Anomalous electric and magnetic dipole moments in τ-lepton pair production

Z. Was*,

*Institute of Nuclear Physics, Polish Academy of Sciences, Krakow

(A) Exp. Data and New Physics (NP): Define most sensitive 1-dim observables, $\leftrightarrow$ multidimensional signatures and Machine Learning techniques

(B) Real conditions: backgrounds selection cuts, models, couplings in fits etc.

(C) New Physics effects $\rightarrow$ tiny/nonexistent, but for large SM background:

precision neded.

(D) KKMC is nowdays a good candidate for FCC-ee $e^+e^- \rightarrow \tau^+\tau^- n(\gamma)$ MC:

1. Flexible τ decay channels, for SM and NP. Hope: Belle 2 hadronic currents public?
2. Imprint into KKMC events: anomalous magnetic/electric moments using weights.
3. Imprint into KKMC events: additional NP particles: dark photons or new light scalars.
4. tauola and photos MC's for τ decays and radiative corrections in decays associate activities with KKMC .

- **(E)** My focus: tools their status.

Phase space and Monte Carlo

- **(1)** NP with the help of event weights, calculated at **runtime** or later from production files.
- **(2)** It is convenient if Monte Carlo use explicit and exact phase space parametrization.
- **(3)** and all approximations localized in matrix elements.
- **(4)** I mean approximate matrix elements, but of explicit and fully controlled approximations.
- **(5)** New Physics effects approximations are usually OK, but not(!) for SM often interfering and large background.

- **Backbone:** KKMC **Monte Carlo for** $e^+e^- \to \tau^+\tau^-(n\gamma)$ **process (with** τ **decays).**

- KKMC has well established precision tag and is installed in Belle 2 collaboration software. Changes for NP, bring minor changes to the code only **(straigtforward to extend and use in FCCee variant of KKMC)**:
- S. Banerjee, D. Biswas, T. Przedzinski and Z. Was, "The tau lepton Monte Carlo Event Generation - imprinting New Physics models with exotic scalar or vector states into simulation samples," [arXiv:2112.07330 [hep-ph]].
- S. Banerjee, A.Yu. Korchin and Z. Was "Anomalous magnetic and electric dipole moments and spin correlations in production of τ-lepton pairs , IFJ-PAN-IV-2022-12, Sept. 2022.

(A) New physics signatures

Start: new physics model builders identify potentially detetctable signatures.

First step luminosity, NP cross section, idealized observable ...

Difficulties: orders of magnitude larger background, detection deformations, all need to be taken into account one day.

Challenge:

precision background estimation and flexibility for tools modifications/extensions is fundamental.

Efforts:

Work is on-going for FCCee KKMC τ **pair production**

Hope: one day better τ **decay parametrizations from Belle 2 data**

My today talk is devoted to re-weighting SM events for New Physics.

SM can not be avoided but let's explore efforts of somebody else.

(B) precondition: preserve SM precision.

Phase space and Monte Carlo

(1) For the precision predictions and integration over acceptance regions for realistic observables, Monte Carlo techniques are indispensable.

(2) It is convenient to use explicit and exact phase space parametrization,

(3) and have all approximations localized in matrix elements.

(4) I mean approximate but explicit matrix elements.

(5) New Physics effects can be then injected into such programs in unambiguous way.

(6) Approximations for New Physics effects are OK. For SM, often interfering background:

no precision compromise!

- **I will use KKMC Monte Carlo for $e^+e^- \to \tau^+\tau^-(n\gamma)$ process (with τ decays).**

- KKMC has well established (and improving) precision tag. It is installed in Belle 2 collaboration software too. Changes for NP, bring minor ones for the code only:

- S. Banerjee, D. Biswas, T. Przedzinski and Z. Was, “The tau lepton Monte Carlo Event Generation - imprinting New Physics models with exotic scalar or vector states into simulation samples,” [arXiv:2112.07330 [hep-ph]].

- S. Banerjee, A.Yu. Korchin and Z. Was “Anomalous magnetic and electric dipole moments and spin correlations in production of τ-lepton pairs , IFJ-PAN-IV-2022-12, Sept. 2022.

(C) How to add extra interactions:

To start: M^{SM} and M^{SM+NP} are needed.

- Check if factorization properties for NP match with what is in SM. Precision requirements for New Physics implementation are not high. Use of interpolated Born configurations in presence of hard bremsstrahlung photons is OK.
- Seem trivial, but one has to keep in mind practical details.
- I will say little about reliability proofs, even though they are essential.
- Important is to preserve SM (interfering-)background bulk of the process!
- OK, for **anomalous magnetic/electric dipole** moments implementation in $e^+e^- \to \tau^+\tau^-(n\gamma)$ process (τ decays included).
- Next slide: condition of work with KKMC Monte Carlo.

(C) How to add extra interactions:

Formalism for $\tau^+\tau^-$: phase space $\times$ M.E. squared

- Because narrow τ width (τ propagator works as Dirac δ), cross-section for $f\bar{f} \to \tau^+\tau^- Y; \tau^+ \to X^+\bar{\nu}; \tau^- \to \nu\nu$ reads (norm. const. dropped):

$$d\sigma = \sum_{spin} |\mathcal{M}|^2 d\Omega = \sum_{spin} |\mathcal{M}|^2 d\Omega_{prod}\; d\Omega_{\tau^+}\; d\Omega_{\tau^-}$$

$$\mathcal{M} = \sum_{\lambda_1\lambda_2=1}^{2} \mathcal{M}^{prod}_{\lambda_1\lambda_2}\; \mathcal{M}^{\tau^+}_{\lambda_1} \mathcal{M}^{\tau^-}_{\lambda_2}$$

- Pauli matrices orthogonality $\delta_\lambda^{\lambda'}\delta_{\bar{\lambda}}^{\bar{\lambda}'} = \sum_\mu \sigma^\mu_{\lambda\bar{\lambda}}\sigma_\mu^{\lambda'\bar{\lambda}'}$ completes condition for production/decay separation with τ spin states.

- **core formula of spin algorithms, wt is product of density matrices of production and decays**, $0 < wt < 4$, $< wt >= 1$ useful properties.

$$d\sigma = \Big(\sum_{spin} |\mathcal{M}^{prod}|^2\Big)\Big(\sum_{spin} |\mathcal{M}^{\tau^+}|^2\Big)\Big(\sum_{spin} |\mathcal{M}^{\tau^-}|^2\Big) wt\; d\Omega_{prod}\; d\Omega_{\tau^+}\; d\Omega_{\tau^-}$$

(C) How to add extra interactions:

Simplified kinematic for NP implementation. Cross section:

$$wt_{ME} = \Big(\sum_{spin} |\mathcal{M}^{prod\ SM+NP}|^2\Big)/\Big(\sum_{spin} |\mathcal{M}^{prod\ SM}|^2\Big)$$

Complicated is spin weight

$$wt_{spin} = \Big(\sum_{ij} R_{ij}^{SM+NP} h_+^i h_-^j\Big)/\Big(\sum_{ij} R_{ij}^{SM} h_+^i h_-^j\Big)$$

The R_{ij} depend on kinematic of τ-pair production, $h_\pm^i$ on $\tau^\pm$ decays.

Spin quantization frames orientation must be the same for production and decay.

We use KKMC routines to transfer $h_\pm^i$ to lab frame and another routines to transfer back to $\tau^\pm$ but oriented as in New Physics calculation.

In this way reference frames are OK and impact of (not too hard) photons is under control.

Solution works for all τ decay channels! After further tests: weight caluctation from event record only, and for hard photon events.

(C) How to add extra interactions:

a- magnetic dipole moment, b- electric dipole moment couplings.

$$
\begin{aligned}
R_{11} &= \frac{e^4}{4\gamma^2}\left(4\gamma^2\,\mathrm{Re}(a) + \gamma^2 + 1\right)\sin^2(\theta), \\
R_{12} &= -R_{21} = \frac{e^4}{2}\beta\sin^2(\theta)\,\mathrm{Re}(b), \\
R_{13} &= R_{31} = \frac{e^4}{4\gamma}\left[(\gamma^2+1)\mathrm{Re}(a) + 1\right]\sin(2\theta), \\
R_{22} &= -\frac{e^4}{4}\beta^2\sin^2(\theta), \\
R_{23} &= -R_{32} = -\frac{e^4}{4}\beta\,\gamma\sin(2\theta)\,\mathrm{Re}(b), \\
R_{33} &= \frac{e^4}{4\gamma^2}\left[\left(4\gamma^2\,\mathrm{Re}(a) + \gamma^2 + 1\right)\cos^2(\theta) + \beta^2\gamma^2\right], \\
R_{14} &= -R_{41} = \frac{e^4}{4}\beta\,\gamma\sin(2\theta)\,\mathrm{Im}(b), \\
R_{24} &= R_{42} = \frac{e^4}{4}\beta^2\,\gamma\sin(2\theta)\,\mathrm{Im}(a), \\
R_{34} &= -R_{43} = -\frac{e^4}{2}\beta\sin^2(\theta)\,\mathrm{Im}(b), \\
R_{44} &= \frac{e^4}{4\gamma^2}\left[4\gamma^2\,\mathrm{Re}(a) + \beta^2\gamma^2\cos^2(\theta) + \gamma^2 + 1\right].
\end{aligned}
\tag{1}
$$

Tests and example results

1) Check that formula/algorithm for spin effects can be used instead of the native KKMC one.

2) Same as -1) but for configurations when hard photons are requested to be present. Test of interpolation to bremsstrahlung configurations.

3) Both $\tau^\pm$ decay to $\pi^\pm\pi^0\nu$. Test distribution: acoplanarity of the visible decay products oriented half- planes. All momenta in the visible decay products system rest frame.

$$y_1 = \frac{E_{\pi^-} - E_{\pi^0}}{E_{\pi^-} + E_{\pi^0}}, \qquad y_2 = \frac{E_{\pi^+} - E_{\pi^0}}{E_{\pi^+} + E_{\pi^0}} \tag{2}$$

4) Observable does not rely on decay vertex position. In future, interesting extensions with $\tau \to 3\pi\nu$ decays, like for Higgs CP observables.

(C) How to add extra interactions:

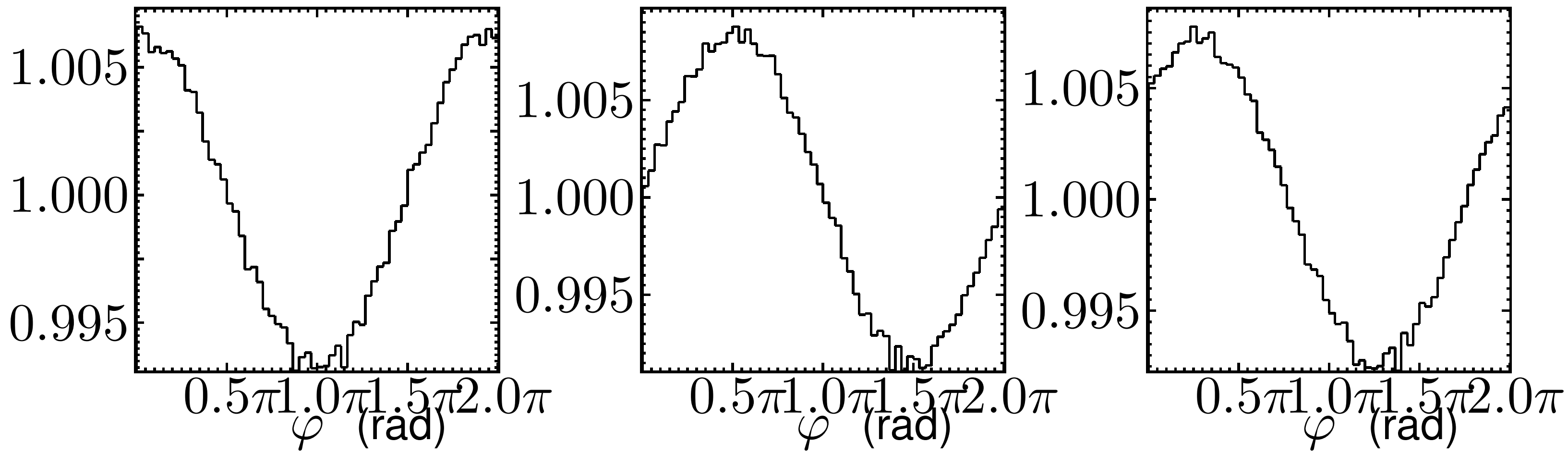


Figure 1: Distribution over acoplanarity angle φ of the ratio $wt^{anomalous}_{spin}$ for $\sqrt{s} = 10.5$ GeV. Constraint $y_1 y_2 > 0$ is imposed. Left: $\mathrm{Re}(a_{NP}) = 0.04$ and other couplings are zero, Center: $\mathrm{Re}(b_{NP}) = 0.04$ and other couplings are zero, Right: $\mathrm{Re}(a_{NP}) = 0.04\,\cos(\pi/4)$, $\mathrm{Re}(b_{NP}) = 0.04\,\sin(\pi/4)$ and other couplings are zero. This is idealized (test of the principle) observable. In practice Machine Learning approach, helpful to combine impact from all τ decay channels will be more appropriate.

- PHOTOS (by E.Barberio, B. van Eijk, Z. W., P. Golonka) is used since 1989 to simulate the effects of radiative corrections in decays.

Full events of complicated mother-daughter tree structure of consecutive decays are generated earlier. PHOTOS eventually modify decay (tree branching).

- Web pages of TAUOLA, PHOTOS and MC-TESTER projects:
- Phase-space is again exact and parametrization under full control
- Matrix element: from factorization and with simplifications. Required lots of work.
- For lepton pair emission algorithm works similarly.
- It can be used not only for QED but for New Physics too. Dark photon, extra scalar/pseudo-scalar imprinting into final state. New Physics particles with consecutive decays to lepton pairs.

Phase Space Formula of Photos

$$dLips_{n+1}(P \to k_1...k_n, k_{n+1}) = dLips_n^{+1\ tangent} \times W_n^{n+1},$$

$$dLips_n^{+1\ tangent} = dk_\gamma d\cos\theta d\phi \times dLips_n(P \to \bar{k}_1...\bar{k}_n),$$
$$\{k_1, \ldots, k_{n+1}\} = \mathbf{T}\big(k_\gamma, \theta, \phi, \{\bar{k}_1, \ldots, \bar{k}_n\}\big). \qquad (3)$$

1. One can verify that if $dLips_n(P)$ was exact, then this formula lead to exact parametrization of $dLips_{n+1}(P)$
2. Practical implementation: Take the configurations from n-body phase space.
3. Turn it back into some coordinate variables.
4. construct new kinematical configuration from all variables.
5. **Forget about temporary** $k_\gamma \theta \phi$. **From now on, only weight and four vectors count.**
6. A lot depend on $\mathbf{T}$. Options depend on matrix element: must tangent at singularities. Simultaneous use of several $\mathbf{T}$ is possible and necessary/convenient if more than one charge is present in final state.

Phase Space: (main formula)

If we choose

$$G_n \;:\; M^2_{2\ldots n}, \theta_1, \phi_1, M^2_{3\ldots n}, \theta_2, \phi_2, \ldots, \theta_{n-1}, \phi_{n-1} \;\to\; \bar{k}_1 \ldots \bar{k}_n \tag{4}$$

and

$$G_{n+1} \;:\; k_\gamma, \theta, \phi, M^2_{2\ldots n}, \theta_1, \phi_1, M^2_{3\ldots n}, \theta_2, \phi_2, \ldots, \theta_{n-1}, \phi_{n-1} \;\to\; k_1 \ldots k_n, k_{n+1} \tag{5}$$

then

$$\mathbf{T} = G_{n+1}(k_\gamma, \theta, \phi, G_n^{-1}(\bar{k}_1, \ldots, \bar{k}_n)). \tag{6}$$

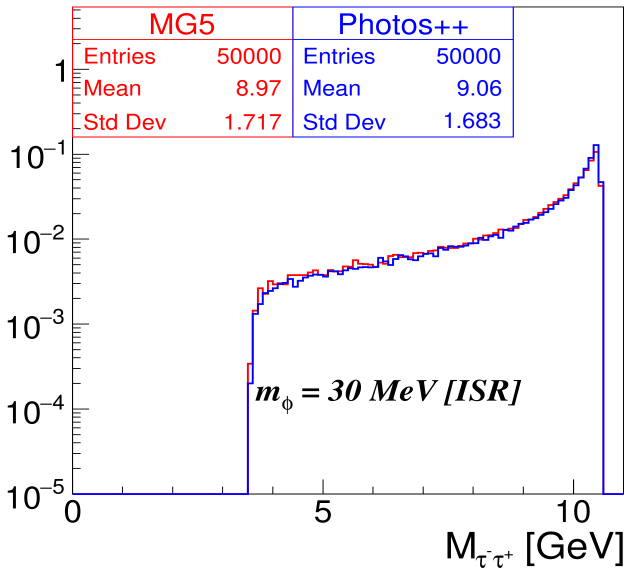
The ratio of the Jacobians form the phase space weight W_n^{n+1} for the transformation. Such solution is universal and valid for any choice of G's. However, G_{n+1} and G_n has to match matrix element, otherwise algorithm will be inefficient (factor 10^{10} ...).

In case of PHOTOS G_n's

$$W_n^{n+1} = k_\gamma \frac{1}{2(2\pi)^3} \times \frac{\lambda^{1/2}(1, m_1^2/M^2_{1\ldots n}, M^2_{2\ldots n}/M^2_{1\ldots n})}{\lambda^{1/2}(1, m_1^2/M^2, M^2_{2\ldots n}/M^2)}, \tag{7}$$

(D) Add extra particles

once phase-space adjusted, again $M^{SM} \to M^{SM+NP}$ is enough.



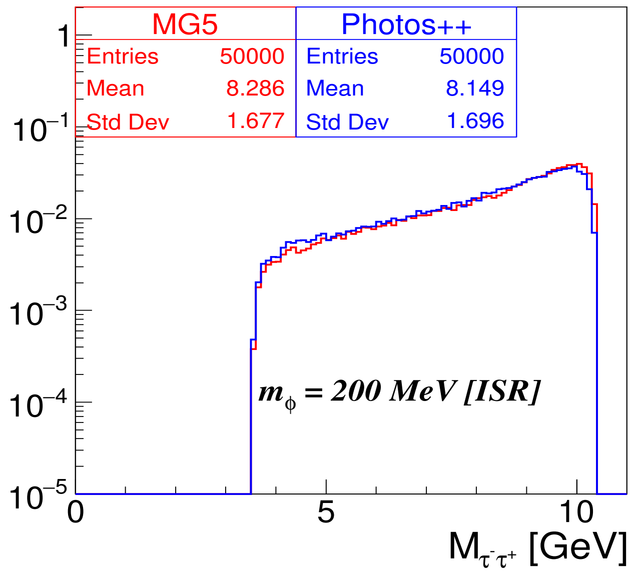


Figure 2: Belle 2 center of mass energies $e^-e^+ \to \tau^-\tau^+\phi_{\mathrm{Dark\ Scalar}}(\to e^-e^+)$ Case of dark scalar of 30 and 200 MeV. Simulation of KKMC+Photos is compared with the one based on MadGraph. Emission kernel was inspired from that comparison. At start, QED pair emission kernel was used. Spin correlations of τ-s modified by rotation of τ^- decay products.

(E) Outlook software side

- Motivation of FCC useful effort: 2021 2022 requests from Belle 2 collaboration.
- From FCC perspective: presented numerical results, just an illustration.
- Small New Physics effects on top of large SM background. Example and prototype?
- Correlated SM and SM+NP event wt-samples: useful for evaluation of ambiguities.
- Avoid time expense if new programs or even new program versions are needed.
  1. For electric/magnetic dipole moment KKMC internal functions for boost from τ rest-frame to lab frame and internal variables $h^i_\pm$ were used.
     No need to libraries recompilation, event sample not modified, just weights added.
  2. For dark photon emission, it was a bit more complicated. Events kinematic is then modified, but change could be introduced at fully constructed event and with the new program photospp. No need to change and test new version of the main code.
- It is important that some of the internal functions or data of main simulation program are not private but can be accessed by the user. Keep that in mind for C++ programs.

(E) Outlook: physics input

Theory side: why it could work.

- Required input. This aspect was not covered in my talk.
- Symmetries helped spin amplitudes separation into parts valid all over the phase space.
- In some cases comparisons with `MadGraph` simulation was used instead.
- It is also helpful for New Physics imprinting into precision Monte Carlo programs.
- My strategy for work:
  1. Calculate spin amplitudes by hand, using algebraic manipulation program to check.
  2. Identify (remaining) most singular terms, check if NP does not damage the patterns.
  3. Combine dominant terms into symmetry invariant set and if allowed iterate for higher orders.
  4. Continue work on remaining terms.
  5. Return to point 2, until nothing is left.
- I concentrated on applications for: Anomalous Magnetic/Electric dipole moment and dark scalar/pseudoscalar/photon imprinting into SM event sample.

(E) Outlook: physics input

- The way how amplitudes are calculated was important.
- Kleiss-Stirling techniques, numerical stability. How to use amplitudes when energy-momentum conservation is corrupted by further photons.
- For New Physics instead of formal work, approximations obtained from educative guess and validated by comparison with `MadGraph` simulations only, were used.
- For dipole moments interpolation to Born kinematic was used.
- **Now extend presented applications for other processes and other experiments like LHC, FCC, ... applications: effort is on-going**
- Implementation of New Physics processes into kinematic configurations with bremsstrahlung photons requires checks if (approximately) basic spin amplitudes structures remain, once New Physics effects are introduced.
- Statement *minor modification of the code* seem unimportant. But it is not. One has to introduce New Physics modification in such a way that interfaces to detector simulation and experimental validation remain valid.

(E) Skipped but critically important aspects

- Symmetries help separation of spin amplitudes into parts. Separation valid all over the phase space.
- Is important for Monte Carlo and calculations of multidimensional distributions:
- separate dominant SM predictions of sub-theories like QED (eikonal QED), QCD.
- Those are parts which need to be taken to higher orders.
- Calculation of small higher order terms present in full SM may be then avoided.
- Similarities between amplitudes (often identical dominant parts) of distinct processes, useful. Nothing new. Basis of of YFS exponentiation, factorization theorems. Also fake supersymmetric-like relations can be seen.
- All this is useful for technicalities of Monte Carlo constructions.
- Disadvantage: pressure on automated methods and higher orders validity proofs.
- Anyway: at LEP separation was indispensable, at LHC too.
- What future will decide? Keep these methods in tool-box.
- **Message: Do not drop the topic out.** It bring sometimes pain, but fun too.

(E) On work strategies

- My strategy was as follows:
  1. Calculate spin amplitudes by hand, using algebraic manipulation program to check for misprints.
  2. Identify most singular (or most peculiar terms).
  3. Localize terms which combine them into symmetry invariant set.
  4. Continue work on remaining terms.
  5. Return to point 2, until nothing is left..
- Starting point, choice of the way how amplitudes are calculated is important.
- But more often than not, I could identify required pattern.
- In fact pattern was usually of more details than what was needed.
- Think this talk as invitation for common efforts
- Thank you.

(E) On work strategies

• **I was talking about** new applications of `KKMC Tauola, photos`. That is OK but.

• How to keep expertise for future users/developers?

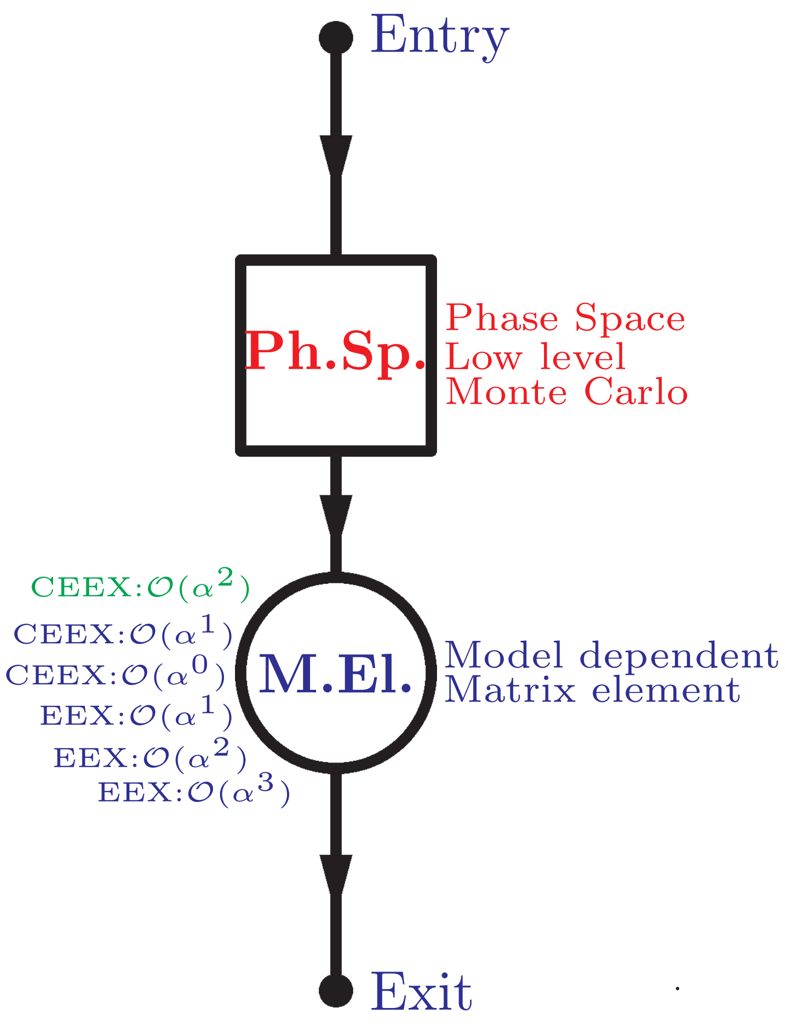
• How to get new people into projects development: challenge?

• Work with new people goes well for some time, but seem challenging to keep them for long, especially if perspective like that of FCC is in mind.

• Educated/trained people choose other careers; often outside the topic, outside Poland/Europe, outside physics.

(E) On work strategies

KKMC follow textbook principle "matrix element × full phase space"



- Phase-space Monte Carlo simulator is a module producing "raw events" (including importance sampling for possible intermediate resonances/singularities)
- Library of Matrix Elements; input for "model weight"; independent module
- This was used extensively for LEP precision Monte Carlos. It is true for KKMC as used in Belle collaboration for τ lepton pair production with decays and multiphoton radiation.
- Correlated samples techniques. Lots of technicalities collected in Phys. Rev. D41 (1990) 1425.
- Solutions useful for New Physics event weights!