Two-loop mixed QCD-electroweak amplitudes for Z+jet production

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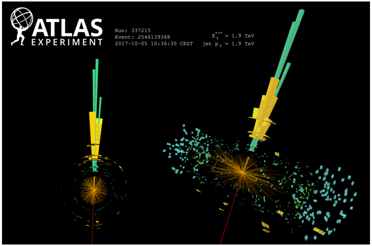
Presentation plan

Motivation

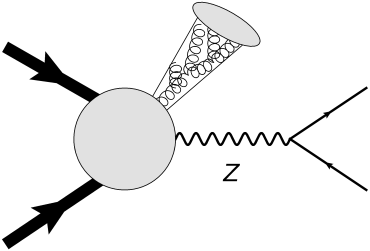
Dark Matter searches at the LHC

[*Lindert et al.* *arXiv:1705.04664*]

experiment
monojet + **missing** transverse energy



theory
$p\,p \to \text{jet} + Z\,(\to \nu\bar{\nu}/\chi\bar{\chi})$



Introduction

- statistical uncertainty : few % for $p_T \in (200, 2000)$ GeV at $\sqrt{s_H} = 13$ TeV
- systematic improvement : perturbative corrections

$$\sigma = \sigma^0 \left(1 + \alpha_s\, \delta^{(1,0)} + \alpha_s^2\, \delta^{(2,0)} + \alpha\, \delta^{(0,1)} + \alpha_s\, \alpha\, \delta^{\mathbf{(1,1)}} + \mathcal{O}(\alpha_s^2, \alpha^2)\right)$$

- Sudakov enhancement : $\alpha \sim 1\% \quad \longrightarrow \quad \frac{\alpha}{4\pi s_w^2} \log^2\left(\frac{s}{m_Z^2}\right) \sim 10\% \sim \alpha_s$

  $\Rightarrow$ **mixed QCD−EWK** corrections important : $\delta^{\mathbf{(1,1)}} \sim \delta^{(2,0)} \sim$ few %
- lower order corrections :
  - NLO QCD [*Giele et al. arXiv:9302225*]
  - NLO EWK [*Denner et al. arXiv:1103.0914*]
  - NNLO QCD [*Gehrmann-De Ridder arXiv:1507.02850*]

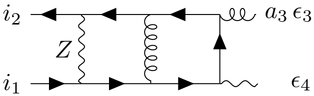
  (t ↓)
- cross section ∼ **scattering amplitude** ⊗ subtraction scheme
- on-shell Z approximation :

$$\mathcal{M}(p\,p \to Z(\to \nu\bar{\nu}) + \text{jet}) \approx \mathcal{A}_\mu(p\,p \to Z\,j)\frac{1}{s - m_Z^2 + i\Gamma_Z m_Z}\mathcal{L}^\mu(Z \to \nu\bar{\nu})$$

- for now : $n_f = 0$, no top

Amplitude structure

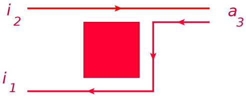
the process $u(p_1) + \bar{u}(p_2) \to g(-p_3) + Z(-p_4)$

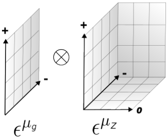


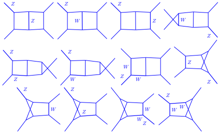
$$= -\frac{i}{2\sqrt{2}}\, g_s^3\, e^3\, g_{L/R}^3\, \frac{1}{N_c}\, T^{a_3}_{i_1 i_2} \int \frac{d^d k_1}{(2\pi)^d}\, \frac{d^d k_2}{(2\pi)^d}$$

$$\times \frac{\bar{u}_2 \gamma_\mu (\not{k}_1 + \not{p}_2)\gamma_\nu (\not{k}_2 + \not{p}_3)\not{\epsilon}_3(\not{k}_2)\not{\epsilon}_4(\not{k}_2 + \not{p}_{123})\gamma^\nu(\not{k}_1 - \not{p}_1)\gamma^\mu u_1}{((k_1)^2 - m_Z^2)(k_2)^2(k_1-p_1)^2(k_1+p_2)^2(k_2+p_3)^2(k_2+p_{123})^2(k_{12}+p_{23})^2}$$

color tensors integrals

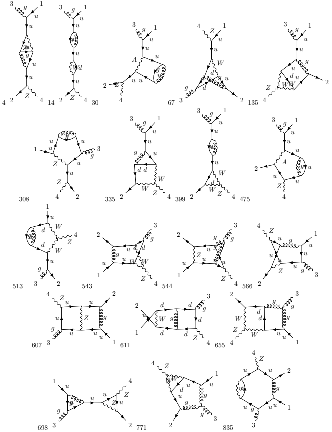






$$\mathcal{A}^{\vec{a},\vec{\lambda}} = \sum_{c,t,i} \mathcal{C}_c^{\vec{a}}\, T_t^{\vec{\lambda}}\, \mathcal{I}_i\, r_{c,t,i}$$

Complexity



× 47 pages

factoring out an overall color singlet structure

↓

projecting all the Lorentz tensor structures onto the 6 physical ones

↓

numerically evaluating all the required 60968 scalar Feynman integrals to 42 digits

↓

evaluating the amplitude in all channels on a 40×41 kinematic **grid**

↓

performing the nontrivial EWK renormalization

computational flow

(QCD,EWK) order	(0,0)	(1,0)	(0,1)	(1,1)
Number of diagrams	2	13	35	900
Number of integral topologies	0	1	4	18
Number of scalar integrals	0	105	275	60968
Number of master integrals	0	7	26	1202
Size of the Feynman diagrams list [kB]	1	6	17	595
Size before IBP reduction [kB]	1	288	1180	422636
Size of the numerical result on the grid [kB]	908	9684	9592	3784

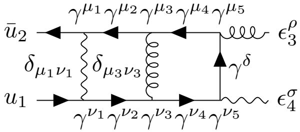
complexity summary

Tensors in d=4−2ϵ dimensions

for further steps, scalar integrals required

$$\int \frac{d^dk_1}{(2\pi)^d}\frac{d^dk_2}{(2\pi)^d}\,\frac{\bar{u}_2\gamma_\mu(\not{k}_1+\not{p}_2)\gamma_\nu(\not{k}_2+\not{p}_3)\not{\epsilon}_3(\not{k}_2)\not{\epsilon}_4(\not{k}_2+\not{p}_{123})\gamma^\nu(\not{k}_1-\not{p}_1)\gamma^\mu u_1}{\mathcal{D}_1\ldots\mathcal{D}_7} = \sum_{i=1}^{?}\mathcal{F}_i T_i$$

for example, consider vector current



Σ(diagrams) # Lorentz indices > # all invariant structures

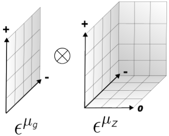
$\#T_i = 39$ (Lorentz invariant tensors) − 22 (by Dirac $\bar{u}_2\not{p}_2 = 0 = \not{p}_1 u_1$)
− 4 (by transversality $\epsilon_3\cdot p_3 = 0$)
− 6 (by gauge fixing $\epsilon_i \cdot p_{i-1} = 0$)
= 7 (independent in d dimensions)
= ? (independent in 4 dimensions)

$$T_i = \epsilon_{3,\mu}(p_3)\,\epsilon_{4,\nu}(p_4)\,\bar{u}(p_2)\quad \big(p_1^\nu\gamma^\mu\,,\, p_1^\mu p_1^\nu \not{p}_3\,,\; p_2^\nu\gamma^\mu\,,\; p_1^\mu\gamma^\nu\,,\; p_1^\mu p_2^\nu\not{p}_3\,,\; g^{\mu\nu}\not{p}_3\,,\; \gamma^\mu\not{p}_3\gamma^\nu\big)\quad u(p_1)$$

Tensors in 4 dimensions

recent loop-univeral **claim** in the 'tHV scheme [*Peraro, Tancredi arXiv:2012.00820*] :
tensors indpt in 4-dim = # indpt helicity states (here $= 3 \times 2^2/2 = 6$)

$$A = \sum_{i=1}^{7} \mathcal{F}_i T_i = \sum_{i=1}^{6} \overline{\mathcal{F}}_i \overline{T}_i$$



orthogonalization : projects out $\overline{T}_7$ from the **physical** 4-dim subspace

$$\sum_{pol} \overline{T}_i^\dagger \overline{T}_j = \left(\begin{array}{c|c} 6\times 6 \text{ (4-dim)} & 0 \\ \hline 0 & 1\times 1 \ (-2\epsilon\text{-dim}) \end{array} \right)$$

gain : 1-1 **correspondence** between form factors and helicity amplitudes

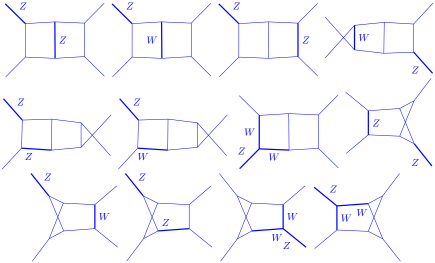
$$\overline{\mathcal{F}}_i \Longleftrightarrow A_{\vec{\lambda}_i}$$

$\Rightarrow$ unphysical information removed

Feynman integrals

$$A_{2,\overline{T}_i} = \sum_{f\in\text{fam}} \sum_{\vec{n}\in\text{int}} c_{f,\vec{n}}(d, m_k, s_{ij}) \int \frac{d^d k_1}{(2\pi)^d} \frac{d^d k_2}{(2\pi)^d} \frac{1}{\mathcal{D}_{f,1}^{n_1} \dots \mathcal{D}_{f,9}^{n_9}}$$

example integral **families**

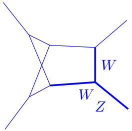


- multiple **scales** : $\{s_{23}, s_{13}, m_Z, m_W\}$ + ($\{m_t, m_H\}$ with top)
- usual approach : Integration By Parts reduction ($6 \times 10^4 \to 1 \times 10^3$)

$$\mathcal{I} = \sum_n \text{rat}(d, m_k, s_{ij})_n \, \text{MI}(d, m_k, s_{ij})_n$$

IBP ineffective

- IBP with `kira-2.2` [*Klappert et al. arXiv:2008.06494*]
- most involved topology : 2-loop non-planar with the W^+W^-Z vertex
- number of integrals to reduce : 1181 , $(\text{ISP})^4$
- number of master integrals : 95



$$\text{amplitude}_{(\text{NPL,WWZ})} = \text{integrand (5.4MB) /. IBPs (640MB) = simpler ?}$$

physical pole motivation ⇒ **partial fraction** coefficients of Master Integrals

- algebraic geometry ⇒ **Groebner basis**
- denominators $\mathcal{P}(d, m_k, s_{ij})$: #=131, deg≤37, #terms≤20282, ≤9-digit coeffs
- `Singular` ineffective

⇓⇓⇓

either choose better Master basis [*Bonetti et al. arXiv:2203.17202*]

see [*talk by Lorenzo, William, Gabriele*]

or evaluate integrals numerically without fully analytic IBPs our strategy :

numerical because 2 → 2 process easy to grid for phenomenology

& final goal = internal top

Numerical strategy

$$m_Z = m_{Z,\text{PDG}} = 91.1876\,\text{GeV}$$

$$m_W^2 = \frac{7}{9} m_Z^2 = (80.4199)^2\,\text{GeV}^2$$

$$\epsilon_n = 10^{-7} + n \times 10^{-9}\,, \quad n = 1, \ldots, 10$$

fix m_Z, m_W, and ϵ numerically

↓

perform the IBP reduction analytically in u and t with `LiteRed`

↓

derive the DEQs in u and t for the MIs with `LiteRed`

↓

evaluate MIs to 120 digits, and so all the required 60968 Feynman integrals to 42 digits with `AMFlow`

↓

evaluate the amplitude in all channels on a 40×41 grid

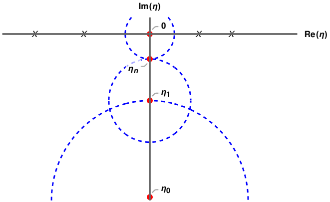
$$p_{T,Z,n} = 200 \cdot 10^{n/39}\,, \qquad n \in [0, 39] \cap \mathbb{Z}$$

$$y_{Z,m} = \frac{m}{4} - 5\,, \qquad m \in [0, 40] \cap \mathbb{Z}$$

Auxiliary Mass Flow method

numerical evaluation

- auxiliary mass : $\frac{1}{\mathcal{D}_k + i0^+} \to \frac{1}{\mathcal{D}_k - \eta}$
  [*Xiao Liu et al.* arXiv:1711.09572]
- differential equations : $\frac{\partial}{\partial \eta}\mathcal{I}(\eta) = A(\eta)\,\mathcal{I}(\eta)$
  easy to solve
- boundary conditions at $\eta = -i\infty$:
  Taylor expansion



$$\frac{1}{((l+p)^2 - m^2 - \eta)^\nu} \simeq \frac{1}{(l^2 - \eta)^\nu} \sum_{i=0}^{N} \frac{\Gamma(\nu + i)}{i!\,\Gamma(\nu)} \left(- \frac{2l.p + p^2 - m^2}{l^2 - \eta} \right)^i$$

- iterative strategy : reduction to **vacuum** bubbles
- **analytic continuation** : path $\{-i\infty, \eta_0, \eta_1, ..., \eta_n, -i0^-\}$

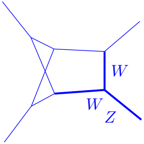
$\Downarrow\Downarrow\Downarrow$

- MIs(η) : #$\leq$4 for 1 prop(η)
- MI precision : 120 digits at $N = 1$ and $n = 20$ with `AMFlow.m`

AMFlow example

example evaluation in the most complicated topology to 50 digits

$$\frac{3.0906596244443939233735868243720876301611376323059 \cdot 10^{-33}}{\epsilon^4}$$
$$- \frac{8.6982149550765304704269203265996575847651084073412 \cdot 10^{-32} + i\,1.9544714863999450257859339734280327013888767054745 \cdot 10^{-32}}{\epsilon^3}$$
$$+ \frac{1.2687966449353006102942224933760780663861340457210 \cdot 10^{-30} + i\,5.6436126481241928121313821894114179219856434323968 \cdot 10^{-31}}{\epsilon^2}$$
$$- \frac{1.2789521457906368621593037090148843633179169961641 \cdot 10^{-29} + i\,8.8196094924668858121129028788908603839793775903 70 \cdot 10^{-30}}{\epsilon}$$
$$+ 9.9835670078122847622686695600404906739206066410 26 \cdot 10^{-29} + i\,9.7580362662760833952300753588731871224505062624 48 \cdot 10^{-29} =$$



$$\text{at} \qquad \frac{u}{m_Z^2} = -\frac{4212389009}{875622495}, \qquad \frac{t}{m_Z^2} = -\frac{185568373013477}{1751244990}$$

- IBPs(u,t,#) : degree=28 rational coefficient functions
- large cancellations : 50 digits ⇒ 120 digits enough to avoid them

UV and IR structure

$$A_b^{(2)} = \frac{c_{4,\mathrm{IR}}}{\epsilon^4} + \frac{c_{3,\mathrm{IR}}}{\epsilon^3} + \frac{c_{2,\mathrm{IR}} + c_{2,\mathrm{UV}}}{\epsilon^2} + \frac{c_{1,\mathrm{IR}} + c_{1,\mathrm{UV}}}{\epsilon^1} + \mathbf{fin}$$

UV

QCD : $\overline{\mathrm{MS}}$ scheme

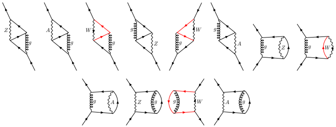
$$A^{(1,QCD)} = (1 + g_s^2 \beta_0)\, A_b^{(1,QCD)}$$

EWK : on-shell G_μ scheme [*Denner arXiv:0709.1075*]

$$A^{(1,EWK)} = (1 + e^2 \delta_Z^{(1)})\, A_b^{(1,EWK)}$$

mixed : [*Buccioni et al. arXiv:2203.11237*]

$$A = (1 + g_s^2 \beta_0 + e^2 \delta_Z^{(1)} + e^2 g_s^2 \delta_Z^{(2)})\, A_b$$



IR

$$A^{(2)} = \mathcal{I}_2\, A^{(0)} + \mathcal{I}_{1,QCD}\, A^{(1,EWK,\mathrm{fin})} + \mathcal{I}_{1,EWK}\, A^{(1,QCD,\mathrm{fin})} + A^{(2,\mathrm{fin})}$$

Fixed kinematics result

$$
\begin{aligned}
A_L^{(1,1,\text{fin})} &= \overline{T}_1 \left(-1.67762319126822917 \times 10^{-4} - \left(8.9124020261485948 \times 10^{-5}\right) i\right) \\
&+ \overline{T}_2 \left(-6.14083995697083227 \times 10^{-10} - \left(1.15078349742596179 \times 10^{-10}\right) i\right) \\
&+ \overline{T}_3 \left(4.9873142274505437 + 2.6245263772911926 i\right) \\
&+ \overline{T}_4 \left(-4.3253096831118915 - 2.4559371163486791 i\right) \\
&+ \overline{T}_5 \left(2.361437106860579067 \times 10^{-7} + \left(5.30188993175405567 \times 10^{-8}\right) i\right) \\
&+ \overline{T}_6 \left(4.6415462269520851 + 2.5128183816883992 i\right) , \\
A_R^{(1,1,\text{fin})} &= \overline{T}_1 \left(-1.9707761016083767 \times 10^{-5} + \left(1.0965436511377151 \times 10^{-5}\right) i\right) \\
&+ \overline{T}_2 \left(1.65144181401856366 \times 10^{-11} + \left(2.08626789202156372 \times 10^{-11}\right) i\right) \\
&+ \overline{T}_3 \left(6.1391040692819743 \times 10^{-1} - \left(4.1040250855245174 \times 10^{-1}\right) i\right) \\
&+ \overline{T}_4 \left(-5.9029051802906398 \times 10^{-1} + \left(4.0662570124472741 \times 10^{-1}\right) i\right) \\
&+ \overline{T}_5 \left(5.4350465607404134 \times 10^{-9} - \left(6.527583146744599 \times 10^{-10}\right) i\right) \\
&+ \overline{T}_6 \left(5.8134142630821290 \times 10^{-1} - \left(4.1777025512005486 \times 10^{-1}\right) i\right) ,
\end{aligned}
$$

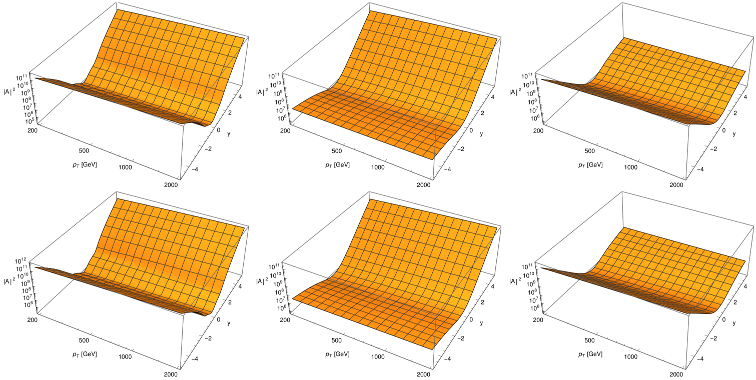
at kinematic point

$$
s_{12} = \frac{200343109174296505501}{188240625000} , \qquad s_{23} = -\frac{2206428746331193800294949}{2073235183593750} ,
$$

$$
s_{13} = -\frac{663464134484282958883}{165858814687500000} , \qquad \mu^2 = s_{12} ,
$$

Full kinematics result

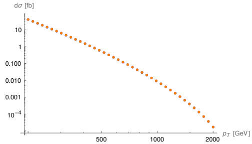
$$\left|2\Re\left(\mathcal{A}_L^{(0,0,\text{fin})*}\mathcal{A}_L^{(1,1,\text{fin})}+\mathcal{A}_R^{(0,0,\text{fin})*}\mathcal{A}_R^{(1,1,\text{fin})}\right)\right|$$

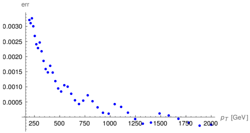


$$u\,\bar{u}\to g\,Z,\; u\,g\to u\,Z,\; g\,u\to u\,Z$$
$$d\,\bar{d}\to g\,Z,\; d\,g\to d\,Z,\; g\,d\to d\,Z$$

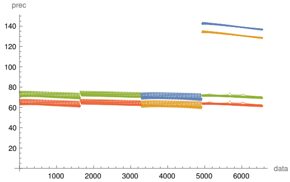
Checks

- grid : hadronic $\frac{d\sigma}{dp_{T,j}}$ VS MFCM6.8 [*Campbell et al. arXiv:1503.06182*]





- 1-loop results : $2\Re(\mathcal{A}^{0*}\mathcal{A}^{1,\text{fin}})$ VS OpenLoops [*Buccioni et al. arXiv:1907.13071*]
  $\geq$12 digits in all gridded channels
- 2-loop framework : QCD-QCD VS analytic [*Gehrmann et al. arXiv:2211.13596*]
  16 digits at fixed kinematic point
- 2-loop poles

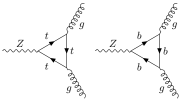


$u\bar{u} \to gZ$:

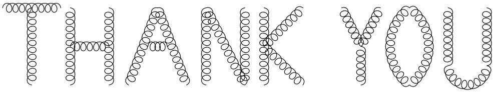
$\mathcal{A}^{(0,0,\text{fin})*}\mathcal{A}^{(1,0,\text{fin})}$, $\mathcal{A}^{(0,0,\text{fin})*}\mathcal{A}^{(0,1,\text{fin})}$,
$\mathcal{A}^{(0,0,\text{fin})*}\mathcal{A}^{(1,1,\text{fin})}$, $\mathcal{A}^{(1,0,\text{fin})*}\mathcal{A}^{(0,1,\text{fin})}$

Outlook

- discussed methods generalizable for the **top** loop
- careful treatment of anomalies



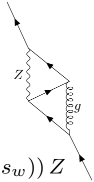
- **hadronic** cross section



Details of the G_μ scheme

- input parameters : $\{G_\mu, m_Z, m_W\}$
- $\alpha(0)\,(1+\Delta r) = \frac{\sqrt{2}G_\mu m_W^2}{\pi}\left(1-\frac{m_W^2}{m_Z^2}\right)$
- amplitude renormalization

$$A(g_{s,0}, g_{L/R,0}(e_0, c_{w,0}, s_{w,0}))\sqrt{Z_u Z_{\bar{u}} Z_g Z_Z} = A(g_s, g_{L/R}(e, c_w, s_w))\,Z$$



$$Z = 1 + \delta_Z = 1 + \frac{1}{2}(\delta_{Z_g} + \delta_{Z_u} + \delta_{Z_{\bar{u}}} + \delta_{Z_{ZZ}} - \frac{Q_f}{g_{L/R}}\delta_{Z_{AZ}}) + \delta_{g_{L/R}} + \delta_{g_s}$$

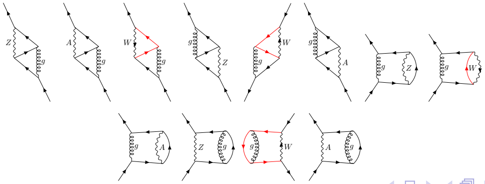
all SM particles contribute ⇒ much more involved then $\overline{\text{MS}}$

UV renormalization

$\overline{\text{MS}}$ QCD $\otimes$ G_μ EWK

$$\mathcal{A} = \sqrt{\frac{\alpha_s}{2\pi}}\sqrt{\frac{\alpha}{2\pi}}\,T^{a_3}_{i_1 i_2}\left(A^{(0,0)} + \frac{\alpha_s}{2\pi}A^{(1,0)} + \frac{\alpha}{2\pi}A^{(0,1)} + \frac{\alpha_s}{2\pi}\frac{\alpha}{2\pi}A^{(1,1)} + \mathcal{O}\left(\alpha_s^2, \alpha^2\right)\right)$$

$$\begin{aligned}
A^{(0,0)} &= S_\epsilon^{-1/2} A_b^{(0,0)}\,, \\
A^{(1,0)} &= S_\epsilon^{-1/2}\left(S_\epsilon^{-1}\mu^{2\epsilon}A_b^{(1,0)} - \frac{\beta_0}{2\epsilon}A_b^{(0,0)}\right)\,, \\
A^{(0,1)} &= S_\epsilon^{-1/2}\left(\mu^{2\epsilon}A_b^{(0,1)} + \delta_{\mathcal{A}}^{(0,1)}A_b^{(0,0)}\right)\,, \\
A^{(1,1)} &= S_\epsilon^{-1/2}\left(S_\epsilon^{-1}\mu^{4\epsilon}A_b^{(1,1)} - \frac{\beta_0}{2\epsilon}\mu^{2\epsilon}A_b^{(0,1)} + \delta_{\mathcal{A}}^{(0,1)}S_\epsilon^{-1}\mu^{2\epsilon}A_b^{(1,0)}\right) \\
&\quad + S_\epsilon^{-1/2}\left(S_\epsilon^{-1}\delta_{\mathcal{A}}^{(1,1)} - \frac{\beta_0}{2\epsilon}\delta_{\mathcal{A}}^{(0,1)}\right)A_b^{(0,0)}\,, \qquad \text{at} \qquad \mu = \mu_0 = \mu_R
\end{aligned}$$



IR regularization

Catani operators

$$
\begin{aligned}
\frac{2\Gamma(1-\epsilon)}{e^{\gamma_E \epsilon}} \mathcal{I}_{1,0}(\epsilon) &= \left(\frac{\mu^2}{-s-i\varepsilon}\right)^{\epsilon} (C_A - 2C_F) \left(\frac{1}{\epsilon^2} + \frac{3}{2\epsilon}\right) \\
&\quad - \left(\left(\frac{\mu^2}{-u-i\varepsilon}\right)^{\epsilon} + \left(\frac{\mu^2}{-t-i\varepsilon}\right)^{\epsilon}\right) \left(C_A \left(\frac{1}{\epsilon^2} + \frac{3}{4\epsilon}\right) + \frac{\beta_0}{2\epsilon}\right), \\
-\frac{\Gamma(1-\epsilon)}{e^{\gamma_E \epsilon}} \mathcal{I}_{0,1}(\epsilon) &= \left(\frac{\mu^2}{-s-i\varepsilon}\right)^{\epsilon} S_\epsilon Q_{up}^2 \left(\frac{1}{\epsilon^2} + \frac{3}{2\epsilon}\right), \\
\mathcal{I}_{1,1}(\epsilon) - \mathcal{I}_{1,0}(\epsilon)\,\mathcal{I}_{0,1}(\epsilon) &= \frac{e^{\gamma_E \epsilon}}{\Gamma(1-\epsilon)} \left(\frac{\mu^2}{-s-i\varepsilon}\right)^{2\epsilon} \frac{1}{\epsilon} S_\epsilon C_F Q_{up}^2 \left(\frac{\pi^2}{2} - 6\zeta_3 - \frac{3}{8}\right),
\end{aligned}
$$

finite part of the amplitude

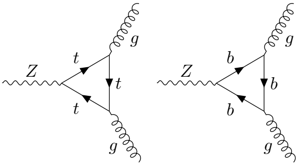
$$
\begin{aligned}
A^{(0,0)} &= A^{(0,0,\text{fin})}, \\
A^{(1,0)} &= \mathcal{I}_{1,0}\, A^{(0,0)} + A^{(1,0,\text{fin})}, \\
A^{(0,1)} &= \mathcal{I}_{0,1}\, A^{(0,0)} + A^{(0,1,\text{fin})}, \\
A^{(1,1)} &= \mathcal{I}_{1,1}\, A^{(0,0)} + \mathcal{I}_{1,0}\, A^{(0,1,\text{fin})} + \mathcal{I}_{0,1}\, A^{(1,0,\text{fin})} + A^{(1,1,\text{fin})}.
\end{aligned}
$$

The top loop

- closed chiral fermion loops
- new tensors $\overline{T}_i$ with $\gamma_\mu\gamma_5$
- Larin's prescription

[Larin arXiv:9302240]

$$\gamma_\mu\gamma_5 = \frac{i}{3!}\epsilon_{\mu\mu_1\mu_2\mu_3}\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}$$



- massive top quark

$\Rightarrow$ "anomaly" noncancellation :

$$\sim f(m_t)\,\mathcal{O}(\epsilon^0)$$