~~Towards~~ **NNLO QCD** corrections for the production of a **heavy-quark pair** in association with a **massive boson**

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based on Phys.Rev.Lett. 130 (2023) and Phys.Rev.D 107 (2023)
in collaboration with *L.Buonocore, S.Catani, S.Devoto, M.Grazzini, S.Kallweit, J.Mazzitelli, L.Rottoli*

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Universität Zürich[UZH]

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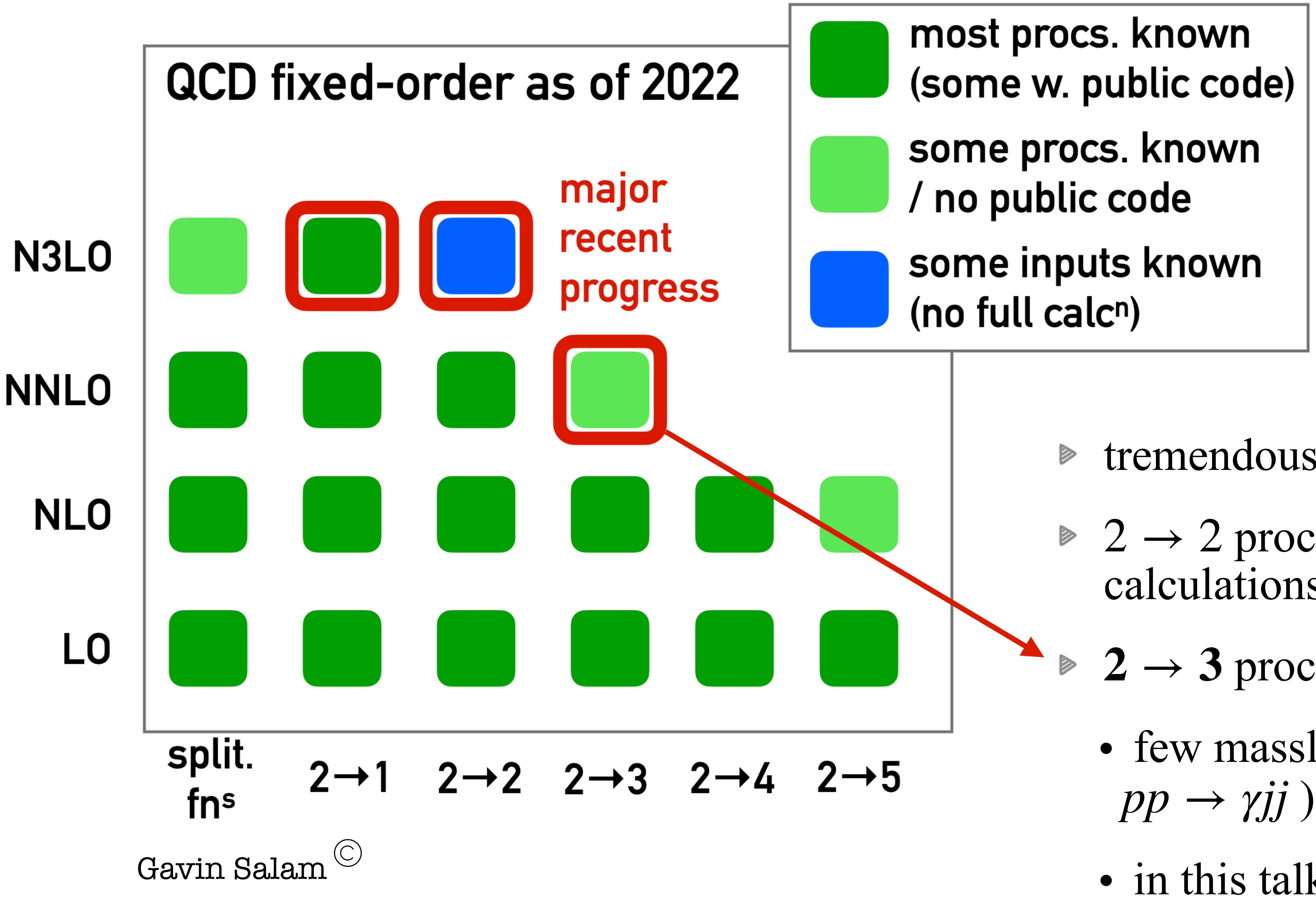
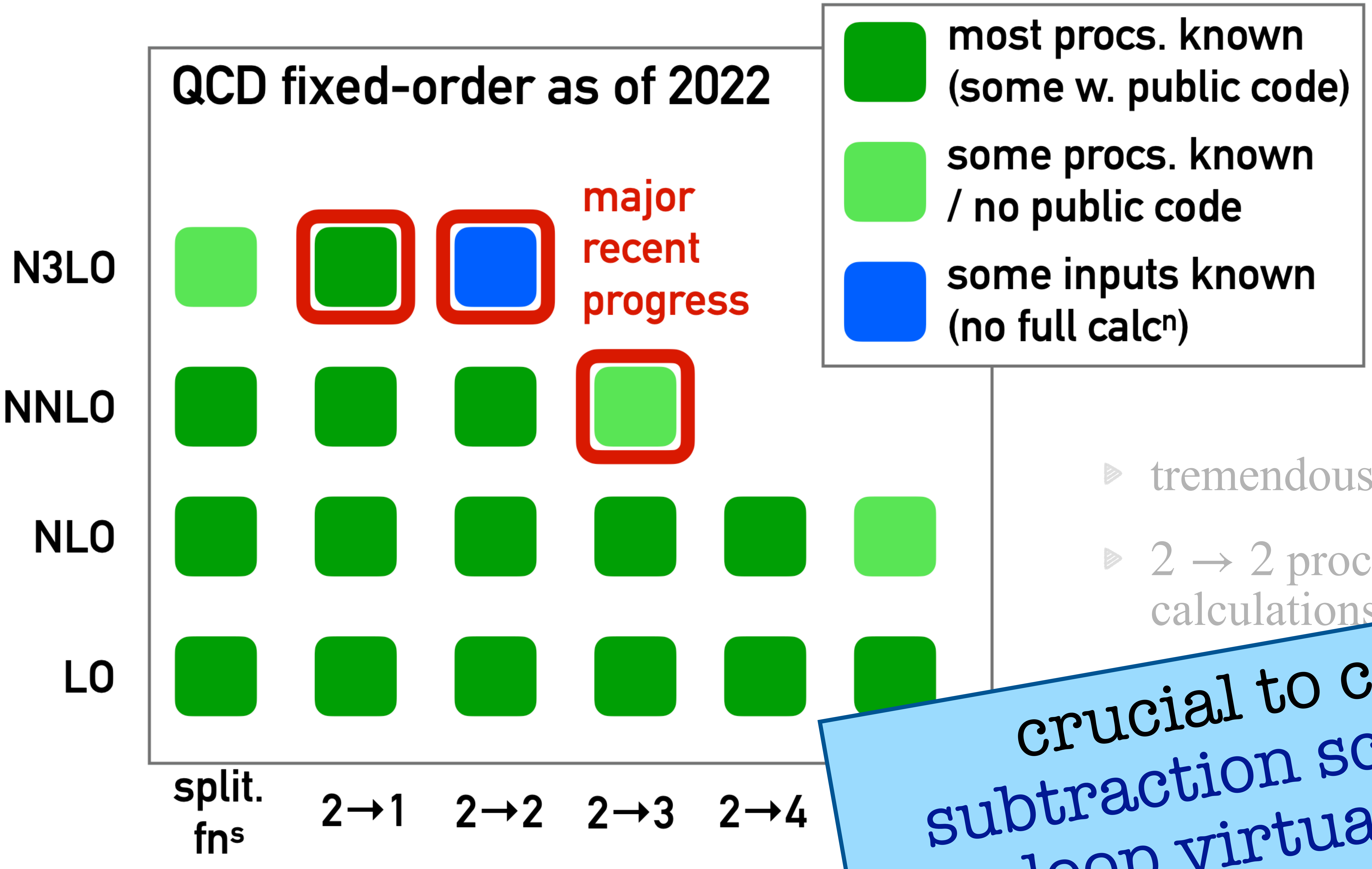
Introduction



- tremendous progress in the past ∼10 years!
- $2 \to 2$ processes at NNLO are under control (independent calculations)
- $\mathbf{2} \to \mathbf{3}$ processes at NNLO represent the **current frontier**
  - few massless computations ($pp \to \gamma\gamma\gamma$, $pp \to \gamma\gamma j$, $pp \to jjj$, $pp \to \gamma jj$)
  - in this talk we will focus on $\mathbf{2} \to \mathbf{3}$ processes with **external massive legs**

$t\bar{t}H$ $Wb\bar{b}$ $t\bar{t}W$

Introduction



Gavin Salam ©

- tremendous progress in the past ~10 years!
- 2 → 2 processes at NNLO are … endent calculations)
- … ntier
- … $p \to jjj$,
- … will focus on **2 → 3** processes with **external massive legs**

crucial to construct an NNLO subtraction scheme and to have the two-loop virtual amplitudes in order to complete an NNLO calculation

The framework: q_T -subtraction

[Catani, Grazzini (2007)]

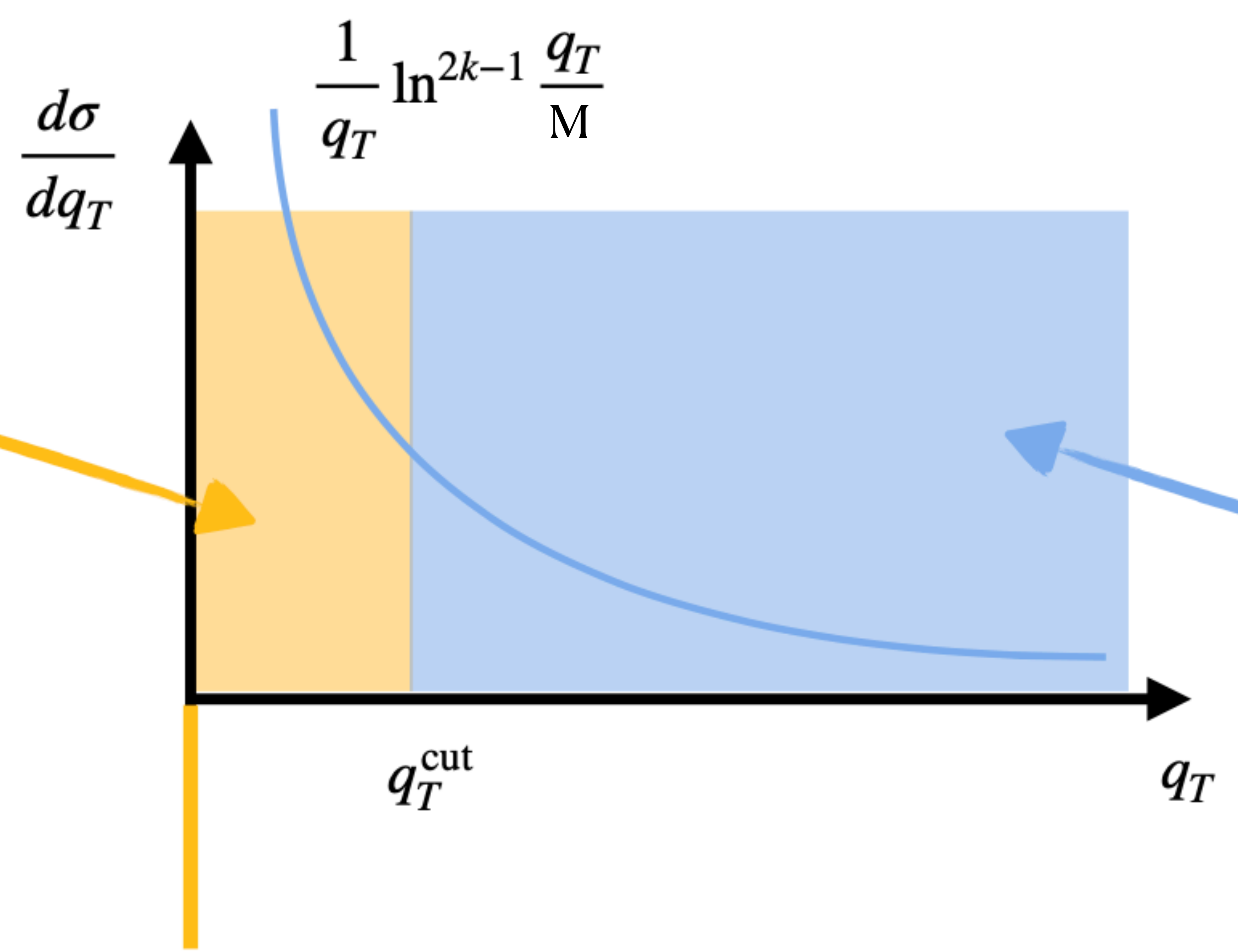
- cross section for the production of a triggered final state at N^{k}LO (in our case the triggered final state is $Q\bar{Q}F$)

crucial to keep the mass of the heavy quark m_Q

all emissions are unresolved

we can exploit the **QCD factorisation** of the matrix elements in the singular soft and/or collinear limits

ingredients from q_T - resummation



1 emission is always **resolved**

the complexity of the calculation is reduced by 1 order

logarithmic IR sensitivity to the cut

q_T and M are the transverse momentum and the invariant mass of the $Q\bar{Q}F$ system

$$d\sigma_{N^kLO} = \mathscr{H}_{N^kLO} \otimes d\sigma_{LO} + [d\sigma^R_{N^{k-1}LO} - d\sigma^{CT}_{N^kLO}]_{q_t > q_t^{\rm cut}} + \mathcal{O}((q_t^{\rm cut})^p)$$

The framework: q_T -subtraction [Catani, Grazzini (2007)]

- master formula at NNLO

$$d\sigma_{NNLO} = \mathscr{H}_{NNLO} \otimes d\sigma_{LO} + [d\sigma^{R}_{NLO} - d\sigma^{CT}_{NNLO}]_{q_t > q_t^{\rm cut}} + \mathcal{O}((q_t^{\rm cut})^p)$$

☑ the required matrix elements can be computed with **automated tools** like OpenLoops2

[Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang, Zoller (2019)]

☑ the remaining NLO-type singularities can be removed by applying a **local subtraction** method

[Catani, Seymour (1998)] [Catani, Dittmaier, Seymour, Trocsanyi (2002)]

☑ automatised implementation in the MATRIX framework, which relies on the efficient multi-channel Monte Carlo integrator MUNICH [Grazzini, Kallweit, Wiesemann (2017)]

The framework: q_T -subtraction [Catani, Grazzini (2007)]

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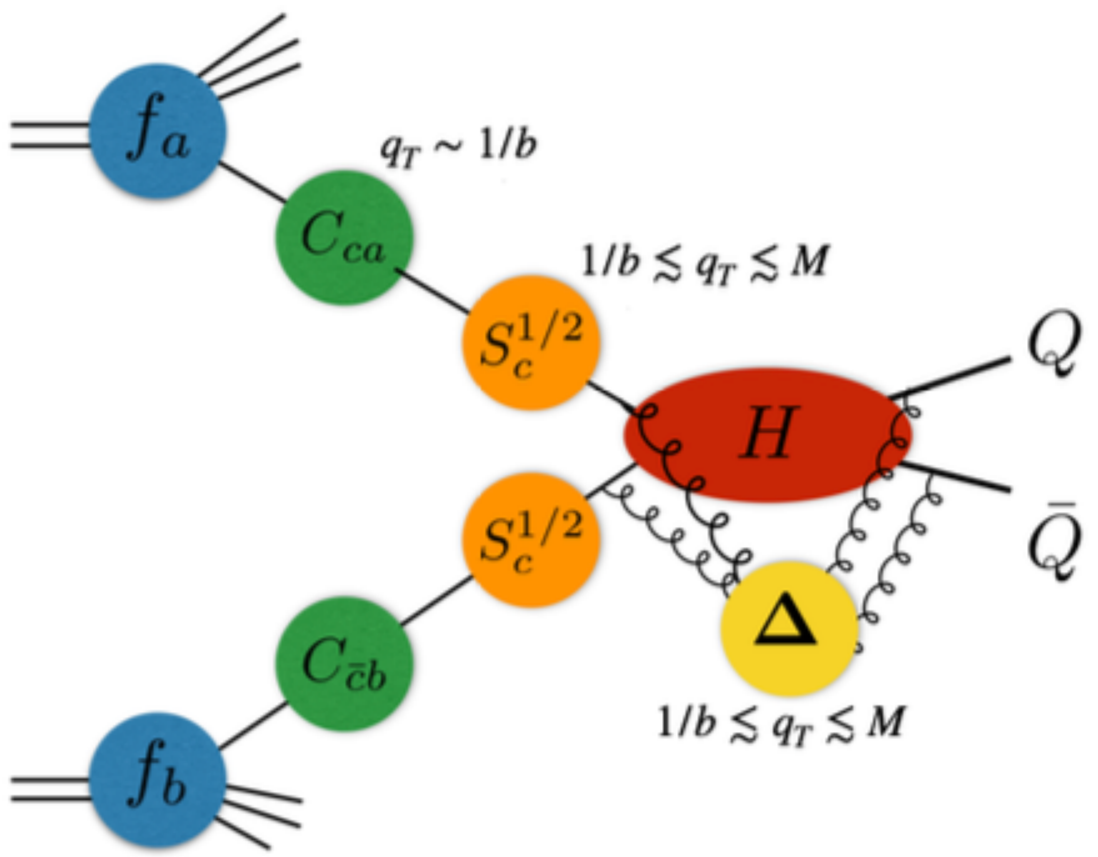
☑ non trivial ingredient: **two-loop soft function** for an **arbitrary kinematics** of the heavy quarks

[Catani, Devoto, Grazzini, Mazzitelli (2023)] [Devoto, Mazzitelli (in preparation)]



- the resummation formula shows a **richer structure** due to the additional soft singularities
- the factor Δ (operator in colour space) is specific for heavy-quark production and it encodes the **soft wide-angle radiation** from the $Q\bar{Q}$ pair and from initial-state final-state interference
- the log-enhanced contributions are controlled by the **transverse momentum anomalous dimension Γ_t**

The framework: q_T-subtraction [Catani, Grazzini (2007)]

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$$d\sigma_{NNLO} = \mathcal{H}_{NNLO} \otimes d\sigma_{LO} + [d\sigma^{R}_{NLO} - d\sigma^{CT}_{NNLO}]_{q_t > q_t^{\text{cut}}} + \mathcal{O}((q_t^{\text{cut}})^p)$$

- ☑ non trivial ingredient: **two-loop soft function** for an arbitrary kinematics of the heavy quarks
- ☑ all ingredients are known except for the **two-loop virtual amplitudes** contributing to the the hard-collinear coefficient

$$\mathcal{H}_{NNLO} = H^{(2)}\delta(1-z_1)\delta(1-z_2) + \delta\mathcal{H}^{(2)}(z_1, z_2)$$

where $$H^{(2)} = \left.\frac{2\Re(\mathcal{M}^{(2)}_{fin}(\mu_{IR}, \mu_R)\mathcal{M}^{(0)*})}{|\mathcal{M}^{(0)}|^2}\right|_{\mu_R=\mu_{IR}=M}$$

UV renormalised and IR subtracted amplitude at scale μ_{IR} (overall normalisation $(4\pi)^\epsilon e^{-\gamma_E \epsilon}$)

Remark: analogous definition for the hard-collinear coefficient at NLO $$H^{(1)} = \left.\frac{2\Re(\mathcal{M}^{(1)}_{fin}(\mu_{IR}, \mu_R)\mathcal{M}^{(0)*})}{|\mathcal{M}^{(0)}|^2}\right|_{\mu_R=\mu_{IR}=M}$$

The framework: q_T -subtraction

[Catani, Grazzini (2007)]

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→ main bottleneck:
$2 \to 3$ two-loop amplitudes
with internal and external
massive legs are currently out
of reach!

The framework: q_T-subtraction

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main bottleneck:
two-loop amplitudes
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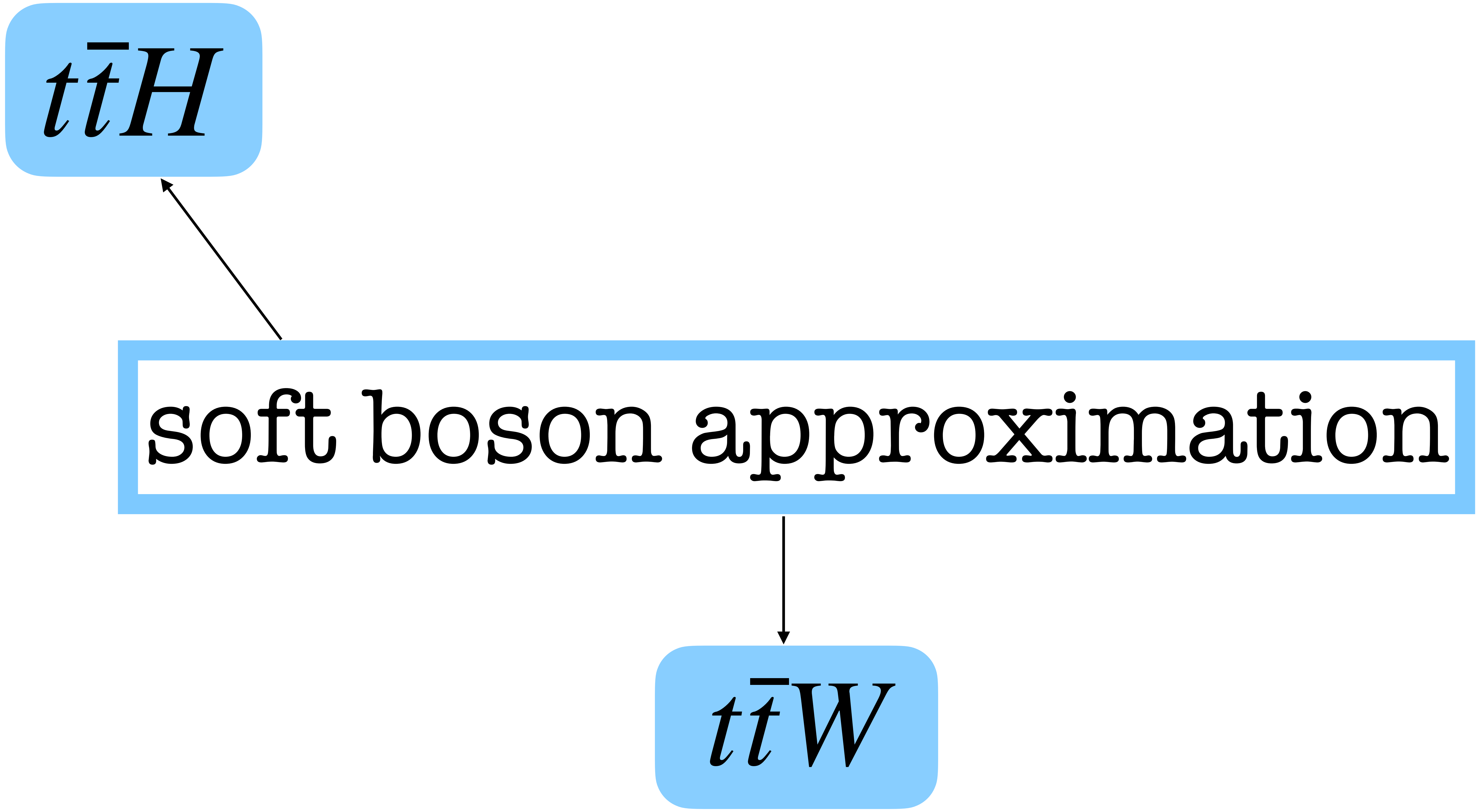
crucial to find one (or more) reasonable approximation

[Catani, Devoto, Grazzini, Kallweit, Mazzitelli, **CS** (2022)]



[Buonocore, Devoto, Grazzini, Kallweit, Mazzitelli, Rottoli, **CS** (in preparation)]

Soft Higgs boson approximation

bottleneck: the two-loop amplitudes are at the frontier of the current techniques

solution: development of a soft boson approximation

- the main idea is to find an analogous formula to the well known factorisation in the case of **soft gluons**

$$\lim_{k\to 0} \mathcal{M}^{bare}(\{p_i\},k) = J(k)\,\mathcal{M}^{bare}(\{p_i\})$$

see e.g. [Catani, Grazzini (2000)]

$$J(k) = g_s\mu^{\epsilon}(J^{(0)}(k) + g_s^2 J^{(1)}(k) + \ldots)$$

purely non abelian

- for a **soft scalar Higgs** radiated off a heavy quark with momentum p_j , we have that

$$\lim_{k\to 0} \mathcal{M}^{bare}(\{p_i\},k) = J^{(0)}(k)\,\mathcal{M}^{bare}(\{p_i\})$$

bare mass of the heavy quark

soft insertion rules, only external legs matter!

$$J^{(0)}(k) = \sum_j \frac{m_{j,0}}{v}\frac{m_{j,0}}{p_j\cdot k}$$

- the naïve factorisation formula does NOT hold at the level of renormalised amplitudes!

Soft Higgs boson approximation

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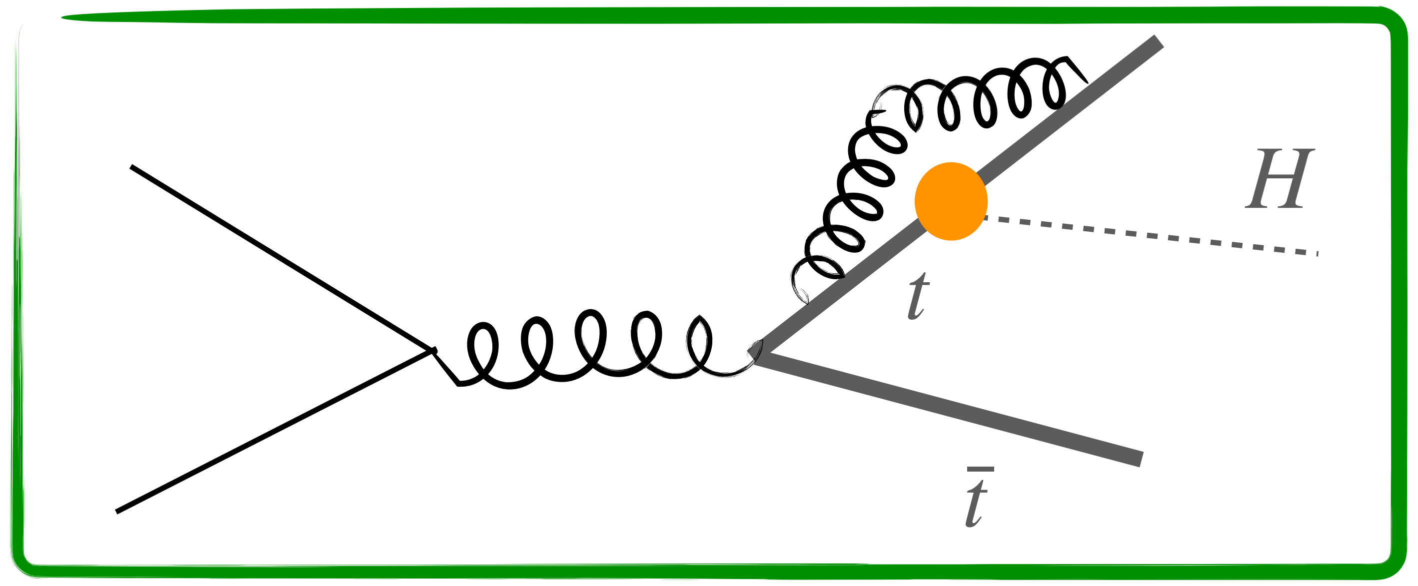
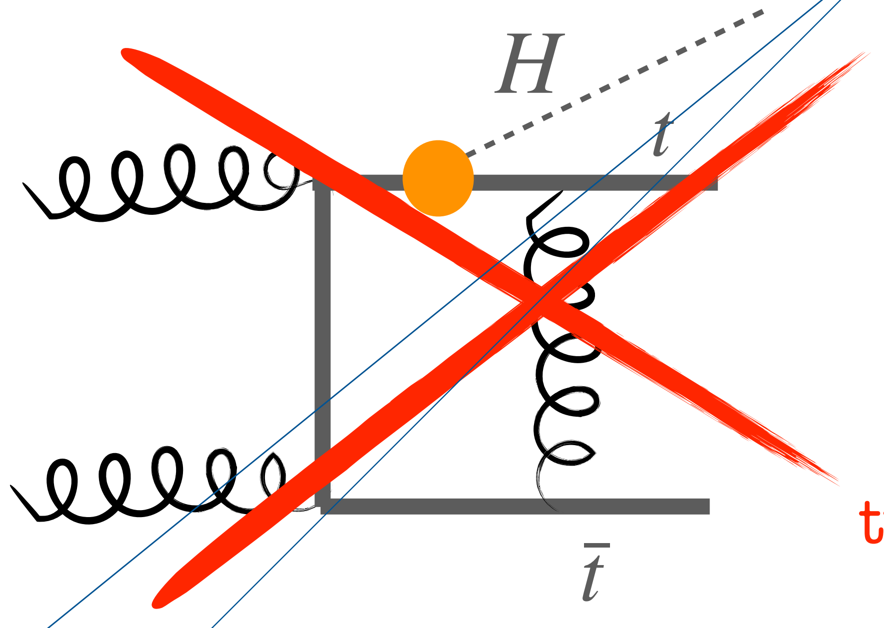
solution: development of a soft boson approximation

- already at one-loop, diagrams that are not captured by the naïve factorisation formula can give an **additional leading contribution** in the soft Higgs limit

one-particle diagrams





two-particle diagrams

- each 2P diagram contributes to the leading behaviour of the matrix element in the soft Higgs limit, if also the loop momentum is soft
- by considering all possible insertions of the Higgs boson on the top quark line, no additional contributions arise wrt the naïve factorisation formula

Soft Higgs boson approximation

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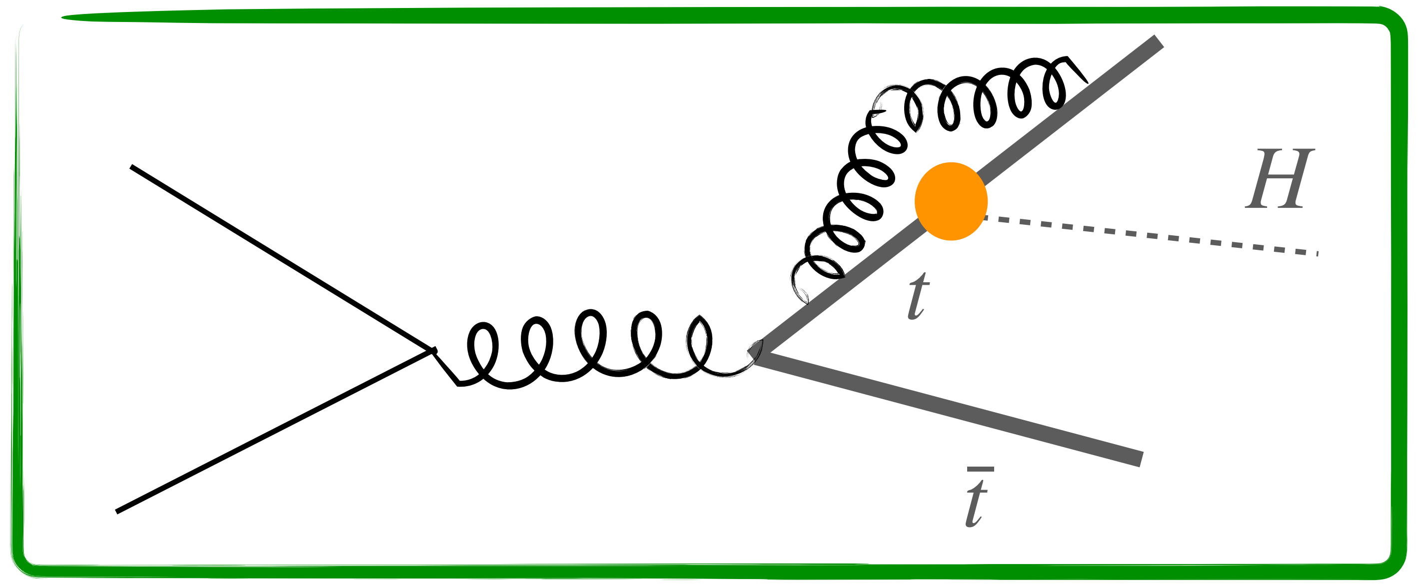

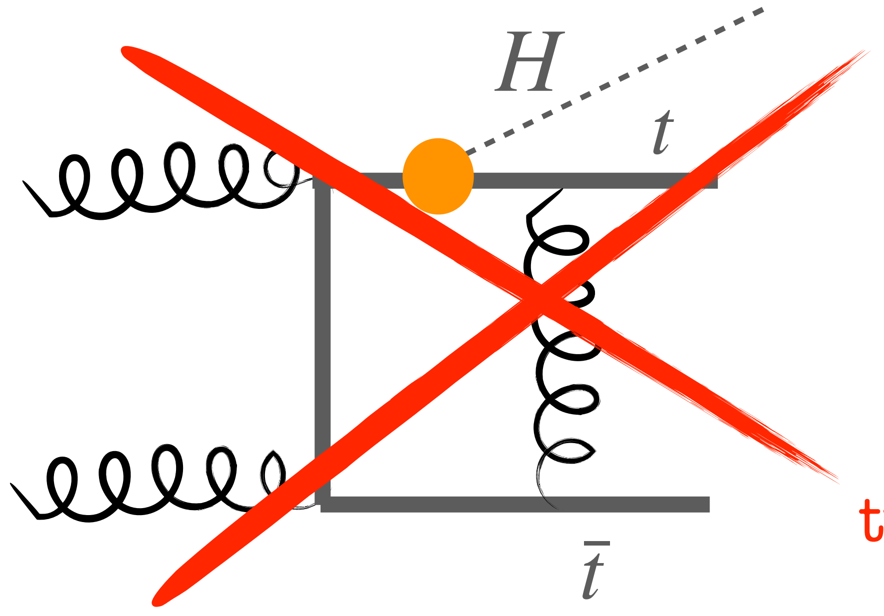

two-particle diagrams

- they give an additional contribution to the the naïve factorisation formula
- in other words, the renormalisation of the heavy-quark mass and wave function induces a modification of the Higgs coupling to the heavy quark

$$\lim_{k\to 0} \mathcal{M}(\{p_i\},k) = F(\alpha_s(\mu_R); m/\mu_R)\, J^{(0)}(k)\, \mathcal{M}(\{p_i\})$$

overall normalisation, finite, gauge-independent and perturbatively computable

renormalised mass

$$J^{(0)}(k) = \sum_j \frac{m}{v} \frac{m}{p_j \cdot k}$$

Soft Higgs boson approximation

- **master formula** in the soft Higgs limit ($k \to 0$, $m_H \ll m_t$)

$$\lim_{k\to 0} \mathcal{M}_{t\bar{t}H}(\{p_i\}, k) = F(\alpha_s(\mu_R); m_t/\mu_R)\, J^{(0)}(k) \mathcal{M}_{t\bar{t}}(\{p_i\})$$

[Bärnreuther, Czakon, Fiedler (2013)]

soft limit of the scalar form factor for the heavy quark [Bernreuther et al. (2005)] [Blümlein et al. (2017)]

$$F(\alpha_s(\mu_R); m_t/\mu_R) = 1 + \frac{\alpha_s(\mu_R)}{2\pi}(-3C_F) + \left(\frac{\alpha_s(\mu_R)}{2\pi}\right)^2 \left(\frac{33}{4}C_F^2 - \frac{185}{12}C_F C_A + \frac{13}{6}C_F(n_L+1) - 6C_F\beta_0 \ln\frac{\mu_R^2}{m_t^2}\right) + \mathcal{O}(\alpha_s^3)$$

up to two-loop order

we assume that all heavy quarks involved in the process have the same mass

- **NEW**: ongoing check of the soft factorisation formula at **three-loop order**, based on
  - three-loop on-shell renormalisation constants Z_m and Z_2 [Melnikov, Ritbergen (2000)]
  - decoupling relations at $\mathcal{O}(\alpha_s^3)$ [Chetyrkin, Kniehl, Steinhauser (1997)]
  - three-loop massive form factors [Fael, Lange, Schönwald, Steinhauser (2022, 2023)]

Soft W-boson approximation

bottleneck: the two-loop amplitudes are at the frontier of the current techniques

solution: development of a soft boson approximation

- goal: compute NNLO QCD corrections for $t\bar{t}W$
- the idea is to follow a similar approach used in the case of $t\bar{t}H$: develop a **soft factorisation formula** also in the case of a W boson (only coupling to massless quarks, masses break the factorisation…)
- for a **soft gauge** W **boson** radiated off a massless quark with momentum p_j, we find that

$$\lim_{k\to 0} \mathcal{M}(\{p_i\}, k) = \frac{g_W}{\sqrt{2}} \sum_j \left(\sigma_j \frac{p_j \cdot \epsilon^*(k)}{p_j \cdot k} \mathcal{M}_{j_L}(\{p_i\}) \right)$$

valid at all perturbative orders

$$\sigma_j = \begin{cases} +1 & \text{incoming } \bar{q}\,, \text{ outgoing } q \\ -1 & \text{incoming } q\,, \text{ outgoing } \bar{q} \end{cases}$$

amplitude where the massless quark with momentum p_j is LEFT-HANDED

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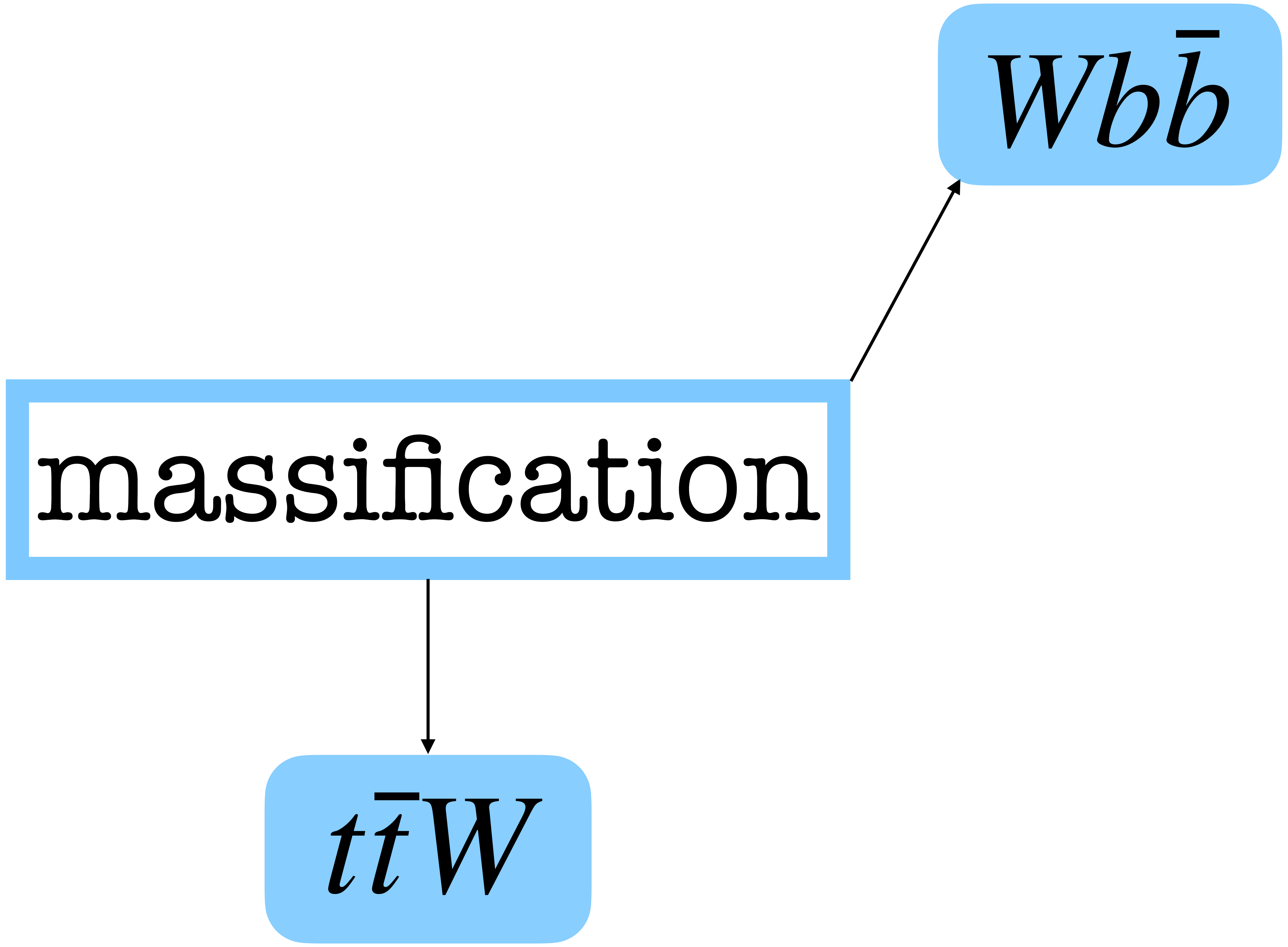
main differences between W boson and Higgs :
- vectorial vs scalar current
- massless vs massive emitters
- no renormalisation effects
- selection of the polarisation state of the emitter

[Buonocore, Devoto, Kallweit, Mazzitelli, Rottoli, **CS** (2022)]



[Buonocore, Devoto, Grazzini, Kallweit, Mazzitelli, Rottoli, **CS** (in preparation)]

Massification

[Moch, Mitov (2007)]
[Becher, Melnikov (2007)]

- idea: exploit the recently computed leading-colour **massless two-loop 5-point amplitudes** for $q\bar{q}' \to WQ\bar{Q}$ production
  [Abreu at al. (2021)] [Badger at al. (2021)]
- and apply the **massification** technique to reconstruct the corresponding massive amplitudes up to power corrections in the mass m_Q
- massification relies on the **factorisation** properties of massless QCD amplitudes (into jet, hard and soft functions)

$$|\mathcal{M}_{\rm p}\rangle = \mathcal{J}_0^{[\rm p]}\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \varepsilon\right) \mathcal{S}_0^{[\rm p]}\left(\{k_i\}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \varepsilon\right) |\mathcal{H}_{\rm p}\rangle$$

- when the mass is introduced, some of the collinear singularities are screened

$1/\epsilon$ poles are traded into $\log m_Q$

- in the limit $m_Q \ll Q$, the massive amplitude "shares" essential properties with the corresponding massless amplitude

change in the renormalisation scheme

- factorisation of massive QCD amplitudes (up to $\mathcal{O}(m_Q/Q)$)

$$|\mathcal{M}_{\rm p}\rangle = \mathcal{J}^{[\rm p]}\left(\frac{Q^2}{\mu^2}, \frac{m_i^2}{\mu^2}, \alpha_s(\mu^2), \varepsilon\right) \mathcal{S}_0^{[\rm p]}\left(\{k_i\}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \varepsilon\right) |\mathcal{H}_{\rm p}\rangle$$

ASSUMPTION: being the process-dependent soft and hard functions insensitive to collinear dynamics, they are assumed not to change, up to power corrections in the mass

Massification

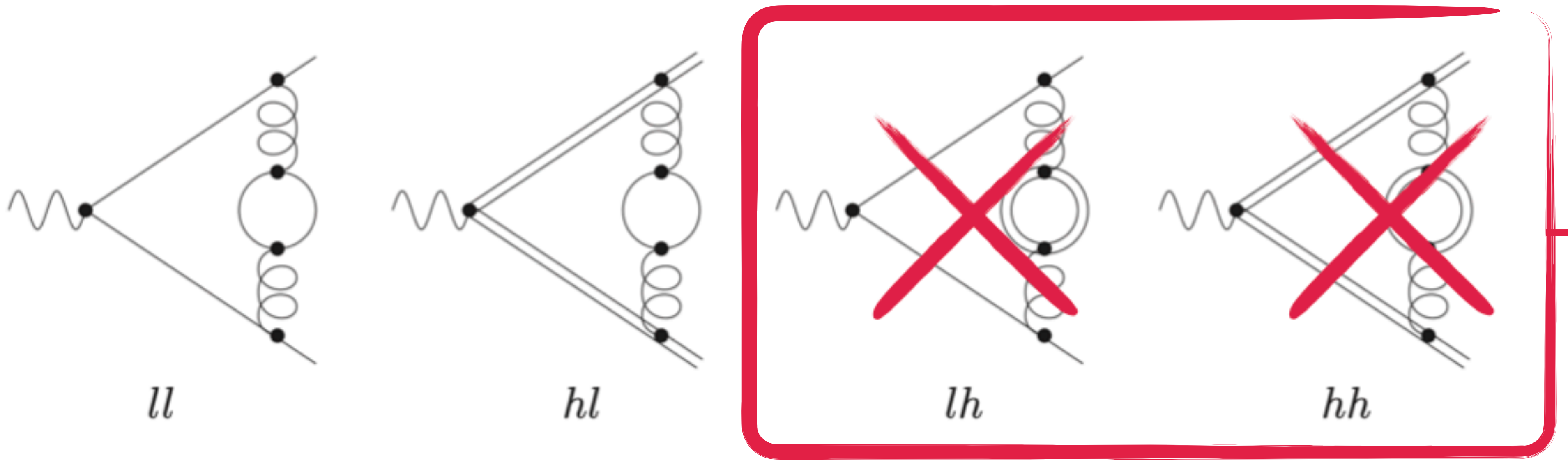
- the **master formula** is

$$\mathcal{M}^{[\mathrm{p}],(m)}\left(\{k_i\},\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\varepsilon\right)=\prod_{i\in\,\{\text{all legs}\}}\left(Z_{[i]}^{(m|0)}\left(\frac{m^2}{\mu^2},\alpha_s(\mu^2),\varepsilon\right)\right)^{\frac{1}{2}}\times\mathcal{M}^{[\mathrm{p}],(m=0)}\left(\{k_i\},\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\varepsilon\right)$$

universal, perturbatively computable, ratio of massive and massless form factors

$$Z_{[i]}^{(m|0)}\left(\frac{m^2}{\mu^2},\alpha_s,\varepsilon\right)=\mathcal{F}^{[i]}\left(\frac{Q^2}{\mu^2},\frac{m^2}{\mu^2},\alpha_s,\varepsilon\right)\left(\mathcal{F}^{[i]}\left(\frac{Q^2}{\mu^2},0,\alpha_s,\varepsilon\right)\right)^{-1}$$

- in the case of $WQ\bar{Q}$, the function $Z_{[q]}^{(m_Q|0)}$ is related to $\gamma^* q\bar{q}$ form factor



excluded, their description requires process-dependent information

Massification

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take-home message:
- the massification procedure predicts the correct ϵ poles, logarithms of the mass and mass independent terms of the massive amplitude
- power corrections in the mass and heavy-quark loop contributions cannot be retrieved

[Catani, Devoto, Grazzini, Kallweit, Mazzitelli, **CS** (2022)]

$t\bar{t}H$

[Buonocore, Devoto, Kallweit, Mazzitelli, Rottoli, **CS** (2022)]

$Wb\bar{b}$

Phenomenological results

$t\bar{t}W$

[Buonocore, Devoto, Grazzini, Kallweit, Mazzitelli, Rottoli, **CS** (in preparation)]

$t\bar{t}H$

Results

setup: NNLO NNPDF31, $m_H = 125 GeV$, $m_t = 173.3 GeV$, $\mu_R = \mu_F = (2m_t + m_H)/2$

- direct probe of the **top Yukawa coupling**
- HL-LHC projection: $\mathcal{O}(2\%)$ [CERN Yellow Report (2019)]
- current theoretical predictions: $\mathcal{O}(10\%)$
  [LHC cross section WG (2016)]
- mandatory to include **NNLO QCD** corrections!
- missing ingredient: 2loop $2 \rightarrow 3$ (2 masses) amplitudes
- prescription: **soft Higgs boson approximation**

all ingredients are computed exactly
except the two-loop contribution

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[Catani, de Florian, Ferrera, Grazzini (2015)]

- we construct a **mapping** to project a $t\bar{t}H$ event onto a $t\bar{t}$ one
- we test the **quality** of the approximation at Born and one-loop level: the observed deviation at NLO is used to estimate the uncertainty at NNLO
- at NNLO, the hard contribution is about **1%** of the LO cross section in gg and **2-3%** in $q\bar{q}$
- it is clear that the quality of the final result depends on the size of the contribution we are approximating

FINAL UNCERTAINTY:

$\pm 0.6\,\%$ on σ_{NNLO}, $\pm 15\,\%$ on $\Delta\sigma_{NNLO}$

$t\bar{t}H$

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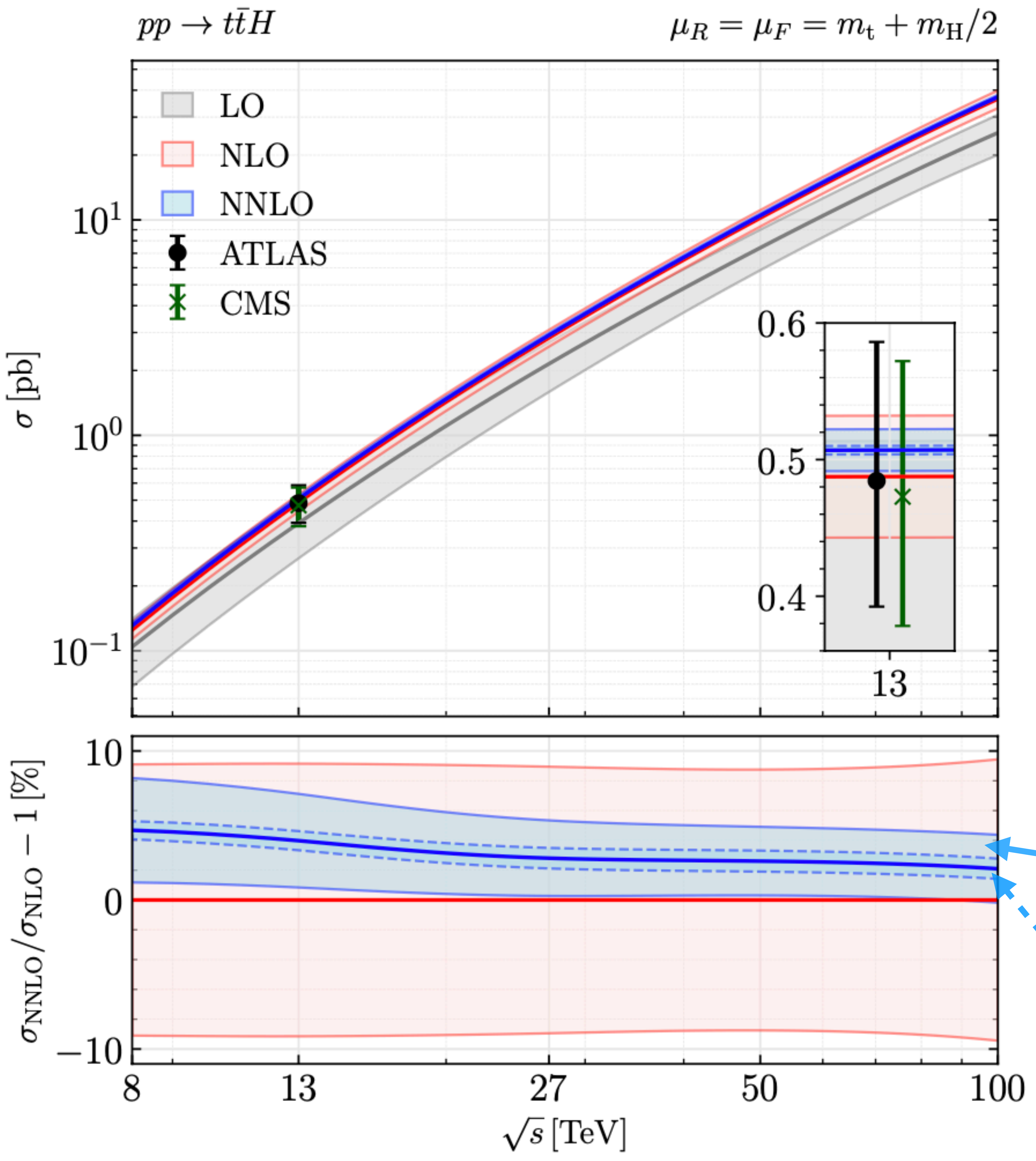

symmetrised 7-point scale variation

systematic + soft-approximation

σ [pb]	$\sqrt{s} = 13$ TeV	$\sqrt{s} = 100$ TeV
σ_{LO}	$0.3910^{+31.3\%}_{-22.2\%}$	$25.38^{+21.1\%}_{-16.0\%}$
σ_{NLO}	$0.4875^{+5.6\%}_{-9.1\%}$	$36.43^{+9.4\%}_{-8.7\%}$
σ_{NNLO}	$0.5070\,(31)^{+0.9\%}_{-3.0\%}$	$37.20(25)^{+0.1\%}_{-2.2\%}$

- @NLO: **+25 (+44)%** at $\sqrt{s} = 13\,(100)\,TeV$
- @NNLO: **+4 (+2)%** at $\sqrt{s} = 13\,(100)\,TeV$
- **significant reduction** of the perturbative uncertainties

$Wb\bar{b}$

Results

[CMS: arXiv 1608.07561]

setup: NNLO NNPDF31 4F, $\sqrt{s} = 8\,TeV$, $\mu_R = \mu_F = E_T(l\nu) + p_T(b_1) + p_T(b_2)$

$p_{T,l} > 30\,GeV \;\; |\eta_l| < 2.1\,, \;\; p_{T,b} > 25\,GeV \;\; |\eta_l| < 2.4\,, \;\; p_{T,j} > 25\,GeV \;\; |\eta_l| < 2.4$

- irreducible background to VH, single top production, BSM searches
- test of perturbative QCD: 4FS vs 5FS, **modelling of flavoured jets**
- large NLO QCD corrections
- mandatory to include **NNLO QCD** corrections!
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- mandatory to include **NNLO QCD** corrections!
- missing ingredient: 2loop $2 \to 3$ (2 masses) amplitudes
- prescription: **massification technique**

all ingredients are computed exactly except the two-loop contribution

- we construct a **mapping** to project the massive bottom momenta to the massless ones (preserve the four momentum of the $b\bar{b}$ pair)
- we rely on the leading-colour two-loop massless amplitudes for $W + 4$ partons [Abreu at al. (2021)] [Badger at al. (2021)]
- **reliability** of the procedure:
  - the discrepancy between the exact and massified virtual contribution at NLO is only **3%** of the NLO correction
  - the part of the two-loop virtual amplitude computed in LCA contributes at the **2%** level of the full NNLO correction

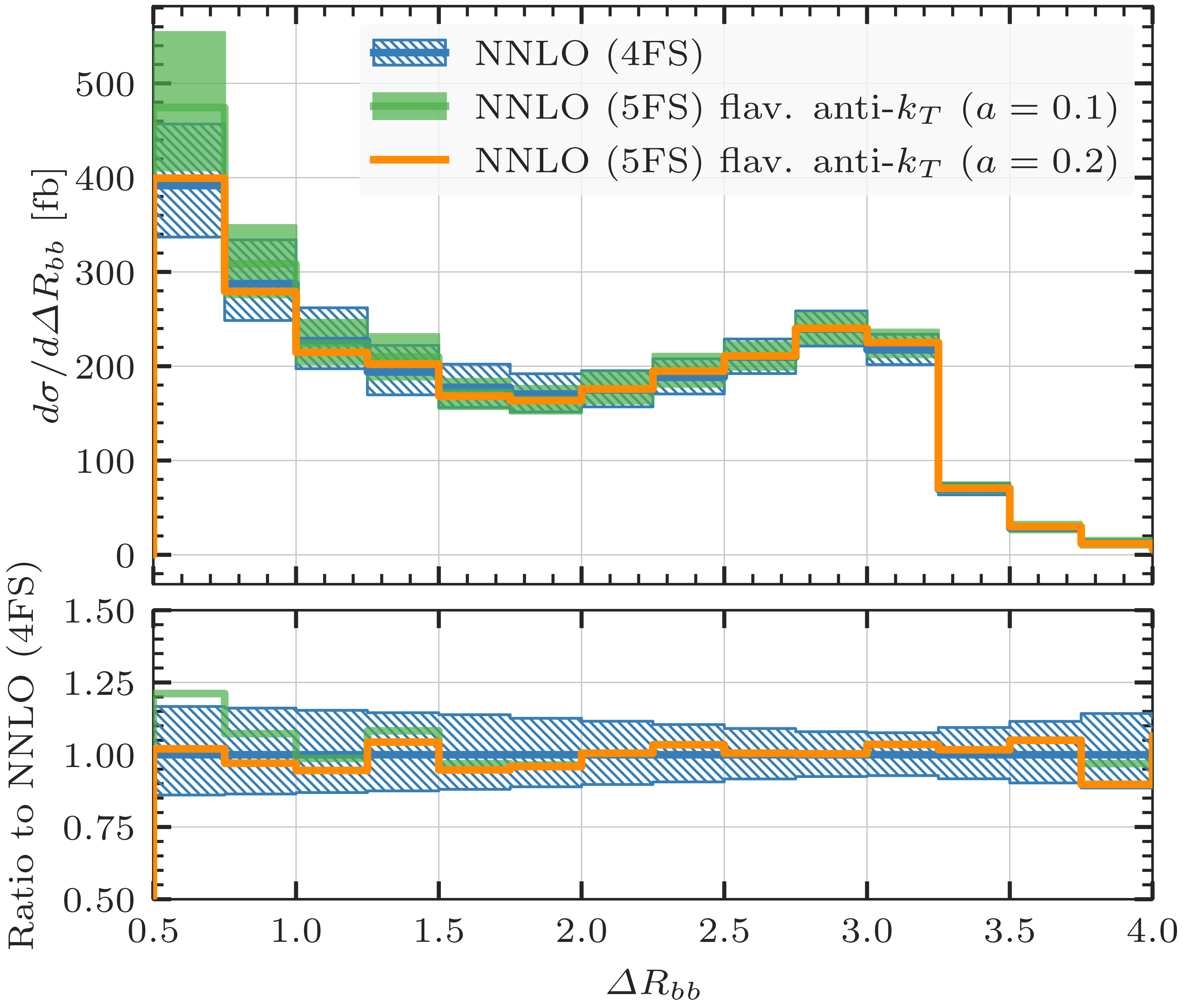
$Wb\bar{b}$

Results

[CMS: arXiv 1608.07561]

setup: NNLO NNPDF31 4F , $\sqrt{s} = 8\,TeV$, $\mu_R = \mu_F = E_T(l\nu) + p_T(b_1) + p_T(b_2)$

$p_{T,l} > 30\,GeV \ |\eta_l| < 2.1$, $p_{T,b} > 25\,GeV \ |\eta_l| < 2.4$, $p_{T,j} > 25\,GeV \ |\eta_l| < 2.4$



order	σ^{4FS} [fb]	$\sigma^{5FS}_{a=0.05}$ [fb]	$\sigma^{5FS}_{a=0.1}$ [fb]	$\sigma^{5FS}_{a=0.2}$ [fb]
LO	$210.42(2)^{+21.4\%}_{-16.2\%}$	$262.52(10)^{+21.4\%}_{-16.1\%}$	$262.47(10)^{+21.4\%}_{-16.1\%}$	$261.71(10)^{+21.4\%}_{-16.1\%}$
NLO	$468.01(5)^{+17.8\%}_{-13.8\%}$	$500.9(8)^{+16.1\%}_{-12.8\%}$	$497.8(8)^{+16.0\%}_{-12.7\%}$	$486.3(8)^{+15.5\%}_{-12.5\%}$
NNLO	$649.9(1.6)^{+12.6\%}_{-11.0\%}$	$690(7)^{+10.9\%}_{-9.7\%}$	$677(7)^{+10.4\%}_{-9.4\%}$	$647(7)^{+9.5\%}_{-9.4\%}$

- comparison against the 5F massless computation [Poncelet et al. (2022)]
  - overall **good agreement** within the scale uncertainties
  - the uncertainties due to variation of $m_b \in [4.2, 4.92]\ GeV$ are at **2%** level (smaller than the ones due to the variation of a, ~**7%**)
- large positive NNLO corrections: **+40%**
- still **large** perturbative **uncertainties**

$t\bar{t}W$

Results

setup: NNLO NNPDF31 luxqed, $\sqrt{s} = 13\,TeV$, $m_W = 80.385 GeV$, $m_t = 173.2 GeV$, $\mu_R = \mu_F = (2m_t + m_W)/2$

- relevant background for SM processes ($t\bar{t}H$, $t\bar{t}t\bar{t}$)
- multi-lepton signature relevant for BSM sources
- "**special**": large NLO QCD and EW corrections
- well known **tension** between theory and experiments (excess at **1-2σ level**)

  [ATLAS-CONF-2023-019]
  [CMS: arXiv 2208.06485]

- current **NLO QCD + EW** predictions, supplemented with **multi-jet merging** are affected by relatively large uncertainties
- mandatory to include **NNLO QCD** corrections!
- missing ingredient: 2loop $2 \to 3$ (2 masses) amplitudes

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▷ we exploit both approximations: **soft approximation** and **massification technique**

all ingredients are computed exactly except the two-loop contribution

$t\bar{t}W$

Results

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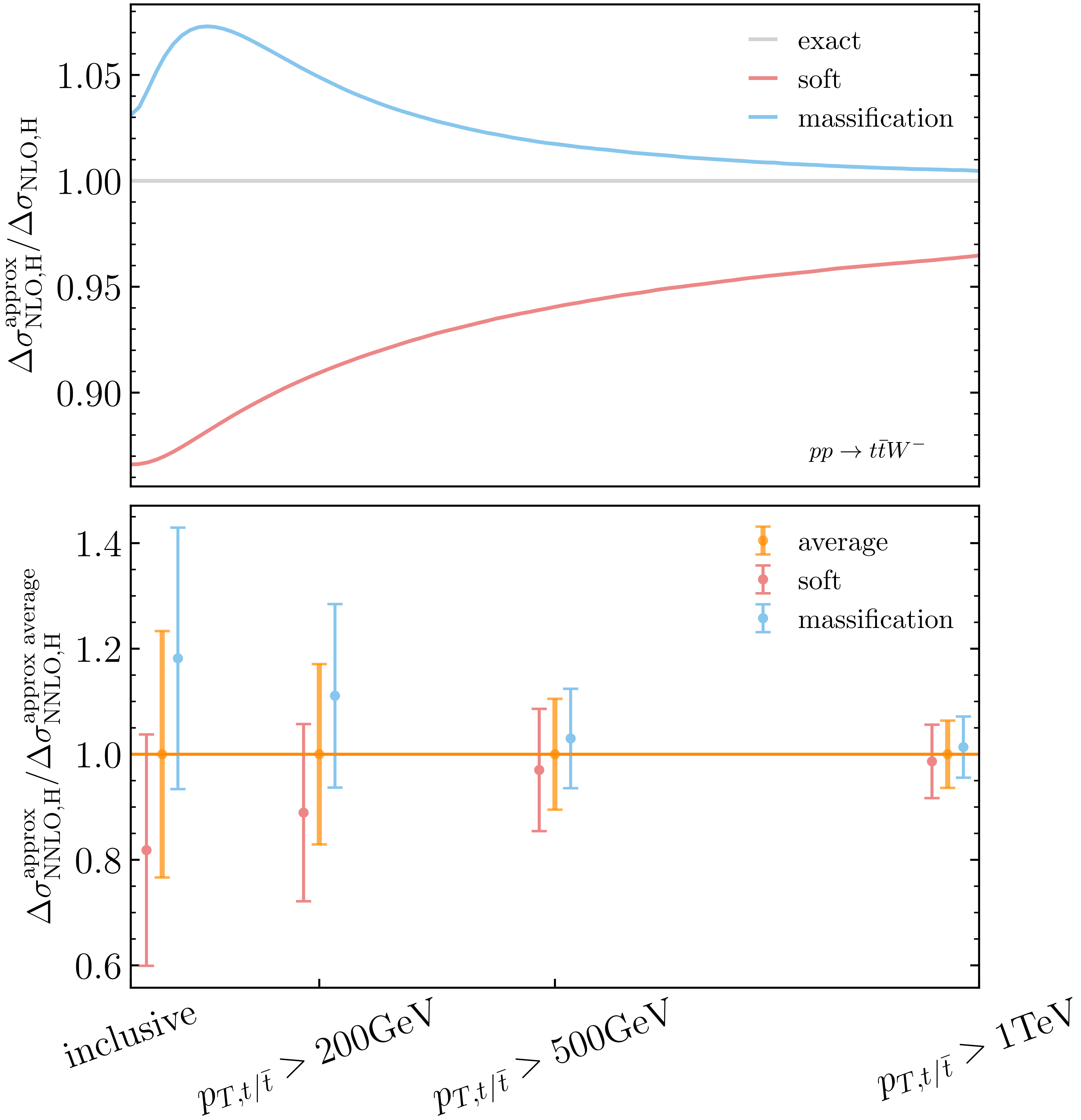
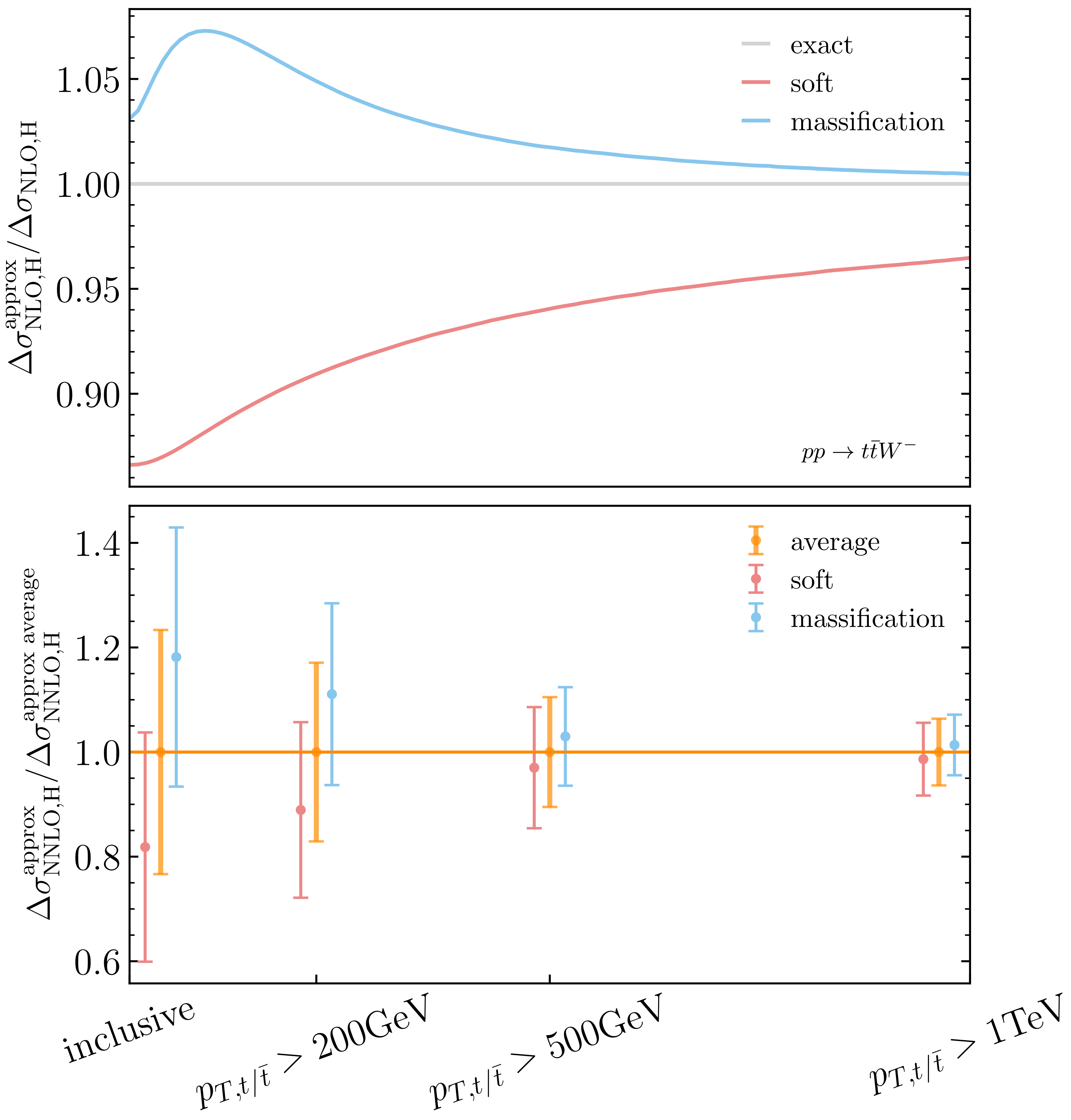


- validation at NLO:
  - both approaches provide a **good quantitative approximation** of the exact virtual coefficient (discrepancy of 5-15%)
  - the soft approximation tends to **undershoot** the exact result while the massification **overshoots** it
  - clear **asymptotic behaviour** towards the exact result for high $p_{T,t}$ where both approximations are expected to perform better (faster convergence of the massification)

$t\bar{t}W$

Results

setup: NNLO NNPDF31 luxqed, $\sqrt{s} = 13\,TeV$, $m_W = 80.385GeV$, $m_t = 173.2GeV$, $\mu_R = \mu_F = (2m_t + m_W)/2$



- based on the validation at NLO, we define **our best prediction** at NNLO as the **average** of the two approximated results
- the **conservative uncertainty** on the approximated two-loop contribution is defined by linearly combining the uncertainties on the two approximations
- the two-loop contribution turns out to be **6-7%** of the NNLO cross section (both for $t\bar{t}W^+$ and $t\bar{t}W^-$)

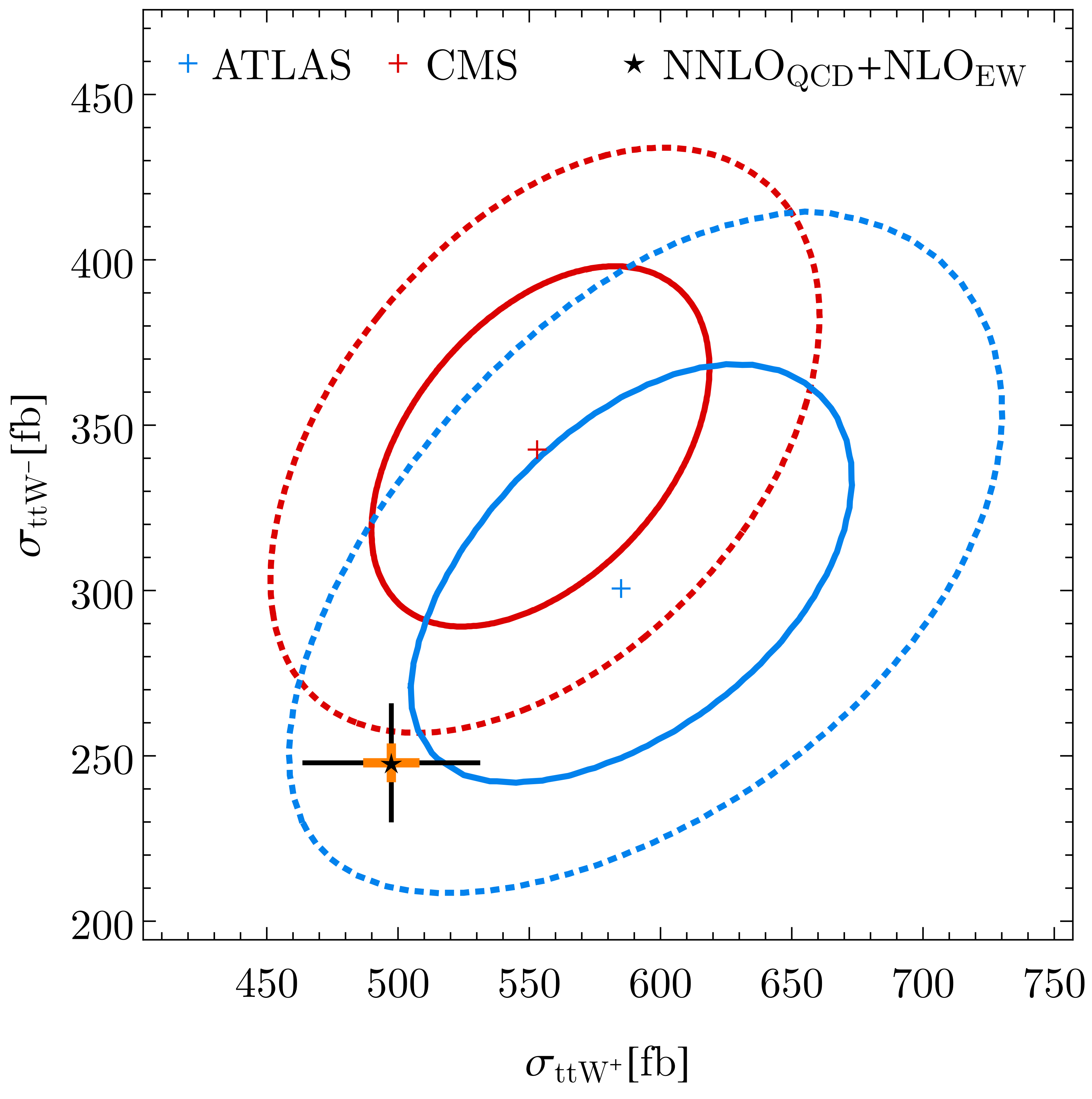
FINAL UNCERTAINTY:

$\pm 1.8\,\%$ on σ_{NNLO}, $\mathcal{O}(25\%)$ on $\Delta\sigma_{NNLO,H}$

$t\bar{t}W$

Results

setup: NNLO NNPDF31 luxqed, $\sqrt{s} = 13\,TeV$, $m_W = 80.385 GeV$, $m_t = 173.2 GeV$, $\mu_R = \mu_F = (2m_t + m_W)/2$



- comparison against the most recent ATLAS and CMS data:
  - the **agreement is at the 1σ and 2σ level** respectively
  - reduction of the perturbative scale uncertainties
  - systematic uncertainties due the two-loop approximation are under control and much smaller than the scale uncertainties

take-home message:
two completely different approximations lead to compatible results for the missing two-loop virtual contribution!!

[ATLAS-CONF-2023-019]
[CMS: arXiv 2208.06485]

Summary & Outlook

summary:

- the current and expected precision of LHC data requires **NNLO QCD predictions**
- the actual frontier is represented by NNLO corrections for $2 \to 3$ processes with **several massive external legs**
- the IR divergencies are regularised within the q_T **-subtraction** framework: two-loop soft function for arbitrary kinematics
- the only missing ingredient is represented by the **two-loop amplitudes**:
  - first approximation based on a soft boson factorisation formula
  - second approximation based on the massification procedure of the corresponding massless amplitudes
- for all three processes considered ($t\bar{t}H$, $Wb\bar{b}$, $t\bar{t}W$), we have a **good control of the systematic uncertainties** associated to the approximation (much smaller than the perturbative uncertainties)

outlook:

- test the performance of the soft approximation in a fiducial setup and at the differential level
- match the $Wb\bar{b}$ fixed order calculation to parton shower
- explore other processes of the same class!

BACKUP SLIDES

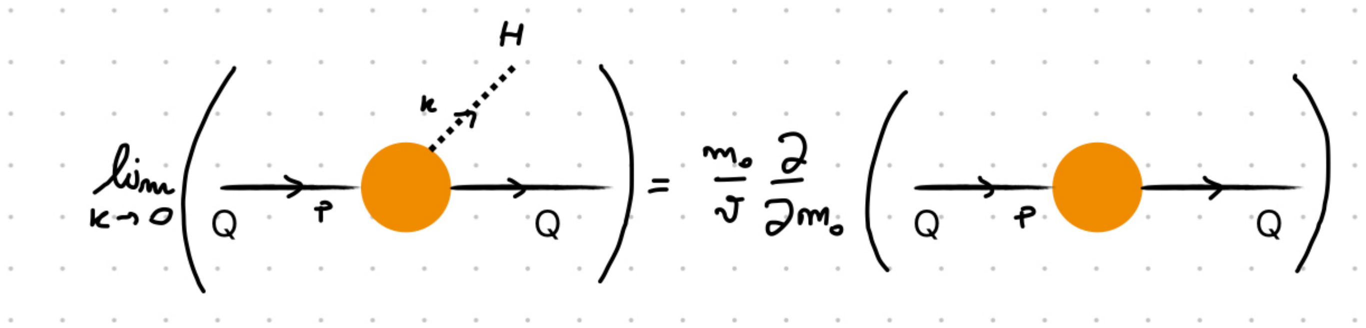
Soft Higgs approximation: more details

[Shifman, Vainshtein, Voloshin, Zakharov (1979)]
[Kniehl, Spira (1995)]

- the effective coupling can also be derived by exploiting Higgs **low-energy theorems** (LETs)

$$\lim_{k\to 0} \mathscr{M}^{bare}_{Q\to QH}(p,k) = \frac{1}{v}\frac{\partial}{\partial \log m_0} \mathscr{M}^{bare}_{Q\to Q}(p)\Big|_{p^2=m^2}$$

heavy-quark self-energy



In the soft limit, the Higgs boson is not a dynamical d.o.f.

Its effect is to shift the mass of the heavy quark:

$$m_0 \to m_0\left(1+\frac{H}{v}\right)$$

$$\mathscr{M}^{bare}_{Q\to Q}(p) = \bar{Q}_0\left\{m_0[-1+\Sigma_S(p)] + \not{p}\,\Sigma_V(p)\right\} Q_0$$

[Broadhurst, Grafe, Gray, Schilcher (1990)]
[Broadhurst, Gray, Schilcher (1991)]

$$\Sigma_S(p) = -\sum_{n=1}^{+\infty}\left[\frac{g_0^2}{(4\pi)^{D/2}(p^2)^\epsilon}\right]^n \left(A_n(m_0^2/p^2) - B_n(m_0^2/p^2)\right)$$

$$\Sigma_V(p) = -\sum_{n=1}^{+\infty}\left[\frac{g_0^2}{(4\pi)^{D/2}(p^2)^\epsilon}\right]^n B_n(m_0^2/p^2)$$

- renormalisation of the quark mass and wave function $m_0\bar{Q}_0 Q_0 = m\bar{Q}QZ_mZ_2$
- $\overline{MS}$ renormalisation of the strong coupling + decoupling of the heavy quark

[Chetyrkin, Kniehl, Steinhauser (1997)]