Precise predictions for the trilinear Higgs self-coupling in the Standard Model and beyond

predicting κ_λ in *any* model at the one-loop order

Henning Bahl, Johannes Braathen, **Martin Gabelmann**, Georg Weiglein

KUTS, Feb 2023

The SM scalar sector

> V_{SM} fixed at tree-level by $m_h \approx 125\,\mathrm{GeV}$ and $v = (\sqrt{2}G_F)^{-1/2} \approx 246\,\mathrm{GeV}$:

$$V_{\mathrm{SM}}(h) = \frac{m_h^2}{2}h^2 + 3\frac{m_h^2}{v}h^3 + 3\frac{m_h^2}{v^2}h^4$$
$$= \frac{m_h^2}{2}h^2 + \lambda_{hhh}h^3 + \lambda_{hhhh}h^4$$

> However: λ_{hhh} (and λ_{hhhh}) experimentally unknown.

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$$= \frac{m_h^2}{2}h^2 + \lambda_{hhh}h^3 + \lambda_{hhhh}h^4$$

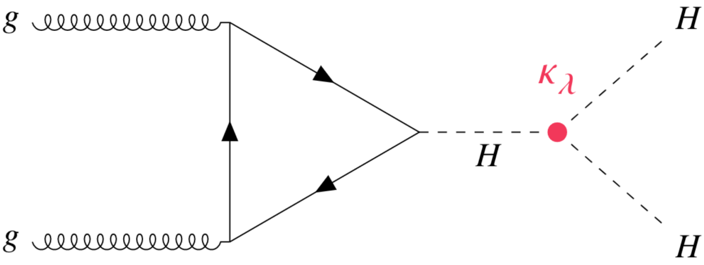
- However: λ_{hhh} (and λ_{hhhh}) experimentally unknown.
- BSM case: deformation of the scalar potential possible!

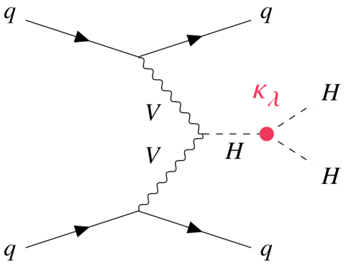
$$V_{\text{BSM}}(h,\dots) = \frac{m_h^2}{2}h^2 + 3\frac{m_h^2}{v}\kappa_\lambda^{\text{BSM}}h^3 + 3\frac{m_h^2}{v^2}\kappa_{2\lambda}h^4 + \dots$$

$\kappa_\lambda^{\text{BSM}}$: describes deviation from SM: $\kappa_\lambda^{\text{BSM}} = \frac{\lambda_{hhh}^{\text{BSM}}}{\lambda_{hhh}^{\text{SM}}}$ in model "BSM".

- $|\kappa_\lambda| \gg 1$ possible even though other (known) couplings behave very SM-like
- Example: "BSM"=THDM (talk by Johannes Braathen)
  $\rightarrow$ **higher-order corrections important**

Higgs pair production (in the alignment limit)





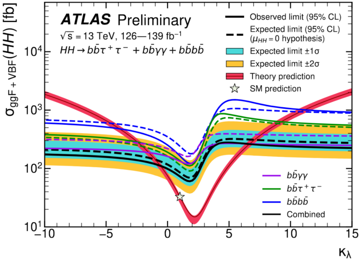
$pp \to hh$: $-0.4 < \kappa_\lambda < 6.3$ [ATLAS-CONF-2022-050]

When to apply the κ_λ-constraint to BSM models?

> only κ_λ is *significantly* modified by BSM physics

> all other couplings SM-like

$\to$ a scenario often enforced by experimental constraints



Higher-order corrections to $\lambda_{hhh}^{\text{BSM}}$

A few studies of the trilinear Higgs self-couplings (i.e. κ_λ) already exists:

- SM [Kanemura et al. '04]
- SM + singlet [Kanemura et al. '16]
- THDM [Kanemura et al. '04][Basler et al. '17][Braathen et al. '19] [Bahl et al. '22]
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- Triplet extensions [Aoki et al. '18] [Chiang et al. '18]
- MSSM [Brucherseifer et al. '13]
- NMSSM [Dao et al. '13] [Dao et al. '15][Dao et al. '22]

of which many can have sizeable deviations from $\kappa_\lambda = 1$ and hence also from $\sigma_{hh}^{\text{SM}}(\kappa_\lambda = 1)$ [Abouabid et al. '21]

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However, many more models to explore:

- THDM-variants
- singlet-variants
- extended gauge sectors
- extended fermion sectors
- non-minimal SUSY, e.g. seesaw extensions, dirac gauginos, SplitSUSY, ...
- (+ combinations)
- ...

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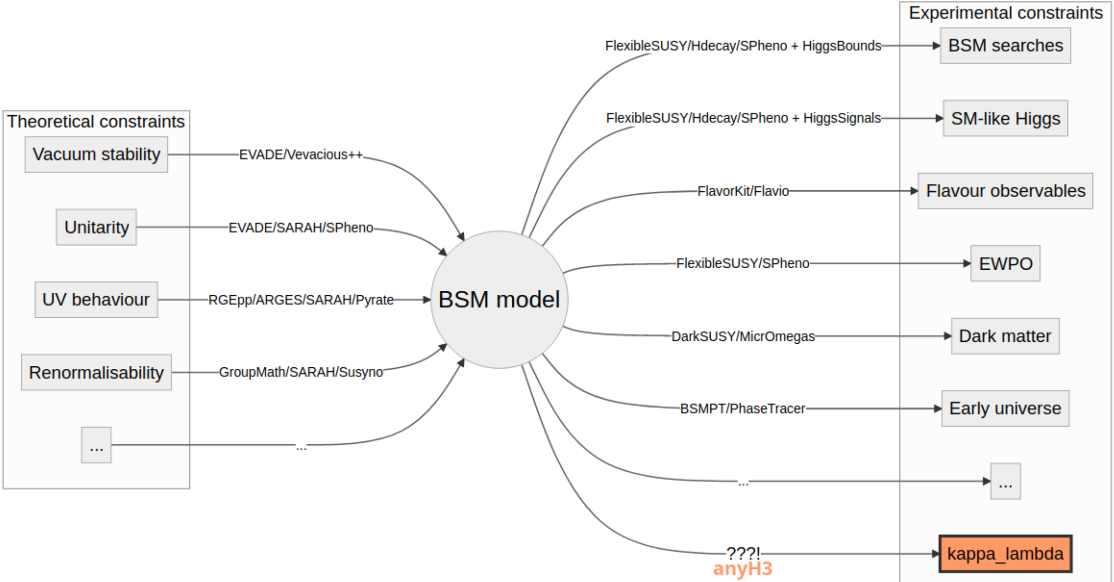
$\rightarrow$ **framework to calculate $\lambda_{hhh}^{\text{BSM}}$ for large class of "BSM's"**

$\lambda_{hhh}^{\text{BSM}}$ in the landscape of generic BSM tool-boxes



Strategy

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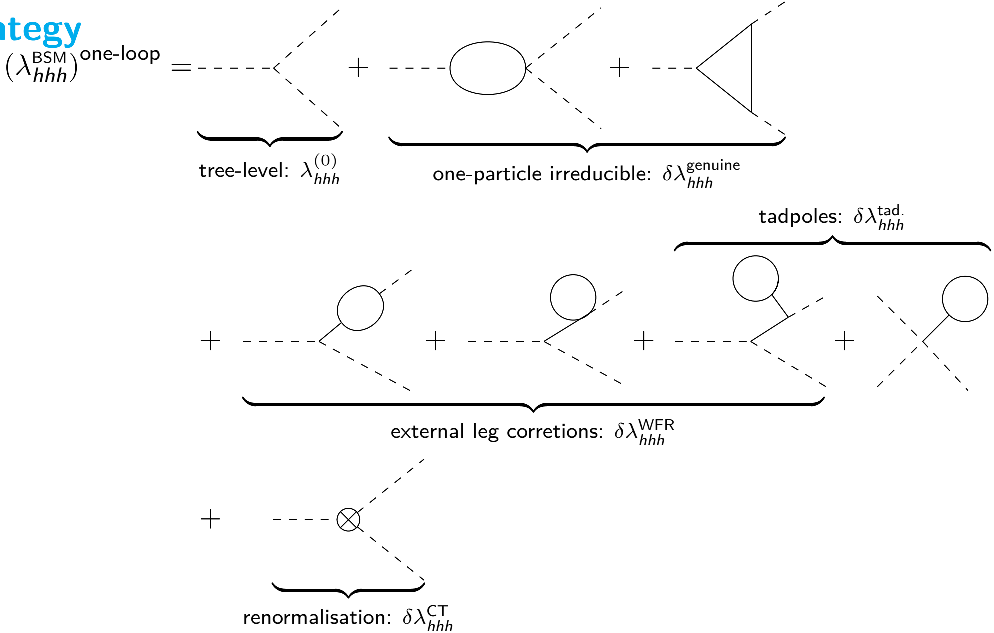
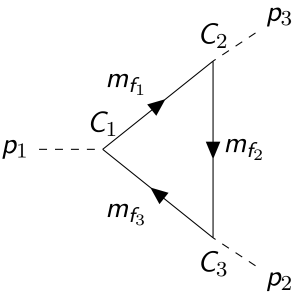


$(\lambda_{hhh}^{\mathrm{BSM}})^{\text{one-loop}} =$
$+$
tree-level: $\lambda_{hhh}^{(0)}$
one-particle irreducible: $\delta\lambda_{hhh}^{\text{genuine}}$
tadpoles: $\delta\lambda_{hhh}^{\text{tad.}}$
external leg corretions: $\delta\lambda_{hhh}^{\text{WFR}}$
renormalisation: $\delta\lambda_{hhh}^{\text{CT}}$

Example: generic fermion triangle

Idea: compute *generic* diagrams i.e. assume most generic



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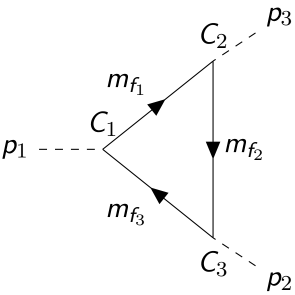

- couplings $C_i = P_L C_i^L + P_R C_i^R, \; P_{R/L} = \frac{1\pm\gamma_5}{2}$
- as well as loop-masses m_{f_i} and
- external momenta p_i, $i = 1, 2, 3$.

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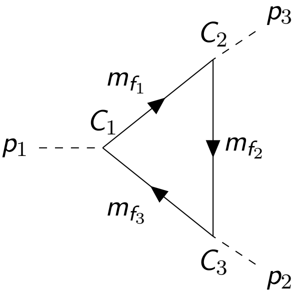

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- external momenta p_i, $i = 1, 2, 3$.

$$= 2\mathbf{B0}(p_3^2, m_2^2, m_3^2)(C_1^L(C_2^L C_3^R m_{f_1} + C_2^R C_3^R m_{f_2} + C_2^R C_3^L m_{f_3}) + C_1^R(C_2^R C_3^L m_{f_1} + C_2^L C_3^L m_{f_2} + C_2^L C_3^R m_{f_3})) + m_{f_1}\mathbf{C0}(p_2^2, p_3^2, p_1^2, m_1^2, m_3^2, m_2^2)((C_1^L C_2^L C_3^R + C_1^R C_2^R C_3^L)(p_1^2 + p_2^2 - p_3^2) + 2(C_1^L C_2^L C_3^L + C_1^R C_2^R C_3^R) m_{f_2} m_{f_3} + 2m_{f_1}(C_1^L(C_2^L C_3^R m_{f_1} + C_2^R C_3^R m_{f_2} + C_2^R C_3^L m_{f_3}) + C_1^R(C_2^R C_3^L m_{f_1} + C_2^L C_3^L m_{f_2} + C_2^L C_3^R m_{f_3}))) + \mathbf{C1}(p_2^2, p_3^2, p_1^2, m_1^2, m_3^2, m_2^2)(2p_2^2(C_1^L C_3^R(C_2^L m_{f_1} + C_2^R m_{f_2}) + C_1^R C_3^L(C_2^R m_{f_1} + C_2^L m_{f_2})) + (p_1^2 + p_2^2 - p_3^2)((C_1^L C_2^L C_3^R + C_1^R C_2^R C_3^L) m_{f_1} + (C_1^L C_2^R C_3^L + C_1^R C_2^L C_3^R) m_{f_3})) + \mathbf{C2}(p_2^2, p_3^2, p_1^2, m_1^2, m_3^2, m_2^2)((p_1^2 + p_2^2 - p_3^2)(C_1^L C_3^R(C_2^L m_{f_1} + C_2^R m_{f_2}) + C_1^R C_3^L(C_2^R m_{f_1} + C_2^L m_{f_2})) + 2p_1^2((C_1^L C_2^L C_3^R + C_1^R C_2^R C_3^L) m_{f_1} + (C_1^L C_2^R C_3^L + C_1^R C_2^L C_3^R) m_{f_3}))$$

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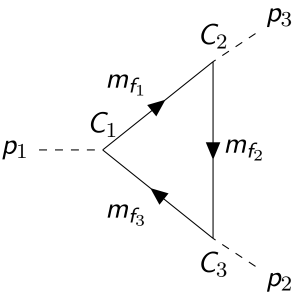


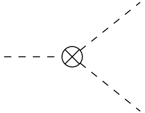
- couplings $C_i = P_L C_i^L + P_R C_i^R,\ P_{R/L} = \frac{1\pm\gamma_5}{2}$
- as well as loop-masses m_{f_i} and
- external momenta p_i, $i = 1, 2, 3$.

$$
\begin{aligned}
&= 2\mathbf{B0}(p_3^2, m_2^2, m_3^2)(C_1^L(C_2^L C_3^R m_{f_1} + C_2^R C_3^R m_{f_2} + C_2^R C_3^L m_{f_3}) + C_1^R(C_2^R C_3^L m_{f_1} + \\
&C_2^L C_3^L m_{f_2} + C_2^L C_3^R m_{f_3})) + m_{f_1}\mathbf{C0}(p_2^2, p_3^2, p_1^2, m_1^2, m_3^2, m_2^2)((C_1^L C_2^L C_3^R + \\
&C_1^R C_2^R C_3^L)(p_1^2 + p_2^2 - p_3^2) + 2(C_1^L C_2^L C_3^L + C_1^R C_2^R C_3^R) m_{f_2} m_{f_3} + \\
&2m_{f_1}(C_1^L(C_2^L C_3^R m_{f_1} + C_2^R C_3^R m_{f_2} + C_2^R C_3^L m_{f_3}) + C_1^R(C_2^R C_3^L m_{f_1} + C_2^L C_3^L m_{f_2} + \\
&C_2^L C_3^R m_{f_3}))) + \mathbf{C1}(p_2^2, p_3^2, p_1^2, m_1^2, m_3^2, m_2^2)(2p_2^2(C_1^L C_3^R(C_2^L m_{f_1} + C_2^R m_{f_2}) + \\
&C_1^R C_3^L(C_2^R m_{f_1} + C_2^L m_{f_2})) + (p_1^2 + p_2^2 - p_3^2)((C_1^L C_2^L C_3^R + C_1^R C_2^R C_3^L) m_{f_1} + \\
&(C_1^L C_2^R C_3^L + C_1^R C_2^L C_3^R) m_{f_3})) + \mathbf{C2}(p_2^2, p_3^2, p_1^2, m_1^2, m_3^2, m_2^2)((p_1^2 + p_2^2 - \\
&p_3^2)(C_1^L C_3^R(C_2^L m_{f_1} + C_2^R m_{f_2}) + C_1^R C_3^L(C_2^R m_{f_1} + C_2^L m_{f_2})) + 2p_1^2((C_1^L C_2^L C_3^R + \\
&C_1^R C_2^R C_3^L) m_{f_1} + (C_1^L C_2^R C_3^L + C_1^R C_2^L C_3^R) m_{f_3}))
\end{aligned}
$$

- insert concrete BSM model (`UFO`[Degrande et al. '11])
- evaluate with the help of `COLLIER`[Denner et al. '16]
- public code `anyH3`

  [Bahl, Braathen, MG, Weiglein tbp]

Renormalisation of λ_{hhh}

$\delta\lambda^{\mathrm{CT}}_{hhh} =$  $= \;\; ?$

- one-loop $\rightarrow$ renormalisation of all parameters entering $\lambda^{(0),\mathrm{BSM}}_{hhh}$
- reminder: $\lambda^{(0),\,\mathrm{SM}}_{hhh} = 3\frac{m_h^2}{v^2}$
- $\rightarrow$ generalization (reminder: quasi-alignment!):

$$\lambda^{(0),\mathrm{BSM}}_{hhh} \equiv \lambda^{(0),\mathrm{BSM}}_{hhh}\big(\underbrace{v^{\mathrm{SM}},\, m_h^{\mathrm{SM}}}_{\text{SM Higgs sector}},\, m_{H_i}^{\mathrm{BSM}},\, v_i^{\mathrm{BSM}},\, \ldots\big)$$

$m_{H_i}^{\mathrm{BSM}}$ → BSM masses; v_i^{BSM} → vevs; $\ldots$ → couplings etc. (that can't be expressed in terms of masses)

- user's choice:
  - SM sector: fully OS **or** $\overline{\mathrm{MS}}/\overline{\mathrm{DR}}$
  - BSM masses: OS **or** $\overline{\mathrm{MS}}/\overline{\mathrm{DR}}$
  - additional couplings/vevs: most likely $\overline{\mathrm{MS}}$ **but also custom ren. conditions possible!**

$$\delta\lambda^{\mathrm{CT}}_{hhh} = \sum_x \frac{\partial\lambda^{(0),\mathrm{BSM}}_{hhh}}{\partial x}\delta^{\mathrm{CT}}x,\;\; x = (m_h^{SM})^{\mathrm{OS}/\overline{\mathrm{MS}}},\, (v^{\mathrm{SM}})^{\mathrm{OS}/\overline{\mathrm{MS}}},\, (m_{H_i}^{\mathrm{BSM}})^{\mathrm{OS}/\overline{\mathrm{MS}}},\, (\ldots)^{\overline{\mathrm{MS}}/\mathrm{custom}}$$

(Default) Renormalization choice of $(v^{\text{SM}})^{\text{OS}}$ and $(m_i^2)^{\text{OS}}$

- $v^{\text{OS}} \equiv \frac{2M_W^{\text{OS}}}{e}\sqrt{1-\frac{M_W^{2\,\text{OS}}}{M_Z^{2\,\text{OS}}}}$ with
  - $\delta^{(1)} M_V^{2\,\text{OS}} = \frac{\text{Re}\Pi_V^{(1),T}}{M_V^{2\,OS}}(p^2 = M_V^{2\,OS})\,, V = W, Z$
  - $\delta^{(1)} e^{\text{OS}} = \frac{1}{2}\Pi_\gamma(p^2=0) + \text{sign}(\sin\theta_W)\frac{\sin\theta_W}{M_Z^2\cos\theta_W}\Pi_{\gamma Z}(p^2=0)$
- attention *(i)*: $\rho^{\text{tree-level}} \neq 1 \to$ further CTs needed (depends on the model)

  $\to$ ability to define *custom* renormalisation conditions
- scalar masses: $m_i^{\text{OS}} = m_i^{\text{pole}}$
  - $\delta^{\text{OS}} m_i^2 = -\text{Re}\Sigma_{h_i}^{(1)}|_{p^2=m_i^2}$
- attention *(ii)*: scalar mixing may also require further CTs/tree-level relations

All bosonic one- & two-point functions and their derivatives for general QFTs are required for flexible OS renormalisation.

Treatment of external leg corrections

default treatment of external legs:

$$\delta^{(1)\,\text{,ext.-legs}}\lambda_{hhh} = -\sum_i \left[\frac{1}{2}\Sigma'_{hh}(p_i^2)\lambda^{(0)}_{hhh} + \underbrace{\sum_{j,h_j\neq h} \frac{\Sigma_{hh_j}(p_i^2)}{p_i^2 - m_{h_j}^2}\lambda^{(0)}_{h_j hh}}_{=0,\text{ for alignment}} \right]$$

- Attention: insert into di-Higgs production:
  need one off- and two on-shell Higgses:

$$\delta^{(1)\text{,ext.-legs}}\lambda_{hhh} = -\left(\frac{1}{2} + \frac{1}{2}\right)\Sigma'_{hh}(m_h^2)\lambda^{(0)}_{hhh}$$

- possible to turn-off default behaviour and specify ext.-leg contributions in terms of selfenergies

Treatment of tadpole corrections

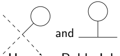
w/o specifying a concrete scheme, nor the vacuum (in the alignment limit):

$$\lambda_{hhh}^{\text{tadpoles}} = -\underbrace{\frac{3t_h}{v^2}}_{\text{tree-level}} - \underbrace{\frac{6}{v^2}\delta^{(1)}_{\text{CT}} t_h}_{\text{CT-inserterd diagrams}} + \underbrace{\delta^{(1)}_{\text{tadpoles}}\lambda_{hhh}}_{\text{tadpole diagrams}} + \underbrace{\frac{3}{v}\delta^{(1)}_{\text{CT, tadpoles}} m_h^2 - \frac{3m_h^2}{v^2}\delta^{(1)}_{\text{CT, tadpoles}} v}_{\text{tad. contr. to input parameters}}$$

- In the SM (and BSM+alignment): once λ_{hhh} is expressed in terms of *physical* input parameters, its result is independent of the treatment (OS, FJ, ...) of the tadpoles (up to higher orders):

$$\delta^{(1)}\lambda_{hhh} \supset \frac{3}{v^2}\delta^{(1)} t_h|_{\text{finite}}$$

- However: `UFO` models do (often) **not** contain the explicit dependence on the tree-level/one-loop/one-loop-CT tadpoles.
- Thus: we choose the Fleischer-Jegerlehner treatment $t_h^{\text{tree-level}} = 0$ and renormalize $\delta^{(1)} t_h^{\text{CT}}|_{\text{finite}} = 0$ in the $\overline{\text{MS}}$ scheme per default (can also turn-off automatic tadpoles and implement own scheme).
- only need to take into account tadpole contributions

  to all two- and three-point functions: 

Feature list (so far) of `anyH3`

- import/convert arbitrary `UFO` models
- definition of renormalisation schemes

```
# schemes.yml
renormalization_schemes:
  OS:
      mass_counterterms:
        h1: OS
        h2: OS
      VEV_counterterm: OS
  MS:
      mass_counterterms:
        h1: MS
        h2: MS
      VEV_counterterm: MS
```

- optional: full p^2 dependence
- numerical / analytical / LaTeX outputs
- restrict to certain topologies
- restrict to certain particles in the loop
- python-library with command-line- and Mathematica-interface

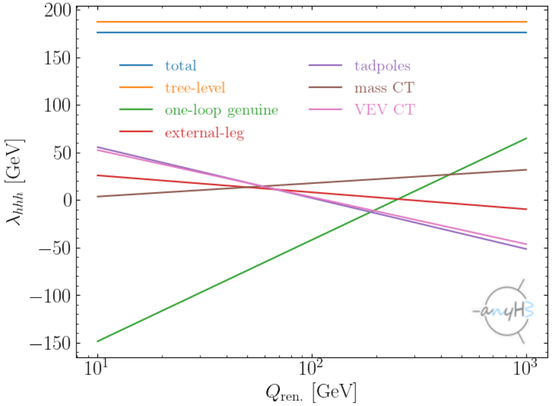
```
from anyBSM import anyH3
myfancymodel = anyH3(
    'path/to/UFO/model',
    scheme = 'OS')
result = myfancymodel.lambdahhh()
```

- ...

Validation/checks and some results

Simple cross-check: UV-finiteness in the SM

Numerically:



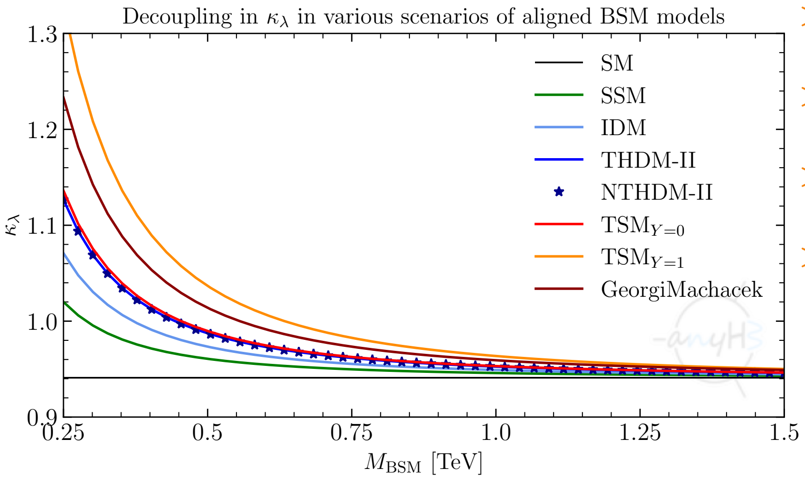
two-loop $\mathcal{O}(\alpha_t(\alpha_t+\alpha_s))^{\text{OS}}$: $\mathcal{O}(+1.4\%)$

[Senaha '18] [Braathen et al. '19]

Analytically:

```
<< anyBSM`
LoadModel["SM"]
lam = lambdahhh[];
(lam["total"] - lam["treelevel"] //. UVparts // Simplify) == 0
True
```

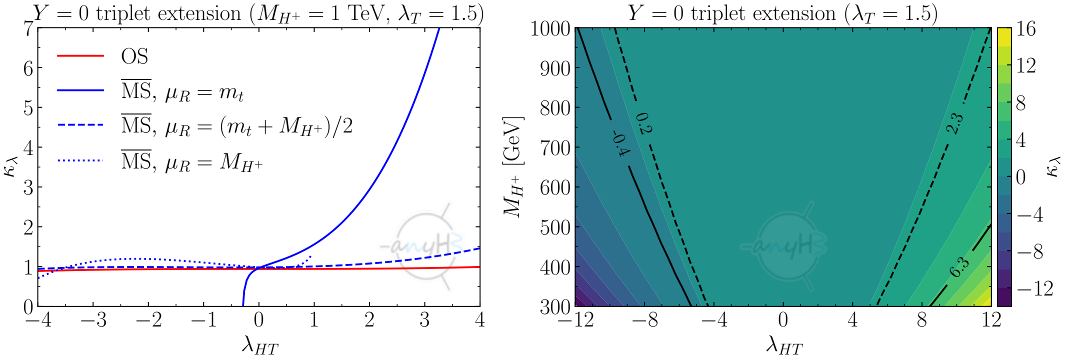
Another check: decoupling in the alignment limit



- alignment means $\kappa_\lambda^{\text{tree-level}} = 1$
- recover SM result for $M_{\text{BSM}} \to \infty$
- many models built-in and cross-checked
- easy to implement new models (`UFO`)

Renormalization scheme comparison for a real Triplet)

$$V(\Phi, T) = \mu^2|\Phi|^2 + \frac{\lambda}{2}|\Phi|^4 + \frac{M_T^2}{2}|T|^2 + \frac{\lambda_T}{2}|T|^4 + \frac{\lambda_{HT}}{2}|T|^2|\Phi|^2, \;\; \langle T\rangle = 0, \, \langle\Phi\rangle = v_{SM}$$



→ natural choice of μ_R closest to OS result

Summary

- developed computer code `anyH3` (`anyBSM`) for λ_{hhh} in arbitrary ren. QFTs
  - at the full one-loop order
  - with arbitrary choice of renormalization schemes
- uses `UFO` input (generate with `SARAH`, `FeynRules` or use a custom one)
- analytical results; fast numerical results; SM: $\mathcal{O}(0.2\,\mathrm{s})$, MSSM: $\mathcal{O}(0.5\,\mathrm{s})$
- many models already implemented:
  SM, SM+**singlet, THDM, NTHDM**, various **triplet extensions** and **MSSM**
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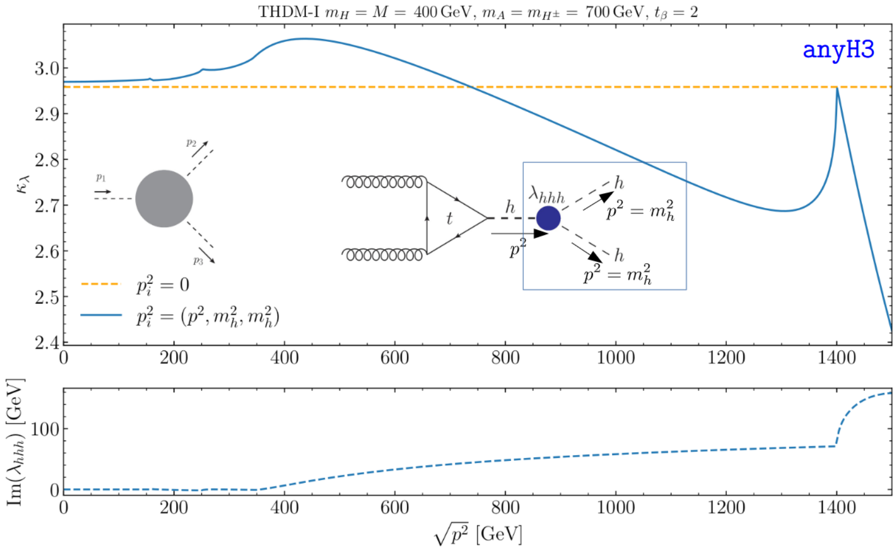
Future todos:

- publish
- more models / cross-checks
- go beyond one-loop (see Johannes' talk)
- non-SM self-couplings (e.g. $\kappa_{\lambda_{hhH}}$)
- κ_t and κ_{tt}
-

Backup

Example: Momentum dependence in the THDM



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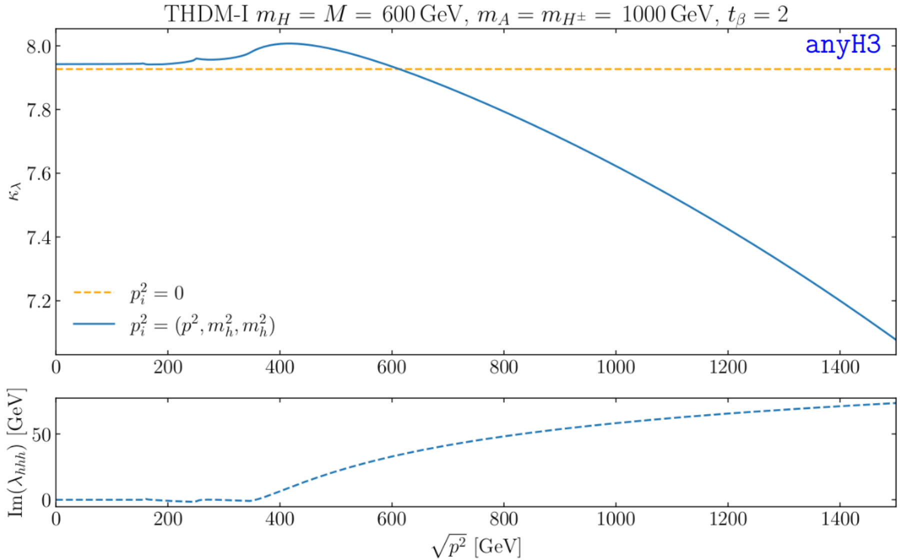
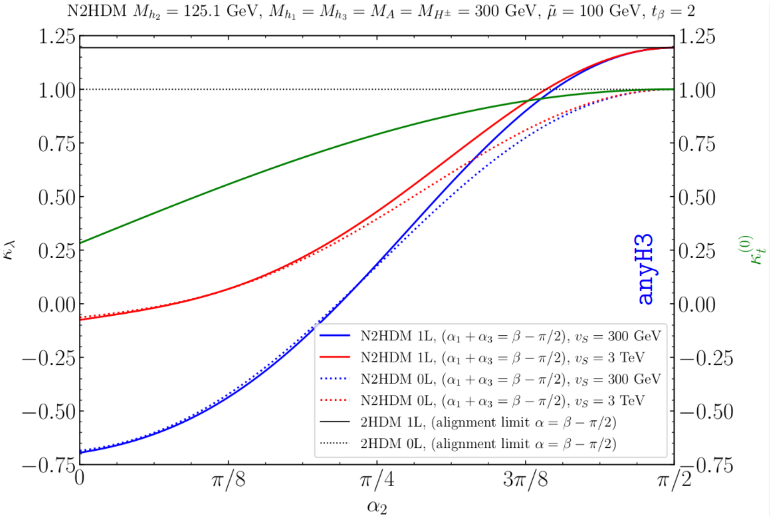


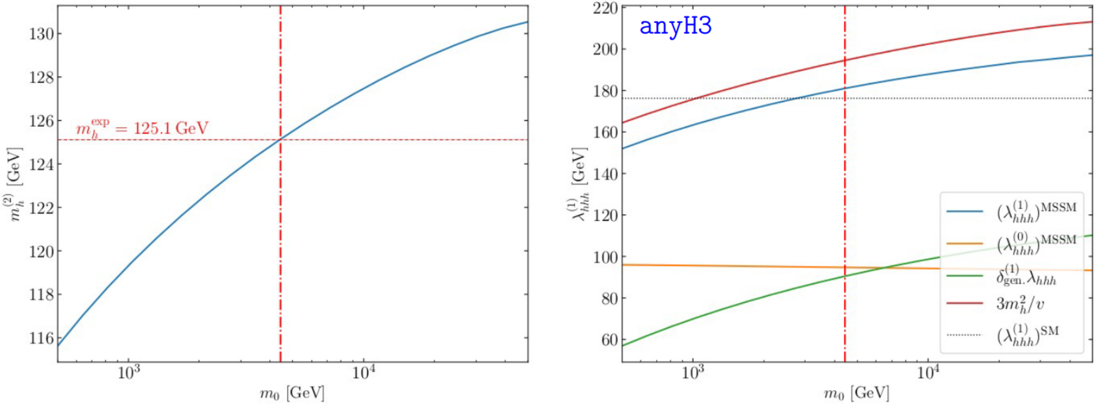
The sign of κ_λ



- **N2HDM = 2HDM + real singlet**
- CP-even sector: 3 states mixing, with 3 mixing angles α_1, α_2, α_3
- Here $\alpha_2 \to \pi/2 \to$ recover 2HDM (itself in alignment limit)
- We can study e.g. the relative sign of κ_λ and κ_t → affects double-Higgs production
- κ_t too far away from 1 excluded

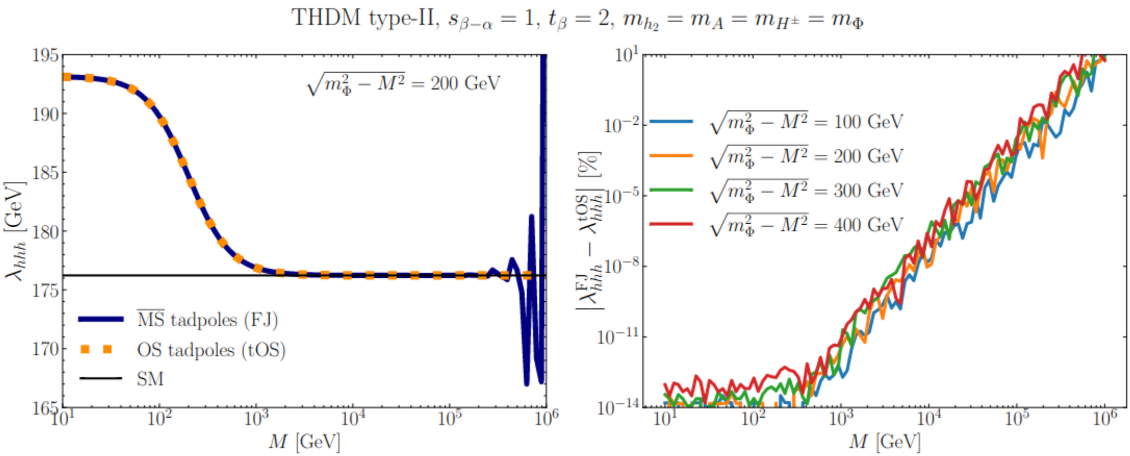
Full MSSM result: interface to SPheno

CMSSM, $m_0 = m_{1/2} = -A_0$, $\tan\beta = 10$, $\text{sgn}(\mu) = 1$, with m_h computed at 2L in SPheno

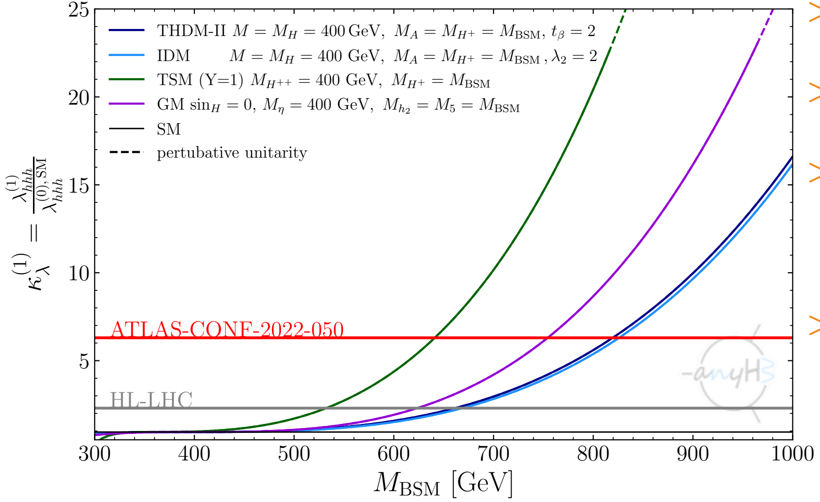


- Example for a very simple version of the constrained MSSM → BSM parameters m_0, $m_{1/2}$, A_0, sgn(μ), tanβ
- For each point, M_h computed at 2L with SPheno, and SLHA output of SPheno used as input of anyH3

OS vs FJ tadpole treatment

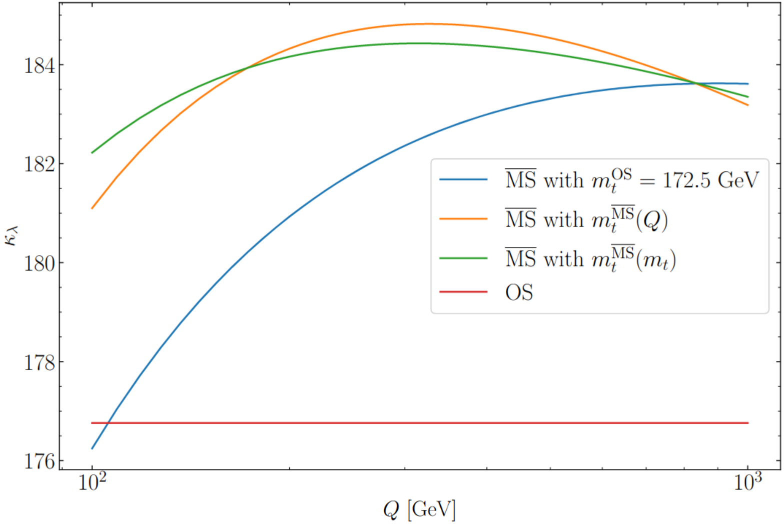


Non-decoupling in the alignment limit

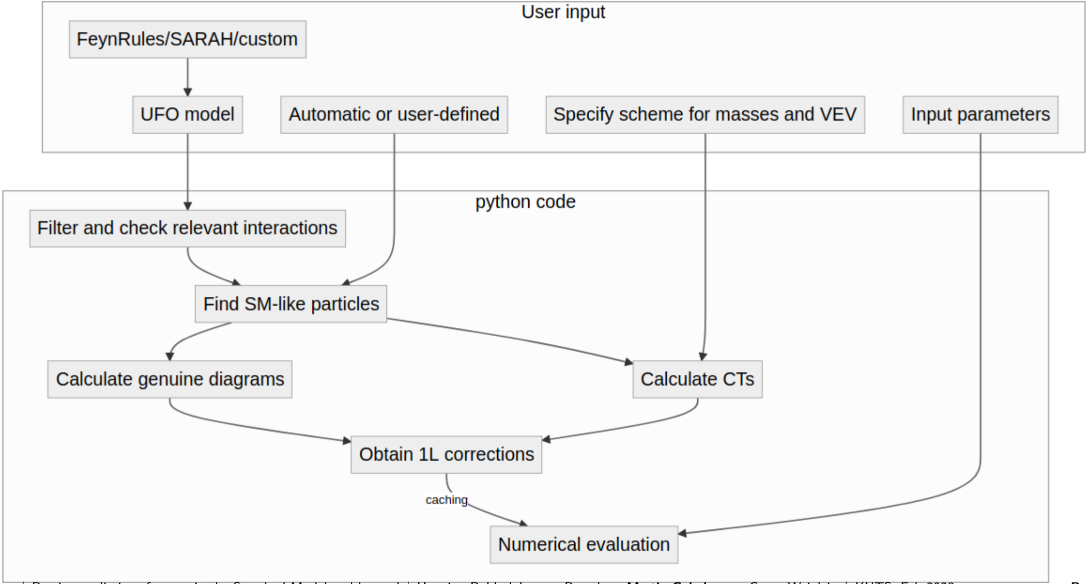


- mass spliting within the same multiplet
- induces large couplings for $M_{\text{BSM}} \to \infty$
- corrections large-enough to exclude otherwise unconstrained parameter space
- (see also previous talk by Johannes)

Scheme- and top-mass- uncertainty



Workflow



User input
FeynRules/SARAH/custom
UFO model
Automatic or user-defined
Specify scheme for masses and VEV
Input parameters
python code
Filter and check relevant interactions
Find SM-like particles
Calculate genuine diagrams
Calculate CTs
Obtain 1L corrections
caching
Numerical evaluation

W Mass

- start with HO corrections to muon decay: $M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)=\frac{\pi\alpha_{\text{em}}}{\sqrt{2}G_F}\left[1+\Delta r\right]$
- and solve for: $M_W^2=M_Z^2\left[\frac{1}{2}+\sqrt{\frac{1}{4}-\frac{\pi\alpha_{\text{em}}}{\sqrt{2}G_F M_Z^2}(1+\Delta r)}\right]$
- with: $\Delta r^{(1)}=2\delta^{(1)}e+\frac{\Pi_W^{(1),T}(0)-\delta^{(1)}M_W^2}{M_W^2}-\frac{\delta^{(1)}\sin^2\theta_W}{\sin^2\theta_W}+\delta_{\text{vertex+box}}$
- and: $\frac{\delta^{(1)}\sin^2\theta_W}{\sin^2\theta_W}=\frac{\cos^2\theta_W}{\sin^2\theta_W}\left(\frac{\Pi_W^{(1),T}(M_W^2)}{M_W^2}-\frac{\Pi_W^{(1),T}(M_W^2)}{M_W^2}\right)$

It's all there but:

- $\delta^{\text{SM}}_{\text{vertex+box}}=-\frac{2\,\text{sign}(\sin\theta_W)}{\cos\theta_W\sin\theta_W M_Z^2}\Pi_{Z\gamma}(p^2=0)+\frac{\alpha_{QED}}{4\pi\sin^2\theta_W}\left(6+\frac{7-4\sin^2\theta_W}{2\sin^2\theta_W}\right)\log(\cos^2\theta_W)$
- $\delta^{\text{BSM}}_{\text{vertex+box}}=$ **needs to be implemented**

However:

- in many models $\Delta r\supset\frac{\delta\sin^2\theta_W}{\sin\theta_W}\approx\delta\rho$ is the dominant effect!