Conformal Window for Asymptotic Safety up to Four Loops

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in collaboration with
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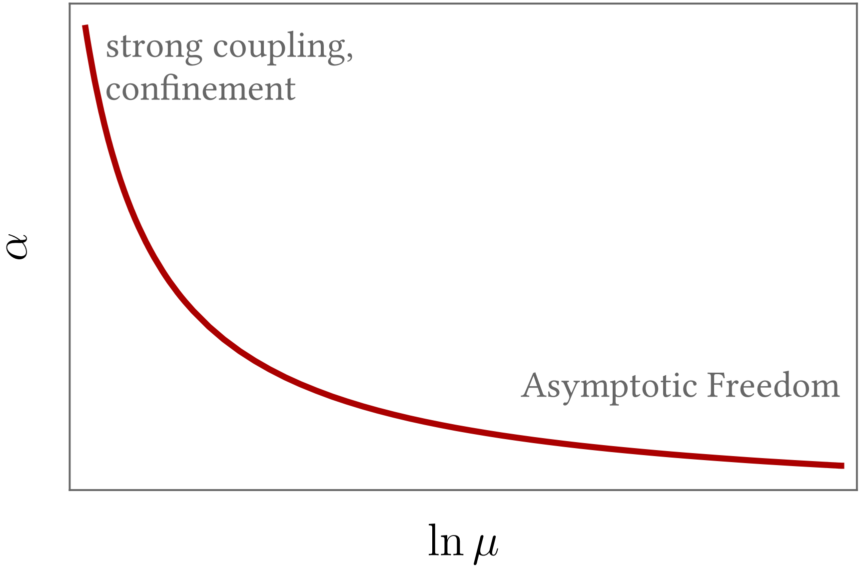
Loopfest XXI, June 28th 2023

Motivation

» QCD: Asymptotic Freedom [Gross,Wilczek,Politzer, (1971)]



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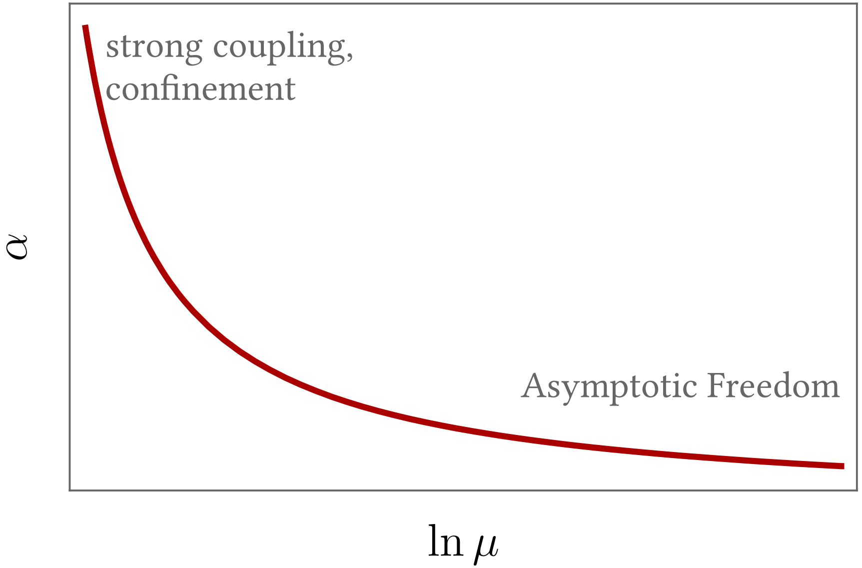

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$\longrightarrow$ theory remains predictive

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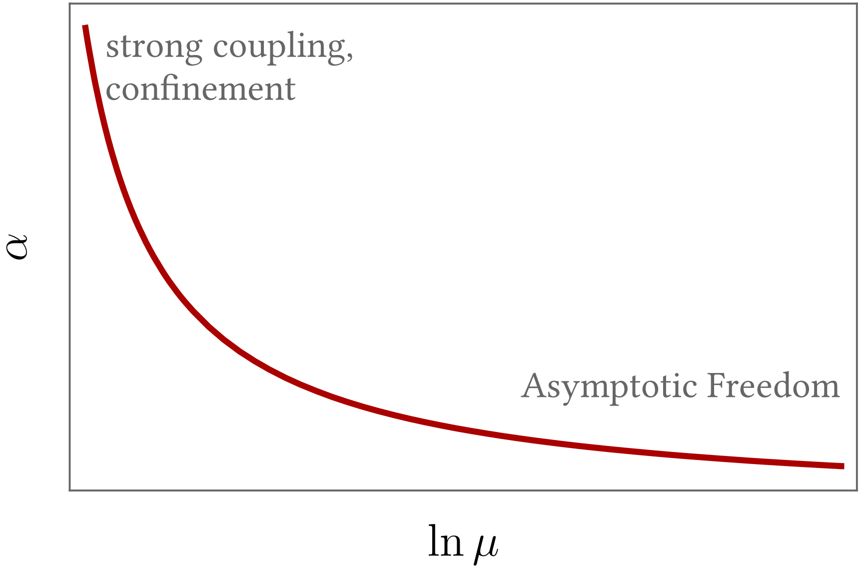

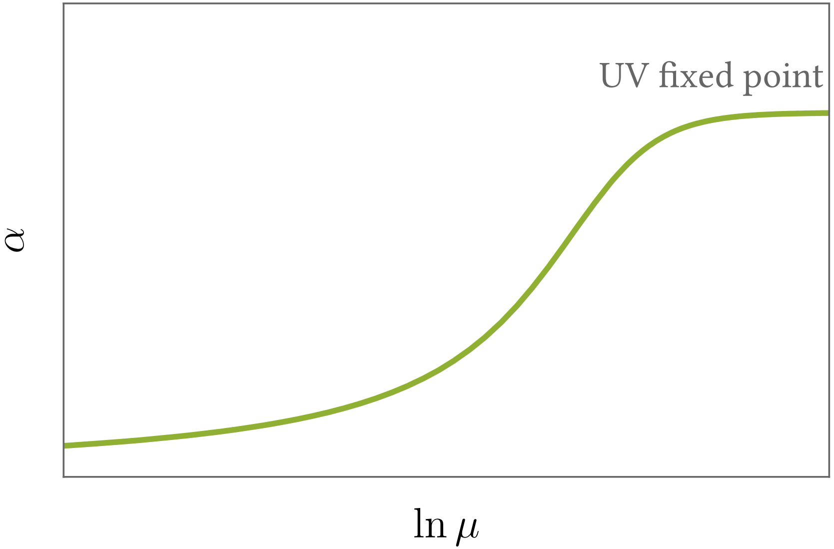

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⟶ known reliable examples away from d=4

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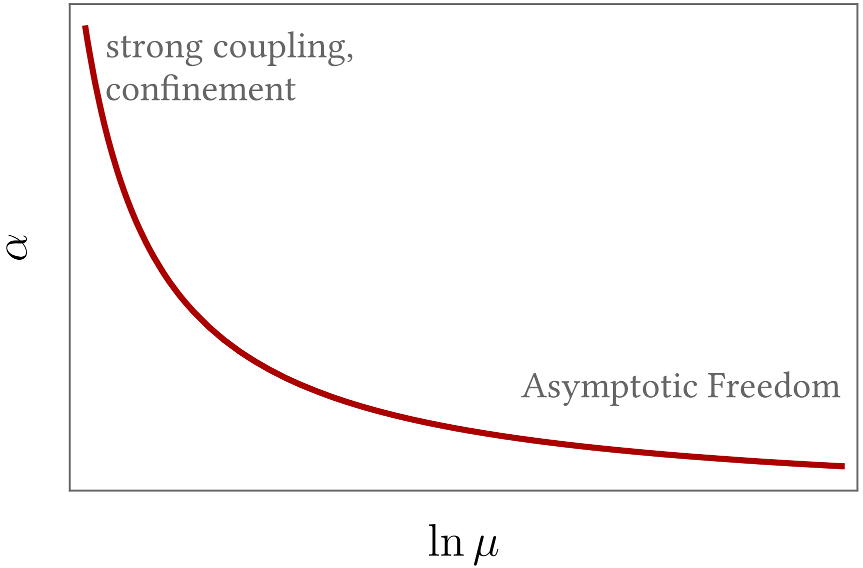

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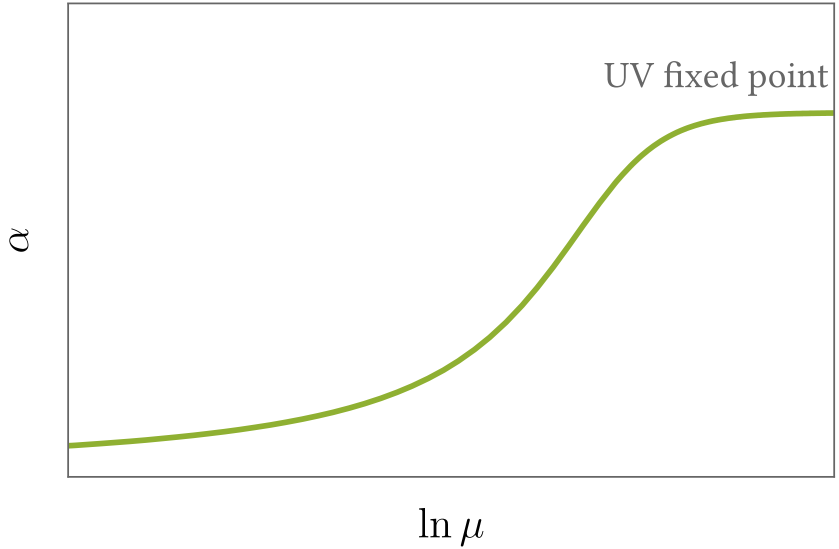

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Single exception:

→ d=4, renormalisable, weakly coupled, FP guaranteed

Litim-Sannino-Model (and equivalent theories)

[Litim, Sannino, (2014)] [Bond, Litim, TS, (2019)]

Litim–Sannino–Model (LiSa)

Field		$SU(N_c)$	$U_L(N_f)$	$U_R(N_f)$
'Quarks'	ψ_L	N_c	N_f	1
	ψ_R	N_c	1	N_f
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\begin{aligned}
\mathcal{L} = & -\frac{1}{4} F^{A\mu\nu} F^{A}{}_{\mu\nu} + \mathcal{L}_{\mathrm{gf}} + \mathcal{L}_{\mathrm{gh}} \\
& + \mathrm{Tr}\left[\overline{\psi} i \not{D} \psi\right] - y\, \mathrm{Tr}\left[\overline{\psi}\left(\phi\, \mathcal{P}_R + \phi^\dagger\, \mathcal{P}_L\right) \psi\right] \\
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\end{aligned}
$$

single trace

double trace

gauge sector (QCD)

$\downarrow\ \psi$

Yukawa (chiral!)

$\uparrow\ \phi$

scalar sector

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$\longrightarrow$ interacting fixed points under perturbative control

Exact perturbative control

» Veneziano limit: $N_{f,c} \to \infty$ but $N_f/N_c = \text{const.}$

» introduce ‘t Hooft couplings:

$$\alpha_g = \frac{N_c\, g^2}{(4\pi)^2} \qquad \alpha_y = \frac{N_c\, y^2}{(4\pi)^2} \qquad \alpha_u = \frac{N_f\, u}{(4\pi)^2} \qquad \alpha_v = \frac{N_f^2\, v}{(4\pi)^2}$$

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» conformal expansion: $\alpha^* = \epsilon\, a_{\text{LO}} + \epsilon^2\, a_{\text{NLO}} + \epsilon^3\, a_{\text{NNLO}} + \ldots$

a_{LO}	a_{NLO}	a_{NNLO}
2-loop gauge 1-loop Yukawa 1-loop quartic	3-loop gauge 2-loop Yukawa 2-loop quartic	4-loop gauge 3-loop Yukawa 3-loop quartic
[Litim,Sannino, 2014]	[Bond, Medina, Litim, TS, 2017]	[Litim, Riyaz, Stamou, TS, *soon*]

Conformal Windows of LiSa

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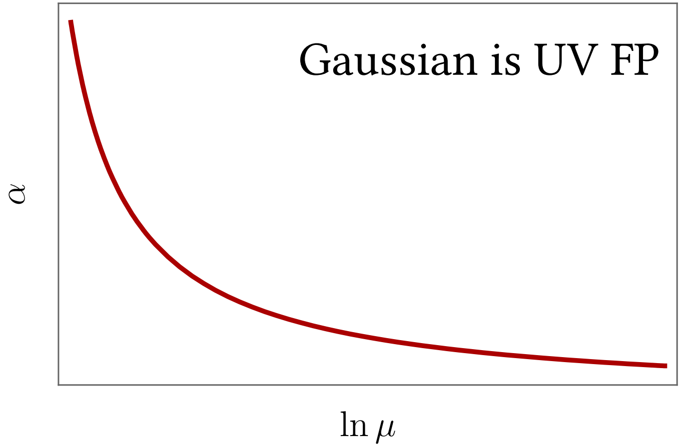

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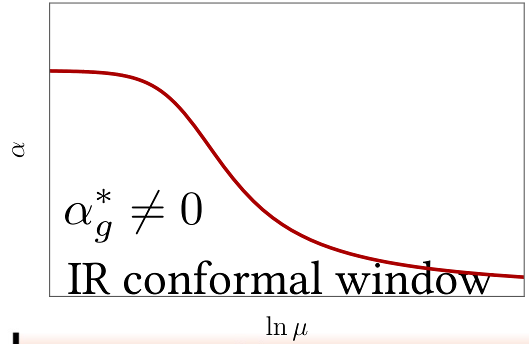

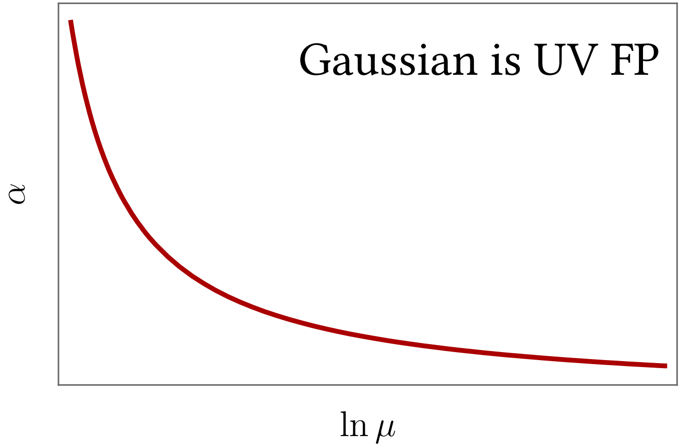

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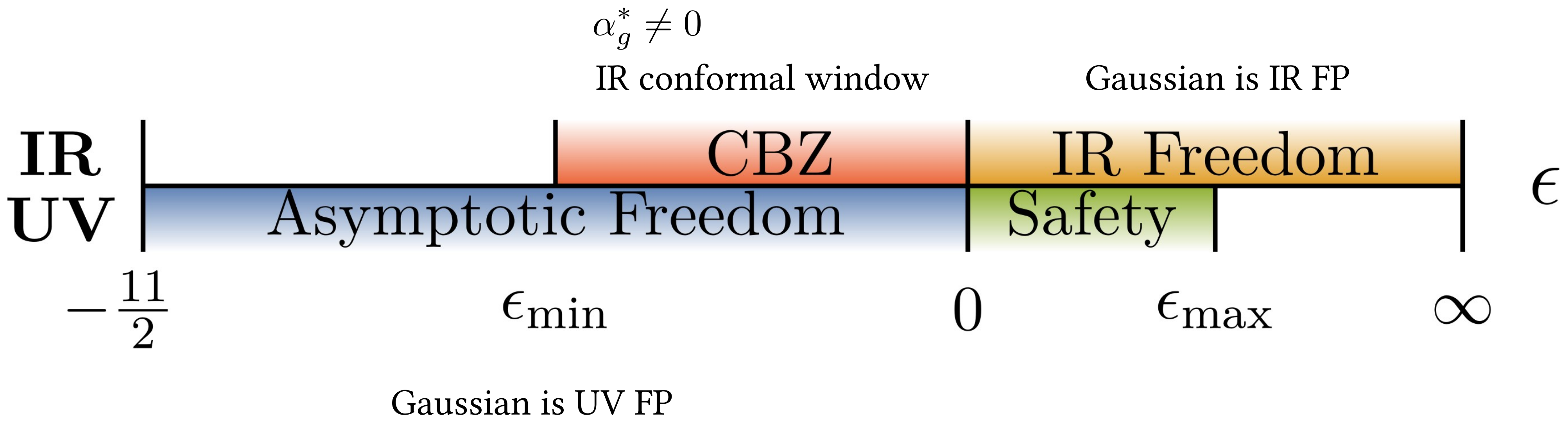

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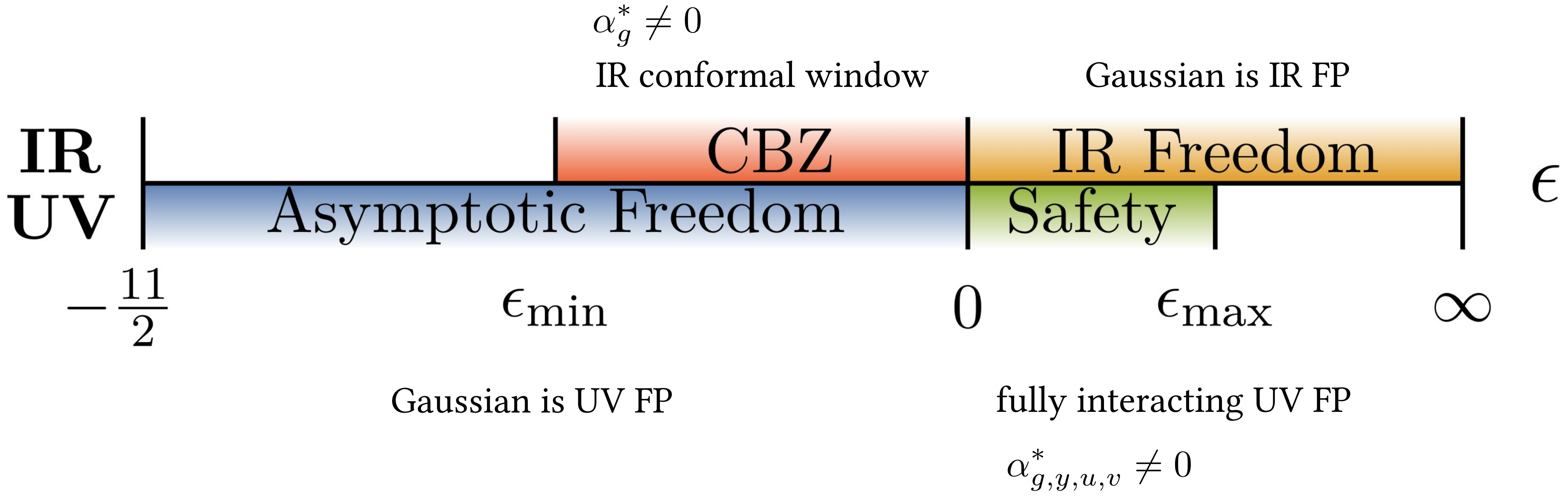


Conformal Windows of LiSa

$\alpha_g^* \neq 0$

IR conformal window

Gaussian is IR FP

Gaussian is UV FP

fully interacting UV FP

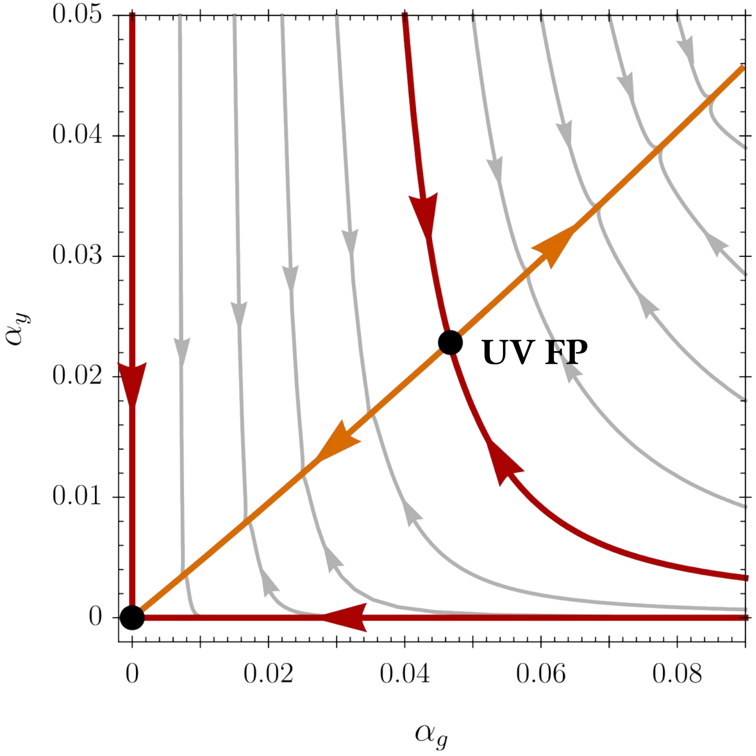
$\alpha_{g,y,u,v}^* \neq 0$

$\longrightarrow$ disappears outside of UV conformal window $[0,\ \epsilon_{\text{max}}]$

$\longrightarrow$ determine why

$\longrightarrow$ determine ϵ_{max} $\longrightarrow$ $(N_f,\ N_c)_{\text{min}}$

UV fixed point



1 relevant and 3 irrelevant directions

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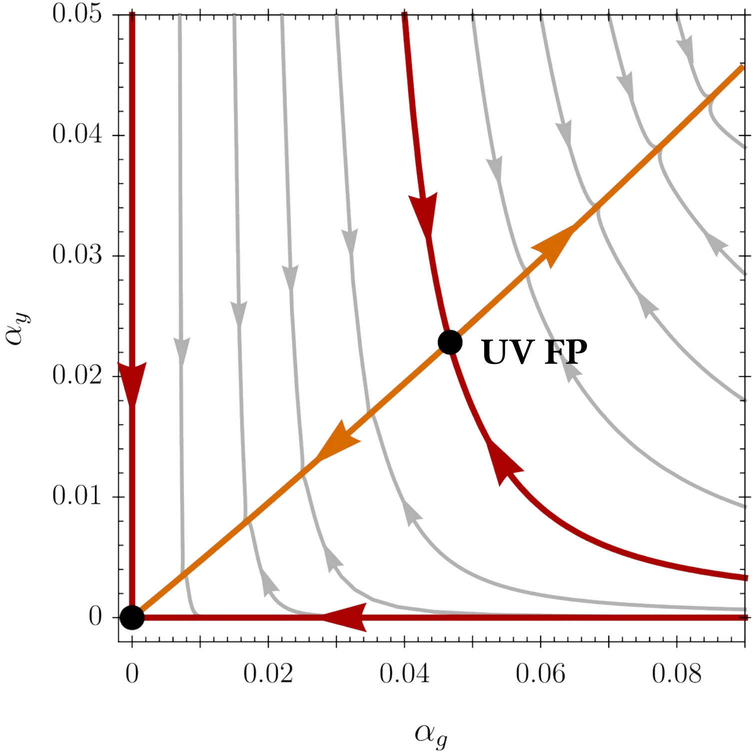

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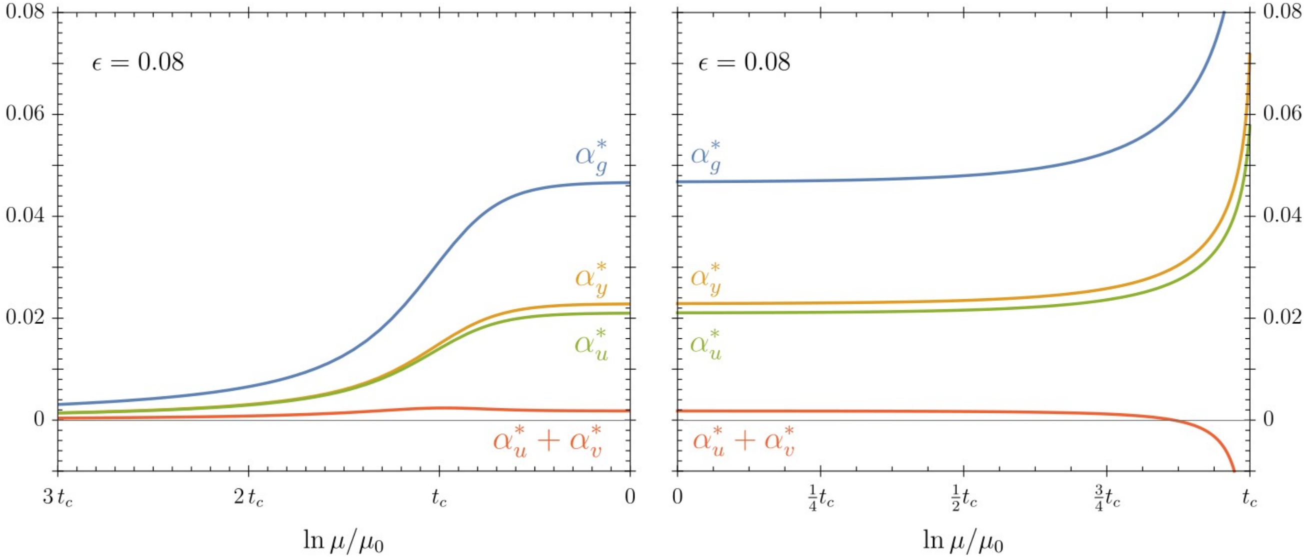

Computation

» obtain β_g at 4 loops, $\beta_{y,u,v}$ at 3 loops and evaluate $\beta_{g,y,u,v} = 0$

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» missing: gauge contributions to 3-loop Quartic beta functions

» determine (finite N) in two different ways

$\longrightarrow$ direct computation in LiSa

$\longrightarrow$ use template RGEs, extract from literature

Direct computation

- everything 3-loop (check against literature), 33.5k diagrams
- own code MaRTIn [Brod, Stamou, Steudtner '22], uses QGRAF [Nogueira '93] and FORM [Vermaseren et al.]
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⟶ no γ_5 ambiguity [Chetyrkin, Zoller '12]

2nd Method: Template RGEs

» template action with generalised fields, couplings:

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» RGE ansatz can be written down for such tensors

$$\beta^\lambda_{abcd} = c_1\,\lambda_{abef}\lambda_{efcd} + c_2\,\mathrm{Tr}\left[y^a y^e\right]\lambda_{ebcd} + c_3\,\mathrm{Tr}\left[y^a y^b y^c y^d\right] + \ldots$$

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» use SM 3L results [Chetyrkin, Zoller '12 –'13] [Bednyakov, Pikelner, Velizhanin '13] and QED-like gauge-Yukawa theory [Marquard, Boyack, Maciejko '18]

⟶ unable to fix all 36 coefficients, but enough to compute LiSa RGEs!

Conformal Window

How to probe the UV conformal window

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$\longrightarrow$ fixed point values $\alpha^*_{g,y,u,v}(\epsilon)$ from $\beta_{g,y,u,v} = 0$

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"single trace"

$$\beta_v = f_0(\alpha_{g,y,u},\epsilon) + f_1(\alpha_{g,y,u},\epsilon)\,\alpha_v + f_2(\alpha_{g,y,u},\epsilon)\,\alpha_v^2$$

$\longrightarrow$ quadratic shape, up to two solutions $\alpha_v^{*\pm}$ for each $\alpha^*_{g,y,u}$

"double trace"

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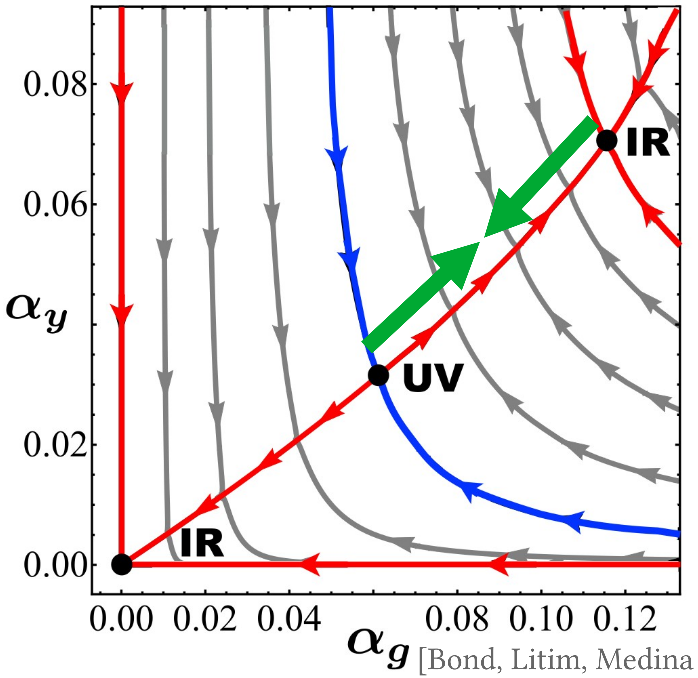

[Bond, Litim, Medina ‘20]

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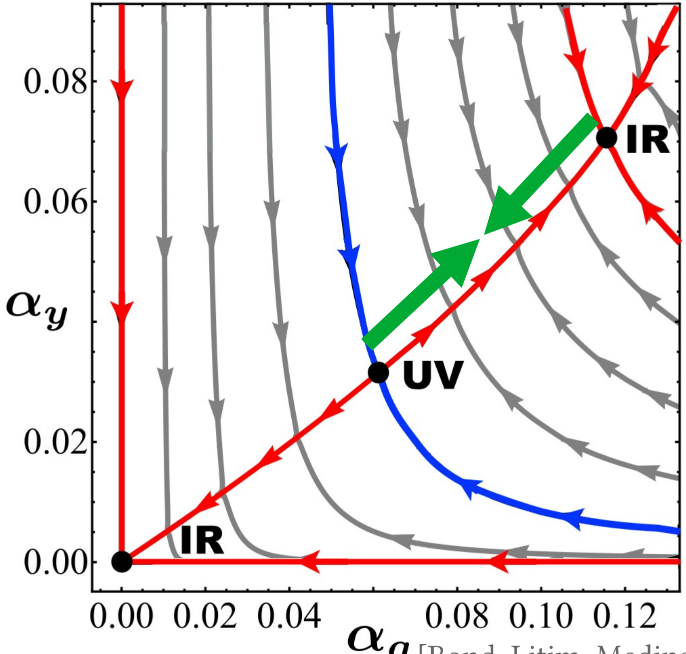

[Bond, Litim, Medina '20]

$\longrightarrow$ single trace merger in $\alpha_{g,y,u}^*$ system

$$\vartheta_1(\epsilon_{\rm max}) = 0$$

$\longrightarrow$ double trace merger: two solutions α_v^* for same $\alpha_{g,y,u}^*$

$$\vartheta_3(\epsilon_{\rm max}) = 0$$

Investigating the conformal window

» ϵ – expansion of $\alpha_i^*(\epsilon)$ and $\vartheta_i(\epsilon)$

$\longrightarrow$ series is exact up to third term

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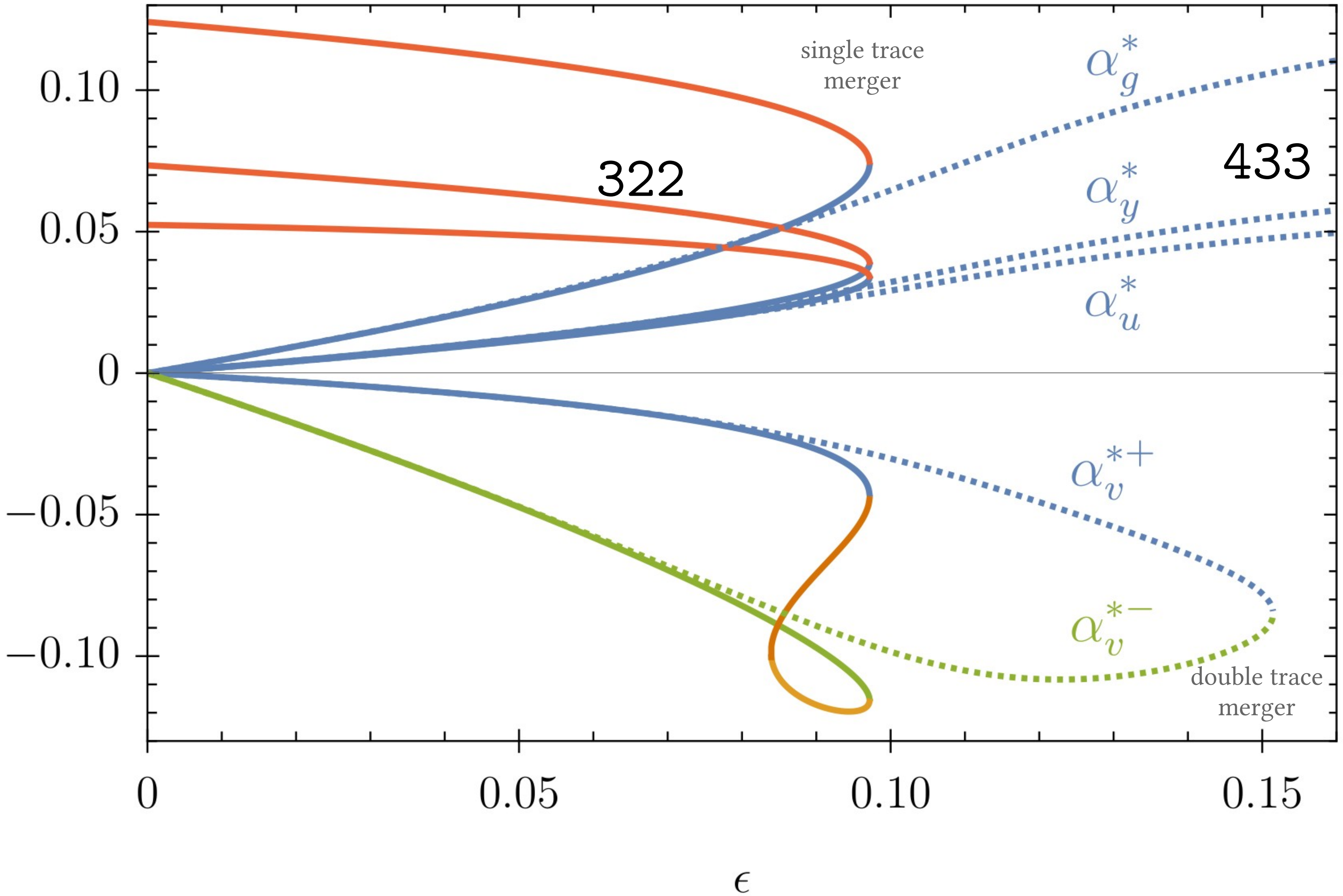
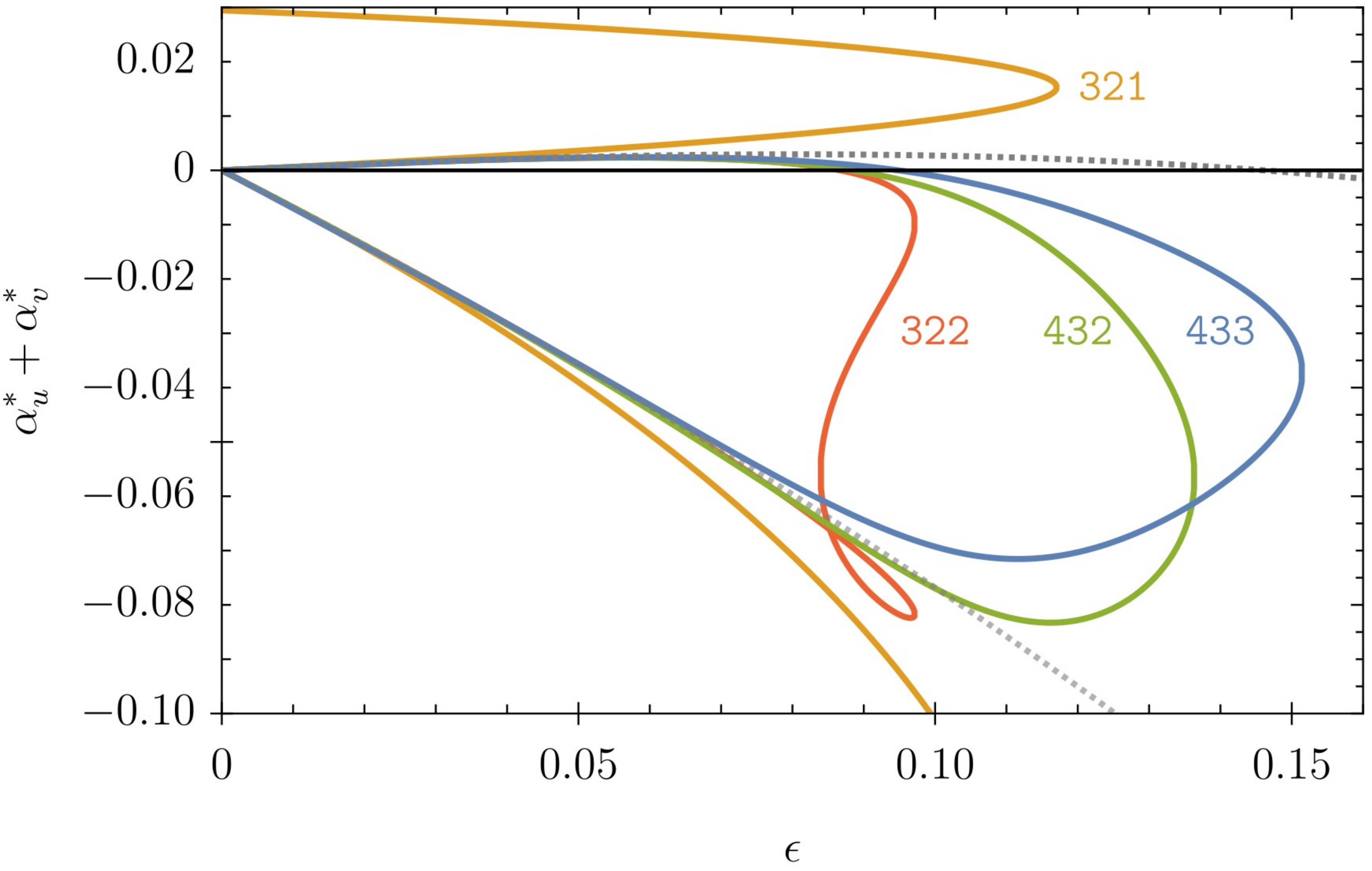

single trace
merger
322
433
α_g^*
α_y^*
α_u^*
α_v^{*+}
α_v^{*-}
double trace
merger
0.10
0.05
0
−0.05
−0.10
0
0.05
0.10
0.15
ϵ


0.02
0
−0.02
−0.04
−0.06
−0.08
−0.10
$\alpha_u^* + \alpha_v^*$
321
322
432
433
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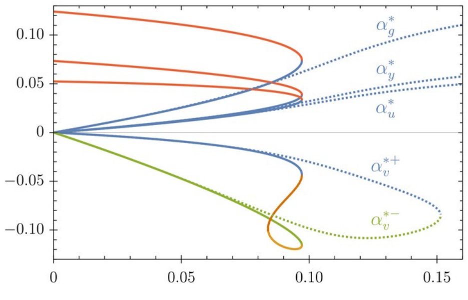

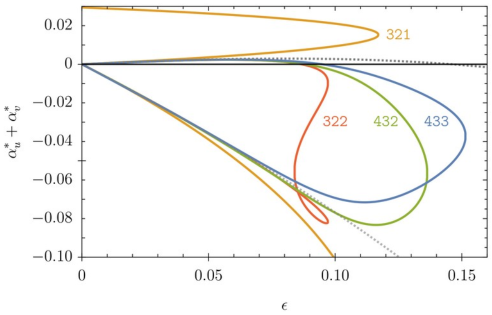

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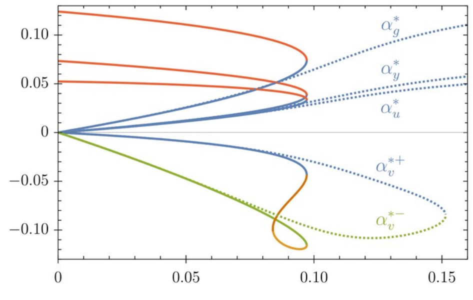

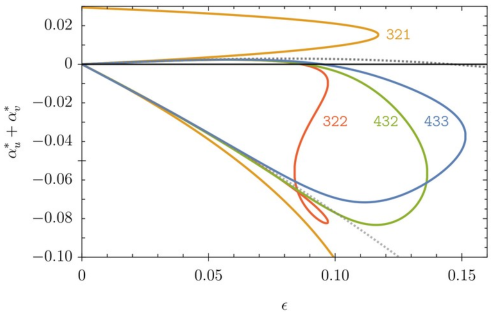

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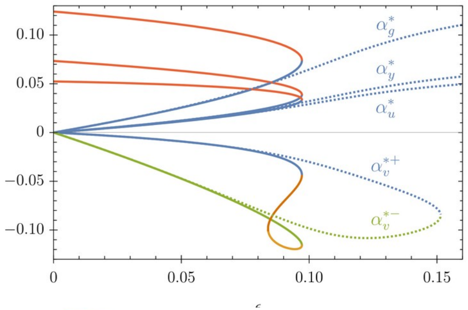

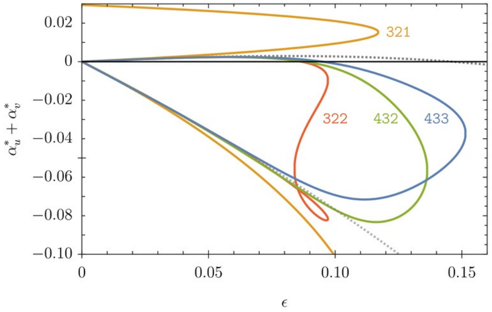

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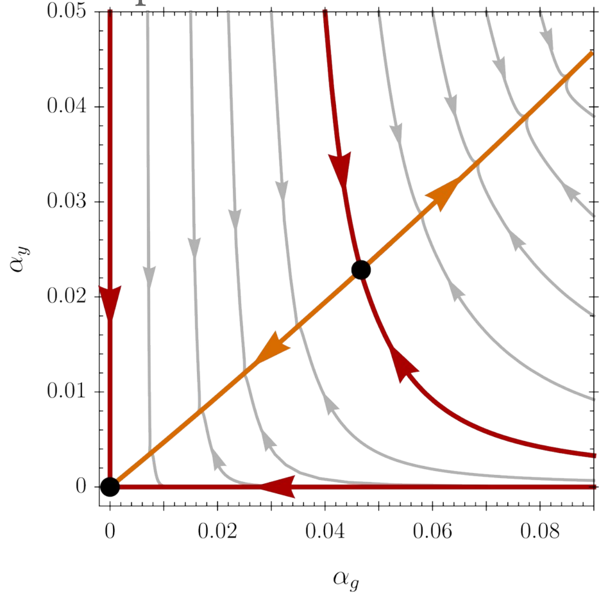
⟶ RGE along relevant separatrix



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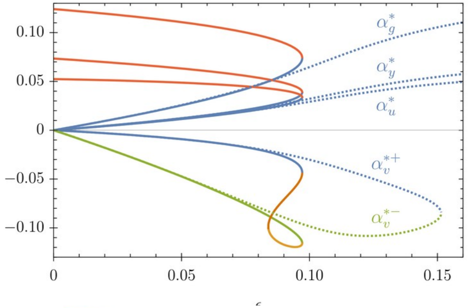

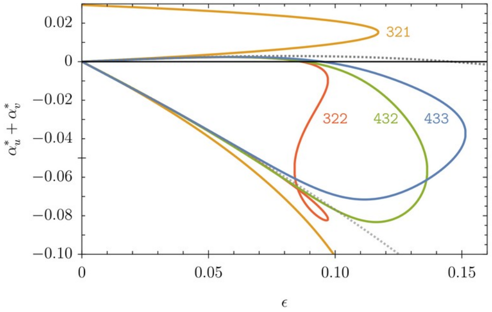

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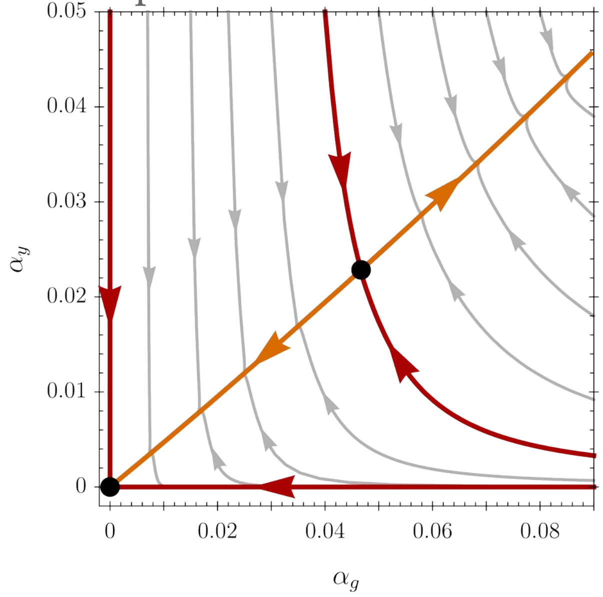
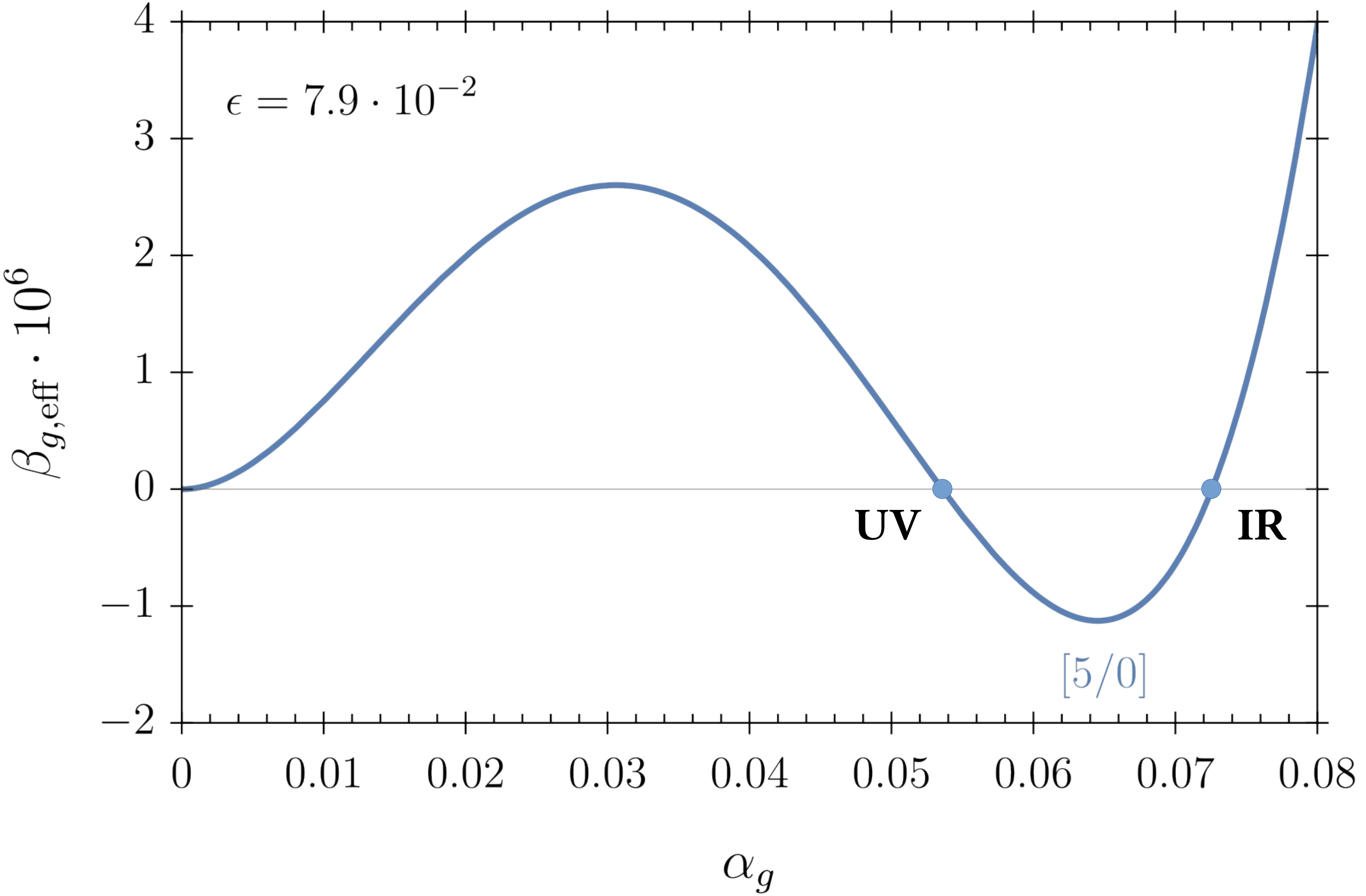


⟶ $\beta_{g,\text{eff}} = \beta_g$ with $\alpha_{y,u,v} = \alpha_{y,u,v}(\alpha_g)$

⟶ Padè and Padè-Borel resummation in α_g

Relevant Separatrix



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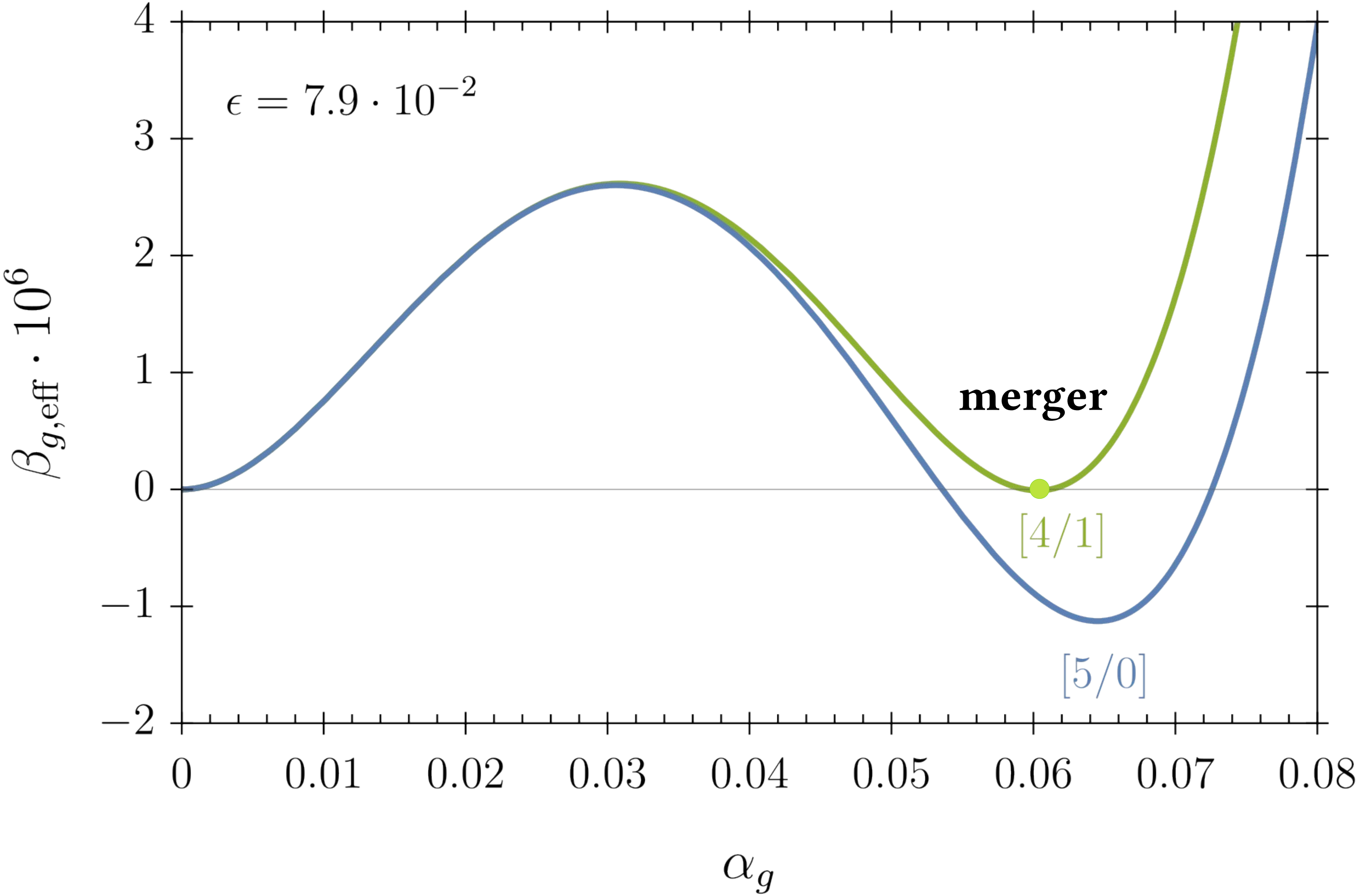

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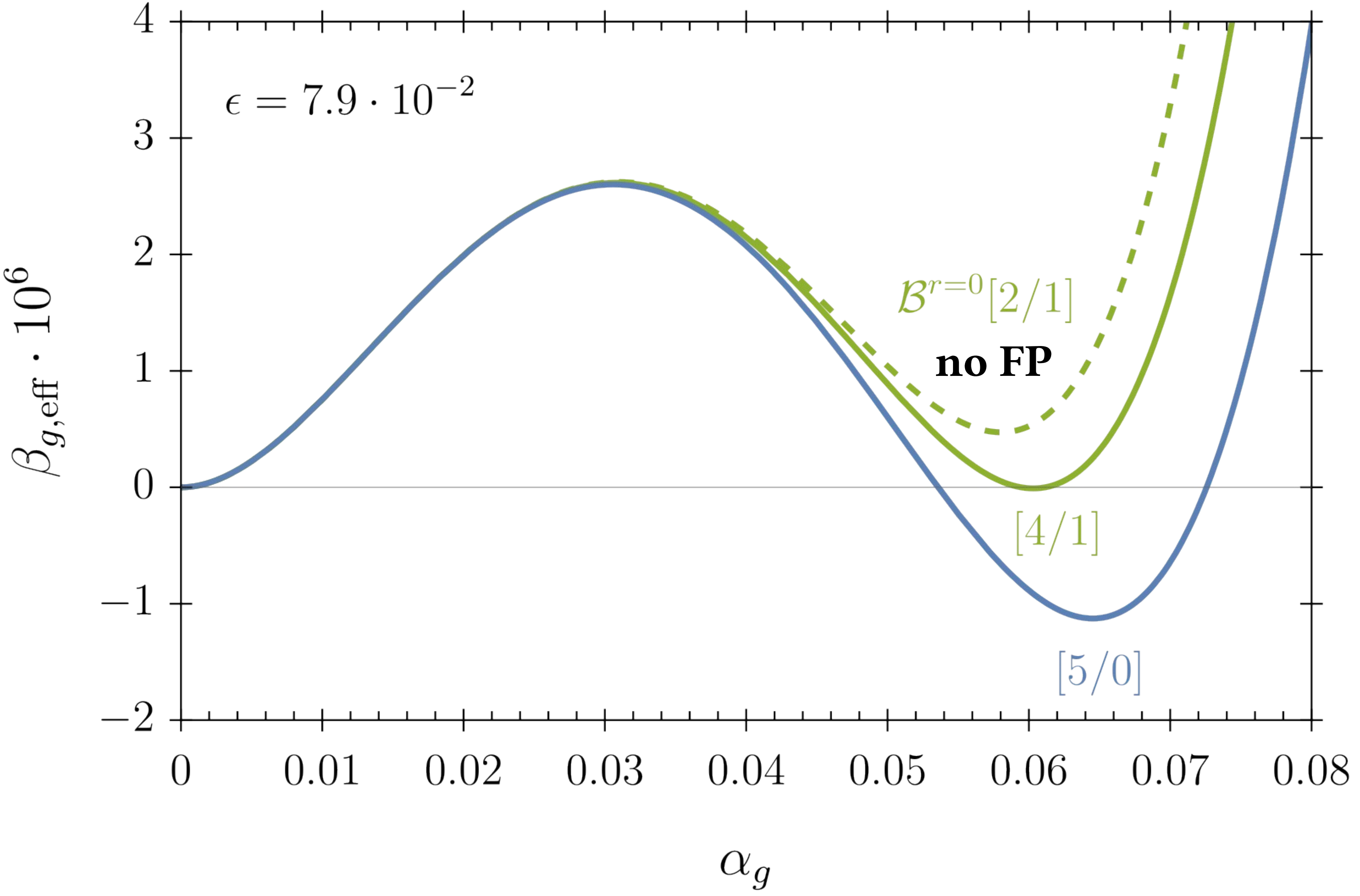

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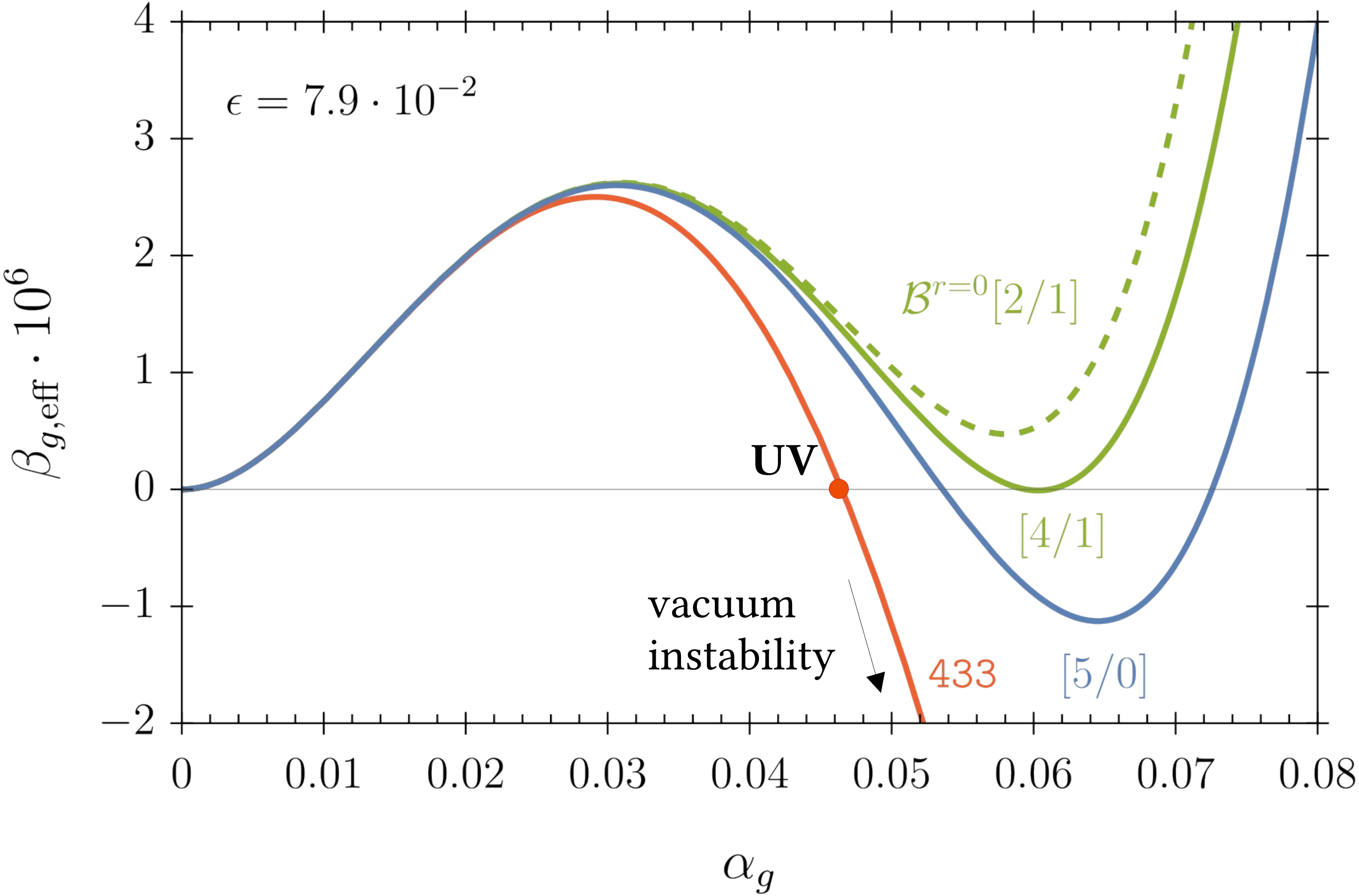

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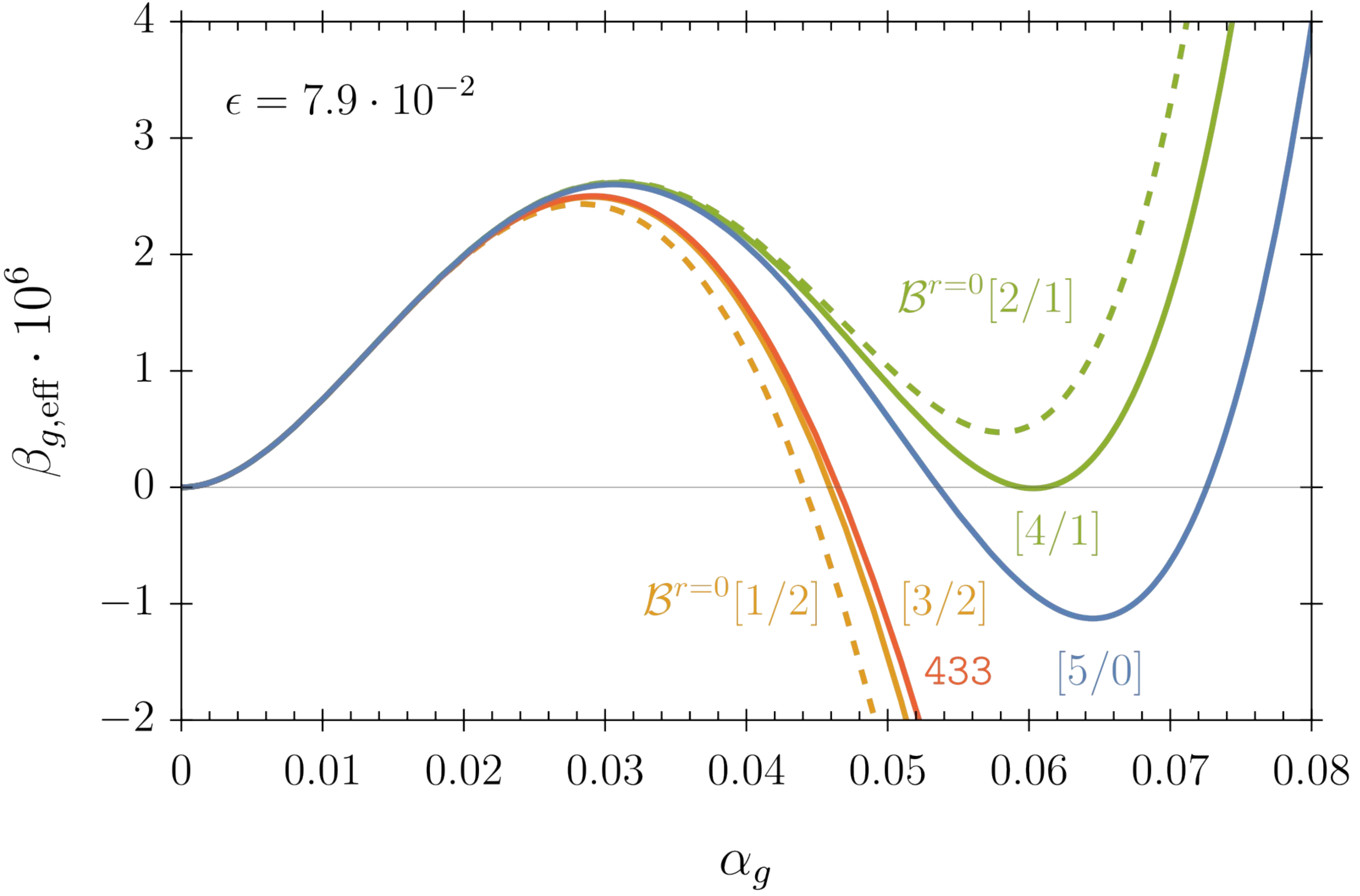

Summary

- increased consistency between conformal expansion and RGEs
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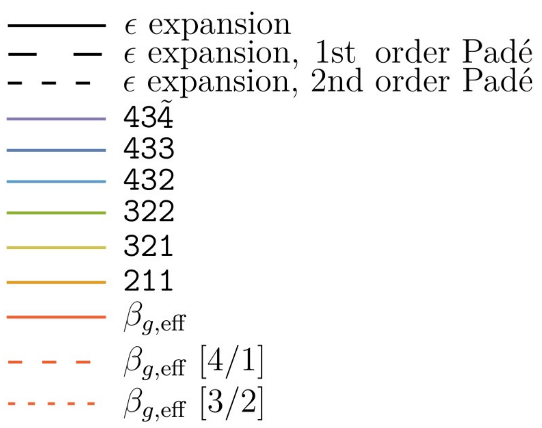

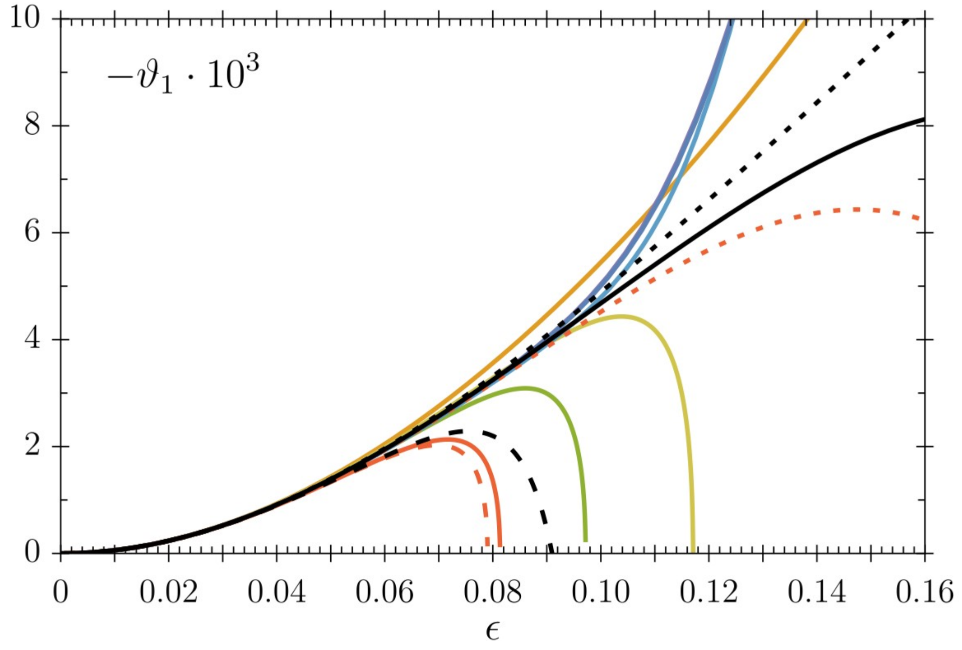

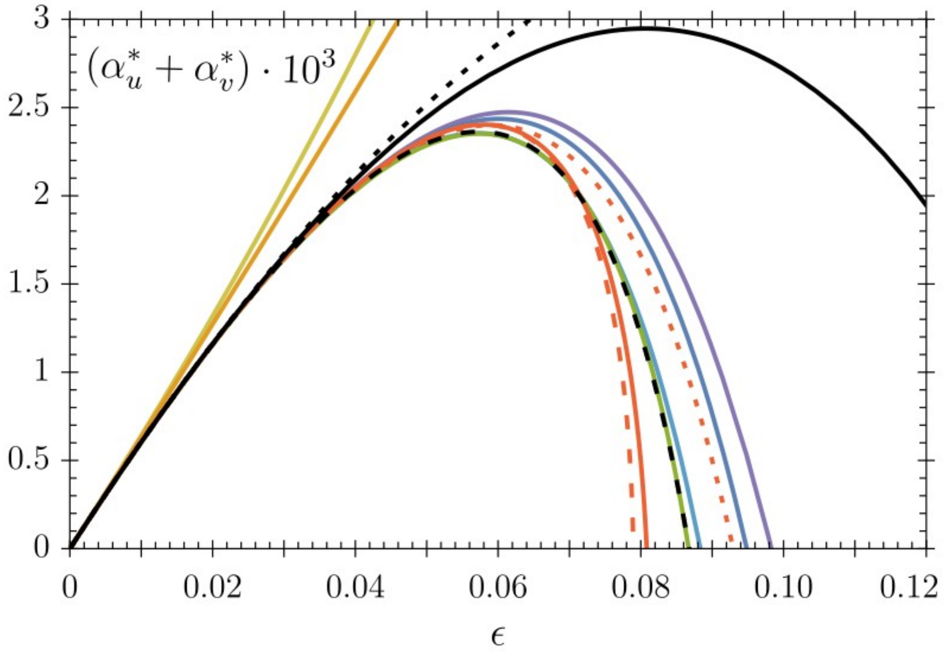

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- » double trace merger after loss of vacuum stability
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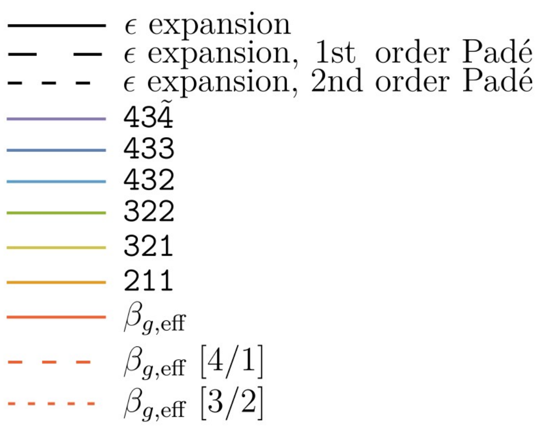
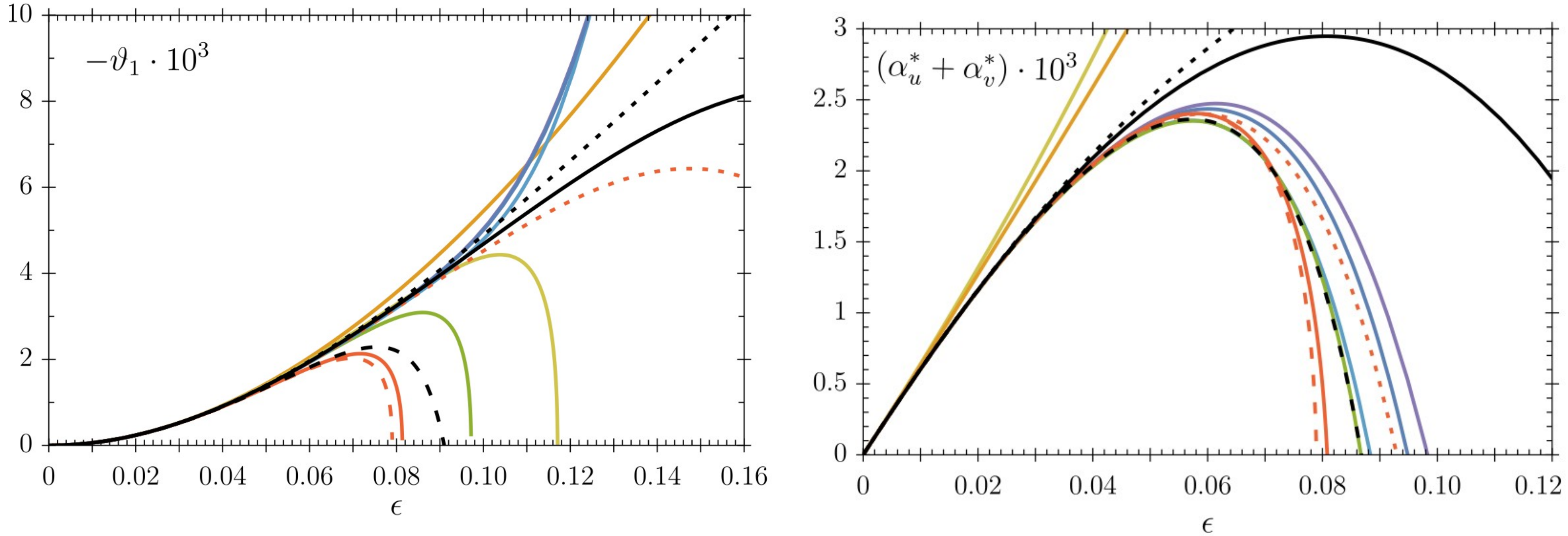
- » estimate: $\epsilon_{\text{max}} \approx 0.09 \pm 0.01$





Summary II

$$\begin{cases} N_c = 2n\,, & N_f^{(z)} = 11n + z\,, & \epsilon_{z,+}(N_c) = \dfrac{z}{N_c}\,, \\ N_c = 2m+1\,, & N_f^{(z)} = 11m + 5 + z\,, & \epsilon_{z,-}(N_c) = \dfrac{z-\frac{1}{2}}{N_c}\,. \end{cases}$$

» safe QFTs

$$(N_c,\, N_f) = (5,26),\ (7,39),\ (9,50),\ (11,61),\ \ldots$$