# Higgs-mass prediction in the NMSSM with heavy BSM particles


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# Detour: how we got there



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If, on the other hand, any significant deviations from the predictions of the SM are detected (e.g., with definitive confirmations of the lepton-flavor anomalies recently observed by LHCb, or of the long-standing $(g-2)_\mu$ anomaly) and/or any BSM particles are discovered, future investigations on the theory side will obviously focus on the classes of models that can accommodate the observed phenomenology. If SUSY models belong to this class, the techniques and results discussed in this report will be crucial to obtain accurate predictions for the Higgs masses, and the case for improving the calculations until the theory uncertainty matches the experimental accuracy of the Higgs-mass measurement will be even stronger.

# Detour: how we got there



Despite all of these developments, the quest for high-precision predictions for the Higgs masses in SUSY models is not by any means concluded. As described at the end of Sects. 3–6 of this report, efforts aimed at improving and extending the calculations, and KUTS meetings to discuss them, are bound to continue. An obvious question in this context is what should be the target for the precision of the theoretical predictions. Ideally, to fully exploit the potential of the Higgs-mass measurement in constraining the parameter space of SUSY models, one would need to bring the theory uncertainty of the prediction for the mass of the SM-like Higgs boson below the level of the current (and future) experimental precision. This would however require a reduction of the theory uncertainty by more than a factor of 10 compared to the current level, which is probably too ambitious a target in the medium term. A more realistic goal is that, in the coming years, the theory uncertainty of the Higgs-mass prediction be reduced by a factor of about $2-3$, down to the level of the current "parametric" uncertainty that stems from the experimental uncertainty with which the SM input parameters are known. As discussed in Sect. 6.4, the dominant contribution to this parametric uncertainty comes from the value of the top mass measured at hadron colliders, whose relation with the theoretically well-defined top-mass parameter that is needed as input for the Higgs-mass calculation is an additional source of uncertainty. It is therefore unlikely that further reductions of the theory uncertainty would bring substantial benefits, at least until an improved measurement of the top mass, e.g. at future $e^+e^-$ colliders, reduces the associated parametric uncertainty down to the level of the experimental precision of the Higgs-mass measurement.

The urgency of further improvements in the accuracy of the Higgs-mass predictions in SUSY models will also depend on the experimental developments concerning the properties of the observed Higgs boson, the electroweak precision observables and the direct searches for BSM particles. In the MSSM, the minimal values of the stop masses that lead to a prediction for the Higgs mass compatible with the measured value lie typically above the current bounds from direct stop searches at the LHC. Therefore, the scenario of a SM-like Higgs boson with mass around 125 GeV and no hints for additional particles from direct BSM searches can still be considered a fully consistent realization of the MSSM. If no deviation from the SM is detected in the coming years, a SUSY model with superparticle masses beyond the kinematic reach of the LHC - or even of future hadron colliders such as the FCC-hh - will continue to be a viable possibility (one could invoke fine-tuning arguments to favor or disfavor certain classes of models). In this case, the requirement that the prediction for the mass of the SM-like Higgs boson agree – within the uncertainties – with the measured value will place constraints on the multi-dimensional space of experimentally inaccessible parameters of the considered model. For example, as can be inferred from Fig. 1 in Sect. 2.1, a lower bound on the stop masses of $\mathcal{O}(10\ \text{TeV})$ from future searches at the FCC-hh would constrain the region of the MSSM parameter space with large $\tan\beta$, which is consistent with a possible SUSY explanation of the $(g-2)_\mu$ anomaly. We also remark that, in the absence of new discoveries, the benefits of any possible improvement in the calculation of the Higgs masses will have to be assessed on a case-by-case basis. For example, in the simplified MSSM scenario with a common SUSY scale and a fixed value of $\tan\beta$, the correlation between $M_S$ and $X_t$ discussed in Sect. 2.1 (see Fig. 2 there) illustrates the ultimate sensitivity on the unknown SUSY parameters that could be reached in the idealized situation where experimental and theory uncertainties are negligible. Even in that idealized situation, however, all correlations get blurred when more SUSY parameters are allowed to vary or non-minimal models are considered.

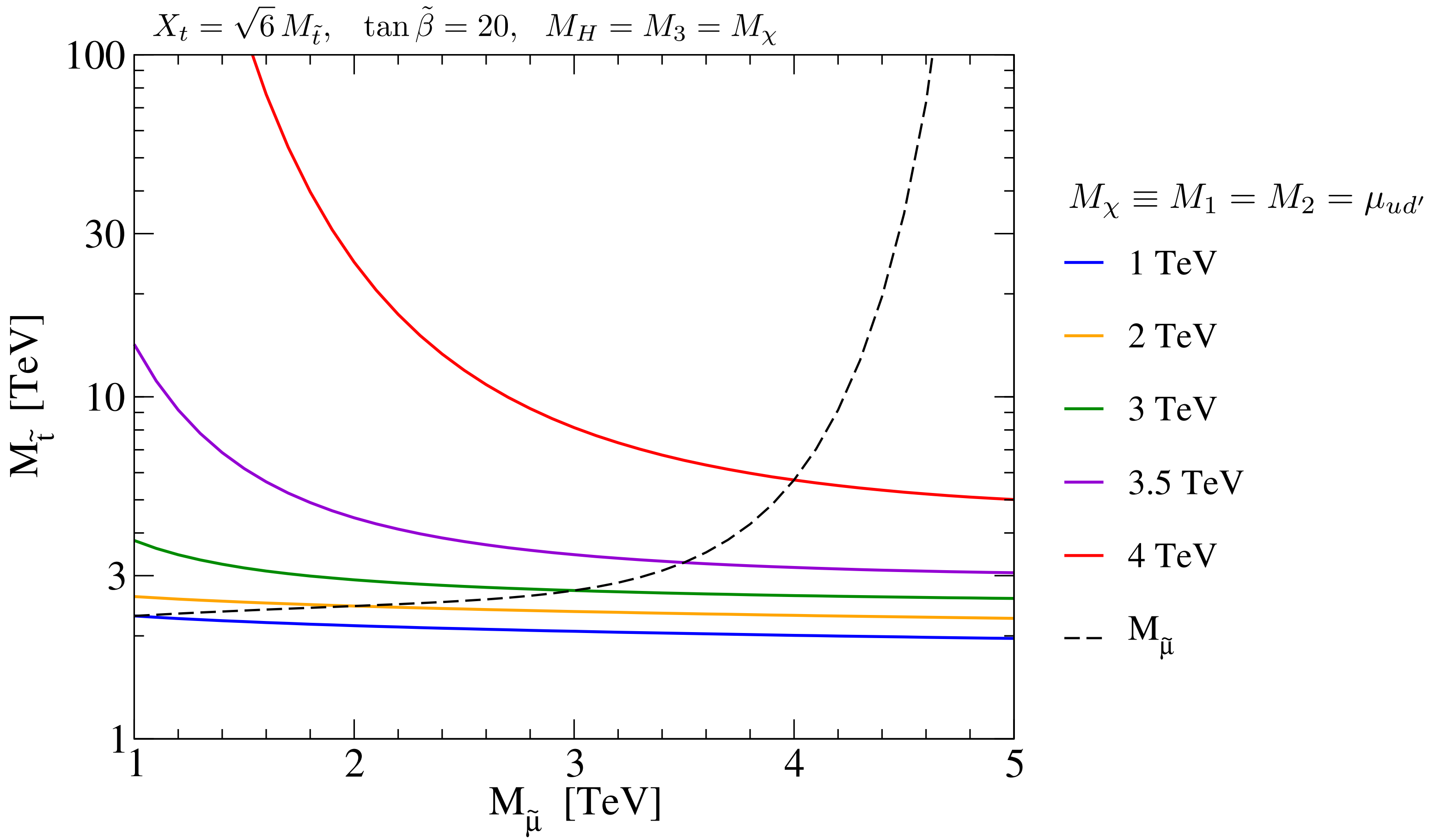
If, on the other hand, any significant deviations from the predictions of the SM are detected (e.g., with definitive confirmations of the lepton-flavor anomalies recently observed by LHCb, or of the long-standing $(g-2)_\mu$ anomaly) and/or any BSM particles are discovered, future investigations on the theory side will obviously focus on the classes of models that can accommodate the observed phenomenology. If SUSY models belong to this class, the techniques and results discussed in this report will be crucial to obtain accurate predictions for the Higgs masses, and the case for improving the calculations until the theory uncertainty matches the experimental accuracy of the Higgs-mass measurement will be even stronger. It is also worth stressing that, while these techniques were developed in the context of SUSY, they can be applied more generally to any BSM model that involves some kind of prediction for the quartic scalar couplings. In combination with direct experimental evidence for BSM physics, the Higgs-mass predictions will be a powerful tool for unraveling the nature of the new phenomena.

**Acknowledgements** The authors of this report wish to thank all of the colleagues who, directly or indirectly, contributed over the past seven years to the progress of the Higgs-mass calculations in SUSY models and to the success of the KUTS initiative. The work of P.S. and M.Go. has been supported in part by the French "Agence Nationale de la Recherche" (ANR), in the context of the LABEX ILP (ANR-11-IDEX-0004-02, ANR-10-LABX-63) and of the grant "HiggsAutomator" (ANR-15-CE31-0002). The work of S.H. has been supported in part by the MEINCOP (Spain) under contract FPA2016-78022-P and under contract PID2019-110058GB-C21, in part by the Spanish Agencia Estatal de Investigación (AEI), in part by the EU Fondo Europeo de Desarrollo Regional (FEDER) through the project FPA2016-78645-P, in part by the "Spanish Red Consolider MultiDark" FPA2017-90566-REDC, and in part by the AEI through the grant IFT Centro de Excelencia Severo Ochoa SEV-2016-0597. H.B., T.B., J.B., P.D., D.M., I.S. and G.W. acknowledge support by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany's Excellence Strategy – EXC 2121 "Quantum Universe" – 390833306. The work of H.E.H. has been partially supported by the U.S. Department of Energy Grant DE-SC0010107. The work of R.H., M.M., A.V.,

# Interplay of $(g\text{-}2)_\mu$ and Higgs-mass prediction in the FSSM *

W. Ke and P.S., 2109.15277

$$\Delta a_\mu \;\approx\; \hat{y}'_\mu \, \frac{m_\mu v_u}{M^2_{\tilde{\mu}}} \, \frac{g^2(g'^{\,2})}{192\pi^2} \, F\left(\frac{M_\chi}{M_{\tilde{\mu}}}\right) \;=\; 251 \times 10^{-11} \qquad\qquad \Delta\lambda^{\tilde{\mu}} \;\approx\; -\frac{\hat{y}'^{\,4}_\mu}{96\pi^2}\left(\frac{M_\chi}{M_{\tilde{\mu}}}\right)^4$$




*Heavier stops are needed in the FSSM for large $y_\mu$ or large $M_\chi/M_{\tilde{\mu}}$*

* SUSY 4HDM introduced in Altmannshofer *et al.*, 2104.08293

# Full one-loop calculation of $\Delta\lambda$ in the FSSM

The contributions from sfermions are analogous to the MSSM, but those from Higgs bosons and from higgsinos/gauginos are much more complicated (4HDM!)

We applied to the FSSM the general results of Braathen, Goodsell & P.S., 1810.09388

E.g. for real-scalar interactions: $\mathcal{L}_S \;=\; -\,\frac{1}{6} a_{ijk}\,\Phi_i\Phi_j\Phi_k - \frac{1}{24}\lambda_{ijkl}\,\Phi_i\Phi_j\Phi_k\Phi_l$

$$
\begin{aligned}
(4\pi^2)\,\delta\lambda_{ijkl} \quad \supset \quad & \frac{1}{16}\,\lambda_{ijxy}\lambda_{klxy}\,P_{SS}(m_x^2, m_y^2) \\
& +\frac{1}{4}\,\lambda_{ijxy}a_{kyz}a_{lzx}\,C_0(m_x^2, m_y^2, m_z^2) \\
& -\frac{1}{8}\,a_{ixy}a_{jyz}a_{kzu}a_{lux}\,D_0(m_x^2, m_y^2, m_z^2, m_u^2) \\
& -\frac{1}{24}\,a_{ixy}a_{rxy}\lambda_{rjkl}\,B'(0; m_x^2, m_y^2) \quad + \quad (ijkl)
\end{aligned}
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(just adapt the general notation to your model; saves the trouble of computing diagrams)

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(just adapt the general notation to your model; saves the trouble of computing diagrams)

**WAIT... THAT'S SO EASY!!!**

# EFT calculation of the SM-like Higgs mass in the NMSSM

*Considering the simplest scenario where the EFT valid below the SUSY scale is just the SM*

Status of the MSSM calculation: *full NLL, partial NNLL* [1407.4081, 1703.08166, 1908.01670] *and partial $N^3LL$* [1807.03509]

- SUSY-scale boundary conditions: full 1-loop + full 2-loop QCD *[including mixed QCD-EW]*
  + 2-loop-Yukawa in the "gaugeless limit"
  + 3-loop of $O(g_t^4\, g_s^4)$ *[assuming hierarchies…]*
- Evolution between the SUSY and EW scales: full 3-loop RGE of the SM
  + 4-loop of $O(g_t^4\, g_s^6)$ for $\lambda$
- EW-scale (=SM) boundary conditions: full 2-loop relation between $\lambda$ and $M_h$
  + 3-loop terms of $O(g_t^4\, g_s^4\, v^2)$

SUSY-scale boundary conditions for the NMSSM calculation:

- FlexibleEFT (numerical) approach [1609.00371, 1710.03760, 2003.04639]
  (also SARAH/SPheno [1703.03267], but only LL )
- Analytical results: only 1-loop in a very constrained scenario [1810.09388]
  *(all BSM masses depending on just one parameter)*

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Can we do any better?

# The Higgs sector of the NMSSM with heavy BSM particles

$$W \supset -\lambda\, \hat{S}\, \hat{H}_1 \hat{H}_2 + \frac{\kappa}{3}\, \hat{S}^3 \,,$$

$$-\mathcal{L}_{\rm soft} \supset m^2_{H_1} H_1^\dagger H_1 + m^2_{H_2} H_2^\dagger H_2 + m^2_S\, S^* S - \Big(\lambda\, A_\lambda\, S\, H_1 H_2 - \frac{\kappa}{3}\, A_\kappa\, S^3 + \text{h.c.}\Big)$$

"Higgs basis" for the doublets:

$$\begin{pmatrix} H \\ A \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} -\epsilon H_1^* \\ H_2 \end{pmatrix} \qquad\qquad S = v_s + \frac{1}{\sqrt{2}}\,(s + i\, a)$$

*We work in the limit v –> 0, as appropriate to the EFT calculation*

Tree-level minimum condition
for the scalar-singlet potential:

$$m_S^2 = -\,\kappa\, v_s\,(A_k + 2\,\kappa\, v_s)$$

$$m_s^2 = \kappa\, v_s\,(A_\kappa + 4\,\kappa\, v_s) \qquad m_a^2 = -\,3\,\kappa\, v_s\, A_\kappa$$

Tree-level masses
of the BSM particles:

$$m_A^2 = \frac{\lambda\, v_s\,(A_\lambda + \kappa\, v_s)}{\sin\beta\cos\beta} \qquad \mu = \lambda\, v_s \qquad m_{\tilde{s}} = 2\,\kappa\, v_s$$

## Tree-level matching condition for the quartic coupling of the SM-like Higgs doublet:

$$\lambda_{\rm SM}^{\rm tree} \;=\; \frac{1}{4}(g^2+g^{\prime\,2})\cos^2 2\beta \;+\; \frac{1}{2}\,\lambda^2\sin^2 2\beta \;-\; \frac{a_{hhs}^2}{m_s^2}$$

$$a_{hhs} \;=\; \frac{\lambda}{\sqrt{2}}\,\left[2\,\lambda\, v_s - (A_\lambda + 2\,\kappa\, v_s)\,\sin 2\beta\right]$$

- The matching condition splits in an MSSM-like part and an NMSSM-specific part controlled by $\lambda^2$. The latter can be positive or negative, depending on *tanβ*

- We need renormalization conditions for all of the parameters entering $\lambda_{\rm SM}^{\rm tree}$:

$$g^\prime\,,\quad g\,,\quad \tan\beta\,,\quad \lambda\,,\quad \kappa\,,\quad v_s\,,\quad A_\lambda\,,\quad A_\kappa\,,\quad (m_S^2)$$

- Even in a fully $\overline{\rm DR}$ calculation, there are several options for $m_s^2$ and $v_s$

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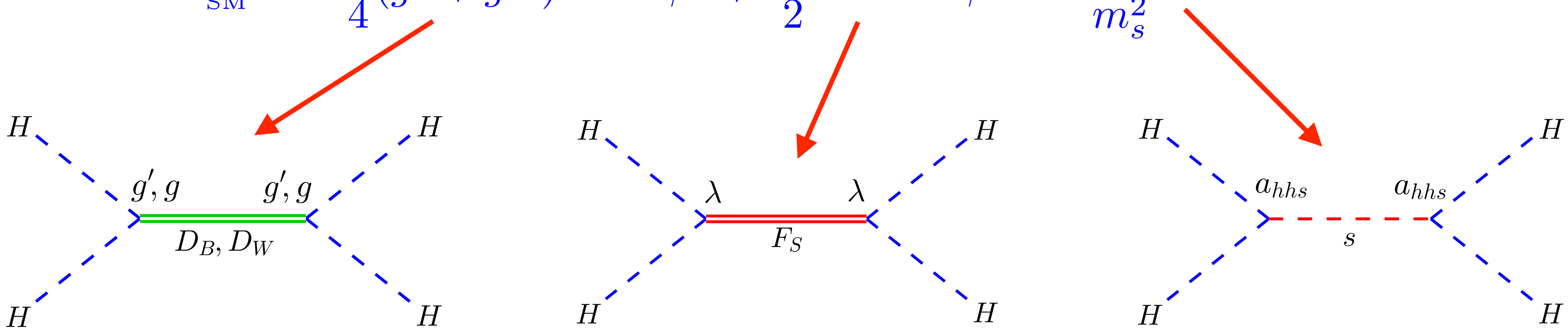


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[ All Feynman diagrams in this talk were drawn with R. Harlander’s `FeynGame` ]

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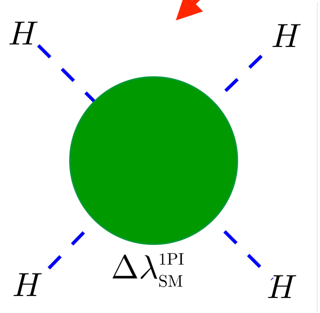

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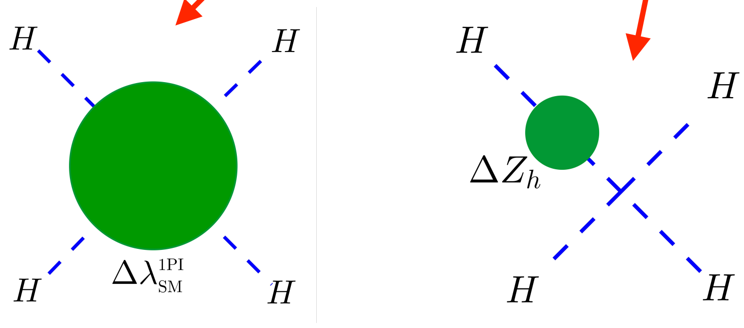

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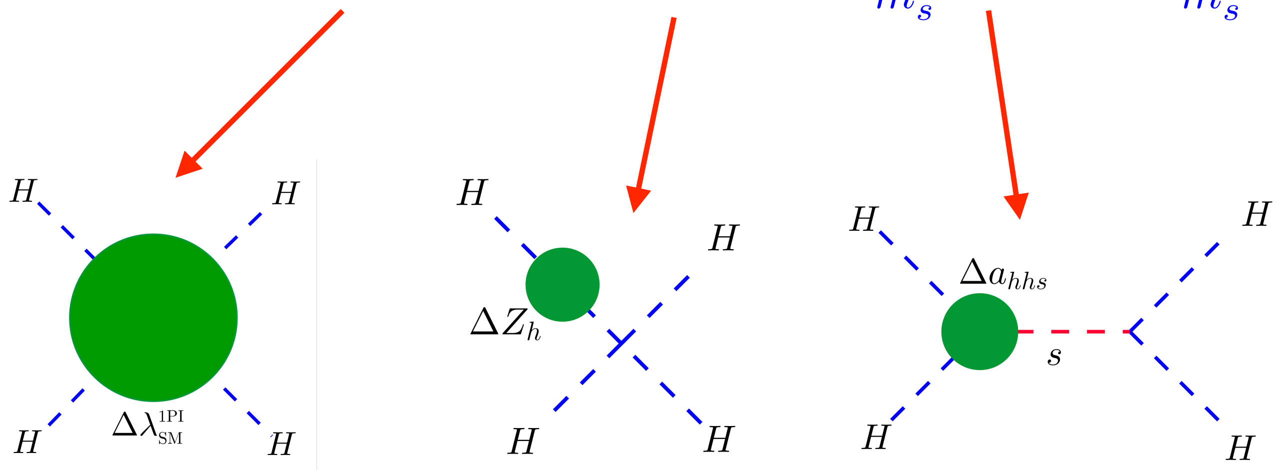

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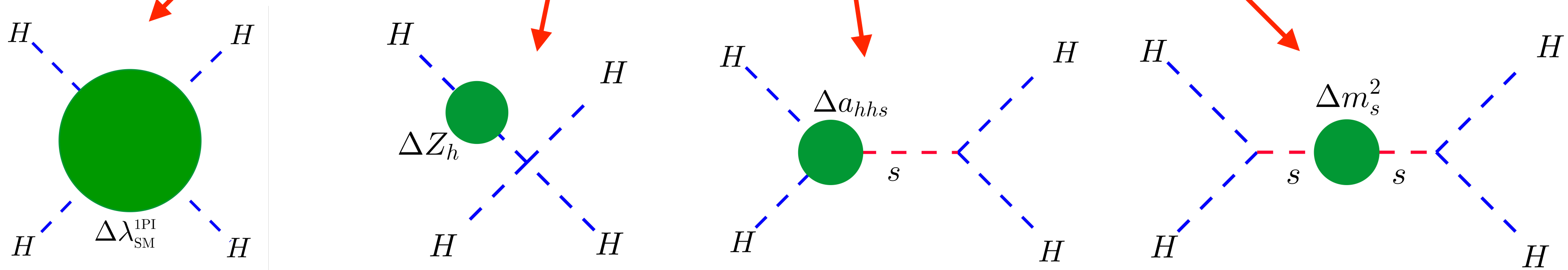

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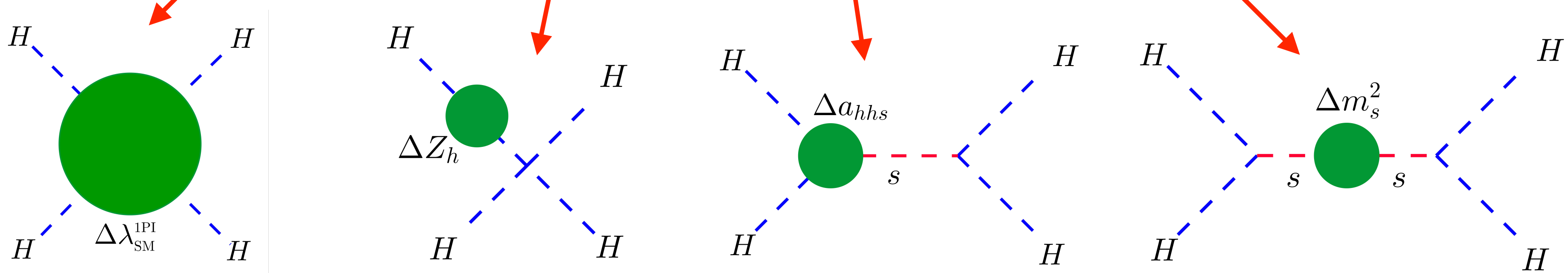

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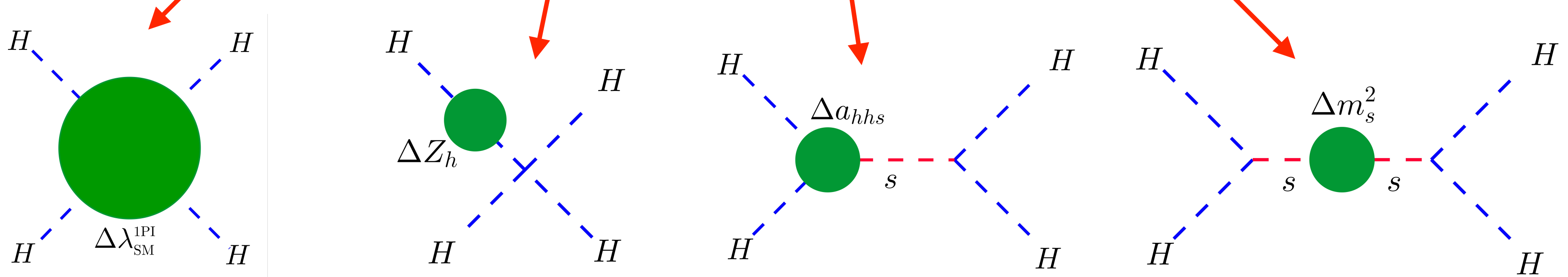


All one-loop 2,3,4-point functions obtained from the general formulas of 1810.09388

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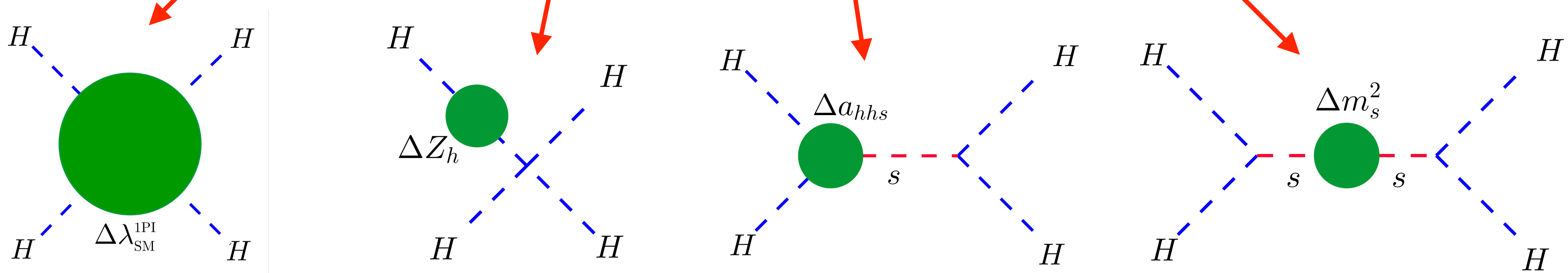


All one-loop 2,3,4-point functions obtained from the general formulas of 1810.09388

Note:

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# One-loop matching condition for the quartic Higgs coupling

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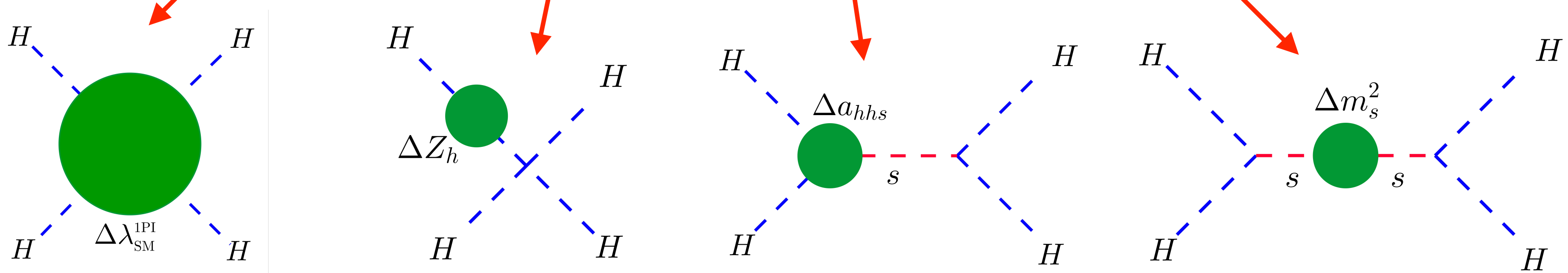


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## Three options for the treatment of the singlet vev and mass entering the tree-level part

The minimum condition
depends on the definition of $v_s$:

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\begin{aligned}
m_S^2 &= -\kappa\, v_s^0\, (A_k + 2\,\kappa\, v_s^0) \\
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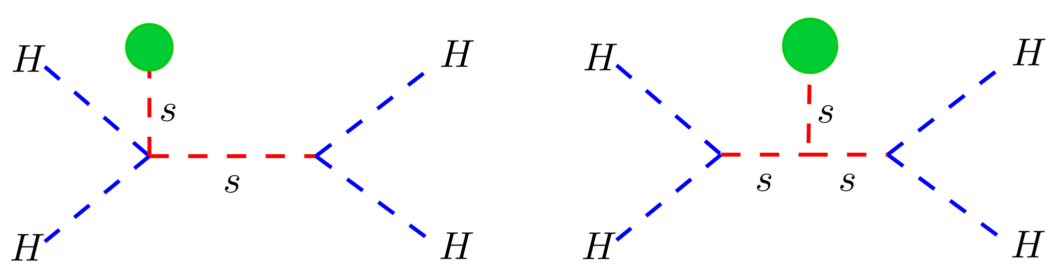


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## In the EFT approach the "MSSM limit" of the NMSSM is manifest:

Once we identify $\mu \equiv \lambda v_s$,
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$$\longrightarrow \quad \left(\Delta\lambda_{\rm SM}^{1\ell,\,\tilde t}\right)_{\lambda} = 2\,(\lambda_{\rm SM}^{\rm tree})_{\lambda}\,\Delta Z_h^{1\ell,\,\tilde t} - 2\,\frac{a_{hhs}}{m_s^2}\,\Delta a_{hhs}^{1\ell,\,\tilde t}$$

$$\Delta Z_h^{1\ell,\,\tilde t} = -\,\frac{g_t^2 N_c}{(4\pi)^2}\,\frac{X_t^2}{6\,m_{Q_3} m_{U_3}}\,\widetilde F_5\left(\frac{m_{Q_3}}{m_{U_3}}\right)$$

(same as MSSM)

$$\Delta a_{hhs}^{1\ell,\tilde t} = \sqrt{2}\,N_c\,\frac{\lambda\, g_t^2}{(4\pi)^2}\,\frac{X_t\,\cot\beta}{m_{Q_3}^2 - m_{U_3}^2}\left[m_{Q_3}^2\left(1 - \ln\frac{m_{Q_3}^2}{Q^2}\right) - m_{U_3}^2\left(1 - \ln\frac{m_{U_3}^2}{Q^2}\right)\right]$$

# Two-loop-QCD contribution to the matching condition

The NMSSM-specific part has the same structure as the one-loop squark contribution:

$$\left(\Delta\lambda_{\rm SM}^{2\ell,\rm QCD}\right)_\lambda \;=\; 2\,\left(\lambda_{\rm SM}^{\rm tree}\right)_\lambda\,\Delta Z_h^{2\ell,\rm QCD} \;-\; 2\,\frac{a_{hhs}}{m_s^2}\,\Delta a_{hhs}^{2\ell,\rm QCD} \;+\; \left(\Delta\lambda_{\rm SM}^{2\ell,\rm RS}\right)_\lambda$$

- $\Delta Z_h^{2\ell,\rm QCD}$ is again the same as in the MSSM and can be borrowed from 1908.01670

*This accounts for 2-loop momentum effects currently missing in FO calculations for the NMSSM*

- $\left(\Delta\lambda_{\rm SM}^{2\ell,\rm RS}\right)_\lambda$ is also obtained from the product of known contributions:

$$\left(\Delta\lambda_{\rm SM}^{2\ell,\rm RS}\right)_\lambda \;=\; \frac{g_s^2\,C_F\,N_c}{(4\pi)^4}\,(g_t^2+g_b^2)\,\left(\lambda_{\rm SM}^{\rm tree}\right)_\lambda \;+\; 2\,\Delta g_t\left(\Delta\lambda_{\rm SM}^{1\ell,\,\tilde t}\right)_\lambda$$

$$\lambda_{\rm SM}^{\overline{\rm DR}} \longrightarrow \lambda_{\rm SM}^{\overline{\rm MS}} \qquad\qquad g_t^{\rm NMSSM,\overline{DR}} \longrightarrow g_t^{\rm SM,\overline{MS}}$$

$$\Delta g_t \;=\; -\,\frac{g_s^2\,C_F}{(4\pi)^2}\left[1+\ln\frac{m_{\tilde g}^2}{Q^2}\;+\;\widetilde F_6\left(\frac{m_{Q_3}}{m_{\tilde g}}\right)\;+\;\widetilde F_6\left(\frac{m_{U_3}}{m_{\tilde g}}\right)\;-\;\frac{X_t}{m_{\tilde g}}\,\widetilde F_9\left(\frac{m_{Q_3}}{m_{\tilde g}},\frac{m_{U_3}}{m_{\tilde g}}\right)\right]$$

The correction to the trilinear Higgs-singlet coupling can be computed with usual methods

$$\Delta a_{hhs}^{2\ell,\,\mathrm{QCD}} = \left.\frac{\partial^3 \Delta V^{2\ell,\,\tilde{g}}}{\partial^2 h\,\partial s}\right|_{v=0}$$

E.g., the stop contribution:

$$\begin{aligned}
\left(\Delta a_{hhs}^{2\ell,\,\mathrm{QCD}}\right)^{\tilde{t}} = & -2\sqrt{2}\,\lambda\,g_t^2\,\cot\beta\,\Bigg\{ X_t \Bigg[-2 + \left(2 - \frac{2\,(1+x_Q)\ln x_Q}{x_Q - x_U}\right)\ln\frac{m_{\tilde{g}}^2}{Q^2} - \frac{1}{2}\ln^2\frac{m_{\tilde{g}}^2}{Q^2} \\
& + \frac{2\,(1+2\,x_Q)\,\ln x_Q}{x_Q - x_U} - \frac{(x_Q^2 + x_Q - x_U)\,\ln^2 x_Q}{(x_Q - x_U)^2} \\
& + \frac{x_Q\,x_U\,\ln x_Q\,\ln x_U}{(x_Q - x_U)^2} - \frac{2\,(1-x_Q)}{x_Q - x_U}\,\mathrm{Li}_2\left(1 - \frac{1}{x_Q}\right)\Bigg] \\
& + m_{\tilde{g}}\left[\frac{5}{2} - \left(2 - \frac{2\,x_Q\,\ln x_Q}{x_Q - x_U}\right)\ln\frac{m_{\tilde{g}}^2}{Q^2} + \frac{1}{2}\ln^2\frac{m_{\tilde{g}}^2}{Q^2} - \frac{4\,x_Q\,\ln x_Q}{x_Q - x_U}\right. \\
& \left. - \frac{(1-x_Q)\,\ln^2 x_Q}{x_Q - x_U} - \frac{2\,(1-x_Q)}{x_Q - x_U}\,\mathrm{Li}_2\left(1 - \frac{1}{x_Q}\right)\right] \\
& + \;(x_Q \longleftrightarrow x_U)\;\Bigg\}
\end{aligned}$$

$$\left(x_Q = m_{Q_3}^2/m_{\tilde{g}}^2\,, \quad x_U = m_{U_3}^2/m_{\tilde{g}}^2\right)$$

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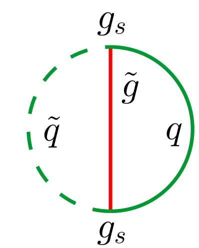


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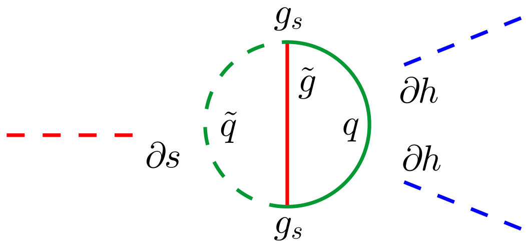


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Non-trivial consistency check of our result:

$$(4\pi)^2 \frac{d}{d\ln Q^2} \left(\lambda_{\rm SM}^{\rm tree} + \Delta\lambda_{\rm SM}^{1\ell} + \Delta\lambda_{\rm SM}^{2\ell,\rm QCD}\right) = \lambda_{\rm SM}^{\rm tree}\left(6\,\lambda_{\rm SM}^{\rm tree} + 6\,g_t^2 + 6\,g_b^2 + 2\,g_\tau^2 - \frac{9}{2}\,g^2 - \frac{3}{2}g'^{\,2}\right)$$

$$-\,6\,g_t^4 - 6\,g_b^4 - 6\,g_\tau^4 + \frac{9}{8}\,g^4 + \frac{3}{8}\,g'^{\,4} + \frac{3}{4}\,g^2 g'^{\,2}$$

$$+ \quad \frac{40\,g_s^2}{(4\pi)^2}\,\lambda_{\rm SM}^{\rm tree}\,(g_t^2 + g_b^2)$$

= the full 1-loop + 2-loop-QCD RGE for $\lambda_{\rm SM}$ in the SM

Combining:

- the explicit Q² dependence of $\Delta\lambda_{\rm SM}^{1\ell}$ and $\Delta\lambda_{\rm SM}^{2\ell,\rm QCD}$
- the full 1-loop RGEs of all parameters in $\lambda_{\rm SM}^{\rm tree}$
- the 2-loop $\mathcal{O}(\alpha_s\alpha_t)$ and $\mathcal{O}(\alpha_s\alpha_b)$ RGEs of $\lambda$, $A_\lambda$, $\tan\beta$ in $\lambda_{\rm SM}^{\rm tree}$
- the 1-loop $\mathcal{O}(\alpha_s)$ RGEs of the relevant parameters in $\Delta\lambda_{\rm SM}^{1\ell}$

# Estimating the theory uncertainty of the Higgs-mass prediction

- *“SM uncertainty”* :

- *“SUSY uncertainty”* :

- *“EFT uncertainty”* :

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| *Action:* | *What it simulates:* |
|---|---|

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| Remove 3-loop QCD corrections in the extraction of $g_t\,(M_t)$ from $M_t$ | $N^3LL$ effects of highest order in QCD |

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$N^3$LL effects of highest order in QCD

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Change 1-loop definition of $g_t\,(M_S)$ :

$$g_t^{\rm NMSSM}(M_S) \;=\; \frac{g_t^{\rm SM}(M_S)}{1-\Delta g_t^{\rm MSSM} - (\Delta g_t)_\lambda}$$

$\mathcal{O}(g_t^4\,\lambda^2)$ terms in $\Delta\lambda_{\rm SM}^{2\ell}$
+ various 3-loop terms

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- *"SUSY uncertainty"* :

Change 1-loop definition of $g_t\,(M_S)$ :

$$g_t^{\rm NMSSM}(M_S) \;=\; \frac{g_t^{\rm SM}(M_S)}{1-\Delta g_t^{\rm MSSM}-(\Delta g_t)_\lambda}$$

$\mathcal{O}(g_t^4\,\lambda^2)$ terms in $\Delta\lambda_{\rm SM}^{2\ell}$ + various 3-loop terms

Alternative definitions for the singlet masses:

$$m_s^2 \;=\; m_S^2 + 2\,\kappa\,v_s\,(A_\kappa + 3\,\kappa\,v_s)$$
$$m_a^2 \;=\; m_S^2 - 2\,\kappa\,v_s\,(A_\kappa - \kappa\,v_s)$$

$\mathcal{O}(\lambda^6)$ terms in $\Delta\lambda_{\rm SM}^{2\ell}$ + various 3-loop terms

- *"EFT uncertainty"* :

# Estimating the theory uncertainty of the Higgs-mass prediction

| *Action:* | *What it simulates:* |
|---|---|
| • *"SM uncertainty"* : | |
| Remove 3-loop QCD corrections in the extraction of $g_t(M_t)$ from $M_t$ | $N^3LL$ effects of highest order in QCD |
| • *"SUSY uncertainty"* : | |
| Change 1-loop definition of $g_t(M_S)$ : $$g_t^{\rm NMSSM}(M_S) \;=\; \frac{g_t^{\rm SM}(M_S)}{1-\Delta g_t^{\rm MSSM}-(\Delta g_t)_\lambda}$$ | $\mathcal{O}(g_t^4\,\lambda^2)$ terms in $\Delta\lambda_{\rm SM}^{2\ell}$ + various 3-loop terms |
| Alternative definitions for the singlet masses: $$m_s^2 \;=\; m_S^2 + 2\,\kappa\,v_s\,(A_\kappa+3\,\kappa\,v_s)$$ $$m_a^2 \;=\; m_S^2 - 2\,\kappa\,v_s\,(A_\kappa-\kappa\,v_s)$$ | $\mathcal{O}(\lambda^6)$ terms in $\Delta\lambda_{\rm SM}^{2\ell}$ + various 3-loop terms |

• *"EFT uncertainty"* : negligible in scenarios with multi-TeV SUSY masses

# Numerical impact of the NMSSM-specific contributions

Heavy-SUSY scenario:

$$M_S \;=\; 5~\text{TeV}, \qquad X_t \;=\; \sqrt{6}\,M_S\,,$$

$$(M_1,\,M_2,\,M_3) = (1,\,2,\,2.5)~\text{TeV}$$

$\overline{\text{DR}}$ parameters in the Higgs/singlet sector:

$(Q = M_S)$

$$m_A \;=\; 3~\text{TeV}$$

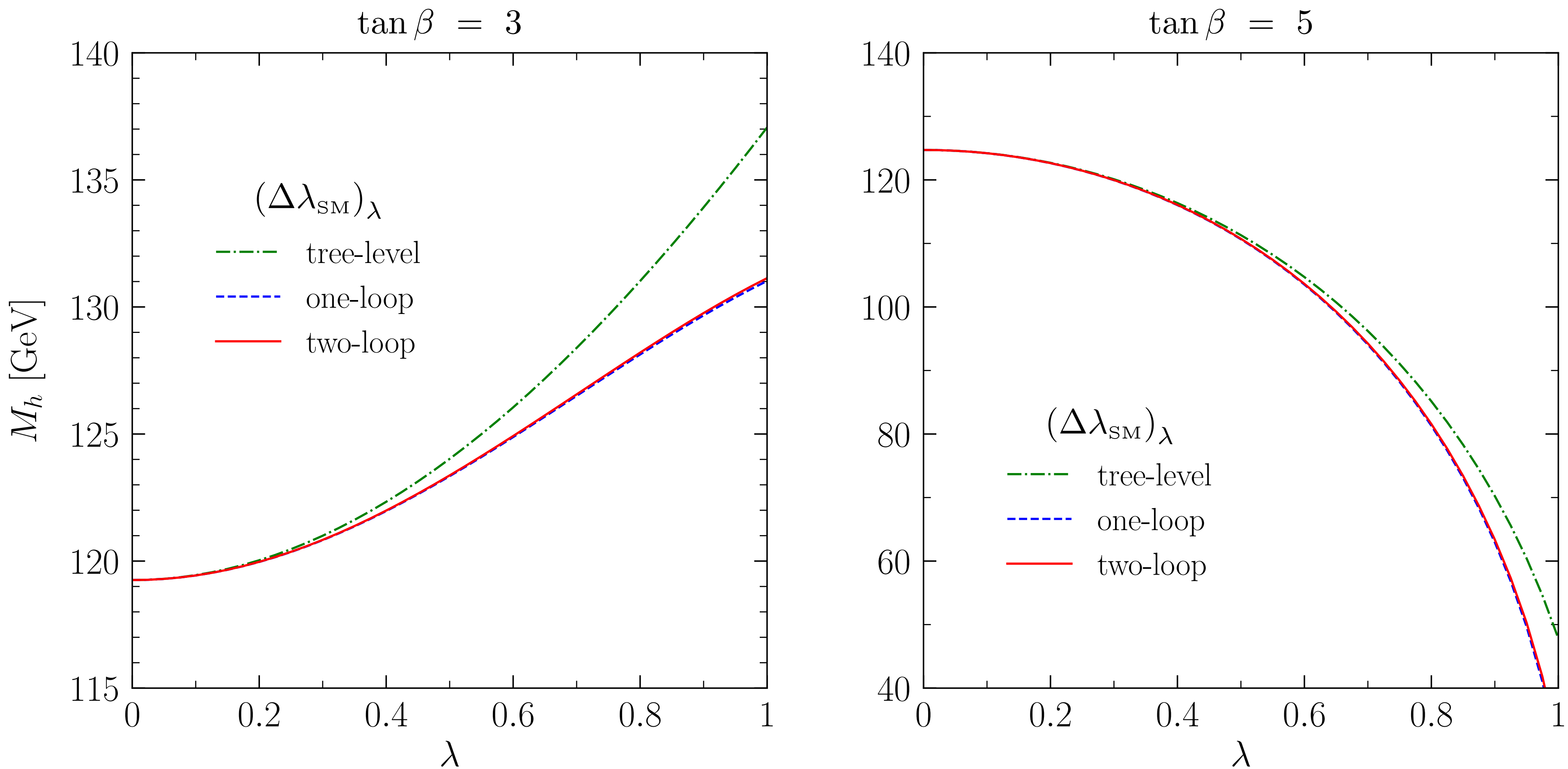
$$\mu \;=\; \lambda\, v_s \;=\; 1.5~\text{TeV}$$

$$\kappa \;=\; \lambda\,, \qquad A_\kappa \;=\; -2~\text{TeV}$$

$$\longrightarrow \quad m_s \;\approx\; 2.45~\text{TeV}, \qquad m_a \;=\; m_{\tilde{s}} \;=\; 3~\text{TeV}$$

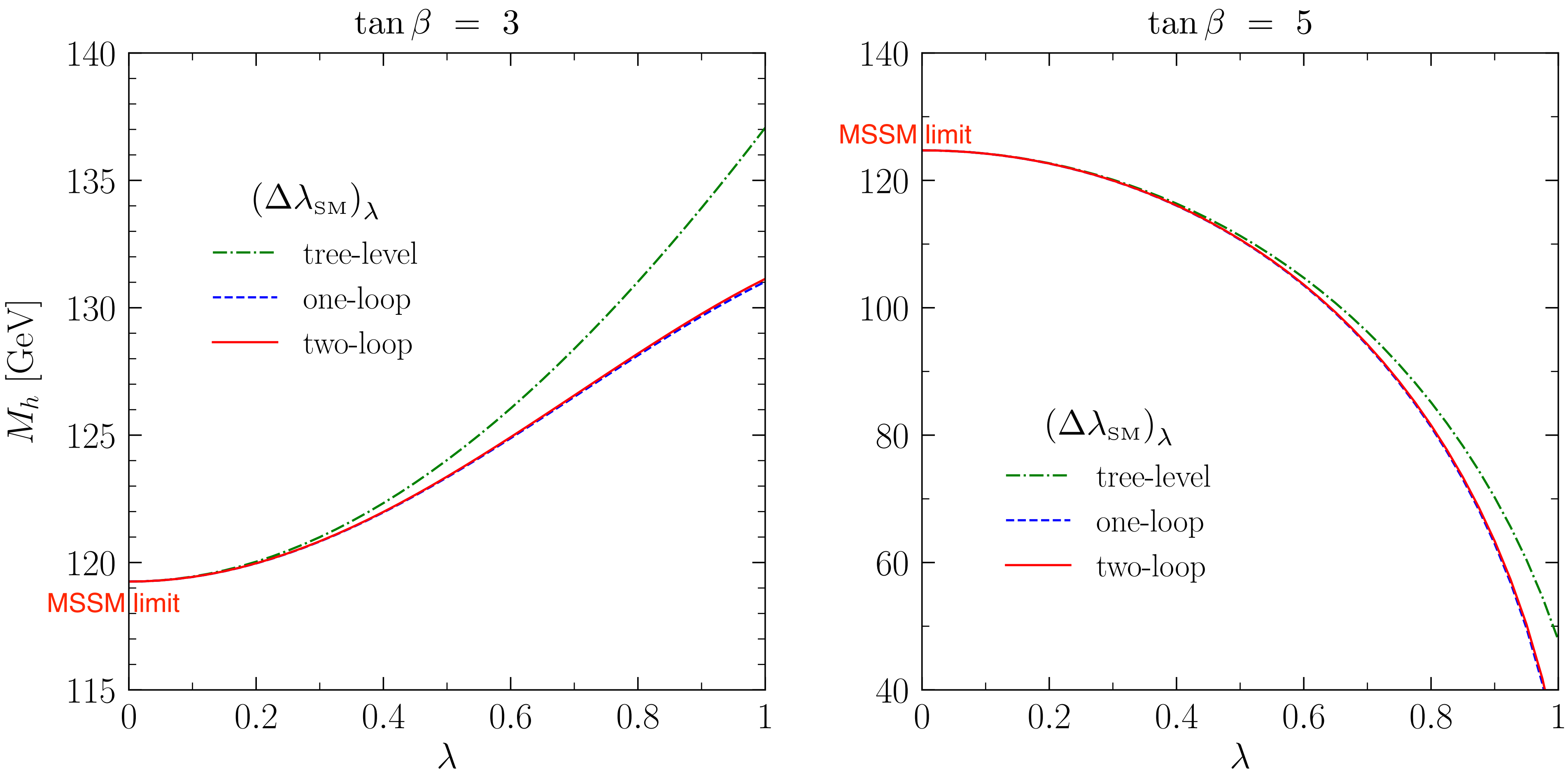
We study the dependence of $M_h$ on $\lambda$ and $\tan\beta$

- We use $\mathtt{mr}$ to extract the SM parameters and evolve up to the SUSY scale
- We scan the input value of $M_h$ until $\lambda_{\rm SM}(M_S)$ matches the NMSSM prediction




Driven by the tree-level dependence on $\lambda$ : $(\lambda^{\rm tree}_{\rm SM})_\lambda = \frac{1}{2}\,\lambda^2 \sin^2 2\beta - \frac{a^2_{hhs}}{m^2_s}$

1-loop contribution significant at large $\lambda$ , 2-loop contribution much smaller




Driven by the tree-level dependence on $\lambda$ : $(\lambda_{\rm SM}^{\rm tree})_\lambda = \frac{1}{2}\,\lambda^2 \sin^2 2\beta - \frac{a_{hhs}^2}{m_s^2}$

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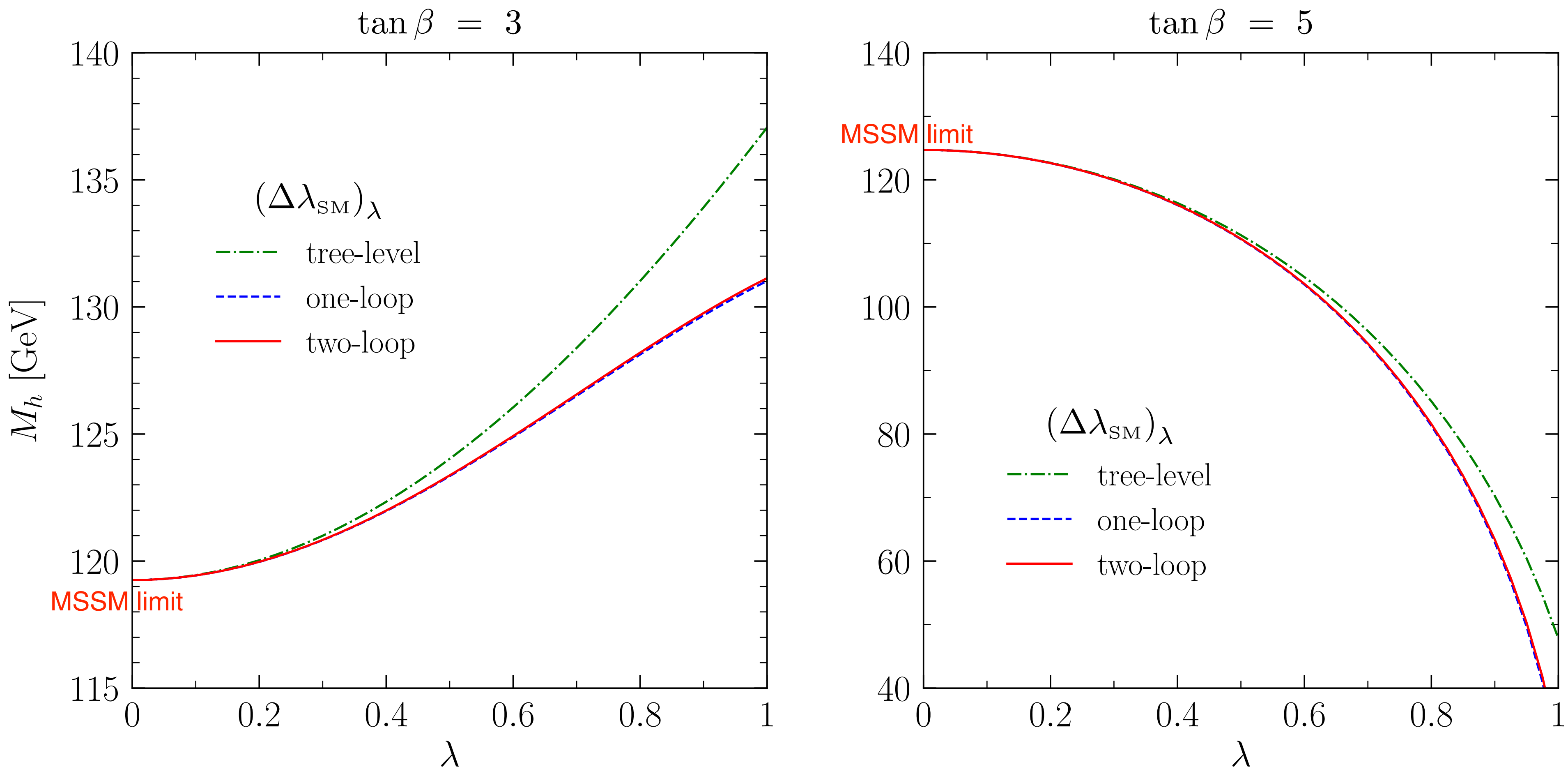
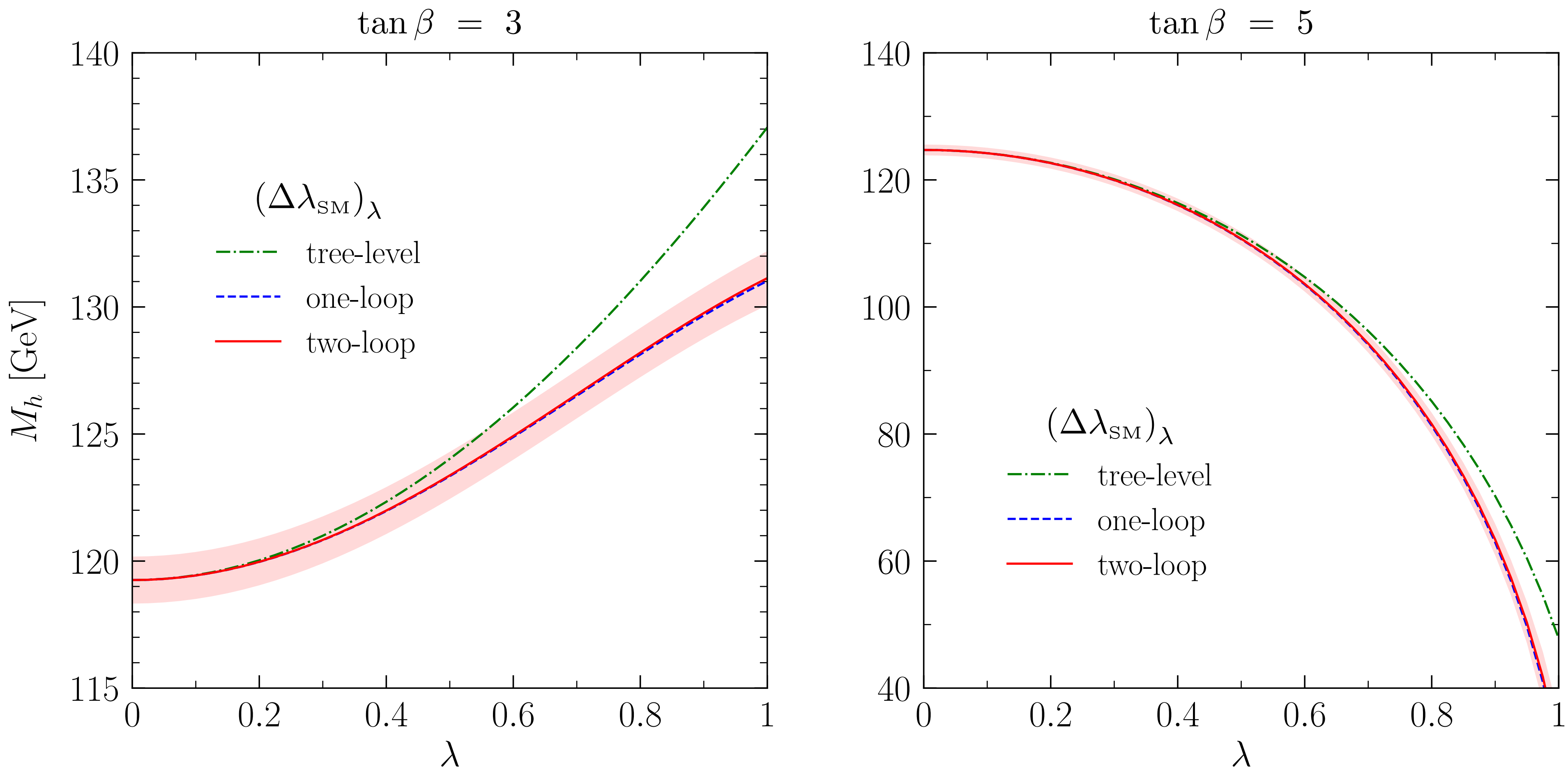



Driven by the tree-level dependence on $\lambda$ : $\left(\lambda_{\rm SM}^{\rm tree}\right)_\lambda \;=\; \frac{1}{2}\,\lambda^2 \sin^2 2\beta \;-\; \frac{a_{hhs}^2}{m_s^2}$

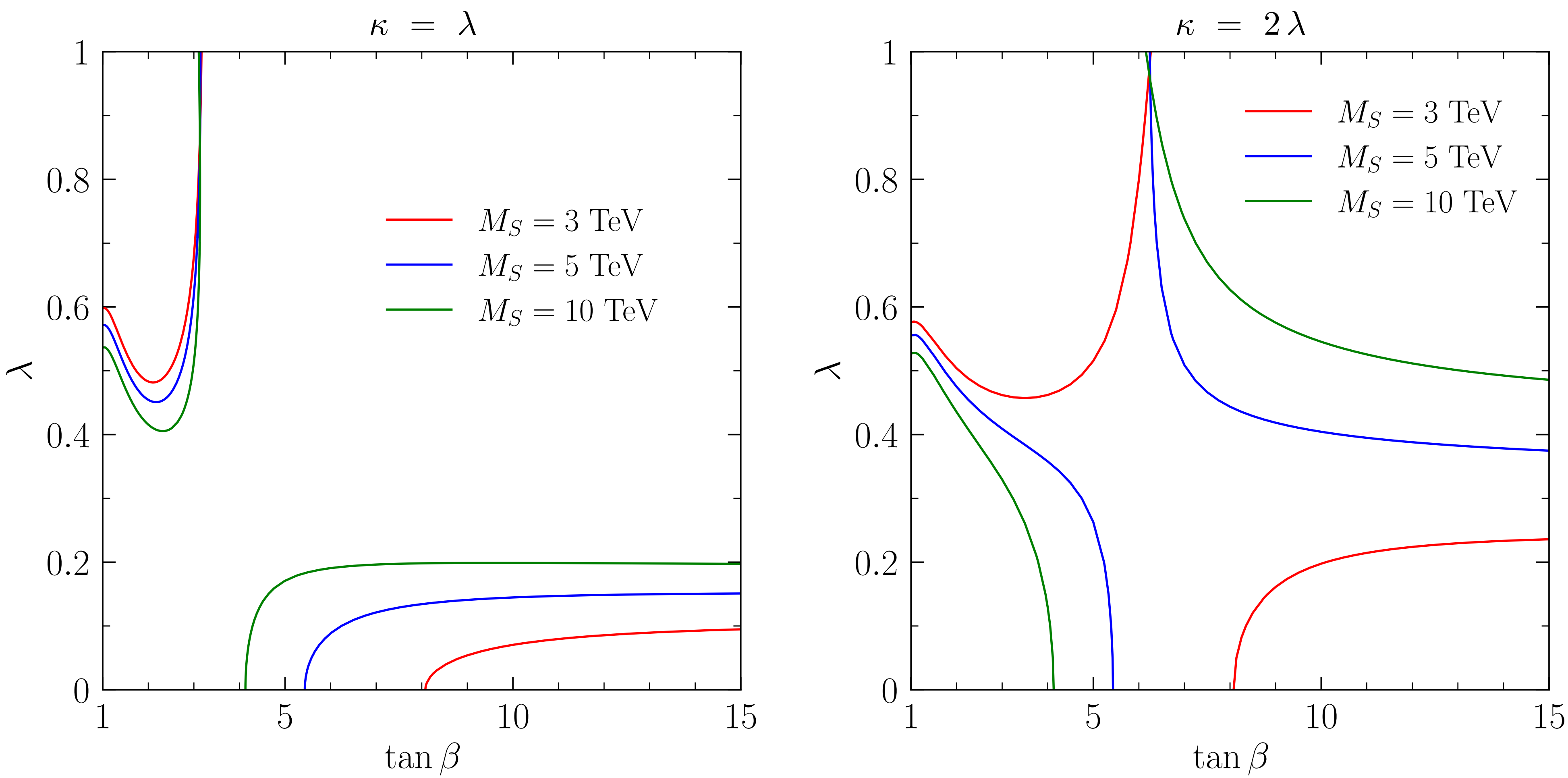
1-loop contribution significant at large $\lambda$ , 2-loop contribution much smaller

The NMSSM-specific stop contribution is already small at 1-loop in this scenario: $\left(\Delta\lambda_{\rm SM}^{1\ell,\,\tilde t}\right)_\lambda \;\approx\; -0.02\,\times\,\lambda_{\rm SM}^{\rm tree}$




*Uncertainty estimate dominated by the SM contribution*

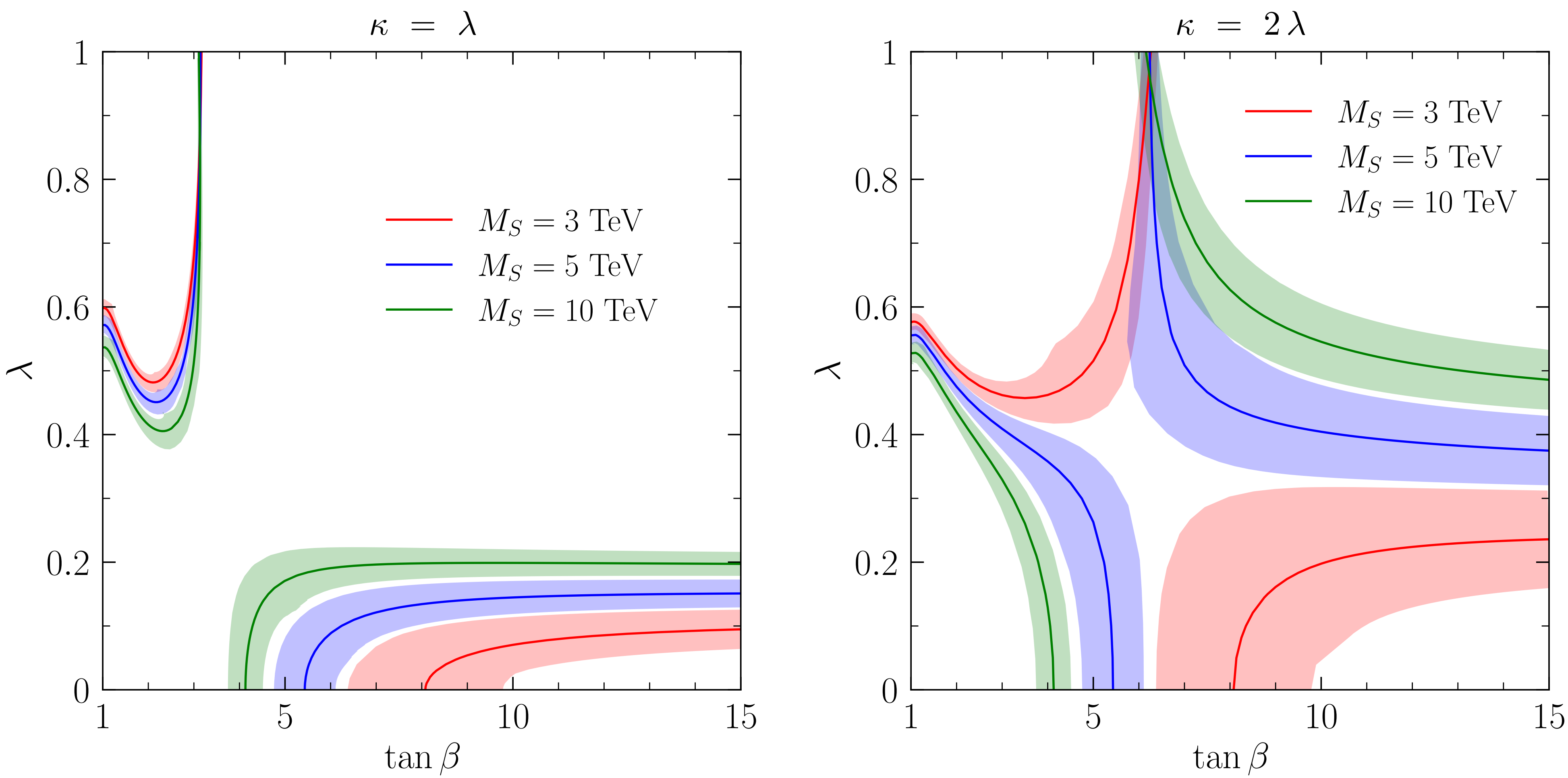
# Constraints on the parameter space from $M_h = 125.25$ GeV




Again driven by the tree-level dependence on $\lambda$ and $\tan\beta$ :

$$\lambda_{\rm SM}^{\rm tree} = \frac{1}{4}(g^2 + g'^2)\cos^2 2\beta + \frac{1}{2}\lambda^2 \sin^2 2\beta - \frac{a_{hhs}^2}{m_s^2}$$

# Constraints on the parameter space from $M_h = 125.25$ GeV




Again driven by the tree-level dependence on $\lambda$ and $\tan\beta$ :

$$\lambda_{\rm SM}^{\rm tree} \; = \; \frac{1}{4}(g^2 + g'^{\,2})\cos^2 2\beta \; + \; \frac{1}{2}\,\lambda^2 \sin^2 2\beta \; - \; \frac{a_{hhs}^2}{m_s^2}$$

# Bonus track: OS scheme for the singlet parameters

Parameters in the tree-level condition:

$$g\,,\ \ g'\,,\ \ \tan\beta\,,\ \ \lambda\,,\ \ \kappa\,,\ \ v_s\,,\ \ A_\lambda\,,\ \ A_\kappa$$

# Bonus track: OS scheme for the singlet parameters

Parameters in the tree-level condition:

$$\underbrace{g\,,\;\; g'\,,\;\; \tan\beta\,,\;\; \lambda\,,\;\; \kappa\,,\;\; v_s\,,\;\; A_\lambda\,,\;\; A_\kappa}_{\overline{\text{MS}}/\overline{\text{DR}}\text{ at } Q\,=\,M_S}$$

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# Bonus track: OS scheme for the singlet parameters

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$$M_{\tilde h} \;\equiv\; \lambda\, v_s^{\rm OS}\,, \qquad M_{\tilde s} \;\equiv\; 2\,\kappa^{\rm OS} v_s^{\rm OS}\,,$$

$$M_A^2 \;\equiv\; \frac{\lambda\, v_s^{\rm OS}\,(A_\lambda^{\rm OS} + \kappa^{\rm OS} v_s^{\rm OS})}{\sin\beta\cos\beta}\,, \qquad M_s^2 \;\equiv\; \kappa^{\rm OS} v_s^{\rm OS}\,(A_\kappa^{\rm OS} + 4\,\kappa^{\rm OS} v_s^{\rm OS})$$

# Bonus track: OS scheme for the singlet parameters

Parameters in the tree-level condition:

$$\underbrace{g\,,\;\; g'\,,\;\; \tan\beta\,,\;\; \lambda\,,}_{\text{MS/DR at } Q = M_S}\;\; \underbrace{\kappa\,,\;\; v_s\,,\;\; A_\lambda\,,\;\; A_\kappa}_{\text{Derived from pole masses}}$$

$$M_{\tilde{h}} \;\equiv\; \lambda\, v_s^{\rm OS}\,, \qquad M_{\tilde{s}} \;\equiv\; 2\,\kappa^{\rm OS} v_s^{\rm OS}\,,$$

$$M_A^2 \;\equiv\; \frac{\lambda\, v_s^{\rm OS}\,(A_\lambda^{\rm OS} + \kappa^{\rm OS} v_s^{\rm OS})}{\sin\beta\cos\beta}\,, \qquad M_s^2 \;\equiv\; \kappa^{\rm OS} v_s^{\rm OS}\,(A_\kappa^{\rm OS} + 4\,\kappa^{\rm OS} v_s^{\rm OS})$$

$$\lambda_{\rm SM}^{\rm tree\,,OS} \;=\; \frac{1}{4}(g^2 + g'^{\,2})\cos^2 2\beta \;+\; \frac{1}{2}\,\lambda^2 \sin^2 2\beta \;-\; \frac{(a_{hhs}^{\rm OS})^2}{M_s^2}\;,$$

$$\begin{aligned} a_{hhs}^{\rm OS} \;&=\; \frac{\lambda}{\sqrt{2}}\,\left[2\,\lambda\, v_s^{\rm OS} - (A_\lambda^{\rm OS} + 2\,\kappa^{\rm OS} v_s^{\rm OS})\,\sin 2\beta\right] \\ &=\; \sqrt{2}\,\lambda\,\left[M_{\tilde{h}} - \frac{1}{4}\left(\frac{M_A^2}{M_{\tilde{h}}}\,\sin 2\beta + M_{\tilde{s}}\right)\sin 2\beta\right] \end{aligned}$$

$$\Delta\lambda_{\rm SM}^{1\ell,\rm OS} \;=\; 2\,\frac{a_{hhs}^{\rm OS}}{M_s^2}\,\delta a_{hhs} - \frac{(a_{hhs}^{\rm OS})^2}{M_s^4}\,\delta M_s^2$$

$$\delta a_{hhs} \;=\; \sqrt{2}\,\lambda\,\left\{\delta M_{\tilde h} - \frac{1}{4}\left[\,\frac{M_A^2}{M_{\tilde h}}\left(\frac{\delta M_A^2}{M_A^2} - \frac{\delta M_{\tilde h}}{M_{\tilde h}}\right)\,\sin 2\beta + \delta M_{\tilde s}\right]\sin 2\beta\right\}$$

$$\begin{aligned}
2\,M_{\tilde h}\,\delta M_{\tilde h} &= -{\rm Re}\,\hat\Pi_{\tilde h\tilde h}(M_{\tilde h}^2)\\
2\,M_{\tilde s}\,\delta M_{\tilde s} &= -{\rm Re}\,\hat\Pi_{\tilde s\tilde s}(M_{\tilde s}^2)\\
\delta M_s^2 &= \frac{\lambda}{\sqrt{2}\,M_{\tilde h}}\hat T_s - {\rm Re}\,\hat\Pi_{ss}(M_s^2)\\
\delta M_A^2 &= -{\rm Re}\,\hat\Pi_{AA}(M_A^2) + \hat\Pi_{HH}(0) + 2\cot 2\beta\,\hat\Pi_{HA}(0)
\end{aligned}$$

$$\Delta\lambda_{\rm SM}^{1\ell,{\rm OS}} \;=\; 2\,\frac{a_{hhs}^{\rm OS}}{M_s^2}\,\delta a_{hhs} - \frac{(a_{hhs}^{\rm OS})^2}{M_s^4}\,\delta M_s^2$$

$$\delta a_{hhs} \;=\; \sqrt{2}\,\lambda\left\{\delta M_{\tilde h} - \frac{1}{4}\left[\frac{M_A^2}{M_{\tilde h}}\left(\frac{\delta M_A^2}{M_A^2} - \frac{\delta M_{\tilde h}}{M_{\tilde h}}\right)\sin 2\beta + \delta M_{\tilde s}\right]\sin 2\beta\right\}$$

*General formulas for the 1-loop self-energies:*

$$\left.\begin{aligned} 2\,M_{\tilde h}\,\delta M_{\tilde h} &= -{\rm Re}\,\hat\Pi_{\tilde h\tilde h}(M_{\tilde h}^2)\\ 2\,M_{\tilde s}\,\delta M_{\tilde s} &= -{\rm Re}\,\hat\Pi_{\tilde s\tilde s}(M_{\tilde s}^2)\end{aligned}\right\}$$

S.P. Martin, hep-ph/0509115

$$\left.\begin{aligned} \delta M_s^2 &= \frac{\lambda}{\sqrt{2}\,M_{\tilde h}}\hat T_s - {\rm Re}\,\hat\Pi_{ss}(M_s^2)\\ \delta M_A^2 &= -{\rm Re}\,\hat\Pi_{AA}(M_A^2) + \hat\Pi_{HH}(0) + 2\cot 2\beta\,\hat\Pi_{HA}(0)\end{aligned}\right\}$$

S.P. Martin, hep-ph/0312092

$$\Delta\lambda_{\rm SM}^{1\ell,\rm OS} \;=\; 2\,\frac{a_{hhs}^{\rm OS}}{M_s^2}\,\delta a_{hhs} - \frac{(a_{hhs}^{\rm OS})^2}{M_s^4}\,\delta M_s^2$$

$$\delta a_{hhs} \;=\; \sqrt{2}\,\lambda\,\left\{\delta M_{\tilde h} - \frac{1}{4}\left[\,\frac{M_A^2}{M_{\tilde h}}\left(\frac{\delta M_A^2}{M_A^2} - \frac{\delta M_{\tilde h}}{M_{\tilde h}}\right)\,\sin 2\beta + \delta M_{\tilde s}\right]\sin 2\beta\right\}$$

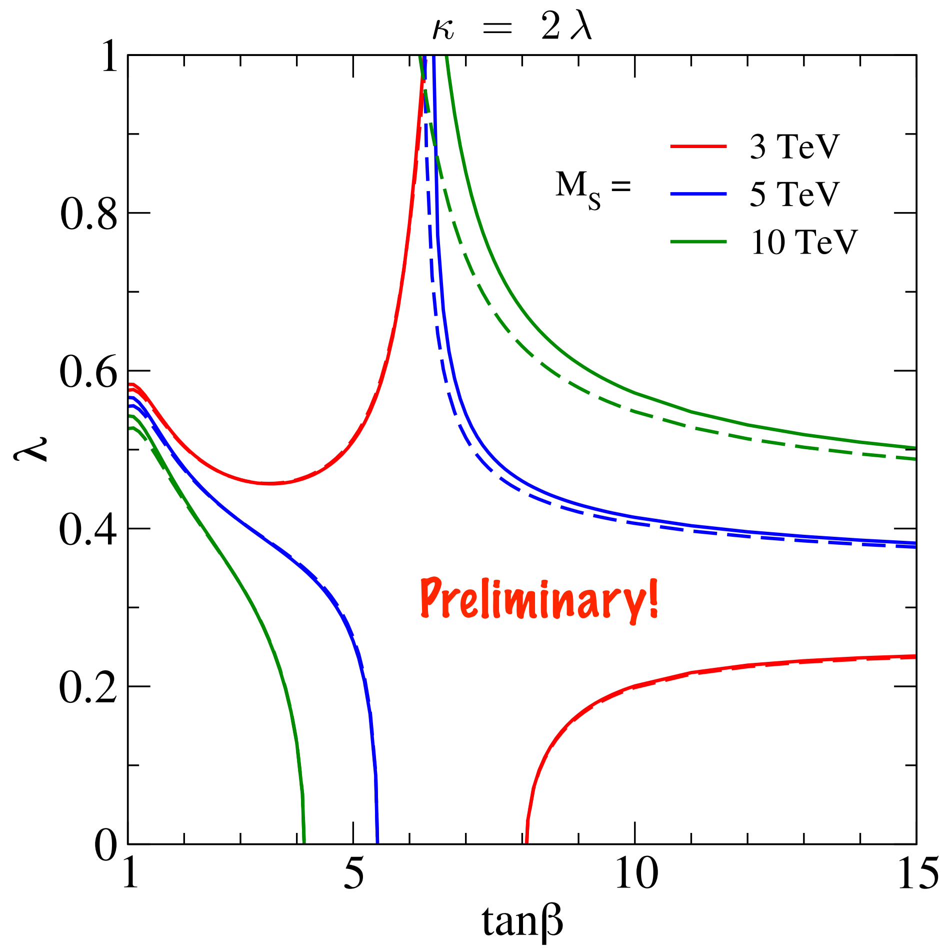
*General formulas for the 1-loop self-energies:*

$$\left.\begin{aligned} 2\,M_{\tilde h}\,\delta M_{\tilde h} &= -{\rm Re}\,\hat\Pi_{\tilde h\tilde h}(M_{\tilde h}^2) \\ 2\,M_{\tilde s}\,\delta M_{\tilde s} &= -{\rm Re}\,\hat\Pi_{\tilde s\tilde s}(M_{\tilde s}^2) \end{aligned}\right\}\quad \text{S.P. Martin, hep-ph/0509115}$$

$$\left.\begin{aligned} \delta M_s^2 &= \frac{\lambda}{\sqrt{2}\,M_{\tilde h}}\hat T_s - {\rm Re}\,\hat\Pi_{ss}(M_s^2) \\ \delta M_A^2 &= -{\rm Re}\,\hat\Pi_{AA}(M_A^2) + \hat\Pi_{HH}(0) + 2\cot 2\beta\,\hat\Pi_{HA}(0) \end{aligned}\right\}\quad \text{S.P. Martin, hep-ph/0312092}$$

*(Note: this receives also 2-loop-QCD contributions)*

# Numerical effect of the change of scheme




$$(M_{\tilde{h}}\,,\; M_{\tilde{s}}\,,\; M_s\,,\; M_A) \;=\; (1.5\,,\; 6\,,\; 5.5\,,\; 3)\ \text{TeV}$$

$$(\mu\,,\; m_{\tilde{s}}\,,\; m_s\,,\; m_A\,)_{Q=M_S} \;=\; (1.5\,,\; 6\,,\; 5.5\,,\; 3)\ \text{TeV}$$

# Summary

- "General formulas" are powerful tools to compute 1-(and 2)-loop corrections, but human input still needed for subtleties related to renormalization choices

- In the NMSSM with heavy BSM particles, the EFT calculation of the Higgs mass requires some care for the definition of the parameters in the singlet sector

- The NMSSM-specific contributions to the matching condition for the quartic Higgs coupling can be neatly grafted onto the MSSM result

- The numerical impact of the 1-loop contributions can be sizable at large $\lambda$, but the qualitative behavior is already driven by the tree-level dependence

- Our full-1-loop + 2-loop-QCD NMSSM-specific contributions to the quartic Higgs coupling are available on request. Shall we implement them in `FeynHiggs`?

Thank you!!!

# Backup slides

How we get $\delta M_A^2 = -\mathrm{Re}\,\hat{\Pi}_{AA}(M_A^2) + \hat{\Pi}_{HH}(0) + 2\cot 2\beta\,\hat{\Pi}_{HA}(0)$

How we get $$\delta M_A^2 \quad = \quad -\mathrm{Re}\,\hat{\Pi}_{AA}(M_A^2) + \sin^2\beta\,\frac{T_1}{v_1} + \cos^2\beta\,\frac{T_2}{v_2}$$

How we get $\delta M_A^2 \quad = \quad -\mathrm{Re}\,\hat{\Pi}_{AA}(M_A^2) + \hat{\Pi}_{HH}(0) + 2\cot 2\beta\,\hat{\Pi}_{HA}(0)$

How we get $\delta M_A^2 \quad = \quad -\mathrm{Re}\,\hat{\Pi}_{AA}(M_A^2) + \hat{\Pi}_{HH}(0) + 2\cot 2\beta\,\hat{\Pi}_{HA}(0)$

Starting point: $\begin{pmatrix} H \\ A \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} -\epsilon H_1^* \\ H_2 \end{pmatrix}$

How we get $\delta M_A^2 \;=\; -\mathrm{Re}\,\hat{\Pi}_{AA}(M_A^2) + \hat{\Pi}_{HH}(0) + 2\cot 2\beta\,\hat{\Pi}_{HA}(0)$

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Route 1:

$$\sin\beta\cos\beta \;=\; \frac{\lambda\, v_s\,(A_\lambda + \kappa\, v_s) - \hat{\Pi}_{12}(0)}{\overline{M}_A^2 - \overline{M}_H^2}$$

$$\begin{aligned} M_A^2 &= \overline{M}_A^2 + \hat{\Pi}_{AA}(0) - \mathrm{Re}\,\hat{\Pi}_{AA}(M_A^2) \\ &= \frac{\lambda\, v_s\,(A_\lambda + \kappa\, v_s)}{\sin\beta\cos\beta} - \frac{\hat{\Pi}_{12}(0)}{\sin\beta\cos\beta} + \hat{\Pi}_{AA}(0) - \mathrm{Re}\,\hat{\Pi}_{AA}(M_A^2) \end{aligned}$$

$$\hat{\Pi}_{12}(0) \;=\; \sin\beta\cos\beta\left[\hat{\Pi}_{AA}(0) - \hat{\Pi}_{HH}(0)\right] + \cos 2\beta\,\hat{\Pi}_{HA}(0)$$

How we get $\delta M_A^2 = -\mathrm{Re}\,\hat{\Pi}_{AA}(M_A^2) + \hat{\Pi}_{HH}(0) + 2\cot 2\beta\,\hat{\Pi}_{HA}(0)$

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$$M_A^2 = \overline{M}_A^2 + \hat{\Pi}_{AA}(0) - \mathrm{Re}\,\hat{\Pi}_{AA}(M_A^2)$$

$$= \frac{\lambda\,v_s\,(A_\lambda + \kappa\,v_s)}{\sin\beta\cos\beta} - \frac{\hat{\Pi}_{12}(0)}{\sin\beta\cos\beta} + \hat{\Pi}_{AA}(0) - \mathrm{Re}\,\hat{\Pi}_{AA}(M_A^2)$$

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How we get $\delta M_A^2 \;=\; -\mathrm{Re}\,\hat\Pi_{AA}(M_A^2) + \hat\Pi_{HH}(0) + 2\cot 2\beta\,\hat\Pi_{HA}(0)$

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Route 1: $$\sin\beta\cos\beta \;=\; \frac{\lambda\,v_s\,(A_\lambda + \kappa\,v_s) - \hat\Pi_{12}(0)}{\overline{M}_A^2 - \cancel{\overline{M}_H^2}}$$

$$\begin{aligned} M_A^2 \;&=\; \overline{M}_A^2 \,+\, \hat\Pi_{AA}(0) \,-\, \mathrm{Re}\,\hat\Pi_{AA}(M_A^2) \\ &=\; \frac{\lambda\,v_s\,(A_\lambda + \kappa\,v_s)}{\sin\beta\cos\beta} \,-\, \frac{\hat\Pi_{12}(0)}{\sin\beta\cos\beta} \,+\, \hat\Pi_{AA}(0) \,-\, \mathrm{Re}\,\hat\Pi_{AA}(M_A^2) \end{aligned}$$

$$\hat\Pi_{12}(0) \;=\; \sin\beta\cos\beta\left[\hat\Pi_{AA}(0) - \hat\Pi_{HH}(0)\right] \,+\, \cos 2\beta\,\hat\Pi_{HA}(0)$$

Route 2: $$\sin\beta_0\cos\beta_0 \;=\; \frac{\lambda\,v_s\,(A_\lambda + \kappa\,v_s)}{m_A^2 - m_H^2}$$

$$\begin{aligned} M_A^2 \;&=\; m_A^2 \,-\, \mathrm{Re}\,\hat\Pi_{AA}(M_A^2) \\ &=\; \frac{\lambda\,v_s\,(A_\lambda + \kappa\,v_s)}{\sin\beta_0\cos\beta_0} \,+\, m_H^2 \,-\, \mathrm{Re}\,\hat\Pi_{AA}(M_A^2) \end{aligned}$$

$$\overline{M}_H^2 \;=\; m_H^2 - \hat\Pi_{HH}(0) \;=\; 0\,, \qquad \beta_0 \;=\; \beta \,+\, \frac{\hat\Pi_{HA}(m_H^2)}{m_H^2 - m_A^2}$$

How we get $\delta M_A^2 \;=\; -\mathrm{Re}\,\hat{\Pi}_{AA}(M_A^2) + \hat{\Pi}_{HH}(0) + 2\cot 2\beta\,\hat{\Pi}_{HA}(0)$

Starting point: $\begin{pmatrix} H \\ A \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} -\epsilon H_1^* \\ H_2 \end{pmatrix}$

Route 1:

$$\sin\beta\cos\beta \;=\; \frac{\lambda\, v_s\,(A_\lambda + \kappa\, v_s) - \hat{\Pi}_{12}(0)}{\overline{M}_A^2 - \cancel{\overline{M}_H^2}}$$

$$\begin{aligned} M_A^2 \;&=\; \overline{M}_A^2 \,+\, \hat{\Pi}_{AA}(0) \,-\, \mathrm{Re}\,\hat{\Pi}_{AA}(M_A^2) \\ &=\; \frac{\lambda\, v_s\,(A_\lambda + \kappa\, v_s)}{\sin\beta\cos\beta} \,-\, \frac{\hat{\Pi}_{12}(0)}{\sin\beta\cos\beta} \,+\, \hat{\Pi}_{AA}(0) \,-\, \mathrm{Re}\,\hat{\Pi}_{AA}(M_A^2) \end{aligned}$$

$$\hat{\Pi}_{12}(0) \;=\; \sin\beta\cos\beta\left[\hat{\Pi}_{AA}(0) - \hat{\Pi}_{HH}(0)\right] \,+\, \cos 2\beta\,\hat{\Pi}_{HA}(0)$$

Route 2:

$$\sin\beta_0\cos\beta_0 \;=\; \frac{\lambda\, v_s\,(A_\lambda + \kappa\, v_s)}{m_A^2 - m_H^2}$$

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Starting point: $\begin{pmatrix} H \\ A \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} -\epsilon H_1^* \\ H_2 \end{pmatrix}$

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$$\hat\Pi_{12}(0) = \sin\beta\cos\beta\left[\hat\Pi_{AA}(0) - \hat\Pi_{HH}(0)\right] + \cos 2\beta\,\hat\Pi_{HA}(0)$$

Route 2:

In our calculation we define the divergent part of the counterterm $\delta\beta$ according to eq. (6), but we choose instead to define the finite part in such a way that it removes entirely the contributions of the off-diagonal WFR of the Higgs doublets from the matching conditions for the effective couplings:

$$\delta\beta^{\,\mathrm{fin}} = \frac{\Pi^{\mathrm{fin}}_{HA}(m_H^2)}{m_H^2 - m_A^2}\,. \tag{7}$$

Loosely speaking, this defines the renormalized $\beta$ as the angle that diagonalizes the radiatively corrected Higgs mass matrix at an external momentum $p^2$ set equal to the light-Higgs mass parameter $m_H^2$ (in fact, the latter can be considered zero in comparison to $m_A^2$).

Bagnaschi, Giudice, Strumia & P.S., 1407.4081

$$\overline{M}_H^2 = m_H^2 - \hat\Pi_{HH}(0) = 0\,, \qquad \beta_0 = \beta + \frac{\hat\Pi_{HA}(\cancel{m_H^2})}{\cancel{m_H^2} - m_A^2}$$

How we get $\delta M_A^2 = -\mathrm{Re}\,\hat\Pi_{AA}(M_A^2) + \hat\Pi_{HH}(0) + 2\cot 2\beta\,\hat\Pi_{HA}(0)$

Starting point: $\begin{pmatrix} H \\ A \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} -\epsilon H_1^* \\ H_2 \end{pmatrix}$

Route 1:

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$$\hat\Pi_{12}(0) = \sin\beta\cos\beta\left[\hat\Pi_{AA}(0) - \hat\Pi_{HH}(0)\right] + \cos 2\beta\,\hat\Pi_{HA}(0)$$

Route 2:

$$\sin\beta_0\cos\beta_0 = \frac{\lambda\, v_s\,(A_\lambda + \kappa\, v_s)}{m_A^2 - m_H^2}$$

$$\begin{aligned} M_A^2 &= m_A^2 - \mathrm{Re}\,\hat\Pi_{AA}(M_A^2) \\ &= \frac{\lambda\, v_s\,(A_\lambda + \kappa\, v_s)}{\sin\beta_0\cos\beta_0} + m_H^2 - \mathrm{Re}\,\hat\Pi_{AA}(M_A^2) \end{aligned}$$

$$\overline{M}_H^2 = m_H^2 - \hat\Pi_{HH}(0) = 0\,, \qquad \beta_0 = \beta + \frac{\hat\Pi_{HA}(\cancel{m_H^2})}{\cancel{m_H^2} - m_A^2}$$