External-leg corrections as an origin of large logarithms

Based on
arXiv:2112.11419 (JHEP), and 2212.11213 in collaboration with Henning Bahl and Georg Weiglein

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Outline

- Introduction (motivation, large logs, external-leg corrections)

- Large logs from external-leg corrections [Bahl, JB, Weiglein ‘21]
  - Discussion for a toy model
    - 1L case
    - IR divergences
    - 2L case
    - Numerical results
  - Examples in MSSM and N2HDM

- Stop mixing parameter X_t [Bahl, JB, Weiglein ‘22]
    - Experimental probes
    - Choices of renormalisation schemes

- Conclusions

Introduction

Motivation

- Numerous reasons to expect BSM physics (e.g. DM, baryon asymmetry, hierarchy problem)

- BSM theories commonly involve **additional scalars**, e.g.
  - ***Extended Higgs sectors*** → bottom-up extensions of the SM (singlet extensions, 2HDM, N2HDM, ...), supersymmetric models (MSSM, NMSSM, ...)
  - ***Scalar partners*** → SUSY, ...

- To correctly determine the viable parameter space of BSM models, and assess discovery sensitivities of BSM scalars, **precise theory predictions for the production and decay processes of the new scalars are needed**

- Lack of experimental results tends to **favour heavier BSM states**
  (light states with small couplings to SM also possible, but we won't consider this in this talk)

Large logarithms

- Calculations in QFT notoriously known to be plagued by (potential) **large logs**, when **widely separated mass scales** are present

- Among the possible types of large logarithms:
  - Logs involving ratio of high and low mass scales, in calculation of quantity/observable at low scale, e.g. $\log(M_{SUSY}/m_t)$ in SUSY Higgs mass calculations (see review [Slavich, Heinemeyer, et al. '20])
    → Solution: *Resummation of logs via Effective Field Theory*

  - Sudakov logarithms in QCD
    → Solution: *exponentiation, or Soft-Collinear Effective Theory (SCET)*

  - Electroweak Sudakov logarithms (related to exchange of Z, W, h)
    → Solution: *SCET*

e.g.

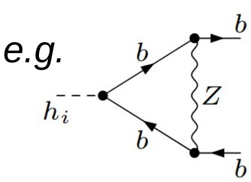


Fig. from [Domingo, Paßehr '20]

Here: we point out a new type of large, Sudakov-like, logarithms appearing in external-leg corrections involving heavy scalars

Large logarithms from external legs: toy model example

A simple toy model

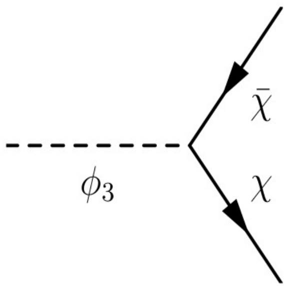
- Three scalars Φ_1, Φ_2, Φ_3, and a Dirac fermion χ
- Z_2-symmetry (unbroken): $\Phi_1 \rightarrow -\Phi_1$, $\Phi_2 \rightarrow -\Phi_2$, $\Phi_3 \rightarrow \Phi_3$, $\chi \rightarrow \chi$
- Consider a hierarchy where $m_1 \ll m_2, m_3$

$$\begin{aligned}\mathcal{L}_{\text{int.}} =& -\frac{1}{2}A_{113}\phi_1^2\phi_3 - A_{123}\phi_1\phi_2\phi_3 - \frac{1}{2}A_{223}\phi_2^2\phi_3 - \frac{1}{6}A_{333}\phi_3^3 \\ & -\frac{1}{24}\lambda_{1111}\phi_1^4 - \frac{1}{6}\lambda_{1112}\phi_1^3\phi_2 - \frac{1}{4}\lambda_{1122}\phi_1^2\phi_2^2 - \frac{1}{6}\lambda_{1222}\phi_1\phi_2^3 - \frac{1}{24}\lambda_{2222}\phi_2^4 \\ & -\frac{1}{4}\lambda_{1133}\phi_1^2\phi_3^2 - \frac{1}{2}\lambda_{1233}\phi_1\phi_2\phi_3^2 - \frac{1}{4}\lambda_{2233}\phi_2^2\phi_3^2 - \frac{1}{24}\lambda_{3333}\phi_3^4 \\ & + y_3\phi_3\bar{\chi}\chi\end{aligned}$$

- Only Φ_3 can couple to the fermions
- Main focus: trilinear couplings, in particular $\mathbf{A_{123}}$ (light-heavy-heavy coupling)

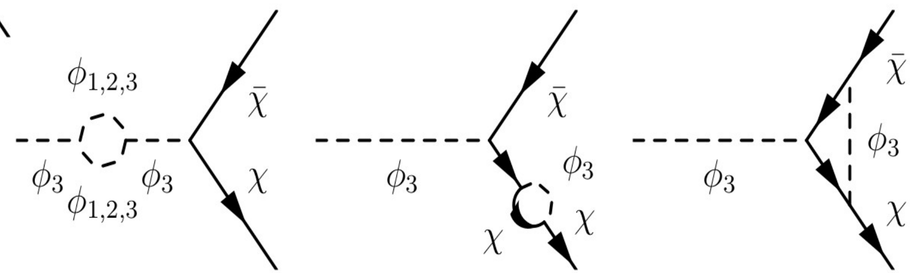
The $\phi_3 \to \chi\bar{\chi}$ decay process

- Consider the decay of Φ_3 into 2 fermions χ (prototype of scalar → 2 fermions, or fermion → scalar-fermion decays)
- Tree level:



$$\Gamma^{(0)}(\phi_3 \to \chi\bar{\chi}) = \frac{1}{8\pi} m_3 y_3^2 \left(1 - \frac{4m_\chi^2}{m_3^2}\right)^{3/2}$$

- 1L virtual corrections:



- ***Corrections involving A_{ijk}*** → no vertex corrections, no mixing contributions

$$\Delta\hat{\Gamma}^{(1)}_{\phi_3\to\bar{\chi}\chi} \supset -\frac{1}{2}k\mathrm{Re}\Bigg[(A_{113})^2\frac{d}{dp^2}B_0(p^2,m_1^2,m_1^2) + 2(A_{123})^2\frac{d}{dp^2}B_0(p^2,m_1^2,m_2^2)$$

$$(k \equiv 1/(16\pi^2))$$

$$+(A_{223})^2\frac{d}{dp^2}B_0(p^2,m_2^2,m_2^2) + (A_{333})^2\frac{d}{dp^2}B_0(p^2,m_3^2,m_3^2)\Bigg]\Bigg|_{p^2=m_3^2} + \cdots,$$

(with $\hat{\Gamma}(\phi_3 \to \chi\bar{\chi}) = \Gamma^{(0)}(\phi_3 \to \chi\bar{\chi})[1 + \Delta\hat{\Gamma}_{\phi_3\to\chi\bar{\chi}}]$)

(B_0: usual Passarino-Veltmann function)

Infrared limits

Derivative of the light-heavy B_0 loop function can become **IR divergent** if:

- Φ_1 is light, and Φ_2,Φ_3 are almost mass-degenerate, i.e. $m_1 \to 0$, $m_2 \to m_3$

$$\frac{d}{dp^2} B_0(p^2, m_1^2, m_2^2)\big|_{p^2=m_3^2} = \frac{1}{m_3^2}\left(\frac{1}{2}\ln\frac{m_3^2}{m_1^2} - 1 + \mathcal{O}\left(\epsilon^{1/2}\right)\right)$$

with $\epsilon = m_3^2 - m_2^2$. IR divergence **regulated by m_1**.

- Φ_1 is massless, and Φ_2,Φ_3 are almost mass-degenerate, i.e. m_1=0, $m_2 \to m_3$

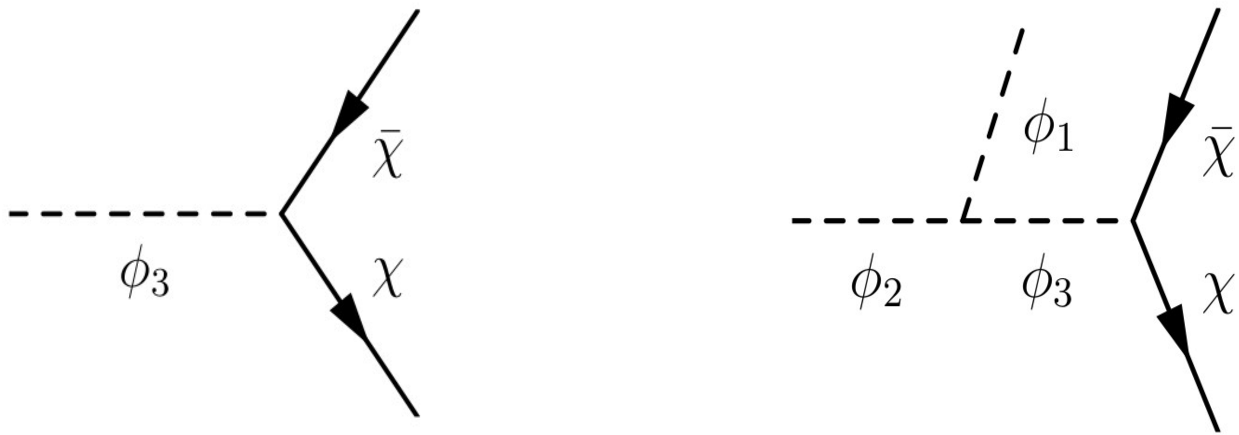
$$\frac{d}{dp^2} B_0(p^2, 0, m_2^2)\big|_{p^2=m_3^2} = \frac{1}{m_3^2}\left(\ln\left(-\frac{m_3^2}{\epsilon}\right) - 1 + \mathcal{O}\left(\epsilon\right)\right)$$

IR divergence **regulated by squared-mass difference** $\epsilon = m_3^2 - m_2^2$

Curing the IR divergences at 1L – inclusion of real radiation

In mass scenario where $m_1 \to 0$, $m_2 = m_3$

- Inclusion of real radiation, following Kinoshita-Lee-Nauenberg theorem [Kinoshita '62], [Lee, Nauenberg '64]
  →IR divergence interpreted as stemming from lack of inclusiveness of observable



- Φ_1 radiation not possible from an initial Φ_3 in $\Phi_3 \to \chi\bar{\chi}$ process (would break Z_2 symmetry)
  … but KLN theorem requires summing on *energy degenerate states* and Φ_2 can radiate a Φ_1

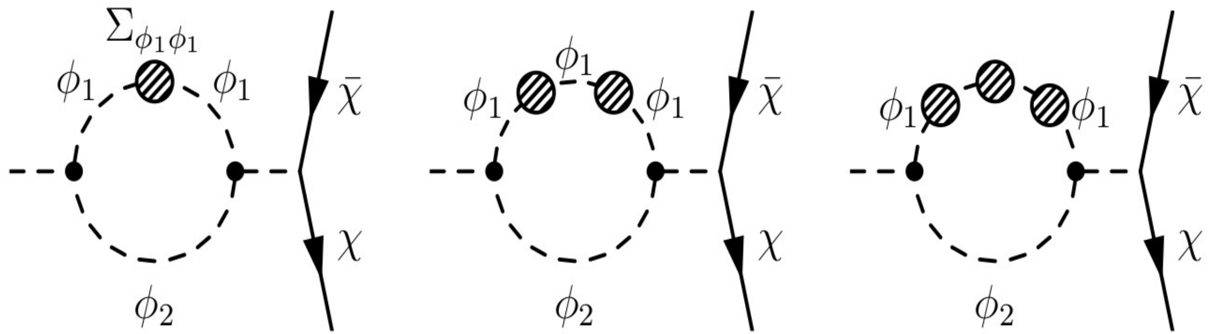
$$\Gamma(\phi_3 \to \chi\bar{\chi}) + \Gamma(\phi_2 \to \phi_1\chi\bar{\chi})|^{\text{soft}} = \text{finite}$$

- $\Gamma(\Phi_2 \to \Phi_1\chi\bar{\chi})|^{\text{soft}}$ contains dependence on energy resolution E_l but this can be removed when including also hard radiation (3-body phase space computed numerically)

Curing the IR divergences at 1L – resummation

In mass scenario where $m_1 \to 0$, $m_2 = m_3$

- Resummation of Φ_1 contributions (inspired by one of the solutions to Goldstone boson catastrophe [Martin '14], [Elias-Miro, Espinosa, Konstandin '14], [JB, Goodsell '16], [Espinosa, Konstandin '17])



→**IR divergence interpreted as stemming from a breakdown of the perturbative expansion**, because in scenarios with large hierarchies, the mass of light scalar Φ_1 receives very significant loop corrections, and thus diagrams with $\Sigma_{\Phi_1\Phi_1}$ subloop insertions are very large

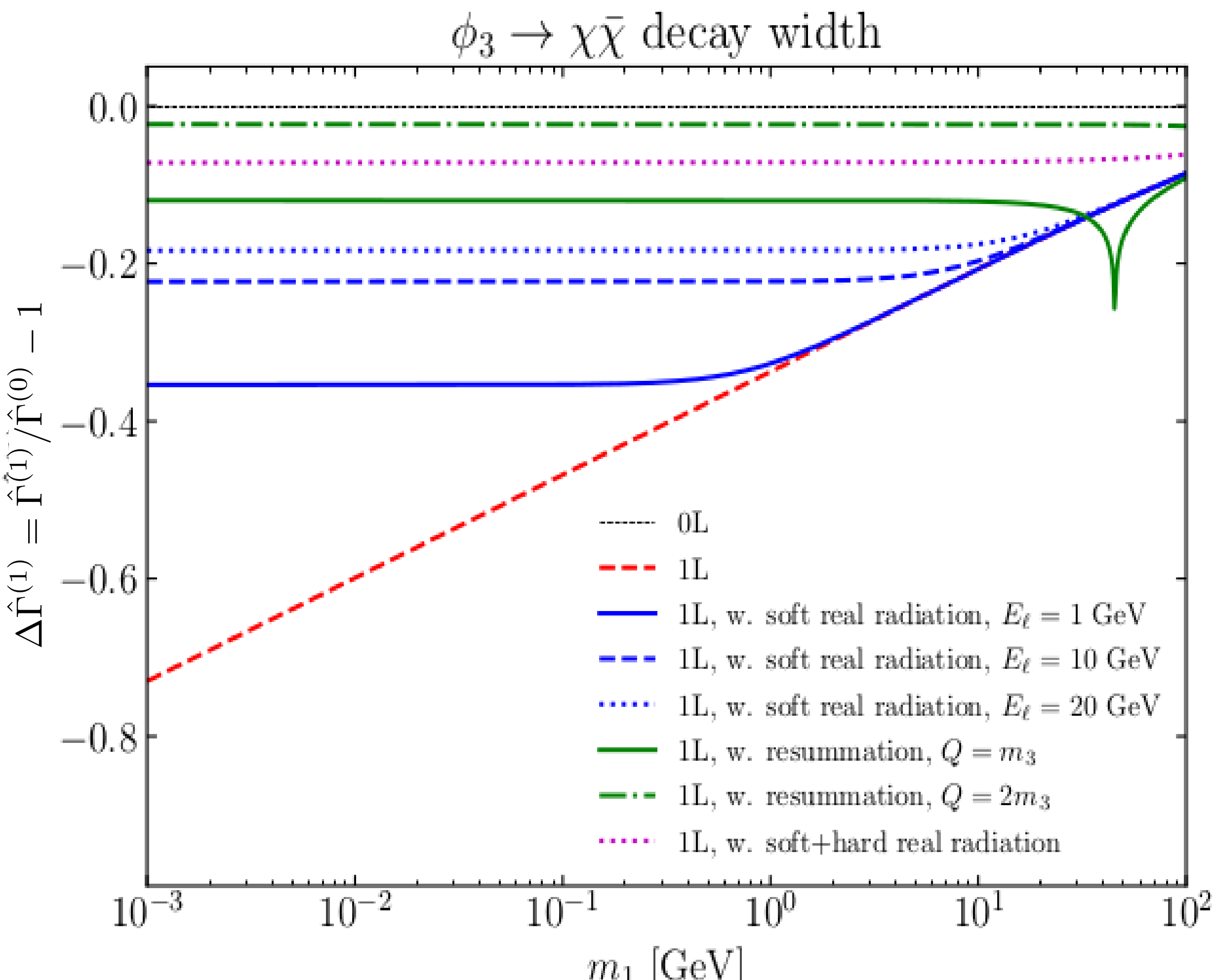
→ resummation produces an effective mass for Φ_1

$$\Delta m_1^2 = \hat{\Sigma}^{(1)}_{\phi_1\phi_1}(p^2=0) = -k\left[\frac{1}{2}\lambda_{1122}A_0(m_2^2) + \frac{1}{2}\lambda_{1133}A_0(m_3^2) + (A_{113})^2 B_0(0,0,m_3^2) + (A_{123})^2 B_0(0,m_2^2,m_3^2)\right]$$
$$= \mathcal{O}(km_3^2)$$

(A_0, B_0: usual Passarino-Veltmann functions)

Curing the IR divergences at 1L – results

In mass scenario where $m_1 \to 0$, $m_2=m_3$



with
A_{123} =3 TeV
(other A_{ijk}=0)
y_3=1,
$m_2 = m_3$ = 1 TeV,
m_χ =200 GeV,
λ_{1122} = 0.25,
λ_{1133} = 0.4.

(NB: at 1L, including the width of ϕ_3 would also cure the IR divergence, but one can devise a model where the width is zero)

Remaining large logarithms

- **Divergences in IR limit can be cured**
  - Resummation (but physical meaning of resummed decay width is ambiguous)
  - Inclusion of (soft) real radiation

- However, if m_1 (or ε) is large enough, then $\Phi_3 \to \chi\bar{\chi}$ and $\Phi_2 \to \chi\bar{\chi}\Phi_1$ can be distinguished!

- 1L corrections to $\Phi_3 \to \chi\bar{\chi}$ decay width contain a term of the form

$$\Delta\hat{\Gamma}^{(1)} \supset -\frac{1}{2} k y_3 \frac{A_{123}^2}{m_3^2} \ln \frac{m_3^2}{m_1^2}$$

- Trilinear couplings involving heavy states Φ_2, Φ_3 typically of the order of the heavy mass $\mathbf{A_{123} \sim m_3}$

→ Large, unsuppressed, logarithm remains in $\Delta\Gamma^{(1)}$!

- What happens at 2L?

External-leg corrections at 2L – setup of the calculation

- For the 2L calculation, we fix the mass scales as

$$m_1^2 = \epsilon \quad \ll \quad m_2^2 = m_3^2 = m^2$$

- Radiative corrections to decay width, from external-leg corrections, up to 2L

$$\hat{\Gamma}(\phi_3 \to \chi\bar{\chi}) = \Gamma^{(0)}(\phi_3 \to \chi\bar{\chi})\Big\{1 - \mathrm{Re}\hat{\Sigma}_{33}^{(1)\prime}(m^2) - \mathrm{Re}\hat{\Sigma}_{33}^{(2)\prime}(m^2) + \big(\mathrm{Re}\hat{\Sigma}_{33}^{(1)\prime}(m^2)\big)^2 - \frac{1}{2}\big(\mathrm{Im}\hat{\Sigma}_{33}^{(1)\prime}(m^2)\big)^2 + \mathrm{Im}\hat{\Sigma}_{33}^{(1)}(m^2) \cdot \mathrm{Im}\hat{\Sigma}_{33}^{(1)\prime\prime}(m^2) + \mathcal{O}(k^3)\Big\}$$

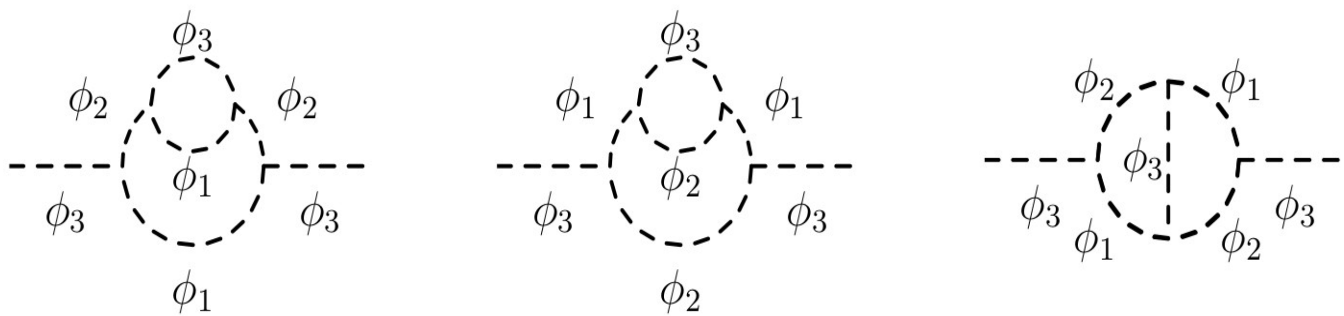
- We consider in the following only terms of $O(A_{123}{}^4)$

External-leg corrections at 2L

$$\hat{\Gamma}(\phi_3 \to \chi\bar{\chi}) = \Gamma^{(0)}(\phi_3 \to \chi\bar{\chi})\Big\{1 - \text{Re}\hat{\Sigma}_{33}^{(1)\prime}(m^2) - \text{Re}\hat{\Sigma}_{33}^{(2)\prime}(m^2) + \left(\text{Re}\hat{\Sigma}_{33}^{(1)\prime}(m^2)\right)^2 - \frac{1}{2}\left(\text{Im}\hat{\Sigma}_{33}^{(1)\prime}(m^2)\right)^2 + \text{Im}\hat{\Sigma}_{33}^{(1)}(m^2)\cdot\text{Im}\hat{\Sigma}_{33}^{(1)\prime\prime}(m^2) + \mathcal{O}(k^3)\Big\}$$

- Genuine 2L O(A_{123}^4) corrections involve derivatives of 2L self-energy diagrams ($m_1^2 = \epsilon \ll m_2^2 = m_3^2 \equiv m^2$)

$$\hat{\Sigma}_{33}^{(2,\text{ genuine})\prime}(p^2 = m^2) = k^2(A_{123})^4 \frac{d}{dp^2}\left[T_{11234}(p^2, m^2, m^2, \epsilon, m^2, \epsilon) + T_{11234}(p^2, \epsilon, \epsilon, m^2, m^2, m^2) + T_{12345}(p^2, m^2, \epsilon, m^2, \epsilon, m^2)\right]\Big|_{p^2=m^2}$$



$T_{11234}(p^2, m^2, m^2, \epsilon, m^2, \epsilon)$ $T_{11234}(p^2, \epsilon, \epsilon, m^2, m^2, m^2)$ $T_{12345}(p^2, m^2, \epsilon, m^2, \epsilon, m^2)$

with

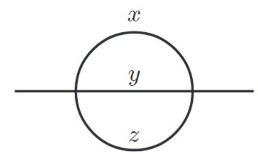
$$\mathbf{T}_{11234}(p^2, x, y, z, u, v) \equiv \left(\frac{(2\pi\mu)^{2\epsilon_{\text{UV}}}}{i\pi^2}\right)^2 \int\int \frac{d^dq_1 d^dq_2}{(q_1^2 - x)(q_1^2 - y)((q_1+p)^2 - z)((q_1-q_2)^2 - u)(q_2^2 - v)}$$

$$\mathbf{T}_{12345}(p^2, x, y, z, u, v) \equiv \left(\frac{(2\pi\mu)^{2\epsilon_{\text{UV}}}}{i\pi^2}\right)^2 \int\int \frac{d^dq_1 d^dq_2}{(q_1^2 - x)((q_1+p)^2 - y)((q_1-q_2)^2 - z)(q_2^2 - u)((q_2+p)^2 - v)}$$

$\overline{\text{MS}}$ scheme results at 2L

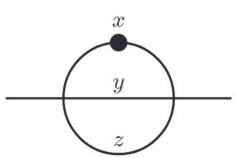
$$\hat{\Sigma}_{33}^{(2,\text{ genuine})\prime}(p^2=m^2) = k^2(A_{123})^4\frac{d}{dp^2}\left[T_{11234}(p^2,m^2,m^2,\epsilon,m^2,\epsilon)+T_{11234}(p^2,\epsilon,\epsilon,m^2,m^2,m^2)+T_{12345}(p^2,m^2,\epsilon,m^2,\epsilon,m^2)\right]\Big|_{p^2=m^2}$$

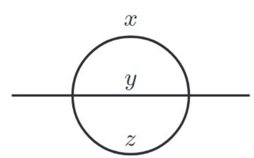
- Analytical evaluation of derivatives of self-energy integrals at finite $p^2=m^2$ using **differential equations** and special limits from [Martin hep-ph/0307101] (in terms of $\overline{\text{MS}}$ quantities)

- For instance, for the (finite part of the) integral $T_{234} \leftrightarrow$ 

$$p^2\frac{d}{dp^2}T_{234}(x,y,z) = T_{234}(x,y,z) - xT_{2234}(x,y,z) - yT_{2234}(y,x,z) - zT_{2234}(z,x,y)$$

$$+x\log\frac{x}{Q^2} + y\log\frac{y}{Q^2} + z\log\frac{z}{Q^2} + \frac{p^2}{2}$$

where $T_{2234} \leftrightarrow$ 

$= \frac{\partial}{\partial x}\Big[$ 

$\Big]$ and Q is the renormalisation scale

- **Expansion in ε** to find IR-dominant terms

$\overline{\text{MS}}$ scheme results at 2L

$$\hat{\Sigma}_{33}^{(2,\text{ genuine})\prime}(p^2=m^2) = k^2(A_{123})^4\frac{d}{dp^2}\left[T_{11234}(p^2,m^2,m^2,\epsilon,m^2,\epsilon)+T_{11234}(p^2,\epsilon,\epsilon,m^2,m^2,m^2)+T_{12345}(p^2,m^2,\epsilon,m^2,\epsilon,m^2)\right]\Big|_{p^2=m^2}$$

- Analytical evaluation of derivatives of self-energy integrals at finite $p^2=m^2$ using **differential equations** and special limits from [Martin hep-ph/0307101] (in terms of $\overline{\text{MS}}$ quantities)
- **Expansion in ε** to find IR-dominant terms
- Results cross-checked numerically with TSIL [Martin, Robertson hep-ph/0501132]

$$\frac{d}{dp^2}T_{11234}(p^2,m^2,m^2,\epsilon,m^2,\epsilon)\Big|_{p^2=m^2} = \frac{\pi(2-\overline{\ln}m^2)}{4\sqrt{\epsilon}m^3}+\frac{-6\overline{\ln}\epsilon\overline{\ln}m^2-3\overline{\ln}^2\epsilon+24\overline{\ln}\epsilon+9\overline{\ln}^2m^2-24\overline{\ln}m^2-\pi^2}{24m^4}+\mathcal{O}(\epsilon)$$

$$\frac{d}{dp^2}T_{11234}(p^2,\epsilon,\epsilon,m^2,m^2,m^2)\Big|_{p^2=m^2} = -\frac{\overline{\ln}m^2}{2m^2\epsilon}+\frac{3\pi\overline{\ln}m^2}{8m^3\sqrt{\epsilon}}+\frac{-50+6\pi^2+3\overline{\ln}\epsilon-12\overline{\ln}m^2+18\overline{\ln}\epsilon\overline{\ln}m^2-18\overline{\ln}^2m^2}{36m^4}+\mathcal{O}(\epsilon)$$

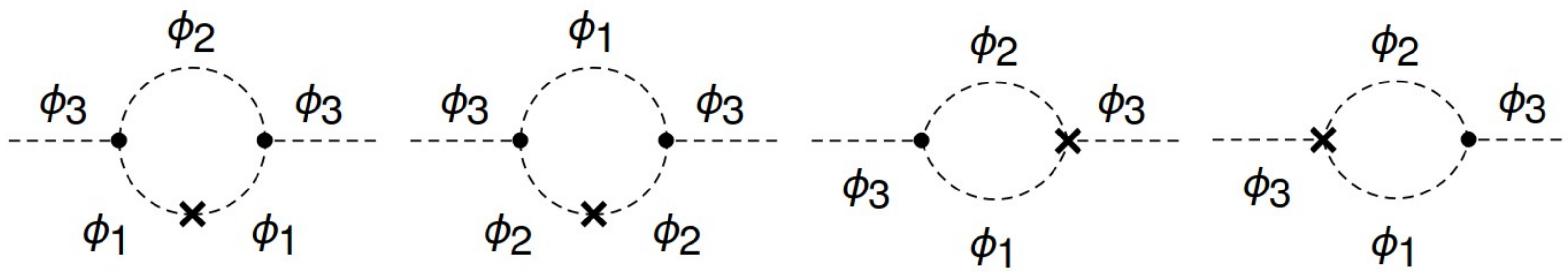
$$\frac{d}{dp^2}T_{12345}(p^2,m^2,\epsilon,m^2,\epsilon,m^2)\Big|_{p^2=m^2} = \frac{1}{4m^4}\left[2+\ln\frac{m^2}{\epsilon}+\ln^2\frac{m^2}{\epsilon}\right]-\frac{\pi^2\ln 2-3/2\zeta(3)}{m^4}+\mathcal{O}(\epsilon) \qquad (\overline{\ln}x\equiv\ln x/Q^2)$$

$$\hat{\Gamma}(\phi_3\to\chi\bar{\chi})=\Gamma^{(0)}(\phi_3\to\chi\bar{\chi})\left\{1-\frac{k(A_{123})^2}{m^2}\left[\frac{1}{2}\ln\frac{m^2}{\epsilon}-1\right]+\frac{k^2(A_{123})^4}{m^4}\left[\frac{m^2\overline{\ln}m^2}{2\epsilon}-\frac{m\pi(4+\overline{\ln}m^2)}{8\sqrt{\epsilon}}+\frac{17}{9}-\frac{\pi^2}{8}+\frac{1}{8}\ln^2\frac{m^2}{\epsilon}+\frac{1}{6}\overline{\ln}\epsilon+\frac{1}{12}\overline{\ln}m^2+\pi^2\ln 2-\frac{3}{2}\zeta(3)\right]\right\}$$

→ **unphysically large 1/ε and 1/√ε terms in addition to logε, log²ε**

Choices of renormalisation schemes at 2L

- Subloop renormalisation in 2L Φ_3 self-energies:



$$\hat{\Sigma}_{33}^{(2,\text{ subloop})}(p^2) = k(A_{123})^2\left[\left(\frac{2\delta^{(1)}A_{123}}{A_{123}} + \delta^{(1)}Z_3\right)B_0(p^2,m_1^2,m_2^2) + \delta^{(1)}m_1^2\frac{\partial}{\partial m_1^2}B_0(p^2,m_1^2,m_2^2) + \delta^{(1)}m_2^2\frac{\partial}{\partial m_2^2}B_0(p^2,m_1^2,m_2^2)\right]$$

- Keep $\overline{\text{MS}}$ renormalisation of wave functions → $\delta^{(1)}Z_3 = 0$

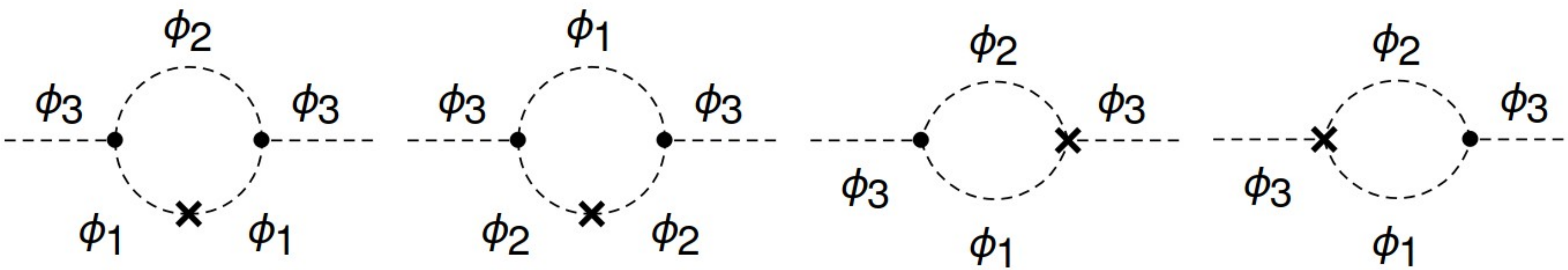
- **OS renormalisation of scalar masses**:

$$\left.\begin{aligned}\delta^{(1)}m_1^2 &= k(A_{123})^2\text{Re}B_0(m_1^2,m_2^2,m_3^2)\\ \delta^{(1)}m_2^2 &= k(A_{123})^2\text{Re}B_0(m_2^2,m_1^2,m_3^2)\end{aligned}\right\} \Rightarrow \Sigma_{33}^{(2,\text{subloop})\prime} \supset B_0(\epsilon,m^2,m^2)\frac{\partial^2}{\partial p^2\partial x}B_0(p^2,x,m^2)\Big|_{p^2=m^2,x=\epsilon}, \text{ and } B_0(m^2,\epsilon,m^2)\frac{\partial^2}{\partial p^2\partial y}B_0(p^2,\epsilon,y)\Big|_{p^2=y=m^2}$$

→ **cancels with $1/\varepsilon$ and $1/\sqrt{\varepsilon}$ terms in $\overline{\text{MS}}$ decay width result!**

Choices of renormalisation schemes at 2L

- Subloop renormalisation in 2L Φ_3 self-energies:



$$\hat{\Sigma}_{33}^{(2,\text{ subloop})}(p^2) = k(A_{123})^2\left[\left(\frac{2\delta^{(1)}A_{123}}{A_{123}} + \delta^{(1)}Z_3\right)B_0(p^2, m_1^2, m_2^2) + \delta^{(1)}m_1^2\frac{\partial}{\partial m_1^2}B_0(p^2, m_1^2, m_2^2) + \delta^{(1)}m_2^2\frac{\partial}{\partial m_2^2}B_0(p^2, m_1^2, m_2^2)\right]$$

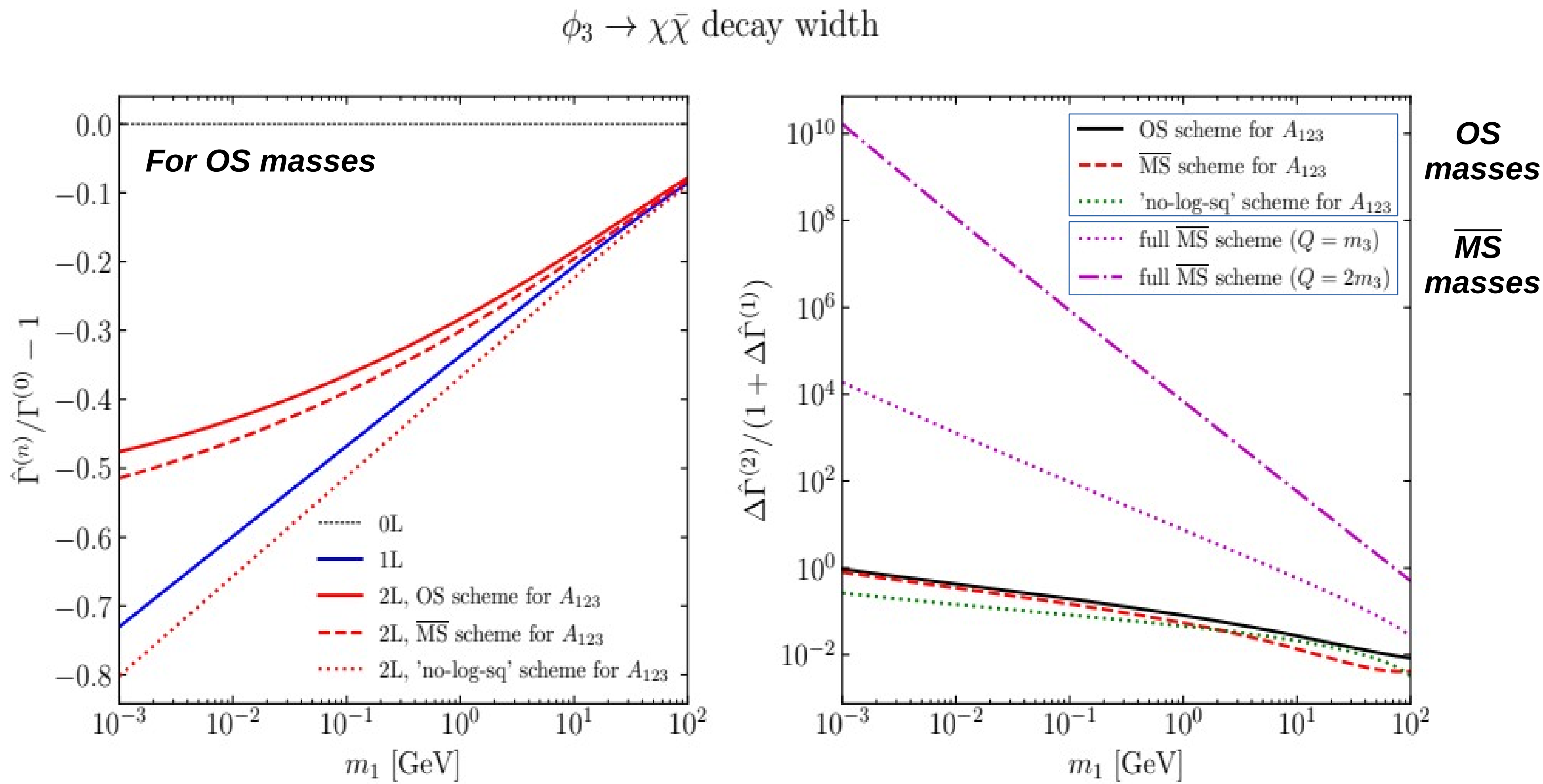
- Different possible choices for renormalisation of A_{123}
  - $\overline{\textbf{MS}} \rightarrow \delta^{\text{fin}}A_{123} = 0$
  - **OS** $\rightarrow$fix $\delta^{\text{fin}}A_{123}$ by demanding that OS-renormalised loop-corrected amplitude for $\Phi_2 \rightarrow \Phi_1\Phi_3$ with momenta on-shell remains equal to its tree-level value
  - **Custom "no-log-sq" scheme**, adjusting $\delta^{\text{fin}}A_{123}$ to cancel the $\log^2$ term in $\Gamma(\Phi_3 \rightarrow \chi\bar{\chi})$

    NB: this only reshuffles the $\log^2$ *into the extraction of* A_{123} *from a physical observable, e.g.* $\Gamma(\Phi_3 \rightarrow \Phi_1\Phi_2)$

- **logε remains at 1L and 2L ($\log^2\varepsilon$ also unless special scheme) !** → full expressions in [Bahl, JB, Weiglein '21]

Numerical results I

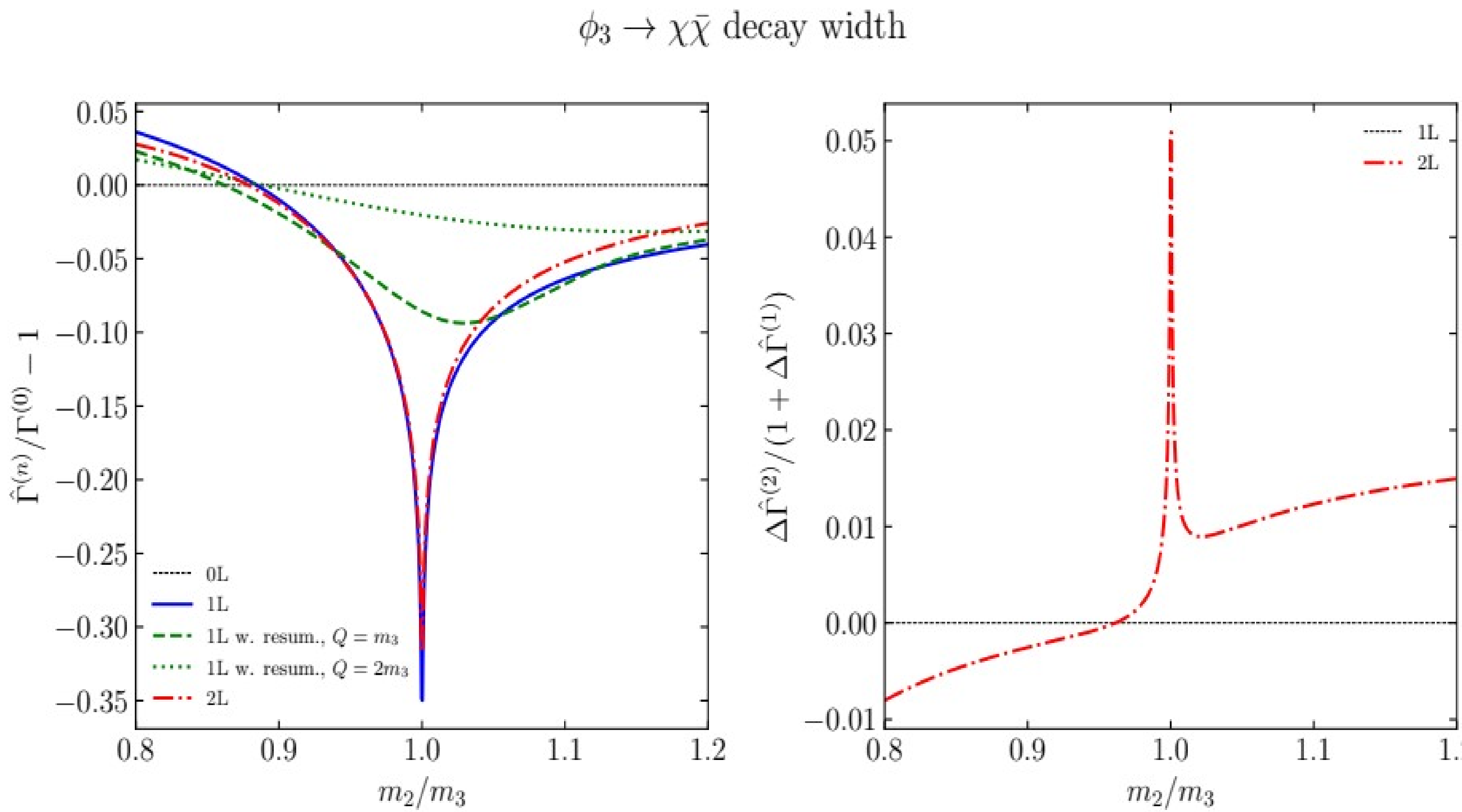
In mass scenario where $m_1 \to 0$, $m_2=m_3$



with $m_2 = m_3 = 1$ TeV, $y_3 = 1$, $A_{123} = 3$ TeV (other $A_{ijk} = 0$)

Numerical results II

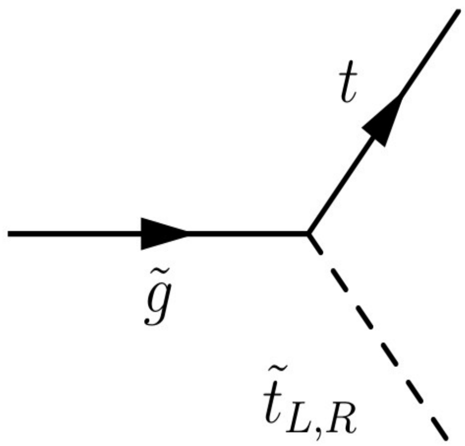
In mass scenario where $m_1=0$, $m_2 \sim m_3$



with m_3 = 500 GeV, m_χ = 200 GeV, λ_{1122} = 1, λ_{1133} = 1.2, and A_{123} = 1.5 TeV (A_{123} renormalised $\overline{\text{MS}}$)

Large logarithms from external legs: MSSM

Decay of a gluino in the MSSM



- **Minimal Supersymmetric extension of the Standard Model**
  - Higgs sector (assuming CP conservation):
    2 CP-even states h,H; CP-odd state A; charged Higgs $H^{\pm}$
    (+ would-be Goldstones)
  - Stops – i.e. scalar partners of top quarks

- Consider the **decay of a gluino** (fermionic partner of gluon) **into a top quark and a stop**
- **Stop-Higgs couplings** important for corrections to this decay
  → involve $X_t \equiv A_t - \mu \cot\beta$ or $Y_t \equiv A_t + \mu \tan\beta$

(with A_t trilinear stop coupling, μ Higgsino mass parameter, and $\tan\beta \equiv v_2/v_1$ ratio of vacuum expectation values of the two Higgs doublets)

- Experimental limits → M_{SUSY} must be large, potentially >> M_A (scale of BSM Higgses)
- Neglect EW gauge couplings and set v~0 (<<M_{SUSY}) for simplicity → no stop mixing!

- Typical mass hierarchy: $M_{SUSY} \gg M_A \gg m_h, m_G, m_{G\pm} \sim 0$

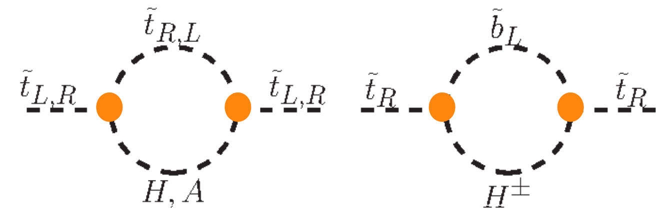
NB: case with v≠0 also considered in [Bahl, JB, Weiglein ‘21]

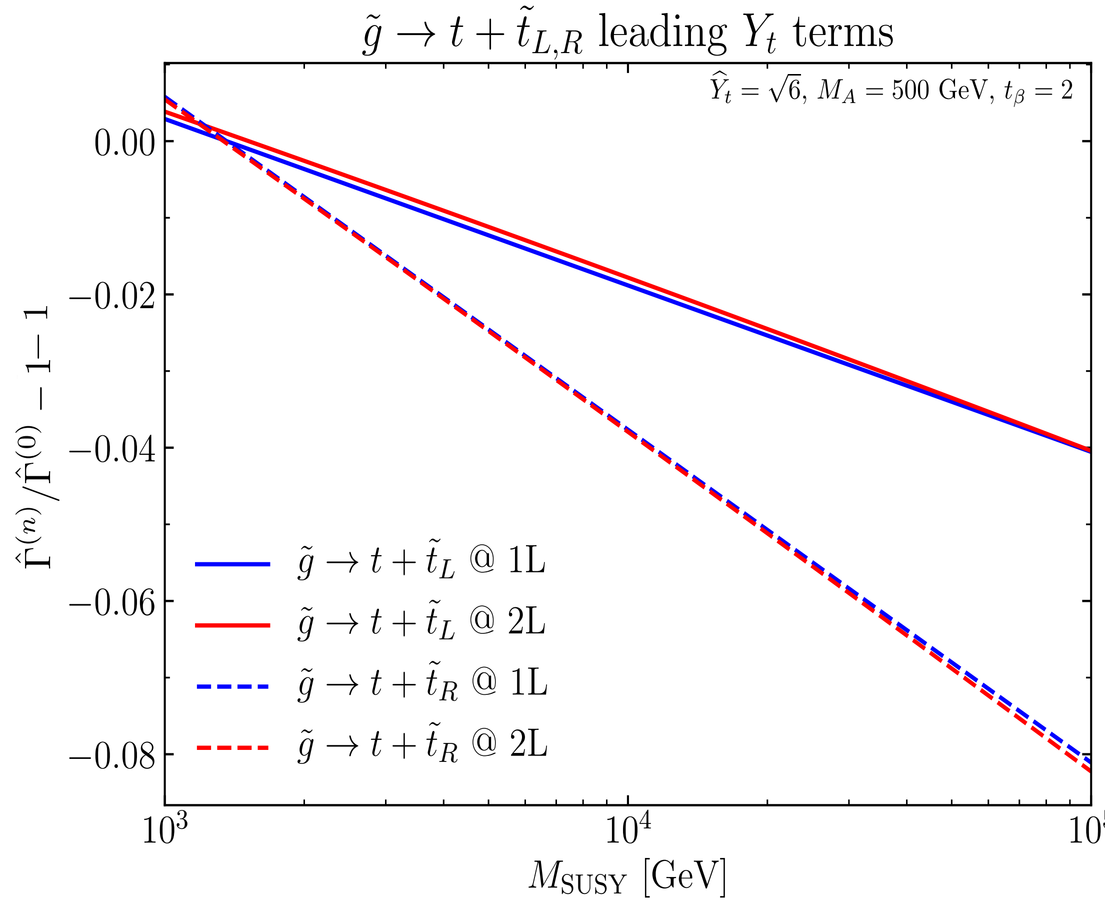
$\tilde{g}\rightarrow t\tilde{t}$ decay – Y_t terms

- Terms involving powers of $Y_t \equiv A_t + \mu \tan\beta$
  → **stop—BSM-Higgs couplings**

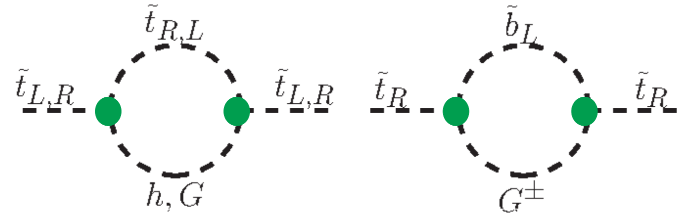
$$c(H\tilde{t}_L\tilde{t}_L) = c(H\tilde{t}_R\tilde{t}_R) = c(A\tilde{t}_L\tilde{t}_L) = c(A\tilde{t}_R\tilde{t}_R) = 0,$$
$$c(H\tilde{t}_L\tilde{t}_R) = -\frac{1}{\sqrt{2}}h_t c_\beta Y_t,$$
$$c(A\tilde{t}_L\tilde{t}_R) = -c(A\tilde{t}_R\tilde{t}_L) = \frac{1}{\sqrt{2}}h_t c_\beta Y_t,$$
$$c(H^+\tilde{t}_R\tilde{b}_R) = c(H^+\tilde{t}_L\tilde{b}_L) = c(H^+\tilde{t}_L\tilde{b}_R) = 0,$$
$$c(H^+\tilde{t}_R\tilde{b}_L) = -h_t c_\beta Y_t,$$

- Light scalars: H, A, $H^\pm$
  $M_A \neq 0$ but << M_{SUSY}
  → e.g. M_A = 500 GeV
- Heavy scalars: $\tilde{t}_L$, $\tilde{t}_R$
  $m_{\tilde{t}L} = m_{\tilde{t}R} = M_{SUSY}$
  (Same as $m_1 \neq 0$, $m_2 = m_3$ in toy model)





$\tilde{g}\to t\tilde{t}$ decay – X_t terms (at v=0)



- Terms involving powers of $X_t \equiv A_t - \mu \cot\beta$

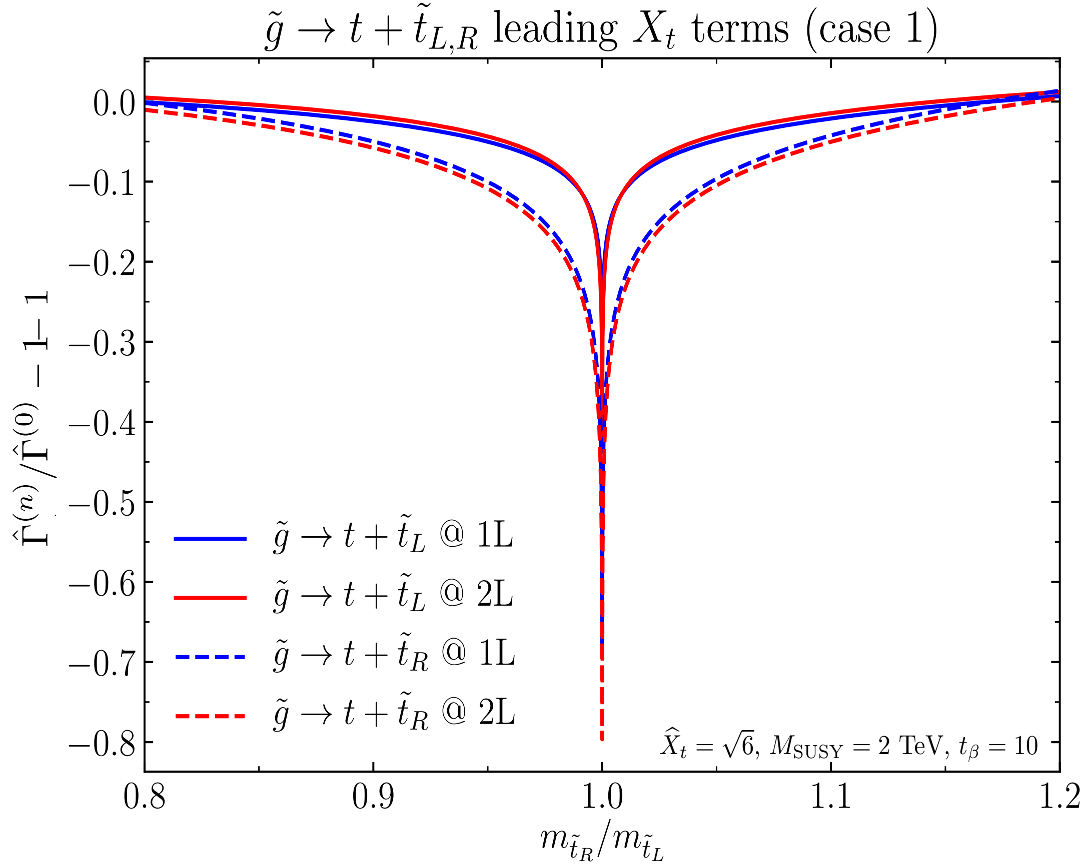
  → **stop—Higgs + Goldstone couplings**

$$c(h\tilde{t}_L\tilde{t}_L) = c(h\tilde{t}_R\tilde{t}_R) = c(G\tilde{t}_L\tilde{t}_L) = c(G\tilde{t}_R\tilde{t}_R) = 0,$$
$$c(h\tilde{t}_L\tilde{t}_R) = \frac{1}{\sqrt{2}}h_t s_\beta X_t,$$
$$c(G\tilde{t}_L\tilde{t}_R) = -c(G\tilde{t}_R\tilde{t}_L) = \frac{1}{\sqrt{2}}h_t s_\beta X_t,$$
$$c(G^+\tilde{t}_R\tilde{b}_R) = c(G^+\tilde{t}_L\tilde{b}_L) = c(G^+\tilde{t}_L\tilde{b}_R) = 0,$$
$$c(G^+\tilde{t}_R\tilde{b}_L) = -h_t s_\beta X_t.$$

- Light scalars: h, G, $G^\pm$
  - m_h = 0 in gaugeless limit
  - $m_G = m_{G\pm} = 0$
- Heavy scalars: $\tilde{t}_L$, $\tilde{t}_R$

  $m_{\tilde{t}L} \neq m_{\tilde{t}R} \sim M_{SUSY}$

(Same as m_1=0, m_2~m_3 in toy model)



Summary of Part 1

Precise theory predictions are of paramount importance to properly assess BSM discovery sensitivities, and to constrain parameter space of BSM models

- We pointed out the existence of a **new type of large Sudakov-like logarithms, in external-leg corrections of heavy scalars**, in presence of mass hierarchy
- Can be further **enhanced by large trilinear couplings**
- At 1L, we showed how these logs are related to singularities in IR limit, and we discussed how to address these divergences
- **Computed large logs at 2L** (derivatives of self-energies with non-zero masses and at finite p^2)
- Showed the importance of OS renormalisation of masses
- In MSSM and N2HDM (in backup) examples: large effects at 1L; size of 2L effects well below that of 1L ones → SCET resummation doesn't seem compulsory
- Similar large logs can appear in scheme conversions of parameters (e.g. trilinear couplings like X_t) [Bahl, JB, Weiglein '22] → **Part 2**

Stop mixing parameter X_t: experimental probes and choices of renormalisation schemes

(In relation to item 2 of Pietro's shopping list for hybrid calculations)

Stop sector and stop mixing parameter

- Stop mass matrix (in gauge eigenstate basis $\tilde{t}_L$, $\tilde{t}_R$):

$$\mathbf{M}_{\tilde{t}} = \begin{pmatrix} m^2_{\tilde{t}_L} + m_t^2 + \cos(2\beta)(\frac{1}{2} - \frac{2}{3}s_W^2)M_Z^2 & m_t\, X_t^* \\ m_t\, X_t & m^2_{\tilde{t}_R} + m_t^2 + \frac{2}{3}\cos(2\beta)s_W^2 M_Z^2 \end{pmatrix}$$

- $m_{\tilde{t}L}$, $m_{\tilde{t}R}$: stop soft SUSY-breaking masses; $X_t \equiv A_t - \mu^* \cot\beta$: stop mixing parameter
- Diagonalise the stop mass matrix

$$\mathbf{U}_{\tilde{t}}\mathbf{M}_{\tilde{t}}\mathbf{U}_{\tilde{t}}^\dagger = \mathrm{diag}(m^2_{\tilde{t}_1}, m^2_{\tilde{t}_2})$$

with $m^2_{\tilde{t}_{1,2}} = m_t^2 + \dfrac{1}{2}\left\{m^2_{\tilde{t}_L} + m^2_{\tilde{t}_R} \mp \sqrt{\left[m^2_{\tilde{t}_L} - m^2_{\tilde{t}_R} + M_Z^2 c_{2\beta}\left(\dfrac{1}{2} - \dfrac{4}{3}s_W^2\right)\right]^2 + 4m_t^2|X_t|^2}\right\}$

and $\mathbf{U}_{\tilde{t}} = \begin{pmatrix} c_{\tilde{t}} & s_{\tilde{t}}e^{-i\phi_{X_t}} \\ -s_{\tilde{t}}e^{i\phi_{X_t}} & c_{\tilde{t}} \end{pmatrix}$ where $\phi_{X_t} = \arg(X_t)$

$$\cos(2\theta_{\tilde{t}}) = \frac{m^2_{\tilde{t}_R} - m^2_{\tilde{t}_L} - M_Z^2 c_{2\beta}(\frac{1}{2} - \frac{4}{3}s_W^2)}{m^2_{\tilde{t}_2} - m^2_{\tilde{t}_1}}$$

(stop mixing angle)

- In the following, we assume X_t to be real for simplicity ($\rightarrow \phi_{X_t}=0$)

Accessing X_t experimentally

→ via stop masses

→ via stop mixing angle

→ via stop decay

→ via Higgs mass

Note: we define $M_{\rm SUSY} = \sqrt{m_{\tilde{t}_L} m_{\tilde{t}_R}}$

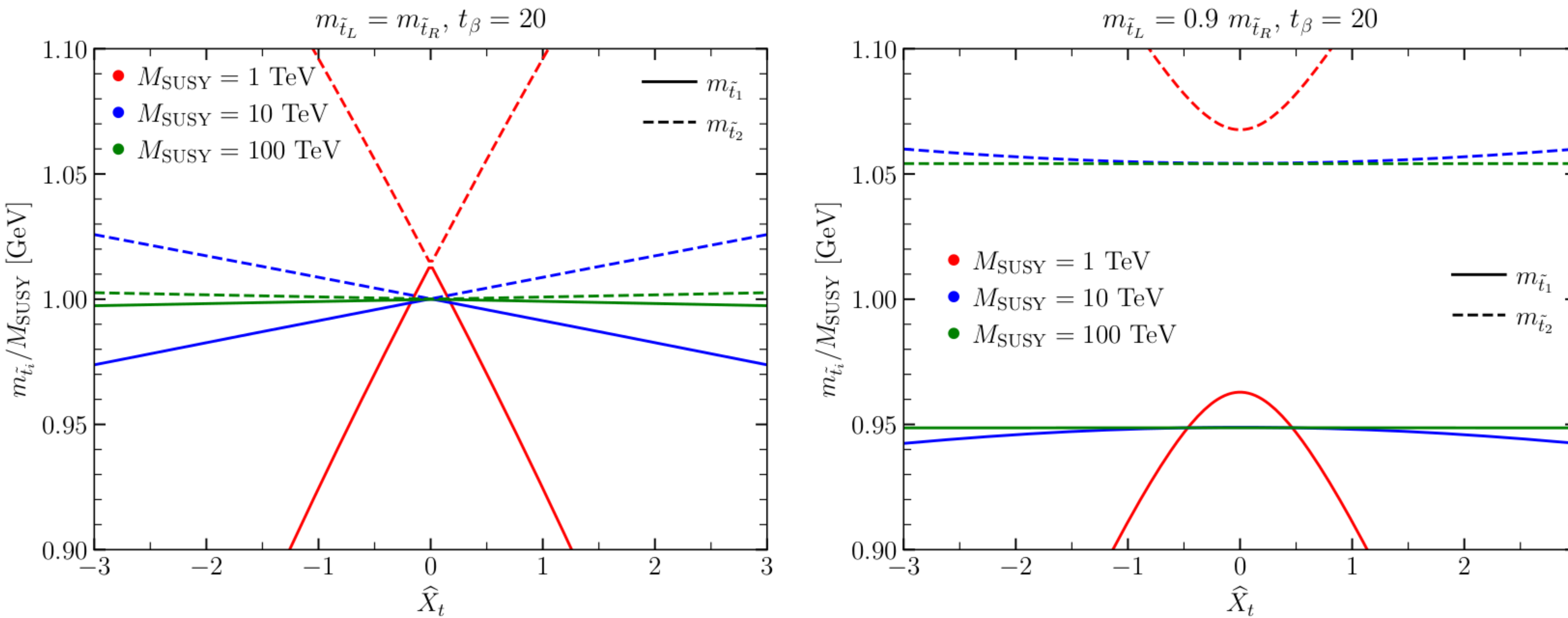
and $\widehat{X}_t = X_t / M_{\rm SUSY}$

Accessing X_t via stop mass measurements

Recall

$M_{\text{SUSY}} = \sqrt{m_{\tilde{t}_L} m_{\tilde{t}_R}}$

$\widehat{X}_t = X_t / M_{\text{SUSY}}$



- Assumption on relation between soft masses is necessary (as 2 inputs to determine X_t, $m_{\tilde{t}L}$, $m_{\tilde{t}R}$)
- Not possible in general to disentangle X_t from measurement of $m_{\tilde{t}1}$, $m_{\tilde{t}2}$ only
- Sensitivity lost as stop masses increase

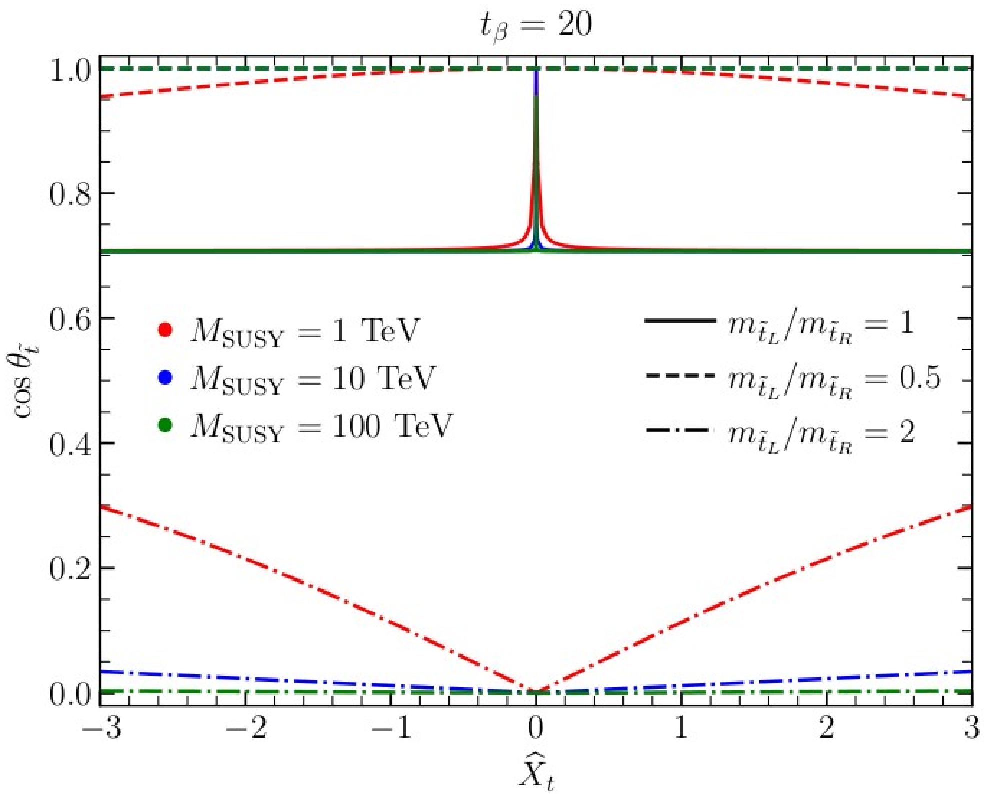
Accessing X_t via a measurement of the stop mixing angle

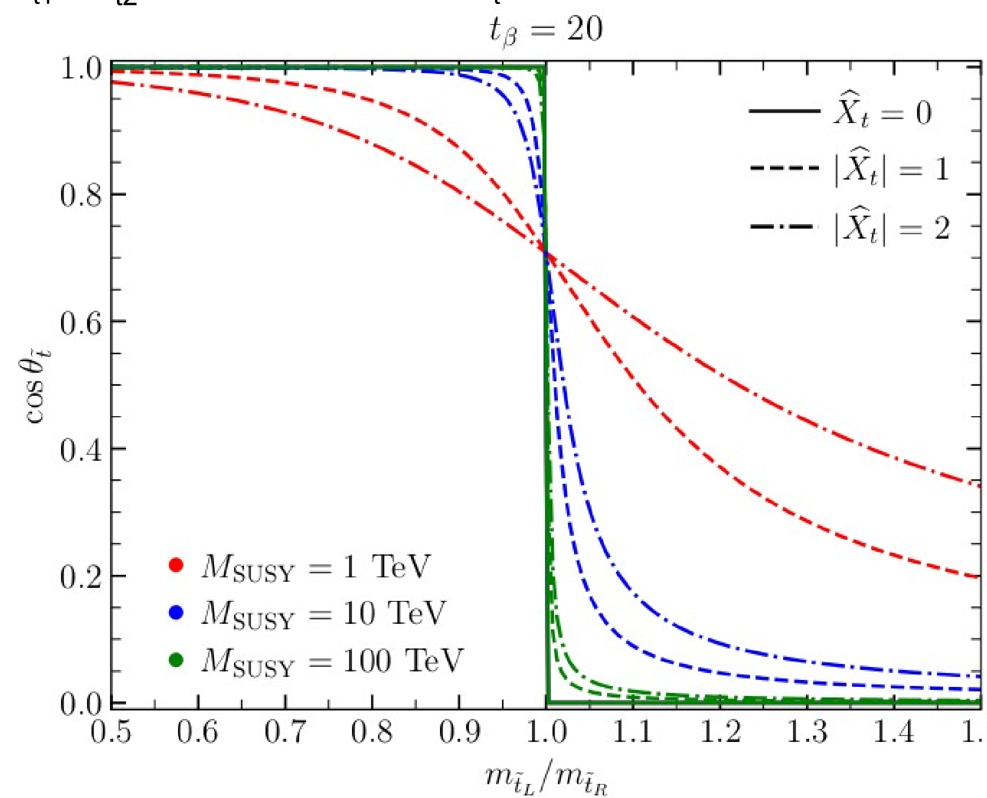
Recall

$M_{\rm SUSY} = \sqrt{m_{\tilde{t}_L} m_{\tilde{t}_R}}$

$\widehat{X}_t = X_t / M_{\rm SUSY}$

- Measurement of stop mixing angle already challenging
- But supposing it can be done (+ measurement of $m_{\tilde{t}1}$, $m_{\tilde{t}2}$), can we derive X_t?





- Again sensitivity lost as stop masses increase, as well as if $m_{\tilde{t}L} \sim m_{\tilde{t}R}$

Accessing X_t via stop decays

- Decay $\tilde{t}_2 \to \tilde{t}_1 h$ depends on X_t at tree level

$$d\Gamma_{\tilde{t}_2\to\tilde{t}_1 h} = \frac{1}{64\pi^2}\frac{\sqrt{(m^2_{\tilde{t}_2}-(m_{\tilde{t}_1}+m_h)^2)(m^2_{\tilde{t}_2}-(m_{\tilde{t}_1}-m_h)^2)}}{m^3_{\tilde{t}_2}}|\underbrace{\mathcal{M}(\tilde{t}_2\to\tilde{t}_1 h)}_{\propto X_t}|^2 d\cos\theta$$

- Limit $m_{\tilde{t}1}$, m_h << $m_{\tilde{t}2}$

$$d\Gamma_{\tilde{t}_2\to\tilde{t}_1 h} \xrightarrow{m_h, m_{\tilde{t}_1}\ll m_{\tilde{t}_2}} \frac{1}{64\pi^2}\frac{1}{m_{\tilde{t}_2}}|\mathcal{M}(\tilde{t}_2\to\tilde{t}_1 h)|^2 d\cos\theta \propto \frac{|X_t|^2}{m_{\tilde{t}_2}} \underset{X_t\sim\mathcal{O}(M_\text{SUSY})}{\sim} \mathcal{O}(M_\text{SUSY})$$

- Limit m_h << $m_{\tilde{t}1}$, $m_{\tilde{t}2}$

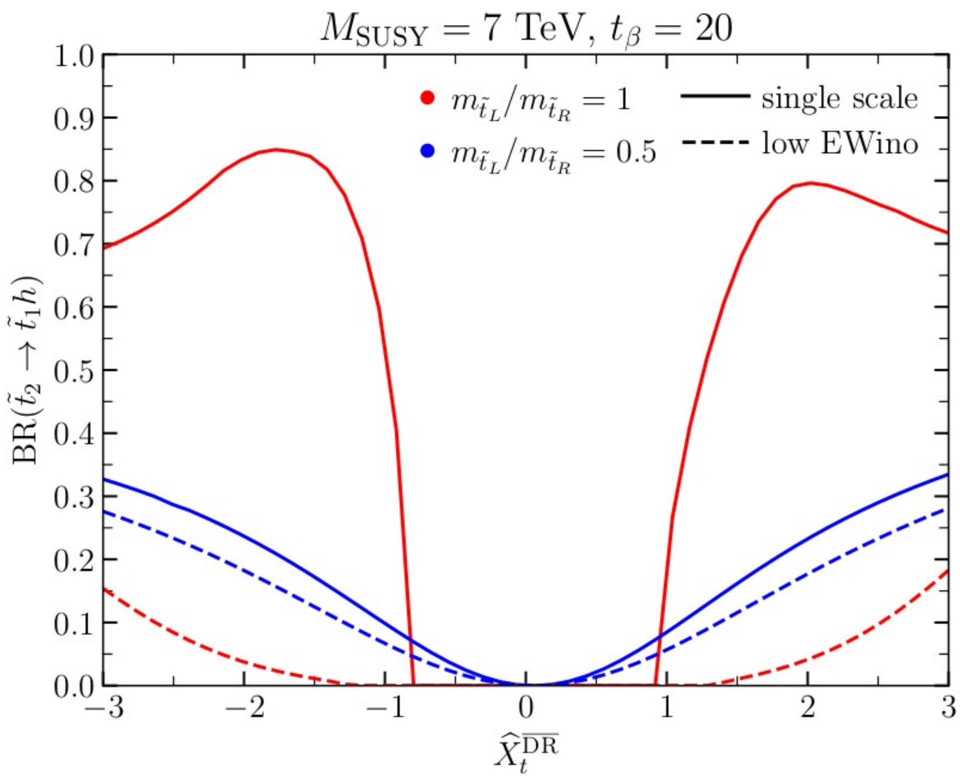
$$d\Gamma_{\tilde{t}_2\to\tilde{t}_1 h} \xrightarrow{m_h\ll m_{\tilde{t}_1}\sim m_{\tilde{t}_2}} \frac{1}{64\pi^2}\frac{|m^2_{\tilde{t}_2}-m^2_{\tilde{t}_1}|}{m^3_{\tilde{t}_2}}|\mathcal{M}(\tilde{t}_2\to\tilde{t}_1 h)|^2 d\cos\theta \simeq$$

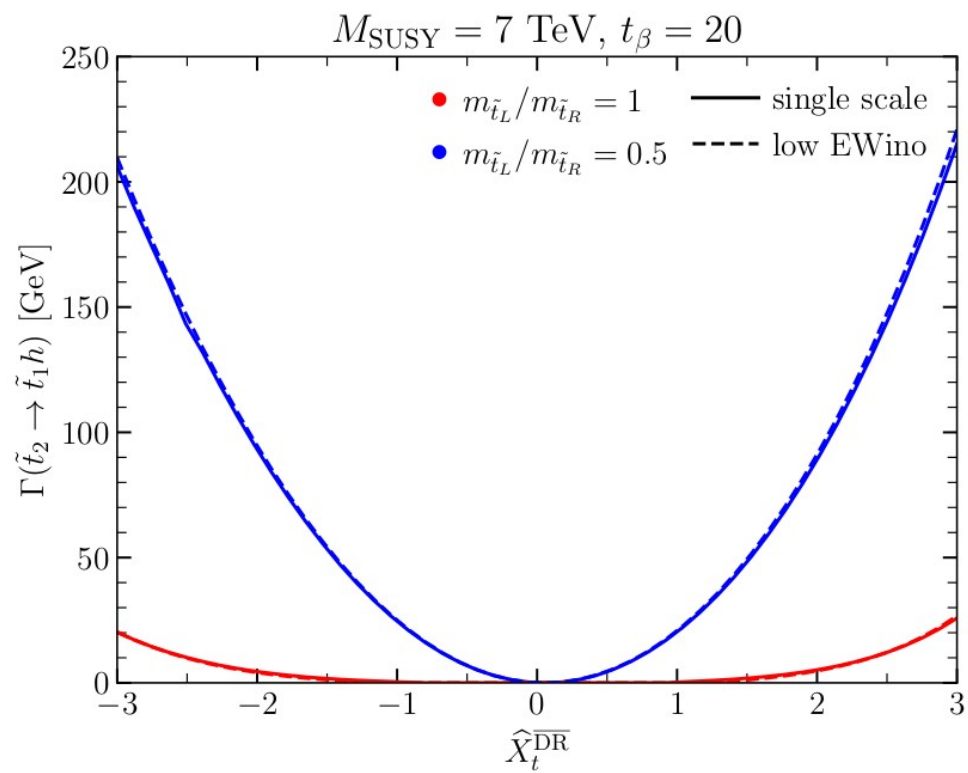
$$\simeq \frac{1}{64\pi^2}\frac{2m_t|X_t|}{m^3_{\tilde{t}_2}}|\mathcal{M}(\tilde{t}_2\to\tilde{t}_1 h)|^2 d\cos\theta \propto \frac{m_t|X_t|^3}{m^3_{\tilde{t}_2}} \underset{X_t\sim\mathcal{O}(M_\text{SUSY})}{\sim} \mathcal{O}(m_t)$$

(phase space suppression)

Accessing X_t via stop decays II

- With SUSY-HIT, investigate 2 scenarios
  - Single scale: all SUSY-breaking masses = M_{SUSY} = 7 TeV
  - Set instead $M_1 = M_2 = \mu = M_{SUSY}/2$ → light Ewkinos





- Usefulness depends highly on sparticle spectrum:
  If $m_{\tilde{t}L} \sim m_{\tilde{t}R}$ or if other decay channels are open (e.g. to quark+EWkino), it becomes more difficult to extract X_t from stop decay

Accessing X_t via the Higgs boson mass

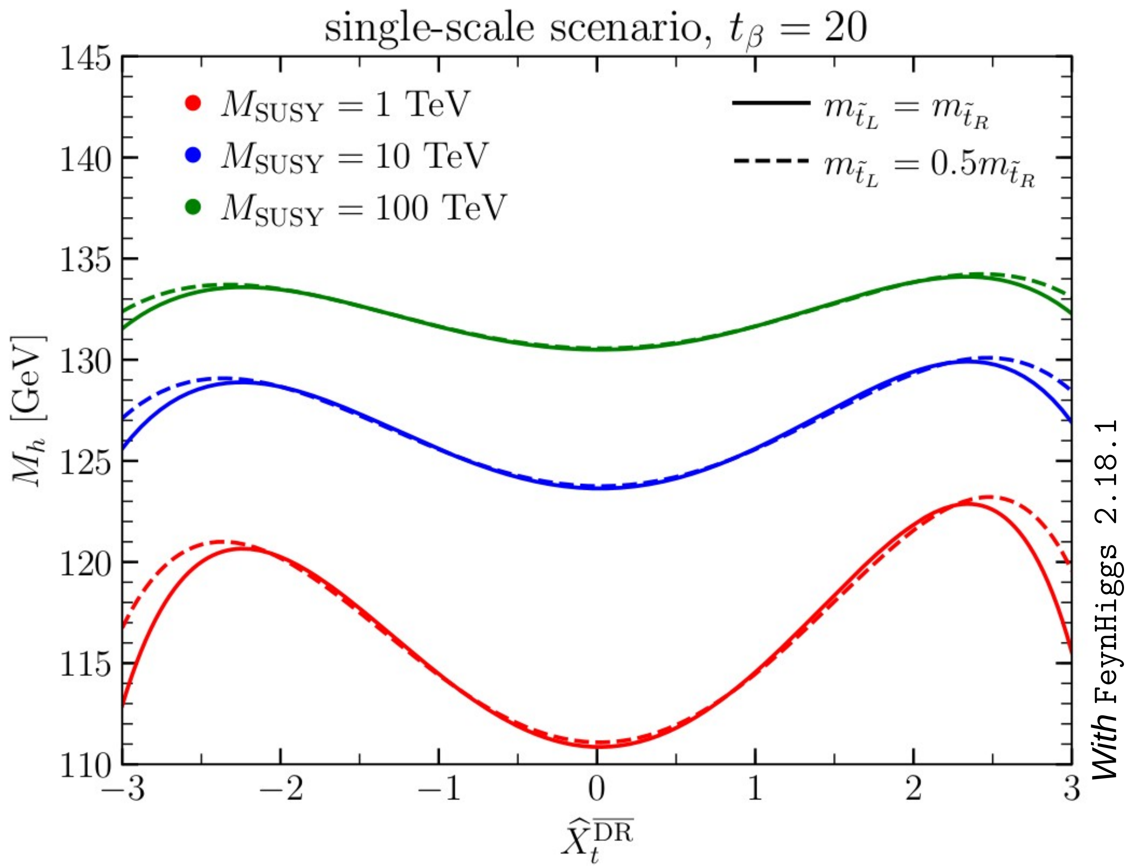
- Another observable where X_t enters is M_h, from 1L

$$M_h^2 \simeq \underbrace{m_h^2}_{M_Z^2 c_{2\beta}^2} + \frac{3m_t^4}{4\pi^2 v^2}\left(\ln\frac{M_{\text{SUSY}}^2}{m_t^2} + |\widehat{X}_t|^2 - \frac{1}{12}|\widehat{X}_t|^4\right) + \dots$$

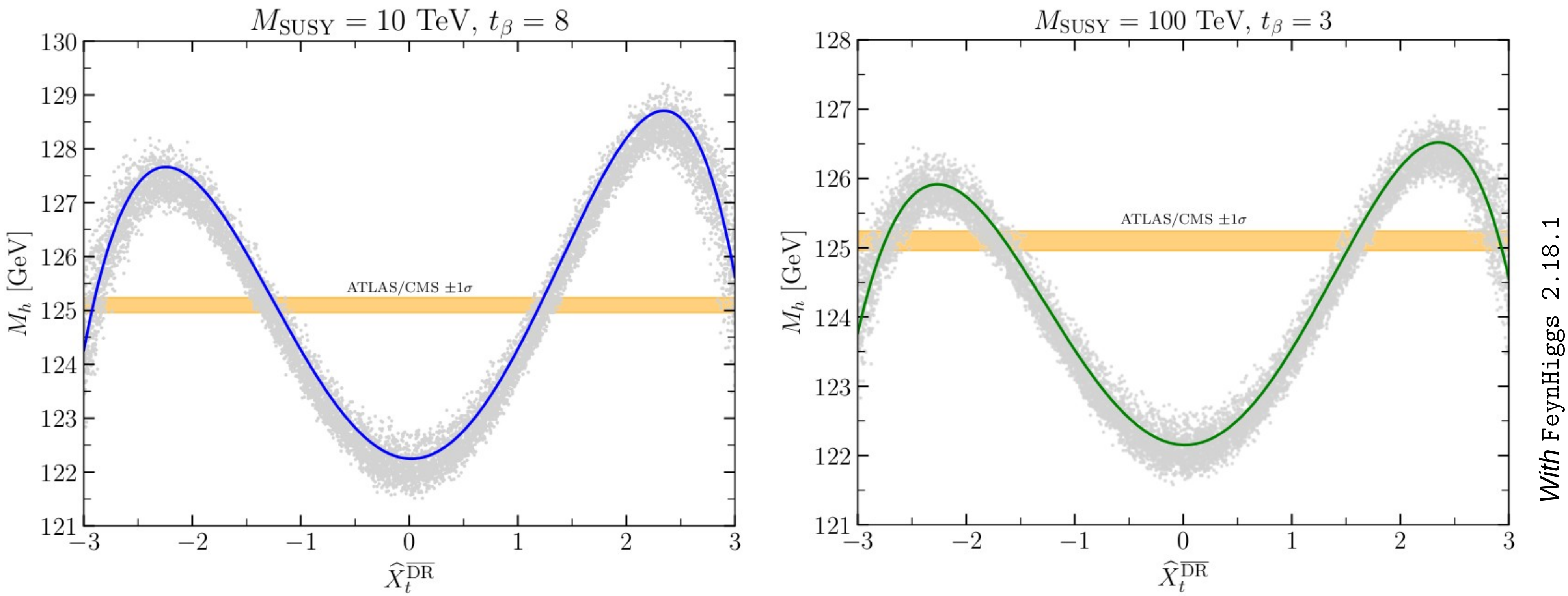
- Single scale scenario (all soft SUSY-breaking masses = M_A = μ = M_{SUSY})

- Significant dependence of M_h on X_t, no matter if $m_{\tilde{t}L} \sim m_{\tilde{t}R}$ or not, and even for high SUSY scale (10 or 100 TeV)

- X_t could be extracted from M_h, if stop masses are known



Accessing X_t via the Higgs boson mass II



- **Blue/green lines**: all non-SM masses = M_{SUSY}, $A_{f\neq t} = 0$
- **Grey points**: scan over SUSY parameters (masses and trilinears) between $M_{SUSY}/2$ and 2 M_{SUSY}
- If stop masses and tanβ known → can extract X_t

How to define X_t theoretically – i.e. possible choices of renormalisation schemes

Renormalisation of the stop/top sector

- One choice of parameters: m_t, $m_{\tilde{t}L}$,$m_{\tilde{t}R}$, X_t
- Counter terms: $$m^2_{\tilde{t}_{L/R}} \to m^2_{\tilde{t}_{L/R}} + \delta^{(1)} m^2_{\tilde{t}_{L/R}}, \quad X_t \to X_t + \delta^{(1)} X_t, \quad m_t \to m_t + \delta^{(1)} m_t$$
- Stop mass matrix counterterm: $$\delta^{(1)} \mathbf{M}_{\tilde{t}} = \begin{pmatrix} \delta^{(1)} m^2_{\tilde{t}_L} + \delta^{(1)} m_t^2 & X_t^* \, \delta^{(1)} m_t + m_t \, \delta^{(1)} X_t^* \\ X_t \, \delta^{(1)} m_t + m_t \, \delta^{(1)} X_t & \delta^{(1)} m^2_{\tilde{t}_R} + \delta^{(1)} m_t^2 \end{pmatrix}$$
- Rotate to mass eigenstate basis $$\mathbf{U}_{\tilde{t}} \, \delta^{(1)} \mathbf{M}_{\tilde{t}} \, \mathbf{U}^\dagger_{\tilde{t}} = \begin{pmatrix} \delta^{(1)} m^2_{\tilde{t}_1} & \delta^{(1)} m^2_{\tilde{t}_{12}} \\ (\delta^{(1)} m^2_{\tilde{t}_{12}})^* & \delta^{(1)} m^2_{\tilde{t}_2} \end{pmatrix}$$
- Relate counterterms in gauge eigenstate basis to those in mass eigenstate basis (easier to impose conditions on)

$$\begin{aligned} \delta^{(1)} X_t = & \frac{1}{m_t} \left[\mathbf{U}_{\tilde{t}_{11}} \mathbf{U}^*_{\tilde{t}_{12}} \left(\delta^{(1)} m^2_{\tilde{t}_1} - \delta^{(1)} m^2_{\tilde{t}_2} \right) \right. \\ & \left. + \delta^{(1)} m^2_{\tilde{t}_{12}} \mathbf{U}_{\tilde{t}_{21}} \mathbf{U}^*_{\tilde{t}_{12}} + \delta^{(1)} m^2_{\tilde{t}_{21}} \mathbf{U}_{\tilde{t}_{11}} \mathbf{U}^*_{\tilde{t}_{22}} - X_t \delta^{(1)} m_t \right], \\ \delta^{(1)} m^2_{\tilde{t}_L} = & \delta^{(1)} m^2_{\tilde{t}_1} |\mathbf{U}_{\tilde{t}_{11}}|^2 + \delta^{(1)} m^2_{\tilde{t}_2} |\mathbf{U}_{\tilde{t}_{12}}|^2 \\ & + \delta^{(1)} m^2_{\tilde{t}_{12}} \mathbf{U}_{\tilde{t}_{21}} \mathbf{U}^*_{\tilde{t}_{11}} + \delta^{(1)} m^2_{\tilde{t}_{21}} \mathbf{U}_{\tilde{t}_{11}} \mathbf{U}^*_{\tilde{t}_{21}} - 2 m_t \, \delta^{(1)} m_t, \\ \delta^{(1)} m^2_{\tilde{t}_R} = & \delta^{(1)} m^2_{\tilde{t}_1} |\mathbf{U}_{\tilde{t}_{12}}|^2 + \delta^{(1)} m^2_{\tilde{t}_2} |\mathbf{U}_{\tilde{t}_{22}}|^2 \\ & + \delta^{(1)} m^2_{\tilde{t}_{12}} \mathbf{U}_{\tilde{t}_{22}} \mathbf{U}^*_{\tilde{t}_{12}} + \delta^{(1)} m^2_{\tilde{t}_{21}} \mathbf{U}_{\tilde{t}_{12}} \mathbf{U}^*_{\tilde{t}_{22}} - 2 m_t \, \delta^{(1)} m_t \, . \end{aligned}$$

Renormalisation of the stop/top sector II

- Alternative choice of parameters: m_t, $m_{\tilde{t}L}$,$m_{\tilde{t}R}$, θ_t, ϕ_{Xt}
- Counter terms:

$$m^2_{\tilde{t}_{L/R}} \to m^2_{\tilde{t}_{L/R}} + \delta^{(1)} m^2_{\tilde{t}_{L/R}}, \quad \theta_t \to \theta_t + \delta^{(1)}\theta_t, \quad \phi_{X_t} \to \phi_{X_t} + \delta^{(1)}\phi_{X_t}, \quad m_t \to m_t + \delta^{(1)} m_t$$

- Reexpress stop mass matrix → obtain counterterm matrix elements:

$$\begin{aligned}
\delta^{(1)}\mathbf{M}_{\tilde{t}_{11}} &= \cos^2\theta_{\tilde{t}}\,\delta^{(1)}m^2_{\tilde{t}_1} + \sin^2\theta_{\tilde{t}}\,\delta^{(1)}m^2_{\tilde{t}_2} + (m^2_{\tilde{t}_2} - m^2_{\tilde{t}_1})\sin 2\theta_{\tilde{t}}\,\delta^{(1)}\theta_{\tilde{t}},\\
\delta^{(1)}\mathbf{M}_{\tilde{t}_{12}} &= (\delta^{(1)}m^2_{\tilde{t}_1} - \delta^{(1)}m^2_{\tilde{t}_2})\sin\theta_{\tilde{t}}\cos\theta_{\tilde{t}}\,e^{-i\phi_{X_t}}\\
&\quad + (m^2_{\tilde{t}_1} - m^2_{\tilde{t}_8 2})(\delta^{(1)}\theta_{\tilde{t}}\ \cos 2\theta_{\tilde{t}} - i\delta^{(1)}\phi_{X_t}\sin\theta_{\tilde{t}}\ \cos\theta_{\tilde{t}})\ e^{-i\phi_{X_t}},\\
\delta^{(1)}\mathbf{M}_{\tilde{t}_{21}} &= (\delta^{(1)}m^2_{\tilde{t}_1} - \delta^{(1)}m^2_{\tilde{t}_2})\sin\theta_{\tilde{t}}\cos\theta_{\tilde{t}}\,e^{i\phi_{X_t}}\\
&\quad + (m^2_{\tilde{t}_1} - m^2_{\tilde{t}_{22}})(\delta^{(1)}\theta_{\tilde{t}}\ \cos 2\theta_{\tilde{t}} + i\delta^{(1)}\phi_{X_t}\sin\theta_{\tilde{t}}\ \cos\theta_{\tilde{t}})\ e^{i\phi_{X_t}},\\
\delta^{(1)}\mathbf{M}_{\tilde{t}_{22}} &= \cos^2\theta_{\tilde{t}}\,\delta^{(1)}m^2_{\tilde{t}_2} + \sin^2\theta_{\tilde{t}}\,\delta^{(1)}m^2_{\tilde{t}_1} + (m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2})\sin 2\theta_{\tilde{t}}\,\delta^{(1)}\theta_{\tilde{t}}\ .
\end{aligned}$$

- Obtain for the off-diagonal mass counterterm in mass eigenstate basis

$$\delta^{(1)} m^2_{\tilde{t}_{12}} = e^{-i\phi_{X_t}}(m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2})(\delta^{(1)}\theta_{\tilde{t}} - i\delta^{(1)}\phi_{X_t}\sin\theta_{\tilde{t}}\ \cos\theta_{\tilde{t}})$$

Process-dependent/-independent OS renormalisation schemes

- For stop/top masses, simple interpretation of OS scheme in terms of **physical masses**

$$\delta^{(1)} m^2_{\tilde{t}_i} = \mathrm{Re}\Sigma^{(1)}_{\tilde{t}_i\tilde{t}_i}(m^2_{\tilde{t}_i}), \quad \delta^{(1)} m_t = \mathrm{Re}\Sigma^{(1)}_{tt}(m^2_t)$$

$\Sigma^{(1)}$*: 1L self-energy*

- For X_t (or equivalently θ_t, ϕ_{Xt}), no unique/straightforward choice
- **Process-dependent** definition, e.g. with $\tilde{t}_2 \to \tilde{t}_1 h$ process

→ difficult to access processes involving X_t experimentally (c.f. previous discussion)

→ depends on sparticle spectrum / only reliable in parts of parameter space

- **Process-independent**, like

$$\delta^{(1)} m^2_{\tilde{t}_{12}} = \frac{1}{2}\,\mathrm{Re}\left[\Sigma^{(1)}_{\tilde{t}_1\tilde{t}_2}(m^2_{\tilde{t}_1}) + \Sigma^{(1)}_{\tilde{t}_1\tilde{t}_2}(m^2_{\tilde{t}_2})\right]$$

from which one can obtain $\delta^{(1)} X_t \ (\delta^{(1)}\theta_t), \quad \delta^{(1)} m_{\tilde{t}_{L,R}}$ with relations shown before

→ but not related to physical observable directly

→ potentially gauge dependent

$\overline{\text{DR}}$ / $\overline{\text{MDR}}$ / mixed renormalisation schemes

- **$\overline{\text{DR}}$**: set finite parts of all counterterms to 0
  - No direct physical interpretation of parameters
  - But, convenient e.g. with high-scale SUSY scenarios
  - Can be plagued by unphysical non-decoupling effects if gluinos are much heavier than stops

- **$\overline{\text{MDR}}$**: keep idea of $\overline{\text{DR}}$ scheme, but define finite part of counterterms to absorb unphysical large corrections

$$\left(m_{\tilde{t}_{L,R}}^{\overline{\text{MDR}}}\right)^2(Q) = \left(m_{\tilde{t}_{L,R}}^{\overline{\text{DR}}}\right)^2(Q)\left[1 + \frac{\alpha_s}{\pi}C_F\frac{|M_3|^2}{m_{\tilde{t}_{L,R}}^2}\left(1 + \ln\frac{Q^2}{|M_3|^2}\right)\right]$$

$$X_t^{\overline{\text{MDR}}}(Q) = X_t^{\overline{\text{DR}}}(Q) - \frac{\alpha_s}{\pi}C_F M_3\left(1 + \ln\frac{Q^2}{|M_3|^2}\right)$$

[Bahl, Sobolev, Weiglein ‘19]

- **Mixed**: renormalise stop and top masses OS, but keep X_t in $\overline{\text{DR}}$/$\overline{\text{MDR}}$ scheme (possible problems with 1/ε * ε pieces at higher orders)

What renormalisation scheme to use for X_t in Higgs mass calculations

Renormalisation of X_t for different types of Higgs mass calculations

- **Fixed order**: (process-independent) OS scheme possible/convenient

- **EFT**: if X_t in OS scheme, large $\log(M_{SUSY}^2/m_t^2)$ pieces remain, which would be resummed by running of X_t → $\overline{\text{DR}}$ / $\overline{\text{MDR}}$ scheme preferable for X_t

- **Hybrid**: use OS for fixed-order part; $\overline{\text{DR}}$ / $\overline{\text{MDR}}$ for EFT part

- Both in EFT and hybrid approaches
  → $X_t^{\overline{\text{DR}}}$ must be extracted from physical input / related to X_t^{OS}
  → **large logs**

OS to $\overline{\text{DR}}$ conversion of X_t and large logarithms

- OS → $\overline{\text{DR}}$ conversion of X_t:

$$X_t^{\text{OS}} = X_t^{\overline{\text{DR}}}(M_{\text{SUSY}})\,\frac{m_t^{\overline{\text{DR}},\text{MSSM}}(M_{\text{SUSY}})}{m_t^{\text{OS}}} - \frac{1}{m_t^{\text{OS}}}\delta^{(1)}(m_t X_t)\big|_{\text{fin}}$$

both terms contain large logs!

- First from m_t:

$$m_t^{\overline{\text{DR}},\text{MSSM}}(M_{\text{SUSY}}) = m_t^{\text{OS}} \underbrace{- m_t^{\text{OS}}\left[\left(\frac{\alpha_s}{\pi} - \frac{3\alpha_t}{16\pi}\right)\ln\frac{M_{\text{SUSY}}^2}{m_t^2} + \text{no large logs}\right]}_{=\delta^{(1)} m_t^{\text{OS}}(Q=M_{\text{SUSY}})\big|_{\text{fin}}} + \ldots$$

sub-leading

→ resum the large logs by using $m_t^{\overline{\text{DR}},\text{MSSM}}(Q=M_{\text{SUSY}})$ or $m_t^{\overline{\text{MS}},\text{SM}}(Q=M_{\text{SUSY}})$

- What about the 2nd term?

OS to $\overline{\text{DR}}$ conversion of X_t and large logarithms II

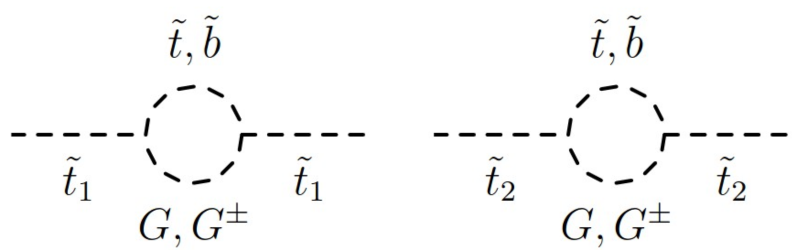
$$X_t^{\text{OS}} = X_t^{\overline{\text{DR}}}(M_{\text{SUSY}}) \frac{m_t^{\overline{\text{DR}},\text{MSSM}}(M_{\text{SUSY}})}{m_t^{\text{OS}}} - \frac{1}{m_t^{\text{OS}}}\delta^{(1)}(m_t X_t)\big|_{\text{fin}}$$

- **Case 1**: $m_{\tilde{t}L} = m_{\tilde{t}R} = M_{SUSY}$ and for $v/M_{SUSY} << 1$ (as in EFT setting)

At $O(\alpha_t)$):

$$\delta^{(1)}(m_t X_t)\Big|_{\text{fin}} = \frac{3\alpha_t}{16\pi} m_t X_t \, |\widehat{X}_t|^2 \ln \frac{M_{\text{SUSY}}^2}{m_t^2} + \text{no large logs}$$

- Caused by diagrams in $\tilde{t}_1, \tilde{t}_2$ mass counterterm of the form



- **Same type of diagrams as in external-leg corrections!** (part 1 of this talk)
  - IR divergence for $m_{\tilde{t}2} \to m_{\tilde{t}1}$, cured by real Higgs radiation (NB: Higgs massless in limit $v/M_{SUSY}<<1$)
  - Large log remains for $m_{\tilde{t}2} \neq m_{\tilde{t}1}$, regulated by squared mass difference
  - Can't be resummed by standard EFT techniques, but size of 2L corrections much smaller than 1L (c.f. part 1)

OS to $\overline{\text{DR}}$ conversion of X_t and large logarithms II

$$X_t^{\text{OS}} = X_t^{\overline{\text{DR}}}(M_{\text{SUSY}})\,\frac{m_t^{\overline{\text{DR}},\text{MSSM}}(M_{\text{SUSY}})}{m_t^{\text{OS}}} - \frac{1}{m_t^{\text{OS}}}\delta^{(1)}(m_t X_t)\big|_{\text{fin}}$$

- **Case 1**: $m_{\tilde{t}L} = m_{\tilde{t}R} = M_{SUSY}$ and for $v/M_{SUSY} << 1$ (as in EFT setting)

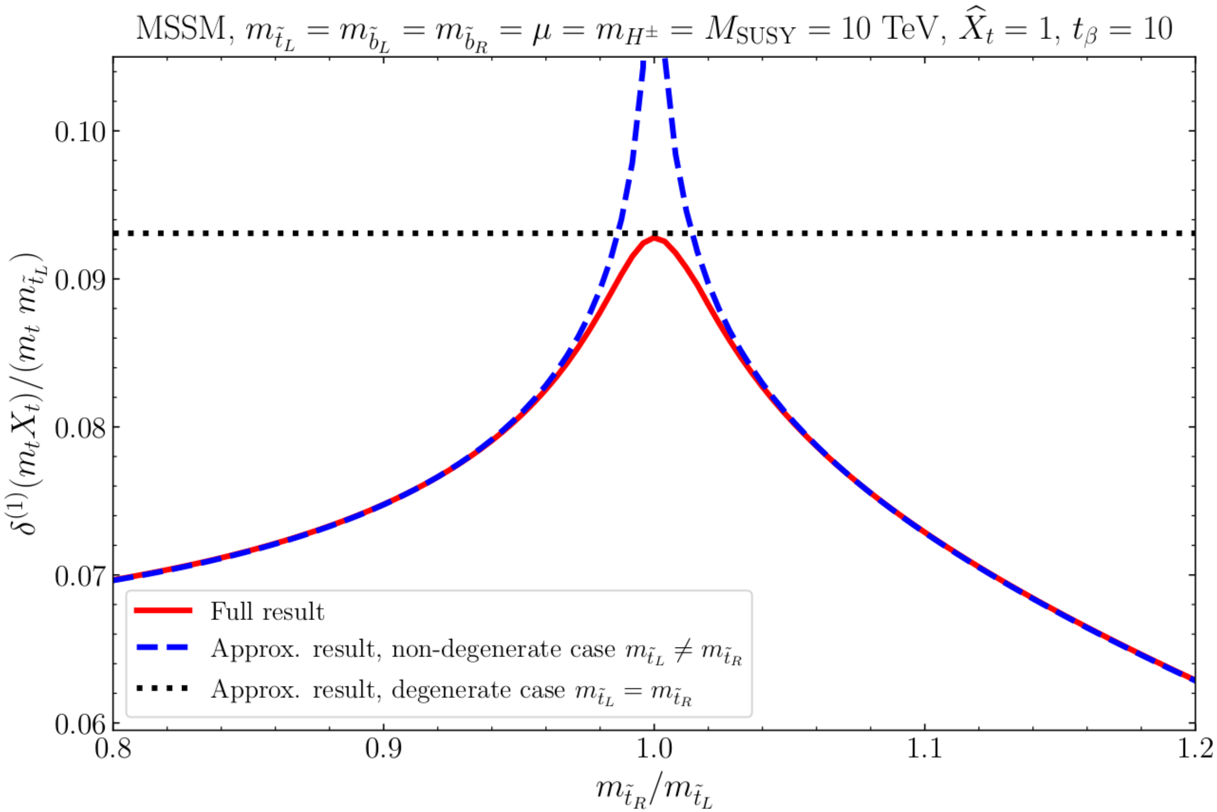
At $O(\alpha_t)$:
$$\delta^{(1)}(m_t X_t)\Big|_{\text{fin}} = \frac{3\alpha_t}{16\pi}\, m_t X_t\, |\widehat{X}_t|^2 \ln\frac{M_{\text{SUSY}}^2}{m_t^2} + \text{no large logs}$$

- **Case 2**: $m_{\tilde{t}L} \neq m_{\tilde{t}R}$ and for $v/M_{SUSY} << 1$

At $O(\alpha_t)$:
$$\delta^{(1)}(m_t X_t)\Big|_{\text{fin}} = \frac{\alpha_t}{8\pi}\, m_t X_t\, |\widehat{X}_t|^2 \left(\frac{2m_{\tilde{t}_L}}{m_{\tilde{t}_R}}\ln\frac{m_{\tilde{t}_L}^2}{|m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2|} + \frac{m_{\tilde{t}_R}}{m_{\tilde{t}_L}}\ln\frac{m_{\tilde{t}_R}^2}{|m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2|}\right)$$

- Once again large logs, again regulated by squared mass difference $|m_{\tilde{t}2}^2 - m_{\tilde{t}1}^2| \sim |m_{\tilde{t}L}^2 - m_{\tilde{t}R}^2|$

OS to $\overline{\text{DR}}$ conversion of X_t and large logarithms III



- Case 1: $m_{\tilde{t}L} = m_{\tilde{t}R}$

$$X_t^{\overline{\text{DR}}}(M_{\text{SUSY}}) = X_t^{\text{OS}} \left\{1 + \left[\frac{\alpha_s}{\pi} - \frac{3\alpha_t}{16\pi}\left(1 - \frac{|X_t|^2}{M_{\text{SUSY}}^2}\right)\right] \ln \frac{M_{\text{SUSY}}^2}{m_t^2}\right\} + \cdots$$

- Case 2: $m_{\tilde{t}L} \neq m_{\tilde{t}R}$

$$X_t^{\overline{\text{DR}}}(M_{\text{SUSY}}) = X_t^{\text{OS}} \left\{1 + \left[\frac{\alpha_s}{\pi} - \frac{3\alpha_t}{16\pi}\right] \ln \frac{M_{\text{SUSY}}^2}{m_t^2}\right\} + \cdots$$

→ for $O(\alpha_t)$ pieces, no transition between the two expanded cases $m_{\tilde{t}L} = m_{\tilde{t}R}$ and $m_{\tilde{t}L} \neq m_{\tilde{t}R}$ (but for $O(\alpha_t)$ there is)

→ full result is well behaved, but one is then mixing order in EFT expansion (in v/M_{SUSY})

→ **keep X_t in $\overline{\text{DR}}$ / $\overline{\text{MDR}}$ scheme even in fixed-order calculation**, to avoid conversion

Summary of Part 2

- We discussed, for the example of the MSSM stop mixing parameter X_t, possible experimental probes, and theoretical definitions (choices of renormalisation scheme) of the parameter

- **Experimental probes**:
  - **M_h seems the best avenue to determine X_t** (once stop masses and tanβ are known) → sensitivity to X_t no matter the stop mass hierarchy, and even to high SUSY scales

  - Stop decays also an option, but highly dependent on sparticle spectrum (i.e. what decay channels are open) → only useful for parts of parameter space

- **Renormalisation scheme choices**:
  - Choice of scheme for X_t in M_h calculation crucial, as M_h is best way to access X_t
  - **No ideal choice**, but given that $\overline{\text{DR}}/\overline{\text{MDR}}$ is preferable for EFT (and hybrid) → use also **$\overline{\text{DR}}/\overline{\text{MDR}}$ for X_t (mixed scheme) in fixed-order part of hybrid calculation**, to avoid large log in conversion (would however reappear in extraction of X_t from experimental input + issue of 1/ε*ε pieces at higher orders)

- Results in principle applicable more broadly to BSM trilinear couplings

Thank you very much for your attention!

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External-leg corrections

- **LSZ formalism** [Lehmann, Symanzik, Zimmermann '55]→ to obtain a reliable prediction for an observable, need to ensure correct on-shell properties of external particles → **LSZ factor**

- External scalar Φ, without mixing: include for each external leg a factor

$$\sqrt{Z_\phi} = \left(1 + \hat{\Sigma}'_{\phi\phi}(p^2 = \mathcal{M}^2_\phi)\right)^{-1/2}$$

$\hat{\Sigma}'_{\phi\phi}$ Derivative of renormalised self-energy w.r.t p^2

$\mathcal{M}_\phi$ Complex pole mass

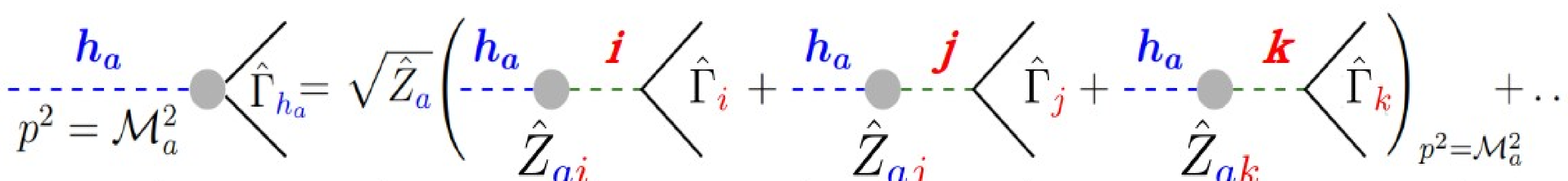
Up to 2L order:

$$\sqrt{Z_\phi} = 1 - \text{Re}\hat{\Sigma}^{(1)\prime}_{\phi\phi}(m^2) - \text{Re}\hat{\Sigma}^{(2)\prime}_{\phi\phi}(m^2) + \left(\text{Re}\hat{\Sigma}^{(1)\prime}_{\phi\phi}(m^2)\right)^2$$
$$-\frac{1}{2}\left(\text{Im}\hat{\Sigma}^{(1)\prime}_{\phi\phi}(m^2)\right)^2 + \text{Im}\hat{\Sigma}^{(1)}_{\phi\phi}(m^2)\cdot \text{Im}\hat{\Sigma}^{(1)\prime\prime}_{\phi\phi}(m^2) + \mathcal{O}(3\text{L})$$

- Case with mixing → we employ the **Z-matrix formalism** [Frank et al. '06, Fuchs and Weiglein '16, '17]

$$\hat{\Gamma}_{h_a^{\text{physical}}} = \sum_j \hat{\mathbf{Z}}_{aj}\hat{\Gamma}_j \qquad \text{with} \qquad \hat{\mathbf{Z}}_{aj} = \sqrt{\hat{Z}^a_i}\,\hat{Z}^a_{ij}$$

e.g. with 3 scalars i,j,k:



(more details also in [Bahl, JB, Weiglein '21])

$\tilde{g}\to t\tilde{t}$ decay – X_t terms (at $v\neq 0$)

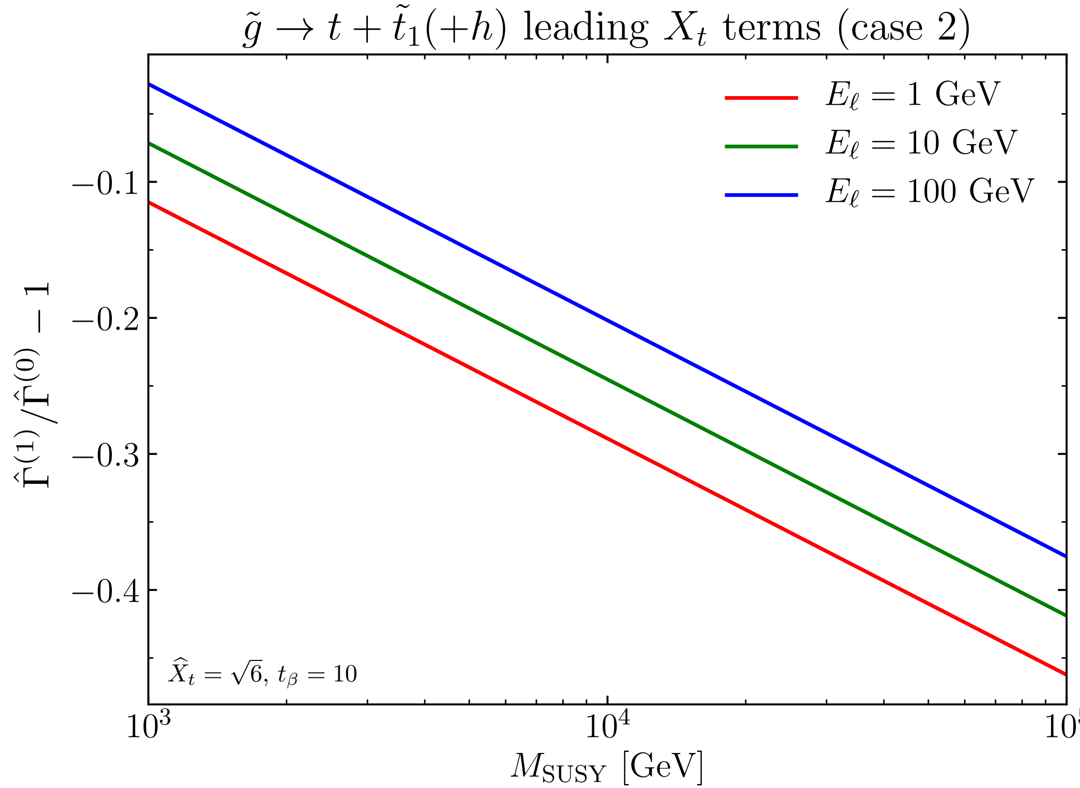
- $v\neq 0 \to$ stop mixing
- Heavy scalars: $\tilde{t}_1$, $\tilde{t}_2$
  - Assume $m_{\tilde{t}L} = m_{\tilde{t}R} = M_{SUSY}$
  - $m_{\tilde{t}2}^2 - m_{\tilde{t}1}^2 = 2\, m_t X_t$

$$c(h\tilde{t}_1\tilde{t}_1) = -c(h\tilde{t}_2\tilde{t}_2) = \frac{1}{\sqrt{2}} h_t s_\beta X_t,$$
$$c(h\tilde{t}_1\tilde{t}_2) = c(h\tilde{t}_2\tilde{t}_1) = 0,$$
$$c(G\tilde{t}_1\tilde{t}_1) = c(G\tilde{t}_2\tilde{t}_2) = 0,$$
$$c(G\tilde{t}_1\tilde{t}_2) = -c(G\tilde{t}_2\tilde{t}_1) = \frac{1}{\sqrt{2}} h_t s_\beta X_t,$$
$$c(G^+\tilde{t}_1\tilde{b}_1) = c(G^+\tilde{t}_2\tilde{b}_1) = -\frac{1}{\sqrt{2}} h_t s_\beta X_t,$$
$$c(G^+\tilde{t}_1\tilde{b}_2) = c(G^+\tilde{t}_2\tilde{b}_2) = 0.$$

- Light scalars:
  - $m_h \neq 0$ but $<< M_{SUSY}$
  - Set $m_h \sim m_G \sim m_{G\pm} \sim m_{IR}$ (IR regulator mass)

(Same as m_1~0, m_2~m_3 in toy model)

IR divergence cured by real radiation



Large logarithms from external legs: N2HDM

Decay of a heavy Higgs boson in the N2HDM

- Extend SM scalar sector by an **additional Higgs doublet** (→2HDM) plus a **real singlet Φ_S**

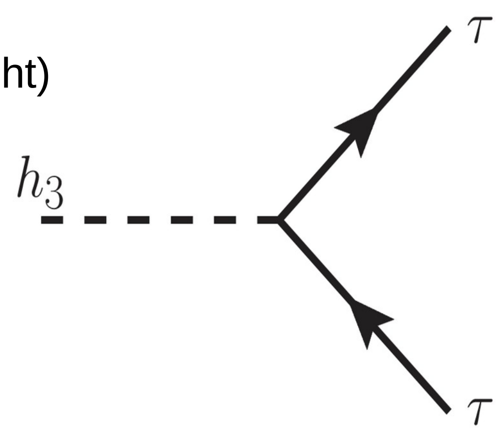
$$V^{(0)} = m_{11}^2|\Phi_1|^2 + m_{22}^2|\Phi_2|^2 - m_{12}^2\left(\Phi_1^\dagger\Phi_2 + \text{h.c.}\right) + \frac{1}{2}\lambda_1|\Phi_1|^4 + \frac{1}{2}\lambda_2|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_2^\dagger\Phi_1|^2 + \frac{1}{2}\lambda_5\left((\Phi_1^\dagger\Phi_2)^2 + \text{h.c.}\right)$$
$$+ \frac{1}{2}m_S^2\Phi_S^2 + \boxed{\frac{1}{6}a_S\Phi_S^3} + \frac{1}{24}\lambda_S|\Phi_S|^4 + \boxed{\frac{1}{2}a_{1S}|\Phi_1|^2\Phi_S + \frac{1}{2}a_{2S}|\Phi_2|^2\Phi_S} + \frac{1}{6}\lambda_{1S}|\Phi_1|^2\Phi_S^2 + \frac{1}{2}\lambda_{2S}|\Phi_2|^2\Phi_S^2\,.$$

- Z_3 symmetry often imposed to forbid trilinear couplings in Lagrangian, *but not in our case*
- For convenience, define $X_a \equiv \frac{1}{4}(a_{1S} - a_{2S})\,, Y_a \equiv \frac{1}{4}a_{1S}s_\beta^2 + a_{2S}c_\beta^2\,, Z_a \equiv \frac{a_S}{4} - Y_a$

- ***Physical spectrum*** (assuming CP-conservation):
  3 CP-even states, h_1,h_2,h_3; 1 CP-odd state A; 1 charged Higgs boson $H^\pm$; (G, $G^\pm$ would-be Goldstones)

- Consider a scenario with mass hierarchy $m_{h_1} \sim m_{h_2} \sim m_G \sim m_{G^\pm} \sim \sqrt{\epsilon}$ (light) and $m_{h_3} = m_A = m_{H^\pm} = m$ (heavy)

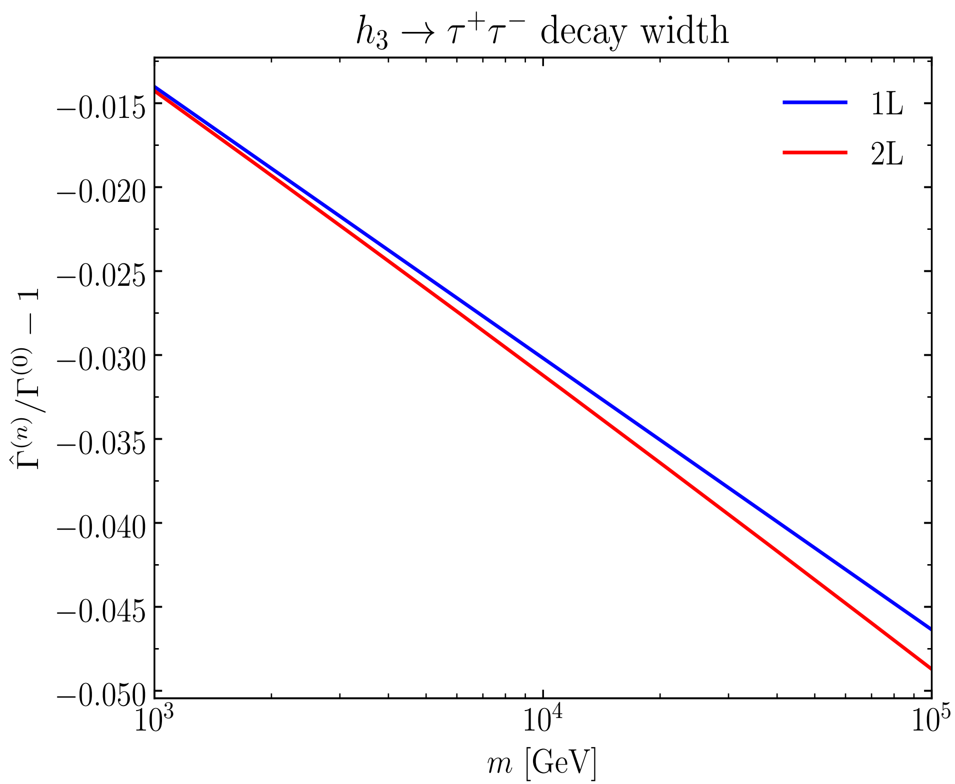
- Investigate trilinear-enhanced contributions to **$h_3 \to \tau^+\tau^-$ decay** process (h_3 being doublet-like), involving $\mathbf{X_a}$



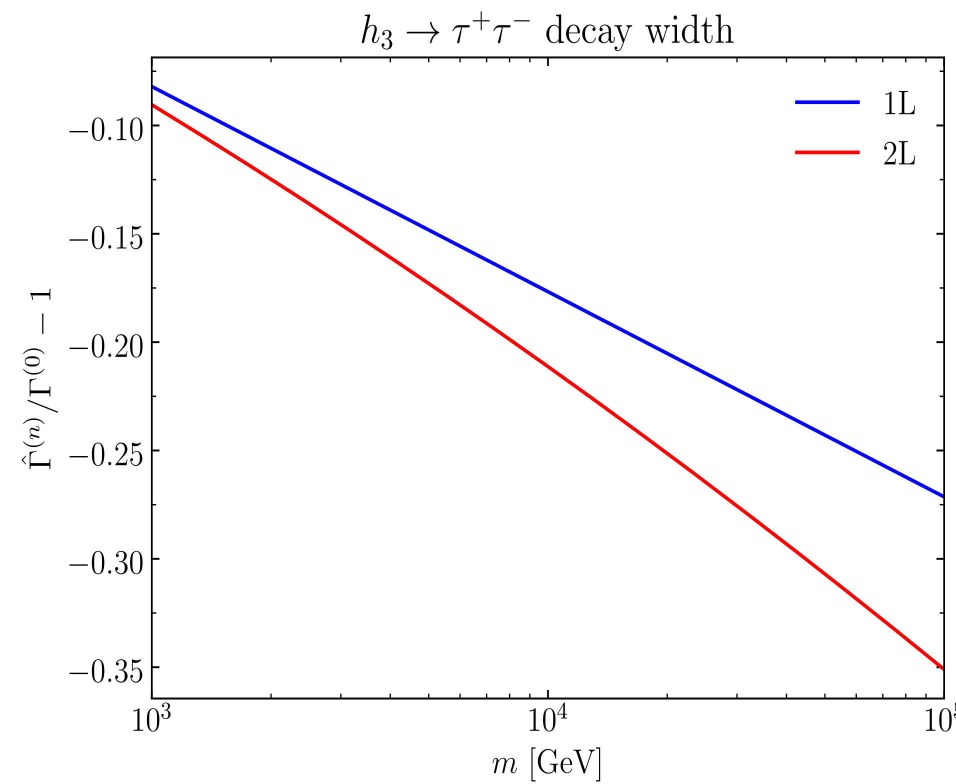
$h_3 \rightarrow \tau^+\tau^-$ decay – trilinear-coupling enhanced X_a terms

Set ε=(50 GeV)^2, X_a=3m, vary m between 1 and 100 TeV

(Same as $m_1 \neq 0$, $m_2 = m_3$ in toy model)



tanβ=1.4, sinα_3=0.99



tanβ=1.26, sinα_3=0.94

- Effects can be significant! (enhanced by deviation from alignment and by multiplicity of diagrams)
- 2L corrections always well smaller than 1L ones