Higgs Boson Production at the Next-to-Next-to-Leading Power

Alexander Penin

University of Alberta

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Topics discussed

- High-energy or small-mass limit of $gg \to H$
  - *leading logs at* $\mathcal{O}(m_q^2/s)$

    T. Liu, A.A. Penin, Phys.Rev.Lett. **119**, 262001 (2017)
  - *next-to-leading logs at* $\mathcal{O}(m_q^2/s)$

    C. Anastasiou, A.A. Penin, JHEP **07**, 195 (2020)
  - *leading logs at* $\mathcal{O}(m_q^4/s^2)$

    T. Liu, S. Modi, A.A. Penin, JHEP **02**, 170 (2022)

The advent of power corrections

- Power *vs* perturbative corrections
  - Λ_X^2/Q^2 *vs* α_X^n
  - *e.g.* $\Lambda_X = m_b,\ Q = m_H,\ \alpha_X = \alpha_s$ ⇨ $n = 3$
- Logarithmically enhanced power corrections
  - *phenomenologically relevant*
  - *intriguing from QFT point of view*

  ↪ *weird RG structure, magic relations, etc.*
- Recent stream of the NLP results for
  - *mass, angle, soft momentum, threshold, jetiness, ...*

Higgs production at the LHC

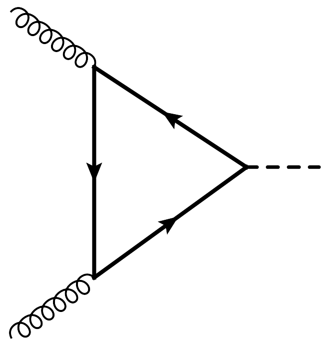
- Total cross section at 13 GeV $\sigma_{pp\to H+X} = 48.68$ *pb*

- Dominant theory uncertainties
  - *scale choice* $^{+0.10}_{-1.15}$ *pb*
  - *PDF N^{3}LO* ± 0.56 *pb*
  - $m_b > 0$ *NNLO+* ± 0.40 *pb*

Anastasiou et al. JHEP **1605**, 058 (2016)

Bottom quark mass effect

- Leading contribution

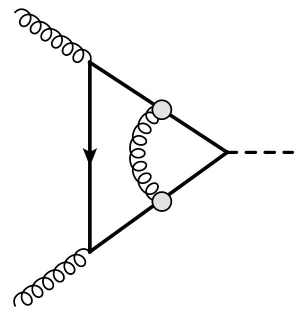


$$\propto \alpha_s \ln^2(m_H^2/m_b^2)\frac{m_b^2}{m_H^2}$$

- ➥ *large logs at subleading power*
- *effective expansion parameter* $\alpha_s \ln^2(m_H^2/m_b^2) \sim 40\alpha_s$
- ➥ *resummation is mandatory*

$gg \to H$ amplitude at NLP LL

- **Non-Sudakov logs** T. Liu, A.A. Penin, Phys.Rev.Lett. **119**, 262001 (2017)



- **Factorization formula** $\mathcal{M}^b_{gg\to H} = Z_g^{2\,LL}\, g(z)\, \mathcal{M}^{b\,(0)}_{gg\to H}$

  - *gluon Sudakov factor* $Z_g^{2\,LL} = \exp\left[-\frac{C_A}{\varepsilon^2}\frac{\alpha_s}{2\pi}\frac{\mu^{2\varepsilon}}{Q^{2\varepsilon}}\right]$
  - *non-Sudakov double logarithms*

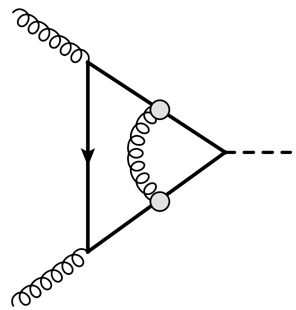
$$g(z) = 2\int_0^1 \mathrm{d}\xi \int_0^{1-\xi} \mathrm{d}\eta\, e^{2z\eta\xi} = {}_2F_2\,(1,1;3/2,2;z/2)$$

  - *double-log variable* $z = \ (C_A - C_F)\ x,\ \ x = \frac{\alpha_s}{4\pi}L^2,\ L = \ln(m_H^2/m_q^2)$

eikonal color nonconservation

$gg \to H$ amplitude at NLP LL

- Non-Sudakov logs   T. Liu, A.A. Penin, Phys.Rev.Lett. **119**, 262001 (2017)



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➥ *Magic #1: same function for NLP QED scattering in Regge limit*
color (Sudakov, soft) $\Leftrightarrow$ kinematics (Regge, Glauber)

$gg \to H$ amplitude at NLP NLL

C. Anastasiou, A.A. Penin, JHEP **07**, 195 (2020)

$$\mathcal{M}^{b\,NLL}_{gg\to H} = C_b \left(\frac{\alpha_s(m_H)}{\alpha_s(m_b)}\right)^{\gamma_m^{(1)}/\beta_0} Z_g^{2\,NLL} \left[-\frac{3}{2}\frac{m_b^2}{m_H^2}L^2\,\mathcal{M}^{t\,(0)}_{gg\to H}\right]$$

Yukawa RG factor *gluon Sudakov form factor* *LO amplitude*

$$C_b = \left[g(z) + \frac{\alpha_s L}{4\pi}\left(2\gamma_q^{(1)} g_\gamma(z) - \beta_0 g_\beta(z)\right)\right] = 1 + \sum_{n=1}^{\infty} c_n$$

$$c_1 = \frac{z}{6} + C_F\frac{\alpha_s L}{4\pi}\,, \quad c_2 = \frac{z^2}{45} + \frac{z}{5}\frac{\alpha_s L}{4\pi}\left[\frac{3}{2}C_F - \beta_0\left(\frac{5}{6}\frac{L_\mu}{L} - \frac{1}{3}\right)\right],$$

$$c_3 = \frac{z^3}{420} + \frac{z^2}{5}\frac{\alpha_s L}{4\pi}\left[\frac{5}{21}C_F - \beta_0\left(\frac{2}{9}\frac{L_\mu}{L} - \frac{2}{21}\right)\right], \quad \ldots$$

$$L = \ln(s/m_q^2),\ L_\mu = \ln(s/\mu^2)$$

c_2 *agrees with 3-loop num. calc.* Czakon, Niggetiedt, JHEP **05**, 149 (2020)

Top-bottom interference in threshold cross section

C. Anastasiou, A.A. Penin, JHEP **07**, 195 (2020)

	LO	NLO	NNLO	N^{3}LO
$\delta\sigma^{\rm LL}_{pp\to H+X}$	-1.420	-1.640	-1.667	-1.670
$\delta\sigma^{\rm NLL}_{pp\to H+X}$	-1.420	-2.048	-2.170	-2.189
$\delta\sigma_{pp\to H+X}$	-1.023	-2.000		

- NLL K-factors with full threshold $\delta\sigma^{NLO}_{pp\to H+X}$

$$\delta\sigma^{\rm NNLO}_{gg\to H+X} \approx -0.12\ pb$$

$$\delta\sigma^{\rm N^3LO}_{gg\to H+X} \approx -0.02\ pb$$

- *New uncertainty interval* -0.32 *to* 0.08 *pb* *(factor 2 reduction)*

Next-to-next-to leading power: why?

- Theory
  - *proof of principle*
  - *structure of RG: terra incognita*
- Phenomenology
  - *validate the quark mass expansion*
  - *NNLO Higgs pair production at high $p_\perp$, etc.*

Next-to-next-to leading power: how?

- Brut-force approach

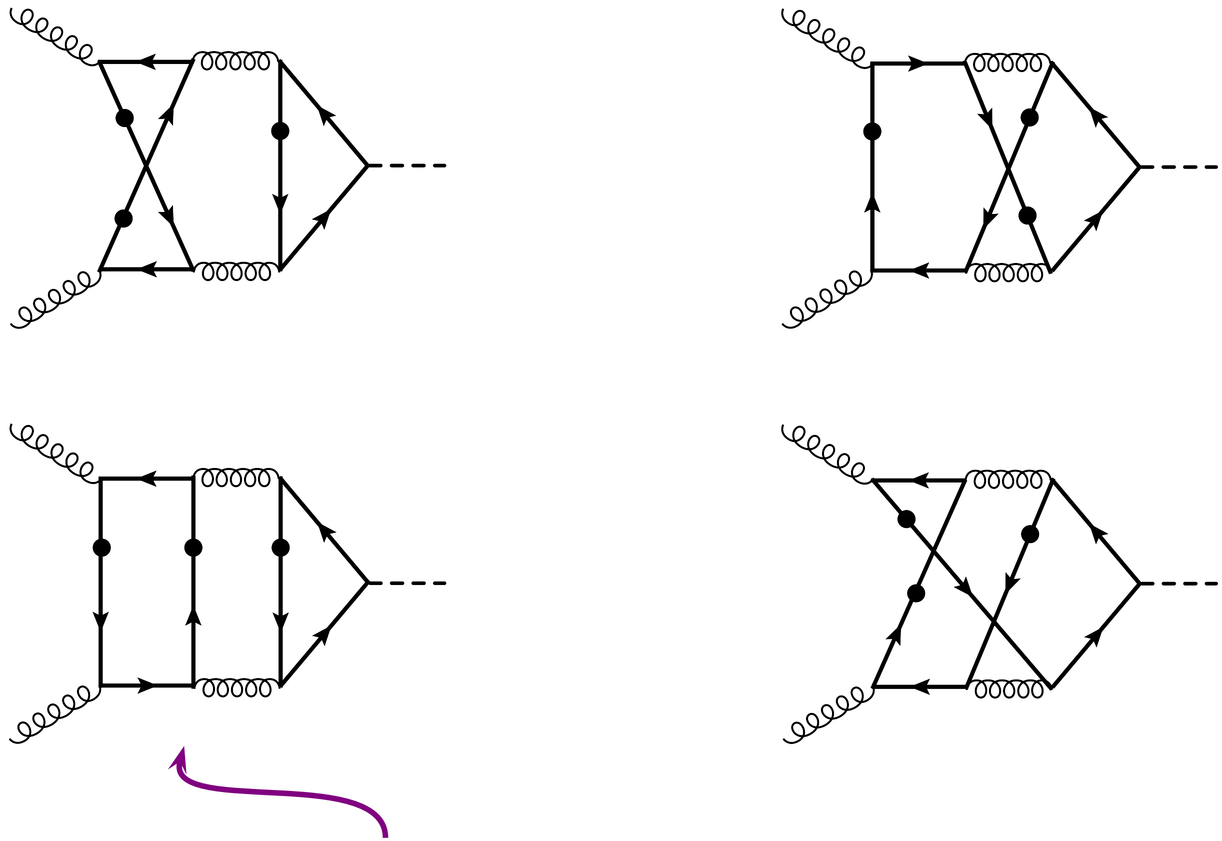
  A.A. Penin, Phys.Lett. B **119** (2015) 262001

  - *expansion by regions* ⇨ *homogeneous integrals* ⇨ *log corrections*
  - *Ward identities + momentum shifts + eikonal factorization* ⇨ *resummation*

- Classification of NNLP terms

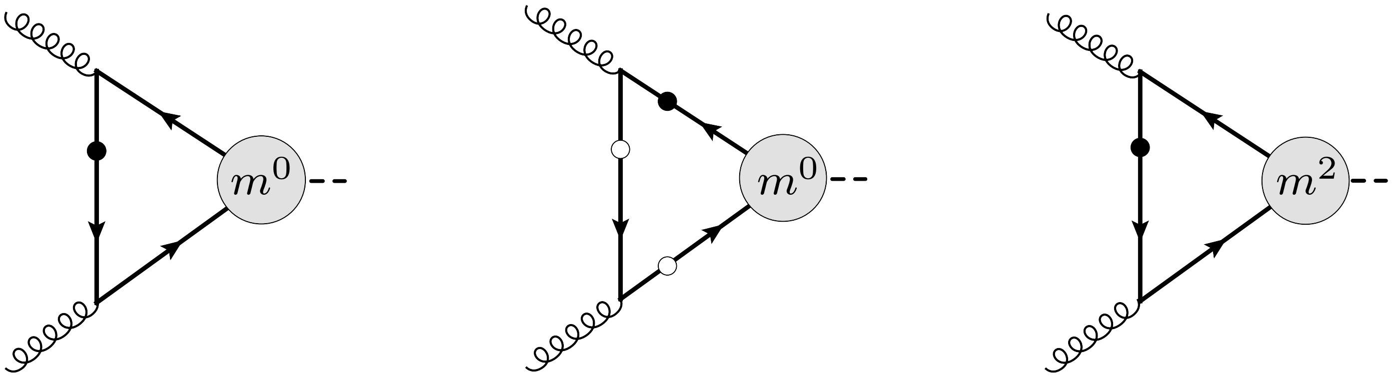
  - *soft quarks exchange: single/triple*
  - *topology: factorizable/nonfactorizable*

Triple soft quark exchange



only planar graph contributes ⇨ factorizable topology

Factorizable contribution



NLP $\quad$ $NNLP\,(a)$ $\quad$ $NNLP\,(b)$

- *(a) power corrections to $gg \to q\bar{q}$ amplitude*
  - *non helicity-flip contribution for the first time*
- *(b) power corrections to the off-shell FF*
  - *can be inferred from on-shell result*

Scalar form factor NLP LL

- On-shell FF T. Liu, A.A. Penin, Phys.Rev.Lett. **119**, 262001 (2017)

$$F_S = Z_q^2 \sum_{n=0}^{\infty} \frac{m_q^2}{Q^2}^n F_S^{(n)}, \qquad F_S^{(1)} = -\frac{C_F T_F}{3} x^2 f(-z)$$

- *Magic #2: universal function for all NLP form factors*

$$f(z) = 1 + \frac{z}{5} + \frac{11}{420} z^2 + \frac{z^3}{378} + \dots$$

confirmed to 3 loops in M. Fael, et al. Phys.Rev.D **106**, 034029 (2022)

Scalar form factor NLP LL

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- *Magic #2: universal function for all NLP form factors*

$$f(z) \sim 6\left[\ln\left(\frac{z}{2}\right) + \gamma_E\right]\left(\frac{2\pi e^z}{z^5}\right)^{1/2}$$

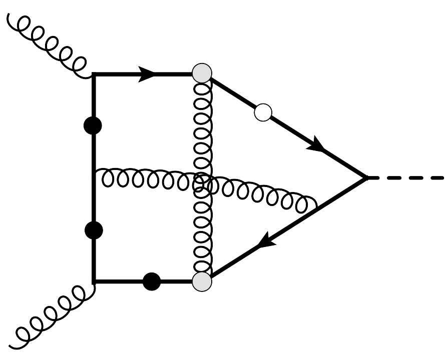
$$f(-z) \sim \left[(\ln(2z) + \gamma_E)^2 - \frac{\pi^2}{2}\right]\frac{3}{z^2},$$

asymptotic behaviour at $z \to \infty$

Nonfactorizable contribution

- *Abelian diagram*
  - *eikonal gluon coupled to soft quark*

- *Non-Abelian case* $C_F \to C_F - C_A$



Results

T. Liu, S. Modi, A.A. Penin, JHEP **02**, 170 (2022)

- Scalar amplitude

$$M^q_{ggH} = Z_g^2 \ln^2\left(\frac{m_q^2}{m_H^2}\right)\sum_{n=0}^{\infty}\left(\frac{m_q^2}{m_H^2}\right)^n M^{(n)}_{ggH}, \qquad M^{(0)}_{ggH} = g(z)$$

- NNLP

$$M^{(1)}_{ggH} = \left[-4g(z) + \left(\frac{T_F C_F}{45}h(z) - \frac{(C_A - C_F)(C_A - 2C_F)}{9}j(z)\right)x^2\right]$$

large-N_c

  - three loops

$$M^{(1)}_{ggH} = \left[-4 - \frac{2}{3}(C_A - C_F)x + \left(\frac{T_F C_F}{45} - \frac{14}{45}C_F^2 + \frac{23}{45}C_F C_A - \frac{9}{45}C_A^2\right)x^2\right]$$

  - *agrees with num. calc.* M.Czakon, M.Niggetiedt, JHEP **2005**, 149 (2020)

Results

- All-order
  - *factorizable single soft quark:* $g(z) = {}_2F_2\,(1,1;3/2,2;x/2)$
  - *factorizable triple soft quark*

$$h(z) = 6!\int_0^1 \mathrm{d}\eta \int_0^{1-\eta} \mathrm{d}\xi \int_0^{\eta} \mathrm{d}\eta_2 \int_0^{\xi} \mathrm{d}\xi_2 \int_0^{\eta_2} \mathrm{d}\eta_1 \int_0^{\xi_2} \mathrm{d}\xi_1\, e^{2z(\eta\xi-\eta_2\xi_2+\eta_1\xi_1)}$$

  - *nonfactorizable*

$$j(z) = 72\int_0^1 \mathrm{d}\eta \int_0^{1-\eta} \mathrm{d}\xi \int_0^{1-\xi} \mathrm{d}\eta_1 \int_0^{1-\eta_1-\xi} \mathrm{d}\xi_1\, \eta\xi_1 e^{2z\eta(\xi+\xi_1)}$$

$$\times\left[1+\frac{e^{-2z\eta\xi}-1}{2}+\frac{e^{-2z\eta\xi}-1+2z\eta\xi}{4z\eta\xi_1}\right],$$

- $C_F \to C_F - C_A$ *rule is still to be proven beyond three loops*

Summary

- Bottom effect in Higgs boson production in gluon fusion *(NLP-NLL-threshold)*

  ↪ *uncertainty interval reduced to* $0.40\ pb$ *(factor 2)*

- Power corrections in mass
  - *first ever NNLP LL result*
  - *small quark mass expansion parameter* $4m_q^2/s$