Five-point QCD amplitudes in Full Colour

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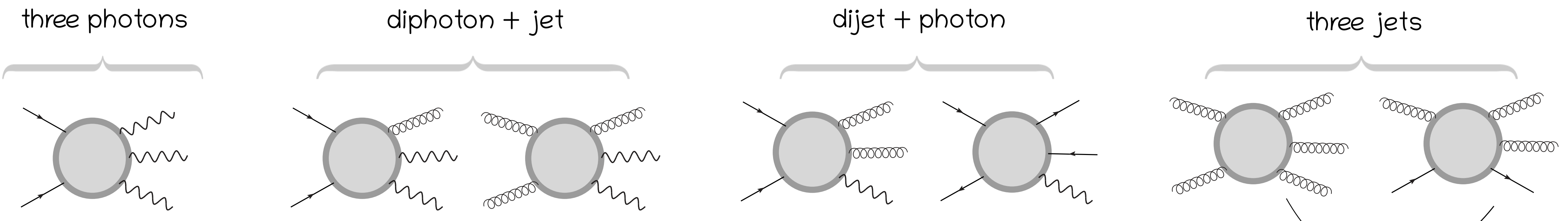
In collaboration with:

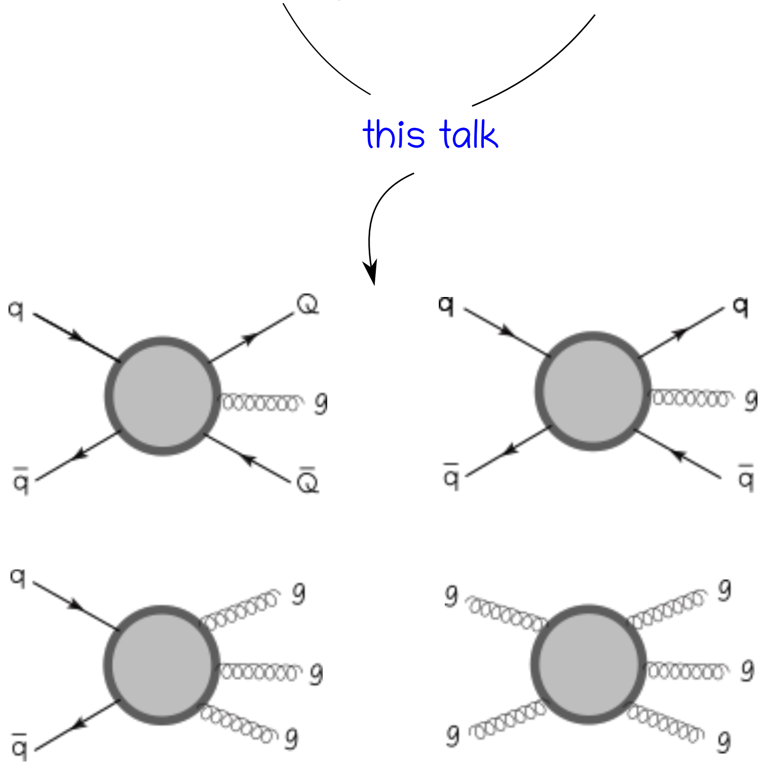
A. Agarwal, F. Devoto, G. Gambuti, A. von Manteuffel and L. Tancredi

What this talk is about

Contributors: [Abreu, Agarwal, Badger, FB, Chawhdry, Chicherin, Czakon, Cordero Febres, De Laurentis, Gehrmann, Brønnum-Hansen, Hartanto, Henn, Ita, Klinkert, Kryś, Marcoli, Mitov, Moodie, Page, Pascual, Peraro, Poncelet, Sotnikov, Tancredi, Manteuffel von, Zoia]

Two-loop massless 5-point amplitudes in QCD:





This talk is about the complete calculation of two-loop 5-point amplitudes in massless QCD for the scattering of 5 coloured particles (last missing in full colour)

Keywords:

Challenges from colour

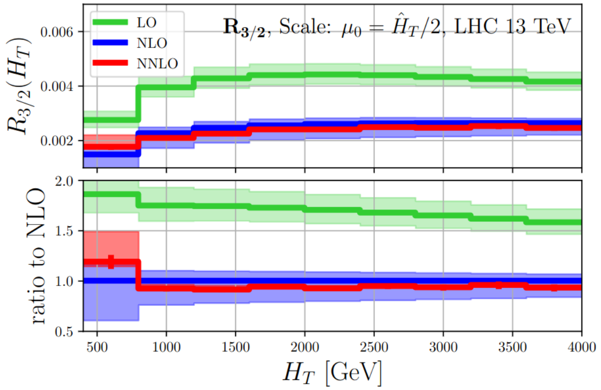
non-planar & subleading colour

algebraic complexity

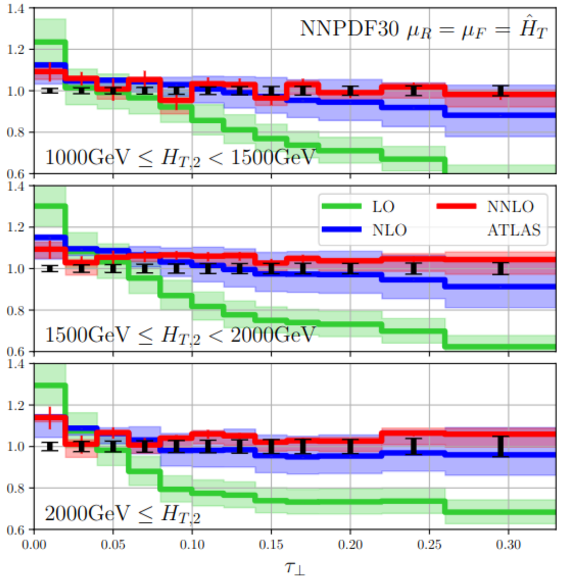
strategies to tackle all of the above

Why working so hard to get SLC contributions

First calculation of fully differential three-jet cross-sections@LHC [Czakon, Mitov, Poncelet]



[Czakon, Mitov, Poncelet 2106.05331]



[Czakon, Mitov, Poncelet 2301.01086]

Calculation carried out in full colour except for two-loop hard function:
evaluated in Leading Colour approximation

Hard to imagine a big shift in predictions caused by 2-loop sub-leading colour contributions, but...

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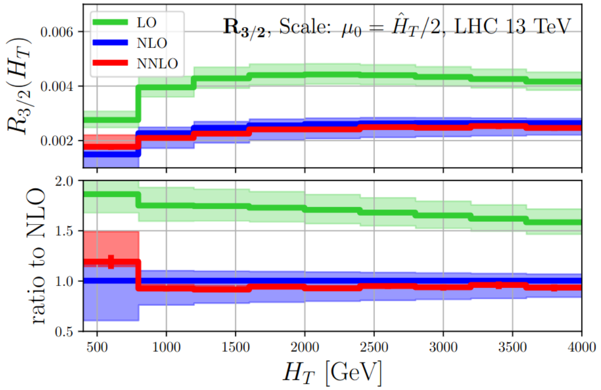

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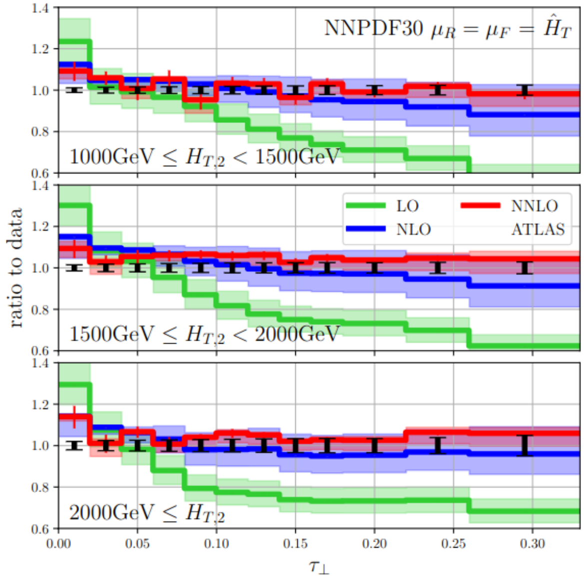

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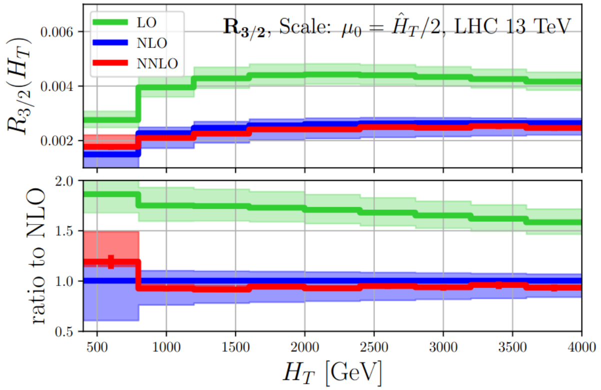

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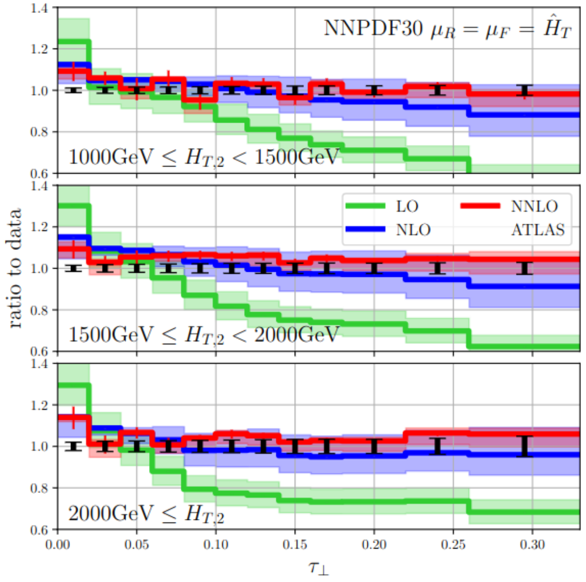

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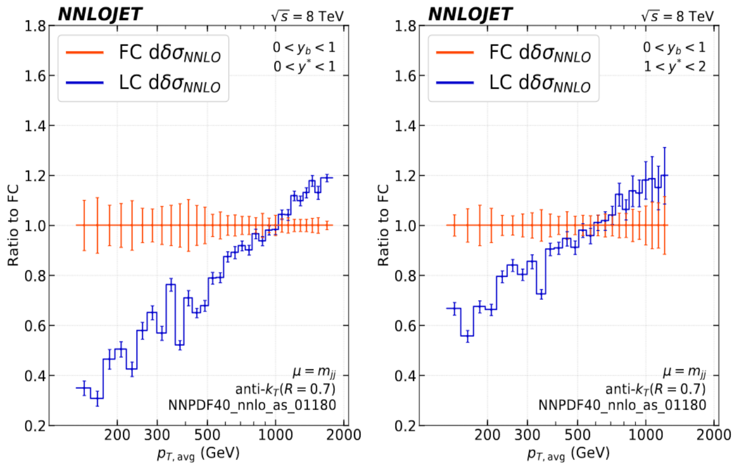

[Chen, Gehrmann, Glover, Huss, Mo 2204.10173]

Sizeable impact of sub-leading colour effect in dijet@NNLO for triple differential distributions

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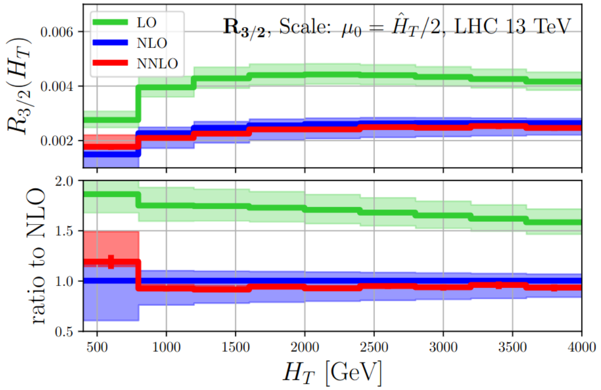

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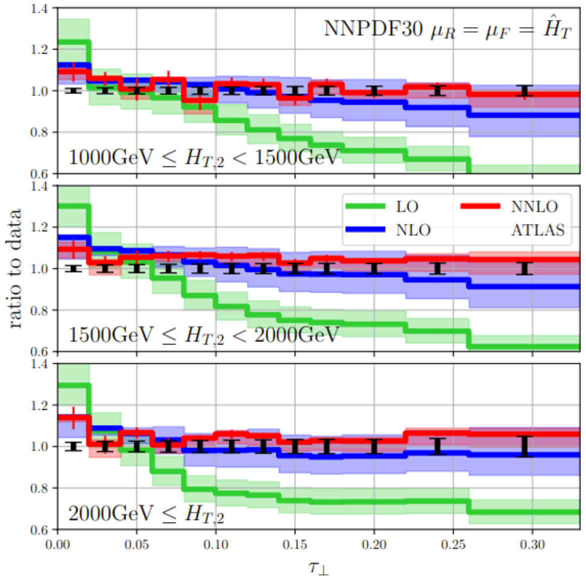

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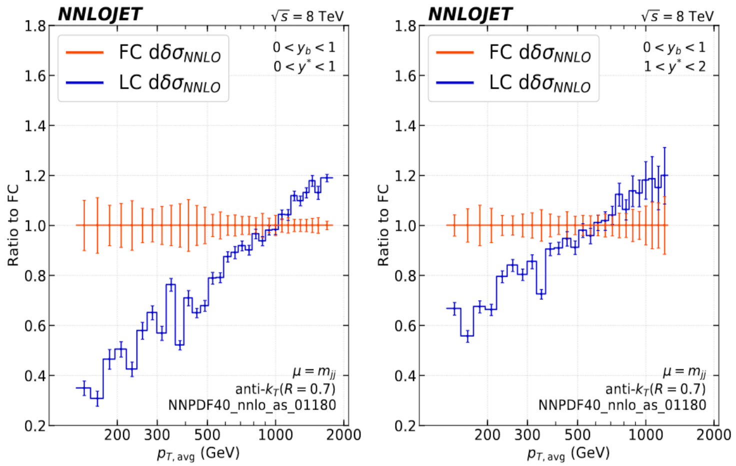

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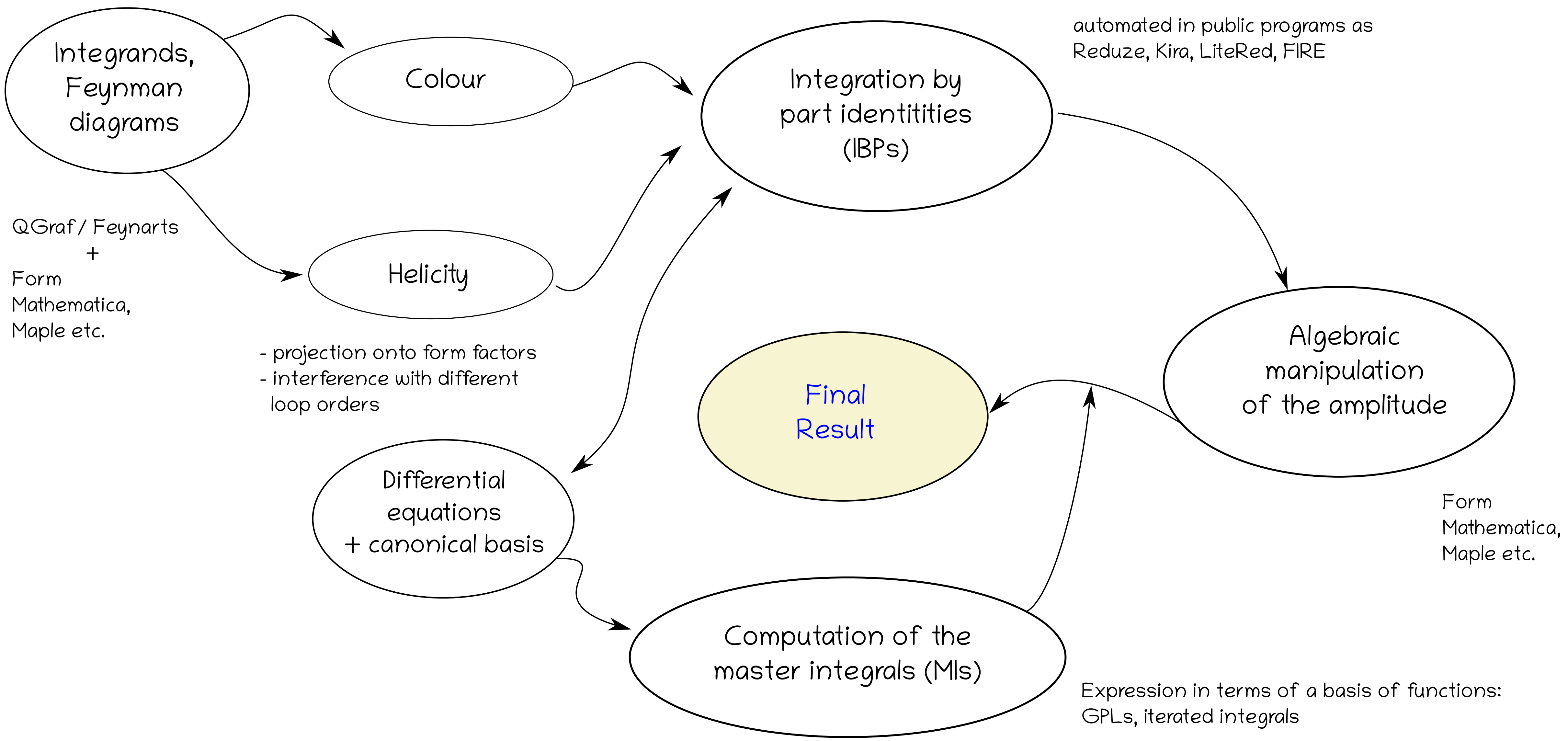
Sizeable impact of sub-leading colour effect in dijet@NNLO for triple differential distributions

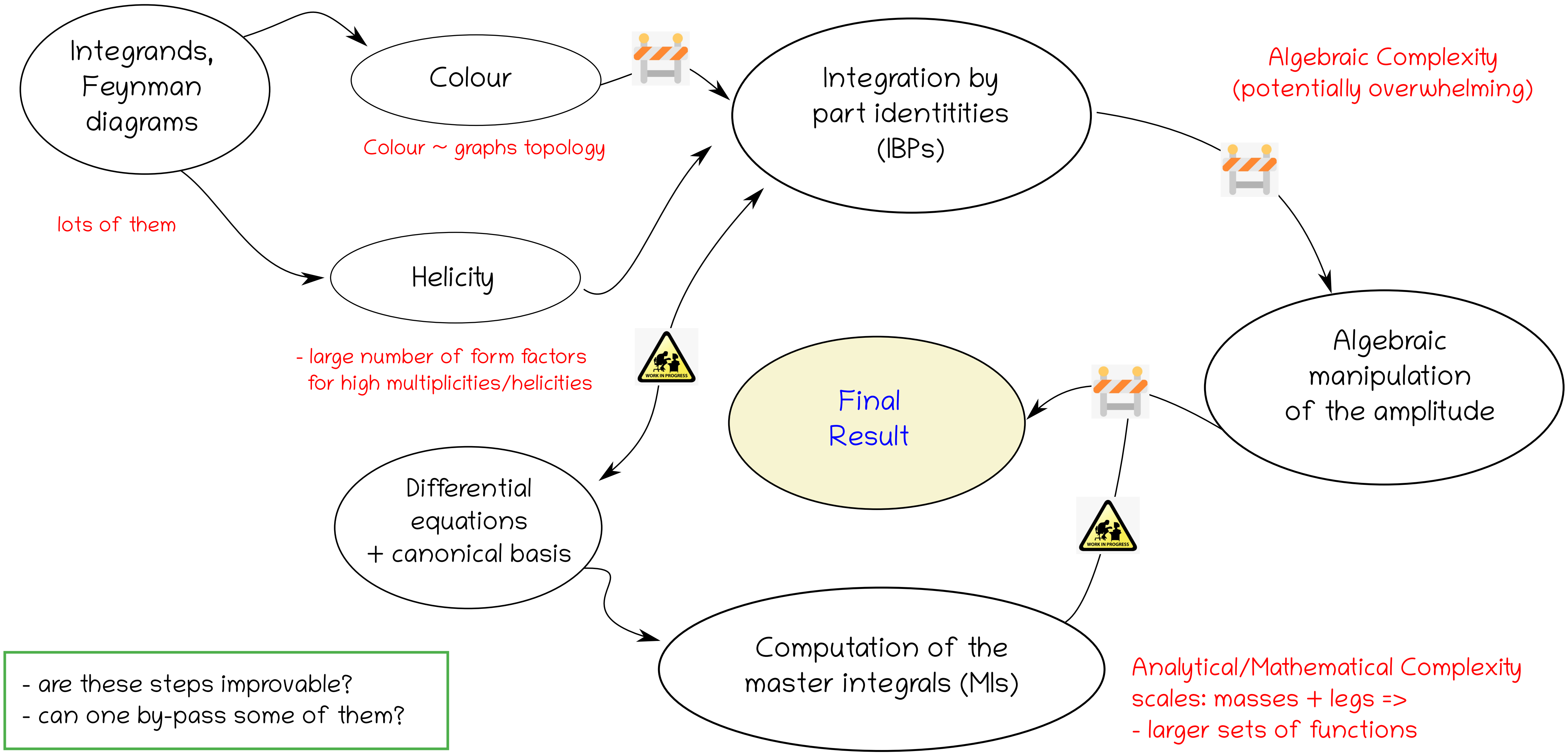
More generally at the amplitude level:

- structure of non-planar QCD sectors
- RVV contribution to N^3LO dijet
- IR limits 5 → 4
- multi-Regge kinematics

Traditional approach to multiloop calculation

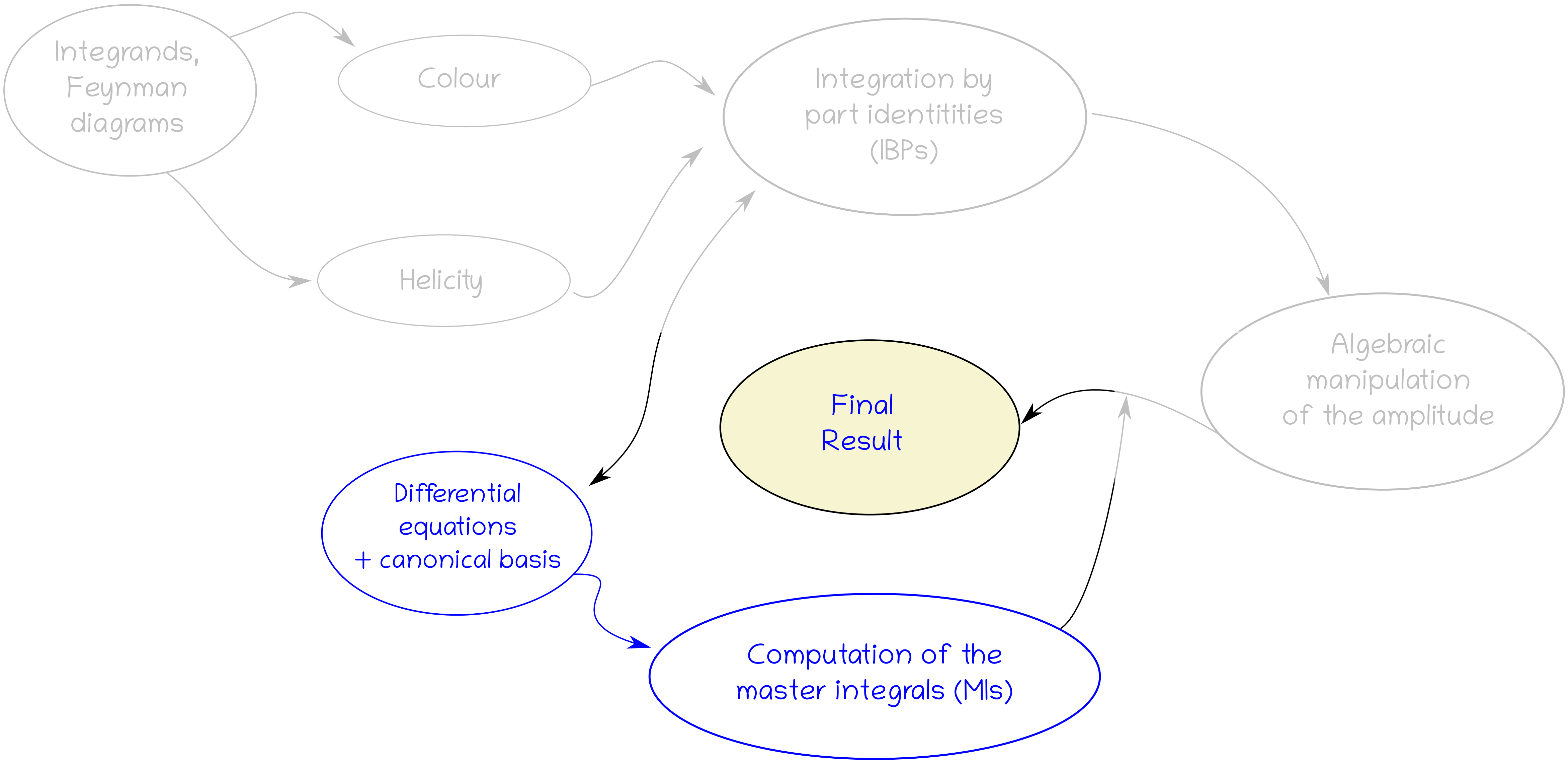


Challenges and complexity



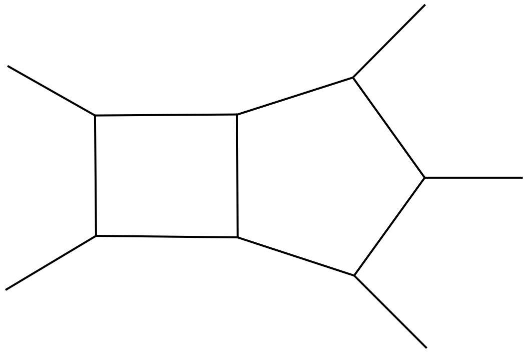
- are these steps improvable?
- can one by-pass some of them?

Traditional approach to multiloop calculation



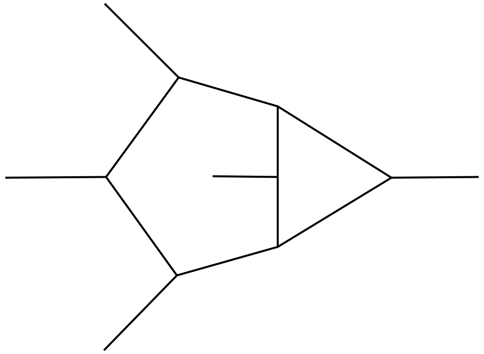
Pentagon functions for 2→3 masless scattering amplitudes

Pentagon-Box



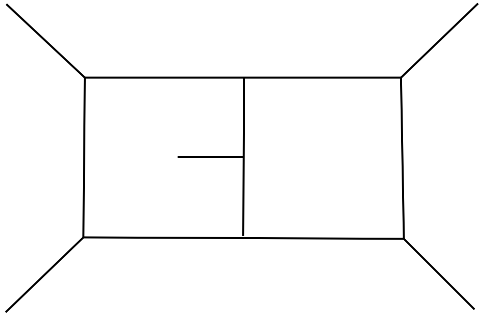
[Gehrmann, Henn, Lo Presti 1511.05409, 1807.09812],
[Papadopoulos, Tommasini, Wever 1511.09404]

Hexagon-Box



[Boehm, Georgoudis, Larsen, Schoenemann, Zhang],
[Abreu, Page, Zeng, 1807.11522]
[Chicherin, Gehrmann, Henn, Lo Presti, Mitev, Wasser 1809.06240]

Double-Pentagon



[Abreu, Dixon, Herrmann, Page, Zeng 1901.08563],
[Chicherin, Gehrmann, Henn, Wasser, Zhang, Zoia 1812.11160]

MIs through Pentagon Functions
Expressed (and evaluated) as iterated Chen integrals along a path γ

$$f^{(\omega)}(\vec{x}) = \int_\gamma \mathrm{d}\log W_{i_1} \dots \mathrm{d}\log W_{i_n}$$

ω integrations

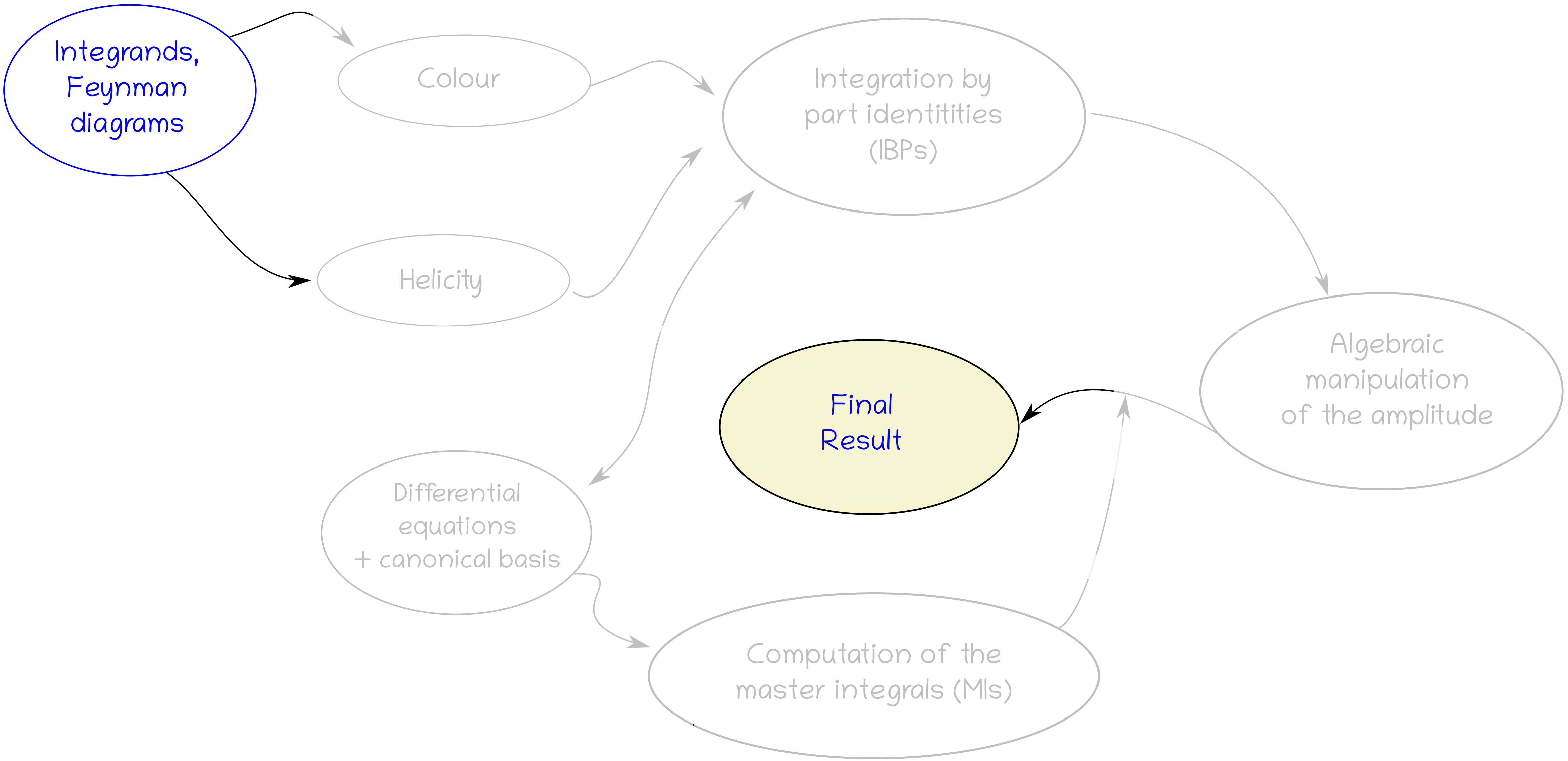
Full set made available [Chicherin, Sotnikov 2009.07803]

Results in the whole physical region

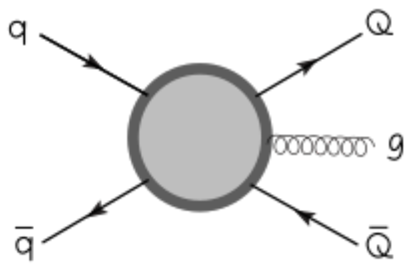
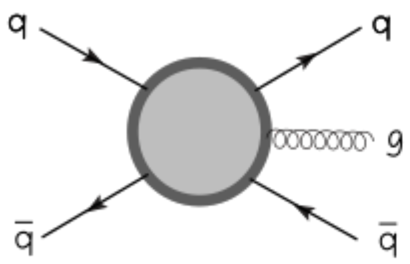
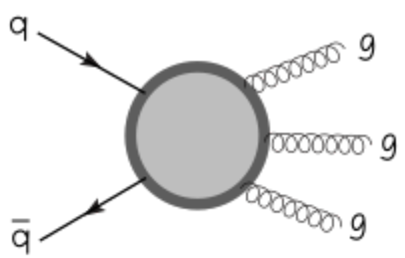
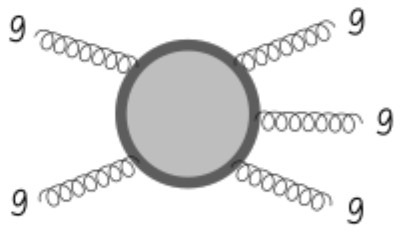
They can be used for *all* massless 5-pt amplitudes

Evaluation in ~ 1s

Traditional approach to multiloop calculation



Two-loop five-point QCD amplitudes

				
Feynman Diagrams	2522	4258	9136	28020
Helicities	8	8	16	32
Dimension colour space	4	4	11	22
Tot # colour structures	24	24	54	75

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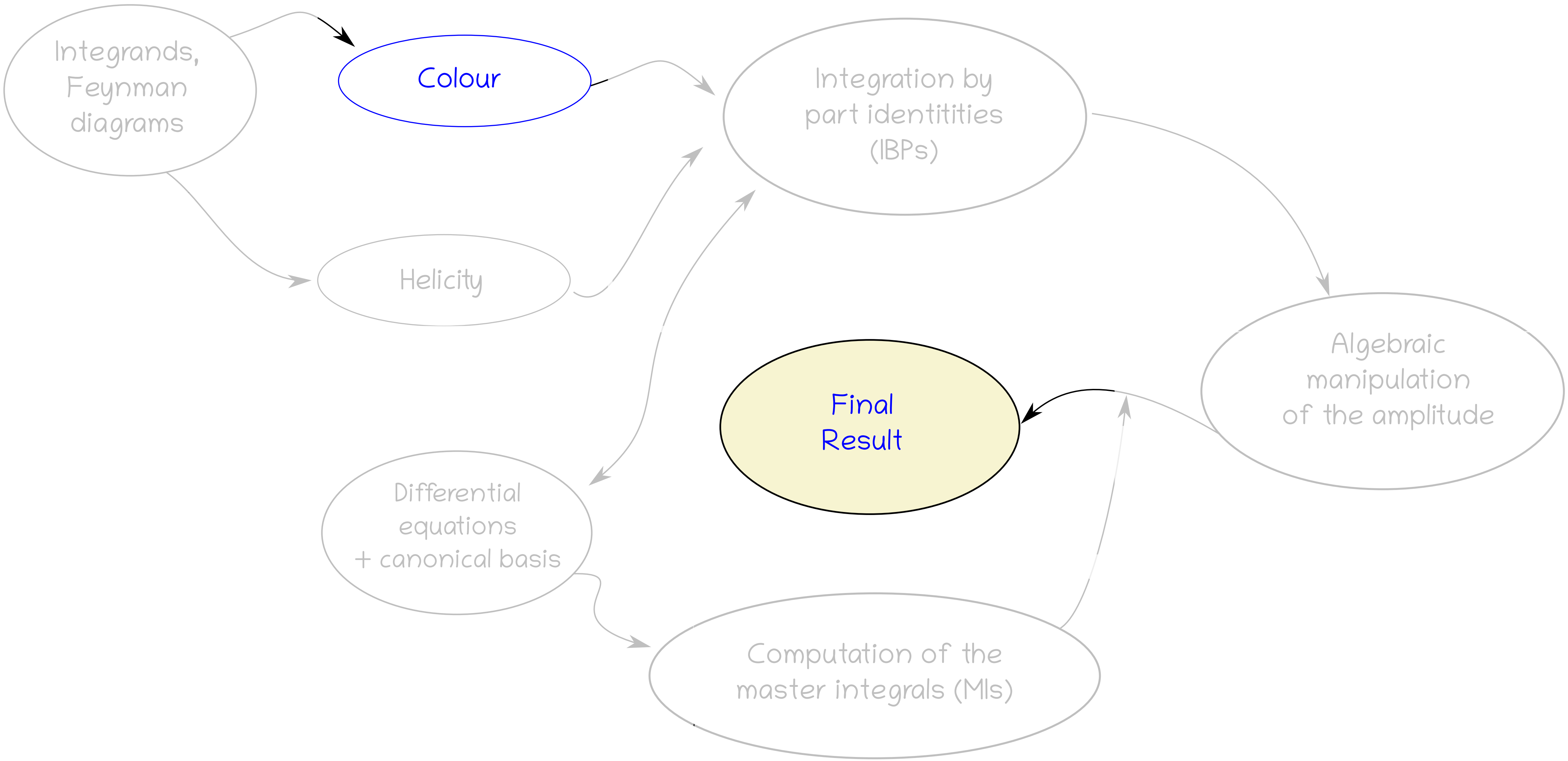

Amplitudes in colour space

$$\mathcal{A}^{c_1 \dots c_5} = \sum_i^n A_i | c_1 \dots c_5 \rangle_i \equiv \sum_i^n A_i | \mathcal{C}_i \rangle$$

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Partial amplitudes:
polynomials in N_c, n_f

Basis elements
of the colour space

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Basis elements
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full colour + matter content: @2loop corrections on top of tree as N_c^k with k in [-2,2] and n_f^k with k in [0,2]

$$A_i = a_i^{(2,0)} N_c^2 + a_i^{(1,0)} N_c + a_i^{(0,0)} 1 + a_i^{(-1,0)} N_c^{-1} + a_i^{(0,-2)} N_c^{-2} + a_i^{(1,1)} N_c n_f + a_i^{(0,2)} n_f^2 + \dots$$

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Subleading colour

Amplitudes in colour space

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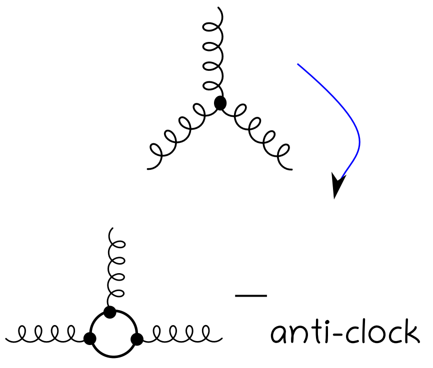
Subleading colour

$$\mathcal{A}^{(L)} = \sum_{\lambda=1}^{12} \left(\sum_{k=0}^{\lfloor \frac{L}{2} \rfloor} N^{L-2k} A_\lambda^{(L,2k)} \right) T_\lambda + \sum_{\lambda=13}^{22} \left(\sum_{k=0}^{\lfloor \frac{L-1}{2} \rfloor} N^{L-2k-1} A_\lambda^{(L,2k+1)} \right) T_\lambda$$

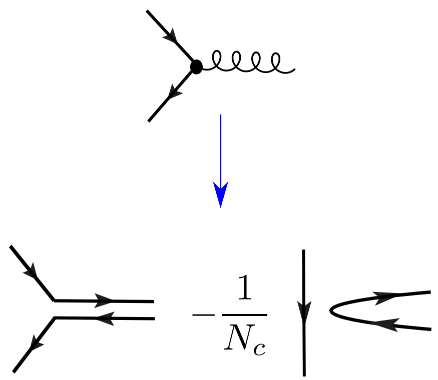
0→99999

Amplitudes in leading N_c

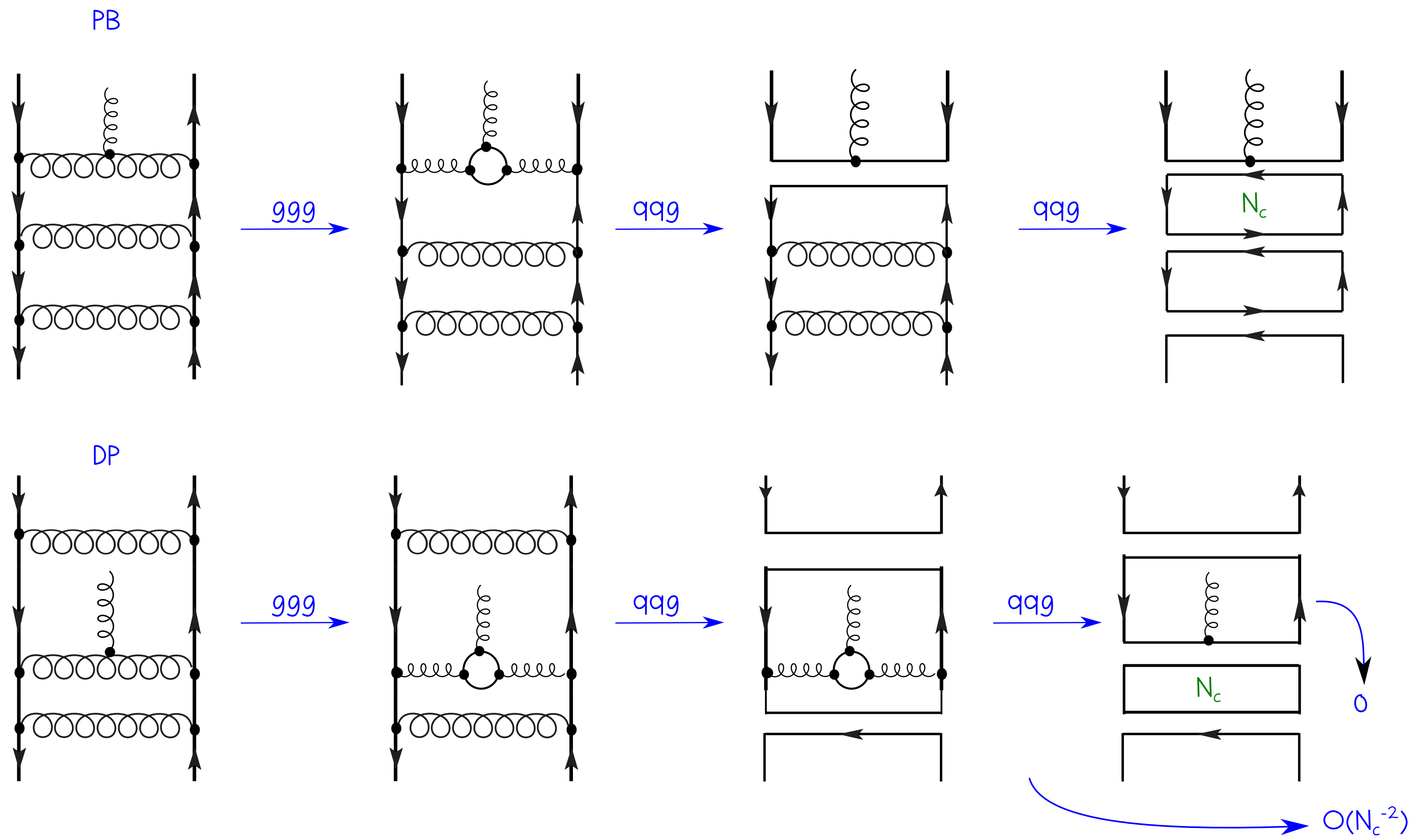
ggg interaction



qqg interaction



Leading colour contributions only from planar diagrams [t'Hooft '73]



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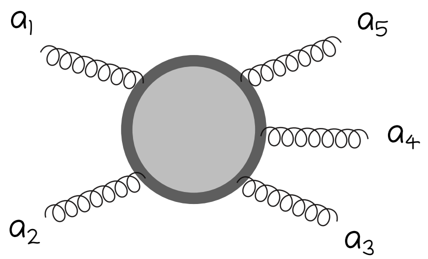
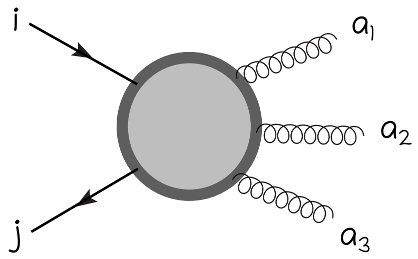
In leading colour approximation the basis is the same at all loop orders

Beyond leading colour the full colour space is spanned

$0 \to ggggg$



$0 \to \bar{q}qggg$



At tree-level:

$|\mathcal{C}_i\rangle$ $\quad \mathrm{Tr}\,(T^{a_1}T^{a_2}T^{a_3}T^{a_4}T^{a_5}) - \mathrm{Tr}\,(T^{a_5}T^{a_4}T^{a_3}T^{a_2}T^{a_1})$

+ 11 permutations → $[S_5/Z_5] \times 1/2$

From 1-loop on: tree-level +

$$[\mathrm{Tr}\,(T^{a_1}T^{a_2}) \cdot (\mathrm{Tr}\,(T^{a_1}T^{a_2}T^{a_3}) - \mathrm{Tr}\,(T^{a_3}T^{a_2}T^{a_1}))]$$

+ 9 permutations

At tree-level:

$|\mathcal{C}_i\rangle$ $\quad (T^{a_1}T^{a_2}T^{a_3})_{ji}$ + 5 permutations of (a_1,a_2,a_3)

From 1-loop on: tree-level +

$T^{a_1}_{ji}\,\mathrm{Tr}\,(T^{a_2}T^{a_3})$ + 2 permutations

$\mathrm{Tr}\,(T^{a_1}T^{a_2}T^{a_3}) - \mathrm{Tr}\,(T^{a_3}T^{a_2}T^{a_1}) \sim f^{a_1a_2a_3}$

$\mathrm{Tr}\,(T^{a_1}T^{a_2}T^{a_3}) + \mathrm{Tr}\,(T^{a_3}T^{a_2}T^{a_1}) \sim d^{a_1a_2a_3}$

Infrared structure

$$\mathcal{A}_{\text{ren}}(\epsilon,\{p\}) = \mathbf{Z}(\epsilon,\{p\},\mu)\mathcal{A}_{\text{fin}}(\mu,\{p\})$$

$$\mathbf{Z}^{-1} = \mathbf{1} - \left(\frac{\alpha_s}{2\pi}\right)\mathbf{I}_1 - \left(\frac{\alpha_s}{2\pi}\right)^2\mathbf{I}_2 + \mathcal{O}(\alpha_s^3)$$

[Catani 9802439, Gardi, Magnea: 0901.1091, 0908.3273; Becher, Neubert: 0903.1126]

$$\boldsymbol{I}^{(1)}(\epsilon) = \frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)}\sum_i\left(\frac{1}{\epsilon^2} - \frac{\gamma_0^i}{2\epsilon}\frac{1}{\boldsymbol{T}_i^2}\right)\sum_{j\neq i}\frac{\boldsymbol{T}_i\cdot\boldsymbol{T}_j}{2}\left(\frac{\mu^2}{-s_{ij}}\right)^\epsilon$$

$I_{1,2}$ operators are diagonal in leading colour

$$\boldsymbol{I}^{(2)}(\epsilon) = \frac{e^{-\epsilon\gamma_E}\,\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)}\left(\frac{\gamma_1^{\text{cusp}}}{8} + \frac{\beta_0}{2\epsilon}\right)\boldsymbol{I}^{(1)}(2\epsilon) - \frac{1}{2}\boldsymbol{I}^{(1)}(\epsilon)\left(\boldsymbol{I}^{(1)}(\epsilon) + \frac{\beta_0}{\epsilon}\right) + \boldsymbol{H}^{(2)}_{\text{R.S.}}(\epsilon)$$

Beyond leading colour:

- off-diagonal elements in $I_{1,2}$ operators
- within this definition of I_1 need to be taken into account appearing only beyond LC with n > 3 coloured particles

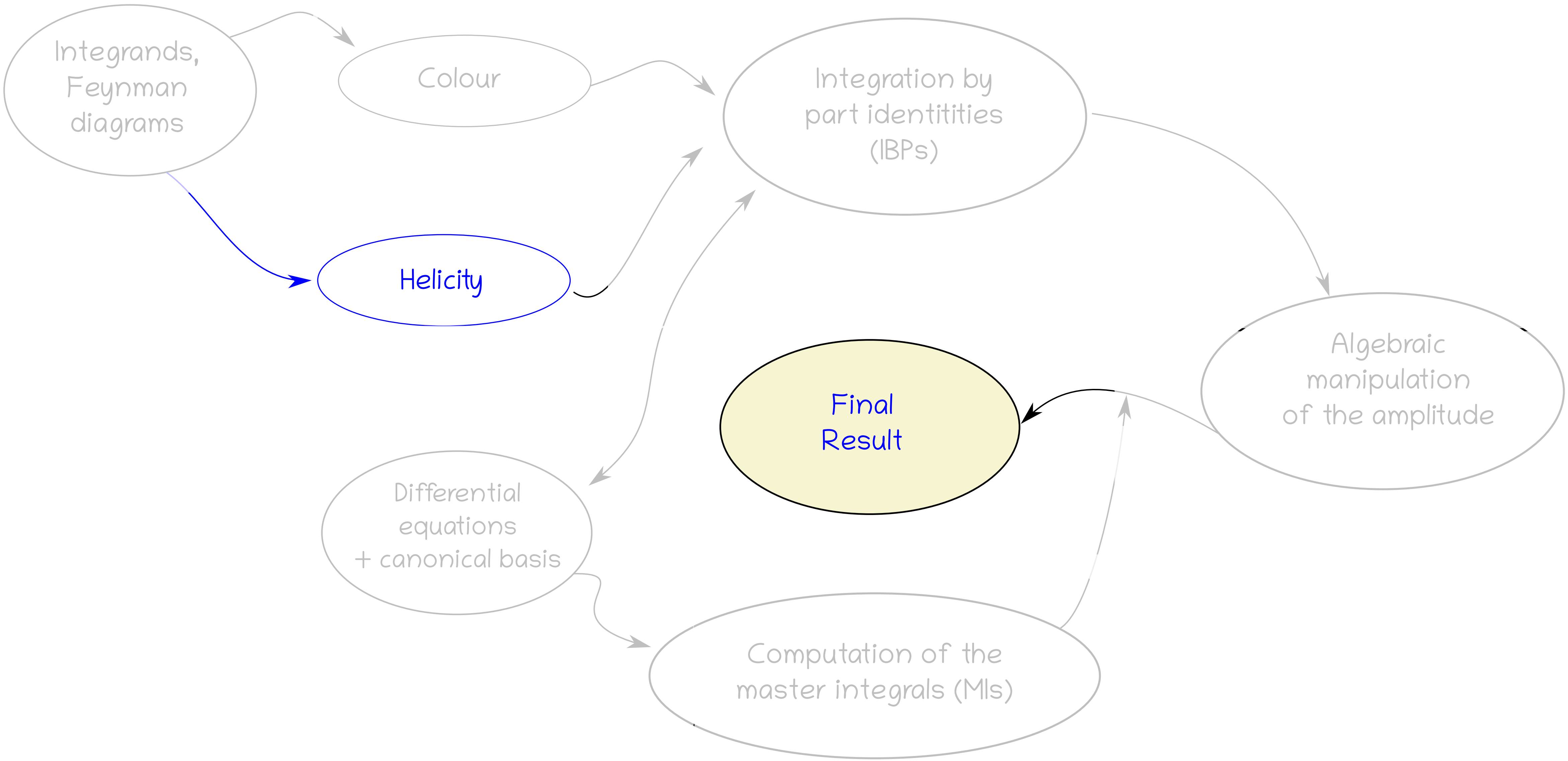
$$\boldsymbol{H}^{(2)}_{\text{R.S.}}(\epsilon) = \frac{1}{16\epsilon}\sum_i\left(\gamma_1^i - \frac{1}{4}\gamma_1^{\text{cusp}}\gamma_0^i + \frac{\pi^2}{16}\beta_0\gamma_0^{\text{cusp}}C_i\right)$$

$$+\frac{if^{abc}}{24\epsilon}\sum_{(i,j,k)}\boldsymbol{T}_i^a\boldsymbol{T}_j^b\boldsymbol{T}_k^c\ln\frac{-s_{ij}}{-s_{jk}}\ln\frac{-s_{jk}}{-s_{ki}}\ln\frac{-s_{ki}}{-s_{ij}}$$

$$-\frac{if^{abc}}{128\epsilon}\gamma_0^{\text{cusp}}\sum_{(i,j,k)}\boldsymbol{T}_i^a\boldsymbol{T}_j^b\boldsymbol{T}_k^c\left(\frac{\gamma_0^i}{C_i} - \frac{\gamma_0^j}{C_j}\right)\ln\frac{-s_{ij}}{-s_{jk}}\ln\frac{-s_{ki}}{-s_{ij}}$$

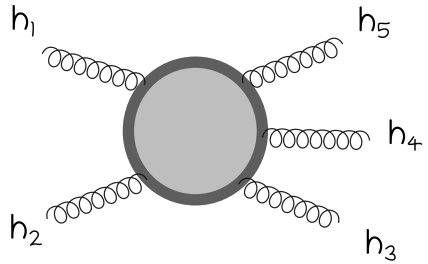
[Aybat, Dixon, Sterman 0607309] [Bern, Dixon, Kosower 0404293]

Traditional approach to multiloop calculation



Helicity and form factors

$$\mathcal{A}^{h_1\ldots h_5} = \mathcal{A}^{\mu_1\ldots\mu_5}\epsilon^{h_1}_{\mu_1}\cdots\epsilon^{h_1}_{\mu_5}$$



t'Hooft-Veltman scheme: external state live in d=4

Decompose the amplitude into Lorentz structures which are independent in d=4 [Peraro, Tancredi 1906.03298,2012.00820]

Transversality of on-shell bosons + reference choice: # of independent tensors in (d=4) = # of helicity configurations

$$\mathcal{A}^{h_1\ldots h_5} = \sum_{i=1}^{32} \mathcal{F}^j T_j^{\mu_1\ldots\mu_5}\epsilon^{h_1}_{\mu_1}\cdots\epsilon^{h_1}_{\mu_5}$$

avoid evanescent form factors throughout

Each form factor F_j can then be extracted via suitable projectors:

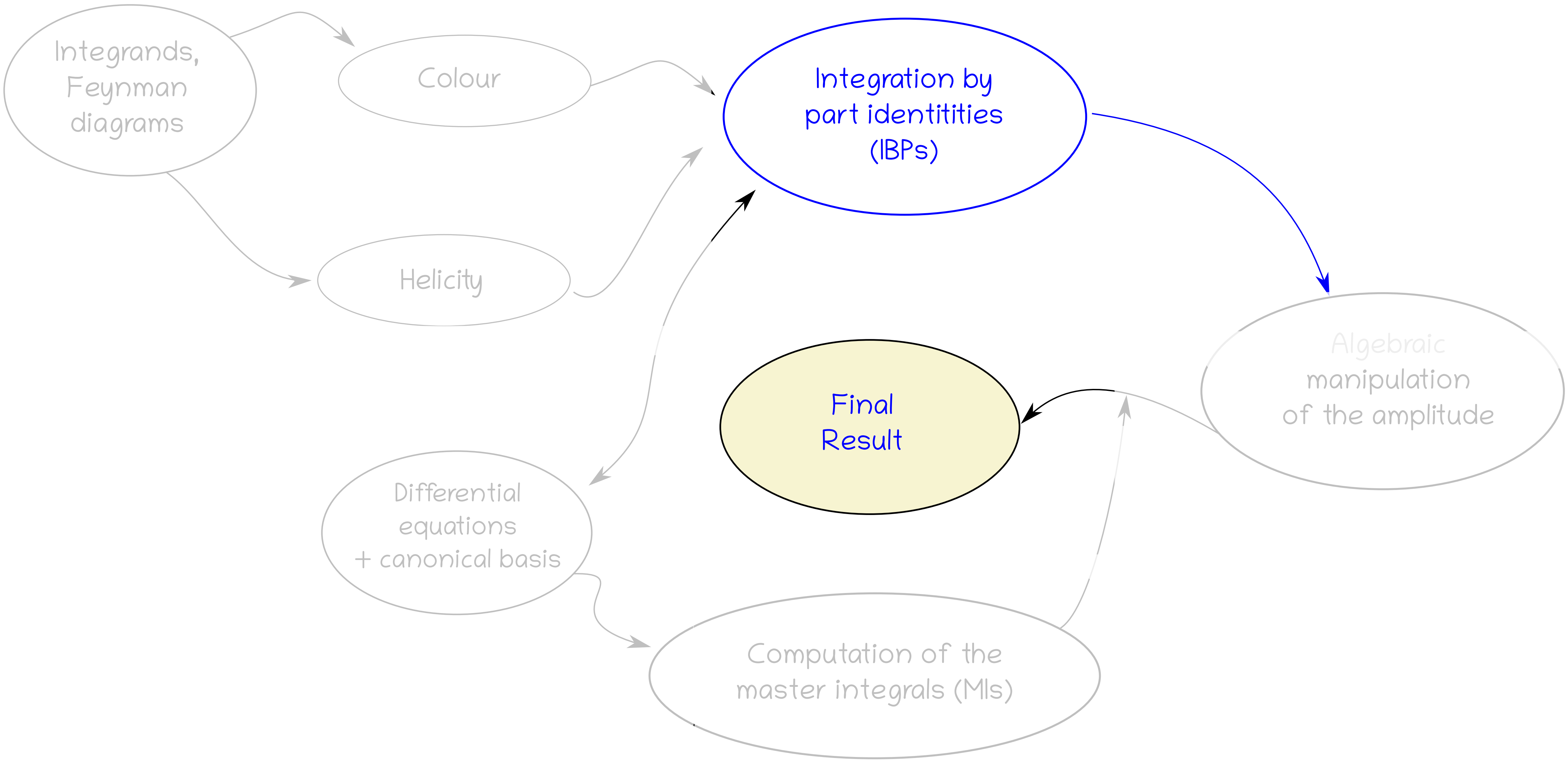
$$\mathcal{F}_j = \sum_{\text{pol}} \mathcal{P}_j(\mathbf{h})\mathcal{A}(\mathbf{h})$$

$$\mathcal{P}_j(\mathbf{h}) = \sum_{k=1}^{32} c_{jk} T_k^{\dagger,\mu_1\ldots\mu_5}\epsilon^{h_1}_{\mu_1}\cdots\epsilon^{h_5}_{\mu_5}$$

- Plenty of freedom in the choice of T_j
- For n>4, Tj can be fully built using external momenta

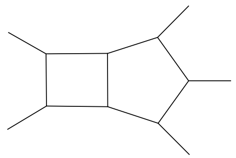
Identical arguments hold for external quark lines

Traditional approach to multiloop calculation

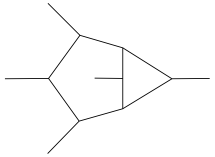


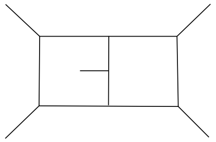
Reduction to master integrals

IBP identities obtained using FinRed [A.v.Manteuffel]: complete reduction to MIs of all integral topologies



- Finite-fields arithmetics [von Manteuffel, Schabinger 1406.4513; Peraro 1905.08019]
- Syzygy techniques [Gluza, Kadja, Kosower 1009.0472, Ita 1510.05626; Larsen, Zhang, 1511.01071, Agarwal, Jones, von Manteuffel 2011.15113]
- Denominators guessing [Abreu, Dormans, Febres Cordero, Ita, Page 1812.04586; Heller, von Manteuffel 2101.08283]





A good choice of MIs basis is crucial: canonical basis/UT weight integrals [Chicherin, Sotnikov 2009.07803].

- Exposes physical cuts of the integrals
- simpler rational coefficients
- extra bonus: d-dependence factorised

$$a_k(s_{ij};d) = \frac{\mathcal{N}(s_{ij};d)}{\mathcal{Q}(d)\mathcal{D}(s_{ij})} \qquad \mathcal{D}(s_{ij}) = \prod_{n=1}^{N_d} \mathcal{D}_n^{p_n}(s_{ij})$$

Generic IBP identity

$$I(s_{ij};d) = \sum_{k=1}^{M} a_k(s_{ij},d)\mathcal{J}(s_{ij};d)$$

rational function, over a common denominator

Natural to make the association:

Rational function → partial-fraction decomposition

1) univariate partial-fraction decomposition wrt d (trivial)

2) multivariate partial-fraction decomposition wrt s_{ij} (hard)

Multivariate partial fraction decomposition

It has been long known that a MVPFD simplifies significantly the IBP reductions

$$a_k(s_{ij};d) = \frac{\mathcal{N}(s_{ij};d)}{\mathcal{Q}(d)\mathcal{D}(s_{ij})} \xrightarrow[\text{pf in d}]{\text{after}} a_k(s_{ij};d) = \sum_l g_l(d)\mathcal{R}_l(s_{ij})$$

How should we go about this?

Proposals/approaches for MVPFD:
[Pak 1111.0868], [Abreu et al, 1904.00945],
[Boehm, Wittmann, Wu, Xu, Zhang, 2008.13194]

Systematic study of reduction of IBPs: [2008.13194; Bendle et al 2104.06866]

We employ the algorithm implemented in the package MultivariateApart [Heller, von Manteuffel, 2101.08283]

Big advantages of MultivariateApart:

1) systematically avoids spurious denominator factors
2) produces unique results also when applied to terms of a sum separately

we exploit both

Drastic reduction of algebraic complexity. IBPs tractable in a fully symbolic fashion

Examples:

		common den.		MVPFD
PB:	INT[TA,8,255,8,5,{1,1,1,1,1,1,1,1,-5,0,0}]	162 mb	→	3.9 mb
HB:	INT[TB,8,255,8,5,{1,1,1,1,1,1,1,1,-4,0,-1}]	513 mb	→	9.9 mb
DP:	INT[TB,8,510,8,5,{0,1,1,1,1,1,1,1,1,-5,0}]	2.9 gb	→	24 mb

The largest simplifications occur for the most complicated integrals: up to a factor ~ 100 in reduction size!

Crossing of IBP identities

For the complete reduction we need (potentially) all permutations of the external momenta

Being able to treat the IBPs in a fully symbolic fashion, this becomes extremely cheap (wrt other steps)

Start from IBPs in partial-fraction decomposed form → Cross invariants (x120 times) → Obtain all IBPs needed

You don't want to spoil what done so far

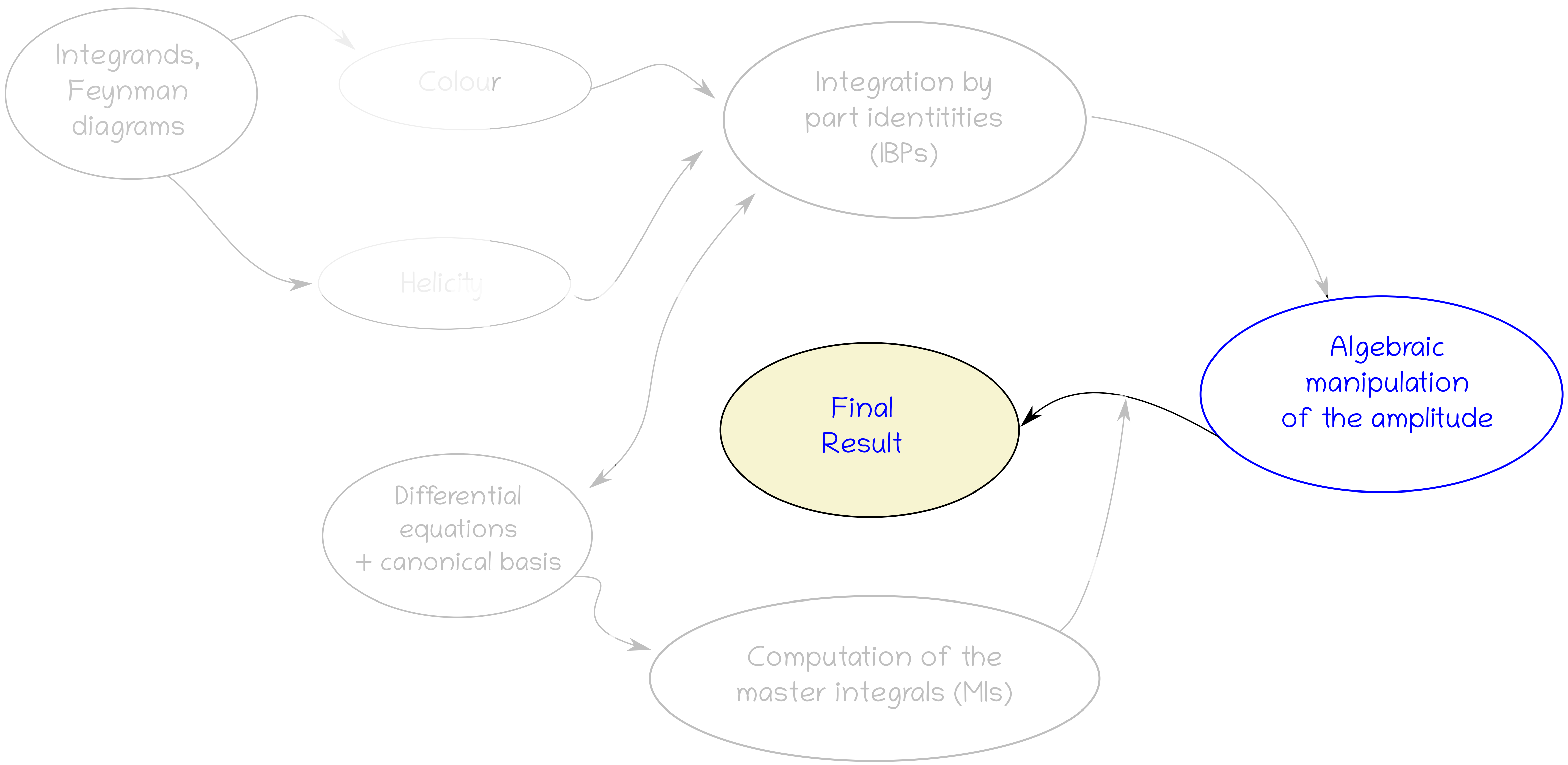
After crossing the invariants: second partial fraction decomposition according to a prefixed global Groebner basis

In practice: all terms in the sum decomposed locally but a unique representation of the rational functions across all IBP identities guaranteed

Crucial for the many (very many indeed) cancellations in the final result.

No need for expensive operations (back to common denominator)

Traditional approach to multiloop calculation



Final expression of the amplitude

Insert IBPs into the amplitude + partial fraction decomposition: Multivariate Apart + Singular [Decker, Greuel, Pfister, Schoenemann]

Cancellations happen wo expensive manipulations

think of a linear combination
with f_k elements of the basis

$$A_i(\mathbf{h}) = \sum_k r_k(\{s_{ij}, \epsilon_5\}) f_k(\{s_{ij}, \epsilon_5\})$$

in partial fraction decomposed form:
i.e. sum of a large number of monomials

Observed also in:
[Abreu, Dormans, Febres Cordero, Ita, Page, Sotnikov 1904.00945]
[De Laurentis, Maitre, arXiv:2010.14525]
[Chawdhry, Czakon, Mitov, Poncelet, 2012.13553, 2103.04319]
[Abreu, Cordero, Ita, Page,Sotnikov, 2102.13609]

rational functions are not independent

$$r_k = \sum_{m_1+\ldots+m_n \leq p} a_{k,m_1\ldots m_{30}} M_{m_1\ldots m_{30}},$$

$$M_{m_1\ldots m_{30}} \equiv q_1^{m_1} \cdots q_{25}^{m_{25}} s_{12}^{m_{26}} \cdots s_{51}^{m_{30}}$$

monomials are independent objects

We look for linear relations among the various rational functions:
[Agarwal, F.B., A.v.Manteuffel and Tancredi 2103.02671, 2105.04585]

$$0 = \sum_k r_k b_k$$

as many equations
as independent monomials

$$0 = \sum_k a_{k,m_1\ldots m_{30}} b_k$$

monomials >> # rational functions
but this linear system is over constrained thus admits a solution

(similar in spirit to IBP reduction)

drastic reduction of
final expressions

Complete result in colour/helicity spaces

After UV renormalisation and IR subtraction:

$$\mathcal{R}(\mathbf{h}) = \sum_k r_k(\{s_{ij}, \epsilon_5\}) f_k(\{s_{ij}, \epsilon_5\})$$

[Agarwal, F.B., v.Manteuffel, Tancredi 2105.04585]

Crossing r_k is trivial

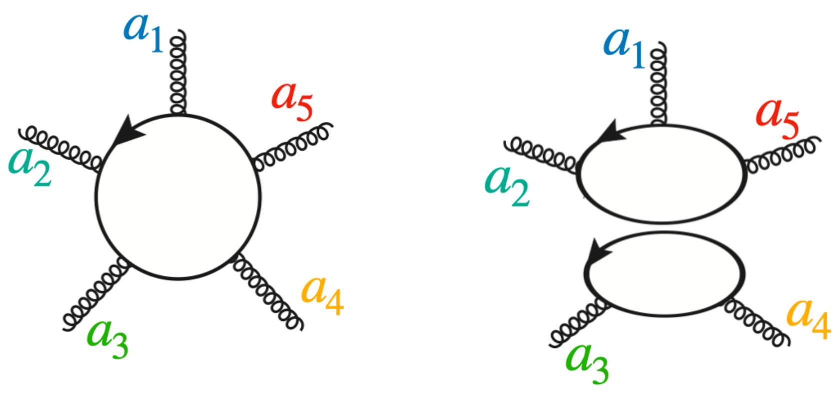
Crossing, aka analytic continuation, of f_k more involved

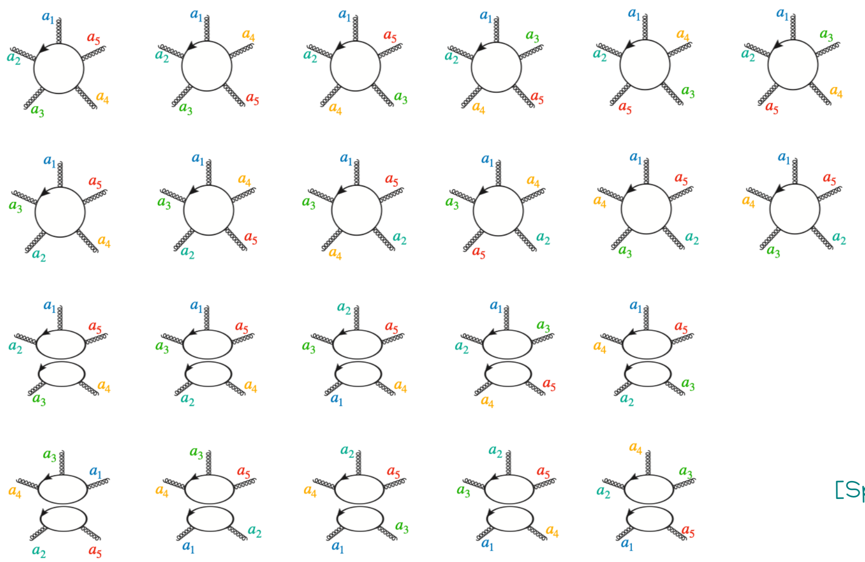
1) Express 2-loop MIs and crossings thereof in terms of pentagon functions

2) Exploit the fact that the full set of MIs is mapped onto itself under permutations

3) Obtain a formal system of linear equations for crossed pentagon functions

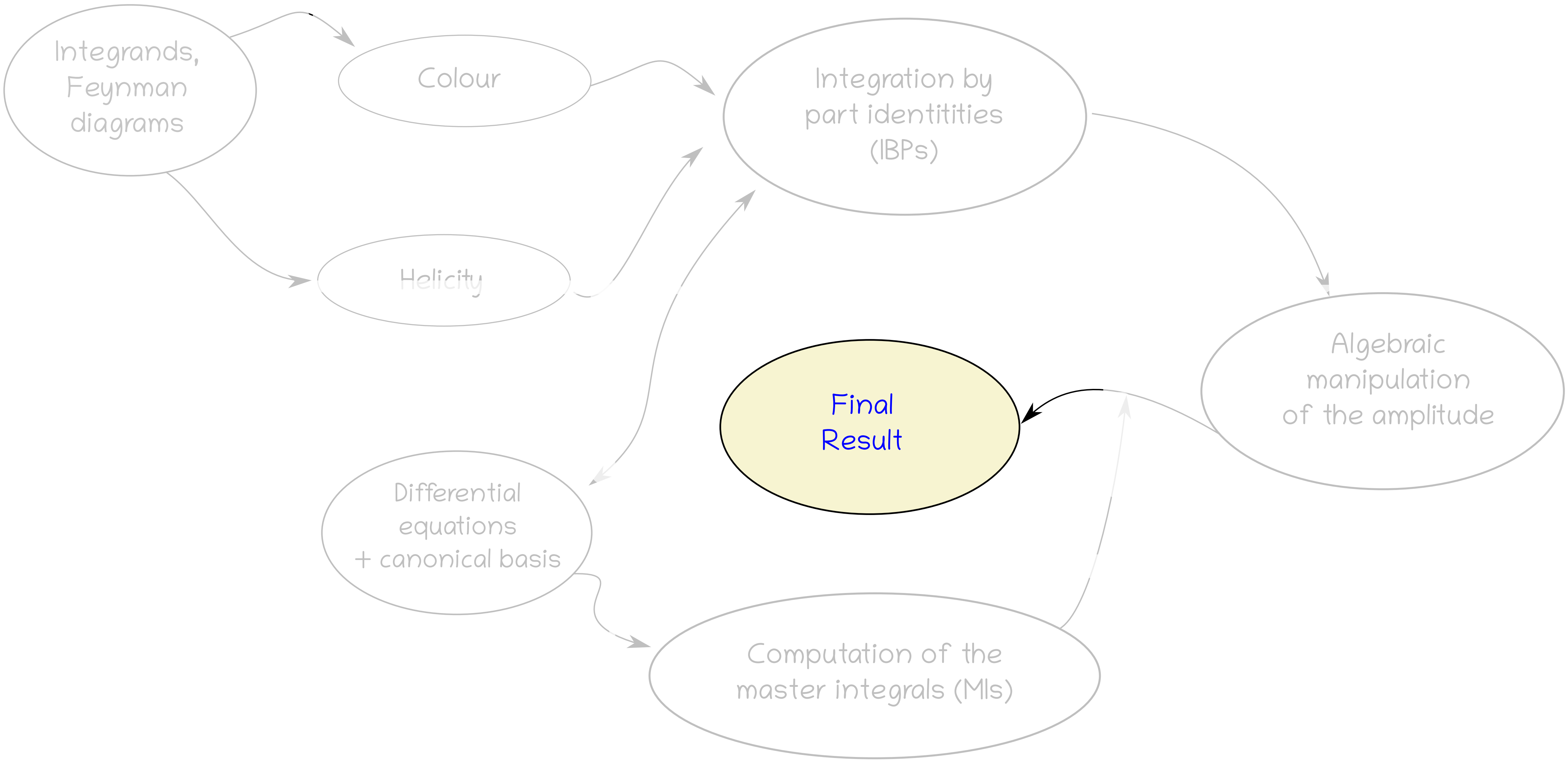
4) Solve the system (using FinRed). Solutions are enough to cross the whole amplitude





[Special thanks to G.Gambuti for the artwork]

Traditional approach to multiloop calculation



Checks and status of the results

1 loop:

- match prediction for IR poles @NLO
- for five gluon amplitude: check colour trace identities [Bern, Kosower Nucl.Phys.B 362 (1991)]
- reproduce available results [Bern, Dixon, Kosower hep-ph/9302280] and checked vs public codes (OpenLoops)

2 loop:

$\times 2$

- $ggggg\to 0$ (✓) $q\bar{q}Q\bar{Q}g\to 0$ (✓) $q\bar{q}q\bar{q}g\to 0$ (WIP) $q\bar{q}ggg\to 0$ (WIP)
- match prediction for IR poles @NNLO
- for ggggg: agreement with LC result [Abreu, Febres Cordero, Ita, Page, Sotnikov: 2102.13609]
- for ggggg agreement with all-plus full colour Yang-Mills result [Badger, Chicherin, Gehrmann, Heinrich, Henn, Peraro, Wasser, Zhang, Zoia 1905.03733]

TODO:

- check against LC result for all channels [Abreu, Febres Cordero, Ita, Page, Sotnikov 2102.13609]
- in ggggg case, check colour trace identities [Edison, Naculich 1111.3821]

Summary and outlook

Calculation of two-loop 5-point amplitudes in full colour QCD with 5 coloured partons

For a selection of partonic channels in principle everything to study numerical impact and analyse structure

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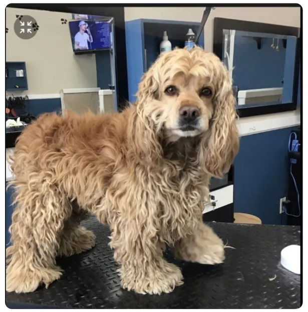
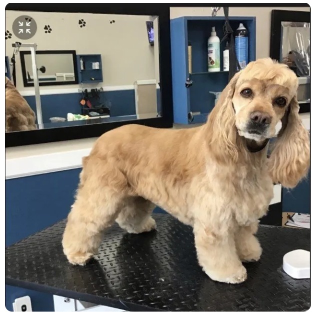


What we would like:



Need to groom and massage the final result to expose genuine simplicity/complexity, e.g.

- understand/remove spurious singularities
- improve representation of rational functions: overcomplete s_{ij} (?) or spinor products (?)
- test stability when partons goes soft/collinear

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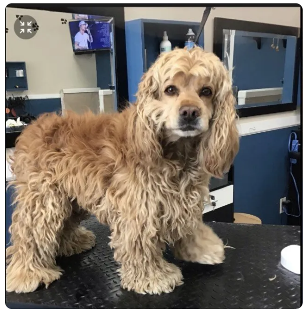

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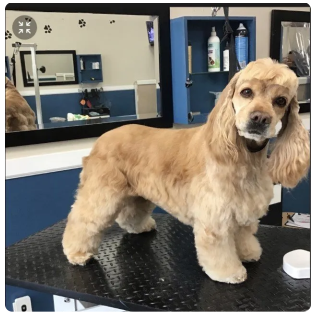

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For the future, many exciting avenues:

- Make amplitudes available and deploy them for pheno studies and consistently assess impact of SLC contributions
- study multi-Regge kinematics of QCD amplitudes
- IR limits of N^3LO: tripole contribution in soft-gluon current

  computed in full-colour up to finite contributions [Dixon, Herrmann, Yan, Zhu 1912.09370]

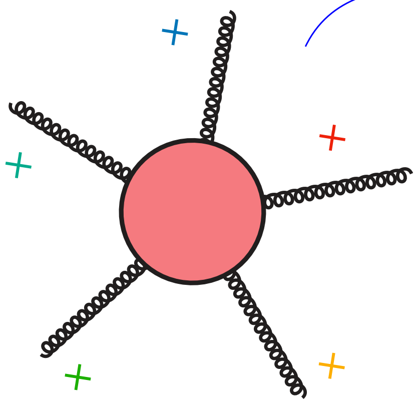
  check our results against prediction ~ studies towards N^3LO (subtraction scheme)

Backup

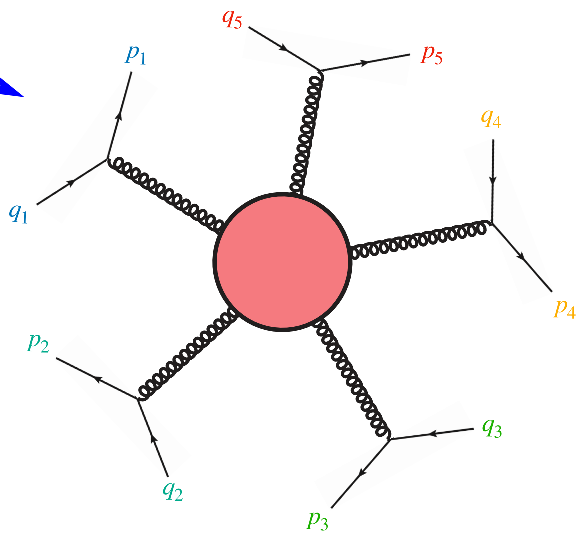
Spinor helicity for five-gluon amplitude

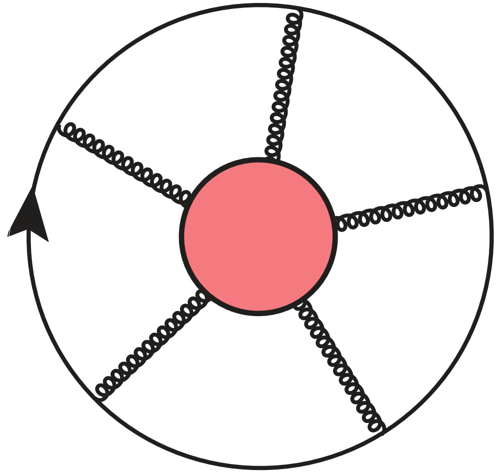
[Slide by Giulio Gambuti Radcor 2023]

Example



$$\epsilon^{+}_{\mu_i} = \frac{[q_i|\mu_i|p_i\rangle}{\sqrt{2}[p_i q_i]}$$





$$\epsilon^{+}_{\mu_1}\ldots\epsilon^{+}_{\mu_5} = \frac{1}{2^{5/2}}\frac{[q_1|\mu_1|1\rangle}{[q_1 p_1]}\frac{[q_2|\mu_2|2\rangle}{[q_2 p_2]}\frac{[q_3|\mu_3|3\rangle}{[q_3 p_3]}\frac{[q_4|\mu_4|4\rangle}{[q_4 p_4]}\frac{[q_5|\mu_5|5\rangle}{[q_5 p_5]}$$

$$\epsilon^{+}_{\mu_1}\ldots\epsilon^{+}_{\mu_5} = \frac{1}{2^{5/2}}\frac{\mathrm{Tr}_{-}\left\{\gamma_{\mu_1}p_1\gamma_{\mu_2}p_2\gamma_{\mu_3}p_3\gamma_{\mu_4}p_4\gamma_{\mu_5}p_5\right\}}{[12]\,[23]\,[34]\,[45]\,[51]}$$

$$\mathrm{Tr}_{\pm}\{\ldots\} = \frac{1}{2}\mathrm{Tr}\left\{(1\pm\gamma_5)\ldots\right\}$$

Spinor helicity for five-gluon amplitude

[Slide by Giulio Gambuti Radcor 2023]

$$\underbrace{A^{\mu_1\ldots\mu_5}}_{D}\ \underbrace{\epsilon^{h_1}_{\bar{\mu}_1}\ldots\epsilon^{h_5}_{\bar{\mu}_5}}_{4}$$

't Hooft-Veltman scheme

p^i_μ ✓

$$\bar{g}_{\bar{\mu}\bar{\nu}} = g_{\mu\nu} - \frac{1}{G(1234)} G\begin{pmatrix} \mu & p_1 & p_2 & p_3 & p_4 \\ \nu & p_1 & p_2 & p_3 & p_4 \end{pmatrix}$$ ✓

$$\epsilon_{\mu\nu\rho\sigma} = \frac{1}{\epsilon_{1234}}\,\epsilon_{\mu\nu\rho\sigma}\epsilon_{1234} = \frac{1}{\epsilon_{1234}}\,(\ldots + p_i^\mu p_j^\nu p_k^\rho p_h^\sigma + \ldots)$$ ✓