A gauge choice for infrared singularities

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Interpolating gauge

- I describe *interpolating gauge*, invented by Doust (1987) and by Baulieu and Zwanziger (1999).
- Our interest is in simplifying the description of the soft and collinear singularities of QCD.
- This may be useful for defining subtractions that remove the soft and collinear singularities in perturbative QCD calculations.
- Our particular interest is in defining the splitting functions for a parton shower at order α_s^2.

- Interpolating gauge interpolates between Feynman gauge (or Lorenz gauge) and Coulomb gauge.

- Doust and Baulieu and Zwanziger were interested in providing a better definition of Coulomb gauge.

- With our different goal, we adopt a different notation and emphasize different features of the gauge.

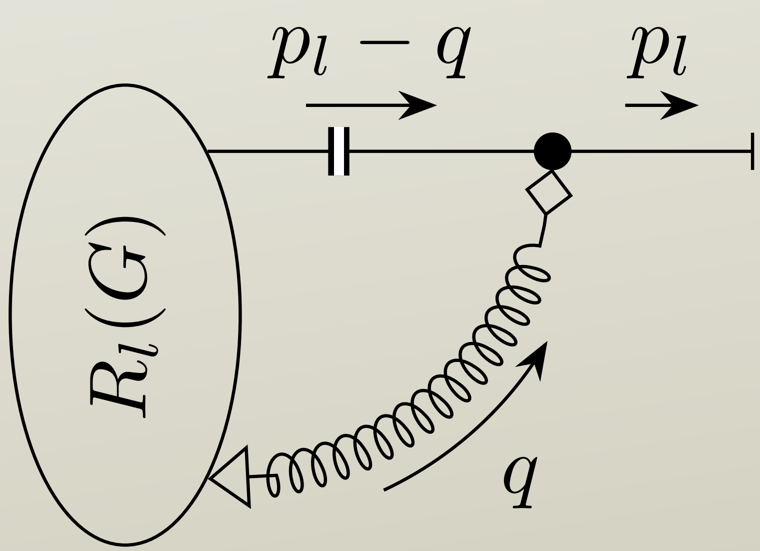
- We also explore technical issues in some detail.

Why not Feynman gauge?

- The gluon propagator in Feynman gauge is very simple.

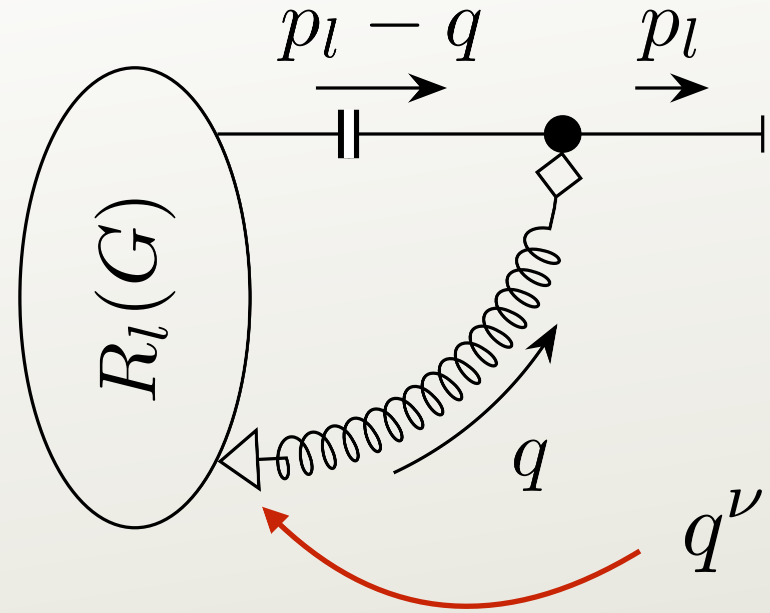
- But consider a virtual gluon with momentum q that couples to an external line with momentum p_l.

$$\sum_l \int \frac{d^d q}{(2\pi)^d} \sum_G$$



- There are collinear singularities that give a logarithmic divergences from $q \to x p_l$.

$$\sum_l \int \frac{d^d q}{(2\pi)^d} \sum_G$$

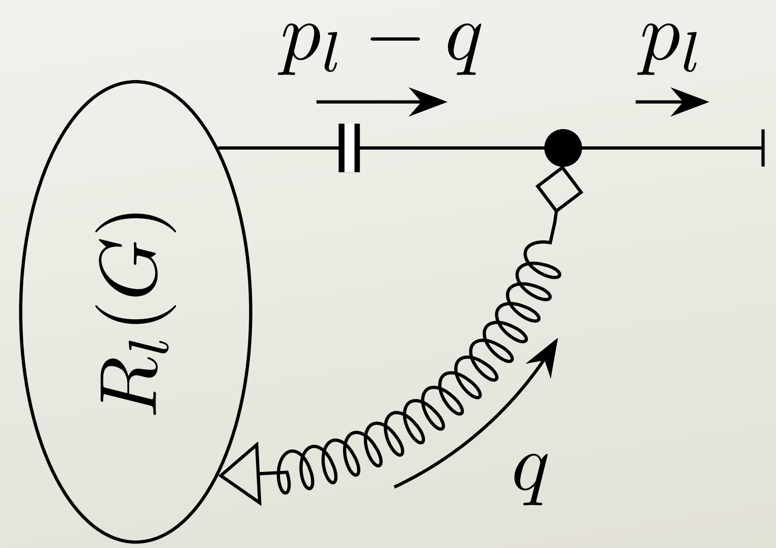


- The collinear divergences appear even when the gluon couples to an off-shell internal line in the graph.
- The gluon has an unphysical polarization:

$$J_\mu D^{\mu\nu}(q) \propto q^\nu$$

- We can use Ward identities to get rid of these.
- But this is more complicated when there are multiple gluons collinear to different external partons.
- *Cf.* C. Anastasiou and G. Sterman, *Locally finite two-loop QCD amplitudes from IR universality for electroweak production*, JHEP 05 (2023) 242.

$$\sum_l \int \frac{d^d q}{(2\pi)^d} \sum_G$$



- It might be better if these unphysical collinear singularities did not occur.

Definition of interpolating gauge

- Use a special reference frame defined by a vector n, with $n^2 = 1$.
- Define a tensor $h^{\mu\nu}$ with components in the $\vec{n} = 0$ frame

$$h^{\mu\nu} = \begin{pmatrix} 1/v^2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- For any vector q we define an associated vector $\tilde{q}$ by

$$\tilde{q}^\mu = h^\mu_\nu \, q^\nu$$

$$\tilde{q} = \left(\frac{q^0}{v^2}, \vec{q} \right)$$

- Use gauge fixing condition $G[A] = 0$ with

$$G[A]_c(x) = \tilde{\partial}_\mu A_c^\mu(x) - \omega_c(x)$$

- Compare this to

$$G'[A]_c(x) = \partial_\mu A_c^\mu(x) - \omega_c(x)$$

for a covariant gauge.

- The gauge fixing Lagrangian is

$$\mathcal{L}_{\mathrm{GF}}(x) = -\frac{v^2}{2\xi}\,(\tilde{\partial}_\mu A_a^\mu(x))(\tilde{\partial}_\nu A_a^\nu(x))$$

- Parameters ξ and v (and n) determine the gauge choice.

- The gluon propagator is then

$$D^{\mu\nu}(q) = \frac{1}{q^2 + \mathrm{i}0}\left[- g^{\mu\nu} + \frac{q^\mu\, \tilde{q}^\nu + \tilde{q}^\mu\, q^\nu}{q\cdot\tilde{q} + \mathrm{i}0} - \left(1 + \frac{1}{v^2}\right) \frac{q^\mu\, q^\nu}{q\cdot\tilde{q} + \mathrm{i}0}\right] - \frac{\xi - 1}{v^2}\, \frac{q^\mu\, q^\nu}{(q\cdot\tilde{q} + \mathrm{i}0)^2}$$

- Usually we choose $\xi = 1$.

The gluon propagator

- We divide the propagator into two parts:

$$D^{\mu\nu}(q) = D_{\mathrm{T}}^{\mu\nu}(q) + D_{\mathrm{L}}^{\mu\nu}(q)$$

- Choose $\xi = 1$. Then

$$D_{\mathrm{T}}^{\mu\nu}(q) = \frac{1}{q^2 + \mathrm{i}0} \sum_{s=1,2} \varepsilon^{\mu}(q,s)\,\varepsilon^{\nu}(q,s)$$

- Here $\varepsilon(q,s) \cdot \varepsilon(q,s') = -\delta_{s,s'}$ and

$$\varepsilon(q,s) \cdot n = 0$$

$$\varepsilon(q,s) \cdot q = 0$$

- This describes the propagation of transversely polarized gluons.

- The propagator for the L-gluons is

$$D^{00}_{\mathrm{L}}(q) = -\frac{1}{q\cdot\tilde{q} + \mathrm{i}0}\ ,$$

$$D^{0i}_{\mathrm{L}}(q) = D^{i0}_{\mathrm{L}}(q) = 0\ ,$$

$$D^{ij}_{\mathrm{L}}(k) = \frac{1}{v^2}\,\frac{1}{q\cdot\tilde{q} + \mathrm{i}0}\,\frac{q^i q^j}{\vec{q}^{\,2}}$$

- Note the denominators

$$\frac{1}{q\cdot\tilde{q} + \mathrm{i}0}$$

- $q\cdot\tilde{q} = (q^0)^2/v^2 - \vec{q}^{\,2}$, the condition for on-shell propagation is

$$q^0 = \pm v|\vec{q}\,|$$

- The L-gluons propagate with speed v in the $\vec{n} = 0$ frame.

- If we take $v = 1$, we get Feynman gauge for $\xi = 1$ and Lorenz gauge for $\xi = 0$.

- If we take $v \to \infty$, we get Coulomb gauge.

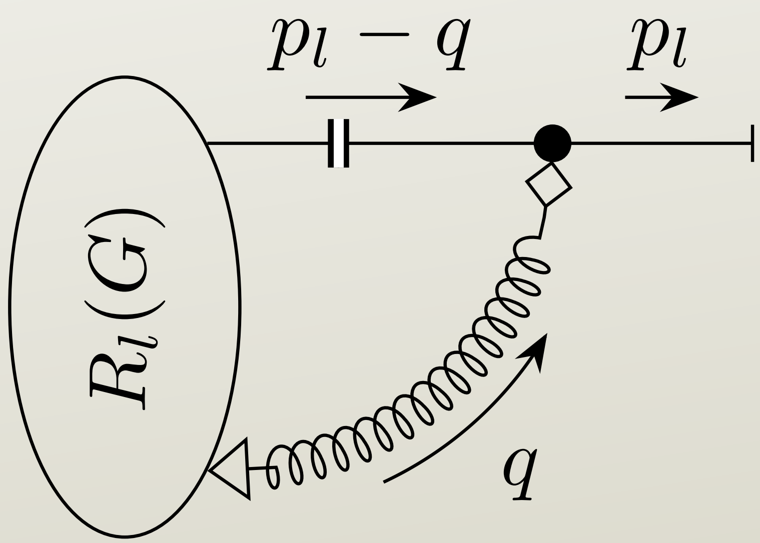
- The L-gluons give the Coulomb force, which propagates with infinite speed in the $\vec{n} = 0$ frame.

- Now Coulomb gauge is defined as a limit.

- We do not need $v \to \infty$: $v = 2$ is fine.

Why might interpolating gauge be useful?

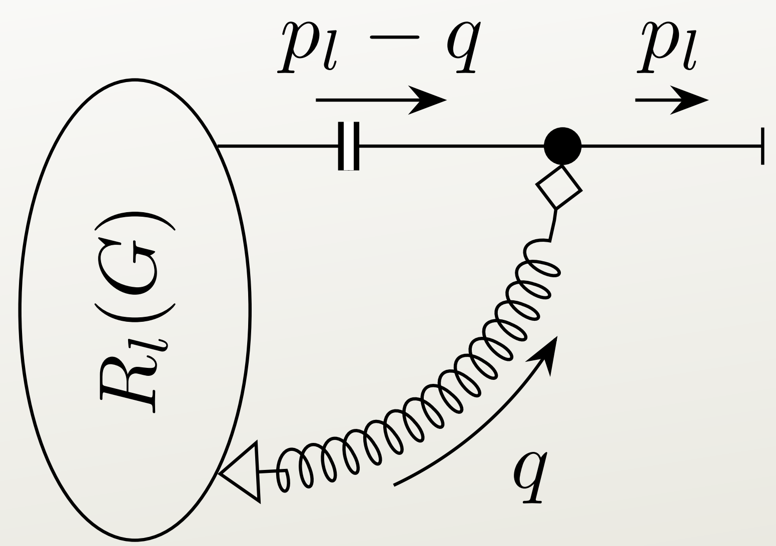
$$\sum_l \int \frac{d^d q}{(2\pi)^d} \sum_G$$



- T-gluons do not give collinear divergences except for self-energy insertions on an external leg.

- That is because $q \cdot \varepsilon(q, s) = 0$.

$$\sum_l \int \frac{d^d q}{(2\pi)^d} \sum_G$$



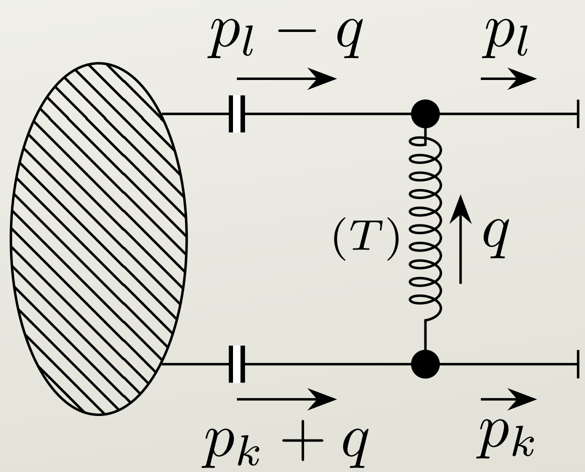
- L-gluons do not give collinear divergences.

- That is because if $p_l^2 = 0$ and $q = x p_l$ then $(p_l - q)^2 = 0$ but $q \cdot \tilde{q} \neq 0$.

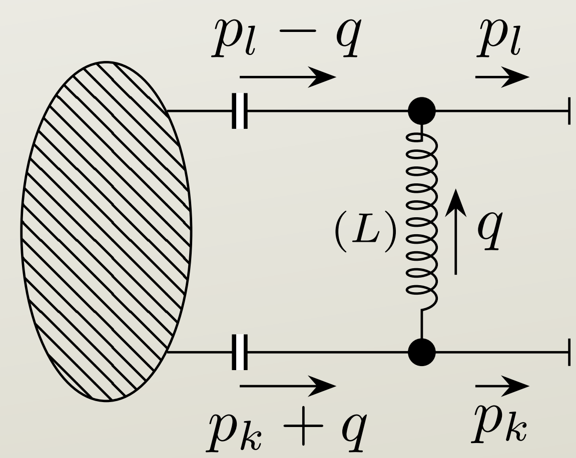
- Thus interpolating gauge is like a physical gauge with respect to collinear divergences.

- Both T-gluons and L-gluons create soft $(q \to 0)$ divergences when they couple to two external legs.

$$\sum_{l<k} \int \frac{d^d q}{(2\pi)^d}$$



$$\sum_{l<k} \int \frac{d^d q}{(2\pi)^d}$$



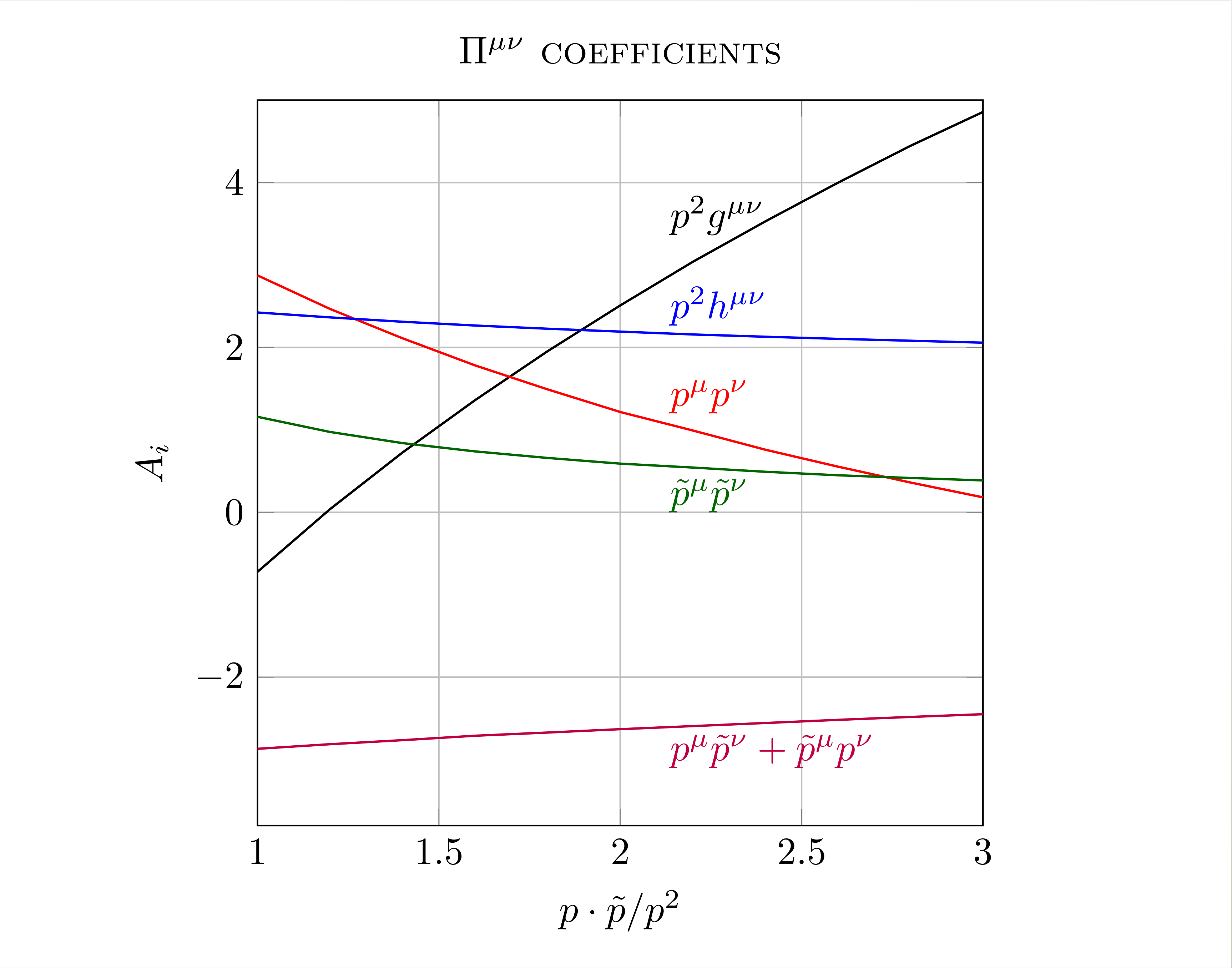
- These are soft divergences, but without collinear divergences.

Technical issues

- Renormalization works. We calculate $[Z_A^{1/2}]^{\mu}_{\nu}$, $Z_\psi^{1/2}$, Z_η, Z_g, Z_v, and Z_ξ at order $\alpha_{\rm s}$.

- BRST invariance shows that the S-matrix is independent of v, ξ, and n.

- One can calculate loop integrals with the help of Feynman parameterization and then numerical integration:



Conclusion

- Interpolating gauge may be useful for calculations that aim to isolate soft and collinear singularities of QCD.