Azimuthal Asymmetries in Drell-Yan and Semi-Inclusive DIS Beyond Leading Order

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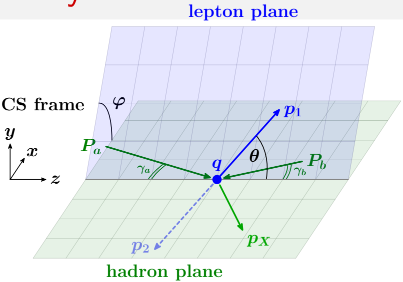
Intro: Azimuthal Asymmetries in Drell-Yan



[notation from 2006.11382]

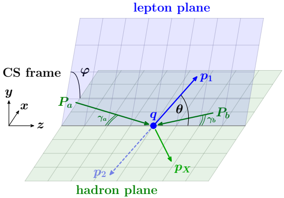
- $pp \to \gamma^*/Z \to \ell^+\ell^-$, angular coefficient $A_i = W_i/W_{\rm unpol}$

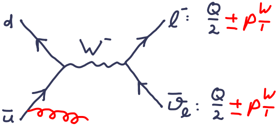
$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4q\,\mathrm{d}\!\cos\theta\,\mathrm{d}\varphi}$$
$$\sim L_+\Big[(1+\cos^2\theta)W_{\rm unpol} + \frac{1}{2}\sin^2\theta\cos(2\varphi)W_2 + \sin(2\theta)\cos\varphi W_1 + \sin(2\theta)\sin\varphi W_6\Big]$$
$$+ L_-\Big[\sin\theta\cos\varphi W_3 + \sin\theta\sin\varphi W_7\Big] + \cdots$$

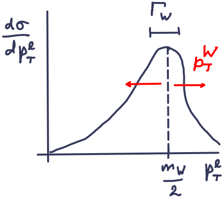
- Azimuthal Asymmetries: structure functions odd under $\varphi \to \varphi + \pi$
- This talk: $\lambda = \frac{q_T}{Q} \to 0$ limit of azimuthal asymmetries

Motivation: m_W Measurement at the LHC

- $W \to \ell\nu$: need theory input since neutrino is lost







$$m_W^{\text{LHCb}} = 80354 \pm 23_{\text{stat.}} \pm 10_{\text{exp.syst.}} \pm 17_{\text{theory}} \pm 9_{\text{PDF}} \text{ MeV}$$ [LHCb, 2109.01113]

Most theory uncertainty comes from their model for W_3!

- Jacobian peak $\sim \lambda = \frac{q_T}{Q} \to 0$: W_i's have interesting pert. structure

$$q_T \frac{\mathrm{d}\sigma^{(0)}}{\mathrm{d}q_T} \sim \alpha_s (L+1) + \alpha_s^2 \left(L^3 + L^2 + L + 1\right) + \cdots\,, \qquad \text{where } L = \ln(q_T/Q)\,,$$

$$q_T \frac{\mathrm{d}\sigma^{(1)}}{\mathrm{d}q_T} \sim \frac{q_T}{Q} \Big\{ \alpha_s + \alpha_s^2 (L^2 + L + 1) + \alpha_s^3 (L^4 + \cdots) + \cdots \Big\}\,,$$

Azimuthal Asymmetries: structure functions start at $\mathcal{O}(\lambda)$ as $q_T \to 0$,

$\Rightarrow$ Factorization theorems $W_i \sim H \otimes \bar{B} \otimes B \otimes S$ needed to resum large logs; B, S: transverse-momentum-dependent (TMD) beam, soft functions

Motivation: Improve Slicing Method for q_T Subtraction

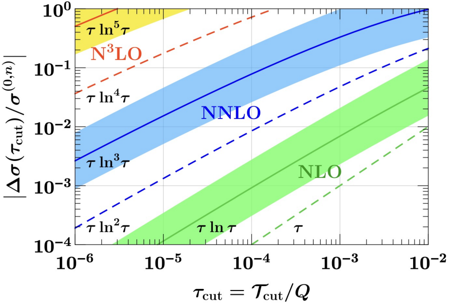
for observables w./o. $\varphi \to \varphi + \pi$ symmetry, e.g. p_T^ℓ

$$\sigma(X) = \sigma^{\rm sub}\left(X, q_T^{\rm cut}\right) + \int_{q_T^{\rm cut}}^{\infty} {\rm d}q_T \frac{{\rm d}\sigma(X)}{{\rm d}q_T} + \Delta\sigma\left(X, q_T^{\rm cut}\right)$$

- In current applications: $\sigma^{\rm sub} = \sigma^{\rm LP}$
- Numerical impact of Logarithms in error term

$$\Delta\sigma \sim \alpha_s^n \left(q_T^{\rm cut}/Q\right) \ln^m q_T^{\rm cut}/Q$$

becomes worse at higher orders



[Illustration from 1612.00450]

⇒ With analytic NLP logs predicted in $\Delta\sigma$, one can increase $q_T^{\rm cut}$ (reduce computing time for Monte-Carlo calculations)

Motivations

- $W_{6,7}$ start at $\mathcal{O}(\alpha_s^2)$ from real-virtual contributions:

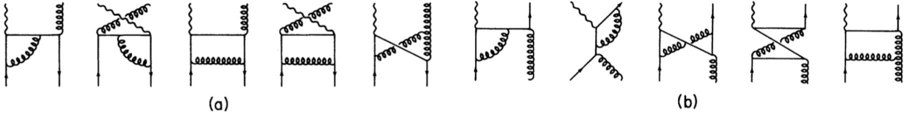


FIG. 1. Feynman diagrams contributing to the absorptive part of the parton scattering amplitude, (a) quark + antiquark → W + gluon and (b) quark + gluon → W + quark, in one-loop order. Wavy lines denote W and curly lines denote gluons.

[Analytic expressions in Phys.Rev.Lett. 52 (1984) 1076]

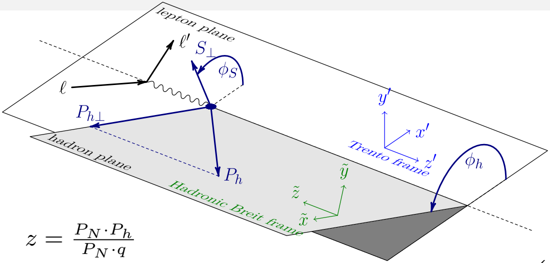
[For angular coeffs from MC: 1708.00008 NNLO$_1$ QCD, 2204.12394 NNLO$_1$ QCD + NLO$_1$ EW]

$\Rightarrow$ Powerful near-term analytic checks of subl. power fact. theorems

Motivation for factorizing $\mathcal{O}(q_T/Q)$ structure functions in Drell-Yan

- Universal functions (same pert. or nonpert. objects) in different obs.
- Making pheno predictions that incorporate fixed order and large log resummation
- Improve slicing methods

Azimuthal Asymmetries in Semi-Inclusive DIS (SIDIS)



- $y = \frac{P_N \cdot q}{P_N \cdot p_\ell}\,, \quad z = \frac{P_N \cdot P_h}{P_N \cdot q}$
- Different polarization contributions of lepton/hadron $\left(\epsilon = \frac{1-y}{1-y+\frac{1}{2}y^2}\right)$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}^2\vec{P}_{hT}} = \frac{\pi\alpha^2}{Q^2}\frac{y}{z}\frac{\kappa_\gamma}{1-\epsilon}\Big[(L{\cdot}W)_{UU} + \lambda_\ell (L{\cdot}W)_{LU}$$

$$+ S_L (L{\cdot}W)_{UL} + \lambda_\ell S_L (L{\cdot}W)_{LL} + S_T (L{\cdot}W)_{UT} + \lambda_\ell S_T (L{\cdot}W)_{LT}\Big]$$

⇒ In total 18 structure functions [Bacchetta et al '06]

$$(L{\cdot}W)_{UU} = W_{UU,T} + \epsilon W_{UU,L} + \sqrt{2\epsilon(1+\epsilon)}\cos(\phi_h) W_{UU}^{\cos(\phi_h)} + \epsilon\cos(2\phi_h) W_{UU}^{\cos(2\phi_h)}\,,$$

$$(L{\cdot}W)_{LU} = \sqrt{2\epsilon(1-\epsilon)}\sin(\phi_h) W_{LU}^{\sin(\phi_h)}\,,$$

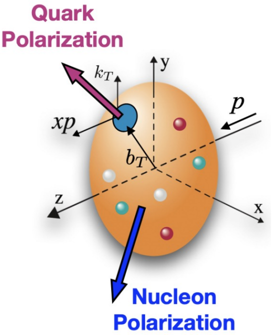
$$(L{\cdot}W)_{LT} = \sqrt{1-\epsilon^2}\cos(\phi_h-\phi_S) W_{LT}^{\cos(\phi_h-\phi_S)}$$

$$+ \sqrt{2\epsilon(1-\epsilon)}\left[\cos(\phi_S) W_{LT}^{\cos(\phi_S)} + \cos(2\phi_h-\phi_S) W_{LT}^{\cos(2\phi_h-\phi_S)}\right]\,,$$

......

Motivations for SIDIS

- Relevant for the Electron-Ion Collider
- Effect can sometimes be large, e.g. Cahn effect
- 3D structure of hadrons
- Spin dependence
- Universality of TMD functions
- Long-standing/challenging/interesting problem
- ...



[TMD handbook '23]

History and Current Status

- Intrinsic transverse momentum of partons inside hadrons ⇒ $W_{UU}^{\cos\phi_h} \sim \mathcal{F}\left[\frac{k_{Tx}}{Q} f_1 D_1\right]$ [Cahn '78, '79]
- A more careful parton model calculation [Mulders, Tangerman '95]

$$\frac{x}{2z} W_{UU}^{\cos\phi_h} = \frac{2M_N}{Q}\mathcal{F}\left\{\frac{-k_{Tx}}{M_N}\left[(f_1 + x\tilde{f}^{\perp})D_1 + \frac{M_h}{M_N} x h_1^{\perp}\frac{\tilde{H}}{z}\right] - \frac{p_{Tx}}{M_h}\left[\left(x\tilde{h} - \frac{k_T^2}{M_N^2} h_1^{\perp}\right) H_1^{\perp} + \frac{M_h}{M_N} f_1 \frac{\tilde{D}^{\perp}}{z}\right]\right\}$$

  - ▷ Doesn't contain radiative corrections
  - ▷ Mismatch with perturbative results at tree level [Bacchetta et al '08]
  - ▷ Resolved by more recent work
- Systematic derivation of bare factorization for SIDIS to all order using soft-collinear effective theory (SCET) [our work '21]
- Recent progress from other groups

  [Balitsky, Tarasov '17] [Bacchetta et al '19] [Inglis-Whalen, Luke, Roy, Spourdalakis 21']

  [TMD operator expansion by Moos, Rodini, Scimemi, Vladimirov '21 '22 '23]

  [CSS formalism by Gamberg, Kang, Shao, Terry, Zhao 22']

Outline

I. Derivation of all-order bare factorization from SCET (including Z/W production for Drell-Yan)

II. Nontrivial renormalization of TMD quark-gluon-quark correlators

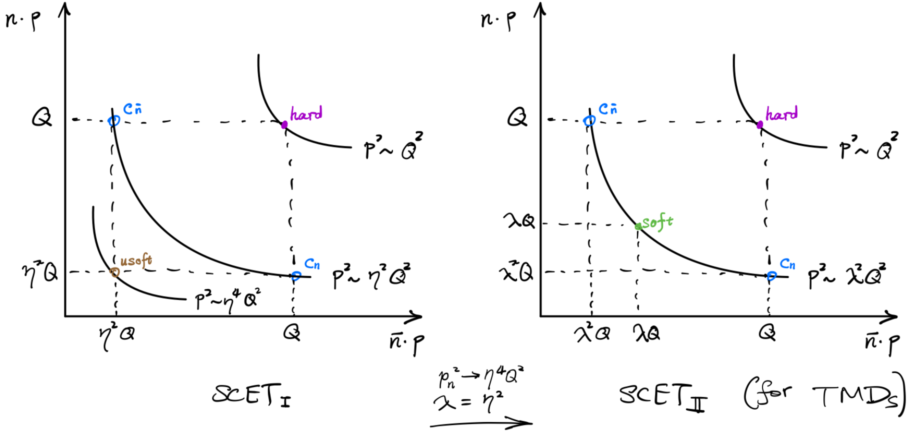
Part I

Factorization

Review of SCET

[Bauer, Fleming, Luke, Pirjol, Stewart '00, '01, '02]

- EFT for collinear/soft/ultrasoft d.o.f.s with power counting parameter $\lambda \ll 1$

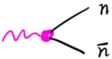


- $\mathrm{SCET_{II}}$ constructed through matching: QCD → $\mathrm{SCET_I}$ → $\mathrm{SCET_{II}}$

Review of SCET

[Bauer, Fleming, Luke, Pirjol, Stewart '00, '01, '02]

- SCET Lagrangian $\mathcal{L}_{\mathsf{SCET}_{\text{II}}} = \Big(\sum_{i\geq 0} \mathcal{L}_{\text{hard}}^{(i)}\Big) + \Big(\sum_{i\geq 0} \mathcal{L}_{\text{dyn}}^{(i)} + \mathcal{L}_{G}^{(0)}\Big)$
  - ▷ $\mathcal{L}_{\text{hard}}^{(i)} = \sum_k C_k^{(i)} \mathcal{O}_k^{(i)} = \frac{\mathrm{i}e^2}{Q^2} J_{\ell\ell'\mu} \sum_k J_k^{(i)\mu}$, Hard scattering operators



constructed by $\int$out offshell modes above $\lambda^2 Q^2$

  - ▷ $\mathcal{L}_{\text{dyn}}^{(0)} = \mathcal{L}_{n}^{(0)} + \mathcal{L}_{\bar{n}}^{(0)} + \mathcal{L}_{s}^{(0)}$, Collinear and soft dynamics factorize

while $\mathcal{L}_{\text{dyn}}^{(i)}$ with $i \geq 1$ can be more involved

  - ▷ $\mathcal{L}_{\text{G}}^{(0)}$, Glauber $(\lambda^2, \lambda^2, \lambda)$: connect different sectors

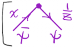
Factorization = Total Glauber contribution vanishes [Rothstein, Stewart '16]

  - Here we assume $\mathcal{L}_{G}^{(0)}$ doesn't spoil factorization (left for future work)
- Building blocks: Collinear fields $\chi_n = W_n^\dagger \xi_n$, $\mathcal{B}_{n\perp}^{\mu} = \frac{1}{g}\Big[W_n^\dagger \, \mathrm{i}D_{n\perp}^{\mu} W_n\Big] \sim \lambda$

  Soft quark and gluon $\psi_{s(n)} \sim \lambda^{3/2}$, $\mathcal{B}_{s(n)}^{\mu} \sim \lambda$

  Momentum operators $\mathcal{P}_\perp$, $n \cdot \partial_s$, $\bar{n} \cdot \partial_s \sim \lambda$, ...

Warm Up: Leading Power (LP) TMD Factorization

LP current $J^{(0)\mu} \sim \sum_f (\gamma_\perp^\mu)^{\alpha\beta}\, C_f^{(0)}(Q)\, \bar\chi^\alpha_{\bar n,\omega_b} [S_{\bar n}^\dagger S_n]\, \chi^\beta_{n,\omega_a} \sim C_f^{(0)}(Q) \Big[$  (+ Wilson lines) $\Big]$

- Plug it into $W^{(0)\mu\nu} \sim \langle N | J^{(0)\dagger\,\mu} | h, X\rangle \langle h, X | J^{(0)\nu} | N\rangle$
- Collinear fields yield quark correlators

$$\hat B_f^{\beta'\beta}(x, \vec b_T) = \Big\langle N \Big| \bar\chi_n^\beta(b_\perp)\, \delta(\omega_a - \overline{\mathcal{P}}_n)\, \chi_n^{\beta'}(0) \Big| N \Big\rangle$$

- Soft Wilson lines $\Rightarrow \mathcal{S}(b_T) = \frac{1}{N_c}\,\mathrm{tr}\Big\langle 0 \Big| \big[S_n^\dagger S_{\bar n}\big](b_\perp)\, \big[S_{\bar n}^\dagger S_n\big](0) \Big| 0 \Big\rangle$
- Combine into the quark correlators $B_f^{\beta'\beta} = \hat B_f^{\beta'\beta} \sqrt{\mathcal{S}}$

$\Rightarrow$ Factorized LP hadronic tensor where $\mathcal{H}_f^{(0)}(Q) = \big|C_f^{(0)}(Q)\big|^2$

$$W^{(0)\mu\nu} \sim \frac{2z}{N_c} \sum_f \int \mathrm{d}^2 b_T\, e^{\mathrm{i}\vec q_T\cdot\vec b_T}\, \mathcal{H}_f^{(0)}(Q)\, \mathrm{Tr}\Big[B_f(x_a, \vec b_T)\, \gamma_\perp^\mu\, \bar B_{\bar f}(x_b, \vec b_T)\, \gamma_\perp^\nu\Big]$$

$$\Rightarrow W^{(0)}_{\text{unpol.}} = \mathcal{F}\Big[\mathcal{H}^{(0)}\, f_1 f_1\Big]\,, \qquad W_2^{(0)} = \mathcal{F}\left[-\frac{2\,p_{Tx}\,k_{Tx} - \vec p_T\cdot\vec k_T}{M_N^2}\mathcal{H}^{(0)}\, h_1^\perp h_1^\perp\right]$$

- For Z/W boson, replace $\gamma_\perp^\mu C_f^{(0)}$ by $\gamma_\perp^\mu (v_q + a_q\gamma_5) C_f^{(0)}$ or $\gamma_\perp^\mu (1-\gamma_5) C_{ff'}^{(0)}$

TMD Factorization at Next-to-Leading Power (NLP)

- Kinematic corrections: trivial, proportional to LP structure functions (show up in SIDIS, but do not show up in the CS frame for DY)
- NLP dynamic Lagrangian contributions vanish
- NLP current contributions
  - Soft currents: vanish, but play an important role in ren.
    e.g. Currents involving $\mathcal{B}_{s\perp}^{(n_i)\mu}$ yield
    $$\frac{1}{N_c}\,\mathrm{tr}\left\langle 0\middle|\left[S_n^\dagger\,S_{\bar n}\right](b_\perp)\left[S_{\bar n}^\dagger\,S_n g\mathcal{B}_{s\perp}^{(n)\rho}+g\mathcal{B}_{s\perp}^{(\bar n)\rho}S_{\bar n}^\dagger\,S_n\right](0)\middle|0\right\rangle=0$$
  - Collinear currents
    $$J_{\mathcal{P}}^{(1)\mu}\sim\frac{C_f^{(0)}}{2\omega_a}\bar\chi_{\bar n,\omega_b}\left[S_{\bar n}^\dagger S_n\right]\gamma^\mu\,\not{\mathcal{P}}_\perp\,\not{\bar n}\chi_{n,\omega_a}+\text{h.c.}$$
    $$J_{\mathcal{B}}^{(1)\mu}\sim(n^\mu+\bar n^\mu)\int\mathrm{d}\omega_a\mathrm{d}\omega_b\mathrm{d}\omega_c\,C_f^{(1)}(Q,\xi=\omega_c/Q)$$
    $$\times\Big[\delta(\omega_a+\omega_c-Q)\,\delta(\omega_b-Q)\,\bar\chi_{\bar n,\omega_b}\left[S_{\bar n}^\dagger S_n\right]\not{\mathcal{B}}_{\perp n,-\omega_c}\chi_{n,\omega_a}$$
    $$+\,\delta(\omega_a-Q)\,\delta(\omega_b+\omega_c-Q)\,\bar\chi_{\bar n,\omega_b}\not{\mathcal{B}}_{\perp\bar n,\omega_c}\left[S_{\bar n}^\dagger S_n\right]\chi_{n,\omega_a}\Big]$$

Subleading Current: $\mathcal{P}_\perp$ Acting on the Collinear Fields

- $\mathcal{P}_\perp$ current $J_{\mathcal{P}}^{(1)\mu} \sim \frac{C_f^{(0)}}{2\omega_a} \bar{\chi}_{\bar{n},\omega_b} \left[S_{\bar{n}}^\dagger S_n \right] \gamma^\mu \not{\mathcal{P}}_\perp \not{\bar{n}} \chi_{n,\omega_a}$ + h.c.
- $\left\langle J^{(0)\mu} J_{\mathcal{P}}^{(1)\nu} \right\rangle$ gives

$$W_{\mathcal{P}}^{(1)\mu\nu} \equiv \frac{2z}{N_c} \sum_f \int \mathrm{d}^2\vec{b}_T \, \mathcal{H}_f^{(0)}(Q)$$
$$\times \left\{ \mathrm{Tr}\left[B_{\mathcal{P}}(x_a, \vec{b}_T)\, \gamma^\mu \, \bar{B}(x_b, \vec{b}_T)\, \gamma^\nu \right] + \mathrm{Tr}\left[B(x_a, \vec{b}_T)\, \gamma^\mu \, \bar{B}_{\mathcal{P}}(x_b, \vec{b}_T)\, \gamma^\nu \right] \right\} .$$

- $\mathcal{P}_\perp \sim \mathrm{i}\partial_\perp \Rightarrow$ Same leading power functions appear

$$B_{\mathcal{P}}(x, \vec{b}_T) = \frac{-\mathrm{i}}{2Q} \frac{\partial}{\partial b_\perp^\rho} \left[\gamma_\perp^\rho \not{\bar{n}} \, , \, B_f(x, \vec{b}_T) \right]$$
$$= \frac{M_N}{2Q} \left\{ -\mathrm{i} M_N f_1^{(1)} \not{b}_\perp - \frac{\mathrm{i}}{4} h_1^{\perp\,(0')} [\not{n}, \not{\bar{n}}] \right\} + \cdots .$$

Subleading Current: with $\mathcal{B}_{n_i\perp}$ Insertion

$$J_{\mathcal{B}}^{(1)\mu} \sim (n^\mu + \bar{n}^\mu) \int \mathrm{d}\omega_a \mathrm{d}\omega_b \mathrm{d}\omega_c \, C_f^{(1)}(Q,\xi) \Big[\delta(\omega_a + \omega_c - Q)\, \delta(\omega_b - Q)\, \bar{\chi}_{\bar{n},\omega_b} [S_{\bar{n}}^\dagger S_n] \not{\mathcal{B}}_{\perp n, -\omega_c} \chi_{n,\omega_a}$$

$$+ \delta(\omega_a - Q)\, \delta(\omega_b + \omega_c - Q)\, \bar{\chi}_{\bar{n},\omega_b} \not{\mathcal{B}}_{\perp \bar{n}, \omega_c} [S_{\bar{n}}^\dagger S_n] \chi_{n,\omega_a} \Big]$$

- $\left\langle J^{(0)\dagger\mu} J_{\mathcal{B}}^{(1)\nu} \right\rangle$ gives ($\xi = \omega_c/Q$, energy fraction of the collinear gluon)

$$W_{\mathcal{B}}^{(1)\mu\nu} = \frac{2z}{Q} \sum_f \int \mathrm{d}^2 b_T \, e^{\mathrm{i}\vec{q}_T \cdot \vec{b}_T} \int \mathrm{d}\xi \, \mathcal{H}^{(1)}(Q,\xi) (n^\mu + \bar{n}^\mu)$$

$$\times \operatorname{Tr} \Big[\tilde{B}_{\mathcal{B}}^{\rho}(x_a, \xi, \vec{b}_T)\, \gamma_\rho \, \bar{B}(x_b, \vec{b}_T)\, \gamma_\perp^\nu + B(x_a, \vec{b}_T)\, \gamma_\perp^\nu \, \tilde{\bar{B}}_{\mathcal{B}}^{\rho}(x_b, \xi, \vec{b}_T)\, \gamma_\rho \Big] + \text{h.c.}\,.$$

- Hard function $\mathcal{H}^{(1)}(Q,\xi) = C_f^{(1)}(Q,\xi)\, C_f^{(0)}(Q)$
- The TMD qgq correlators are defined as

$$\tilde{\bar{B}}_{\mathcal{B}}^{\rho\, \beta' \beta}(x, \xi, \vec{b}_T) \equiv Q \, \langle N | \left[\bar{\chi}_{n,\omega_a}^{\beta} \, \mathcal{B}_{\perp n, -\omega_c}^{\rho} \right] (b_\perp^\mu) \, \chi_n^{\beta'}(0) \, | N \rangle \sqrt{S(b_T)}\,,$$

can be decomposed as [Boer, Mulders, Pijlman '03; Bacchetta, Mulders, Pijlman '04]

$$\tilde{B}_{\mathcal{B}}^{\rho}(x, \xi, \vec{b}_T) = \frac{M_N}{4P_N^-} \Big\{ -\mathrm{i} M_N \, \underline{\tilde{f}}^{\perp(1)} \, b_{\perp\sigma} (g_\perp^{\rho\sigma} - \mathrm{i}\epsilon_\perp^{\rho\sigma}\gamma_5) + \underline{\tilde{h}} \, \mathrm{i}\gamma_\perp^\rho \Big\} + \dots$$

$$= \frac{M_N}{4P_N^-} \Big\{ -\mathrm{i} M_N \Big(\tilde{f}^{\perp(1)} - \mathrm{i}\tilde{g}^{\perp(1)} \Big) b_{\perp\sigma} (g_\perp^{\rho\sigma} - \mathrm{i}\epsilon_\perp^{\rho\sigma}\gamma_5) + \Big(\tilde{h} + \mathrm{i}\tilde{e} \Big) \mathrm{i}\gamma_\perp^\rho \Big\} + \dots\,.$$

Results: Examples of Factorized Azimuthal Asymmetries

SIDIS (including kinematic corrections, $\mathcal{P}_\perp$, $\mathcal{B}_{n_i\perp}$ operator contributions)

$$W_{UU}^{\cos\phi_h} = \mathcal{F}\Big\{ \frac{q_T}{Q}\,\mathcal{H}^{(0)}\left[-f_1 D_1 + h_1^{\perp(1)} H_1^{\perp(1)}\right]$$

$$+\mathcal{H}^{(0)}\left[-\frac{M_N}{Q} f_1^{(1)} D_1 - \frac{M_h}{Q} f_1 D_1^{(1)} + \frac{M_N}{Q} h_1^{\perp(0')} H_1^{\perp(1)} + \frac{M_h}{Q} h_1^{\perp(1)} H_1^{\perp(0')}\right]\Big\}$$

$$-\,\Re\left[\mathcal{H}^{(1)}\left[\frac{2xM_N}{Q}\left(\underline{\tilde{f}}^{\perp(1)} D_1 + \underline{\tilde{h}}\, H_1^{\perp(1)}\right) + \frac{2M_h}{zQ}\left(f_1 \underline{\tilde{D}}^{\perp(1)} + h_1^{\perp(1)} \underline{\tilde{H}}\right)\right]\right]\Big\}$$

$$\mathcal{F}[\mathcal{H}\, g^{(n)} D^{(m)}] = 2z \sum_f \int \mathrm{d}\xi\, \mathcal{H}_f(q^+ q^-, \xi) \int_0^\infty \frac{\mathrm{d}b_T\, b_T}{2\pi} (M_N b_T)^n (-M_h b_T)^m J_{n+m}(b_T q_T)$$

$$\times\; g_f^{(n)}(x, (\xi), b_T)\, D_f^{(m)}(z, (\xi), b_T) + (f \to \bar{f})$$

- D_1: TMD fragmentation function; $H_1^\perp$: Collins Functions
- $\underline{\tilde{f}}^\perp$ is complex qgq correlator, an NLP analog of f_1.
  Similar for $\underline{\tilde{h}}$, $\underline{\tilde{D}}_1^\perp$, $\underline{\tilde{H}}_1^\perp$: NLP analogs of $h_1^\perp$, D_1, $H_1^\perp$

Examples of Factorized Azimuthal Asymmetries

Drell-Yan in the CS frame (including $\mathcal{P}_\perp$, $\mathcal{B}_{n_i\perp}$ operator contributions)

$$W_3 = \mathcal{F}\Big\{ -\frac{M_N}{Q}\Big[\mathcal{H}_4^{(0)} f_1^{(1)} f_1 - \mathcal{H}_4^{(0)} f_1 f_1^{(1)} - \mathcal{H}_5^{(0)} h_1^{\perp(1)} h_1^{\perp(0')} + \mathcal{H}_5^{(0)} h_1^{\perp(0')} h_1^{\perp(1)}\Big]$$

$$-\frac{2M_N}{Q}\,\Re\Big[x_a \mathcal{H}_\times^{(1)} \underline{\tilde{f}}^{\perp(1)} f_1 - x_b \mathcal{H}_\times^{(1)} f_1 \underline{\tilde{f}}^{\perp(1)} + x_b \mathcal{H}_\otimes^{(1)} h_1^{\perp(1)} \underline{\tilde{h}} - x_a \mathcal{H}_\otimes^{(1)} \underline{\tilde{h}}\, h_1^{\perp(1)}\Big]\Big\},$$

$$W_6 = \mathcal{F}\Big\{ -\frac{2M_N}{Q}\,\Im\Big[x_a \mathcal{H}_\times^{(1)} \underline{\tilde{f}}^{\perp(1)} f_1 - x_b \mathcal{H}_\times^{(1)} f_1 \underline{\tilde{f}}^{\perp(1)} + x_b \mathcal{H}_\otimes^{(1)} h_1^{\perp(1)} \underline{\tilde{h}} - x_a \mathcal{H}_\otimes^{(1)} \underline{\tilde{h}}\, h_1^{\perp(1)}\Big]\Big\}$$

- For photon, $W_3 = W_6 = 0$ (P-odd hard functions vanish)
- For $W^\pm$, $\mathcal{H}_5^{(0)} = \mathcal{H}_\otimes^{(1)} = 0$ (Boer-Mulders effects & NLP analog vanish)

$$\mathcal{H}_{4\,W^+W^+\,f\bar{f}'} = \frac{4\pi\alpha_{\rm em}}{N_c}\frac{|V_{ff'}|^2}{\sin^2\theta_w}|C_q|^2\,, \quad \mathcal{H}^{(1)}_{\times\,W^+W^+\,f\bar{f}'} = \frac{4\pi\alpha_{\rm em}}{N_c}\frac{|V_{ff'}|^2}{\sin^2\theta_w}C_q^* C_q^{(1)}$$

- W_6 starts at $\mathcal{O}(\alpha_s^2)$: probe the most non-trivial part of qgq, Interesting to check with fixed-order calculation

Part II

Renormalization

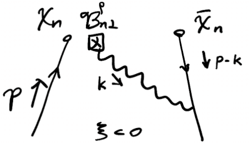
Novel Rapidity Divergence

$$W^{(1)\mu\nu}_{\mathcal{B}\ \mathrm{DY}} \sim \hat{z}^{\nu}\int \mathrm{d}\xi\, \mathcal{H}^{(1)}(\xi)\mathrm{Tr}\Big[\overset{+\frac{1}{\eta}}{\tilde{B}^{\rho}_{\mathcal{B}}(x_a,\xi,\vec{b}_T)}\,\gamma^{\mu}_{\perp}\,\bar{B}(x_b,\vec{b}_T)\,\gamma_{\perp\rho} + B(x_a,\vec{b}_T)\,\gamma^{\mu}_{\perp}\,\overset{-\frac{1}{\eta}}{\bar{\tilde{B}}^{\rho}_{\mathcal{B}}(x_b,\xi,\vec{b}_T)}\,\gamma_{\perp\rho}\Big]$$

where the quark-gluon-quark (qgq) correlators are defined as

$$\tilde{B}^{\rho\,\beta'\beta}_{\mathcal{B}\,f/N}(x,\xi,\vec{b}_T) \equiv \langle N|\,\bar{\chi}^{\beta}_{n}(b^{\mu}_{\perp})\,\big[\mathcal{B}^{\rho}_{\perp n,-\xi Q}\,\chi^{\beta'}_{n,(1-\xi)Q}\big](0)\,|N\rangle\,\sqrt{S(b_T)}\,,$$

At leading order (LO), matching onto quark PDF $f_{q/N}(\frac{x}{z})$,



$$= -\alpha_s 2C_F \underbrace{\frac{\theta(-\xi)}{(-\xi)^{1+\eta}}}_{= -\frac{\delta(\xi)}{\eta} + \mathcal{L}_0(-\xi)} \left(\frac{Q}{\nu}\right)^{-\eta} \delta\left(\frac{1-z}{z}+\xi\right) \underbrace{\mu^{2\epsilon}\int d^{2-2\epsilon}k_{\perp}(i\,k^{\rho}_{\perp})}_{\equiv \mathcal{I}^{\rho}_{NLP}(b_{\perp}) = \frac{b^{\rho}_{\perp}}{b^{2}_{\perp}}\frac{1}{2\pi} + \mathcal{O}(\epsilon)} + \mathcal{O}(\xi^{0})$$

$$\mathcal{L}_0(x) \equiv \left[\frac{\theta(x)}{x}\right]_{+}$$

- The two terms in $[\ldots]$ are individually rapidity divergent, although divergence cancels in $W^{(1)\mu\nu}_{\mathcal{B}}$ [first observed at LO in Rodini, Vladimirov '22]

Novel Rapidity Divergence

$$W^{(1)\mu\nu}_{\mathcal{B}\ \mathrm{DY}} \sim \hat{z}^\nu \int \mathrm{d}\xi\, \mathcal{H}^{(1)}(\xi) \mathrm{Tr}\Big[\overset{+\frac{1}{\eta}}{\tilde{B}^{\rho}_{\mathcal{B}}(x_a, \xi, \vec{b}_T)}\, \gamma^\mu_\perp\, \bar{B}(x_b, \vec{b}_T)\, \gamma_{\perp\rho} + B(x_a, \vec{b}_T)\, \gamma^\mu_\perp\, \overset{-\frac{1}{\eta}}{\tilde{\bar{B}}^{\rho}_{\mathcal{B}}(x_b, \xi, \vec{b}_T)}\, \gamma_{\perp\rho}\Big]$$

where the quark-gluon-quark (qgq) correlators are defined as

$$\tilde{B}^{\rho\ \beta'\beta}_{\mathcal{B}\ f/N}(x, \xi, \vec{b}_T) \equiv \langle N|\, \bar{\chi}^\beta_n(b^\mu_\perp)\, \big[\mathcal{B}^\rho_{\perp n, -\xi Q}\, \chi^{\beta'}_{n,(1-\xi)Q}\big](0)\, |N\rangle\, \sqrt{S(b_T)}\,,$$

- Rapidity poles start at LO, which is $\mathcal{O}(\alpha_s)$

$\Rightarrow$ Can never be canceled by a multiplicative counterterm $Z = 1 + \mathcal{O}(\alpha_s)$

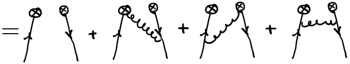
$\Rightarrow$ Need additive counterterm to get separately finite matrix elements

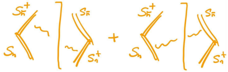
[LO counterterm involving $\partial^\rho \gamma_\zeta^{1-\mathrm{loop}}$ proposed in Rodini, Vladimirov '22]

- For work on such additive counterterms see [dijet: Moult et al '18], [threshold: Beneke et al '18], [q_T: Ebert et al '18], [EEC: Moult et al '19], [$h \to \gamma\gamma$: Liu et al '19], …

Rap. Div. Cancellation: Use Soft Matrix Elements (ME)

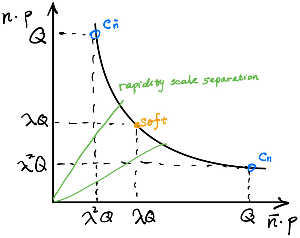
- At LP, rap. div. cancels multiplicatively (eg. η reg.)

$\hat{B}_{q/q} =$  $+\mathcal{O}(\alpha_s^2) = 1 - \alpha_s 4C_F \frac{1}{\eta}\mathcal{I}_{\rm LP} + \mathcal{O}(\eta^0) + \mathcal{O}(\alpha_s^2)$

$S = 1+$ 

$+\mathcal{O}(\alpha_s^2) = 1 + \alpha_s 8C_F \frac{1}{\eta}\mathcal{I}_{\rm LP} + \mathcal{O}(\eta^0) + \mathcal{O}(\alpha_s^2)$

$$\Rightarrow \hat{B}_{q/q}\sqrt{S} \sim \mathcal{O}(\eta^0)$$



- Idea: exploit individual pieces of vanishing NLP soft contribution

Construction of the NLP "Counterterm"

$$W^{(1)\mu\nu}_{\mathcal{B}\ \mathrm{DY}} \sim \hat{z}^{\nu}\int \mathrm{d}\xi\, \mathcal{H}^{(1)}(\xi)\mathrm{Tr}\Big[\tilde{B}^{\rho}_{\mathcal{B}\ f}(x_a,\xi,\vec{b}_T)\,\gamma^{\mu}_{\perp}\,\bar{B}(x_b,\vec{b}_T)\,\gamma_{\perp\rho} + B(x_a,\vec{b}_T)\,\gamma^{\mu}_{\perp}\,\tilde{\bar{B}}^{\rho}_{\mathcal{B}}(x_b,\xi,\vec{b}_T)\,\gamma_{\perp\rho}\Big]$$

- Use the fact that rap. div. cancels between the two terms in $W^{(1)}_{\mathcal{B}}$
- Can take $\mathcal{B}^{\rho}_{\bar{n}\perp}$ to soft $\mathcal{B}^{(n)\rho}_{s\perp}$ in SCET without changing the rap. div.

$$B(x_a,\vec{b}_T)\tilde{\bar{B}}^{\rho}_{\mathcal{B}}(x_b,\xi,\vec{b}_T) \to \delta(\xi)B\bar{B}\,\frac{S^{(n)\rho}_{\mathcal{B}}}{S}$$

$$S^{(n)\rho}_{\mathcal{B}}(b_\perp) \equiv \frac{1}{N_c}\,\mathrm{tr}\,\Big\langle 0\Big|\big[S^{\dagger}_{n}(b_\perp)\,S_{\bar{n}}(b_\perp)\big]\big[S^{\dagger}_{\bar{n}}(0)\,S_n(0)g\mathcal{B}^{(n)\rho}_{s\perp}(0)\big]\Big|0\Big\rangle\,,$$

- By construction, $\boxed{\tilde{B}'^{\rho}_{\mathcal{B}}(\xi) \equiv \tilde{B}^{\rho}_{\mathcal{B}}(\xi) + \delta(\xi)\frac{S^{(n)\rho}_{\mathcal{B}}}{S}B \sim \mathcal{O}(\eta^0)}$,

- Property of the counterterm: $S^{(n)\rho}_{\mathcal{B}}/S = \frac{\mathrm{i}}{2}\partial^{\rho}_{\perp}\ln S$, see backup slide

Construction of the NLP "Counterterm"

- Note that $S_{\mathcal{B}}^{(n)\rho} + S_{\mathcal{B}}^{(\bar{n})\rho}$

$$= \frac{1}{N_c} \operatorname{tr} \Big\langle 0 \Big| \big[S_n^\dagger S_{\bar{n}}\big](b_\perp) \Big[S_{\bar{n}}^\dagger S_n g\mathcal{B}_{s\perp}^{(n)\rho} + g\mathcal{B}_{s\perp}^{(\bar{n})\rho} S_{\bar{n}}^\dagger S_n\Big](0) \Big| 0 \Big\rangle = 0$$

  due to C & P & Poincaré invariance of the vacuum!

  [Ebert, AG, Stewart 21']

- $$\tilde{B}_{\mathcal{B}}^\rho \bar{B} + B \tilde{\bar{B}}_{\mathcal{B}}^\rho = \underbrace{\left[\tilde{B}_{\mathcal{B}}^\rho + \delta(\xi)\, B \frac{S_{\mathcal{B}}^{(n)\rho}}{S}\right]}\bar{B} + B \underbrace{\left[\delta(\xi) \bar{B}\, \frac{S_{\mathcal{B}}^{(\bar{n})\rho}}{S} + \tilde{\bar{B}}_{\mathcal{B}}^\rho\right]}$$

  free of $\frac{1}{\eta}$ divergences to all orders (additive + LP multiplicative div.)

$$= \tilde{B}_{\mathcal{B}\,q}^{\prime\rho} \bar{B} + B \tilde{\bar{B}}_{\mathcal{B}\,\bar{q}}^{\prime\rho}$$

- So we can simply replace the old by the new qgq correlators

Checking Cancellation of Rapidity Divergence

Check deepest $(1/\eta^2)$ pole in $\alpha_s^2 C_F^2$ channel:

$$\Big[\hat{\tilde{B}}^{\rho}_{\mathcal{B}}(\xi) + \delta(\xi)\underbrace{\frac{S^{(n)\rho}_{\mathcal{B}}}{S}}_{\mathcal{O}(C_F/\eta)}\overbrace{\hat{B}\Big]\sqrt{S}}^{\mathcal{O}(C_F\eta^0)} = \mathcal{O}(\eta^0)$$

$\hat{\tilde{B}}^{(2)}_{\mathcal{B}}$

$$= -\,\alpha_s^2\, 8C_F^2\,\frac{1}{\eta^2}\,\delta(1-z)\,\delta(\xi)\,\mathcal{I}^{\rho}_{\rm NLP}\mathcal{I}_{\rm LP} + \mathcal{O}(\frac{1}{\eta})$$

$\hat{\tilde{B}}^{(1)}_{\mathcal{B}}$

$$= +\,\alpha_s\, 2C_F\,\frac{1}{\eta}\,\delta(1-z)\,\delta(\xi)\,\mathcal{I}^{\rho}_{\rm NLP} + \mathcal{O}(\eta^0)$$

$\sqrt{S}$

$\frac{1}{2}$

$$= +\,\alpha_s\, 4C_F\,\frac{1}{\eta}\,\delta(1-z)\,\delta(\xi)\,\mathcal{I}_{\rm LP} + \mathcal{O}(\eta^0)$$ ✓

Checking Cancellation of Rapidity Divergence

Check deepest $(1/\eta^2)$ pole in $\alpha_s^2 C_F C_A$ channel:

$$\Bigg[\underbrace{\hat{\tilde{B}}^{\rho}_{\mathcal{B}}(\xi)}_{\text{have to cancel within}} + \delta(\xi)\underbrace{\frac{S^{(n)\rho}_{\mathcal{B}}}{S}}_{\mathcal{O}(C_F/\eta)}\overbrace{\hat{B}\Bigg]\underbrace{\sqrt{S}}_{1+\alpha_s C_F/\eta}}^{\mathcal{O}(\eta^0)} = \mathcal{O}(\eta^0)$$

$A =$  $= \mathcal{O}(1/\eta^2)$

$B =$  $= \mathcal{O}(1/\eta^2)$

$A + B = \mathcal{O}(1/\eta)$ ✓

Renormalization and Evolution

$$W^{(1)\mu\nu}_{\mathcal{B}\,\mathrm{DY}} \sim \hat{z}^\nu \int \mathrm{d}\xi\, \mathcal{H}^{(1)}(\xi)\, \mathrm{Tr}\Big[\tilde{B}^\rho_{\mathcal{B}}(x_a,\xi,\vec{b}_T)\,\gamma_\perp^\mu\,\bar{B}(x_b,\vec{b}_T)\,\gamma_{\perp\rho} + B(x_a,\vec{b}_T)\,\gamma_\perp^\mu\,\tilde{\bar{B}}^\rho_{\mathcal{B}}(x_b,\xi,\vec{b}_T)\,\gamma_{\perp\rho}\Big]$$

- Renormalization

$$W^{(1)\mu\nu}_{\mathcal{B}\,\mathrm{DY}} \sim \int \mathrm{d}\xi'\, \mathcal{H}^{(1)\mathrm{ren}}(\xi')\, Z^{(0)} Z^{(1)}(\xi',\xi) \otimes \Big[\tilde{B}'^{\rho}_{\mathcal{B}}(\xi)\ \bar{B} + B\,\tilde{\bar{B}}'^{\rho}_{\mathcal{B}}(\xi)\Big]$$

$Z^{(1)}(\xi',\xi)$: counterterm for $C^{(1)}$, calculated in [Freedman, Goerke '14, Goerke, Inglis-Whalen '17; Beneke et al '17; Vladimirov, Moos, Scimemi '21]

- Due to charge conjugation $\tilde{B}'^{\rho}_{\mathcal{B}}(\xi) \leftrightarrow \tilde{\bar{B}}'^{\rho}_{\mathcal{B}}(\xi)$, we know that $Z(\xi',\xi)$ renormalizes $\tilde{B}'^{\rho}_{\mathcal{B}}(\xi) = \tilde{B}^{\rho}_{\mathcal{B}}(\xi) + \delta(\xi) B\, \frac{\mathrm{i}}{2}\partial^\rho_\perp \ln S$,

$$\tilde{B}^{\mathrm{ren}\,\rho}_{\mathcal{B}}(\xi) \equiv Z^{(1)}(\xi',\xi) \otimes \tilde{B}'^{\rho}_{\mathcal{B}}(\xi)$$

- μ RGE and ζ RGE (equivalently: ν RGE)

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}\tilde{B}^{\mathrm{ren}\,\rho}_{\mathcal{B}}(\xi) = \gamma_\mu(\xi,\xi') \otimes \tilde{B}^{\mathrm{ren}\,\rho}_{\mathcal{B}}(\xi')$$

$$2\zeta\frac{\mathrm{d}}{\mathrm{d}\zeta}\tilde{B}^{\mathrm{ren}\,\rho}_{\mathcal{B}} = -\delta(\xi)\big(\mathrm{i}\partial^\rho_\perp \gamma_\zeta\big) B^{\mathrm{ren}} + \gamma_\zeta \tilde{B}^{\mathrm{ren}\,\rho}_{\mathcal{B}}$$

⇒ Can integrate to get the series of $\alpha_s^k L^j$ terms

Are we done?

Are we done? NO!

Endpoint Divergence

$$W^{(1)\mu\nu}_{\mathcal{B}\ \mathrm{DY}} \sim \hat{z}^{\nu} \int \mathrm{d}\xi\, \mathcal{H}^{(1)}(\xi) \mathrm{Tr}\Big[\tilde{B}^{\mathrm{ren}\,\rho}_{\mathcal{B}}(x_a, \xi, \vec{b}_T)\, \gamma^{\mu}_{\perp}\, \bar{B}(x_b, \vec{b}_T)\, \gamma_{\perp\rho} + B(x_a, \vec{b}_T) \gamma^{\mu}_{\perp}\, \bar{\tilde{B}}^{\mathrm{ren}\,\rho}_{\mathcal{B}}(x_b, \xi, \vec{b}_T)\, \gamma_{\perp\rho}\Big]$$

- $C^{(1)}$ calculated in [J. Strohm's master thesis '20, Vladimirov, Moos, Scimemi '21]
  where $L_Q = \ln \frac{-q^2}{\mu^2}$

$$C^{(1)}(q^2, \xi) = 1 + \frac{\alpha_s}{4\pi}\Bigg[C_F\Big(-L_Q^2 + L_Q - 3 + \frac{\pi^2}{6}\Big) - C_A \frac{\ln \xi}{1-\xi}$$
$$- \Big(C_F - \frac{C_A}{2}\Big) \frac{\ln(1-\xi)}{\xi}\Big(2L_Q + \ln(1-\xi) - 4\Big)\Bigg] + \mathcal{O}(\alpha_s^2),$$

- SCET$_{\mathrm{I}}$ ME $\sim \mathcal{O}(\alpha_s \xi^{-1/2}) \Rightarrow$ convergent,
  SCET$_{\mathrm{II}}$ ME $\sim \mathcal{O}(\alpha_s \xi^{-1}) \Rightarrow$ endpoint divergence

$\Rightarrow$ Divergent integral as $\xi \to 0$ at $\mathcal{O}(\alpha_s^2)$

$$\alpha_s^2 \int \mathrm{d}\xi \ln \xi\, \mathcal{L}_0(\pm\xi) = ???$$

- Special: hard coefficient and rapidity divergence conspire to give an endpoint divergence in a soft gluon limit with back-to-back $n \,\&\, \bar{n}$
- Need refactorization for endpoint divergences

  [Z. L. Liu et al, M. Beneke et al, ...]

Solution: Hard-Collinear Matching to $\mathcal{B}_{s\perp}$

Q^2 λQ^2

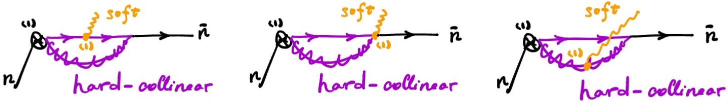
- The full $\mathcal{B}_s^\perp$ current receives hard and hard-collinear contributions ($\hat{p}_s = p_s^\pm$) [Ebert, AG, Stewart 21']

$$J^{(1)\mu}_{\mathcal{B}_s^\perp}(0) = J^{(1)\mu}_{\mathrm{h}\,\mathcal{B}_s^\perp}(0) + J^{(1)\mu}_{\mathrm{hc}\,\mathcal{B}_s^\perp}(0)$$

$$J^{(1)\mu}_{\mathrm{hc}\,\mathcal{B}_s^\perp}(0) \sim \int \mathrm{d}\hat{p}_s \int \mathrm{d}\tilde{\xi}\, C^{(1)}(q^2,\tilde{\xi})\, \tilde{J}_{\mathcal{B}_s^\perp}(\hat{p}_s,\tilde{\xi})$$

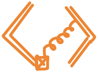
$$\times\, \bar{\chi}_{\bar{n},-\omega_2}\Big\{\big[S^\dagger_{\bar{n}} S_n g\slashed{\mathcal{B}}^{(n)}_{s\perp}\big](p_s^+) + \big[g\slashed{\mathcal{B}}^{(\bar{n})}_{s\perp} S^\dagger_{\bar{n}} S_n\big](p_s^-)\Big\}\chi_{n,-\omega_1}\,,$$

- $T\big[J_{\mathrm{I}}^{(1)\mu}\mathcal{L}_{\mathrm{I}}^{(1)}\big]$ in $\mathrm{SCET_I}$ → hard scattering operators in $\mathrm{SCET_{II}}$



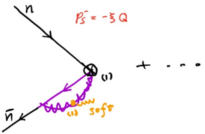
- These $\tilde{J}_{\mathcal{B}_s^\perp}$ graphs reproduce $\xi^{-\epsilon}$ behavior of $C^{(1)}$ $(p_s^- = -\xi Q)$!

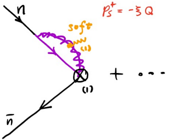
$$\int \mathrm{d}\tilde{\xi}\, \tilde{J}_{\mathcal{B}_s^\perp}(p_s^-,\tilde{\xi}) \sim \sum \text{graphs} = \frac{\alpha_s}{4\pi}\frac{C_A}{\epsilon}\left(\frac{p_s^- Q}{\mu^2}\right)^{-\epsilon} + \mathcal{O}(\epsilon^0) = \frac{\alpha_s}{4\pi}\frac{C_A}{\epsilon}\left(\frac{Q^2}{\mu^2}\right)^{-\epsilon}(1-\epsilon\ln(-\xi)) + \mathcal{O}(\epsilon^0\xi^0$$

Soft ME $\equiv \tilde{\mathcal{S}}_{\mathcal{B}}^{(\bar{n})}(p_s^-) \sim$  $(p_s^-) \sim \dfrac{\alpha_s}{\pi} C_F \dfrac{\theta(p_s^-)}{p_s^-} = \dfrac{\alpha_s}{\pi} C_F \dfrac{\theta(-\xi)}{-\xi Q} \longleftrightarrow \tilde{B}_{\mathcal{B}}^{\mathrm{ren}}(\xi \to 0)$

Solution

- Exploit the vanishing hc soft contribution for Drell-Yan





$$0 = B\bar{B}\left[\int_0^\infty \mathrm{d}p_s^- \,\frac{1}{\epsilon}\left(\frac{-p_s^- Q}{\mu^2}\right)^{-\epsilon}\frac{1}{p_s^-} - \int_0^\infty \mathrm{d}p_s^+ \,\frac{1}{\epsilon}\left(\frac{-p_s^+ Q}{\mu^2}\right)^{-\epsilon}\frac{1}{p_s^+}\right]$$

$$\propto B\bar{B}\int \mathrm{d}\xi\,\frac{1}{\epsilon}\left(\frac{-\xi q^2}{\mu^2}\right)^{-\epsilon}\left[\theta(|\xi|-\xi_{\mathrm{cut}})+\theta(\xi_{\mathrm{cut}}-|\xi|)\right]\left[\frac{\theta(-\xi)}{-\xi}-\frac{\theta(-\xi)}{-\xi}\right]$$

$$= B\bar{B}\left(\frac{-q^2}{\mu^2}\right)^{-\epsilon}\int \mathrm{d}\xi\,\frac{\xi^{-\epsilon}}{\epsilon}\theta(\xi_{\mathrm{cut}}-|\xi|)\Big(\mathcal{L}_0(-\xi)-\mathcal{L}_0(-\xi)\Big)$$

- $\theta(\xi_{\mathrm{cut}}-|\xi|)\ln\xi\,\mathcal{L}_0(-\xi)$ cancels the divergence in qgq as $\xi \to 0$
- The following convolution is finite at $\xi \to 0$!

$$\int \mathrm{d}\xi\left[C_{\mathrm{ren}}^{(1)}(\xi)\tilde{B}_{\mathcal{B}}^{\mathrm{ren}\,\rho}(\xi)+\theta(\xi_{\mathrm{cut}}-|\xi|)\,C_{\mathrm{ren}}^{(1)}\otimes\tilde{J}_{\mathcal{B}_s^\perp}^{\mathrm{ren}}(-\xi Q)\,\frac{\tilde{S}_{\mathcal{B}}^{(n)\mathrm{ren}\,\rho}(-\xi Q)}{S^{\mathrm{ren}}}\,B^{\mathrm{ren}}\right]$$

see backup for solution for SIDIS and a remainder term R_{SIDIS}

Summary

- Studied azimuthal asymmetries in Drell-Yan and SIDIS

▷ Step 1: all-order bare factorization (including Z/W production)

▷ Step 2: all-order renormalized definitions of the TMD qgq corr.

  - ▷ soft subtraction for $\frac{\delta(\xi)}{\eta}$ divergences

$$\tilde{B}'^{\rho}_{\mathcal{B}}(\xi) = \tilde{B}^{\rho}_{\mathcal{B}}(\xi) + \delta(\xi) B \tfrac{\mathrm{i}}{2}\partial^{\rho}_{\perp} \ln S, \quad \tilde{B}^{\mathrm{ren}\,\rho}_{\mathcal{B}}(\xi) \equiv Z^{(1)}(\xi,\xi') \otimes \tilde{B}'^{\rho}_{\mathcal{B}}(\xi')$$

  - ▷ Obtained all order μ and ζ RGEs

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \tilde{B}^{\mathrm{ren}\,\rho}_{\mathcal{B}}(\xi) = \gamma_\mu(\xi,\xi') \otimes \tilde{B}^{\mathrm{ren}\,\rho}_{\mathcal{B}}(\xi'), \quad 2\zeta \frac{\mathrm{d}}{\mathrm{d}\zeta} \tilde{B}^{\mathrm{ren}\,\rho}_{\mathcal{B}} = -\delta(\xi)\big(\mathrm{i}\partial^{\rho}_{\perp}\gamma_\zeta\big) B^{\mathrm{ren}} + \gamma_\zeta \tilde{B}^{\mathrm{ren}\,\rho}_{\mathcal{B}}$$

  - ▷ soft subtraction for removal of endpoint $\ln \xi/\xi$ divergences

$$\int \mathrm{d}\xi \left[C^{(1)}_{\mathrm{ren}}(\xi) \tilde{B}^{\mathrm{ren}\rho}_{\mathcal{B}}(\xi) + \theta(\xi_{\mathrm{cut}} - |\xi|)\, C^{(1)}_{\mathrm{ren}} \otimes \tilde{J}^{\mathrm{ren}}_{\mathcal{B}^{\perp}_{s}}(-\xi Q) \frac{\tilde{S}^{(n)\mathrm{ren}\,\rho}_{\mathcal{B}}(-\xi Q)}{S^{\mathrm{ren}}} B^{\mathrm{ren}} \right] = \mathrm{finite}$$

- Fully renormalized expression (removal of all singularities); yields renormalized factorization theorems for $\mathcal{O}(\lambda)$ Drell-Yan and SIDIS

⇒ Can now exploit RGE to predict logarithms in the NLP series

Are we done?

Summary

- Studied azimuthal asymmetries in Drell-Yan and SIDIS

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  ▷ soft subtraction for removal of endpoint $\ln\xi/\xi$ divergences

$$\int \mathrm{d}\xi \left[C^{(1)}_{\mathrm{ren}}(\xi) \tilde{B}^{\mathrm{ren}\rho}_{\mathcal{B}}(\xi) + \theta(\xi_{\mathrm{cut}} - |\xi|)\, C^{(1)}_{\mathrm{ren}} \otimes \tilde{J}^{\mathrm{ren}}_{\mathcal{B}^{\perp}_{s}}(-\xi Q) \frac{\tilde{S}^{(n)\mathrm{ren}\,\rho}_{\mathcal{B}}(-\xi Q)}{S^{\mathrm{ren}}} B^{\mathrm{ren}} \right] = \mathrm{finite}$$

- Fully renormalized expression (removal of all singularities); yields renormalized factorization theorems for $\mathcal{O}(\lambda)$ Drell-Yan and SIDIS

⇒ Can now exploit RGE to predict logarithms in the NLP series

Are we done? For now!

Summary

- Studied azimuthal asymmetries in Drell-Yan and SIDIS

▷ Step 1: all-order bare factorization (including Z/W production)

▷ Step 2: all-order renormalized definitions of the TMD qgq corr.

  - ▷ soft subtraction for $\frac{\delta(\xi)}{\eta}$ divergences

$$\tilde{B}'^{\rho}_{\mathcal{B}}(\xi) = \tilde{B}^{\rho}_{\mathcal{B}}(\xi) + \delta(\xi) B \, \tfrac{\mathrm{i}}{2}\partial^{\rho}_{\perp} \ln S , \quad \tilde{B}^{\mathrm{ren}\,\rho}_{\mathcal{B}}(\xi) \equiv Z^{(1)}(\xi,\xi') \otimes \tilde{B}'^{\rho}_{\mathcal{B}}(\xi')$$

  - ▷ Obtained all order μ and ζ RGEs

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \tilde{B}^{\mathrm{ren}\,\rho}_{\mathcal{B}}(\xi) = \gamma_{\mu}(\xi,\xi') \otimes \tilde{B}^{\mathrm{ren}\,\rho}_{\mathcal{B}}(\xi') \,, \quad 2\zeta \frac{\mathrm{d}}{\mathrm{d}\zeta} \tilde{B}^{\mathrm{ren}\,\rho}_{\mathcal{B}} = -\delta(\xi) \big(\mathrm{i}\partial^{\rho}_{\perp} \gamma_{\zeta}\big) B^{\mathrm{ren}} + \gamma_{\zeta} \tilde{B}^{\mathrm{ren}\,\rho}_{\mathcal{B}}$$

  - ▷ soft subtraction for removal of endpoint $\ln \xi / \xi$ divergences

$$\int \mathrm{d}\xi \left[C^{(1)}_{\mathrm{ren}}(\xi) \tilde{B}^{\mathrm{ren}\rho}_{\mathcal{B}}(\xi) + \theta(\xi_{\mathrm{cut}} - |\xi|) \, C^{(1)}_{\mathrm{ren}} \otimes \tilde{J}^{\mathrm{ren}}_{\mathcal{B}^{\perp}_{s}}(-\xi Q) \frac{\tilde{S}^{(n)\mathrm{ren}\,\rho}_{\mathcal{B}}(-\xi Q)}{S^{\mathrm{ren}}} B^{\mathrm{ren}} \right] = \mathrm{finite}$$

- Fully renormalized expression (removal of all singularities); yields renormalized factorization theorems for $\mathcal{O}(\lambda)$ Drell-Yan and SIDIS

⇒ Can now exploit RGE to predict logarithms in the NLP series

Thanks for your attention!

Backup: A Neat Property of the "Counterterm"

$$S_{\mathcal{B}}^{(n)\rho}(b_\perp) \equiv \frac{1}{N_c}\,\mathrm{tr}\left\langle 0\middle|\left[S_n^\dagger(b_\perp)\,S_{\bar{n}}(b_\perp)\right]\left[S_{\bar{n}}^\dagger(0)\,S_n(0)g\mathcal{B}_{s\perp}^{(n)\rho}(0)\right]\middle|0\right\rangle$$

$$S_{\mathcal{B}}^{(\bar{n})\rho}(b_\perp) \equiv \frac{1}{N_c}\,\mathrm{tr}\left\langle 0\middle|\left[S_n^\dagger(b_\perp)\,S_{\bar{n}}(b_\perp)\right]\left[g\mathcal{B}_{s\perp}^{(\bar{n})\rho}(0)S_{\bar{n}}^\dagger(0)\,S_n(0)\right]\middle|0\right\rangle$$

- Manipulation of Wilson lines gives

$$\begin{aligned}\left[\mathcal{P}_\perp^\rho S_{\bar{n}}^\dagger S_n\right] &= \left[S_{\bar{n}}^\dagger \mathrm{i}D_{s\perp}^\rho S_n\right] + \left[S_{\bar{n}}^\dagger \mathrm{i}\overleftarrow{D}_{s\perp}^\rho S_n\right] \\ &= S_{\bar{n}}^\dagger S_n\left[S_n^\dagger \mathrm{i}D_{s\perp}^\rho S_n\right] + \left[S_{\bar{n}}^\dagger \mathrm{i}\overleftarrow{D}_{s\perp}^\rho S_{\bar{n}}\right]S_{\bar{n}}^\dagger S_n \\ &= S_{\bar{n}}^\dagger S_n\, g\mathcal{B}_{s\perp}^{(n)\rho} - g\mathcal{B}_{s\perp}^{(\bar{n})\rho}\, S_{\bar{n}}^\dagger S_n\end{aligned}$$

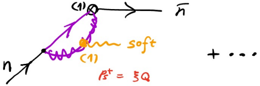
- Since $S_{\mathcal{B}}^{(n)\rho} + S_{\mathcal{B}}^{(\bar{n})\rho} = 0$, we have $S_{\mathcal{B}}^{(n)} = -S_{\mathcal{B}}^{(\bar{n})} = \frac{\mathrm{i}}{2}\partial_\perp^\rho S$ so that

$$S_{\mathcal{B}}^{(n)\rho}/S = \frac{\mathrm{i}}{2}\partial_\perp^\rho \ln S$$

⇒ Due to non-Abelian exponentiation for S, $S_{\mathcal{B}}^{(n)\rho}/S \propto 1/\eta + \mathcal{O}(\eta^0)$ (no ϵ poles, no double poles in η)

Backup: Endpoint Divergence Solution for SIDIS

- Exploit the vanishing hc soft contribution for SIDIS



$$0 = B\mathcal{G}\left[\int_0^\infty \mathrm{d}p_s^- \frac{1}{\epsilon}\left(\frac{p_s^- Q}{\mu^2}\right)^{-\epsilon}\frac{1}{p_s^-} - \int_0^\infty \mathrm{d}p_s^+\left(\frac{-p_s^+ Q}{\mu^2}\right)^{-\epsilon}\frac{1}{p_s^+}\right]$$

$$\propto B\mathcal{G}\int \mathrm{d}\xi\,\frac{1}{\epsilon}\left(\frac{-\xi q^2}{\mu^2}\right)^{-\epsilon}\left[\theta(|\xi|-\xi_{\mathrm{cut}}) + \theta(\xi_{\mathrm{cut}}-|\xi|)\right]\left[\frac{\theta(-\xi)}{-\xi} - \frac{\theta(\xi)}{\xi}\right]$$

$$= B\mathcal{G}\left(\frac{-q^2}{\mu^2}\right)^{-\epsilon}\left[\cancel{-\mathrm{i}\pi\left(\frac{1}{\epsilon} - \ln\xi_{\mathrm{cut}}\right)} - \frac{1}{2}\pi^2 + \int \mathrm{d}\xi\,\frac{\xi^{-\epsilon}}{\epsilon}\theta(\xi_{\mathrm{cut}}-|\xi|)\Big(\mathcal{L}_0(-\xi) - \mathcal{L}_0(\xi)\Big)\right]$$

after including h.c.

- $\theta(\xi_{\mathrm{cut}}-|\xi|)\ln\xi\,\mathcal{L}_0(\mp\xi)$ cancels the divergence in $\int \mathrm{d}\xi C_{\mathrm{ren}}^{(1)}\tilde{B}_{\mathcal{B}}^{\mathrm{ren}\rho}$, $\tilde{\mathcal{G}}_{\mathcal{B}}^{\mathrm{ren}\rho}$

$$\int \mathrm{d}\xi\left[C_{\mathrm{ren}}^{(1)}(\xi)\tilde{B}_{\mathcal{B}}^{\mathrm{ren}\,\rho}(\xi) + \theta(\xi_{\mathrm{cut}}-|\xi|)\,C_{\mathrm{ren}}^{(1)} \otimes \tilde{J}_{\mathcal{B}_s^\perp}^{\mathrm{ren}}(-\xi Q)\tilde{S}_{\mathcal{B}}^{(n)\mathrm{ren}\,\rho}(-\xi Q)/S^{\mathrm{ren}}\,B^{\mathrm{ren}}\right]$$

$$\int \mathrm{d}\xi\left[C_{\mathrm{ren}}^{(1)}(\xi)\tilde{\mathcal{G}}_{\mathcal{B}}^{\mathrm{ren}\,\rho}(\xi) + \theta(\xi_{\mathrm{cut}}-|\xi|)\,C_{\mathrm{ren}}^{(1)} \otimes \tilde{J}_{\mathcal{B}_s^\perp}^{\mathrm{ren}}(\xi Q)\tilde{S}_{\mathcal{B}}^{(\bar{n})\mathrm{ren}\,\rho}(\xi Q)/S^{\mathrm{ren}}\,\mathcal{G}^{\mathrm{ren}}\right]$$

- We are left with a remainder term $\boxed{R_{\mathrm{SIDIS}} \propto \alpha_s C_A \alpha_s \pi^2 C_F B\mathcal{G} + \mathcal{O}(\alpha_s^3)}$
- For $q_T \sim \Lambda_{\mathrm{QCD}}$, $\alpha_s\pi^2 C_F \longrightarrow$ NP function defined by $\tilde{S}_{\mathcal{B}}^{\rho}(\hat{p}_s, b_\perp)$