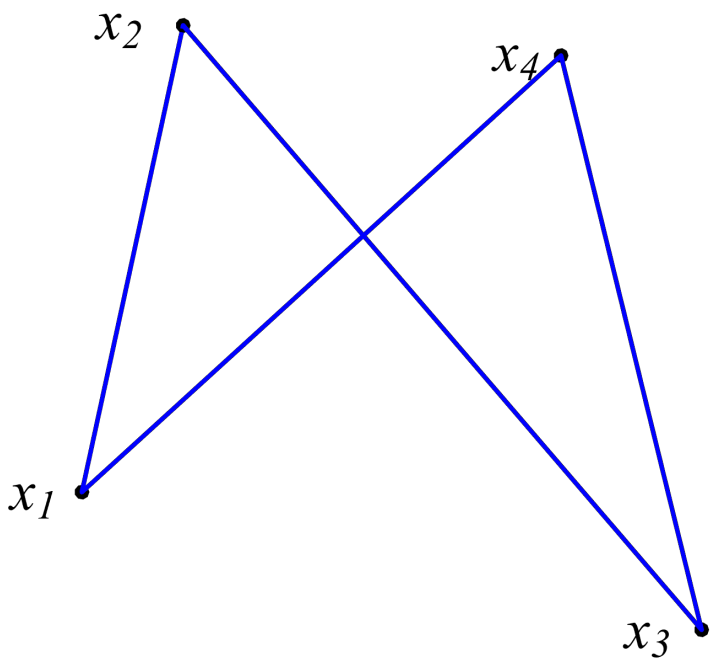
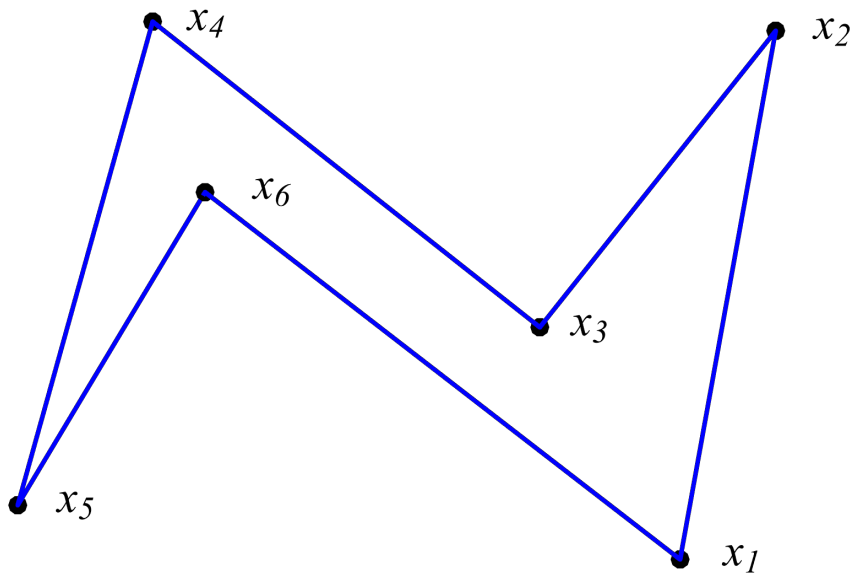
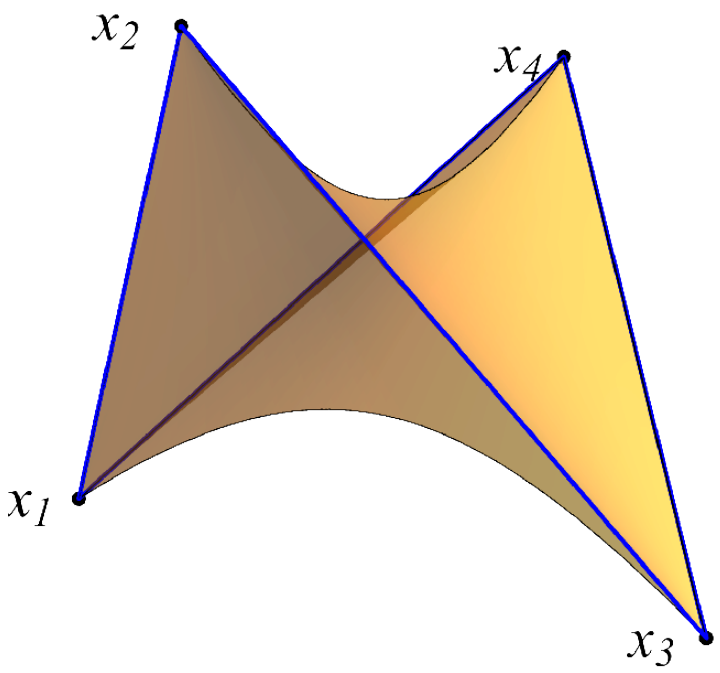
Starting from n 3D momentum vectors, we can find n points in the 3D Minkowski space of dual momenta

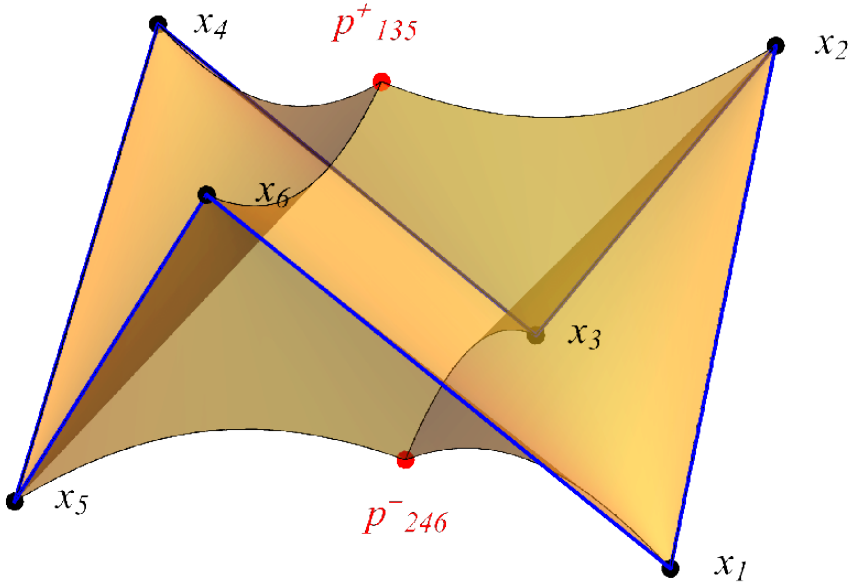
What is the region that is space-like separated from all points?





This defines the (one-loop) *ABJM momentum amplituhedron*
It full characterises one-loop integrands in ABJM
(based on [2306.07312v2])





Massless and Massive 4D Spinor-Helicity

$\underline{p^2 = 0:}$

$$\mathrm{Det}[p] = 0 \implies \mathrm{Rank}[p] = 1 \implies p^{a\dot{a}} =: {}^a|p\rangle[\tilde{p}|^{\dot{a}}$$

$|p\rangle, |\tilde{p}]$ are U(1) covariant *Weyl* spinors

$\underline{p^2 \neq 0:}$

$$\mathrm{Det}[p] \neq 0 \implies \mathrm{Rank}[p] = 2 \implies p^{a\dot{a}} = {}^a|p\rangle[\tilde{p}|^{\dot{a}} + {}^a|q\rangle[\tilde{q}|^{\dot{a}}$$

Rearranging a bit,

$$\begin{aligned} p &= {}^a|p\rangle[\tilde{p}|^{\dot{a}} + {}^a|p]\langle\tilde{p}|^{\dot{a}} \\ &= \begin{pmatrix} |p\rangle & |p] \end{pmatrix} \cdot \begin{pmatrix} [\tilde{p}| \\ \langle\tilde{p}| \end{pmatrix} =: {}^a|\mathbf{p}\rangle^I{}_I[\tilde{\mathbf{p}}|^{\dot{a}} \end{aligned}$$

$|\mathbf{p}\rangle, |\tilde{\mathbf{p}}]$ are SU(2) covariant *Dirac* spinors
Lots to say about the choice of basis, ask me later!

Covariant BCFW for all masses

We upgrade the usual mBCFW shift

$$|\hat{a}\rangle = |a\rangle + z|b\rangle, \qquad |\hat{\tilde{b}}] = |\tilde{b}] - z|\tilde{a}]$$

$$\implies {}^{I}|\hat{\mathbf{a}}\rangle = {}^{I}|\mathbf{a}\rangle + z\Xi^{IJ}{}_{J}|\mathbf{b}\rangle, \qquad {}^{J}|\hat{\tilde{\mathbf{b}}}] = {}^{J}|\tilde{\mathbf{b}}] - z\Xi^{IJ}{}_{I}|\tilde{\mathbf{a}}]$$

Since Ξ^{IJ} is null it suffices to give (with $p^\flat := \lim_{m^2\to 0} p$)

$$\Xi^{11} = 1, \quad \Xi^{12} = \frac{{}^{K}\langle \mathbf{a} b\rangle \; {}_{K}[\tilde{\mathbf{a}}\tilde{b}\rangle}{|a\wedge b| + \frac{1}{2}(a\cdot b^\flat)}, \quad \Xi^{21} = \frac{[a\mathbf{b}\rangle^{K} \; [\tilde{a}\tilde{\mathbf{b}}]_{K}}{|a\wedge b| + \frac{1}{2}(a^\flat\cdot b)}$$

This formula is smooth in all massless limits for a and/or b, yielding the proper BCFW shift for *any* pair of masses

A massive shift is given in 2112.12312 but requires using a special basis, in which our expressions match. <u>The above formula is the first *covariant* massive shift.</u>

Ok, so what?

Applications and related questions:

- Analytic MBCFW (code in-development)
  (coulomb branch, mass regulators, classical scattering, ...)
  (massive amplitudes with *rational* kinematics)
- Geometric understanding of MBCFW
  ($|a \wedge b|$, ...)
- Massless limits and basis choices of MSH expressions
  (NOT always as simple as "unbold the spinors")
  (vDVZ discontinuity in massive SUGRA, ...)

Quarendo invenietes - by seeking you will discover
Details galore in publications to come, stay tuned!

Amplitude bootstap and Double copy in (Anti)de-Sitter space

Jiajie Mei

Durham University

Based on 2305.13894

New representation of AdS amplitude

Amplitude in Minkowski space	Mellin-Momentum amplitude in AdS
Lorentz-Invariance	Conformal Invariance
Translation symmetry $e^{ik_\mu \cdot x^\mu}$ $(2\pi)^{d+1}\delta^{d+1}(\sum_{i=1}^{n} k_i^\mu)$	Boundary translation + Dilatation $e^{ik_\mu \cdot x^\mu} z^{-2s+d/2}$ $(2\pi)^{d}\delta^{d}(\sum_{i=1}^{n} k_i^\mu)$ $\times 2\pi i \delta(d+b+\sum_{i=1}^{n}(2s_i - d/2))$
On-shell condition $k^2 + m^2 = 0$	On-shell condition $z^2 k^2 + (d/2-\Delta)^2 - 4s^2 = 0$
Pole structure $\frac{1}{(k_i+\cdots+k_n)^2}$	Factorization+OPE $\frac{1}{\mathcal{D}_k^\Delta}$ $\frac{1}{(k_I)^2}$

Key feature of the Mellin-Momentum amplitude:

- Field redefinition Invariance and Gauge Invariance.
- Simple pole structure allows us to bootstrap amplitude.
- The same structure as flat space amplitude

Double Copy in AdS_{d+1}

Three point: New Graviton-Dilaton interaction!

Color/Kinematic duality at Four-point

$$n_s + n_t + n_u = 0$$

Double Copy gives the first Four-point pure Einstein Gravity amplitude in AdS_{d+1}.

$$\mathcal{M}_4 = \frac{n_s^2}{\mathcal{D}_{k_s}^d} + \frac{n_t^2}{\mathcal{D}_{k_t}^d} + \frac{n_u^2}{\mathcal{D}_{k_u}^d} - \tilde{\mathcal{M}}_{\mathrm{AdS}}$$

where $\tilde{\mathcal{M}}_{\mathrm{AdS}}$ we subtracted out the dilaton exchange and restore conformal symmetry. We verified the expression when $d = 3$ with literature.

Two Loop Vertex Graphs With Vacuum Polarisation Insertion

Taushif Ahmed, Giulio Crisanti, Federico Gasparotto, Syed Mehedi Hassan, Pierpaolo Mastrolia

The last 20 years of the 2-loop vertex function

μ-e scattering in QED

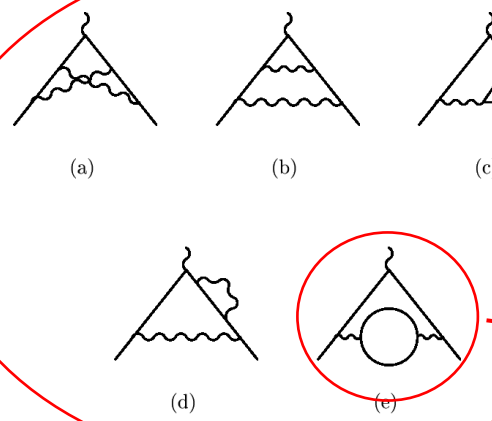
[Passera, Mastrolia, Mandal, Primo et al, 2017-2022]

3-loop Form Factors in QED

[Henn, Smirnov, 2016]

$q\bar{q}$ —> $t\bar{t}$ in QCD

[Mandal, Mastrolia, Ronca, Torres Bobadilla, 2022]



Arbitrary masses case — Numerically

[Fael, Passera, 2018-19]

Our work:

Arbitrary masses case — Analytically

2-loop Vertex in QCD

H to $b\bar{b}$ QCD corrections

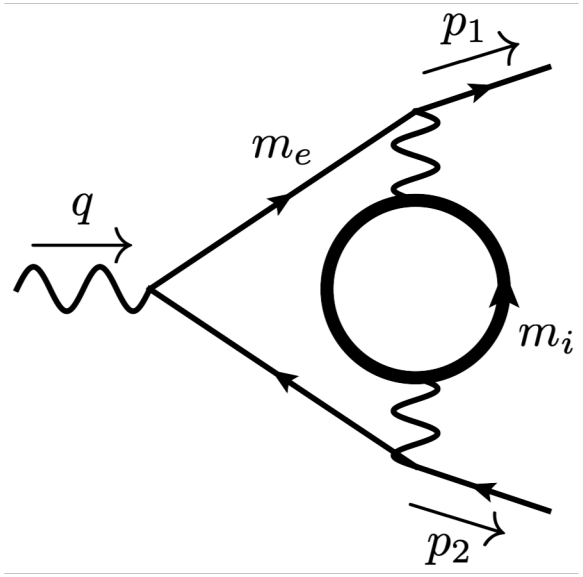
[Gehrmann, Mastrolia, Remiddi et al, 2005]

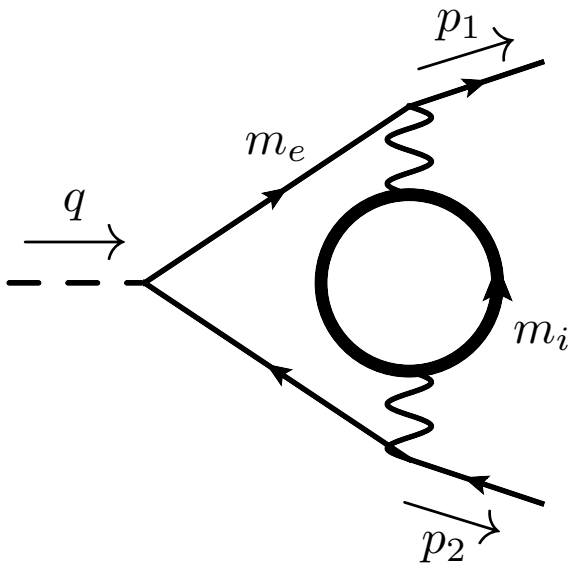
[Gehrmann, Huber, Maître, 2005]

First QED Results using dim reg

[Mastrolia, Bonciani, Remiddi, 2003]

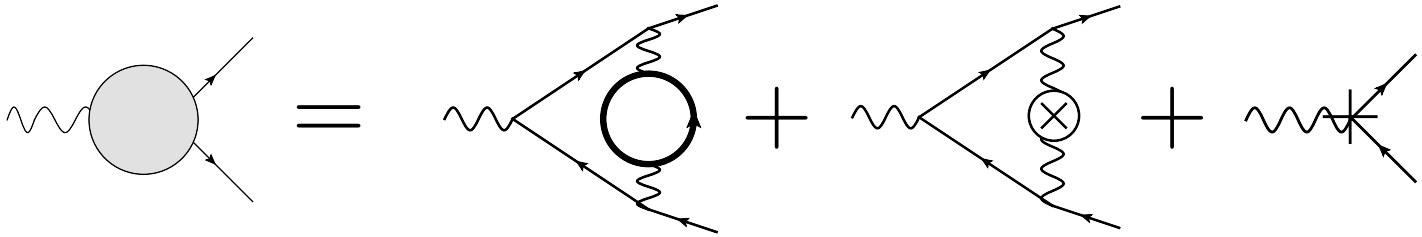
The vertices





Calculation Procedure (Vector Case)

1. Renormalisation: dim reg/on shell scheme

$\Gamma_\mu =$ 

2. Projections

$$\Gamma_\mu = -i\left(g_V F_1(s,m_i,m_e)\gamma_\mu + i\frac{g_V}{2m_e}F_2(s,m_i,m_e)\sigma_{\mu\alpha}q^\alpha + g_A G_1(s,m_i,m_e)\gamma_5\gamma_\mu + \frac{g_A}{2m_e}G_2(s,m_i,m_e)\gamma_5 q_\mu\right)$$

$F_{1,2}$ expressed as a combination of approximately 150 different scalar integrals

3. Reduction to master integrals

The scalar integrals in $F_{1,2}, G_{1,2}$ expressed as a linear combination of 7 master integrals

[Chetyrkin, Tkachov, 1981]

4. Solution of master integrals

The master integrals are expressed as the solution to a system of differential equations

[Kotikov, 1991]

[Remiddi, 1996]

Solution via the Magnus method and Generalised Polylogarithms (GPLs)

[Ghermann, Remiddi, 1999]

[Henn , 2013]

Fixing of boundary conditions

[Argeri, Di Vita, Mastrolia, Mirabella, Schlenk, Schubert, Tancredi, 2013]

Pseudoscalar Higgs boson production studies

JHEP 02 (2020) 121,
Phys. Rev. D 105 (2022) 11, 116015

Dr. Arunima Bhattacharya

Gong Show, CERN Amplitudes 2023

02/08/2023

Processes & Methodologies

1. $\mathbf{gg} \to \mathbf{AA} \Rightarrow$ FO computation till NNLO - **JHEP 02 (2020) 121.**
   - QGRAF, FORM
   - REDUZE2 - **JHEP 08 (2013) 07, JHEP 06 (2014) 032**
   - LiteRed, Mathematica
   - Mathematica, GNUPLOT
2. $\mathbf{gg} \to \mathbf{A} \Rightarrow$ Threshold resummation - **Phys. Rev. D 105 (2022) 11, 116015.**
   - FORM
   - Fortran
   - Mathematica
3. $\mathbf{gg} \to \mathbf{AA} \Rightarrow$ FO + Threshold resummed corrections till NLO - **Ongoing**
   - FORM
   - Fortran
   - Extrapolation

Framework & Observations

1. **EFT:** $\mathcal{L}_{eff}^{A} = \Phi^{A}\left[-\frac{1}{8}C_G O_G(x) - \frac{1}{2}C_J O_J(x)\right]$.
   - The pseudoscalar Higgs boson directly couples to gluons and light quarks through two local composite operators, O_G & O_J.
2. **Explicit computations:**
   - Dim. regularisation of Levi-Civita tensor in O_G and γ_5 in O_J.

     - using the self-consistent prescription proposed by ' t Hooft and Veltman (1972).
3. **Threshold resummed corrections:**
   - Significant contribution from NSV logs compared to SV logs.
   - 7-point scale uncertainties & μ_F scale variations do not show much improvement from gg fusion corrections.
   - For μ_R scale variations, the uncertainties reduce significantly.
     - <u>**Conclusion**</u> $\rightarrow$ The need of NSV contributions from other parton channels, & beyond NSV contributions in the dominant gluon fusion channel.

4 Dimension IBP Relations with Divergent Feynman Integrals

JHEP 03 (2023) 162

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2nd. Aug. 2023