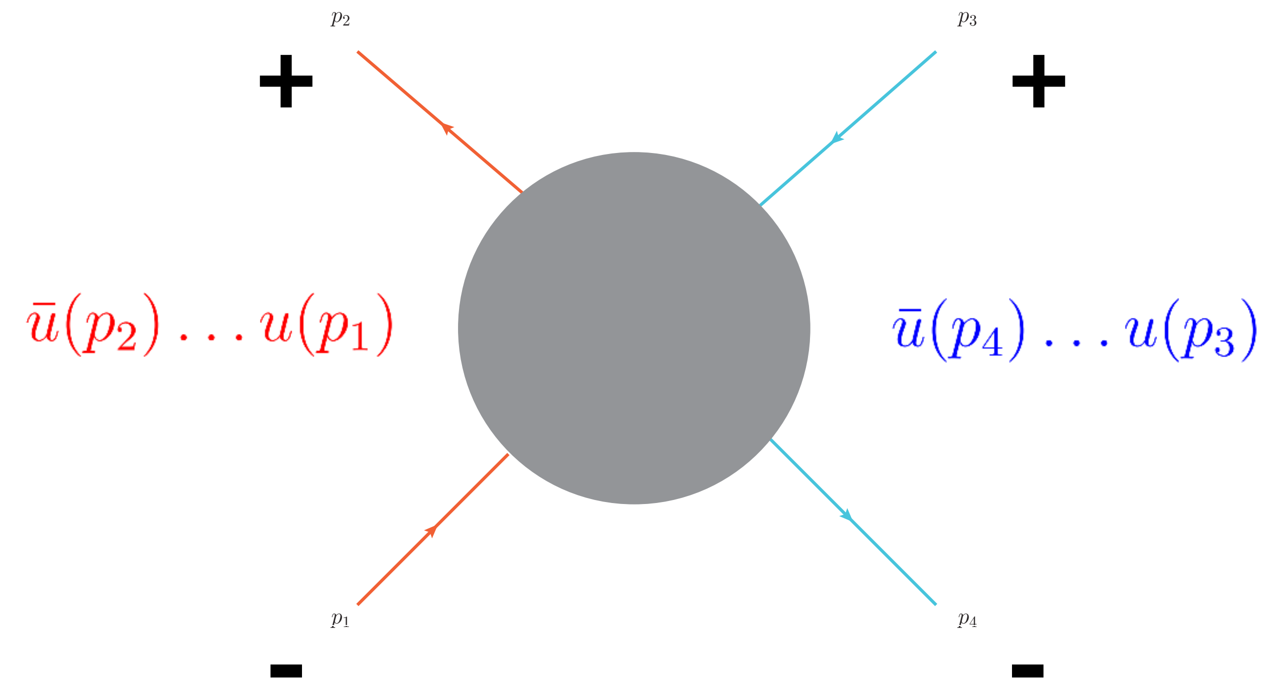

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L. Tancredi, T. Peraro: arXiv:1906.03298, arXiv:2012.00820

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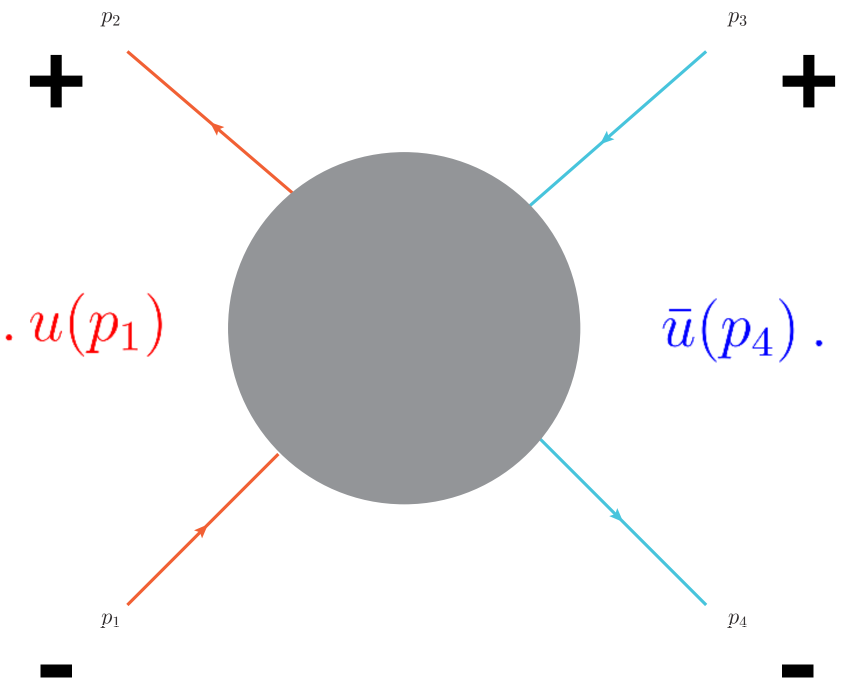

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Orthogonal
&
zero in d=4 !!

L. Tancredi, T. Peraro: arXiv:1906.03298, arXiv:2012.00820

qQ → qQ

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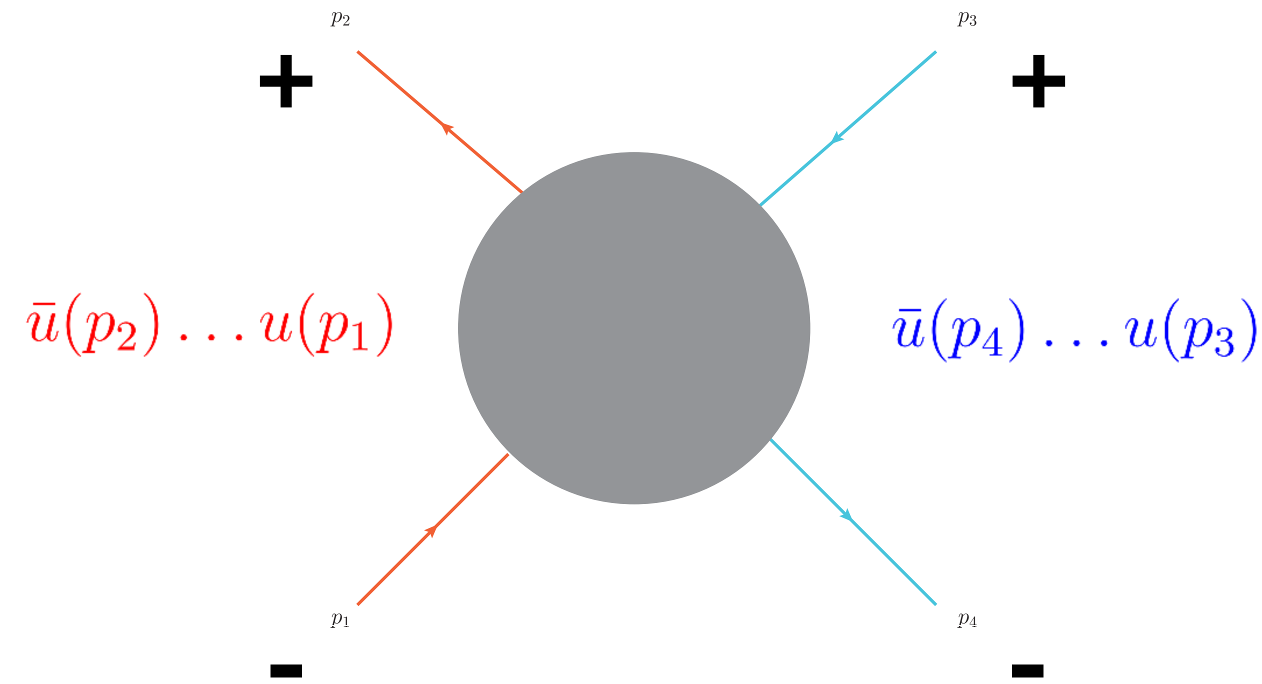

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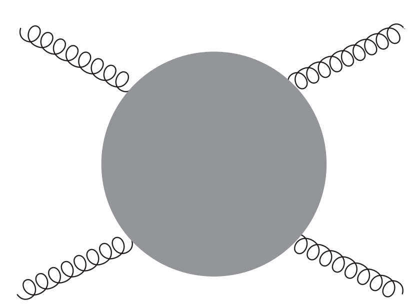
Orthogonal
&
zero in d=4 !!



From 138 to 8 tensors!

L. Tancredi, T. Peraro: arXiv:1906.03298, arXiv:2012.00820