Loopfest 2023, 26-28 June

Master Integrals for electroweak corrections to $gg \to \gamma\gamma$

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Motivations and Introduction

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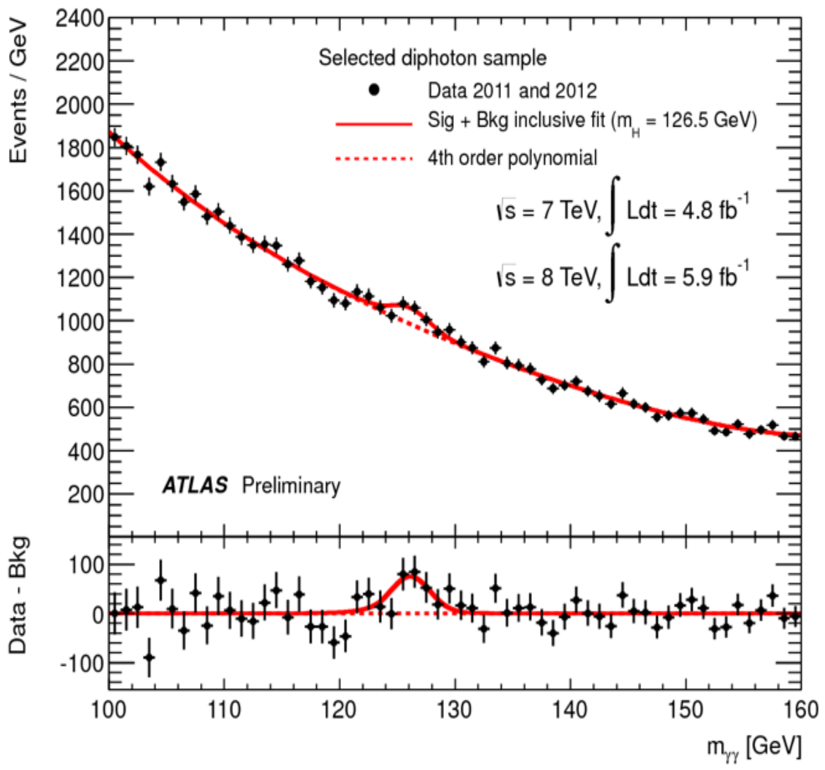
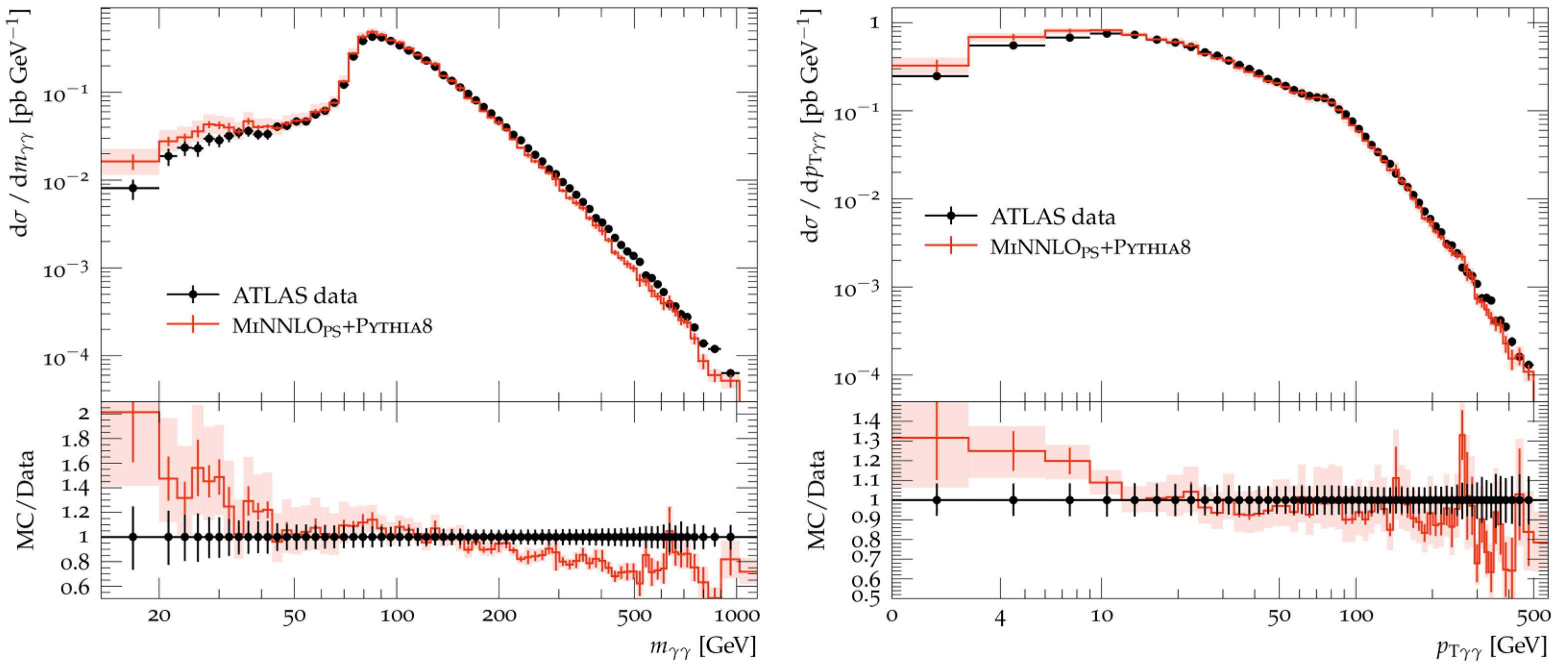
Motivation: Big Picture

Diphoton channel plays a crucial role in Higgs phenomenology. High precision data requires accurate theoretical predictions.



ATLAS Collaboration, 2012



Gavardi, Oleari, Re, 2022

Diphoton Predictions:

- $pp \to \gamma\gamma$

Full NLO:

- *Binoth, Guillet, Pilon, Werlen, '99*

NNLO, QCD:

- *Catani, Cieri, de Florian, Ferrera, Grazzini, 2011*
- *Campbell, Ellis, Li, Williams, 2016*

NNLO, q_t-resummed:

- NNLL: *Cieri, Coradeschi, de Florian, 2015*
- N3LL: *Neumann, 2021*

N3LO amplitude:

- *Caola, Manteuffel, Tancredi, 2020*

NLO EW corrections:

- *Bierweiler, Kasprzik, Kühn, 2013*

QCD corrections + jets:

- *Del Duca, Maltoni, Nagy, Trocsanyi, 2003*
- *Gehrmann, Greiner, Heinrich, 2013*
- *Badger, Guffanti, Yundin, 2013*

- $gg \to \gamma\gamma$

NNLO:

- *Dicus, Willenbrock, '88*

N3LO, QCD (light quarks):

- Light: *Bern, Dixon, Schmidt 2002*
- Heavy: *Maltoni, Mandal, Zhao, 2018*

N3LO + jet, QCD corrections:

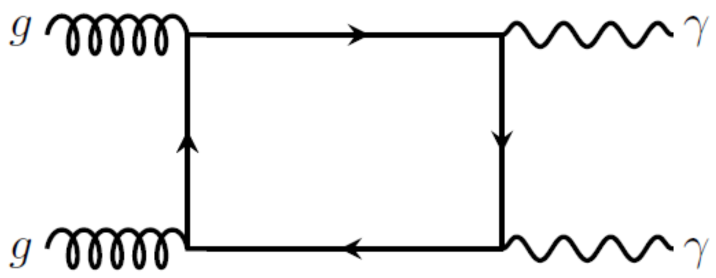
- *Badger, Gehrmann, Marcoli, Moodie, 2021*

3 Loops (N4LO), QCD:

- *Bargiela, Caola, Manteuffel, Tancredi, 2022*

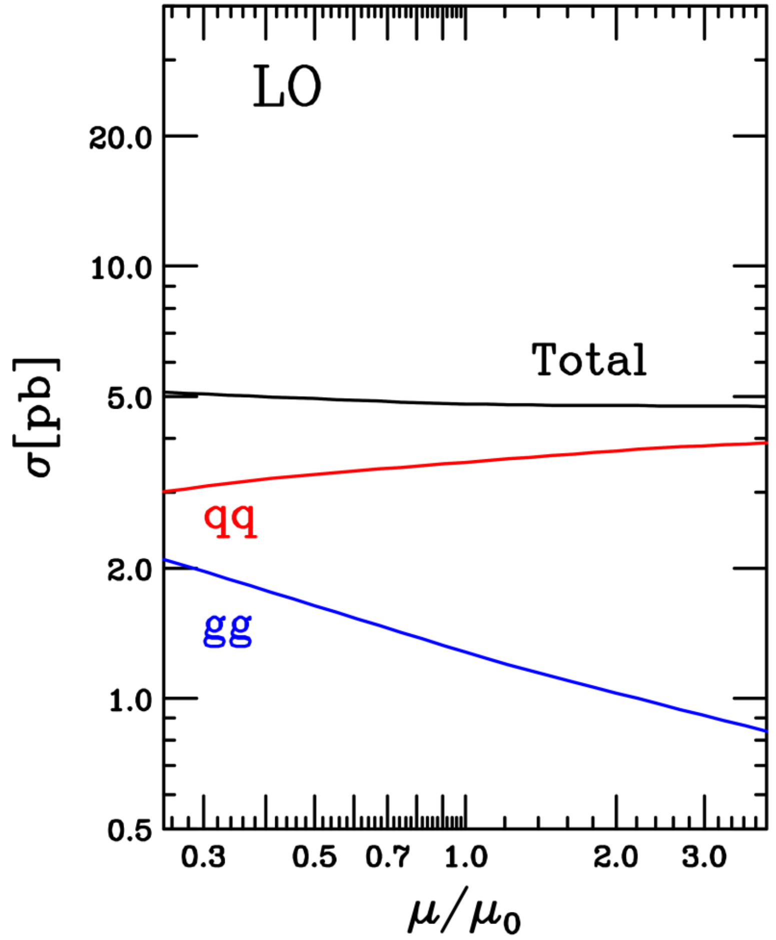
Motivation: gg Channel

- As part of the partonic process $pp \to \gamma\gamma$, the corrections enter at NNLO in QCD.



- Sizable contributions to the cross-section due to large initial-state gluon flux.

- Given the quality of recent LHC data, $\mathcal{O}(\alpha)$ corrections for the $gg \to \gamma\gamma$ channel are required to improve the precision our theoretical predictions.



Campbell, Ellis, Williams, 2011

Motivation: Interference

- Higgs width is too narrow to be directly measured at LHC. Necessity for indirect bounds.

- The interference between the diphoton channel $gg \to H \to \gamma\gamma$ and the QCD background $gg \to \gamma\gamma$ plays a crucial role in that regard(*Dixon, Siu, 2006; Martin, 2012; Dixon, Li, 2013*).

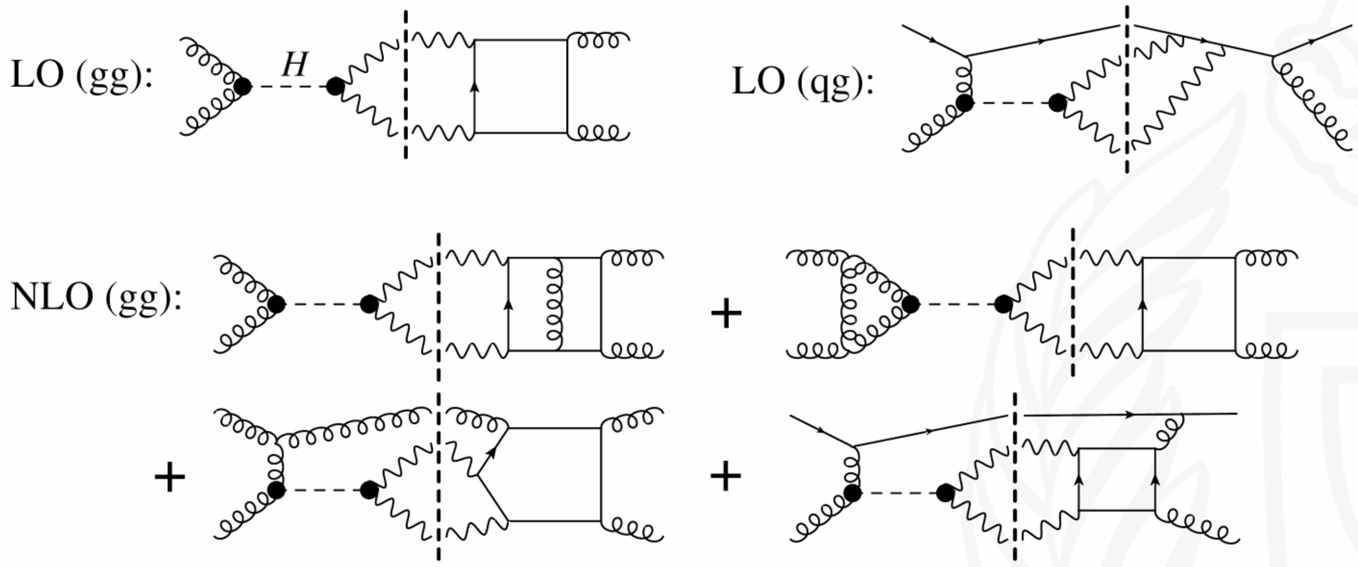
$$\hat{\delta}_{gg\to H\to\gamma\gamma} = -2\left(\hat{s}-m_H^2\right)\frac{\mathrm{Re}(\mathcal{A}_{gg\to H}\mathcal{A}_{H\to\gamma\gamma}\mathcal{A}^*_{\mathrm{cont}})}{\left(\hat{s}-m_H^2\right)^2+m_H^2\Gamma_H^2} - 2m_H\Gamma_H\frac{\mathrm{Im}(\mathcal{A}_{gg\to H}\mathcal{A}_{H\to\gamma\gamma}\mathcal{A}^*_{\mathrm{cont}})}{\left(\hat{s}-m_H^2\right)^2+m_H^2\Gamma_H^2}$$

Anti-symmetric → mass shift

Symmetric → width bounds + cross-section reduction

Motivation: Interference

- NLO analysis shows mass shift effect of ~ 70MeV and width bounded to 20-40 Γ_H^{SM} (Campbell, Carena, Harnik, Liu, 2017).

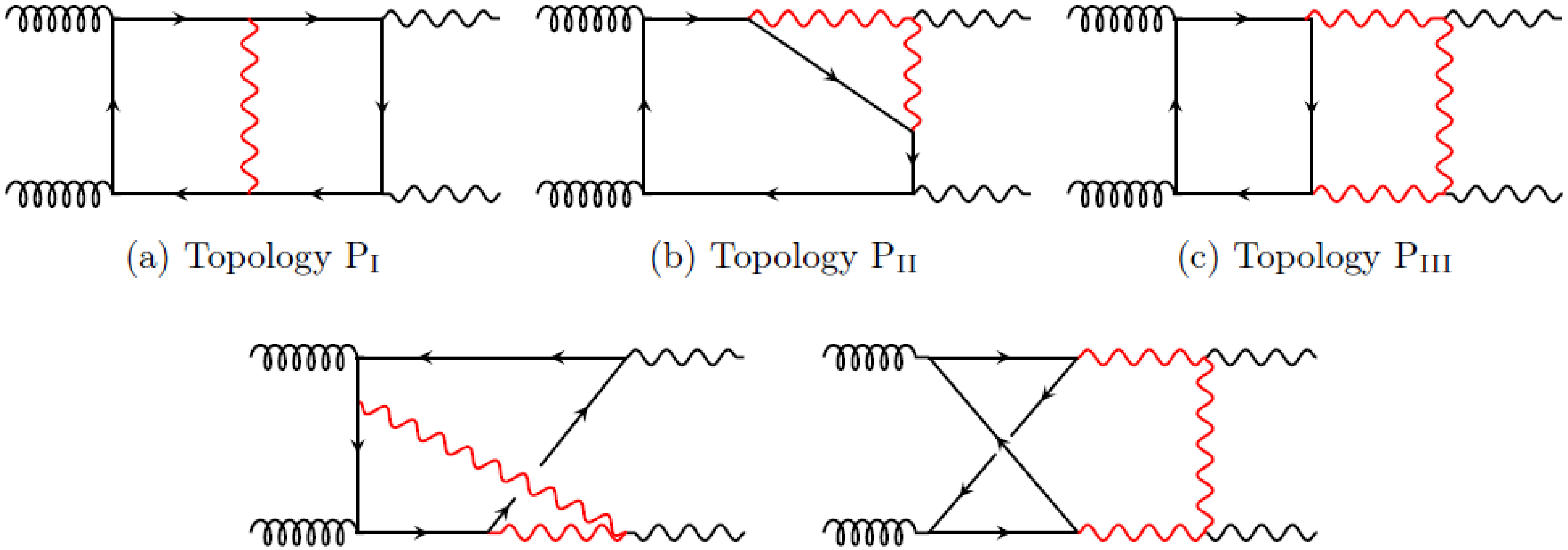


- Ongoing effort to extract the NNLO in QCD(Bargiela, Buccioni, Caola, Devoto, Manteuffel, Tancredi, 2022).

- With the QCD predictions under control, the EW corrections start to become more relevant.

Amplitude and Topologies

- We present the evaluation of the master integrals for massless quark loops.
- Different topologies depending on the vector boson flow. The complexity varies significantly among them.
- To extract the helicity amplitude, we introduce 8 projectors in order to extract the form factors.


(a) Topology P_I (b) Topology P_{II} (c) Topology P_{III} (d) Topology N_I (e) Topology N_{II}

Tensor structures

- We can therefore write the helicity amplitude as combination of form factors.

- The number of independent tensor structures corresponds to the number of physical helicity configurations.

- The projectors are built from the tensor structures.

- The form factors contain thousands of loop integrals → MIs reduction!

$$\mathcal{A}(s,t) = \sum_{i=1}^{8} F_i\, T_i .$$

$$T_1 = \epsilon_1\cdot k_3\ \epsilon_2\cdot k_1\ \epsilon_3\cdot k_1\ \epsilon_4\cdot k_2\ ,$$

$$T_2 = \epsilon_1\cdot k_3\ \epsilon_2\cdot k_1\ \epsilon_3\cdot\epsilon_4, \quad T_3 = \epsilon_1\cdot k_3\ \epsilon_3\cdot k_1\ \epsilon_2\cdot\epsilon_4,$$

$$T_4 = \epsilon_1\cdot k_3\ \epsilon_4\cdot k_2\ \epsilon_2\cdot\epsilon_3, \quad T_5 = \epsilon_2\cdot k_1\ \epsilon_3\cdot k_1\ \epsilon_1\cdot\epsilon_4,$$

$$T_6 = \epsilon_2\cdot k_1\ \epsilon_4\cdot k_2\ \epsilon_1\cdot\epsilon_3, \quad T_7 = \epsilon_3\cdot k_1\ \epsilon_4\cdot k_2\ \epsilon_1\cdot\epsilon_2,$$

$$T_8 = \epsilon_1\cdot\epsilon_2\ \epsilon_3\cdot\epsilon_4 + \epsilon_1\cdot\epsilon_4\ \epsilon_2\cdot\epsilon_3 + \epsilon_1\cdot\epsilon_3\ \epsilon_2\cdot\epsilon_4\ .$$

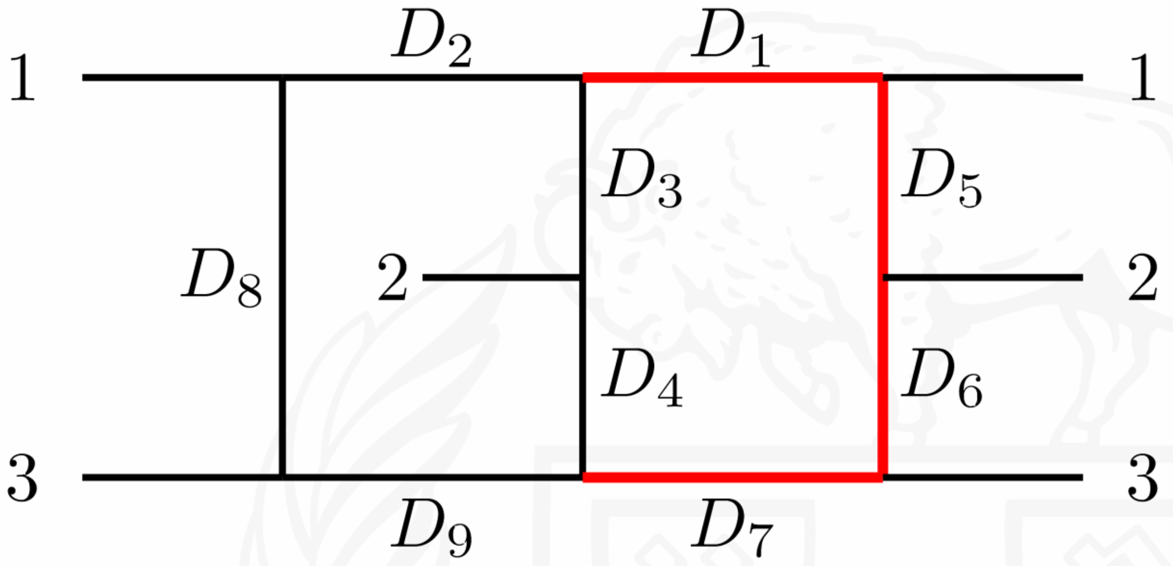
$$\sum_{pol} P_i T_j = \delta_{ij}$$

Reduction to MIs

- We introduce an auxiliary topology to account for all the possible scalar products involving the loop momenta.

- All the scalar integrals can be written in terms of the auxiliary topology's propagators.

- These integrals are reduced to a minimal set of Master Integrals (MIs) by using Kira (*Klappert, Lange, Maierhöfer, Usovitsch, 2023*).



$$I^{\mathrm{N_{II}}}_{a_1\ldots a_9} = \mathcal{C}(\epsilon)\int \frac{d^d\ell_1}{(2\pi)^d}\frac{d^d\ell_2}{(2\pi)^d}\frac{1}{D_1^{a_1}\ldots D_9^{a_9}}$$

DEQ: Overview

- General strategy to evaluate MIs: Differential Equation method (DEQ) (*Remiddi, 1997*).

Laporta Basis $\vec{\mathcal{I}}(\{x_i\})$ ⟶ $$\frac{\partial}{\partial x_i}\vec{\mathcal{I}}(\{x_i\}) = \underbrace{\mathcal{A}(\{x_i\},\epsilon)}\,\vec{\mathcal{I}}(\{x_i\})$$

- The complexity goes up pretty quickly for the Laporta Basis. Solution: Canonical Basis (*Henn, 2013*).

Canonical Basis $\vec{\mathcal{G}}(\{x_i\})$ ⟶ $$\frac{\partial}{\partial x_i}\vec{\mathcal{G}}(\{x_i\}) = \epsilon\underbrace{\mathcal{A}(\{x_i\})}\,\vec{\mathcal{G}}(\{x_i\})$$

- The factorization of the space-time parameter allows to write the solution in an iterative way:

$$\vec{\mathcal{G}}(\{x_i\}) = \left(\mathbb{1} + \epsilon\int_\gamma d\mathcal{A} + \epsilon^2\int_\gamma d\mathcal{A}\,d\mathcal{A} + \ldots\right)\vec{\mathcal{G}}(\{x_i^0\})$$

DEQ: Canonical Basis

- The kinematic information is encoded in the alphabet $\{\eta_k\}$ of the differential equation:

$$d\mathcal{A} = \sum_k \mathcal{M}_k d\log \eta_k$$

- The solution for rational letters is expressed in terms of Goncharov Polylogarithms (GPLs) (*Goncharov, 1998*).

- Efficient numerical evaluation of GPLs using GiNaC(*Vollinga, 2005*)/HandyG(*Naterop, Signer, Ulrich, 2016*).

- Non-rational letters may or may not be represented in terms of GPLs. We implement a generalized approach based on Chen's Iterated Integrals(*Chen, 2005; Caron-Huot, Henn, 2014*).

DEQ: Solution

The solution can be evaluated (up to boundary constants) at any fixed order:

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$$\vec{\mathcal{G}}^{(1)}(\{x_i\}) = \int_\gamma d\mathcal{A}\,\vec{\mathcal{G}}^{(0)}(\{x_i^0\}) + \vec{\mathcal{G}}^{(1)}(\{x_i^0\})$$

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$$\vec{\mathcal{G}}^{(2)}(\{x_i\}) = \int_\gamma d\mathcal{A}\,d\mathcal{A}\,\vec{\mathcal{G}}^{(0)}(\{x_i^0\}) + \int_\gamma d\mathcal{A}\,\vec{\mathcal{G}}^{(1)}(\{x_i^0\}) + \vec{\mathcal{G}}^{(2)}(\{x_i^0\})$$

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$$\vec{\mathcal{G}}^{(3)}(\{x_i\}) = \int_\gamma d\mathcal{A}\, d\mathcal{A}\, d\mathcal{A}\, \vec{\mathcal{G}}^{(0)}(\{x_i^0\}) + \int_\gamma d\mathcal{A}\, d\mathcal{A}\, \vec{\mathcal{G}}^{(1)}(\{x_i^0\}) + \int_\gamma d\mathcal{A}\, \vec{\mathcal{G}}^{(2)}(\{x_i^0\}) + \vec{\mathcal{G}}^{(3)}(\{x_i^0\})$$

DEQ: Solution

The solution can be evaluated (up to boundary constants) at any fixed order:

$$\vec{\mathcal{G}}^{(1)}(\{x_i\}) = \int_\gamma d\mathcal{A}\,\vec{\mathcal{G}}^{(0)}(\{x_i^0\}) + \vec{\mathcal{G}}^{(1)}(\{x_i^0\})$$

$$\vec{\mathcal{G}}^{(2)}(\{x_i\}) = \int_\gamma d\mathcal{A}\,d\mathcal{A}\,\vec{\mathcal{G}}^{(0)}(\{x_i^0\}) + \int_\gamma d\mathcal{A}\,\vec{\mathcal{G}}^{(1)}(\{x_i^0\}) + \vec{\mathcal{G}}^{(2)}(\{x_i^0\})$$

$$\vec{\mathcal{G}}^{(3)}(\{x_i\}) = \int_\gamma d\mathcal{A}\,d\mathcal{A}\,d\mathcal{A}\,\vec{\mathcal{G}}^{(0)}(\{x_i^0\}) + \int_\gamma d\mathcal{A}\,d\mathcal{A}\,\vec{\mathcal{G}}^{(1)}(\{x_i^0\}) + \int_\gamma d\mathcal{A}\,\vec{\mathcal{G}}^{(2)}(\{x_i^0\}) + \vec{\mathcal{G}}^{(3)}(\{x_i^0\})$$

$$\vec{\mathcal{G}}^{(4)}(\{x_i\}) = \int_\gamma d\mathcal{A}\,d\mathcal{A}\,d\mathcal{A}\,d\mathcal{A}\,\vec{\mathcal{G}}^{(0)}(\{x_i^0\}) + \int_\gamma d\mathcal{A}\,d\mathcal{A}\,d\mathcal{A}\,\vec{\mathcal{G}}^{(1)}(\{x_i^0\}) + \int_\gamma d\mathcal{A}\,d\mathcal{A}\,\vec{\mathcal{G}}^{(2)}(\{x_i^0\}) + \int_\gamma d\mathcal{A}\,\vec{\mathcal{G}}^{(3)}(\{x_i^0\}) + \vec{\mathcal{G}}^{(4)}(\{x_i^0\}).$$

DEQ: Solution

The solution can be evaluated (up to boundary constants) at any fixed order:

$$\vec{\mathcal{G}}^{(1)}(\{x_i\}) = \int_\gamma d\mathcal{A}\,\vec{\mathcal{G}}^{(0)}(\{x_i^0\}) + \vec{\mathcal{G}}^{(1)}(\{x_i^0\})$$

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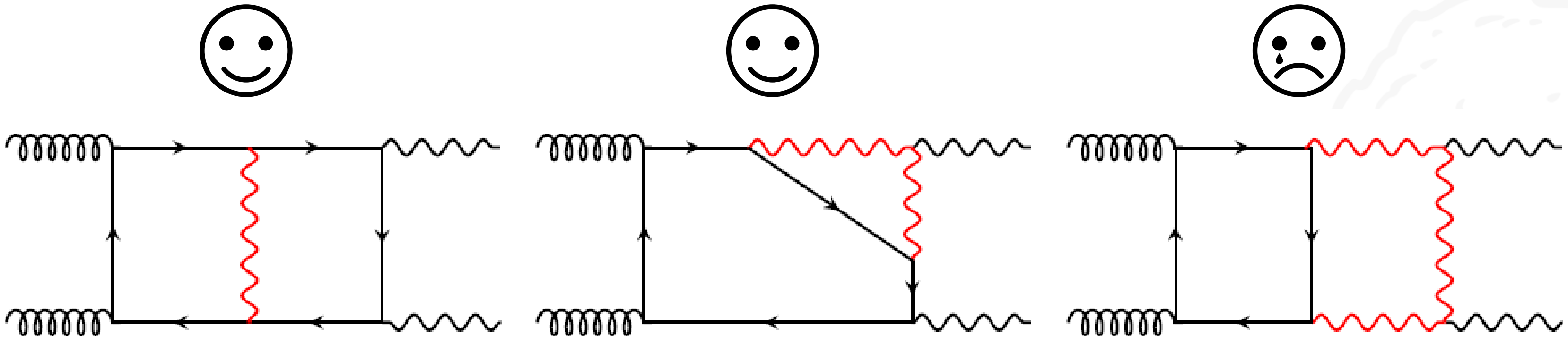
$$\begin{aligned}\vec{\mathcal{G}}^{(4)}(\{x_i\}) = &\int_\gamma d\mathcal{A}\,d\mathcal{A}\,d\mathcal{A}\,d\mathcal{A}\,\vec{\mathcal{G}}^{(0)}(\{x_i^0\}) + \int_\gamma d\mathcal{A}\,d\mathcal{A}\,d\mathcal{A}\,\vec{\mathcal{G}}^{(1)}(\{x_i^0\}) + \int_\gamma d\mathcal{A}\,d\mathcal{A}\,\vec{\mathcal{G}}^{(2)}(\{x_i^0\}) \\ &+ \int_\gamma d\mathcal{A}\,\vec{\mathcal{G}}^{(3)}(\{x_i^0\}) + \vec{\mathcal{G}}^{(4)}(\{x_i^0\}).\end{aligned}$$

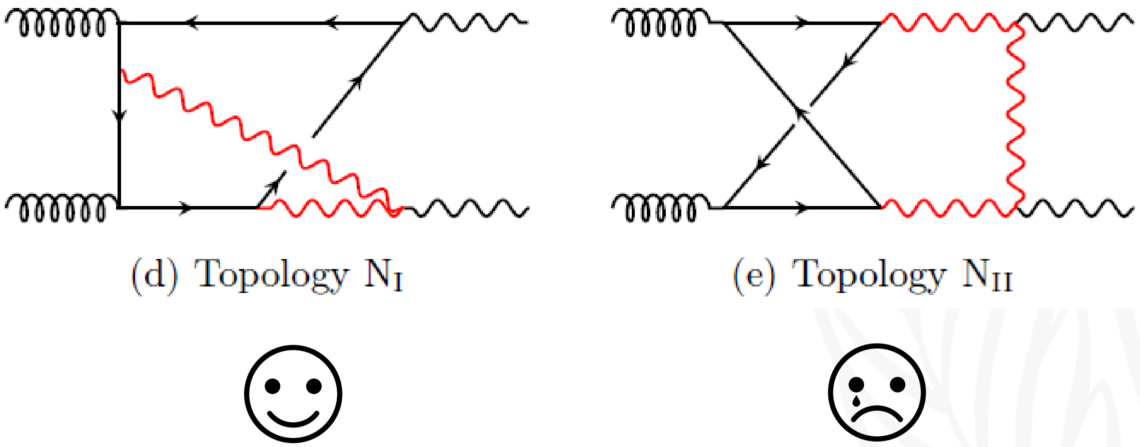
Efficient semi-numerical implementation thanks to the recursive nature of the iterated integrals.

Calculation and Results

Rational vs Non-Rational alphabet



(a) Topology P_I (b) Topology P_{II} (c) Topology P_{III}



(d) Topology N_I (e) Topology N_{II}

Topology $\mathrm{P_I}$

25 Master integrals. The Canonical basis is obtained via Magnus Series Expansion (*Blanes, Casas, Oteo, Ros, 2008*):

$$\mathcal{G}_1^{\mathrm{P_I}} = \mathcal{F}_1, \quad \mathcal{G}_2^{\mathrm{P_I}} = x\mathcal{F}_2, \quad \mathcal{G}_3^{\mathrm{P_I}} = (1+x)\mathcal{F}_3,$$
$$\mathcal{G}_4^{\mathrm{P_I}} = \mathcal{F}_4 - \mathcal{F}_3, \quad \mathcal{G}_5^{\mathrm{P_I}} = y\mathcal{F}_5, \quad \mathcal{G}_6^{\mathrm{P_I}} = (1+y)\mathcal{F}_6,$$
$$\mathcal{G}_7^{\mathrm{P_I}} = \mathcal{F}_7, \quad \mathcal{G}_8^{\mathrm{P_I}} = \epsilon x\mathcal{F}_8, \quad \mathcal{G}_9^{\mathrm{P_I}} = \epsilon x\mathcal{F}_9,$$
$$\mathcal{G}_{10}^{\mathrm{P_I}} = \epsilon y\mathcal{F}_{10}, \quad \mathcal{G}_{11}^{\mathrm{P_I}} = \epsilon y\mathcal{F}_{11}, \quad \mathcal{G}_{12}^{\mathrm{P_I}} = y^2\mathcal{F}_{12},$$
$$\mathcal{G}_{13}^{\mathrm{P_I}} = \epsilon^2 x\mathcal{F}_{13}, \quad \mathcal{G}_{14}^{\mathrm{P_I}} = \epsilon^2 y\mathcal{F}_{14}, \quad \mathcal{G}_{15}^{\mathrm{P_I}} = \epsilon(\epsilon-1)xy\mathcal{F}_{15},$$
$$\mathcal{G}_{16}^{\mathrm{P_I}} = \epsilon y((2\epsilon-1)\mathcal{F}_{16} + \mathcal{F}_{17}), \quad \mathcal{G}_{17}^{\mathrm{P_I}} = \epsilon xy\mathcal{F}_{17}, \quad \mathcal{G}_{18}^{\mathrm{P_I}} = \epsilon((2\epsilon-1)x\mathcal{F}_{18} + \mathcal{F}_{19}),$$
$$\mathcal{G}_{19}^{\mathrm{P_I}} = \epsilon xy\mathcal{F}_{19}, \quad \mathcal{G}_{20}^{\mathrm{P_I}} = \epsilon^2(x+y)\mathcal{F}_{20}, \quad \mathcal{G}_{21}^{\mathrm{P_I}} = -\epsilon(x+y+xy)\mathcal{F}_{21},$$
$$\mathcal{G}_{22}^{\mathrm{P_I}} = \epsilon(1-2\epsilon)y\mathcal{F}_{22}, \quad \mathcal{G}_{23}^{\mathrm{P_I}} = \epsilon^2(1+x)y^2\mathcal{F}_{23}, \quad \mathcal{G}_{24}^{\mathrm{P_I}} = \epsilon^2 y^2(\mathcal{F}_{24} - \mathcal{F}_{23}),$$
$$\mathcal{G}_{25}^{\mathrm{P_I}} = x\left(\frac{1}{4}y\mathcal{F}_{12} + 2\epsilon^2\mathcal{F}_{20} + y\epsilon^2\mathcal{F}_{23}\right) + \epsilon^2 y(\mathcal{F}_{25} - \mathcal{F}_{22}).$$

The differential equation is then expressed in terms of the variables:

$$x = -\frac{2p_1 \cdot p_2}{M_W^2} \qquad y = -\frac{2p_2 \cdot p_3}{M_W^2}$$

- All letters are rational, solution given by pure GPLs.
- Three kinematic regions, depending on the permutation of the external legs.
- Boundary conditions determined by demanding finitness at pseudo-thresholds.

$$\mathcal{G}_1^{\mathrm{P_I}} = -1 - 2\zeta_2\epsilon^2 + 2\zeta_3\epsilon^3 - 9\zeta_4\epsilon^4$$

Topology P_{II}

24 MIs. The canonical basis is extracted the same way as before:

$$\mathcal{G}_1^{\mathrm{P_{II}}} = \mathcal{F}_1, \quad \mathcal{G}_2^{\mathrm{P_{II}}} = x\mathcal{F}_2, \quad \mathcal{G}_3^{\mathrm{P_{II}}} = (1+x)\mathcal{F}_3,$$
$$\mathcal{G}_4^{\mathrm{P_{II}}} = \mathcal{F}_4 - \mathcal{F}_3, \quad \mathcal{G}_5^{\mathrm{P_{II}}} = (1+y)\mathcal{F}_5, \quad \mathcal{G}_6^{\mathrm{P_{II}}} = \mathcal{F}_6,$$
$$\mathcal{G}_7^{\mathrm{P_{II}}} = y\mathcal{F}_7,$$
$$\mathcal{G}_8^{\mathrm{P_{II}}} = \frac{2x^2(2\epsilon-1)\mathcal{F}_8 + 2\epsilon x(\mathcal{F}_1 - \mathcal{F}_9) + x\mathcal{F}_2}{2(x-1)}, \quad \mathcal{G}_9^{\mathrm{P_{II}}} = \epsilon x\mathcal{F}_9,$$
$$\mathcal{G}_{10}^{\mathrm{P_{II}}} = \epsilon y\mathcal{F}_{10}, \quad \mathcal{G}_{11}^{\mathrm{P_{II}}} = \epsilon y\mathcal{F}_{11}, \quad \mathcal{G}_{12}^{\mathrm{P_{II}}} = \epsilon^2 y\mathcal{F}_{12},$$
$$\mathcal{G}_{13}^{\mathrm{P_{II}}} = \epsilon^2 x\mathcal{F}_{13}, \quad \mathcal{G}_{14}^{\mathrm{P_{II}}} = \epsilon x\mathcal{F}_{14}, \quad \mathcal{G}_{15}^{\mathrm{P_{II}}} = \epsilon x((2\epsilon-1)\mathcal{F}_{15} + \mathcal{F}_{16})$$
$$,\mathcal{G}_{16}^{\mathrm{P_{II}}} = \epsilon xy\mathcal{F}_{16}, \quad \mathcal{G}_{17}^{\mathrm{P_{II}}} = \epsilon^2(x+y)\mathcal{F}_{17}, \quad \mathcal{G}_{18}^{\mathrm{P_{II}}} = \epsilon(x+y+xy)\mathcal{F}_{18},$$
$$\mathcal{G}_{19}^{\mathrm{P_{II}}} = \epsilon(\epsilon-1)xy\mathcal{F}_{19}, \quad \mathcal{G}_{20}^{\mathrm{P_{II}}} = \epsilon y((2\epsilon-1)\mathcal{F}_{20} + \mathcal{F}_{21}), \quad \mathcal{G}_{21}^{\mathrm{P_{II}}} = \epsilon xy\mathcal{F}_{21},$$
$$\mathcal{G}_{22}^{\mathrm{P_{II}}} = \epsilon^2 y\mathcal{F}_{22}, \quad \mathcal{G}_{23}^{\mathrm{P_{II}}} = \epsilon^2 xy\mathcal{F}_{23}, \quad \mathcal{G}_{24}^{\mathrm{P_{II}}} = \epsilon^2 xy\mathcal{F}_{24},$$

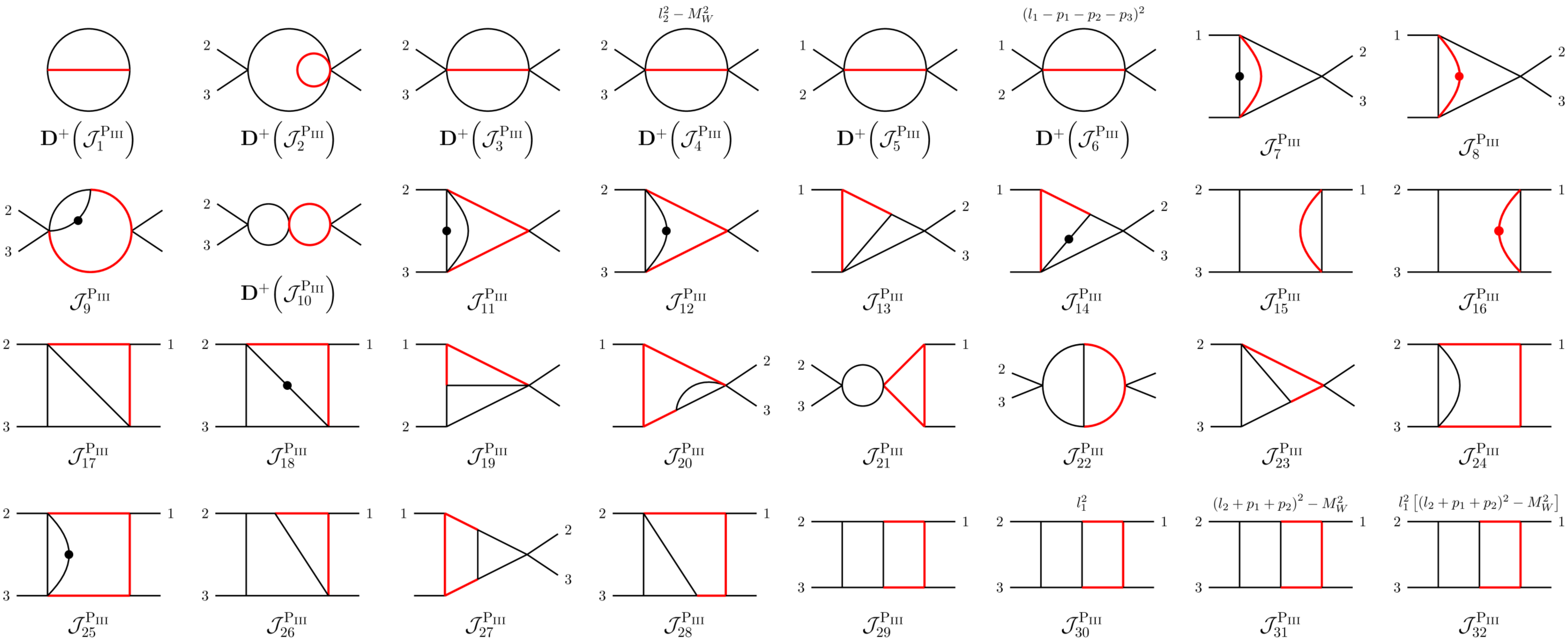
Same auxiliary topology, we reuse the same DEQ variables:

$$x = -\frac{2p_1 \cdot p_2}{M_W^2} \qquad y = -\frac{2p_2 \cdot p_3}{M_W^2}$$

- All letters are rational, solution given by pure GPLs.
- Now we have two kinematic regions, depending on the crossing symmetries.
- Boundary conditions determined by demanding finitness at pseudo-thresholds.

$$\mathcal{G}_1^{\mathrm{P_{II}}} = -1 - 2\zeta_2\epsilon^2 + 2\zeta_3\epsilon^3 - 9\zeta_4\epsilon^4$$

Topology P_{III}: Master Integrals



Topology P_{III}: Canonical Basis

The Canonical basis is obtained again via Magnus Series Expansion. Here we show the 7-propagators family:

$$\mathcal{G}_{29}^{\mathrm{P_{III}}} = \epsilon^2 y\sqrt{r_2}\mathcal{F}_{29},$$

$$\mathcal{G}_{30}^{\mathrm{P_{III}}} = \epsilon^2 \sqrt{r_1}(2x\mathcal{F}_{28} + xy\mathcal{F}_{29} + y\mathcal{F}_{30}),$$

$$\mathcal{G}_{31}^{\mathrm{P_{III}}} = \epsilon^2 y^2(\mathcal{F}_{29} + \mathcal{F}_{31}),$$

$$\mathcal{G}_{32}^{\mathrm{P_{III}}} = -\frac{1}{2}\epsilon\Big(2x(1-2\epsilon)\mathcal{F}_{21} + y^2\epsilon(-\mathcal{F}_{27} + x\mathcal{F}_{29} + \mathcal{F}_{30}) + 2y\epsilon(2\mathcal{F}_{17} + \mathcal{F}_{22} + x\mathcal{F}_{28} + x\mathcal{F}_{29} - \mathcal{F}_{32})\Big).$$

The differential equation is expressed in terms of the variables:

$$x = -\frac{2p_1\cdot p_2}{M_W^2} \qquad y = -\frac{2p_2\cdot p_3}{M_W^2}$$

- Non-Rational letters:

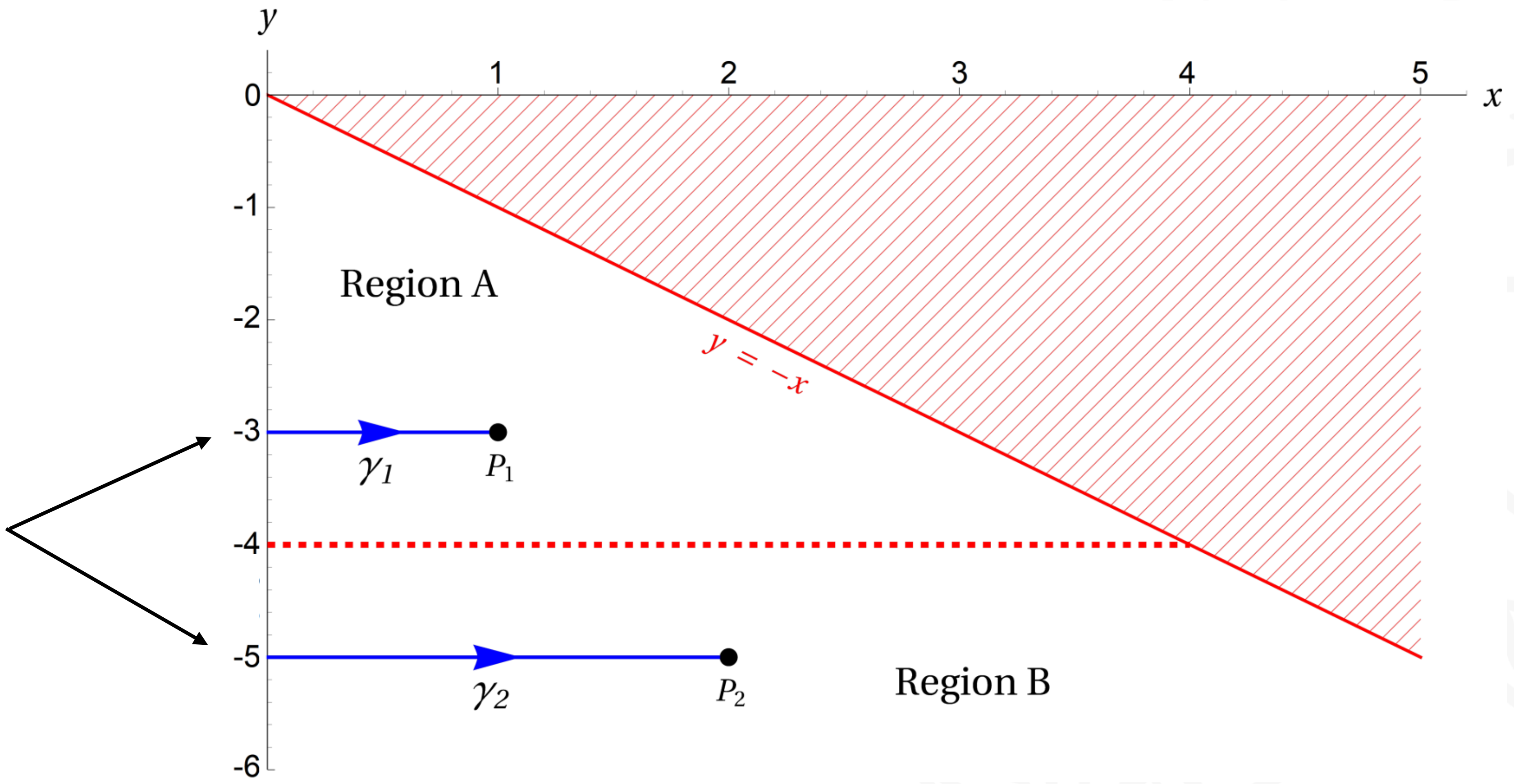
$$r_1 = y(4+y),$$
$$r_2 = y\left(y + 2xy + x^2(4+y)\right),$$
$$r_3 = xy\left(4y + x(4+y)\right).$$

- For efficiency reasons, we only rationalize r_1.
- Need additional mapping for clean dLog form.
- Mixed Chen/GPL representation.

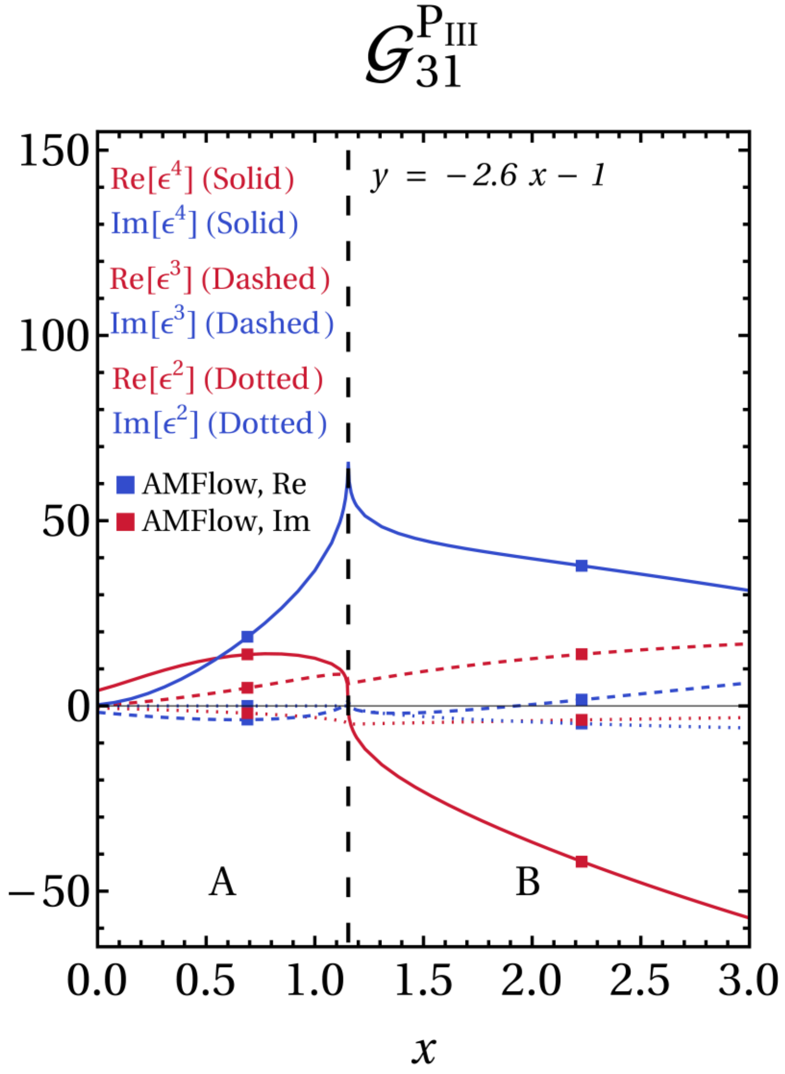
Topology P_{III}: Kinematic Sub-Regions

$$\text{Region A}: [0 \geq y \geq -4,\; x \leq -y],\quad y = -\frac{4z_A^2}{1+z_A^2} \qquad \text{Region B}: [y \leq -4,\; x \leq -y],\quad y = -\frac{4}{1-z_B^2}$$

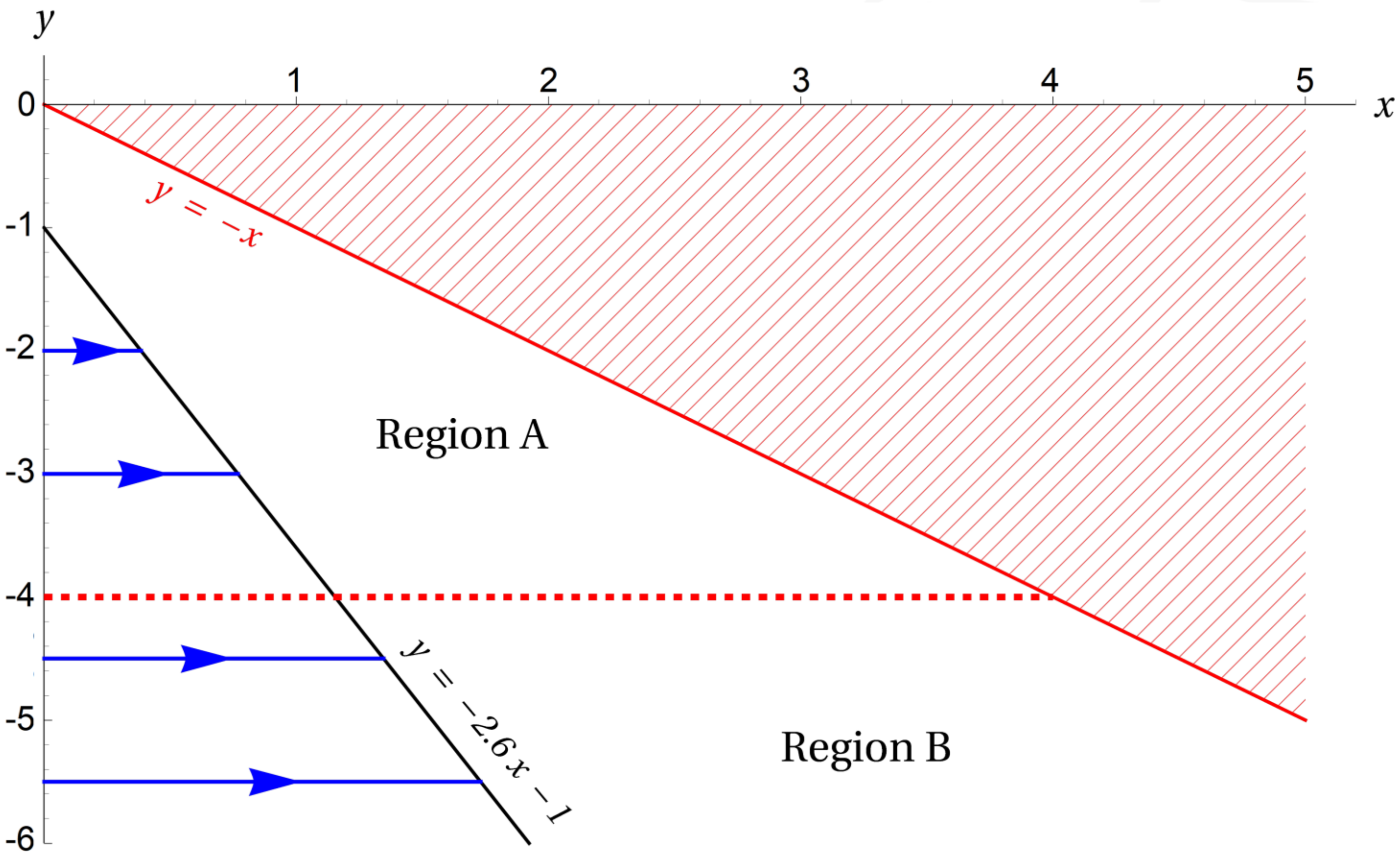
$$\mathcal{G}_i^{\mathrm{P_{III}}}(x=0)$$



Topology P_{III}: Numerical Evaluation



Integration Path:



Topology $\mathrm{N_I}$

40 MIs. For brevity, we show the 7-propagators family:

$$\mathcal{G}_{37}^{\mathrm{N_I}} = \epsilon^2 \frac{(x+y)}{2}\left(x^2\mathcal{F}_{37} + x(\mathcal{F}_{32}+\mathcal{F}_{34}-2\mathcal{F}_{37}+y\mathcal{F}_{37}-\mathcal{F}_{39}) + y(-2\mathcal{F}_{37}+\mathcal{F}_{39})\right),$$

$$\mathcal{G}_{38}^{\mathrm{N_I}} = \frac{x}{2y}\Big(\mathcal{F}_1-\mathcal{F}_2-\mathcal{F}_3-2\mathcal{F}_7-2\epsilon(x+y)\mathcal{F}_9+6\epsilon^2 x\mathcal{F}_{15}-2\epsilon\left(x(-1+y)+y^2\right)\mathcal{F}_{16}$$
$$+2\epsilon y(x+y)\mathcal{F}_{23}+2\epsilon^2 y\mathcal{F}_{27}-2\epsilon^2(x+y)\mathcal{F}_{31}\Big)$$
$$-\epsilon^2\frac{x+y}{2}\Big(x(\mathcal{F}_{32}-\mathcal{F}_{34})+(x+y)((2+x)\mathcal{F}_{37}-2\mathcal{F}_{38}-\mathcal{F}_{39})\Big),$$

$$\mathcal{G}_{39}^{\mathrm{N_I}} = -\epsilon^2\sqrt{r_2}\left(x^2\mathcal{F}_{37}+x(\mathcal{F}_{32}+\mathcal{F}_{34}+y\mathcal{F}_{37}-\mathcal{F}_{39})-y\mathcal{F}_{39}\right),$$

$$\mathcal{G}_{40}^{\mathrm{N_I}} = -\epsilon^2 x(x+y)\mathcal{F}_{37}+\epsilon^2(x+y)\mathcal{F}_{40}+\frac{x}{16y}\Big(10\mathcal{G}_1^{\mathrm{N_I}}+2\mathcal{G}_2^{\mathrm{N_I}}-11\mathcal{G}_3^{\mathrm{N_I}}+3\mathcal{G}_4^{\mathrm{N_I}}-10\mathcal{G}_5^{\mathrm{N_I}}$$
$$-4\mathcal{G}_6^{\mathrm{N_I}}-16\mathcal{G}_9^{\mathrm{N_I}}-12\mathcal{G}_{12}^{\mathrm{N_I}}-2\mathcal{G}_{13}^{\mathrm{N_I}}+48\mathcal{G}_{15}^{\mathrm{N_I}}-16\mathcal{G}_{16}^{\mathrm{N_I}}-16\mathcal{G}_{17}^{\mathrm{N_I}}+8\mathcal{G}_{19}^{\mathrm{N_I}}-12\mathcal{G}_{20}^{\mathrm{N_I}}$$
$$-2\mathcal{G}_{21}^{\mathrm{N_I}}+16\mathcal{G}_{23}^{\mathrm{N_I}}+8\mathcal{G}_{25}^{\mathrm{N_I}}+8\mathcal{G}_{27}^{\mathrm{N_I}}+8\mathcal{G}_{28}^{\mathrm{N_I}}-16\mathcal{G}_{31}^{\mathrm{N_I}}\Big).$$

We find convenient to switch to the following DEQ variables:

$$x = \frac{2p_1\cdot p_2}{M_W^2} \qquad y = \frac{2p_1\cdot p_3}{M_W^2}$$

- Non-rational letters (in x and y):

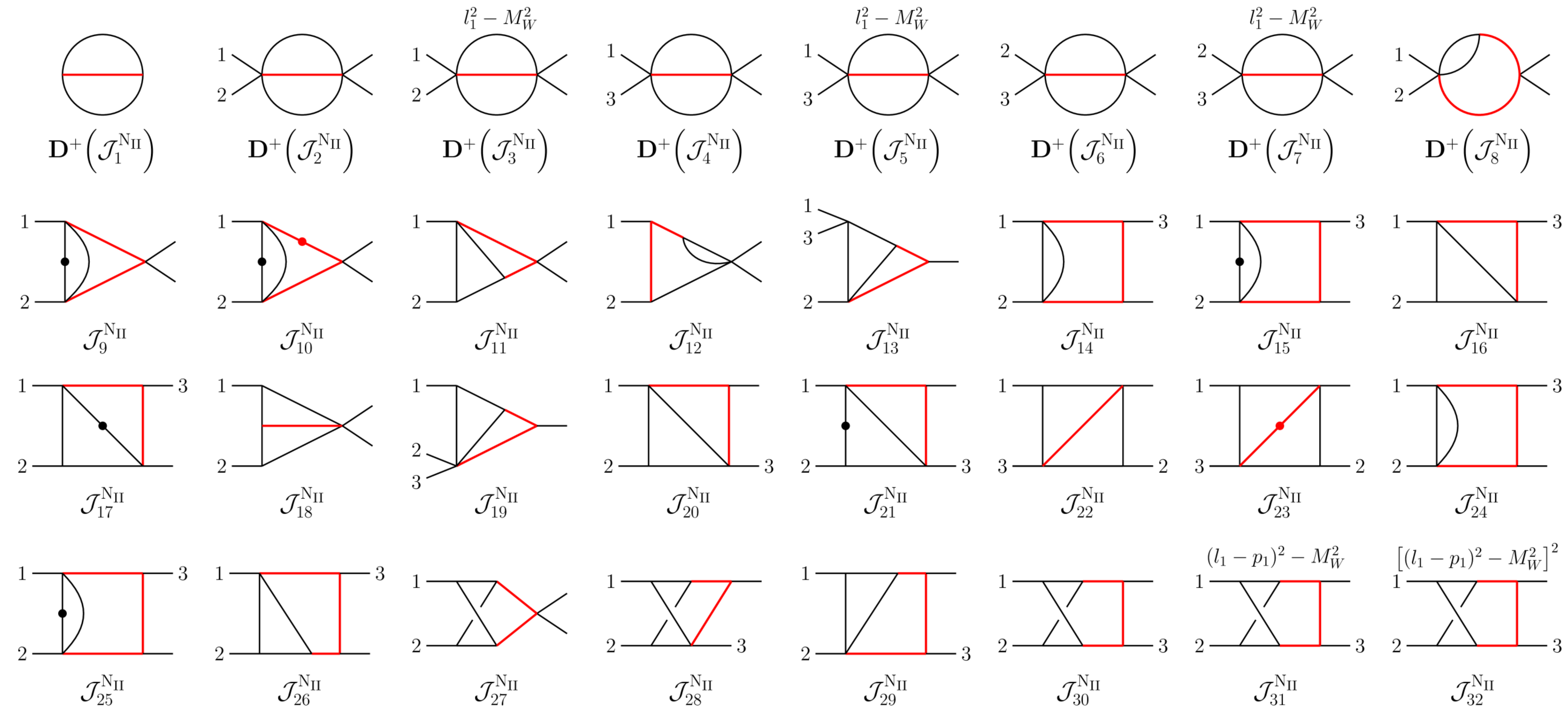
$$r_1 = xy(x+y),$$
$$r_2 = (4-x-y)(x+y)$$

- We can rationalize both. First mapping rationalizes 33 out of 40:

$$x \to \frac{uv}{1+v} \qquad y \to \frac{u}{1+v}$$

- The others need dedicated mapping for each sub-region. Ultimately, every integral can be expressed as GPL.

Topology N_{II}: Master Integrals



Topology N_{II}: Canonical Basis

The Canonical basis is obtained with the usual strategy. The 7-propagators family is given by:

$$\mathcal{G}_{30}^{N_{II}} = -2\epsilon^2\Big(2(x-1)\mathcal{F}_{28} + (1-x-2y+4xy)\mathcal{F}_{30} + \mathcal{F}_{31}\Big),$$

$$\mathcal{G}_{31}^{N_{II}} = -\epsilon^2\frac{\sqrt{r_1}}{2}\Big(2(x-1)\mathcal{F}_{28} + (1-x)\mathcal{F}_{30} + (1-2x)\mathcal{F}_{31}\Big),$$

$$\begin{aligned}\mathcal{G}_{32}^{N_{II}} =&\epsilon^2(y\mathcal{F}_{30} + \mathcal{F}_{31} + \mathcal{F}_{32}) + \frac{1}{8x}(15\mathcal{G}_1^{N_{II}} + 8\mathcal{G}_2^{N_{II}} - 8\mathcal{G}_3^{N_{II}} + 10\mathcal{G}_4^{N_{II}} - 17\mathcal{G}_5^{N_{II}}\\ &+ 4\mathcal{G}_6^{N_{II}} - 4\mathcal{G}_7^{N_{II}} - 24\mathcal{G}_{13}^{N_{II}} - 12\mathcal{G}_{16}^{N_{II}} - 2\mathcal{G}_{17}^{N_{II}} + 8\mathcal{G}_{18}^{N_{II}} - 8\mathcal{G}_{19}^{N_{II}} + 20\mathcal{G}_{20}^{N_{II}}\\ &+ 2\mathcal{G}_{21}^{N_{II}} - 24\mathcal{G}_{22}^{N_{II}} - 8\mathcal{G}_{23}^{N_{II}}).\end{aligned}$$

The differential equation is expressed in terms of the variables:

$$x = -\frac{p_1\cdot p_3}{p_1\cdot p_2} \qquad y = \frac{M_W^2}{2p_1\cdot p_2}$$

- Non-Rational letters:

$$\begin{aligned} r_1 &= 1-4y\\ r_2 &= x^2(1-4y)+2xy+y^2\\ r_3 &= x^2(1-4y)+x(6y-2)+(y-1)^2\\ r_4 &= -x\,(4xy-x-4y)\\ r_5 &= xy\,(x-1)\\ r_6 &= -(x-1)(x(4y-1)+1)\end{aligned}$$

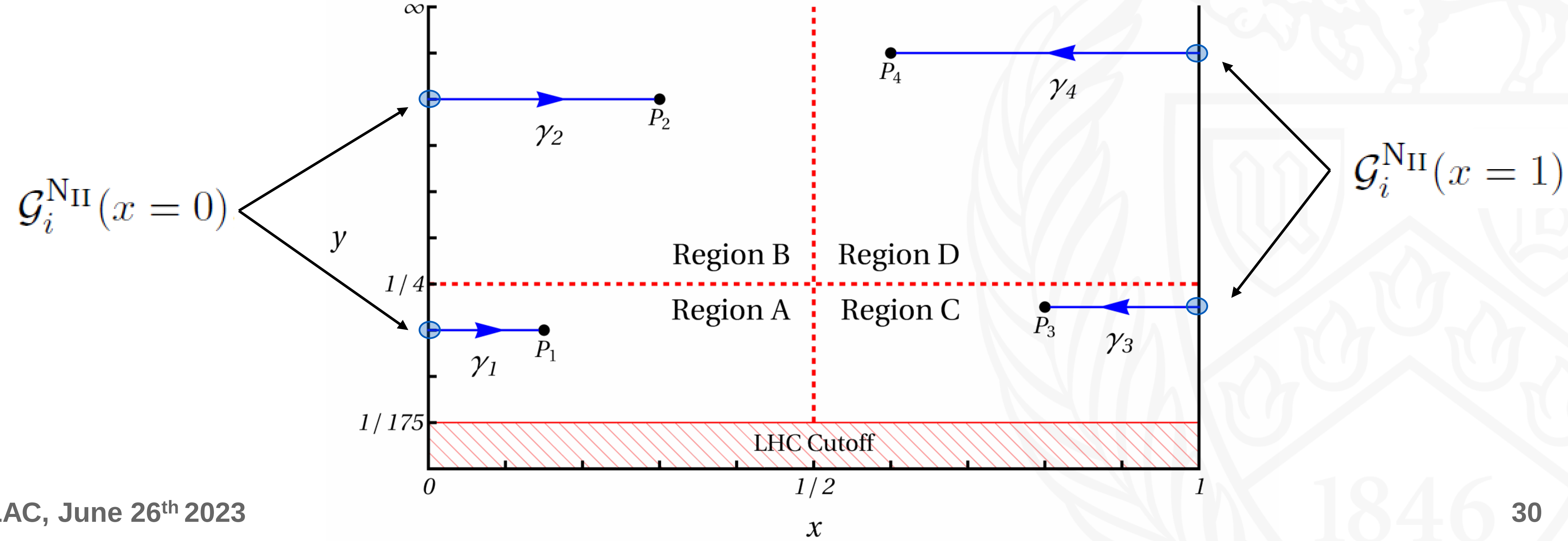
- For efficiency reasons, we only rationalize r_1.

- Mixed Chen/GPL representation.

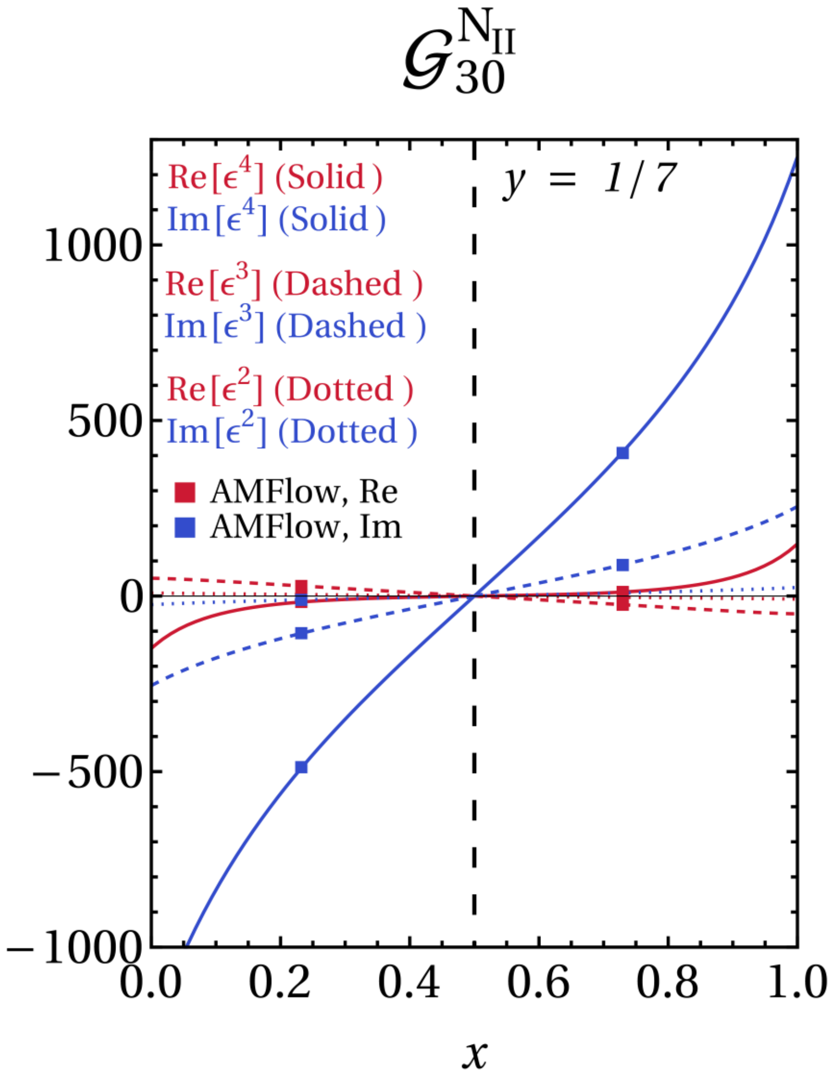
Topology $\mathrm{N_{II}}$: Kinematic Sub-Regions

$$\text{Region A}: \quad [s > 4M_W^2, u < t], \quad y = \frac{z_A}{2}\left(1 - \frac{z_A}{2}\right) \qquad \text{Region B}: \quad [s \leq 4M_W^2, u < t], \quad y = \frac{1 - 2z_B + z_B^2}{4z_B^2}$$

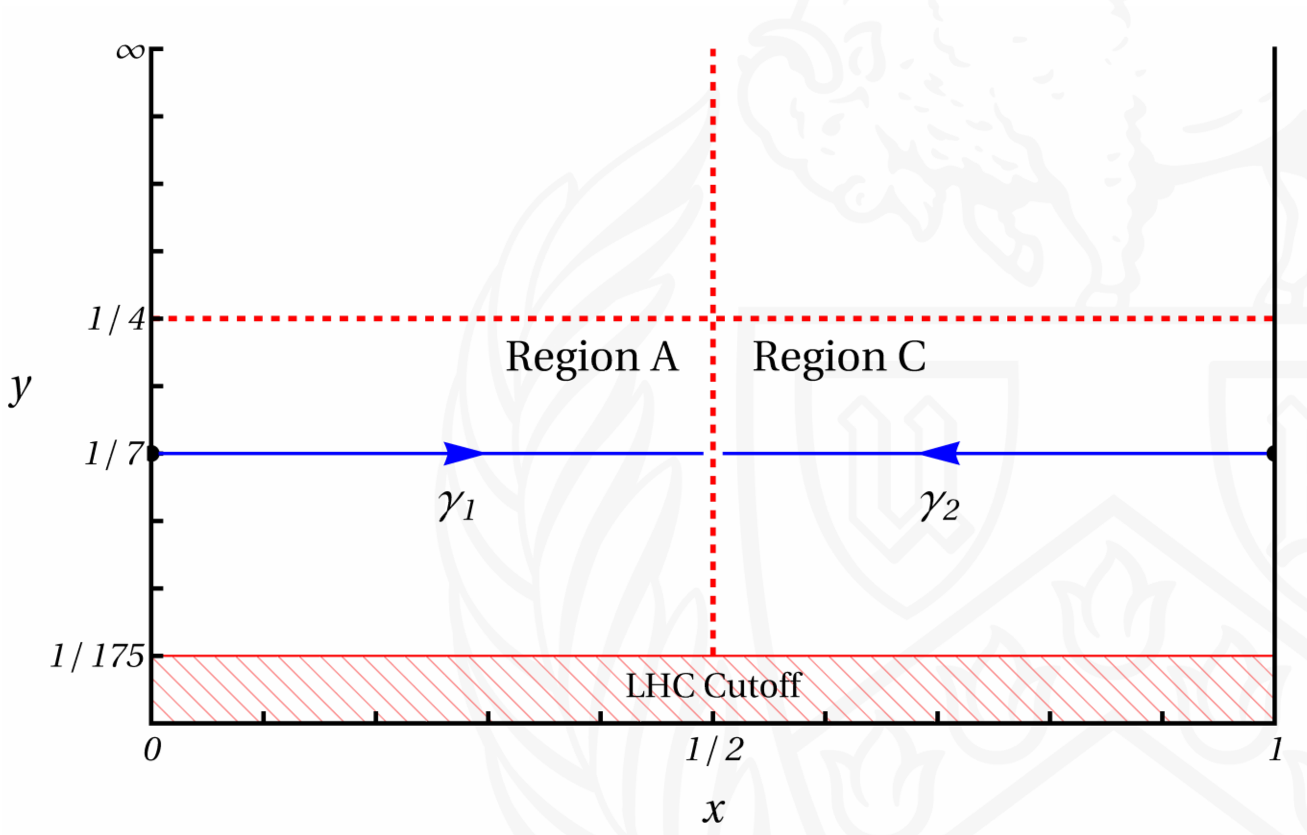
$$\text{Region C}: \quad [s > 4M_W^2, u > t], \quad y = \frac{z_A}{2}\left(1 - \frac{z_A}{2}\right) \qquad \text{Region D}: \quad [s \leq 4M_W^2, u > t], \quad y = \frac{1 - 2z_B + z_B^2}{4z_B^2}$$



Topology N_{II}: Master Integrals



Integration Path:



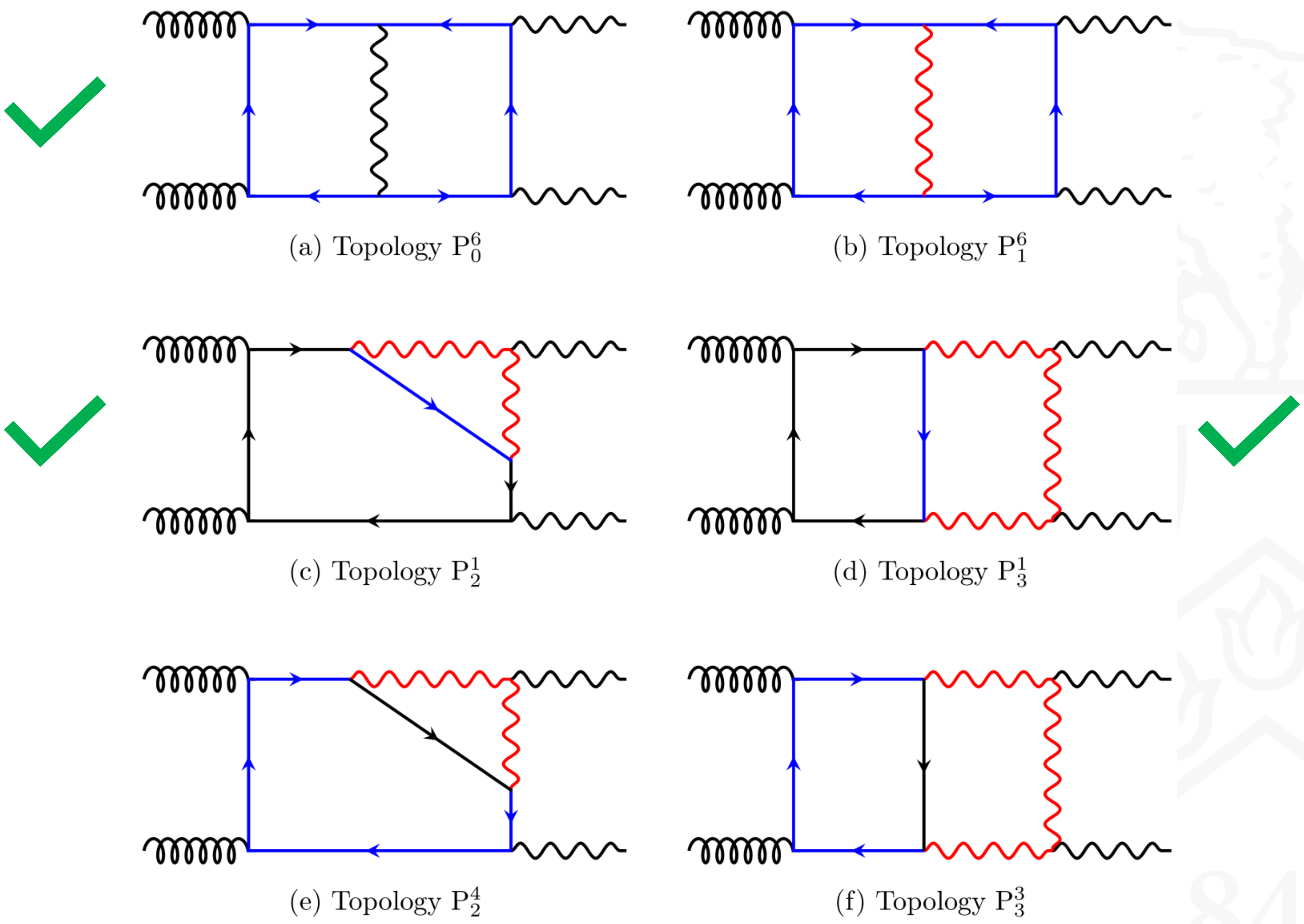
Conclusions

Moving Forward

Top-quark planar Topologies:

Blue propagators:
Top-quark

Red Propagators:
Weak Bosons



Conclusions

Summary:

- Evaluation of the MIs for the electroweak corrections to $gg \to \gamma\gamma$ for massless quark loops.
- Efficient calculation at any kinematic point.
- Versatility on the alphabet: rational vs non-rational letters.
- Solution available for every topology.

Future Directions:

- Evaluation of the amplitude and differential distributions (pheno analysis).
- Inclusion of the top-quark mediated diagrams.
- Complete analysis of the interference order $\mathcal{O}(\alpha)$.

Loopfest 2023, 26-28 June

Grazie!

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