Renormalization of abelian chiral Gauge Theories with non-anticommuting γ_5 at the Multi-Loop Level

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Chiral Gauge Theories and the γ_5-Problem

- Electroweak interactions act on chiral fermions
  - Left-handed and right-handed fermions interact differently with gauge bosons
  - SM and all its extensions for potential new physics are chiral gauge theories
- Abelian chiral gauge theory — Lagrangian of the considered theory

$$\mathcal{L} = i\overline{\psi_{R}}_i \not{D}_{ij}\psi_{Rj} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 - \bar{c}\partial^2 c + \rho^\mu s A_\mu + \bar{R}^i s\psi_{Ri} + R^i s\overline{\psi_{R}}_i$$

with $D^\mu_{ij} = \partial^\mu \delta_{ij} + ieA^\mu \mathcal{Y}_{Rij}$

- Dimensional renormalization of chiral gauge theories leads to the γ_5-problem
  - γ_5 is manifestly 4-dimensional
  - Cannot simultaneously retain the following properties in D dimensions

$$\{\gamma_5, \gamma^\mu\} = 0 \tag{1}$$

$$\mathrm{Tr}(\Gamma_1\Gamma_2) = \mathrm{Tr}(\Gamma_2\Gamma_1) \tag{2}$$

$$\mathrm{Tr}(\gamma_5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = -4i\epsilon^{\mu\nu\rho\sigma} \tag{3}$$

Breitenlohner-Maison/'t Hooft-Veltman (BMHV) scheme

- Abandoning anticommutativity of $\gamma_5 \longrightarrow$ BMHV algebra
  - Decomposing the D dimensional space

$$\mathbb{M} = \mathbb{M}_4 \oplus \mathbb{M}_{-2\epsilon}, \qquad \eta_{\mu\nu} = \overline{\eta}_{\mu\nu} + \hat{\eta}_{\mu\nu}, \qquad X^\mu = \overline{X}^\mu + \hat{X}^\mu$$

  - BMHV algebra

$$\{\gamma^\mu, \gamma_5\} = \{\hat{\gamma}^\mu, \gamma_5\} = 2\,\hat{\gamma}^\mu\,\gamma_5, \qquad \{\overline{\gamma}^\mu, \gamma_5\} = 0, \qquad [\hat{\gamma}^\mu, \gamma_5] = 0$$

- Gauge invariance is broken in intermediate steps by the modified algebraic relations
- Broken symmetry has to be compensated by a more complicated renormalization
  - Counterterms generated by field and parameter renormalization are not sufficient
  - Symmetry-restoring counterterms need to be found and included
  - Leading to a more general counterterm structure

$$S_{\text{ct}} = S_{\text{sct}} + S_{\text{fct}} = S_{\text{sct,inv}} + S_{\text{sct,non-inv}} + S_{\text{fct}}$$

Preview — Finite Symmetry Restoring Counterterm Action

Excerpt: bilinear gauge boson contributions [arXiv:23xx.xxxx]

$$
\begin{aligned}
S_{\mathsf{fct}} = &- \frac{e^2}{16\pi^2} \frac{\mathrm{Tr}\big(\mathcal{Y}_R^2\big)}{3} \int d^4x \, \frac{1}{2} \, \overline{A}_\mu \overline{\partial}^2 \overline{A}^\mu + \dots \\
&+ \frac{e^4}{(16\pi^2)^2} \frac{11}{24} \mathrm{Tr}\big(\mathcal{Y}_R^4\big) \int d^4x \, \frac{1}{2} \, \overline{A}_\mu \overline{\partial}^2 \overline{A}^\mu + \dots \\
&- \frac{1}{(16\pi^2)^3} \frac{e^6}{18} \frac{\big(35242 + 8448\,\zeta_3\big)\mathrm{Tr}\big(\mathcal{Y}_R^6\big) + 1639\,\mathrm{Tr}\big(\mathcal{Y}_R^4\big)\mathrm{Tr}\big(\mathcal{Y}_R^2\big)}{1200} \int d^4x \, \frac{1}{2} \, \overline{A}_\mu \, \overline{\partial}^2 \, \overline{A}^\mu + \dots \\
&+ \dots
\end{aligned}
$$

Symmetries

Slavnov-Taylor Identity

$$\mathcal{S}(\Gamma_{\text{ren}}) = \int \mathrm{d}^4x \, \frac{\delta\Gamma_{\text{ren}}}{\delta\phi(x)} \frac{\delta\Gamma_{\text{ren}}}{\delta K_\phi(x)} \stackrel{!}{=} 0$$

- The Slavnov-Taylor identity reflects symmetries in the full quantum theory
- Abelian gauge theories: Slavnov-Taylor identity $\longrightarrow$ Ward identities, such as
  - Transversality of the gauge boson self energy
  - Fermion self energy to fermion gauge boson interaction current relation
- Require the validity of symmetries as part of the definition of the theory
- Regularization induced symmetry breakings need to be restored

Symmetry Breaking and Restoration

The classical Symmetry can, and may in general, be broken by the Regularization

$$\mathcal{S}(\Gamma_{\text{reg}}) \neq 0$$

- Regularized Quantum Action Principle of Dimensional Regularization

$$\mathcal{S}_D(\Gamma_{\text{DRen}}) = (\widehat{\Delta} + \Delta_{\text{ct}}) \cdot \Gamma_{\text{DRen}}$$

- Possible symmetry breaking can be rewritten as a composite operator insertion

$$\widehat{\Delta} = \mathcal{S}_D(S_0), \qquad \widehat{\Delta} + \Delta_{\text{ct}} = \mathcal{S}_D(S_0 + S_{\text{ct}})$$

- The ultimate symmetry requirement is the Slavnov-Taylor identity

$$\underset{D\to 4}{\text{LIM}} \left(\mathcal{S}_D(\Gamma_{\text{DRen}})\right) = 0$$

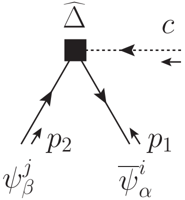
Practical Application in abelian chiral Gauge Theories

- Perturbative requirement from the Slavnov-Taylor identity

$$\underset{D\to 4}{\mathrm{LIM}} \left(\widehat{\Delta} \cdot \Gamma^{n}_{\mathrm{DRen}} + \sum_{k=1}^{n-1} \Delta^{k}_{\mathrm{ct}} \cdot \Gamma^{n-k}_{\mathrm{DRen}} + \Delta^{n}_{\mathrm{ct}} \right) = 0$$

- Tree-level breaking: $\widehat{\Delta}$-operator reflects the breaking of chiral gauge invariance

$$\mathcal{S}_D(S_0) = \widehat{\Delta} = - \int d^D x \, e \, \mathcal{Y}_{Rij} \, c \left\{ \overline{\psi}_i \left(\overleftarrow{\widehat{\partial\!\!\!/}} \mathbb{P}_R + \overrightarrow{\widehat{\partial\!\!\!/}} \mathbb{P}_L \right) \psi_j \right\}$$



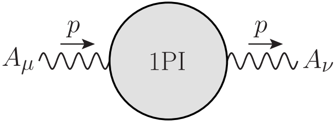
$$= - e \, \mathcal{Y}_{Rij} \left(\widehat{p\!\!\!/}_1 \mathbb{P}_{\mathrm{R}} + \widehat{p\!\!\!/}_2 \mathbb{P}_{\mathrm{L}} \right)_{\alpha\beta}$$

- Compute Feynman diagrams involving an insertion of the composite operator $\widehat{\Delta} + \Delta_{\mathrm{ct}}$

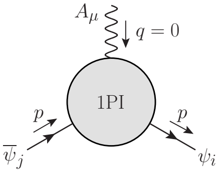
Broken Ward Identities in abelian chiral Gauge Theories

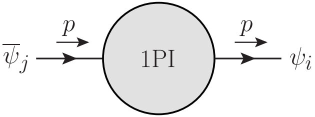
Spurious symmetry breakings induced by the BMHV algebra, e.g.:

- Violation of the transversality of the gauge boson self energy

p_ν  $\neq 0$

- Violation of the fermion self energy to fermion gauge boson interaction current relation



$+\, e\,\mathcal{Y}_R \frac{\partial}{\partial p_\mu}$  $\neq 0$

Broken Ward Identities in abelian chiral Gauge Theories

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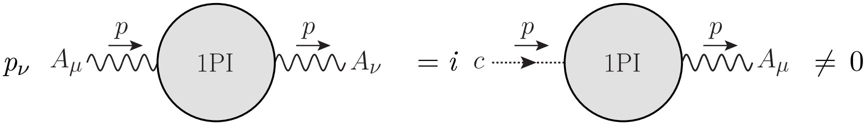
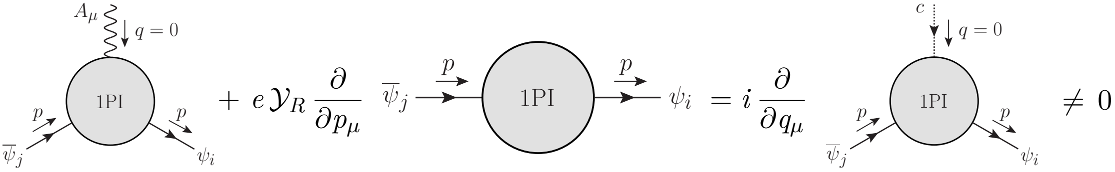
- Violation of the transversality of the gauge boson self energy

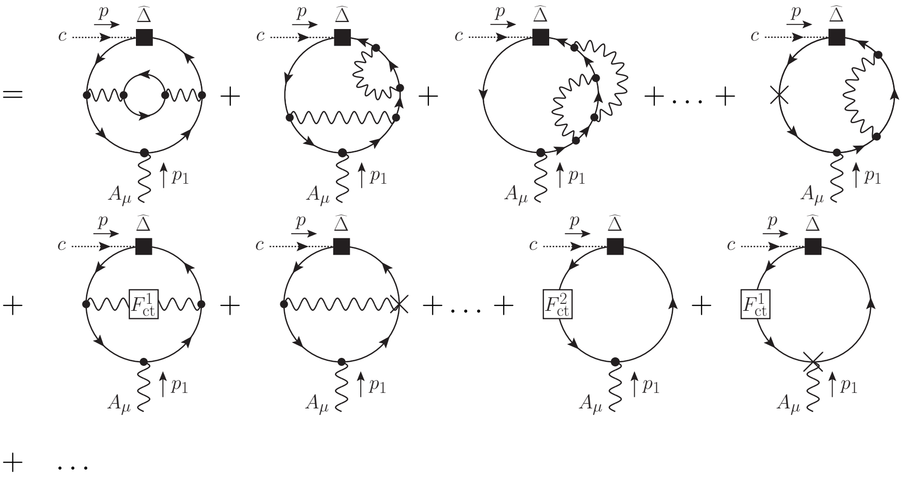


- Violation of the fermion self energy to fermion gauge boson interaction current relation



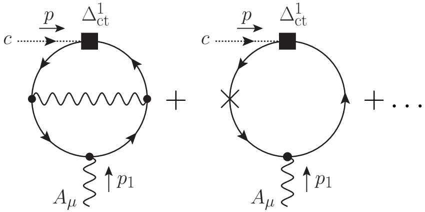
Violation of the Gauge Boson Transversality at the 3-Loop Level

$$i\big(\widehat{\Delta}\cdot\widetilde{\Gamma}^3_{\rm DRen}\big)_{A_\mu c} =$$

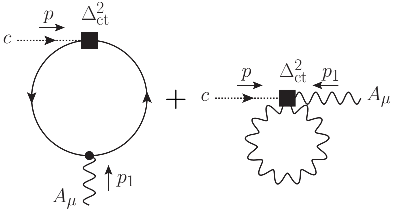


Violation of the Gauge Boson Transversality at the 3-Loop Level

$$i\left(\Delta^1_{\mathsf{ct}} \cdot \widetilde{\Gamma}^2_{\mathrm{DRen}}\right)_{A_\mu c} =$$



$$i\left(\Delta^2_{\mathsf{ct}} \cdot \widetilde{\Gamma}^1_{\mathrm{DRen}}\right)_{A_\mu c} =$$



Computational Set-Up

Computations are performed using `Mathematica` and `C++`

`Mathematica` packages [arXiv:0012260, arXiv:1601.01167, arXiv:2001.04407, arXiv:1611.06793]

- `FeynArts`
- `FeynCalc`
- `FeynHelpers`

`FIRE` (`C++` version) [arXiv:1901.07808]

- Integration by parts reduction of Feynman integrals to master integrals

$$\int \prod_{l=1}^{L} \frac{d^D k_l}{(2\pi)^D} \frac{\partial}{\partial k_i^\mu} \left[p_j^\mu \frac{1}{D_1^{a_1} \cdots D_n^{a_n}} \right] = 0$$

- Master integrals

$$G(n_1, \ldots, n_z) = (e^{\epsilon \, \gamma_E})^L \int \prod_{l=1}^{L} \frac{d^D k_l}{(2\pi)^D} \frac{1}{D_1^{n_1} \cdots D_z^{n_z}}$$

All Massive Tadpoles Method

Extracting UV-divergences utilizing an infrared rearrangement [arXiv:9409454, arXiv:9711266]

$$\frac{1}{(k+p)^2} = \frac{1}{k^2 - M^2} - \frac{p^2 + 2\,k\cdot p + M^2}{k^2 - M^2}\,\frac{1}{(k+p)^2}$$

- Exact decomposition, which can be applied recursively
- Power counting finite terms can be dropped
- Does not affect the UV-divergences after subtraction of subdivergences

Improved tadpole expansion: Introducing M^2 and performing a Taylor-expansion

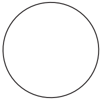
$$\frac{1}{(k+p)^2} \longrightarrow \frac{1}{(k+p)^2 - M^2} = \frac{1}{k^2 - M^2} - \frac{p^2 + 2\,k\cdot p}{(k^2 - M^2)^2} + \frac{(p^2 + 2\,k\cdot p)^2}{(k^2 - M^2)^3} + \dots$$

- Same result as with the exact decomposition when neglecting numerator terms $\propto M^2$
- Auxiliary mass counterterms $\propto M^2$ necessary (not part of the theory)

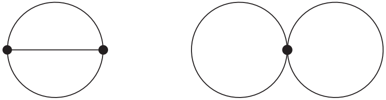
Master Integrals

Single-scale massive vacuum master bubbles

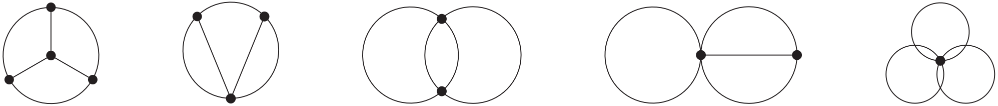
- 1-Loop



- 2-Loop



- 3-Loop



Gauge Boson Self Energy in an abelian chiral Gauge Theory

1-loop result [arXiv:2303.09120, arXiv:2109.11042]

$$i\widetilde{\Gamma}^{\nu\mu}_{AA}(p)\Big|^{1}_{\text{div}} = \frac{i\,e^2}{16\pi^2}\frac{\mathrm{Tr}\left(\mathcal{Y}_R^2\right)}{3}\left[\frac{2}{\epsilon}\left(\overline{p}^\mu\overline{p}^\nu - \overline{p}^2\overline{\eta}^{\mu\nu}\right) - \frac{1}{\epsilon}\widehat{p}^2\overline{\eta}^{\mu\nu}\right]$$

2-loop result

$$i\widetilde{\Gamma}^{\nu\mu}_{AA}(p)\Big|^{2}_{\text{div}} = \frac{i\,e^4}{(16\pi^2)^2}\frac{\mathrm{Tr}\left(\mathcal{Y}_R^4\right)}{3}\left[\frac{2}{\epsilon}\left(\overline{p}^\mu\overline{p}^\nu - \overline{p}^2\overline{\eta}^{\mu\nu}\right) - \left(\frac{1}{2\epsilon^2} - \frac{17}{24\epsilon}\right)\widehat{p}^2\overline{\eta}^{\mu\nu}\right]$$

- Transversality of the gauge boson self energy is violated
- Violation is local
- Symmetry restoration necessary and possible

Gauge Boson Self Energy in an abelian chiral Gauge Theory

3-loop result [arXiv:23xx.xxxx]

$$
\begin{aligned}
i\widetilde{\Gamma}^{\nu\mu}_{AA}(p)\Big|^{3}_{\mathsf{div}} = &-\frac{i}{(16\pi^2)^3}\frac{e^6}{18}\Bigg[\frac{4}{9}\Big(3\,\mathrm{Tr}\big(\mathcal{Y}_R^6\big) - 5\,\mathrm{Tr}\big(\mathcal{Y}_R^4\big)\mathrm{Tr}\big(\mathcal{Y}_R^2\big)\Big)\frac{1}{\epsilon^2} \\
&\qquad - \frac{2552\,\mathrm{Tr}\big(\mathcal{Y}_R^6\big) + 61\,\mathrm{Tr}\big(\mathcal{Y}_R^4\big)\mathrm{Tr}\big(\mathcal{Y}_R^2\big)}{90}\frac{1}{\epsilon}\Bigg]\Big(\overline{p}^\mu\overline{p}^\nu - \overline{p}^2\overline{\eta}^{\mu\nu}\Big) \\
&-\frac{i}{(16\pi^2)^3}\frac{e^6}{18}\Bigg[\frac{\mathrm{Tr}\big(\mathcal{Y}_R^6\big)}{\epsilon^3} - \frac{529\,\mathrm{Tr}\big(\mathcal{Y}_R^6\big) + 122\,\mathrm{Tr}\big(\mathcal{Y}_R^4\big)\mathrm{Tr}\big(\mathcal{Y}_R^2\big)}{60}\frac{1}{\epsilon^2} \\
&\qquad + \frac{(156672\,\zeta_3 - 49427)\,\mathrm{Tr}\big(\mathcal{Y}_R^6\big) - 8374\,\mathrm{Tr}\big(\mathcal{Y}_R^4\big)\mathrm{Tr}\big(\mathcal{Y}_R^2\big)}{3600}\frac{1}{\epsilon}\Bigg]\widehat{p}^2\,\overline{\eta}^{\mu\nu} \\
&+\frac{i}{(16\pi^2)^3}\frac{e^6}{18}\frac{18\,\mathrm{Tr}\big(\mathcal{Y}_R^6\big) + 79\,\mathrm{Tr}\big(\mathcal{Y}_R^4\big)\mathrm{Tr}\big(\mathcal{Y}_R^2\big)}{60}\frac{1}{\epsilon}\,\overline{p}^2\,\overline{\eta}^{\mu\nu}
\end{aligned}
$$

Violation of the Gauge Boson Transversality

3-loop result [arXiv:23xx.xxxx]

$$i\Big(\big[\widehat{\Delta} + \Delta^1_{\text{ct}} + \Delta^2_{\text{ct}}\big]\cdot\widetilde{\Gamma}\Big)^3_{A_\mu c} = i\big(\widehat{\Delta}\cdot\widetilde{\Gamma}^3\big)_{A_\mu c} + i\big(\Delta^1_{\text{ct}}\cdot\widetilde{\Gamma}^2\big)_{A_\mu c} + i\big(\Delta^2_{\text{ct}}\cdot\widetilde{\Gamma}^1\big)_{A_\mu c}$$

$$= -\frac{1}{(16\pi^2)^3}\frac{e^6}{18}\Bigg\{\Bigg[\frac{\text{Tr}\big(\mathcal{Y}_R^6\big)}{\epsilon^3} - \frac{529\,\text{Tr}\big(\mathcal{Y}_R^6\big) + 122\,\text{Tr}\big(\mathcal{Y}_R^4\big)\text{Tr}\big(\mathcal{Y}_R^2\big)}{60}\frac{1}{\epsilon^2}$$
$$+ \frac{\big(156672\,\zeta_3 - 49427\big)\text{Tr}\big(\mathcal{Y}_R^6\big) - 8374\,\text{Tr}\big(\mathcal{Y}_R^4\big)\text{Tr}\big(\mathcal{Y}_R^2\big)}{3600}\frac{1}{\epsilon}\Bigg]\widehat{p}^2\,\overline{p}^\mu$$
$$- \Bigg[\frac{18\,\text{Tr}\big(\mathcal{Y}_R^6\big) + 79\,\text{Tr}\big(\mathcal{Y}_R^4\big)\text{Tr}\big(\mathcal{Y}_R^2\big)}{60}\frac{1}{\epsilon}$$
$$- \frac{\big(35242 + 8448\,\zeta_3\big)\text{Tr}\big(\mathcal{Y}_R^6\big) + 1639\,\text{Tr}\big(\mathcal{Y}_R^4\big)\text{Tr}\big(\mathcal{Y}_R^2\big)}{1200}\Bigg]\overline{p}^2\,\overline{p}^\mu\Bigg\}$$

Renormalization and Symmetry Restoration - Singular Counterterm Action

[arXiv:23xx.xxxx]

$$
\begin{aligned}
S_{\text{sct}} = &- \frac{e^2}{16\pi^2} \frac{2}{3} \mathrm{Tr}\big(\mathcal{Y}_R^2\big) \frac{1}{\epsilon} \left[\overline{S_{AA}} + \frac{1}{2} \int d^D x \, \frac{1}{2} \, \overline{A}_\mu \, \widehat{\partial}^2 \, \overline{A}^\mu \right] + \ldots \\
&- \frac{e^4}{(16\pi^2)^2} \frac{\mathrm{Tr}\big(\mathcal{Y}_R^4\big)}{3} \left[\frac{2}{\epsilon} \overline{S_{AA}} + \left(\frac{1}{2\epsilon^2} - \frac{17}{24\epsilon} \right) \int d^D x \, \frac{1}{2} \, \overline{A}_\mu \, \widehat{\partial}^2 \, \overline{A}^\mu \right] + \ldots \\
&+ \frac{1}{(16\pi^2)^3} \frac{e^6}{18} \left[\frac{12 \, \mathrm{Tr}\big(\mathcal{Y}_R^6\big) - 20 \, \mathrm{Tr}\big(\mathcal{Y}_R^4\big) \mathrm{Tr}\big(\mathcal{Y}_R^2\big)}{9 \, \epsilon^2} - \frac{2552 \, \mathrm{Tr}\big(\mathcal{Y}_R^6\big) + 61 \, \mathrm{Tr}\big(\mathcal{Y}_R^4\big) \mathrm{Tr}\big(\mathcal{Y}_R^2\big)}{90 \, \epsilon} \right] \overline{S_{AA}} \\
&- \frac{1}{(16\pi^2)^3} \frac{e^6}{18} \Bigg\{ \left[\frac{\mathrm{Tr}\big(\mathcal{Y}_R^6\big)}{\epsilon^3} - \frac{529 \, \mathrm{Tr}\big(\mathcal{Y}_R^6\big) + 122 \, \mathrm{Tr}\big(\mathcal{Y}_R^4\big) \mathrm{Tr}\big(\mathcal{Y}_R^2\big)}{60 \, \epsilon^2} \right. \\
&\qquad \left. + \left[\left(\frac{1088}{25} \zeta_3 - \frac{49427}{3600} \right) \mathrm{Tr}\big(\mathcal{Y}_R^6\big) - \frac{4187}{1800} \mathrm{Tr}\big(\mathcal{Y}_R^4\big) \mathrm{Tr}\big(\mathcal{Y}_R^2\big) \right] \frac{1}{\epsilon} \right] \int d^D x \, \frac{1}{2} \, \overline{A}_\mu \, \widehat{\partial}^2 \, \overline{A}^\mu \\
&\qquad - \frac{18 \, \mathrm{Tr}\big(\mathcal{Y}_R^6\big) + 79 \, \mathrm{Tr}\big(\mathcal{Y}_R^4\big) \mathrm{Tr}\big(\mathcal{Y}_R^2\big)}{60 \, \epsilon} \int d^D x \, \frac{1}{2} \, \overline{A}_\mu \, \overline{\partial}^2 \, \overline{A}^\mu \Bigg\} + \cdots \\
&+ \ldots
\end{aligned}
$$

Renormalization and Symmetry Restoration - Finite Counterterm Action

Excerpt: bilinear gauge boson contributions [arXiv:23xx.xxxx]

$$S_{\mathsf{fct}} = -\frac{e^2}{16\pi^2}\frac{\mathrm{Tr}\left(\mathcal{Y}_R^2\right)}{3}\int d^4x\,\frac{1}{2}\,\overline{A}_\mu\overline{\partial}^2\overline{A}^\mu + \dots$$
$$+\frac{e^4}{(16\pi^2)^2}\frac{11}{24}\mathrm{Tr}\left(\mathcal{Y}_R^4\right)\int d^4x\,\frac{1}{2}\,\overline{A}_\mu\overline{\partial}^2\overline{A}^\mu + \dots$$
$$-\frac{1}{(16\pi^2)^3}\frac{e^6}{18}\frac{\left(35242 + 8448\,\zeta_3\right)\mathrm{Tr}\left(\mathcal{Y}_R^6\right) + 1639\,\mathrm{Tr}\left(\mathcal{Y}_R^4\right)\mathrm{Tr}\left(\mathcal{Y}_R^2\right)}{1200}\int d^4x\,\frac{1}{2}\,\overline{A}_\mu\,\overline{\partial}^2\,\overline{A}^\mu + \dots$$
$$+\dots$$

Conclusions

1. γ_5 can be treated rigorously in the BMHV scheme
   - BRST breaking can be restored with symmetry-restoring counterterms
   - These counterterms are guaranteed to exist by Algebraic Renormalization
   - These counterterms may be calculated via Feynman diagrams with $\widehat{\Delta}$-operator insertion
2. Renormalization of an abelian chiral Gauge Theory
   - Results up to the 2-loop level have been reproduced
   - New results at the 3-loop level have been obtained
   - New counterterms arise at higher loop-level, e.g.:

$$4 \text{ dim. singular bilin. gauge boson ct.} \propto \frac{e^6}{(16\pi^2)^3} \frac{1}{\epsilon} \int d^D x \, \frac{1}{2} \overline{A}_\mu \, \overline{\partial}^2 \, \overline{A}^\mu$$

$$\text{singular quartic gauge boson ct.} \propto \frac{e^8}{(16\pi^2)^3} \frac{1}{\epsilon} \int d^D x \, \overline{A}_\mu \, \overline{A}^\mu \, \overline{A}_\nu \, \overline{A}^\nu$$

3. Outlook
   - Upgrading the computational set-up: `Mathematica` $\longrightarrow$ `FORM` $\Longrightarrow$ 4-loop calculation
   - Renormalization of a non-abelian chiral gauge theory
   - Application to the Standard Model

References

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