

# Modern Techniques for Multi-loop Amplitudes

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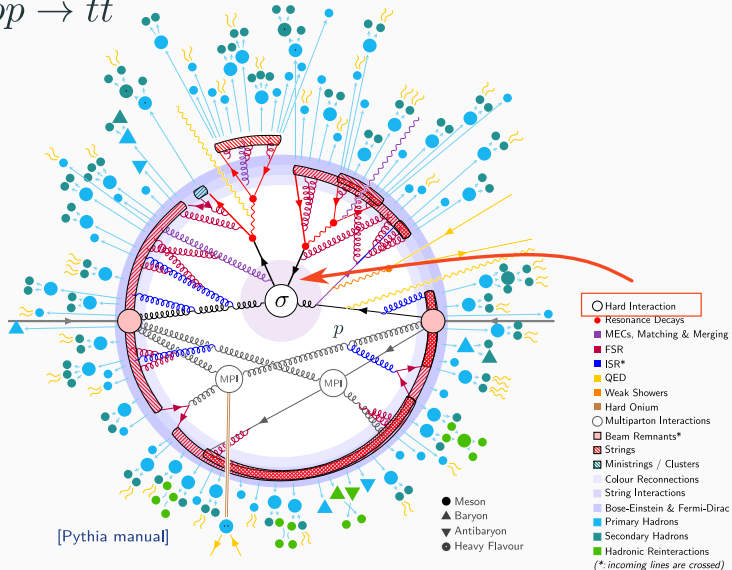
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# Theoretical description of particle collisions

$$pp \rightarrow t\bar{t}$$



Collinear factorization:

$$d\sigma_{h_1 h_2 \rightarrow X}(p_1, p_2) = \sum_{i,j} \int dx_1 dx_2 f_i(x_1, \mu) f_j(x_2, \mu) \boxed{d\hat{\sigma}_{ij \rightarrow X}(x_1 p_1, x_2 p_2, \mu)} + \mathcal{O}(\Lambda_{\text{QCD}}/Q)$$

Parton channels

e.g.  $pp \rightarrow t\bar{t} + X$  :

$q\bar{q} \rightarrow t\bar{t}$

$gg \rightarrow t\bar{t}$

$qg \rightarrow qt\bar{t}$

...

Parton distribution functions

# Perturbation theory

- Properties of fundamental interactions are encoded in patterns of scattered particles.
- **Scattering amplitudes** are theoretical building blocks for these observable quantities, computable **from first principles** of QFT.
- Cannot compute amplitudes exactly  $\implies$  calculation via perturbative expansion in the coupling constants.

$$\alpha_s(M_Z) \sim 0.1$$

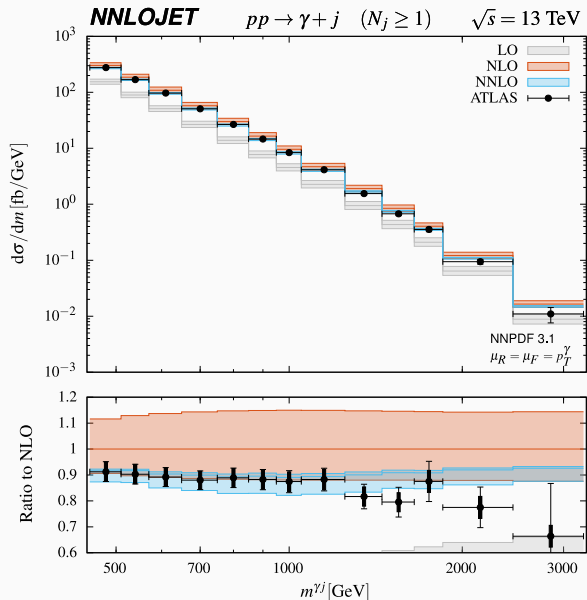
$$\alpha(M_Z) \sim 0.01$$

$$\text{SM} = \text{QCD} \otimes \text{EW}$$

$$\sigma = \sigma_0 \left( 1 + \alpha_s \sigma^{(1,0)} + \alpha \sigma^{(0,1)} + \alpha_s^2 \sigma^{(2,0)} + \alpha \alpha_s \sigma^{(1,1)} + \alpha_s^3 \sigma^{(3,0)} + \alpha_s^2 \alpha \sigma^{(2,1)} + \dots \right)$$

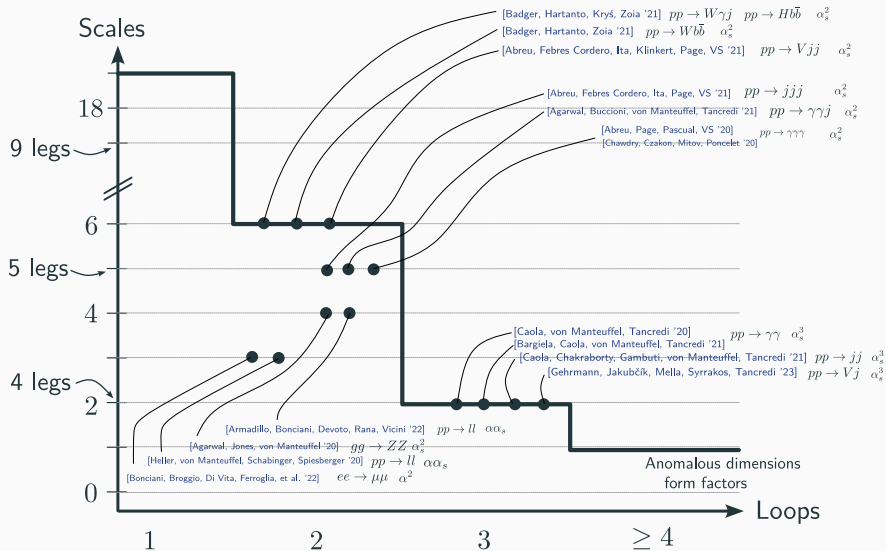
Error due to series truncation

# Example: NNLO predictions vs experimental data



[arXiv:1904.01044]  
data from [1801.00112]

# Loop amplitudes in SM: state of the art



Warning: a biased selection of references!