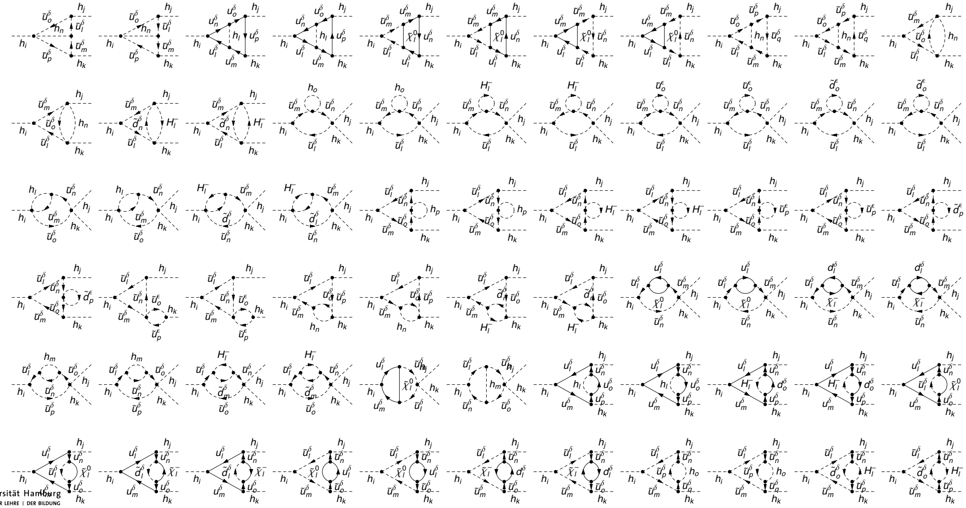
Recent developments in NMSSMCALC (and beyond)

λ_{hhh} (and beyond) in the NMSSM



Martin Gabelmann,
KUTS workshop
Feb. 2023

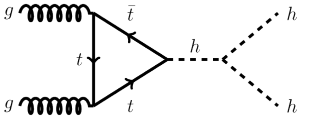
Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

DESY.

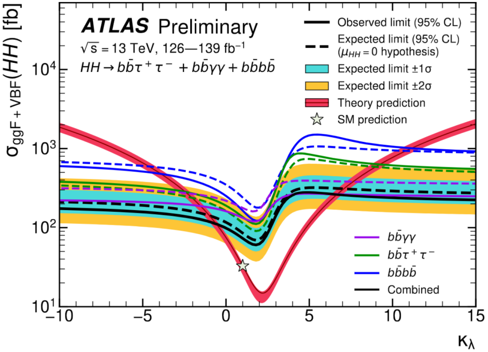
Why the trilinear self-coupling?

- probes electroweak symmetry breaking mechanism
- very sensitive to BSM
- important input for di-Higgs production
- important input for Higgs-to-Higgs decays
- important input for phase transitions

$$V_{SM} \supset \frac{m_h^2}{2} h^2 + \frac{3 m_h^2}{v} \kappa_\lambda h^3 + \frac{3 m_h^2}{v^2} \kappa_{2\lambda} h^4$$



$, \quad \kappa_\lambda^{SM} = 1$



[ATLAS-CONF-2022-050]

Outline

- λ_{hhh} in the SM and SUSY
- status in the NMSSM
- $\mathcal{O}(\alpha_t^2)$ corrections to λ_{hhh} and their phenomenology
- Higgs masses at $\mathcal{O}((\alpha_t + \alpha_\lambda + \alpha_\kappa)^2)$
- outlook

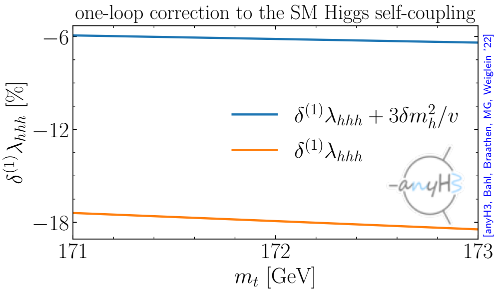
Getting started

In the SM at tree-level:

$$V(h) \supset \frac{m_h^2}{2} h^2 + \frac{m_h^2}{2v} h^3 + \dots \quad \Rightarrow \quad \lambda_{hhh}^{\text{SM}} = \frac{\partial^3 V(h)}{\partial^3 h} = \frac{3 m_h^2}{v}$$

Thus $\lambda_{hhh}^{\text{SM}}$ can be predicted perturbatively as a function of the SM parameters.

- corrections to λ_{hhh} are expected to behave similar to those of the Higgs boson mass
- OS scheme for m_h allows to "absorb" large part of corrections
- in SUSY:
  - $\lambda_{hhh} = 3m_h^2/v$ approximate [Dobado, Herrero, Hollik, Penaranda '02]
  - but m_h not free and $m_h \lesssim m_Z$ at tree-level!
    - $\rightarrow$ requires loop corrections of about 40 GeV (15-30%)
    - $\rightarrow$ can't stop at one-loop; need higher orders ($\rightarrow$KUTS)



$\rightarrow$ the precision of λ_{hhh} (order in perturbation theory) should match those of m_h!

The CP-Violating NMSSM

The Complex Next-to-Minimal Supersymmetric Standard Model

- Singlet extension of the MSSM.
- Theoretically well-motivated (solves μ- and little-hierarchy-problem).
- Rich phenomenology in the Higgs boson sector:

$$H_d = \begin{pmatrix} \frac{v_d + h_d + i a_d}{\sqrt{2}} \\ h_d^- \end{pmatrix}, \; H_u = e^{i\varphi_u} \begin{pmatrix} h_u^+ \\ \frac{v_u + h_u + i a_u}{\sqrt{2}} \end{pmatrix}, \; S = \frac{e^{i\varphi_s}}{\sqrt{2}} (v_S + h_s + i a_s)$$

mix to

$h_1, h_2, h_3, h_4, h_5, G^0$ (mass ordered) and $h^\pm, G^\pm$

- LHC measurements: h_1 **or** h_2 play the role of the Higgs boson h measured at LHC (h_1 or h_2 are "SM-like"). MSSM: no CPV at tree-level and always $h_1 = h$.

Available corrections for m_h and λ_{hhh} in the NMSSM

	Mass corrections δm_h	Coupling corrections $\delta\lambda_{hhh}$
full one-loop	✓ [lots of independent contributions]	✓ [Dao et al. '13]
two-loop $\mathcal{O}(\alpha_t\alpha_s)$	✓ [Dao et al. 14']	✓ [Dao et al. '15]
two-loop $\mathcal{O}(\alpha_t^2)$	✓ [Dao et al. '19]	✓ [Borschensky, Dao, MG, Mühlleitner, Rzehak '22]
two-loop $\mathcal{O}(\alpha_\lambda^2)$	✓ [Goodsell et al. '16] [Dao et al. '21]	×
full two-loop	×	×
three-loop $\mathcal{O}(\alpha_t\alpha_s^2)$	(✓ [Kant. et al. '10] [Reyes, Fazio '19])	×

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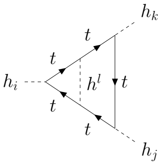
Goal

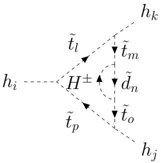
- catch-up on missing higher-orders in the trilinear Higgs couplings
- study their effect on
  - di-Higgs production
  - Higgs-to-Higgs decays (backup slides)
- and the interplay with the m_h-corrections

Calculation of the $\mathcal{O}(\alpha_t^2)$ corrections

- $\mathcal{O}(\alpha_t^2) \hat{=}$ all two-loop diagrams with top/stops and max. one Higgs d.o.f.

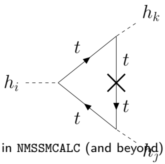


$\sim \mathcal{O}(\frac{m_t^6}{v^5})$

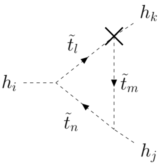


$\sim \mathcal{O}(\frac{A_t^5}{m_{\text{SUSY}}^4} \frac{m_t^5}{v^5})$

- approximations at two-loop:
  - MSSM limit: $\lambda, \kappa \to 0$, $\mu_{\text{eff}} = v_S \lambda / \sqrt{2} e^{i\varphi_S}$ const.
  - gaugeless limit: $g_1, g_2 \to 0$
  - vanishing external momenta: "effective coupling approach"
- OS/$\overline{\text{DR}}$ renormalization allowing to estimate part of the theory-uncertainty:



$\sim \delta^{(1)} m_t^{\text{OS}/\overline{\text{DR}}}$

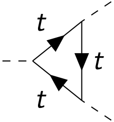


$\sim \delta^{(1)} A_t^{\text{OS}/\overline{\text{DR}}}$

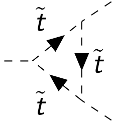
Renormalization

- MSSM- and gaugeless-limit $\rightarrow \lambda_{hhh}^{\text{tree-level}} = \delta\lambda_{hhh}^{\text{counter-terms}} = 0$
- example at $\mathcal{O}(\alpha_t)$

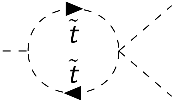
$\delta^{\mathcal{O}(\alpha_t)}\lambda_{hhh}^{\text{MSSM}} = 2\times$



$+2\times$



$+6\times$



$$\approx \frac{3}{32\pi^2}\frac{m_t^4}{v^3}\left(2 + 3\ln\frac{m_t^2}{m_{\tilde{t}_1} m_{\tilde{t}_2}}\right)$$

[Hollik, Penaranda '01]

- at $\mathcal{O}(\alpha_t^2)$: only top / stop and VEV sub-loop renormalization

$$\delta^{(1)} m_t^{\text{OS}/\overline{\text{DR}}}, \quad \delta^{(1)} m_{\tilde{t}_{1,2}}^{\text{OS}/\overline{\text{DR}}}, \quad \delta^{(1)} A_t^{\text{OS}/\overline{\text{DR}}}, \quad \delta^{(1)} Y_t(m_t^{\text{OS}/\overline{\text{DR}}}, v^{\text{OS}}), \quad \delta^{(1)} v^{\text{OS}}$$

$\rightarrow$ UV-finite result ✓

Implementation in NMSSMCALC [(see itp.kit.edu/∼maggie/NMSSMCALC for references)]

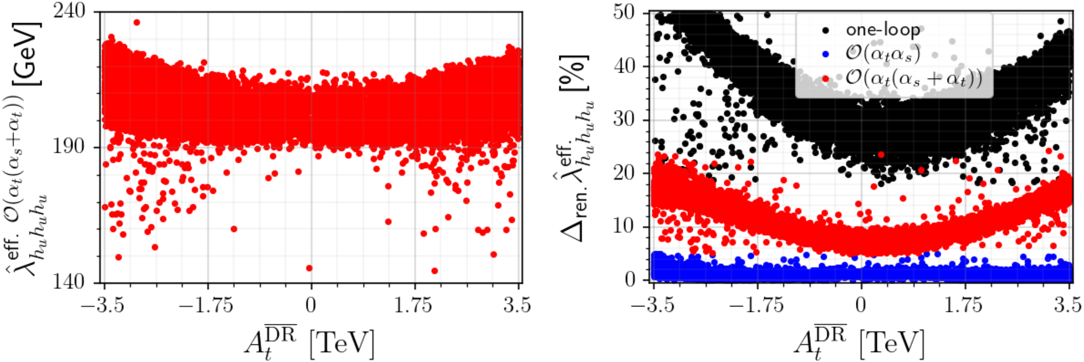
NMSSMCALC is more than a *spectrum generator* for the CP-violating NMSSM:

- takes parameter point using SLHA and calculates:
- Higgs boson masses ($m_{H_i^0}$ and $m_{H^\pm}$) up to two-loop $\mathcal{O}(\alpha_s\alpha_t + (\alpha_t + \alpha_\lambda + \alpha_\kappa)^2)$
- Higgs boson self-couplings $\lambda^{\text{eff}}_{hhh}$ up to two-loop $\mathcal{O}(\alpha_s\alpha_t + \alpha_t^2)$
- $H_i^{0,\pm} \to X_j X_k$ decays at NLO including SUSY QCD+EW corrections (or using $\lambda^{\text{eff}}_{hhh}$)
- electric dipole moments (EDMs) of e and various bound-states
- $(g-2)_e$ and $(g-2)_\mu$
- also for inverse seesaw scenario (NMSSMCALC-nuSS)
- code has been re-structured

example

```
wget https://www.itp.kit.edu/\~maggie/NMSSMCALC/nmssmcalc.tar
tar xf nmssmcalc.tar
cd nmssmcalc-C
make
./run inp.dat
```

Results

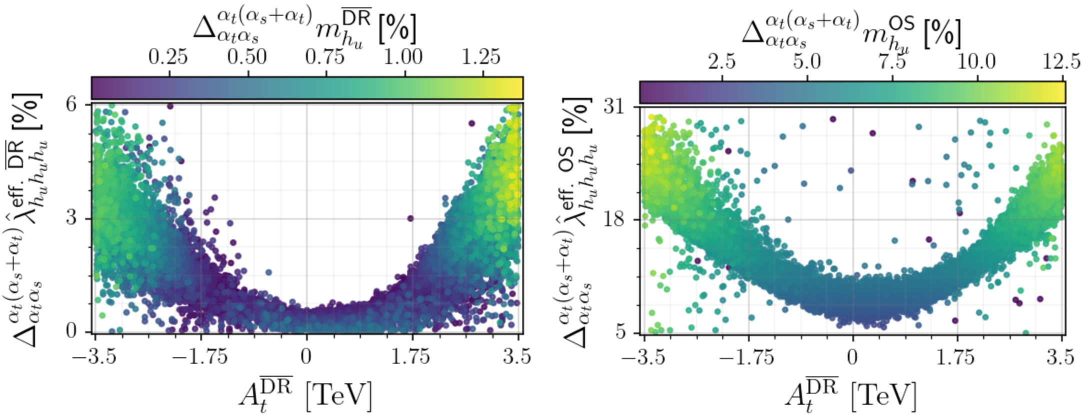


$$\Delta_{\text{ren.}}\lambda_{hhh} = \frac{\lambda_{hhh}(m_t^{\overline{\mathrm{DR}}},A_t^{\overline{\mathrm{DR}}}) - \lambda_{hhh}(m_t^{\mathrm{OS}},A_t^{\mathrm{OS}})}{\lambda_{hhh}(m_t^{\overline{\mathrm{DR}}},A_t^{\overline{\mathrm{DR}}})}$$ $\sim$ higher-orders $\rightarrow$ estimates theory uncertainty

(Points checked against `HiggsSignals 2.6.2` and `HiggsBounds 5.10.2` as well as model-independent constraints on SUSY masses.)

Size of the $\mathcal{O}(\alpha_t^2)$-corrections to λ_{hhh}

...and correlation to $\mathcal{O}(\alpha_t^2)$ m_h-corrections



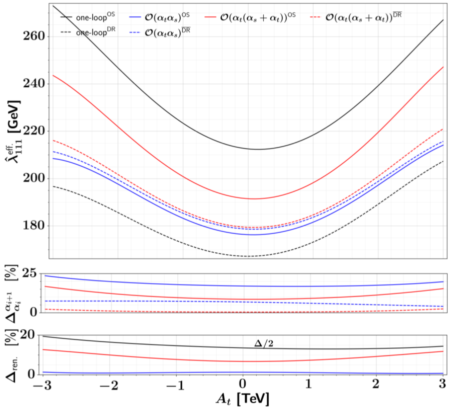
$\lambda^{\text{eff.}}$ in the mass- and gauge basis with small singlet admixture

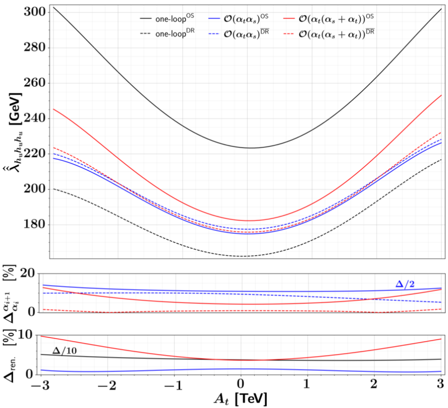
mass basis:

- $\hat{\lambda}^{\text{eff}}_{abc} = \mathcal{R}^{l,\text{eff}}_{an}\,\mathcal{R}^{l,\text{eff}}_{bm}\,\mathcal{R}^{l,\text{eff}}_{cq}\,\hat{\hat{\lambda}}_{nmq}$
- $\mathcal{R}^{l,\text{eff}}_{ij}$ diagonalizes the *loop-corrected* mass matrix at $\mathcal{O}(\alpha_i)$

gauge basis:

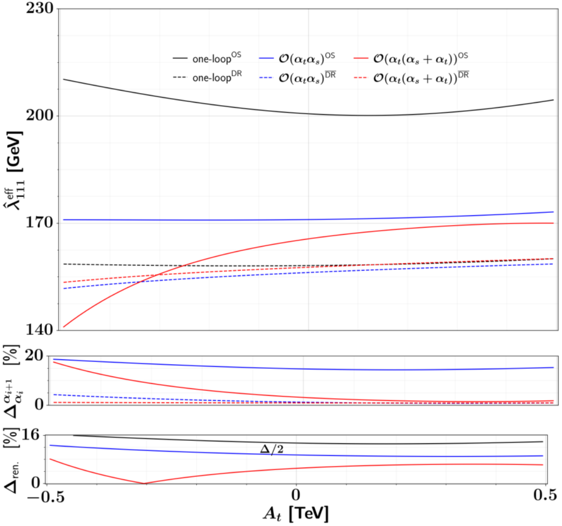
- $\hat{\hat{\lambda}}_{nmq} = \mathcal{R}^{G}_{ni}\mathcal{R}^{G}_{mj}\mathcal{R}^{G}_{qk}\hat{\lambda}_{ijk}$
- $\hat{\lambda} = \lambda + \Delta^{(1)}\lambda + \Delta^{\alpha_t\alpha_s}\lambda + \Delta^{\alpha_t^2}\lambda$
- $\mathcal{R}^{G}_{mj}$ singles-out the Goldstone modes



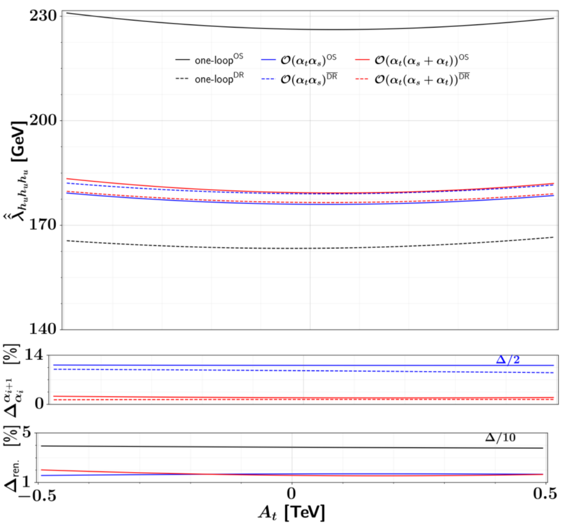


$\lambda^{\text{eff.}}$ in the mass- and gauge basis with large singlet admixture

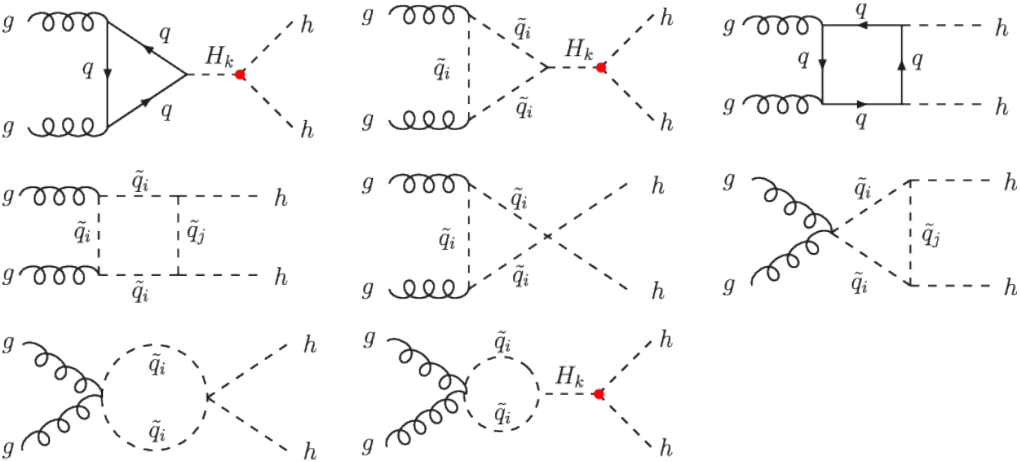
mass basis:



gauge basis:



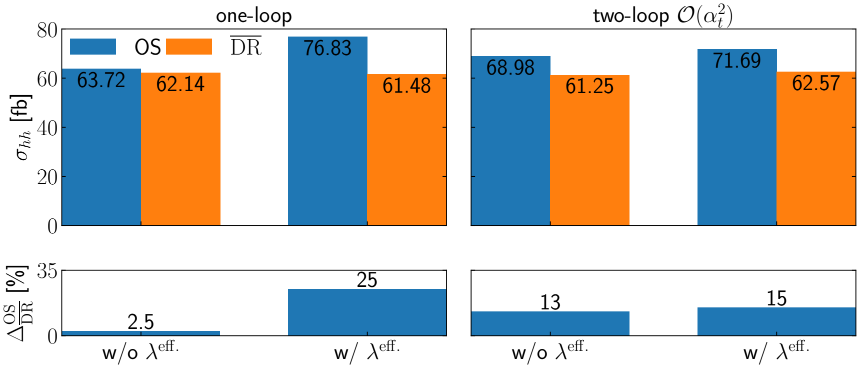
Double Higgs production



Use $\lambda_{hhh}^{\alpha_t^2}$ as input in HPAIR [Spira] to estimate higher-order effects in σ_{hh}.

Double Higgs production

Parameter point with resonant contribution from intermediate BSM Higgs:



- w/o $\lambda^{\text{eff.}}$: loop corrections to masses/mixing angles (and according LSZ-factors)
  $\to$ corrections to the input parameters
- w/ $\lambda^{\text{eff.}}$: additionally use effective coupling at respective order
  $\to$ corrections to the di-Higgs process

(intermediate) Summary

- predicted $\lambda_{h_ih_jh_k}$ in the CP-violation NMSSM at $\mathcal{O}(\alpha_t^2)$
- size of corrections between 0-30(6)% in the OS ($\overline{\text{DR}}$) scheme
- scheme-uncertainty on λ_{hhh} around 10%
- scheme-uncertainty on σ_{hh} reduced by a factor ~ 2 w.r.t one-loop but remains at 15%

trilinear self-coupling in SUSY models remains an interesting field of research:

- still need to catch-up on Higgs mass prediction
  - corrections from $Shh-$coupling
    $\rightarrow$ could further spoil relation between δm_h and $\delta\lambda_{hhh}$
  - corrections to $Shh-$coupling
    $\rightarrow$ enhancement of resonant di-Higgs in case of large singlet mixing

Higgs masses at $(\alpha_t + \alpha_\lambda + \alpha_\kappa)^2$ (slightly less recent)

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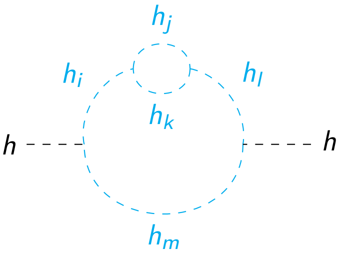
full two-loop	×	×
three-loop $\mathcal{O}(\alpha_t\alpha_s^2)$	(✓ [Kant. et al. '10] [Reyes, Fazio '19])	×

Additional difficulty: Goldstone boson catastrophe

→ Infra-red divergences due to simultaneous application of

- gaugeless limit and
- vanishing external momenta as well as
- having non-zero Higgs self-couplings ($\sim \lambda, \kappa$)

Comparison with MSSM-limit/previous results

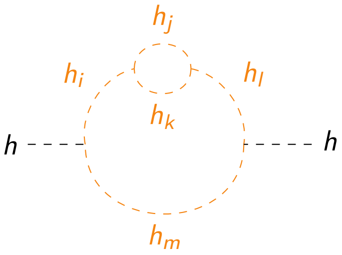


$\xrightarrow{g_1, g_2 \to 0} 0$

In the MSSM, Higgs self-couplings are given by gauge couplings:

$$V_{\mathrm{MSSM}}{}^{\mathrm{quartic}} \propto g_1{}^2(|H_u|^2 - |H_d|^2)^2 + g_2{}^2(H_u \sigma_a H_u + H_d \sigma_a H_d)^2 \xrightarrow{g_1, g_2 \to 0} 0$$

Comparison with MSSM-limit/previous results



$$\xrightarrow{g_1, g_2 \to 0} \neq 0$$

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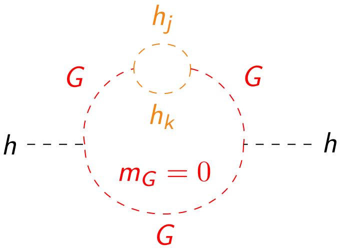
In the NMSSM, there are additional non-zero self-couplings:

$$V^{\mathrm{quartic}}_{\mathrm{NMSSM}} \propto V_{\mathrm{MSSM}}{}^{\mathrm{quartic}} + |\boldsymbol{\lambda} H_u H_d + \boldsymbol{\kappa} S^2|^2 \xrightarrow{g_1, g_2 \to 0} \boldsymbol{\neq 0}$$

$\rightarrow$ Many new **two-loop diagrams with Higgs self-couplings**.

Comparison with MSSM-limit/previous results



$$\propto \log p^2 \xrightarrow{p^2 \to 0} -\infty$$

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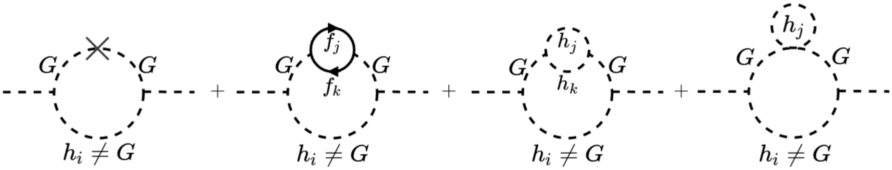
$\rightarrow$ Many new **two-loop diagrams with Higgs self-couplings**.
Massless Goldstones $\rightarrow$ appearance of IR divergences.

IR finite two-loop self-energies

Example of an IR-finite subset with intermediate IR-divergences:



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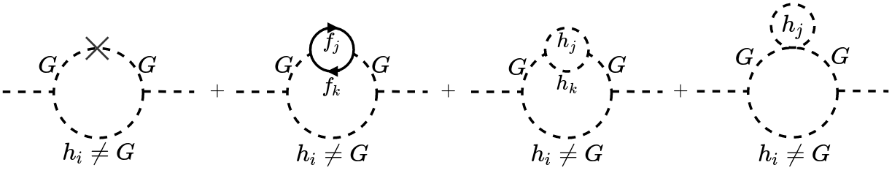
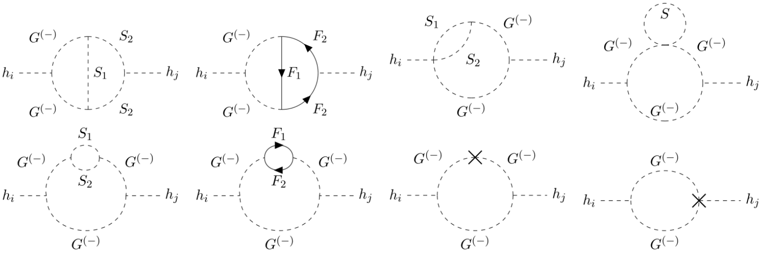
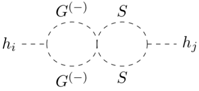


Careful isolation of divergences using mass regulator or dimensional regularisation shows:

- IR-divergence of first diagram cancels against the other three
- cancellation happens only if $M^{\text{1-loop}}_{\text{Goldstone}} \equiv 0$
- $\rightarrow$ working at the *tree-level* minimum is sufficient or alternatively using an OS-condition for the Goldstone mass [Braathen, Goodsell, '16]

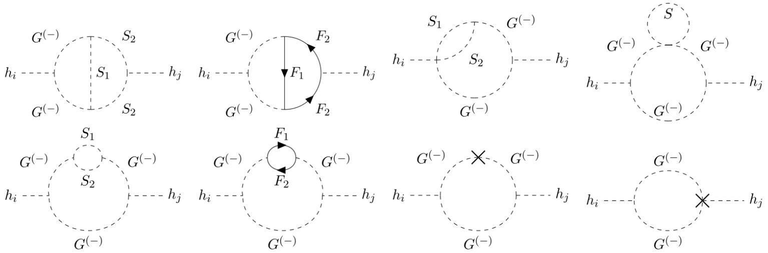
IR divergent two-loop self-energies

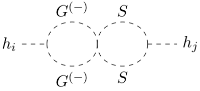




$$\{S_1, S_2\} = \{h, h\}, \{\tilde{t}, \tilde{t}\}(, \{\tilde{d}, \tilde{t}\}, \{h, h^-\}, \{h^-, h\})$$
$$\{F_1, F_2\} = \{\chi, \chi\}, \{t, t\}(, \{d, t\}, \{\chi, \chi^-\}, \{\chi^-, \chi\})$$
$$S = \{h, h^-, \tilde{t}, \tilde{d}\}$$

IR divergent two-loop self-energies





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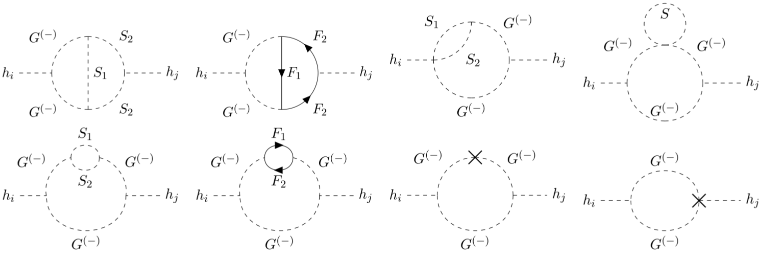
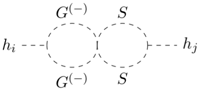
$$S = \{h, h^-, \tilde{t}, \tilde{d}\}$$

- $\propto \log p^2 \xrightarrow{p^2 \to 0} -\infty$
- $\rightarrow$ inclusion of finite external momentum required!

IR divergent two-loop self-energies

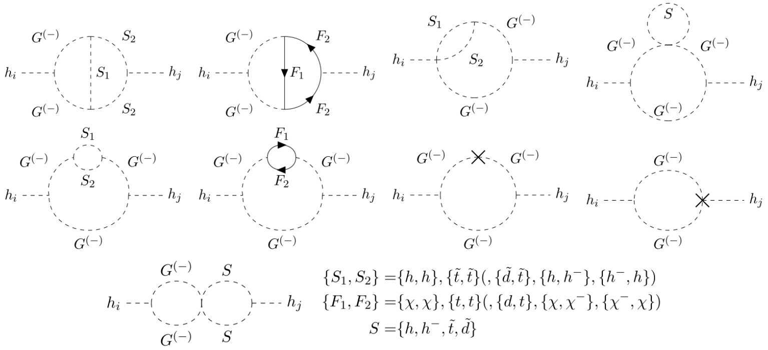




$$\{S_1, S_2\} = \{h, h\}, \{\tilde{t}, \tilde{t}\}(, \{\tilde{d}, \tilde{t}\}, \{h, h^-\}, \{h^-, h\})$$
$$\{F_1, F_2\} = \{\chi, \chi\}, \{t, t\}(, \{d, t\}, \{\chi, \chi^-\}, \{\chi^-, \chi\})$$
$$S = \{h, h^-, \tilde{t}, \tilde{d}\}$$

- $\propto \log p^2 \xrightarrow{p^2 \to 0} -\infty$
- $\to$ inclusion of finite external momentum required!
- **Solutions:**
  - assume $p^2 \neq 0 \to$ multi-scale problem (numerical integration required)

IR divergent two-loop self-energies



$$\{S_1, S_2\} = \{h,h\}, \{\tilde{t},\tilde{t}\}(,\{\tilde{d},\tilde{t}\},\{h,h^-\},\{h^-,h\})$$
$$\{F_1, F_2\} = \{\chi,\chi\}, \{t,t\}(,\{d,t\},\{\chi,\chi^-\},\{\chi^-,\chi\})$$
$$S = \{h, h^-, \tilde{t}, \tilde{d}\}$$

- $\propto \log p^2 \xrightarrow{p^2\to 0} -\infty$
- $\to$ inclusion of finite external momentum required!
- **Solutions:**
  - assume $p^2 \neq 0 \to$ multi-scale problem (numerical integration required)
  - use $M_{\text{Goldstone}} = 0 \to M_{\text{Regulator}}$; test if $\partial m_h / \partial M^2_{\text{Regulator}}$ is small

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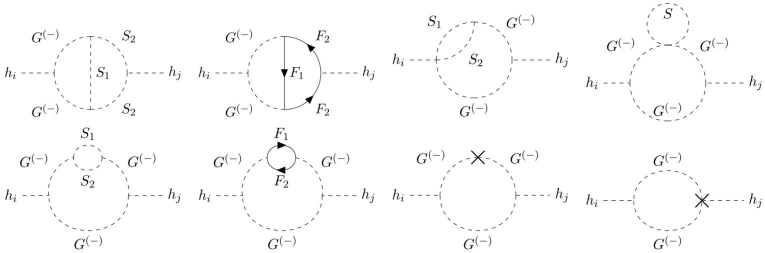

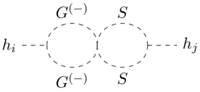

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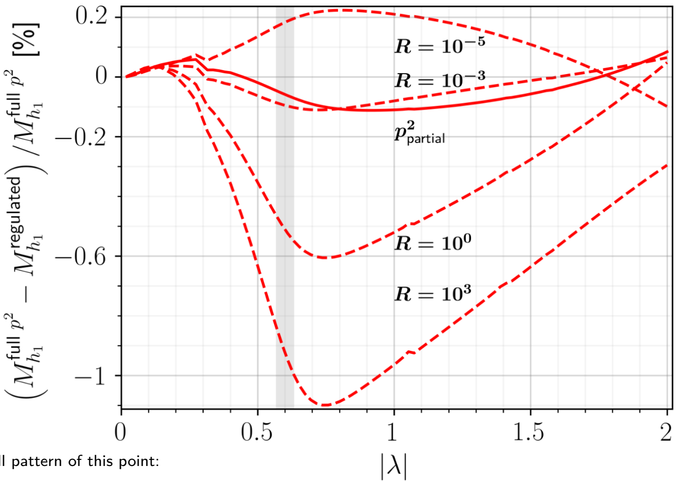
  - assume partial $p^2 \neq 0$; only in IR-divergent diagrams [Braathen, Goodsell, '16]
    $\rightarrow$ avoids numerical integration methods.

Example: momentum dependence

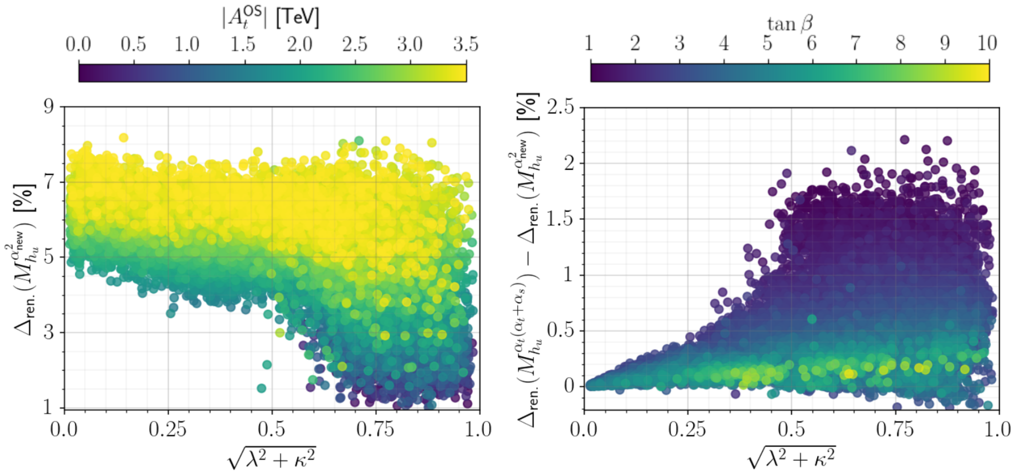
Compare **full-momentum** result with **partial-momentum** (solid) and **mass-regulator** (dashed, $M^2_{\text{Regulator}} = R\mu^2_{\text{Ren.}}$):



to be compared with the overall pattern of this point:

$$\underset{\Delta_{\text{ren.}}}{X} \xrightarrow{\Delta_Y^X} \underset{\Delta_{\text{ren.}}}{Y}: \quad \underset{0\%}{\text{tree-level}} \xrightarrow{25\text{-}50\%} \underset{16\%}{\text{one-loop}} \xrightarrow{5\text{-}10\%} \underset{0\text{-}1\%}{\mathcal{O}(\alpha_t\alpha_s)} \xrightarrow{0\text{-}5\%} \underset{3\text{-}4\%}{\mathcal{O}(\alpha_t(\alpha_t+\alpha_s))} \xrightarrow{0\text{-}1\%} \underset{3\text{-}4\%}{\mathcal{O}(\alpha^2_{\text{new}})}$$

(Scheme) Uncertainty



$\rightarrow$ uncertainty reduction w.r.t. previous results in NMSSM-specific corners

Outlook: beyond $m_h^{\text{fixed-order}}$ and λ_{hhh}

- m_h^{eff}
  - via pole-mass matching as well as $\lambda_{hhh}^{\text{SM, eff}}$
  - one-loop CPC: full agreement between both methods ✓
  - CPV and two-loop: ongoing

- M_W
  - $v^{\text{OS}} \equiv v(M_Z, M_W, \alpha)$
  - $\rightarrow$ vector boson self-energies up to $\mathcal{O}((\alpha_t + \alpha_\kappa + \alpha_\lambda)^2)$ already available in NMSSMCALC ✓
  - $\rightarrow$ can compute $\delta\rho$, which enters M_W, prediction at two-loops ✓
  - vertex/box diagrams at one-loop ✓

- code links NMSSMCALC $\rightarrow$ { micrOMEGAs, HiggsTools, SModelS, EVADE} for more complete pheno studies

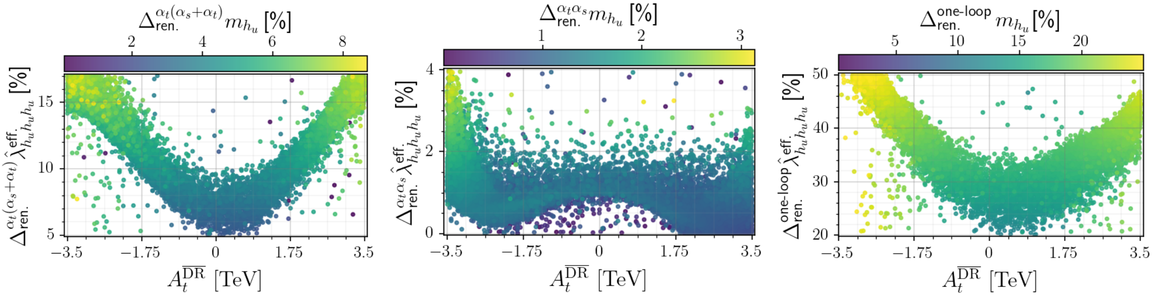
Summary

Progress in `NMSSMCALC` on many frontiers:

- m_h at $\mathcal{O}((\alpha_t + \alpha_\kappa + \alpha_\lambda)^2)$
  - efficient solutions to the GBC
  - reduced theory-uncertainty
- λ_{hhh} at $\mathcal{O}(\alpha_t^2)$
  - necessary input for di-Higgs
    - scheme-uncertainty: cancellation between parameter- and process-corrections

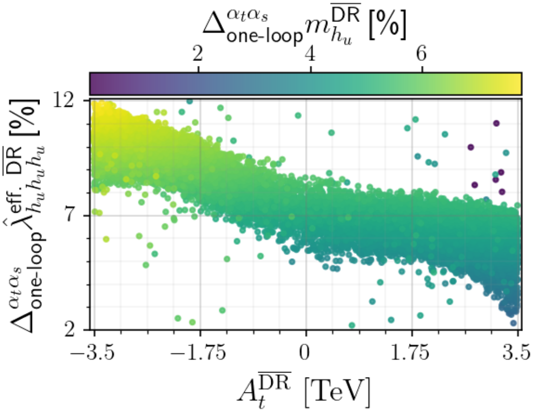
Backup

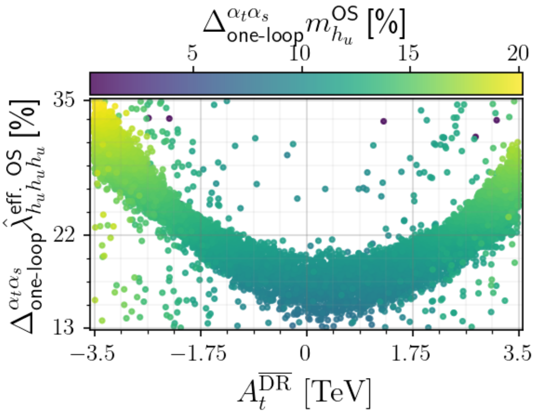
λ_{hhh}: detailed scheme-uncertainty plots



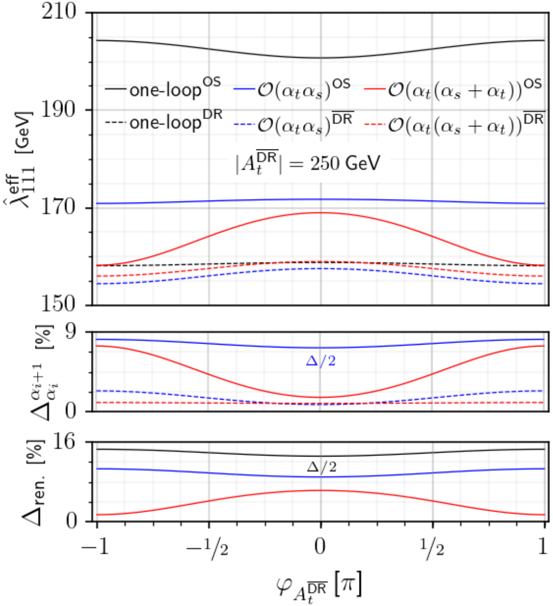
λ_{hhh}: size of the $\mathcal{O}(\alpha_t\alpha_s)$-corrections

...and correlation to $\mathcal{O}(\alpha_t^2)$ m_h-corrections

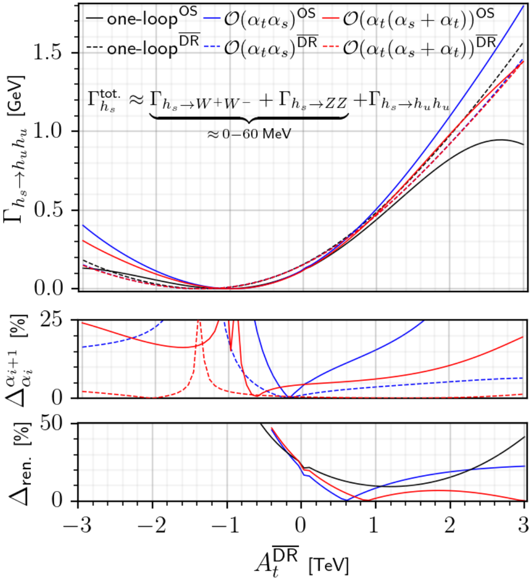




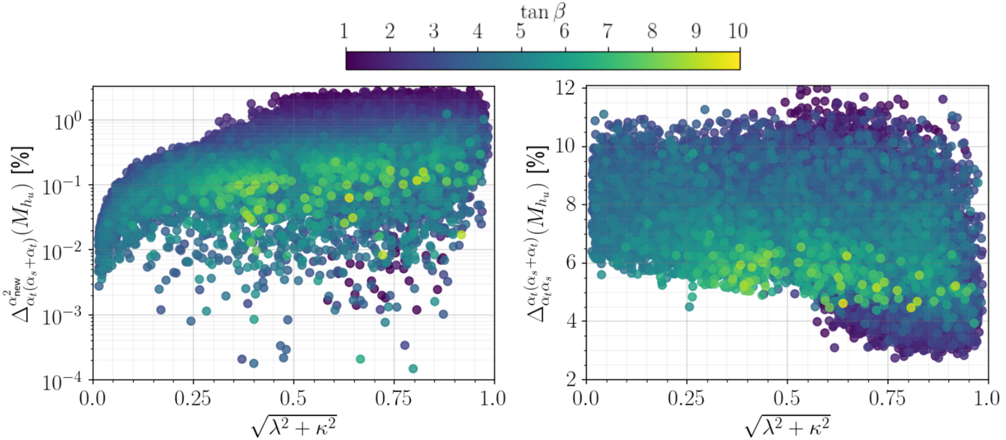
λ_{hhh}: CP-violation



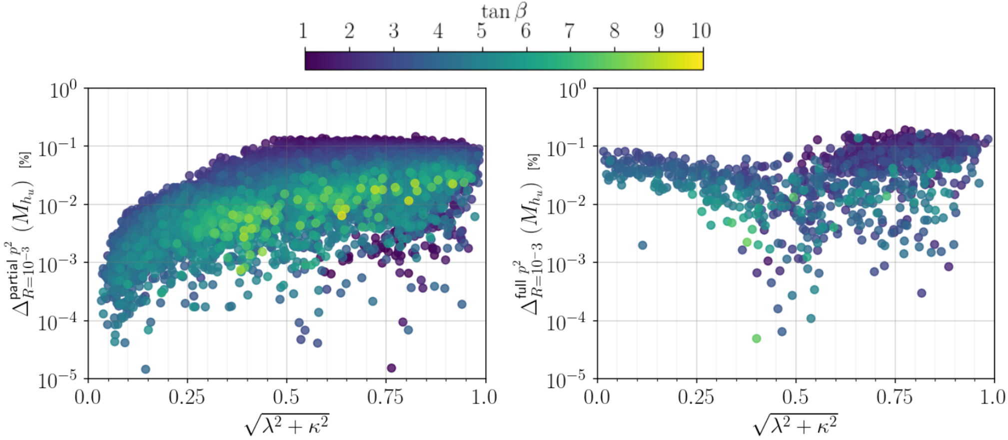
λ_{hhh}: influence on $h_s \to h_u h_u$ decay (same BP as for di-Higgs example)



m_h: size of new two-loop corrections



m_h: size of momentum-dependent corrections



Inputs for shown benchmark points

BP10 (low singlet-mixing):

$$\begin{aligned}
|\lambda| &= 0.65\,,\ |\kappa| = 0.65\,,\ \mathrm{Re}(A_\kappa) = -432\ \mathrm{GeV}\,,\ |\mu_{\mathrm{eff}}| = 225\ \mathrm{GeV}\,,\ \tan\beta = 2.6\,,\\
M_{H^\pm} &= 611\ \mathrm{GeV}\,,\ m_{\tilde{Q}_3} = 1304\ \mathrm{GeV}\,,\ m_{\tilde{t}_R} = 1576\ \mathrm{GeV}\,,\ m_{\tilde{X}\neq\tilde{Q}_3,\tilde{t}_R} = 3\ \mathrm{TeV}\,,\\
A_t &= 46\ \mathrm{GeV}\,,\ A_b = -1790\ \mathrm{GeV}\,,\ A_\tau = -93\ \mathrm{GeV}\,,\ A_c = 267\ \mathrm{GeV}\,,\\
A_s &= -618\ \mathrm{GeV}\,,\ A_\mu = 1851\ \mathrm{GeV}\,,\ A_u = -59\ \mathrm{GeV}\,,\ A_d = -175\ \mathrm{GeV}\,,\\
A_e &= 1600\ \mathrm{GeV}\,,\ |M_1| = 810\ \mathrm{GeV}\,,\ |M_2| = 642\ \mathrm{GeV}\,,\ M_3 = 2\ \mathrm{TeV}\,.
\end{aligned}$$

P2OS (large singlet-mixing):

$$\begin{aligned}
|\lambda| &= 0.59\,,\ |\kappa| = 0.23\,,\ \mathrm{Re}(A_\kappa) = -546\ \mathrm{GeV}\,,\ |\mu_{\mathrm{eff}}| = 397\ \mathrm{GeV}\,,\ \tan\beta = 2.05\,,\\
M_{H^\pm} &= 922\ \mathrm{GeV}\,,\ m_{\tilde{Q}_3} = 1.2\ \mathrm{TeV}\,,\ m_{\tilde{t}_R} = 1.37\ \mathrm{TeV}\,,\ m_{\tilde{X}\neq\tilde{Q}_3,\tilde{t}_R} = 3\ \mathrm{TeV}\,, \qquad (37)\\
A_t &= -911\ \mathrm{GeV}\,,\ A_{i\neq t,\kappa} = 0\ \mathrm{GeV}\,,\ |M_1| = 656\ \mathrm{GeV}\,,\ |M_2| = 679\ \mathrm{GeV}\,,\ M_3 = 2\ \mathrm{TeV}\,.
\end{aligned}$$

Scan ranges

parameter	scan range [TeV]
$M_{H^\pm}$	[0.5, 1]
M_1, M_2	[0.4, 1]
M_3	2
μ_{eff}	[0.1, 1]
$m_{\tilde{Q}_3}, m_{\tilde{t}_R}$	[0.4, 3]
$m_{\tilde{X}\neq\tilde{Q}_3,\tilde{t}_R}$	3

parameter	scan range
$\tan\beta$	[1, 10]
λ	[0.01, 0.7]
κ	$\lambda\cdot\xi$
ξ	[0.1, 1.5]
A_t	$[-3.5, 3.5]$ TeV
$A_{i\neq t}$	$[-2, 2]$ TeV

We neglected parameter points with any of the following mass configurations:

$$(i)\quad m_{\chi_i^{(\pm)}}, m_{h_i} > 1\,\text{TeV},\ m_{\tilde{t}_2} > 2\,\text{TeV}$$

$$(ii)\quad m_{h_i} - m_{h_j} < 0.1\,\text{GeV},\ m_{\chi_i^{(\pm)}} - m_{\chi_j^{(\pm)}} < 0.1\,\text{GeV}$$

$$(iii)\quad m_{\chi_1^\pm} < 94\,\text{GeV},\ m_{\tilde{t}_1} < 1\,\text{TeV}\ .$$

Partial resummation of large logs: treatment of m_t

In the renormalization of the top quark pole mass:

$$m_t^{\text{OS}} \equiv m_t^{\text{pole, NMSSM}} = m_t^{\text{OS, NMSSM}} - \Sigma_{tt} + \delta m_t^{\text{OS}}$$

large logs $\ln {}^{m_t}/_{Q_{\text{ren.}}}$ can lead to artificial large dependence on $Q_{\text{ren.}} \sim m_{\text{SUSY}}$.
This kind of log can be resummed in the following way:

$$\begin{aligned} m_t^{\text{OS}} \equiv m_t^{\text{pole, SM}} &\xrightarrow{\text{conversion}} m_t^{\overline{\text{MS}}\text{, SM}}(m_t) \\ &\xrightarrow{\text{SM RGEs}} m_t^{\overline{\text{MS}}\text{, SM}}(m_{\text{SUSY}}) \\ &\xrightarrow{\text{conversion}} m_t^{\overline{\text{DR}}\text{, SM}}(m_{\text{SUSY}}) \\ &\xrightarrow{\text{SUSY thresholds}} m_t^{\overline{\text{DR}}\text{, NMSSM}}(m_{\text{SUSY}}) \end{aligned}$$

Such that $m_t^{\overline{\text{DR}}\text{, NMSSM}}$ (and consequently $\lambda_{hhh}(m_t^{\overline{\text{DR}}\text{, NMSSM}})$) is free of such large logs.

Renormalization in NMSSMCALC

In NMSSMCALCthe Higgs sector can be renormalized as follows

$$\underbrace{t_{h_d}, t_{h_u}, t_{h_s}, t_{a_d}, t_{a_s}, M_{H^\pm}^2, v, s_{\theta_W}}_{\text{on-shell scheme}}, \underbrace{\tan\beta, |\lambda|, v_s, |\kappa|, \text{Re}A_\kappa}_{\overline{\text{DR}}\text{ scheme}}, \qquad (1)$$

in case $M_{H^\pm}^2$ is used as independent input, or

$$\underbrace{t_{h_d}, t_{h_u}, t_{h_s}, t_{a_d}, t_{a_s}, v, s_{\theta_W}}_{\text{on-shell scheme}}, \underbrace{\tan\beta, |\lambda|, v_s, |\kappa|, \text{Re}A_\lambda, \text{Re}A_\kappa}_{\overline{\text{DR}}\text{ scheme}}, \qquad (2)$$

for $\text{Re}A_\lambda$ as independent input.
In the squark/top sector the OS and optionally the $\overline{\text{DR}}$ renormalisation scheme is used for:

$$m_t,\ m_{\tilde{Q}_3},\ m_{\tilde{t}_R} \quad \text{and} \quad A_t\ .$$

Two-Loop Diagrammatic n-Point Functions ($n \leq 3\,(4)$)

Idea: calculate "generic" diagrams
Assume most general Lorentz-invariant couplings and arbitrary masses.
Calculate to "robust form" and perform specific field-insertions later-on.

Strategy:

- FeynArts: generate generic diagrams *("InsertionLevel→{Generic}")* [Hahn, '01]
- FeynCalc: basic simplifications, Dirac traces [Shtabovenko, '16]
- TARCER: reduction to scalar master integrals [Tarasov, '97] [Mertig, Scharf, '98]
- handle special cases such as vanishing Gram determinants etc.

Then:

- NMSSM FeynArts model file with SARAH [Staub, '08]:
  calculates LO-vertices and NLO-CT-vertices
- generate arbitrary set of diagrams with FeynArts *("InsertionLevel→{Classes or Particles}")*
- iterate over generic amplitudes while applying insertion rules
- evaluate numerics ← `NMSSMCALC`