

Full NNLO QCD corrections to diphoton production

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In collaboration with M. Becchetti, R. Bonciani, L. Cieri, F. Ripani

Outline of the talk

Introduction

-  Motivations
-  State of the art

Double Virtual Contribution

-  Form factors
-  Master Integrals
-  Hard Function

Real-Virtual Contribution

Real-Real Contribution

Final Results

Conclusions

Motivations

- ❖ Diphoton is an experimentally clean final state
- ❖ QCD background for Higgs
- ❖ Important to measure the fundamental parameters within the Standard Model
- ❖ Search for new physics

State of the art

- ❖ Full NLO
- ❖ QCD NNLO
- ❖ $\gamma\gamma + jets$
- ❖ Form factors up to 3 loops

State of art

T. Binoth, J. P. Guillet, E. Pilon and M. Werlen, *A Full next-to-leading order study of direct photon pair production in hadronic collisions*, *Eur. Phys. J. C* **16** (2000) 311–330, [[hep-ph/9911340](#)].

Z. Bern, L. J. Dixon and C. Schmidt, *Isolating a light Higgs boson from the diphoton background at the CERN LHC*, *Phys. Rev. D* **66** (2002) 074018, [[hep-ph/0206194](#)].

S. Catani, L. Cieri, D. de Florian, G. Ferrera and M. Grazzini, *Diphoton production at hadron colliders: a fully-differential QCD calculation at NNLO*, *Phys. Rev. Lett.* **108** (2012) 072001, [[1110.2375](#)].

V. Del Duca, F. Maltoni, Z. Nagy and Z. Trocsanyi, *QCD radiative corrections to prompt diphoton production in association with a jet at hadron colliders*, *JHEP* **04** (2003) 059, [[hep-ph/0303012](#)].

Z. Bern, A. De Freitas and L. J. Dixon, *Two loop amplitudes for gluon fusion into two photons*, *JHEP* **0109** (2001) 037 [[hep-ph/0109078](#)].

J. M. Campbell and C. Williams, *Triphoton production at hadron colliders*, *Phys.Rev.* **D89** (2014) 113001 [[1403.2641](#)].

P. Bargiela, A. Chakraborty and G. Gambuti, *Three-loop helicity amplitudes for photon+jet production*, *Phys. Rev. D* **107** (2023) L051502 [[2212.14069](#)].

P. Bargiela, F. Buccioni, F. Caola, F. Devoto, A. von Manteuffel and L. Tancredi, *Signal-background interference effects in Higgs-mediated diphoton production beyond NLO*, *Eur. Phys. J. C* **83** (2023) 174 [[2212.06287](#)].

J. M. Campbell, R. K. Ellis, Y. Li and C. Williams, *Predictions for diphoton production at the LHC through NNLO in QCD*, *JHEP* **07** (2016) 148, [[1603.02663](#)].

S. Frixione, *Isolated photons in perturbative QCD*, *Phys. Lett. B* **429** (1998) 369–374, [[hep-ph/9801442](#)].

M. Becchetti and R. Bonciani, *Two-Loop Master Integrals for the Planar QCD Massive Corrections to Di-photon and Di-jet Hadro-production*, *JHEP* **01** (2018) 048, [[1712.02537](#)].

F. Maltoni, M. K. Mandal and X. Zhao, *Top-quark effects in diphoton production through gluon fusion at next-to-leading order in QCD*, *Phys. Rev. D* **100** (2019) 071501, [[1812.08703](#)].

V. Del Duca, W. B. Kilgore and F. Maltoni, *Multiphoton amplitudes for next-to-leading order QCD*, *Nucl.Phys.* **B566** (2000) 252–274 [[hep-ph/9910253](#)].

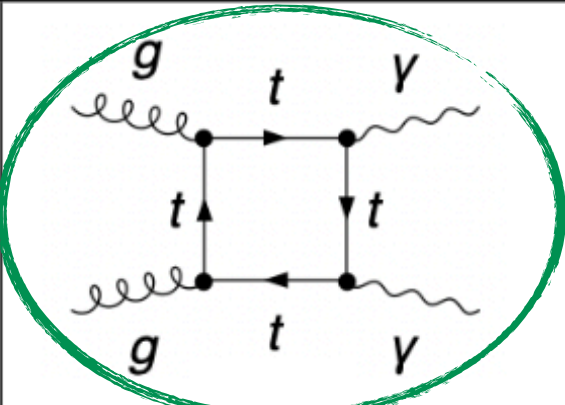

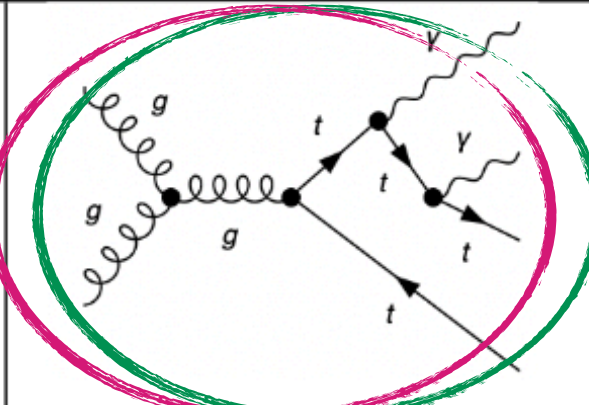
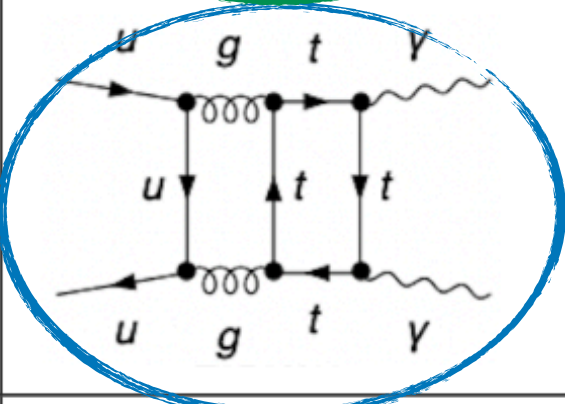
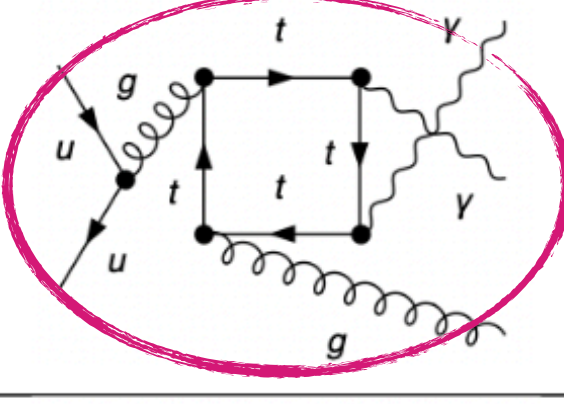
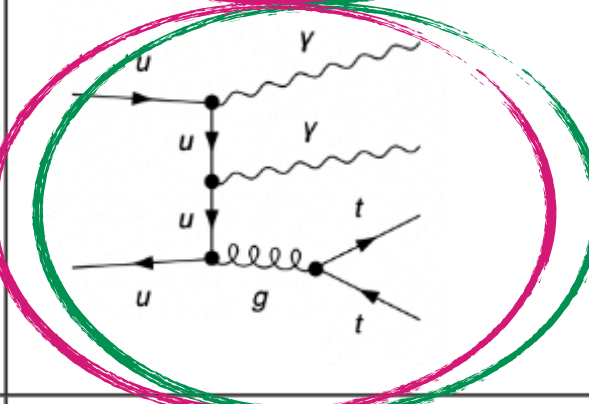

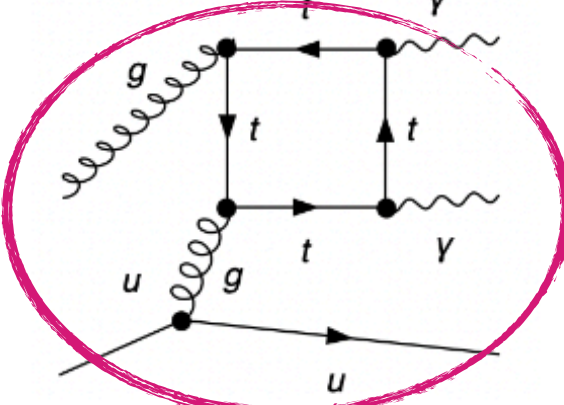

F. Caola, A. Von Manteuffel and L. Tancredi, *Diphoton Amplitudes in Three-Loop Quantum Chromodynamics*, *Phys. Rev. Lett.* **126** (2021) 112004, [[2011.13946](#)].

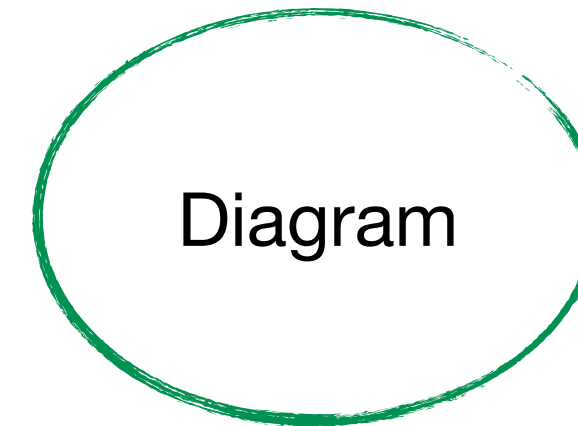
P. Bargiela, F. Caola, A. von Manteuffel and L. Tancredi, *Three-loop helicity amplitudes for diphoton production in gluon fusion*, *JHEP* **02** (2022) 153 [[2111.13595](#)].

H. A. Chawdhry, M. Czakon, A. Mitov and R. Poncelet, *NNLO QCD corrections to diphoton production with an additional jet at the LHC*, *JHEP* **09** (2021) 093 [[2105.06940](#)].

Massive Corrections

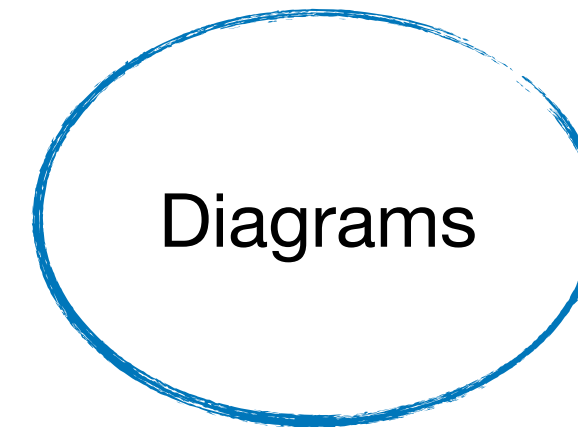
Massive corrections $\mathcal{O}(\alpha_s^2)$

Channels	$\gamma\gamma$	$\gamma\gamma j$	$\gamma\gamma jj$
gg			
$q\bar{q}$			
qg			

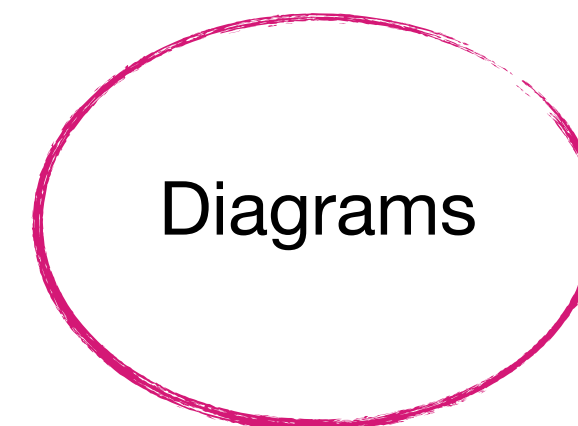


[J.M.Campbell,R.K.Ellis,Y.Li,C.Williams]

[F.Buccioni,J-N.Lang,J.M.Lindert,P.Maierhofer,
S.Pozzorini,H.Zhang,M.Zoller]

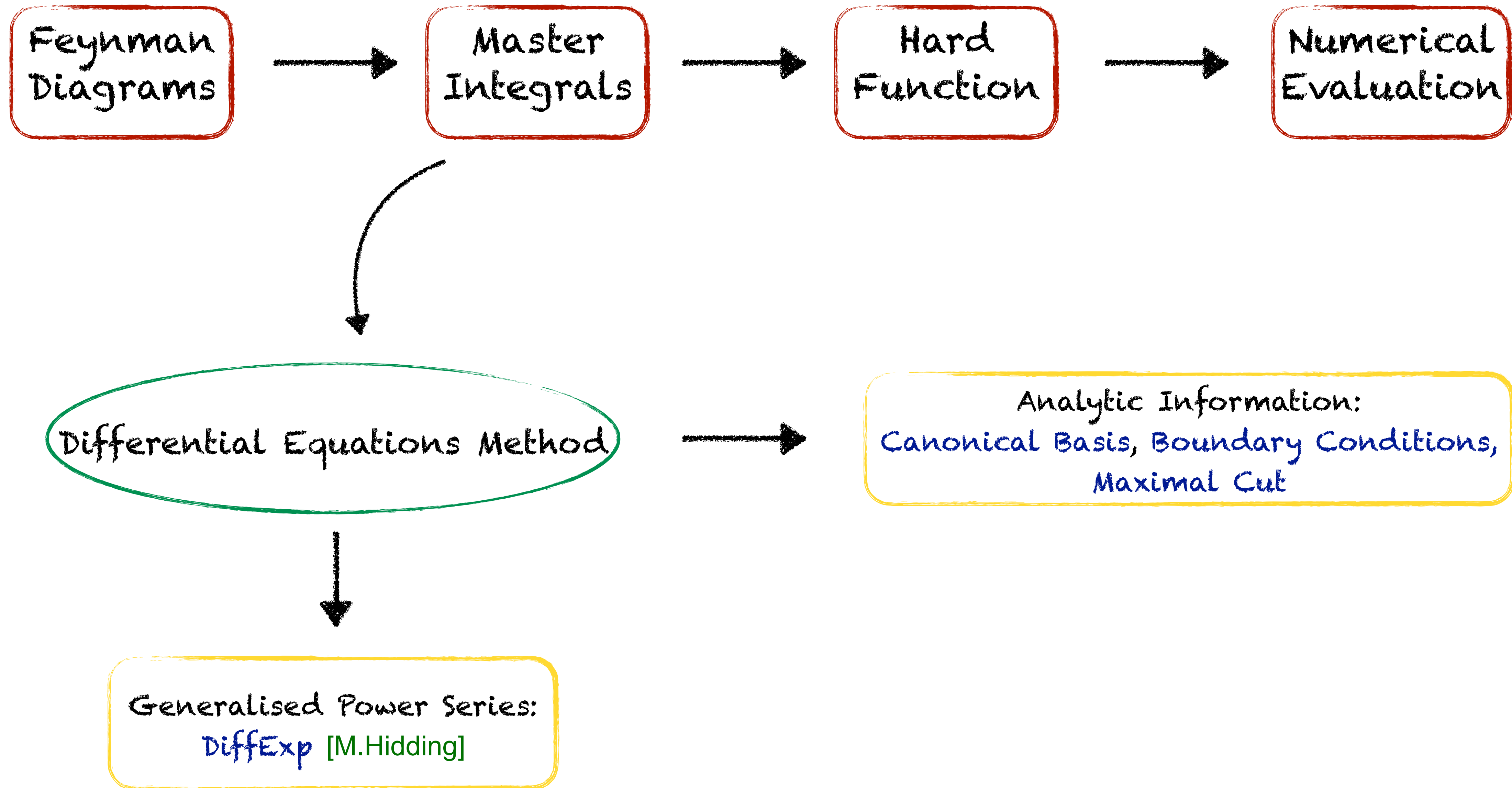


Original results and main focus of the talk



Evaluated for the final result

Computational pipeline



Form factors

At any order in QCD perturbation theory, the amplitude can be decomposed as:

$$\mathcal{A}_{q\bar{q},\gamma\gamma}(s, t, m_t^2) = \sum_{i=1}^4 \mathcal{F}_i(s, t, m_t^2) \bar{v}(p_2) \Gamma_i^{\mu\nu} u(p_1) \epsilon_{3,\mu} \epsilon_{4,\nu}$$

In dimensional regularisation:

$$\Gamma_1^{\mu\nu} = \gamma^\mu p_2^\nu, \quad \Gamma_2^{\mu\nu} = \gamma^\nu p_1^\mu, \quad \Gamma_3^{\mu\nu} = p_{3,\rho} \gamma^\rho p_1^\mu p_2^\nu, \quad \Gamma_4^{\mu\nu} = p_{3,\rho} g^{\mu\nu}$$

[F.Caola,A.Von Manteuffel,L.Tancredi]

The form factors admits a perturbative expansion:

$$\mathcal{F}_i = \mathcal{F}_i^{(0)} + \left(\frac{\alpha_s^B}{\pi}\right) \mathcal{F}_i^{(1)} + \left(\frac{\alpha_s^B}{\pi}\right)^2 \mathcal{F}_i^{(2)} + \dots$$



Massive contribution appears at $\mathcal{O}(\alpha_s^2)$:

$$\mathcal{F}_i^{(2)} = \delta_{kl} C_F (4\pi\alpha_{em}) \left[Q_q^2 \mathcal{F}_{i;0}^{(2)} + Q_t^2 \mathcal{F}_{i;2}^{(2)} \right]$$

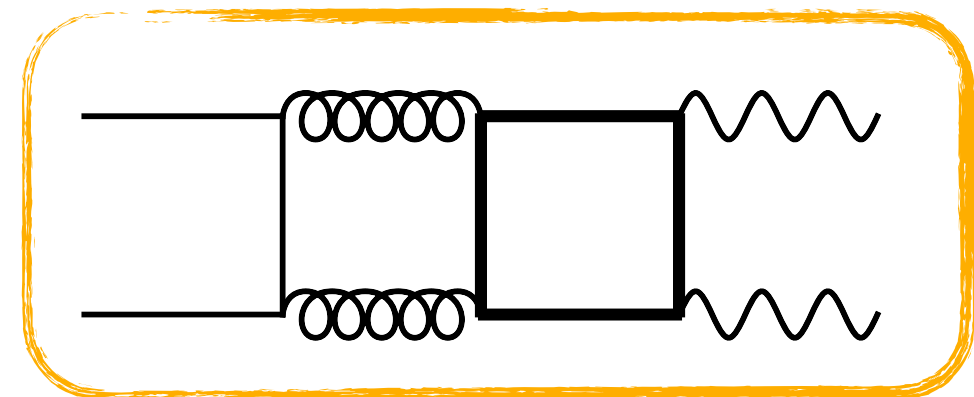
Q_q is the charge of light quark
 Q_t is the charge of heavy quark

Two-loop Feynman diagrams

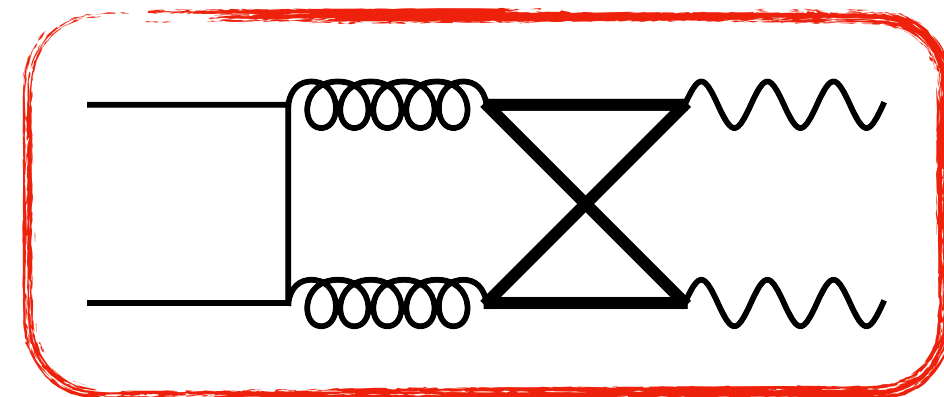
At partonic level the scattering process is: $q(p_1) + \bar{q}(p_2) \rightarrow \gamma(p_3) + \gamma(p_4)$

External particles on-shell and the top quark running in the loop

Feynman diagrams generated with **FeynArts** [T.Hahn]

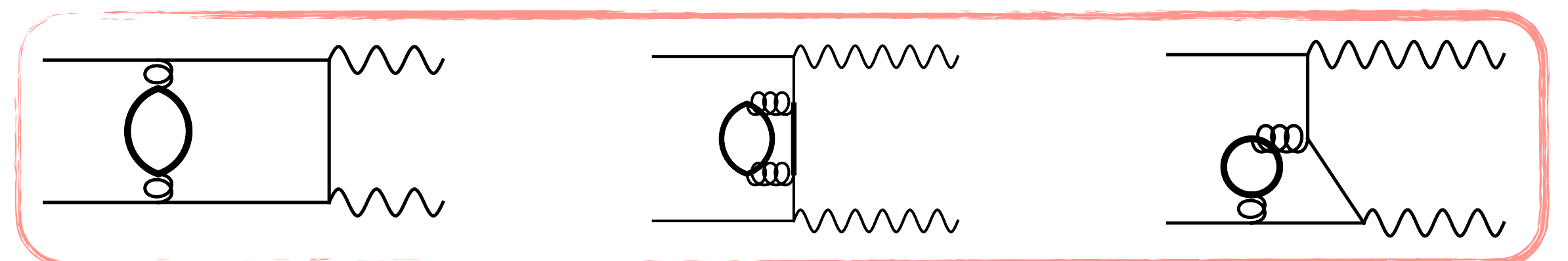


PLA



NPL

The MIs for the top-sector NPL are an original result



PLB

Form factors



Scalar integrals



IBPs reduction



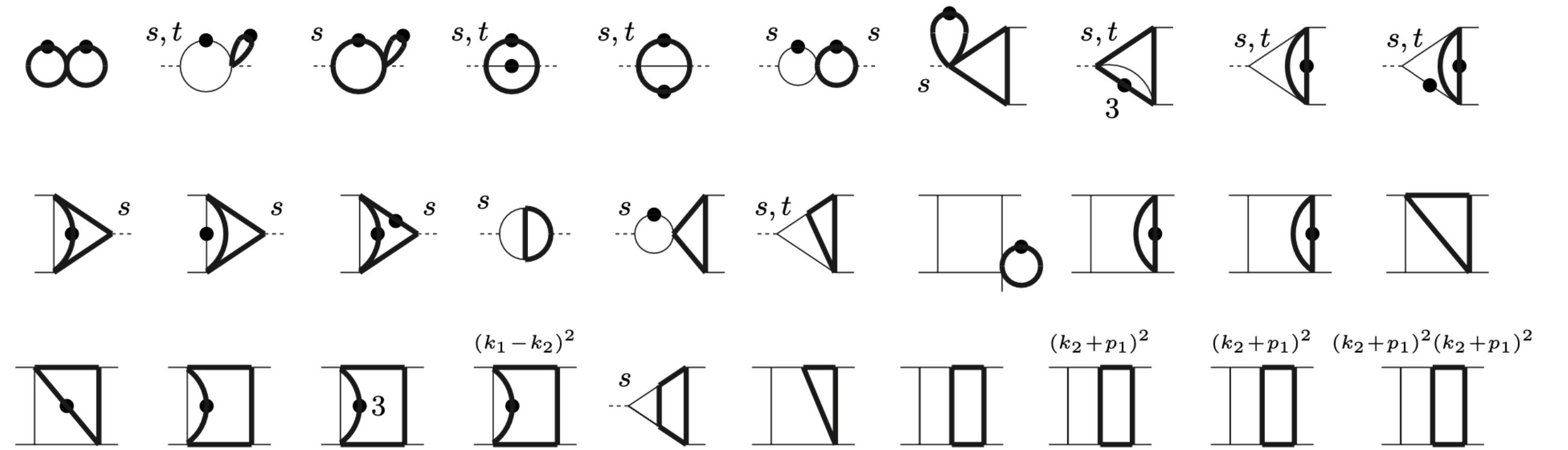
PLA: 32 MIs

NPL: 36 MIs

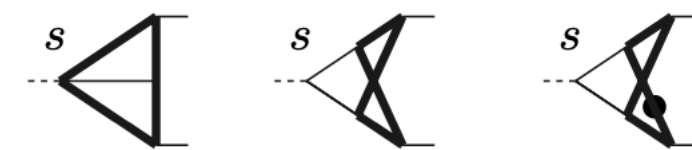
PLB: 10 MIs

Master Integrals

PLA and PLB Master Integrals

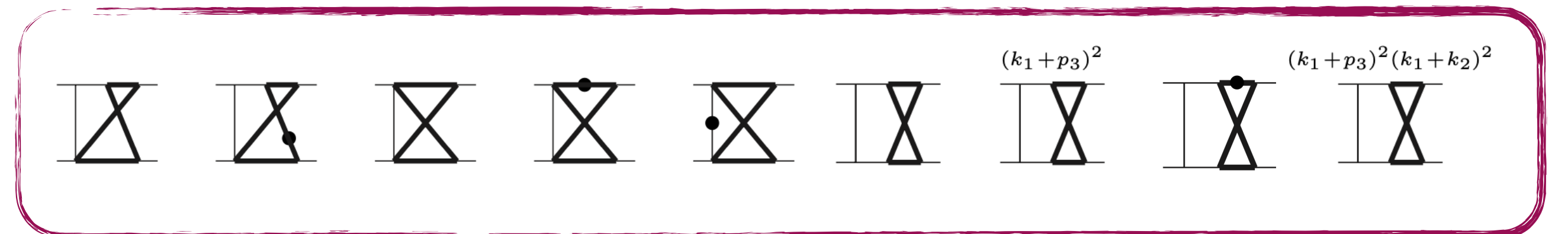


[M.Becchetti,R.Bonciani]



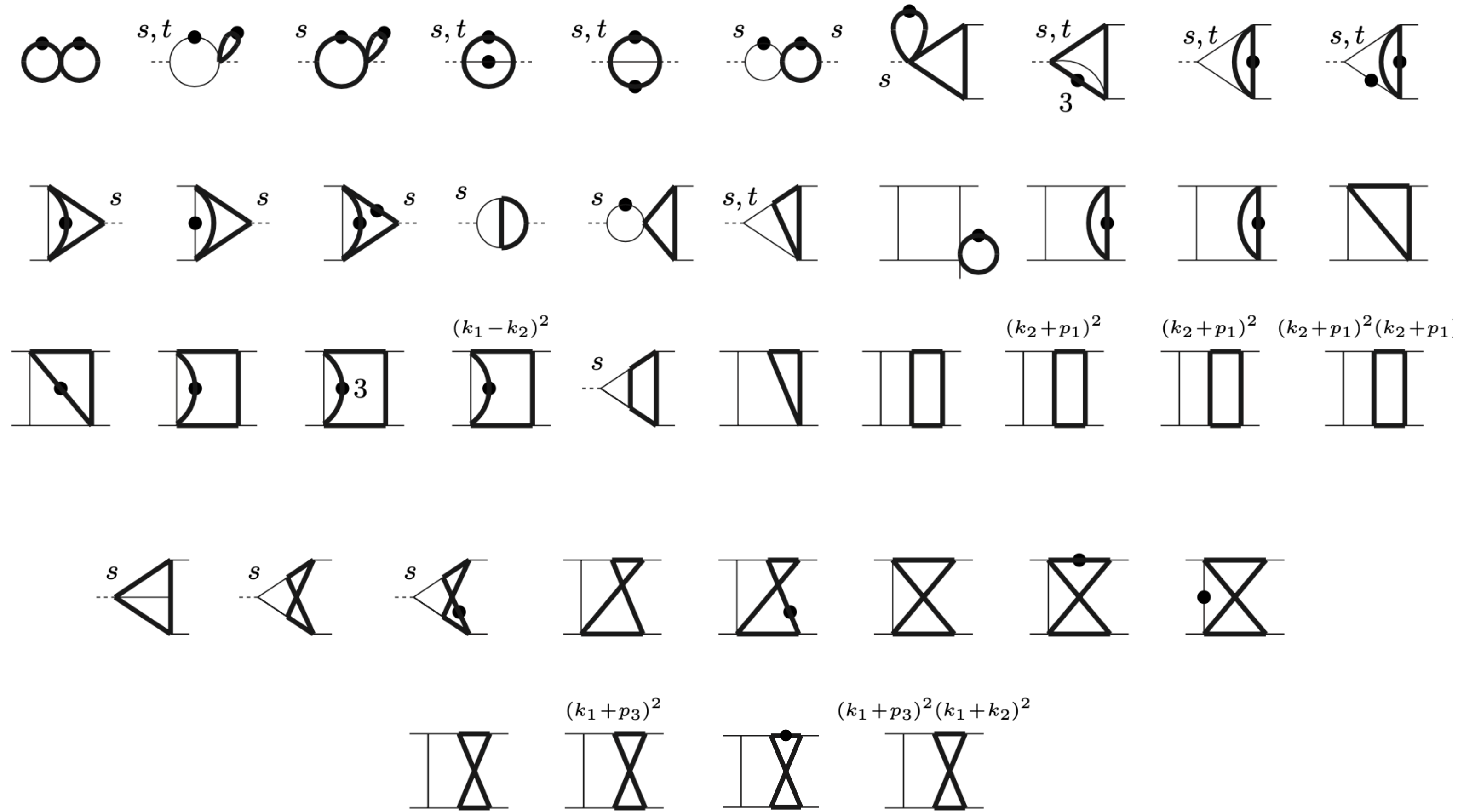
[A.Von Manteuffel,L.Tancredi]

NPL Master Integrals



Original MIs

Master Integrals



Now we have
42 MIs
for all the
process!

Evaluation of the Master Integrals

The MIs are computed through the differential equations method:

PLA family:

$$df(\underline{x}, \epsilon) = \epsilon dA(\underline{x})f(\underline{x}, \epsilon)$$

Canonical logarithmic form!

[J.M.Henn]

with respect to the kinematic invariants: $\underline{x} = \{y, z\}$, $y = \frac{s}{m_t^2}$, $z = \frac{t}{m_t^2}$

Boundary Conditions: $\underline{x} = 0$

Five different square roots in the letters

- ❖ Non linearizable square roots
- ❖ Non trivial solution!
- ❖ Big expressions!

PLB family:

This topology contains only one different MIs from the other two topologies, which was computed analytically

Evaluation of the Master Integrals

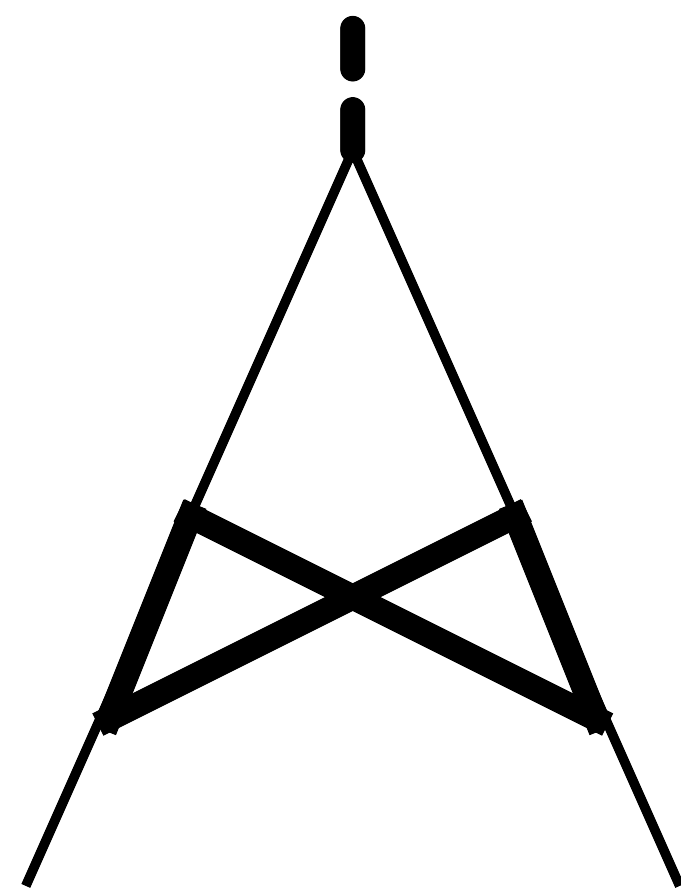
NPL family:

$$df(\underline{x}, \epsilon) = \epsilon dA(\underline{x})f(\underline{x}, \epsilon) + d\tilde{A}(\underline{x}, \epsilon)f(\underline{x}, \epsilon)$$

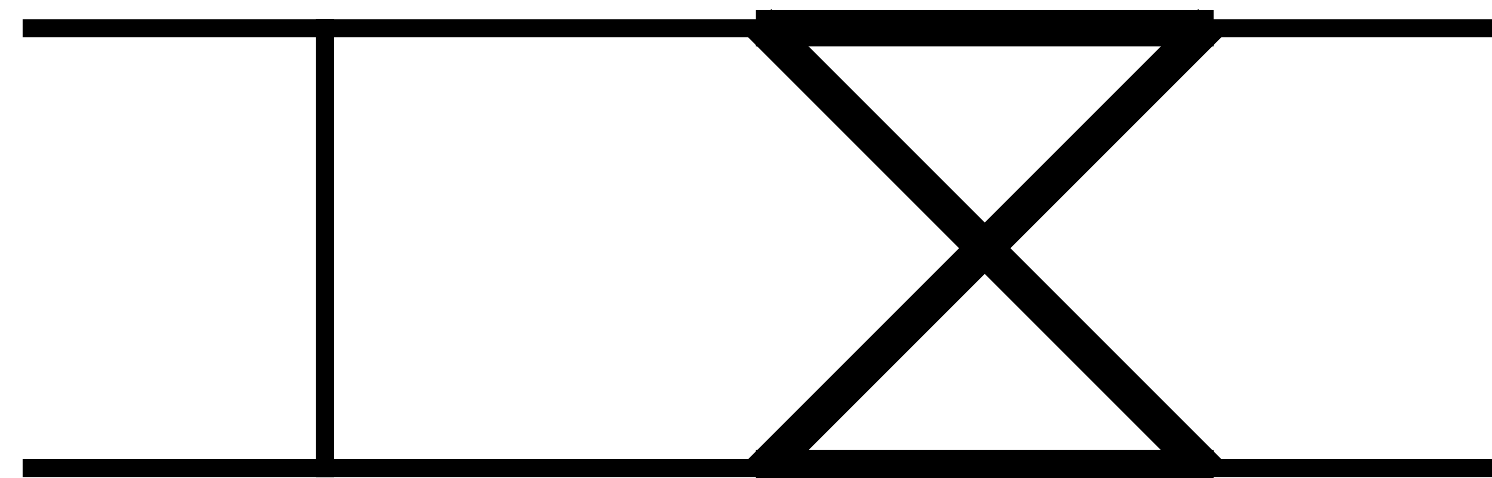
Two different subsets

Canonical
Logarithmic

ELLIPTIC
Sectors



eMPLS

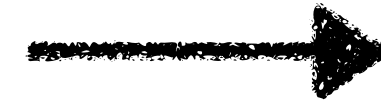


- ❖ Non trivial solution!
- ❖ Nine square roots in the alphabet
- ❖ Integrals involving eMPLs kernels

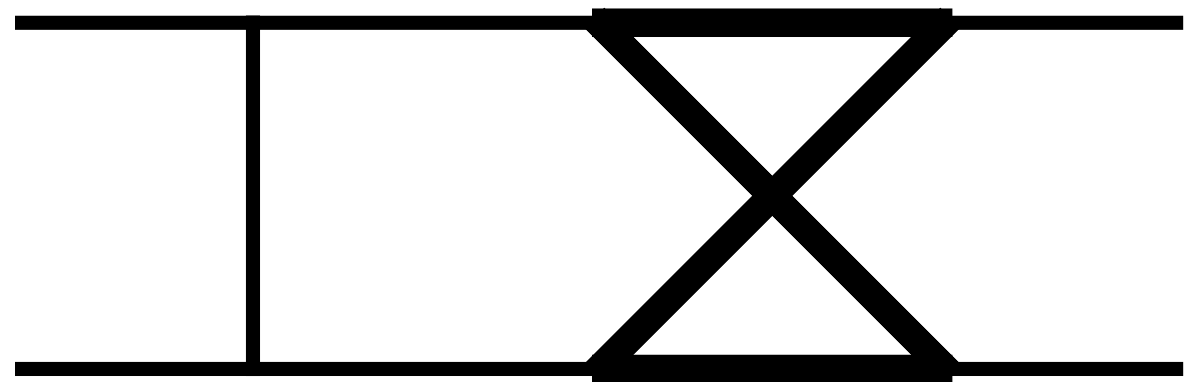
[A.Von Manteuffel, L.Tancredi]

Maximal Cut

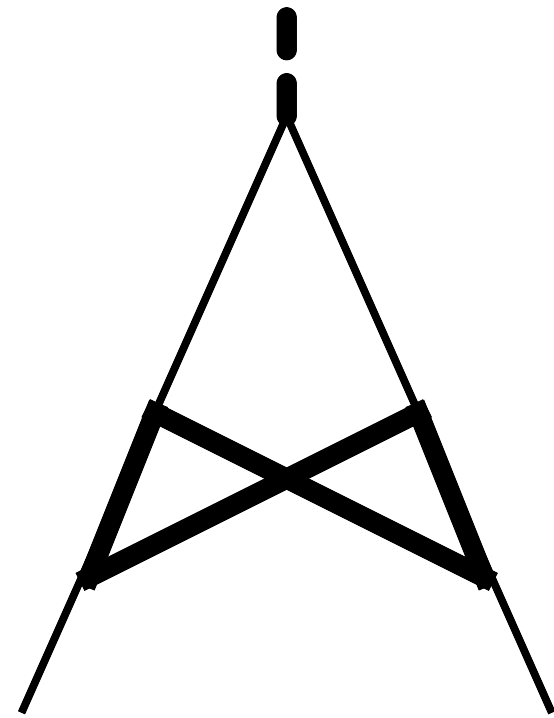
The homogeneous part of the DEs contains elliptic functions



This is verified by the **Maximal Cut**



$$y_c^2 = (z_8 + t)(z_8 + s + t)(z_8 - z_+)(z_8 - z_-)$$



$$y^2 = \bar{x}_2(\bar{x}_2 - 1)(\bar{x}_2 - b_+)(\bar{x}_2 - b_-)$$

[J. Broedel, C. Duhr, F. Dulat, B. Penante, L. Tancredi]

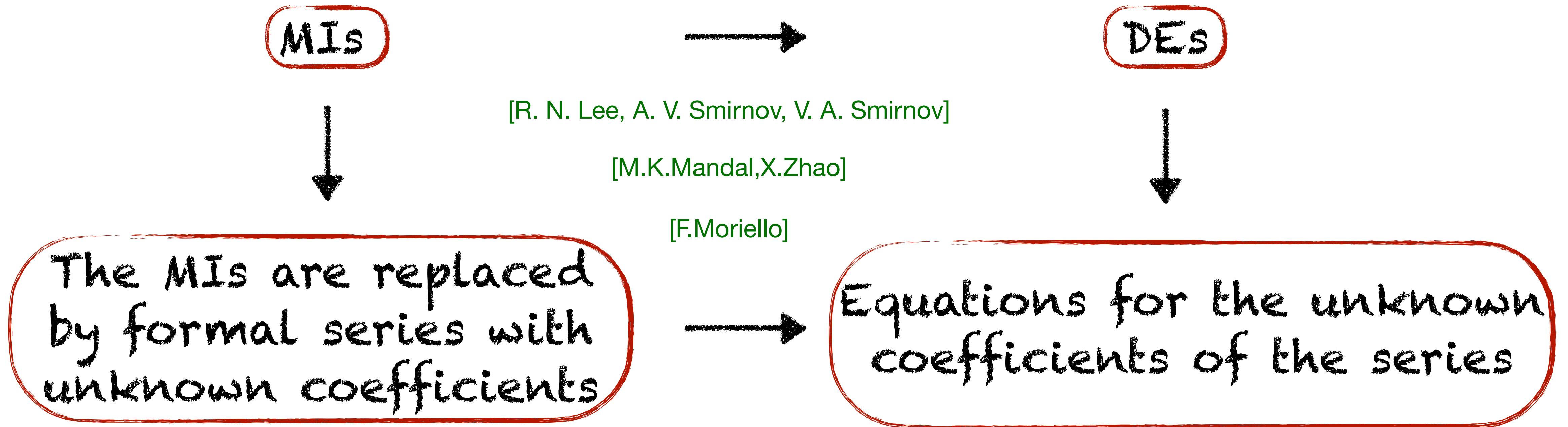
$$z_{\pm} = \frac{1}{2} \left(-s - 2t \pm \sqrt{s} \sqrt{s + 16m_t^2} \right), \quad b_{\pm} = \frac{1}{2} (1 \pm \sqrt{1 - 16a}), \quad a = \frac{m^2}{-(p_1 + p_2)^2}$$



Genus one

The elliptic curve y_c^2 degenerates to y in the forward limit $t = 0$

Generalised power series approach



Pros:

- ❖ It doesn't depend on the function space, so it allows us to avoid elliptic integrals
- ❖ Values at arbitrary phase-space points
- ❖ Can be used to perform phenomenological studies

Numerical evaluation of the Master Integrals

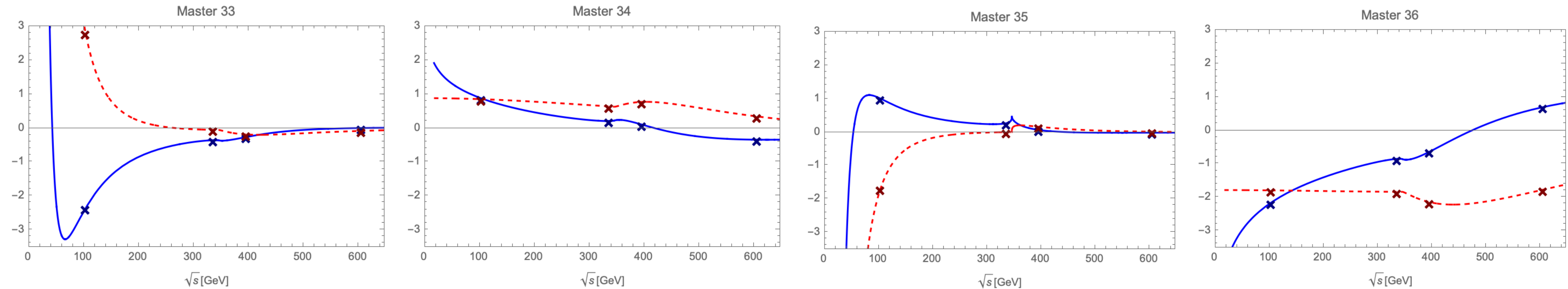
The numerical evaluation of the Master Integrals has been made with DiffExp [M.Hidding]

Several check for the numerical evaluation with AMFLow [X.Liu,Y.Ma]

Correspondance
up to 50 digits!

----- Imaginary Part
————— Real Part

For the four elliptic boxes in NPL Topology, at a fixed angle:



Renormalisation

$\mathcal{F}_i^{(2)}$ does not have IR poles!

We remove the UV poles by adopting a mixed renormalisation scheme:

- ❖ \overline{MS} for α_s
- ❖ On-shell for the external fields

Renormalised form factor: $\overline{\mathcal{F}}_i = Z_q \mathcal{F}_i(\alpha_s^B \rightarrow \alpha_s^R)$ Perturbative expansion \longrightarrow $\overline{\mathcal{F}}_i = \overline{\mathcal{F}}_i^{(0)} + \left(\frac{\alpha_s}{\pi}\right) \overline{\mathcal{F}}_i^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \overline{\mathcal{F}}_i^{(2)} + \dots$

Then, for the two-loop:

$$\overline{\mathcal{F}}_i^{(2)} = C_F N_h \left(\frac{1}{32\epsilon} - \frac{5}{192} \right) \mathcal{F}_i^{(0)} + \frac{N_h}{6\epsilon} \mathcal{F}_i^{(1)} + \mathcal{F}_i^{(2)}$$

Hard Function

In q_T - subtraction scheme:

$$d\sigma_{NNLO}^{\gamma\gamma} = \mathcal{H}_{NNLO}^{\gamma\gamma} \otimes d\sigma_{LO}^{\gamma\gamma} + [d\sigma_{NLO}^{\gamma\gamma+jets} - d\sigma_{NLO}^{CT}]$$

Contains
our massive
contribution

LO cross
section

NLO cross
section for
 $\gamma\gamma + jet$

CT needed to
cancel the IR
singularities

The Hard function admit a perturbative expansion:

$$\mathcal{H}^{\gamma\gamma} = 1 + \frac{\alpha_S}{\pi} \mathcal{H}_{NLO}^{\gamma\gamma} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}_{NNLO}^{\gamma\gamma} + \dots$$

Numerical evaluation of the Hard Function

A numerical grid has been prepared for all the MIs of the PLA and NPL, covering the $2 \rightarrow 2$ physical space:

$$s > 0, \quad t = -\frac{s}{2}(1 - \cos(\theta)), \quad -s < t < 0$$

$-0.99 < \cos(\theta) < +0.99$ 24 different values

$8 \text{ GeV} < \sqrt{s} < 2.2 \text{ TeV}$ 573 different values

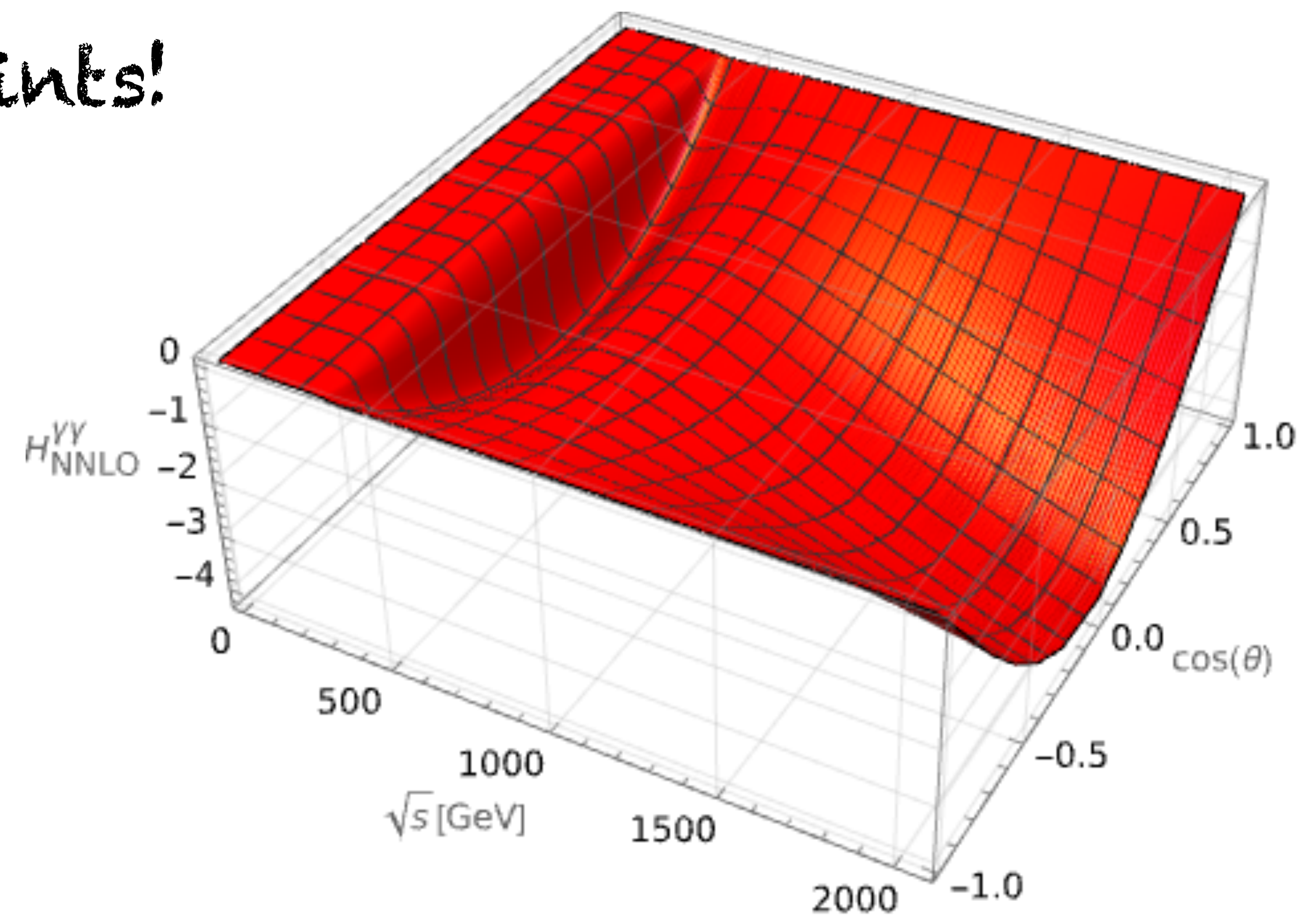
13752 points!

DiffExp time for the $H_{NNLO}^{\gamma\gamma}$ MIs evaluation:

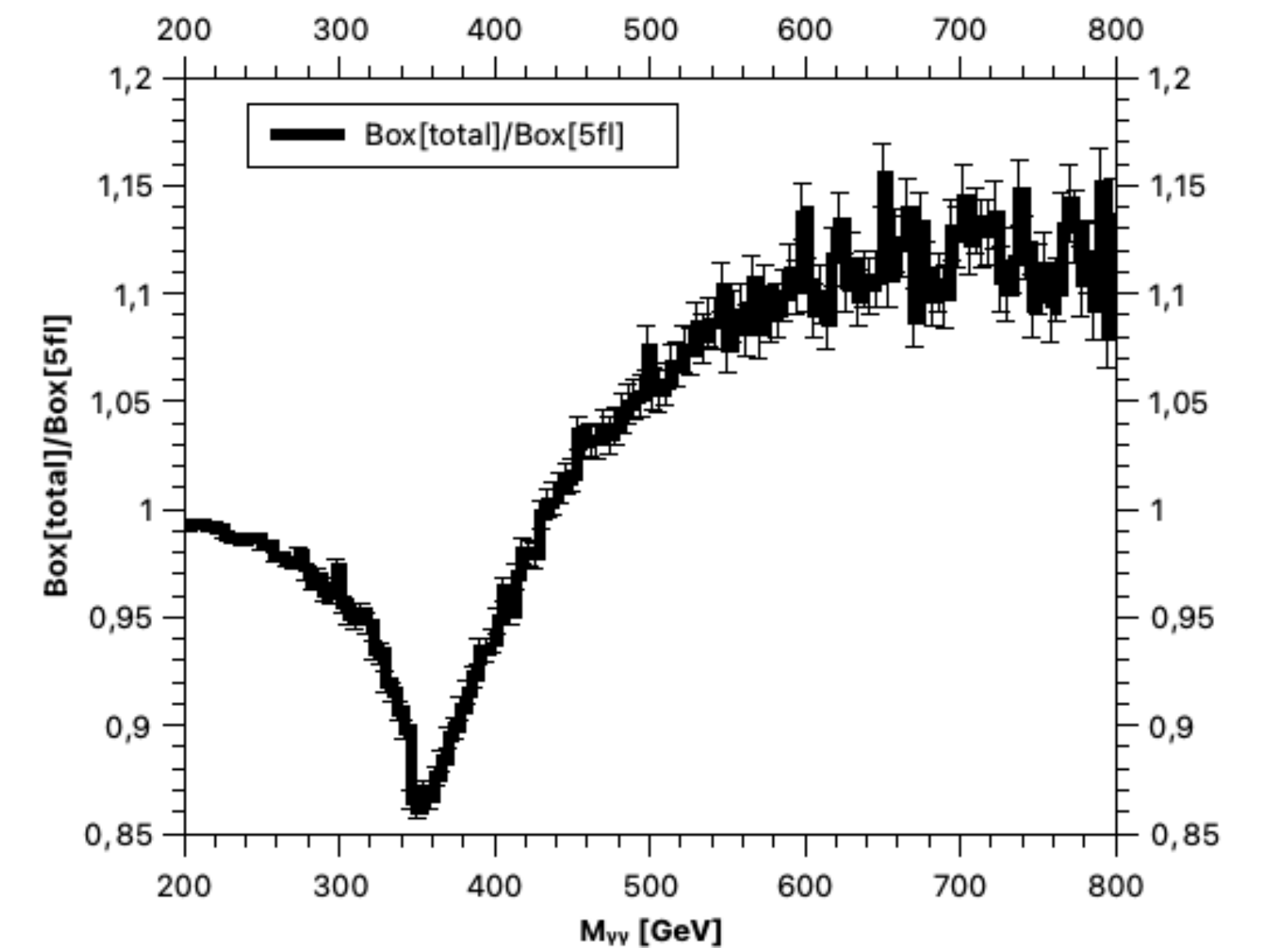
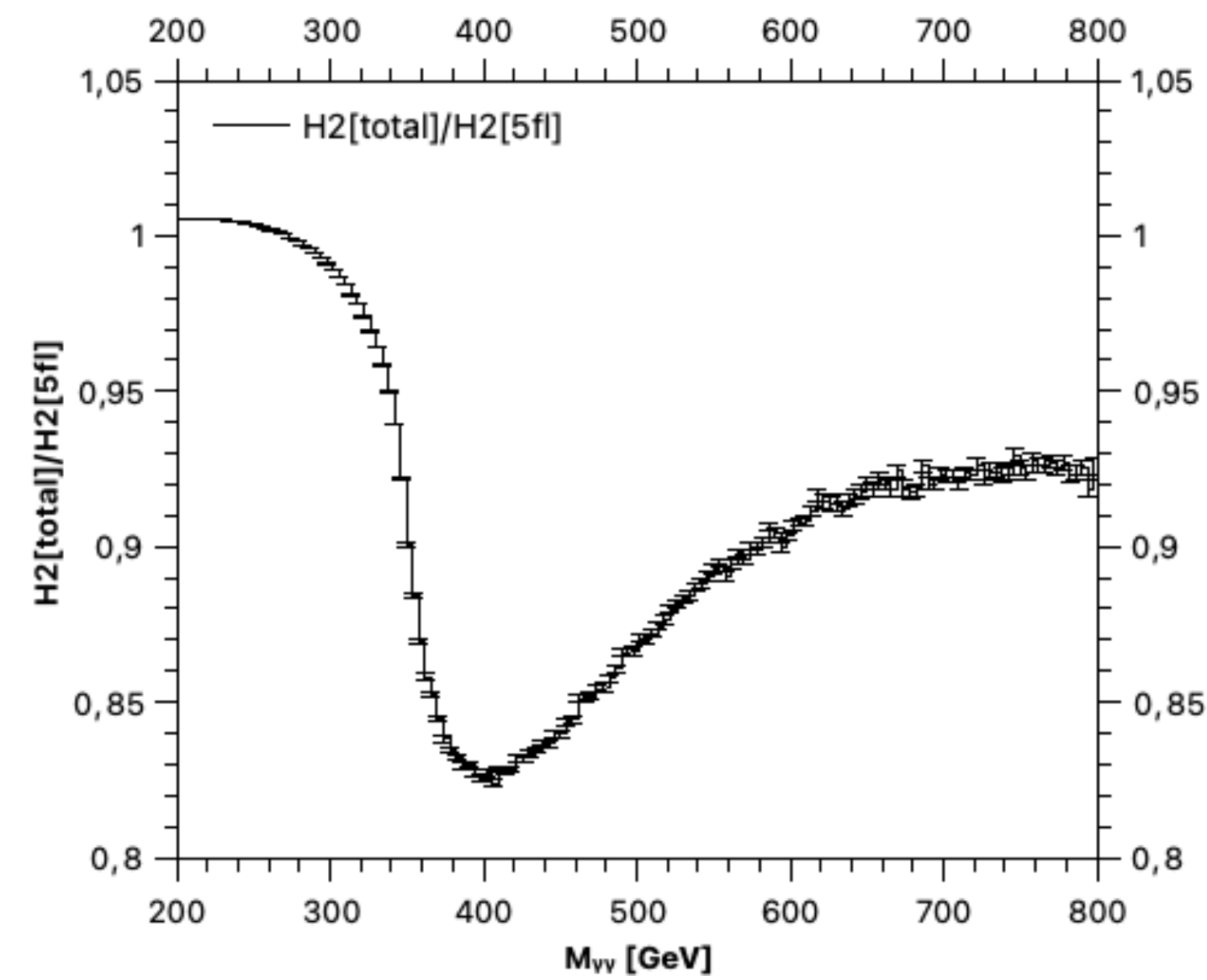
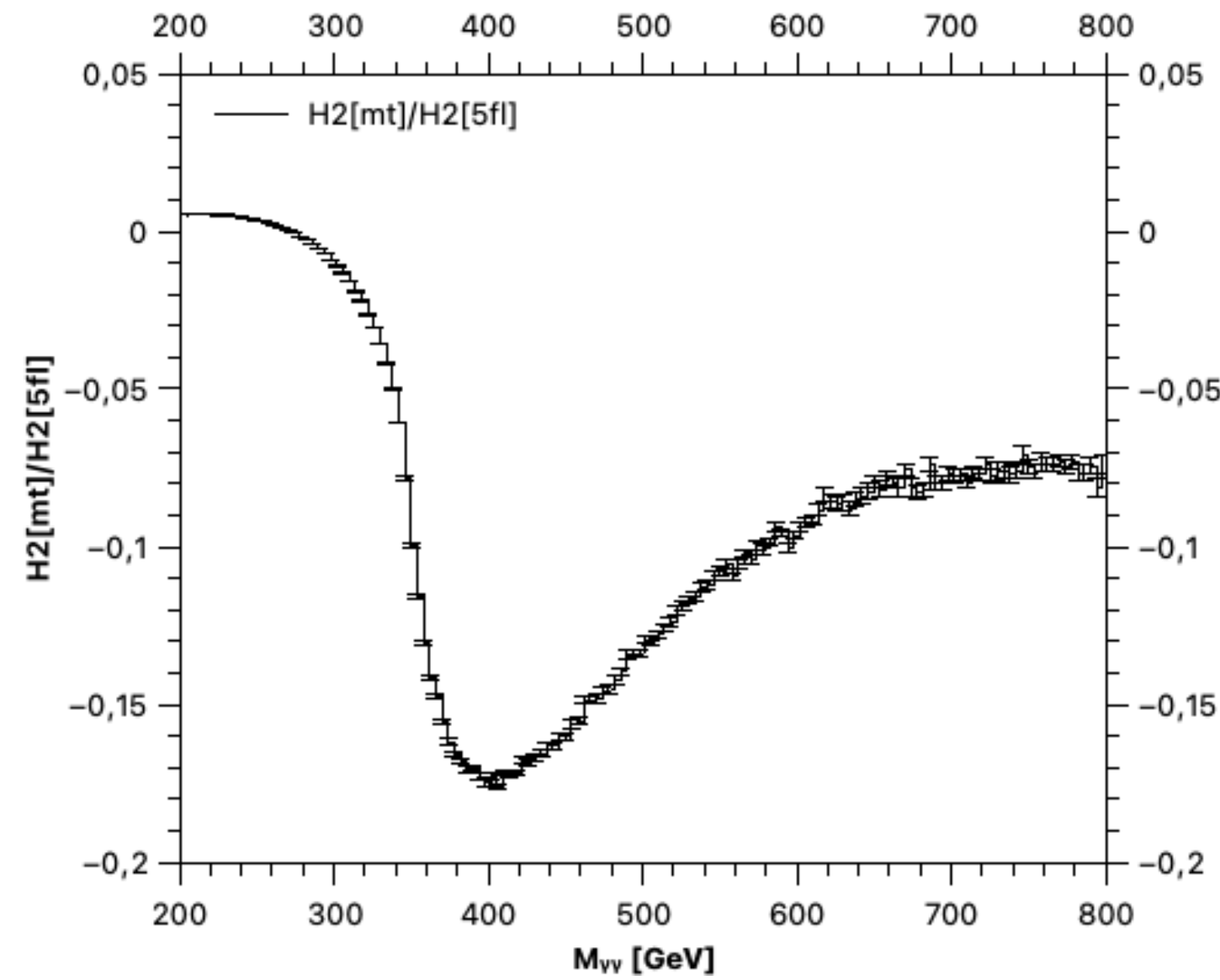
PLA Topology: 32 MIs in $\mathcal{O}(2.5h)$

NPL Topology: 36 MIs in $\mathcal{O}(10.5h)$

On a single core!



Results



Fiducial cuts

Smooth isolation cone

[The ATLAS Collaboration]

❖ $\sqrt{s} = 13 \text{ TeV}$

❖ $p_{T_\gamma}^{\text{Hard}} \geq 40 \text{ GeV}$

❖ $p_{T_\gamma}^{\text{Soft}} \geq 30 \text{ GeV}$

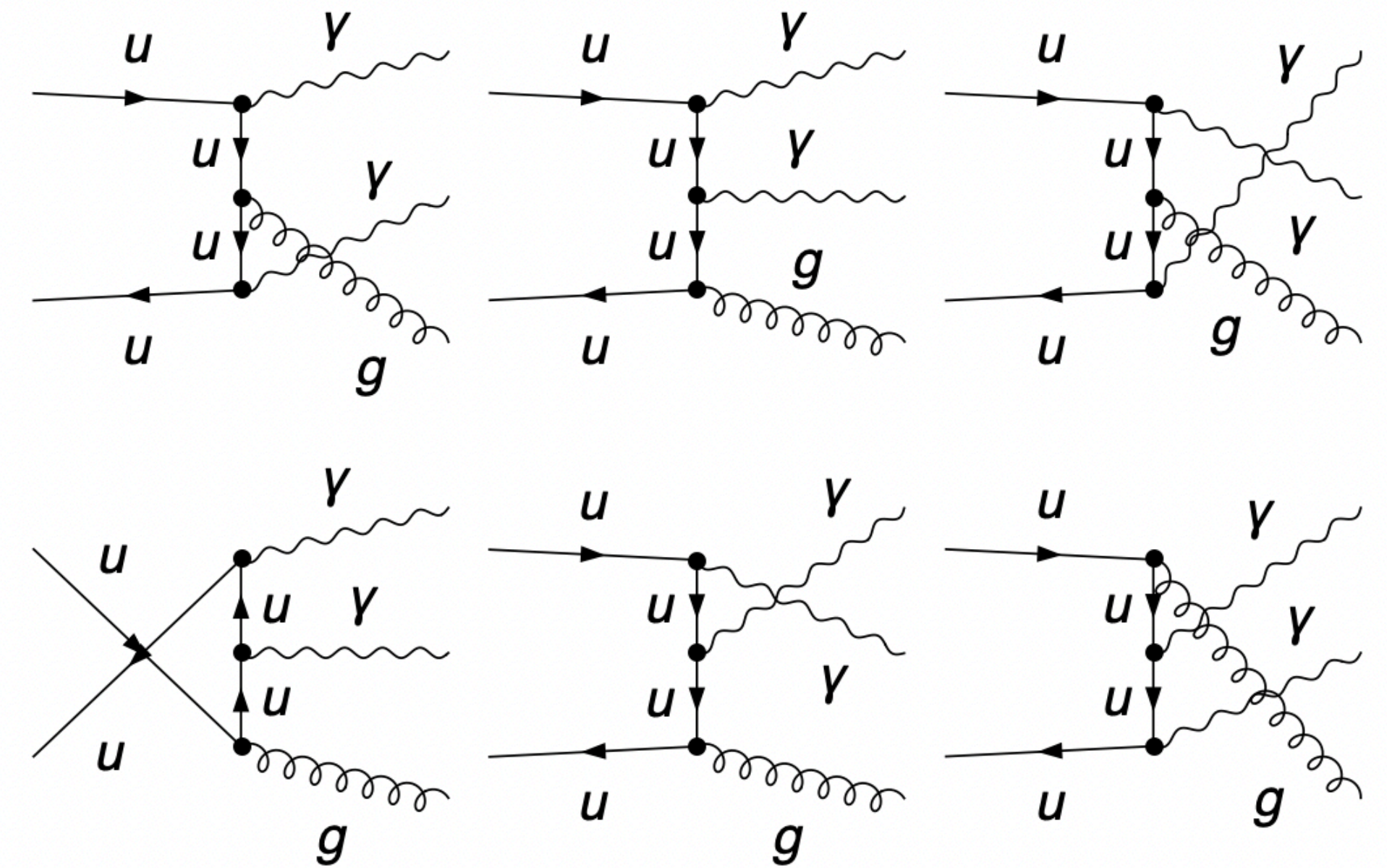
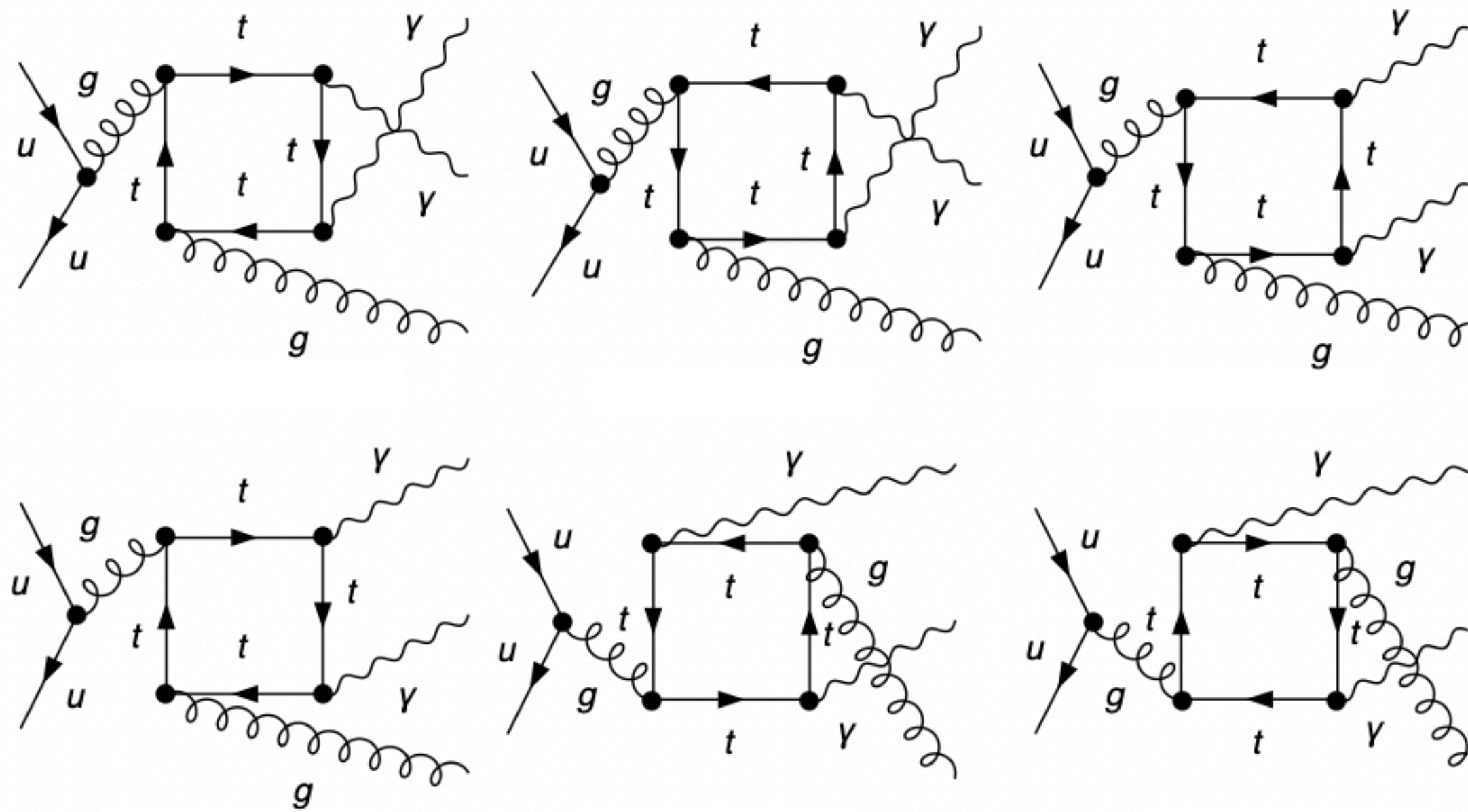
❖ $|\eta_\gamma| < 2.37$ Excluding $1.37 < |\eta_\gamma| < 1.52$

❖ $E_T^{\text{had}}(\delta) \leq E_{T_{\text{max}}}^{\text{had}} \chi(\delta)$

❖ $\Delta R = 0.2$
 $\Delta R_{\gamma\gamma} > 0.4$

Real-Virtual Contributions

Massive corrections $\mathcal{O}(\alpha_s^2)$



$q\bar{q}$ - channel

Analytic Computation of
the Master Integrals

[G.'t Hooft, M. Veltman]

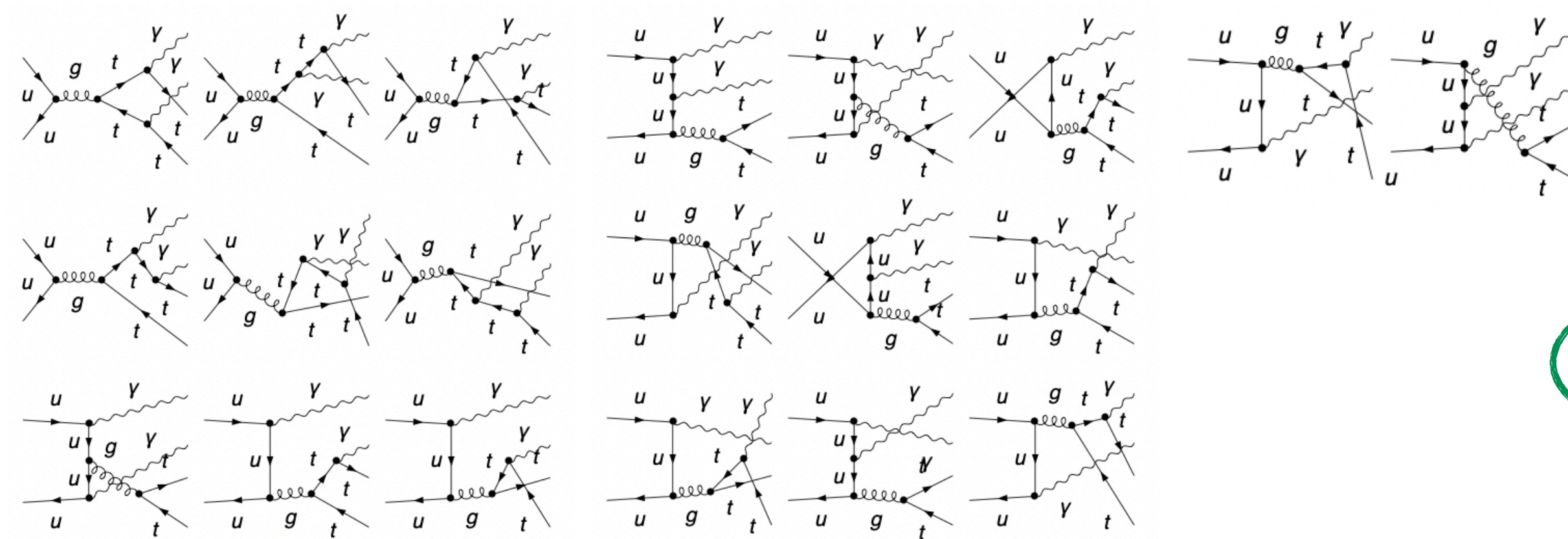
[R.K. Ellis, G. Zanderighi]

[A. Denner, S. Dittmaier]

qg - channel
is considered
as well!

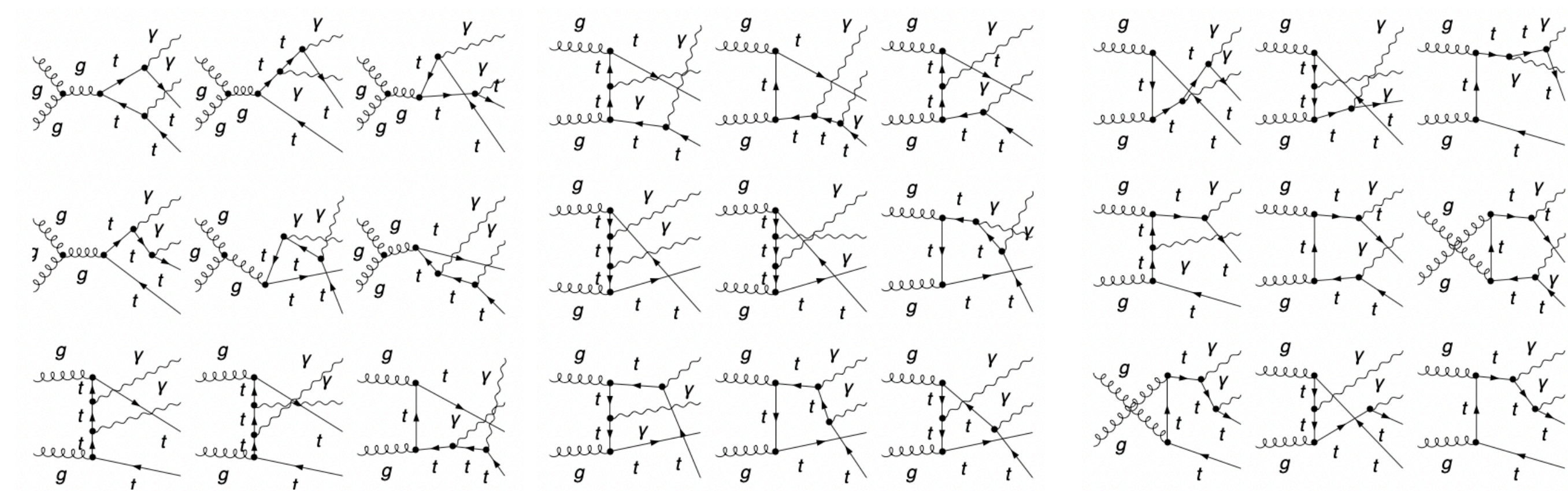
Double Real Contributions

$$q\bar{q} \rightarrow t\bar{t}\gamma\gamma$$



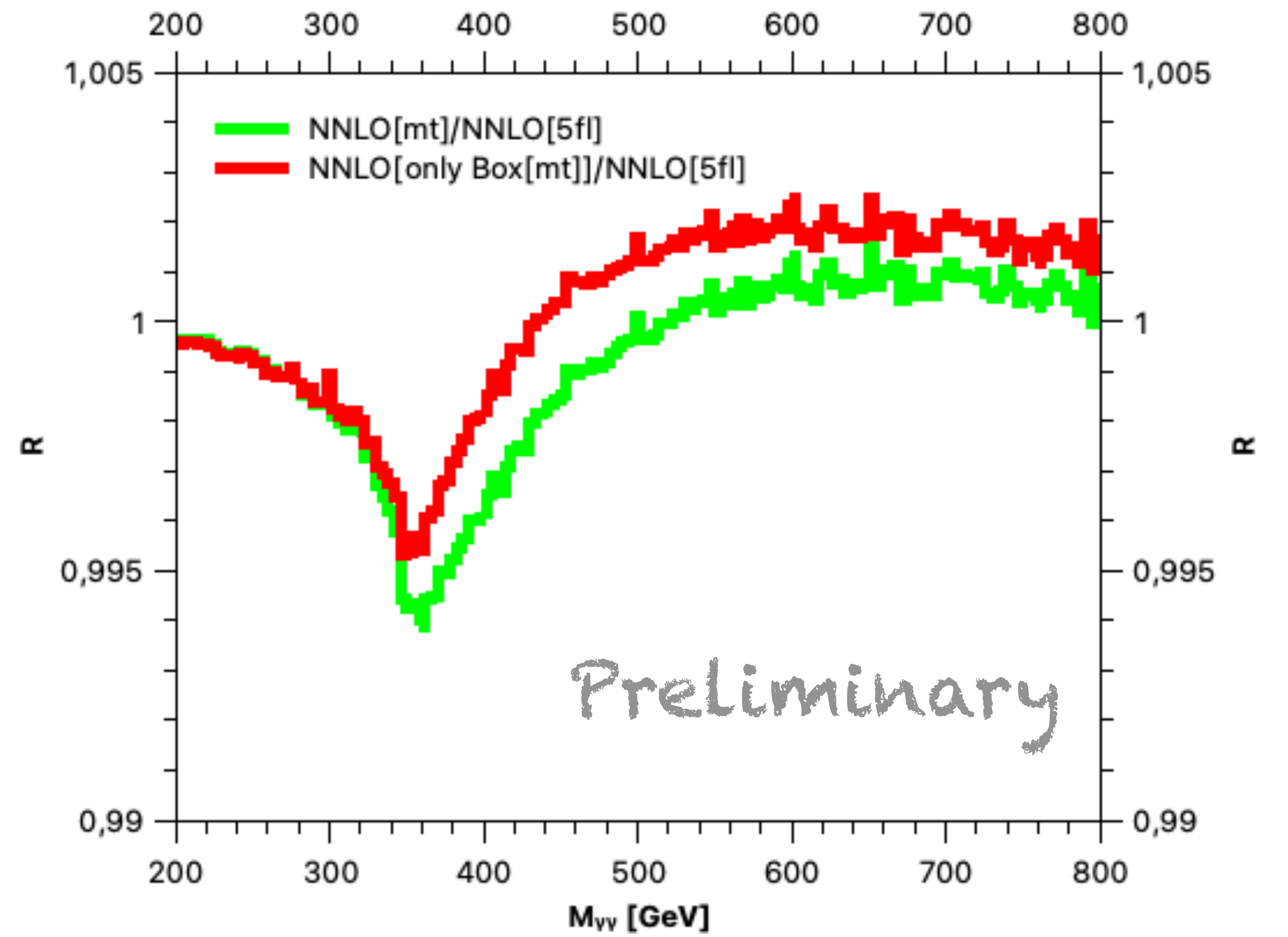
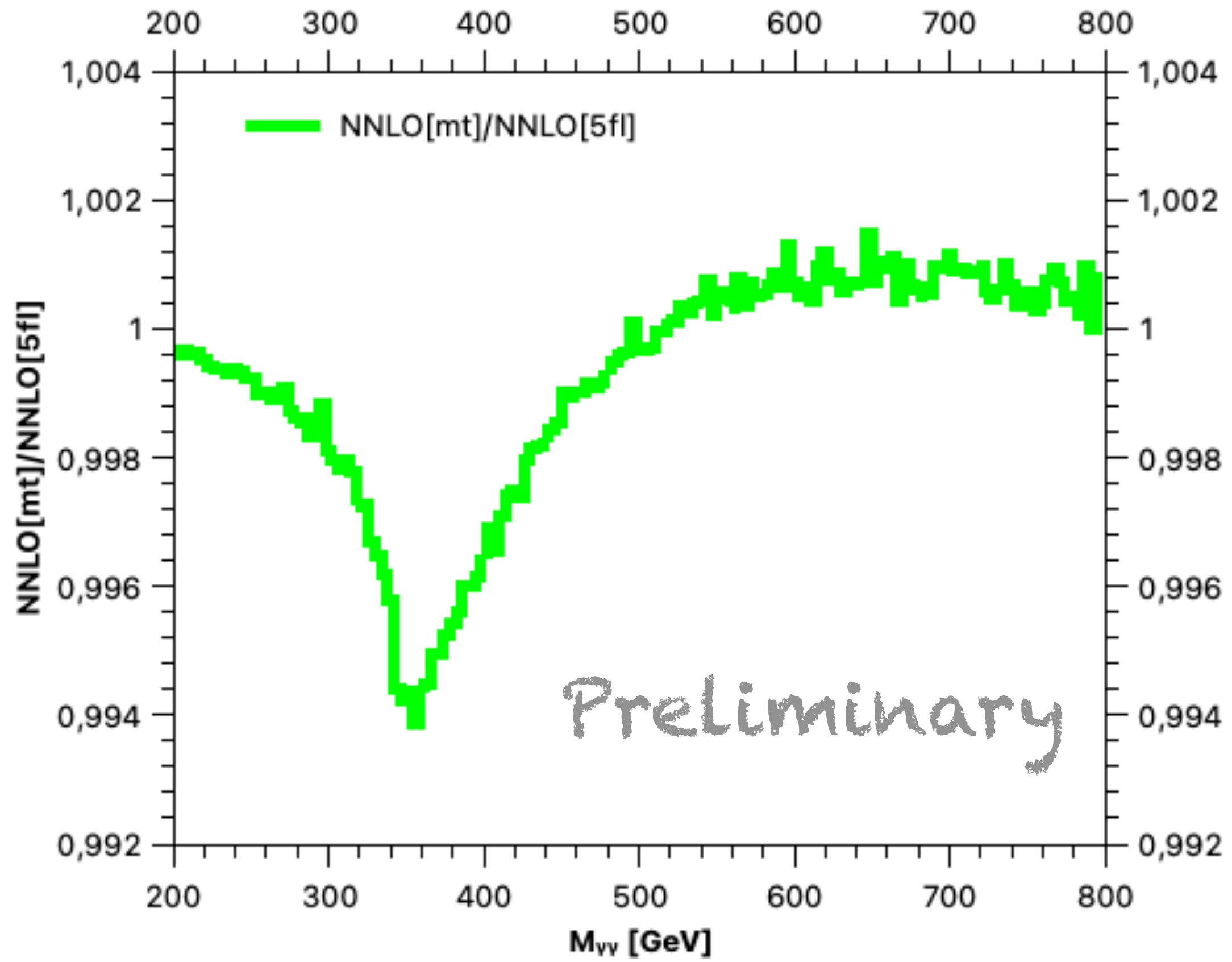
20 diagrams

$$gg \rightarrow t\bar{t}\gamma\gamma$$



30 diagrams

Final Results



Conclusions

- ❖ We computed the massive two-loop form factors
- ❖ The MIs were evaluated using the generalised power series method
- ❖ Computation of the Massive Hard Function NNLO
- ❖ We obtained some preliminary results for the phenomenological part

Outlooks

- ❖ We are about to finish a complete study of the phenomenology with all the massive contributions that until now have not been considered